

## Foundations and Trends in Signal Processing

# A Signal Processing Perspective of Financial Engineering

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### Abstract

Financial engineering and electrical engineering are seemingly different areas that share strong underlying connections. Both areas rely on statistical analysis and modeling of systems; either modeling the financial markets or modeling wireless communication channels. Having a model of reality allows us to make predictions and to optimize the strategies. It is as important to optimize our investment strategies in a financial market as it is to optimize the signal transmitted by an antenna in a wireless link.

This monograph provides a survey of financial engineering from a signal processing perspective, that is, it reviews financial modeling, the design of quantitative investment strategies, and order execution with comparison to seemingly different problems in signal processing and communication systems, such as signal modeling, filter/beamforming design, network scheduling, and power allocation.

### Suggested Citation

Yiyong Feng and Daniel P. Palomar (2016) "A Signal Processing Perspective of Financial Engineering" Foundations and Trends in Signal Processing: Vol. 9: No. 3-4, pp unknown.  
<http://dx.doi.org/10.1561/2000000053>

Foundations and Trends® in Signal Processing  
Vol. XX, No. XX (2015) 1–188  
© 2015 now Publishers Inc.  
DOI: 10.1561/XXXXXXXXXX



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Mr. Feng has several works on financial engineering as listed below.

- Yiyong Feng, Daniel P. Palomar, and Francisco Rubio, “Robust optimization of order execution,” *IEEE Trans. Signal Process.*, 63(4):907-920, Feb. 2015.
- Yiyong Feng and Daniel P. Palomar, “SC RIP: Successive convex optimization methods for risk parity portfolio design,” accepted in *IEEE Trans. Signal Process.*, to appear 2015.
- (in preparation) Yiyong Feng and Daniel P. Palomar, “Asset Selection and Risk Parity Regularized Portfolio Design,” to be submitted, Oct. 2015.

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CATIONS 2007 Special Issue on “Optimization of MIMO Transceivers for Realistic Communication Networks.”

Dr. Palomar has been conducting various research projects on financial engineering and the published and current ongoing financial works are listed below.

- Francisco Rubio, Xavier Mestre, and Daniel P. Palomar, “Performance analysis and optimal selection of large Minimum-Variance portfolios under estimation risk,” *IEEE Journal on Selected Topics in Signal Processing: Special Issue on Signal Processing Methods in Finance and Electronic Trading*, vol. 6, no. 4, pp. 337-350, Aug. 2012.
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- Ying Sun, Prabhu Babu, and Daniel P. Palomar, “Regularized Tyler’s scatter estimator: existence, uniqueness, and algorithms,” *IEEE Trans. Signal Process.*, vol. 62, no. 19, pp. 5143-5156, Oct. 2014.
- Yiyong Feng, Daniel P. Palomar, and Francisco Rubio, “Robust optimization of order execution,” *IEEE Trans. Signal Process.*, 63(4):907-920, Feb. 2015.
- Ying Sun, Prabhu Babu, and Daniel P. Palomar, “Regularized robust estimation of mean and covariance matrix under heavy-

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- Licheng Zhao, Prabhu Babu, and Daniel P. Palomar, “Generalized Robust Low Rank Optimization Against Outliers,” submitted to *IEEE Trans. Signal Process.*, June 2015.
- (in preparation) Yiyong Feng and Daniel P. Palomar, “Asset Selection and Risk Parity Regularized Portfolio Design,” to be submitted, Oct. 2015.
- (in preparation) Ziping Zhao, Prabhu Babu, and Daniel P. Palomar, “Robust maximum likelihood estimation of VECMs under heavy tails and outliers,” to be submitted, Oct. 2015.

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## **Abstract**

Financial engineering and electrical engineering are seemingly different areas that share strong underlying connections. Both areas rely on statistical analysis and modeling of systems; either modeling the financial markets or modeling wireless communication channels. Having a model of reality allows us to make predictions and to optimize the strategies. It is as important to optimize our investment strategies in a financial market as it is to optimize the signal transmitted by an antenna in a wireless link.

