Hierarchical Risk Parity (HRP): Review Questions and Self-Assessment

Test Your Understanding

November 12, 2025

How to Use This Document

These questions are organized by topic and difficulty level:

- **Level 1 (Foundational)**: Basic definitions and concepts
- **Level 2 (Conceptual)**: Understanding and intuition
- **Level 3 (Mathematical)**: Derivations and proofs
- **Level 4 (Applied)**: Problem-solving and calculations
- **Level 5 (Synthesis)**: Critical thinking and integration

Work through questions progressively. If you struggle with Level 1-2 questions, review the corresponding section. Aim to answer Level 3-4 questions without looking at notes, and use Level 5 for deeper exploration.

Contents

1 Part I: Foundations - Portfolio Theory Basics

1.1 Level 1: Foundational Questions

1. What is a portfolio weight vector $\mathbf{w}$? What constraints must it satisfy?
2. Define expected return and variance for a single asset. What do these quantities represent economically?
3. What is covariance? How does it differ from variance?
4. Define the correlation coefficient ρ_{ij}. What are its bounds?
5. What is diversification? Why does it work?

1.2 Level 2: Conceptual Understanding

6. Explain why portfolio return is simply a weighted average of individual returns, but portfolio variance is NOT a weighted average of individual variances.
7. Two assets have the same variance but different correlations with a third asset. Which one would you prefer to add to a portfolio containing the third asset? Why?
8. If you have two assets with correlation $\rho = 0.9$ and another pair with $\rho = 0.1$, which pair offers better diversification benefits? Explain intuitively.
9. Can diversification eliminate all risk? Why or why not? Distinguish between idiosyncratic and systematic risk.
10. Explain why "number of assets" alone is not a good measure of diversification. What matters more?

1.3 Level 3: Mathematical Derivations

11. Derive the formula for portfolio variance: $\sigma_p^2 = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w}$

    Start from: $\sigma_p^2 = \text{Var}\left(\sum_{i=1}^{N} w_i r_i\right)$

12. Prove that correlation $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$ is bounded by $[-1, 1]$.

    Hint: Use the Cauchy-Schwarz inequality or consider the variance of $r_i \pm k r_j$ for appropriate k.

13. Show that for two assets with equal variance σ^2 and correlation ρ, an equally-weighted portfolio has variance:

$$\sigma_p^2 = \frac{\sigma^2}{2}(1 + \rho)$$

    What does this tell you about the benefit of diversification as a function of ρ?

1.4 Level 4: Applied Problems

14. Consider three assets with:

    - $\sigma_1 = 20\%$, $\sigma_2 = 30\%$, $\sigma_3 = 25\%$
    - $\rho_{12} = 0.5$, $\rho_{13} = 0.3$, $\rho_{23} = 0.4$

    Calculate the variance of an equally-weighted portfolio ($w_1 = w_2 = w_3 = 1/3$).

15. Suppose you have two assets with $\sigma_1 = 15\%$, $\sigma_2 = 25\%$, and $\rho_{12} = 0.6$. Find the portfolio weights that minimize variance. Compare the minimum variance to the individual asset variances.

16. An investor holds a portfolio of 10 stocks, all with volatility 30% and pairwise correlation 0.7. What is the portfolio volatility if equally weighted? How does this compare to a single stock?

1.5 Level 5: Synthesis and Critical Thinking

17. During financial crises, correlations tend to increase (often approaching 1). Explain why this is problematic for diversification and foreshadows the problems with Markowitz optimization.

18. Some claim that in modern, globally connected markets, "diversification is dead" because everything moves together. Evaluate this claim using the mathematics of portfolio variance.

19. The formula $\sigma_p^2 = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w}$ is quadratic in weights. What implications does this have for portfolio optimization? Why is optimization more complex than simply choosing low-variance assets?

2 Part II: Markowitz Optimization

2.1 Level 1: Foundational Questions

20. State the Markowitz mean-variance optimization problem. What is being minimized? What are the constraints?

21. What is the efficient frontier? Why is it called "efficient"?

22. What is the minimum variance portfolio (MVP)? Where does it lie on the efficient frontier?

23. What is the Sharpe ratio? What does it measure?

24. Write the formula for the minimum variance portfolio weights. What mathematical operation is required?

2.2 Level 2: Conceptual Understanding

25. Markowitz optimization is "theoretically optimal." Why does it fail in practice? List the three main failures.

26. Explain the difference between in-sample and out-of-sample performance. Why does Markowitz perform well in-sample but poorly out-of-sample?