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# 1

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## Introduction

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Despite the different nature of financial engineering and electrical engineering, both areas are intimately connected on a mathematical level. The foundations of financial engineering lie on the statistical analysis of numerical time series and the modeling of the behavior of the financial markets in order to perform predictions and systematically optimize investment strategies. Similarly, the foundations of electrical engineering, for instance, wireless communication systems, lie on statistical signal processing and the modeling of communication channels in order to perform predictions and systematically optimize transmission strategies. Both foundations are the same in disguise.

This observation immediately prompts the question of whether both areas can benefit from each other. It is often the case in science that the same or very similar methodologies are developed and applied independently in different areas. The purpose of this monograph is to explore such connections and to capitalize on the existing mathematical tools developed in wireless communications and signal processing to solve real-life problems arising in the financial markets in an unprecedented way.

Thus, this monograph is about investment in financial assets treated

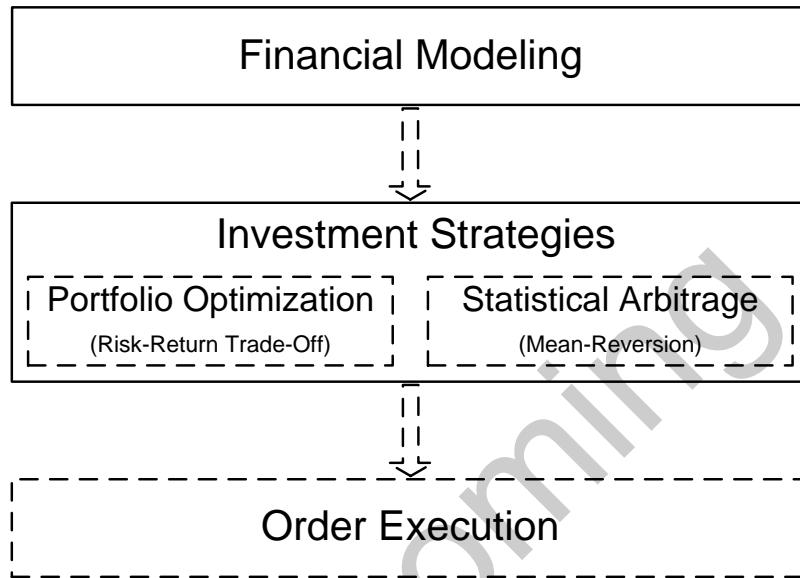
as a signal processing and optimization problem. An investment is the current commitment of resources in the expectation of reaping future benefits. In financial markets, such resources usually take the form of money and thus the investment is the present commitment of money in order to reap (hopefully more) money later [16]. The carriers of money in financial markets are usually referred to as financial assets. There are various classes of financial assets, namely, equity securities (e.g., common stocks), exchange-traded funds (ETFs), market indexes, commodities, exchanges rates, fixed-income securities, derivatives (e.g., options and futures), etc. The detailed description of each kind of asset is well documented in [16]. For different kinds of assets, the key quantities of interest are not the same, for example, for equity securities they are the compounded returns or log-returns; for fixed-income securities they are the changes in yield to maturity; and for options they are changes in the rolling at-the-money forward implied volatility [104].

Roughly speaking, there are three families of investment philosophies: fundamental analysis, technical analysis, and quantitative analysis. Fundamental analysis uses financial and economical measures such as earnings, dividend yields, expectations of future interest rates, and management, to determine the value of each share of the company's stocks and then recommends purchasing the stocks if the estimated value exceeds the current stock price [59, 60]. Probably Warren Buffett of Berkshire Hathaway is the most famous practitioner of fundamental analysis [62]. Technical analysis, also known as "charting," is essentially the search for patterns in one dimensional charts of the prices of a stock. In a way, it pretends to be a scientific analysis of patterns (similar to machine learning) but generally implemented in an unscientific and anecdotal way, and has a low power of prediction as detailed in [94]. Quantitative analysis applies quantitative (namely scientific or mathematical) tools to discover the predictive patterns from financial data [90]. To put this in perspective with the previous approach, technical analysis is to quantitative analysis as astrology is to astronomy. The pioneer of the quantitative investment approach is Edward O. Thorp, who used his knowledge of probability and statistics in the stock markets and has made a significant fortune since the late 1960s

[145]. Quantitative analysis has become more and more widely used since advanced computer science technology has enabled practitioners to apply complex quantitative techniques to reap many more rewards more efficiently and more frequently in practice [3]. In fact, one could even go further to say that algorithmic trading has been one of the main driving forces in the technological advancement of computers. Some institutional hedge fund firms relying on quantitative analysis include Renaissance Technologies, AQR Capital, Winton Capital Management, and D. E. Shaw & Co., to name a few.

In this monograph, we will focus on the quantitative analysis of equity securities since they are the simplest and easiest accessible assets. As we will discover, many quantitative techniques employed in signal processing methods may be applicable for quantitative investment. Nevertheless, the discussion in this monograph can be easily extended to some other tradeable assets such as commodities, ETF, futures, etc.

Thus, to explore the multiple connections between quantitative investment in financial engineering and areas in signal processing and communications, we will show how to capitalize on existing mathematical tools and methodologies developed and widely applied in the context of signal processing applications in order to solve problems in the field of portfolio optimization and investment management in quantitative finance. In particular, we will explore financial engineering in several respects: i) we will provide the fundamentals of market data modeling and asset return predictability, as well as outline state-of-the-art methodologies for the estimation and forecasting of portfolio design parameters in realistic, non-frictionless financial markets; ii) we will present the problem of optimal portfolio construction, elaborate on advanced optimization issues, and make the connections between portfolio optimization and filter/beamforming design in signal processing; iii) we will reveal the theoretical mechanisms underlying the design and evaluation of statistical arbitrage trading strategies from a signal processing perspective based on multivariate data analysis and time series modeling; and iv) we will discuss the optimal order execution and compare it with network scheduling in sensor networks and power allocation in communication systems.



**Figure 1.1:** Block diagram of quantitative investment in financial engineering.

We hope this monograph can provide more straightforward and systematic access to financial engineering for the researchers in signal processing and communication societies so that they can understand problems in financial engineering more easily and may even apply signal processing techniques to handle some financial problems.

In the following content of this introduction, we will first introduce financial engineering from a signal processing perspective and then make connections between problems arising in financial engineering and those arising in different areas of signal processing and communication systems. At the end, the outline of the monograph is detailed.

## 1.1 A Signal Processing Perspective of Financial Engineering

Figure 1.1 summarizes the procedure of quantitative investment. Roughly speaking and oversimplifying, there are three main steps (shown in Figure 1.1):

- financial modeling: modeling the very noisy financial time series to decompose it into trend and noise components;
- portfolio design: designing quantitative investment strategies based on the estimated financial models to optimize some preferred criterion;
- order execution: properly executing the orders to establish or unwind positions of the designed portfolio in an optimal way.

In the following, we will further elaborate the above three steps from a signal processing perspective.

### **1.1.1 Financial Modeling**

For equity securities, the log-prices (i.e., the logarithmic of the prices) and the compounded returns or log-returns (i.e., the differences of the log-prices) are the quantities of interest. From a signal processing perspective, a log-price sequence can be decomposed into two parts: trend and noise components, which are also referred to as market and idiosyncratic components, respectively. The purpose of financial modeling or signal modeling is to decompose the trend components from the noisy financial series. Then based on the constructed financial models, one can properly design some quantitative investment strategies for future benefits [42].