27. What does it mean for a portfolio to be "concentrated"? Why does Markowitz optimization tend to produce concentrated portfolios?

28. Markowitz portfolios often have extreme long and short positions (e.g., 200% in asset A, -150% in asset B). Explain why the optimizer produces such positions.

29. Why does adding a "no short-selling" constraint ($\mathbf{w} \geq 0$) only partially solve the problems with Markowitz optimization?

2.3 Level 3: Mathematical Derivations

30. Derive the minimum variance portfolio weights:

$$\mathbf{w}_{\text{MVP}} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}}$$

    Use the method of Lagrange multipliers with the constraint $\mathbf{w}^T\mathbf{1} = 1$.

31. For two assets, show that the Markowitz minimum variance portfolio has weights:

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}, \quad w_2 = 1 - w_1$$

    What happens when $\sigma_{12} \to \sigma_1\sigma_2$ (correlation approaches 1)?

32. Prove that any portfolio on the efficient frontier can be written as a linear combination of two frontier portfolios (the two-fund theorem).

2.4 Level 4: Applied Problems

33. Consider two assets:

    - Asset 1: $\mu_1 = 8\%$, $\sigma_1 = 15\%$
    - Asset 2: $\mu_2 = 12\%$, $\sigma_2 = 25\%$
    - Correlation: $\rho_{12} = 0.3$

    (a) Find the minimum variance portfolio weights

    (b) Calculate the portfolio's expected return and standard deviation

    (c) Compare to an equally-weighted portfolio

34. An investor uses Markowitz optimization with 5 assets. The optimization suggests weights: [0.45, -0.20, 0.55, 0.15, 0.05]. What concerns would you have about implementing this portfolio?

35. Suppose you estimate a correlation matrix and find that one eigenvalue is very close to zero (e.g., $\lambda_{\text{min}} = 0.001$). What does this imply about using this matrix for Markowitz optimization?

2.5 Level 5: Synthesis and Critical Thinking

36. The efficient frontier is defined using expected returns $\boldsymbol{\mu}$, which are notoriously difficult to estimate. How does estimation error in $\boldsymbol{\mu}$ versus $\boldsymbol{\Sigma}$ affect portfolio construction differently?

37. Some practitioners use "robust optimization" methods that optimize for worst-case scenarios. How does this differ philosophically from Markowitz's approach? What trade-offs are involved?

38. If Markowitz optimization is "optimal," how can simpler methods (like equal weighting) sometimes outperform it? What does this paradox reveal about the nature of optimization in the presence of uncertainty?

3 Part III: The Problems with Classical Optimization

3.1 Level 1: Foundational Questions

39. What is an eigenvalue? What is an eigenvector? Define these in the context of a covariance matrix.

40. Define the condition number of a matrix. What does it measure?

41. What is meant by an "ill-conditioned" matrix?

42. State Markowitz's Curse. Why is it called a "curse"?

43. What is the difference between noise-induced and signal-induced instability?

3.2 Level 2: Conceptual Understanding

44. Explain in intuitive terms why matrix inversion becomes unstable when eigenvalues are very different in magnitude.

45. Why does high correlation between assets lead to numerical instability in optimization? Use the concept of redundant information.

46. Financial data has a low signal-to-noise ratio. What does this mean concretely? Give an example.

47. Explain the tension between data requirements and stationarity: Why can't we just use more historical data to get better covariance estimates?

48. A practitioner says "I'll use Markowitz but with more sophisticated covariance estimation (e.g., shrinkage estimators)." Will this fully solve the instability problem? Why or why not?

3.3 Level 3: Mathematical Derivations

49. Show that for a 2×2 covariance matrix with equal variances σ^2 and correlation ρ:

$$\mathbf{\Sigma} = \sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

The condition number is: $\kappa(\mathbf{\Sigma}) = \frac{1+\rho}{1-\rho}$

What happens as $\rho \to 1$?

50. Prove that adding a small constant to the diagonal of a matrix (Tikhonov regularization) reduces the condition number:

$$\tilde{\mathbf{\Sigma}} = \mathbf{\Sigma} + \epsilon \mathbf{I}$$

Show that $\kappa(\tilde{\mathbf{\Sigma}}) < \kappa(\mathbf{\Sigma})$ for $\epsilon > 0$.

51. For a covariance matrix with eigenvalues $\lambda_1, \ldots, \lambda_N$, show that the inverse has eigenvalues $1/\lambda_1, \ldots, 1/\lambda_N$. Explain why small eigenvalues cause problems.