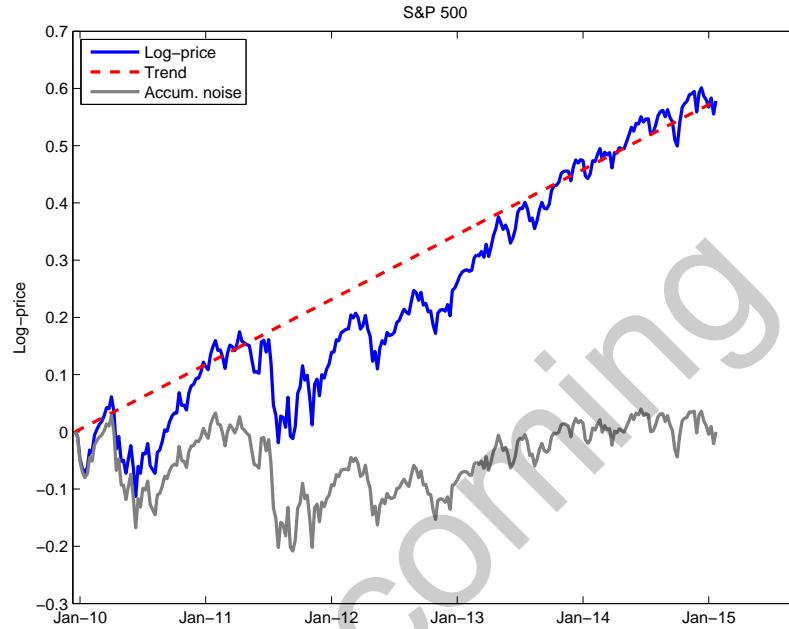
For instance, a simple and popular financial model of the log-price series is the following random walk with drift:

$$y_t = \mu + y_{t-1} + w_t, \quad (1.1)$$

where  $y_t$  is the log-price at discrete-time  $t$ ,  $\{w_t\}$  is a zero-mean white noise series, and the constant term  $\mu$  represents the time trend of the log-price  $y_t$  since  $E[y_t - y_{t-1}] = \mu$ , which is usually referred to as drift.

Based on model (1.1), we can see the trend signal and noise components in the log-prices more clearly by rewriting  $y_t$  as follows:

$$y_t = \mu t + y_0 + \sum_{i=1}^t w_i \quad (1.2)$$



**Figure 1.2:** The decomposition of the log-price sequence of the S&P 500 Index into time trend component and the component without time trend (i.e., the accumulative noise).

where the term  $\mu t$  denotes the trend (e.g., uptrend if  $\mu > 0$ , downtrend if  $\mu < 0$ , or no trend if  $\mu = 0$ ), and the term  $\sum_{i=1}^t w_i$  denotes the accumulative noise as time evolves.

Figure 1.2 shows the weekly log-prices of the S&P 500 index from 04-Jan-2010 to 04-Feb-2015 (the log-prices are shifted down so that the initial log-price is zero, i.e.,  $y_0 = 0$ ), where the estimated drift is  $\mu = 0.0022$ . Obviously, we observe two patterns: first, there exists a significant uptrend since 2010 in the US market (see the red dashed line  $\mu t$ ); and second, the accumulative noise in the log-prices is not steady and looks like a random walk (see the gray solid line for the accumulative noise  $\sum_{i=1}^t w_i = y_t - \mu t$ ).

### 1.1.2 Quantitative Investment

Once the specific financial model is calibrated from the financial time series, the next question is how to utilize such a calibrated financial model to invest. As mentioned before, one widely employed approach is to apply quantitative techniques to design the investment strategies, i.e., the quantitative investment [43, 90, 42, 104].

Figure 1.2 shows that there are two main components in financial series: trend and noise. Correspondingly, there are two main types of quantitative investment strategies based on the two components: a trend-based approach, termed risk-return trade-off investment; and a noise-based approach, termed mean-reversion investment.

The trend-based risk-return trade-off investment tends to maximize the expected portfolio return while keeping the risk low; however, this is easier said than done because of the sensitivity to the imperfect estimation of the drift component and the covariance matrix of the noise component of multiple assets. In practice, one needs to consider the parameter estimation errors in the problem formulation to design the portfolio in a robust way. Traditionally, the variance of the portfolio return is taken as a measure of risk, and the method is thus referred to as “mean-variance portfolio optimization” in the financial literature [96, 99, 98]. From the signal processing perspective, interestingly, the design of mean-variance portfolio is mathematically identical to the design of a filter in signal processing or the design of beamforming in wireless multi-antenna communication systems [86, 110, 163].

The noise-based mean-reversion investment aims at seeking profitability based on the noise component. For clarity of presentation, let us use a simple example of only two stocks to illustrate the rough idea. Suppose the log-price sequences of the two stocks are cointegrated (i.e., sharing the same stochastic drift), at some time if one stock moves up while the other moves down, then people can short-sell the first overperforming stock and long/buy the second underperforming stock<sup>1</sup>, betting that the deviation between the two stocks will eventually diminish.

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<sup>1</sup>In financial engineering, to “long” means simply to buy financial instruments, to “short-sell” (or simply to “short”) means to sell financial instruments that are not currently owned.

This idea can be generalized from only two stocks to a larger number of stocks to create more profitable opportunities. This type of quantitative investment is often referred to as “pairs trading”, or more generally, “statistical arbitrage” in the literature [118, 154].

### 1.1.3 Order Execution

Ideally, after one has made a prediction and designed a portfolio, the execution should be a seamless part of the process. However, in practice, the process of executing the orders affects the original predictions in the wrong way, i.e., the achieved prices of the executed orders will be worse than what they should have been. This detrimental effect is called market impact. Since it has been shown that smaller orders have a much smaller market impact, a natural idea to execute a large order is to partition it into many small pieces and then execute them sequentially [6, 14, 51, 107].

Interestingly, the order execution problem is close to many other scheduling and optimization problems in signal processing and communication systems. From a dynamic control point of view, the order execution problem is quite similar to sensor scheduling in dynamic wireless sensor networks [134, 135, 160]. From an optimization point of view, distributing a large order into many smaller sized orders over a certain time window [6, 52] corresponds to allocating total power over different communication channels in broadcasting networks [149] or wireless sensor networks [164].

## 1.2 Connections between Financial Engineering and Areas in Signal Processing and Communication Systems

We have already briefly introduced the main components of financial engineering from a signal processing perspective. In the following we make several specific connections between financial engineering and areas in signal processing and communication systems.

**Modeling.** One of the most popular models used in financial engineering is the autoregressive moving average (ARMA) model. It models the

current observation (e.g., today's return) as the weighted summation of a linear combination of previous observations (e.g., several previous days' returns) and a moving average of the current and several previous noise components [148]. Actually, this model is also widely used in signal processing and it is referred to as a rational model because its  $z$ -transform is a rational function; or a pole-zero model because the roots of the numerator polynomial of the  $z$ -transform are known as zeros and the roots of the denominator polynomial of the  $z$ -transform are known as poles [95].

**Robust Covariance Matrix Estimation.** After a specific model has been selected, the next step is to estimate or calibrate its parameters from the empirical data. In general, a critical parameter to be estimated is the covariance matrix of the returns of multiple stocks. Usually the empirical data contains noise and some robust estimation methods are needed in practice. One popular idea in financial engineering is to shrink the sample covariance matrix to the identity matrix as the robust covariance matrix estimator [83]. Interestingly, this is mathematically the same as the diagonal loading matrix (i.e., the addition of a scaled identity matrix to the sample interference-plus-noise covariance matrix) derived for robust adaptive beamforming in signal processing and communication systems more than thirty years ago [23, 28, 1]. For large-dimensional data, the asymptotic performance of the covariance matrix estimators is important. The mathematical tool for the asymptotic analysis is referred to as general asymptotics or large-dimensional general asymptotics in financial engineering [84, 85] or random matrix theory (RMT) in information theory and communications [150].

**Portfolio Optimization vs Filter/Beamforming Design.** One popular portfolio optimization problem is the minimum variance problem:

$$\begin{aligned} & \underset{\mathbf{w}}{\text{minimize}} && \mathbf{w}^T \Sigma \mathbf{w} \\ & \text{subject to} && \mathbf{w}^T \mathbf{1} = 1. \end{aligned} \tag{1.3}$$

where  $\mathbf{w} \in \mathbb{R}^N$  is the portfolio vector variable representing the normalized dollars invested in  $N$  stocks,  $\mathbf{w}^T \mathbf{1} = 1$  is the capital budget

**Table 1.1:** Connections between financial engineering and signal processing.

TOOLS	FINANCIAL ENGINEERING	SIGNAL PROCESSING
Modeling	ARMA model [148]	rational or pole-zero model [95]
Covariance Matrix Estimation	shrinkage sample covariance matrix estimator [83]	diagonal loading in beamforming [1, 23, 28]
Asymptotic Analysis	(large-dimensional) general asymptotics [84, 85]	random matrix theory [150]
Optimization	portfolio optimization [96, 99, 133, 163]	filter/beamforming design [110, 163]

constraint, and  $\Sigma \in \mathbb{R}^{N \times N}$  is the (estimated in advance) positive definite covariance matrix of the stock returns.

The above problem (1.3) is really mathematically identical to the filter/beamforming design problem in signal processing [110]:

$$\begin{aligned} & \underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H \mathbf{R} \mathbf{w} \\ & \text{subject to} \quad \mathbf{w}^H \mathbf{a} = 1. \end{aligned} \tag{1.4}$$

where  $\mathbf{w} \in \mathbb{C}^N$  is the complex beamforming vector variable denoting the weights of  $N$  array observations and  $\mathbf{a} \in \mathbb{C}^N$  and  $\mathbf{R} \in \mathbb{C}^{N \times N}$  (estimated in advance) are the signal steering vector (also known as the transmission channel) and the positive definite interference-plus-noise covariance matrix, respectively. The similarity between problems (1.3) and (1.4) shows some potential connections between portfolio optimization and filter/beamforming design, and we will explore more related formulations in detail later in the monograph.

Table 1.1 summarizes the above comparisons in a more compact way and it is interesting to see so many similarities and connections between financial engineering and signal processing.

### 1.3 Outline

The abbreviations and notation used throughout the monograph are provided on pages 173 and 175, respectively.

Figure 1.3 shows the outline of the monograph and provides the recommended reading order for the reader’s convenience. The detailed organization is as follows.

Part I mainly focuses on financial modeling (Chapters 2 and 3) and order execution (Chapter 4).

Chapter 2 starts with some basic financial concepts and then introduces several models, such as i.i.d. model, factor model, ARMA model, autoregressive conditional heteroskedasticity (ARCH) model, generalized ARCH (GARCH) model, and vector error correction model (VECM), which will be used in the later chapters. Thus, this chapter provides a foundation for the following chapters in the monograph.

Chapter 3 deals with the model parameter estimation issues. In particular, it focuses on the estimation of the mean vector and the covariance matrix of the returns of multiple stocks. Usually, those two parameters are not easy to estimate in practice, especially under two scenarios: when the number of samples is small and when there exists outliers. This chapter reviews the start-of-the-art robust estimation of the mean vector and the covariance matrix from both financial engineering and signal processing.

Chapter 4 formulates the order execution as optimization problems and presents the efficient solving approaches.

Once financial modeling and order execution have been introduced in Part I, we move to the design of quantitative investment strategies. As shown in Figure 1.1 there are two main types of investment, namely risk-return trade-off investment and mean-reversion investment, which are documented in Parts II and III, respectively.

Part II entitled “Portfolio Optimization” focuses on the risk-return trade-off investment. It contains Chapters 5-8 and is organized as follows.

Chapter 5 reviews the most basic Markowitz mean-variance portfolio framework, that is, the objective is to optimize a trade-off between the mean and the variance of the portfolio return. However, this frame-

work is not practical due to two reasons: first, the optimized strategy is extremely sensitive to the estimated mean vector and covariance matrix of the stock returns, and second the variance is not an appropriate risk measurement in financial engineering. To overcome the second drawback, some more practical single side risk measurements, e.g., Value-at-Risk (VaR) and Conditional VaR (CVaR), are introduced as the alternatives to the variance.

Chapter 6 presents the robust portfolio optimization to deal with parameter estimation errors. The idea is to employ different uncertainty sets to characterize different estimation errors and then derive the corresponding worst-case robust formulations.

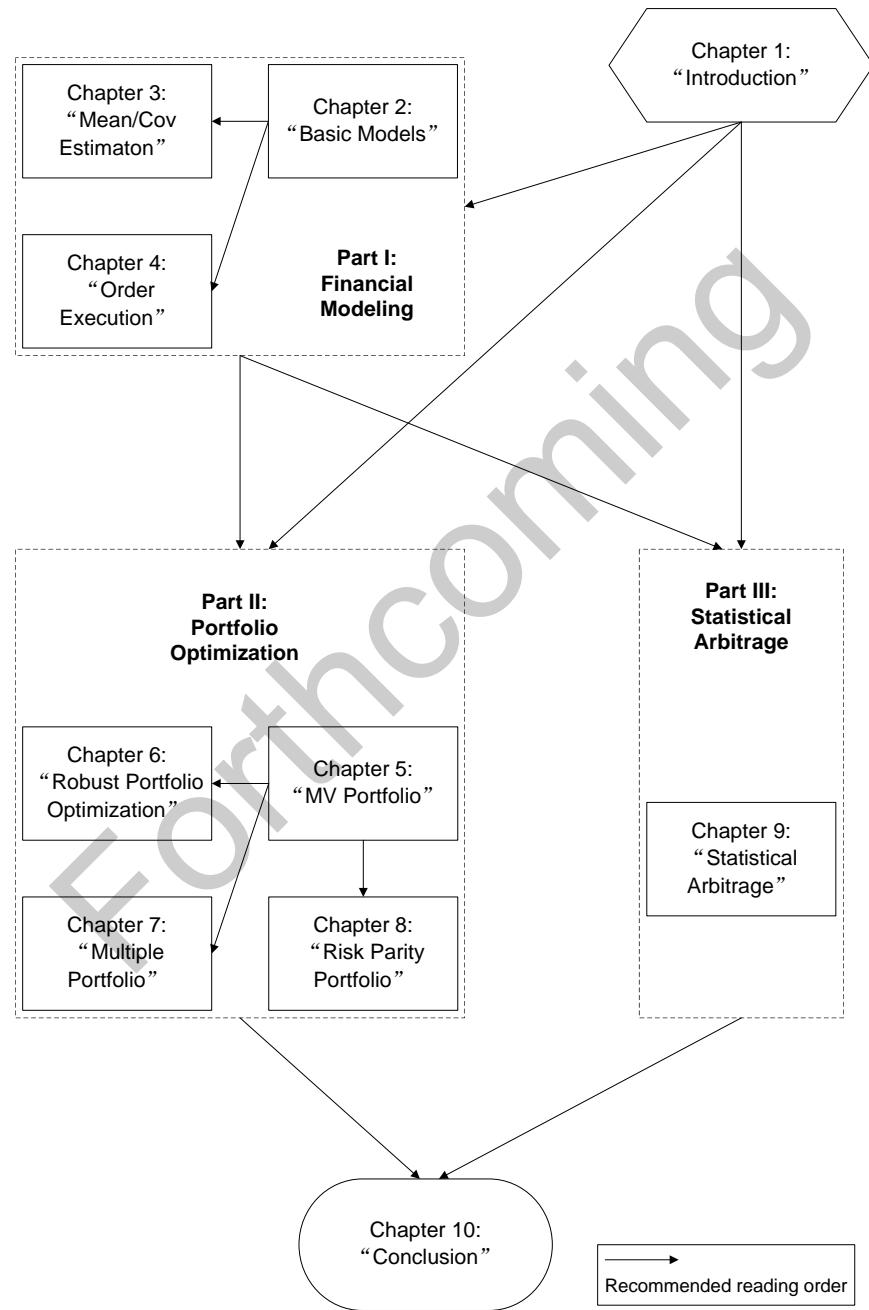
Chapter 7, different from previous Chapters 5 and 6 that consider each portfolio individually, designs multiple portfolios corresponding to different clients jointly via a game theoretic approach by modeling a financial market as a game and each portfolio as a player in the game. This approach is important in practice because multiple investment decisions may affect each other.

Chapter 8 considers a newly developed approach to the portfolio design aiming at diversifying the risk, instead of diversifying the capital as usually done, among the available assets, which is called “risk parity portfolio” in the literature.

Part III, containing Chapter 9, explores the mean-reversion investment that utilizes the noise component in the log-price sequences of multiple assets.

Chapter 9 introduces the idea of constructing a pair of two stocks via cointegration and optimizes the threshold for trading to achieve a preferred criterion. Then it extends further from pairs trading based on only two stocks to statistical arbitrage for multiple stocks.

After covering the main content of the three parts, Chapter 10 concludes the monograph.

**Figure 1.3:** Outline of the monograph.

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