3.4 Level 4: Applied Problems

52. A covariance matrix has eigenvalues: [0.045, 0.032, 0.018, 0.012, 0.003]. Calculate the condition number. Is this matrix well-conditioned?

53. You have 50 assets and 260 daily observations (1 year of data). Is this sufficient for reliable covariance estimation? Use the rule of thumb $T \geq 10N$.

54. Two correlation matrices give nearly identical Markowitz minimum variance portfolios in one simulation, but wildly different portfolios in another. The matrices are:

    Matrix A: $\rho_{12} = 0.850$

    Matrix B: $\rho_{12} = 0.851$

    Explain why such a small difference (0.1%) causes instability. How does this relate to condition number?

55. You observe that a Markowitz portfolio has 80% weight in a single asset. You slightly change the data window (drop 5 days, add 5 new days) and now the weight is only 10%. What does this tell you about the condition number of your covariance matrix?

3.5 Level 5: Synthesis and Critical Thinking

56. Modern portfolio theory assumes returns are stationary (statistical properties don't change over time). Given that markets evolve, how does this fundamental assumption interact with the data requirements for covariance estimation?

57. Some researchers use "realized covariance" from high-frequency data (e.g., 5-minute returns). Does this solve the signal-to-noise problem? What new problems might it introduce?

58. The condition number depends on correlation structure, which itself must be estimated. This creates a circularity: we need good estimates to know if our estimates are reliable. How might we break this circle?

59. Compare the problem of ill-conditioned covariance matrices to the problem of multicollinearity in regression. What are the similarities and differences?

4 Part IV: Mathematical Prerequisites for HRP

4.1 Level 1: Foundational Questions

60. What is a metric space? What four properties must a distance function satisfy?
61. Write the formula for correlation-based distance. Why can't we use correlation directly as a distance?
62. What is a graph? What is a complete graph? What is a tree?
63. Define hierarchical clustering. What is a dendrogram?
64. What is a linkage method? Name three types.

4.2 Level 2: Conceptual Understanding

65. Explain the key difference between working with geometry (Markowitz) versus topology (HRP). Use the subway map analogy.
66. Why does converting correlation to distance require the specific formula $d_{ij} = \sqrt{\frac{1}{2}(1-\rho_{ij})}$? Why not just $d_{ij} = 1 - \rho_{ij}$?
67. A complete graph on N nodes has $\binom{N}{2}$ edges. A tree on N nodes has $N-1$ edges. For $N = 50$, compute both. What does this dramatic reduction tell you about complexity?
68. Explain why hierarchical clustering is more robust to outliers in correlation estimates than using the full correlation matrix.
69. In single linkage clustering, we merge clusters based on their nearest members. Why might this lead to "chaining" (long, stringy clusters)? Would complete linkage be better?

4.3 Level 3: Mathematical Derivations

70. Prove that $d_{ij} = \sqrt{\frac{1}{2}(1-\rho_{ij})}$ satisfies the triangle inequality (the hardest metric axiom to verify).

    Hint: Use the fact that the correlation matrix is positive semi-definite.
71. Show that the triangle inequality would be violated if we used $d_{ij} = 1 - \rho_{ij}$ directly. Provide a counterexample.
72. For a tree with N nodes, prove that there are exactly $N-1$ edges. Use induction.
73. Prove that in a tree, there is exactly one path between any two nodes.

4.4 Level 4: Applied Problems

74. Given correlation matrix:

$$\mathbf{C} = \begin{pmatrix} 1.0 & 0.9 & 0.2 & 0.3 \\ 0.9 & 1.0 & 0.25 & 0.35 \\ 0.2 & 0.25 & 1.0 & 0.8 \\ 0.3 & 0.35 & 0.8 & 1.0 \end{pmatrix}$$

    (a) Compute the distance matrix using $d_{ij} = \sqrt{\frac{1}{2}(1 - \rho_{ij})}$

    (b) Perform hierarchical clustering (single linkage) by hand

    (c) Draw the resulting dendrogram

75. You have 5 assets with distances: $d_{12} = 0.3$, $d_{13} = 0.7$, $d_{14} = 0.8$, $d_{15} = 0.6$, $d_{23} = 0.65$, $d_{24} = 0.75$, $d_{25} = 0.55$, $d_{34} = 0.4$, $d_{35} = 0.5$, $d_{45} = 0.35$.

    Apply single linkage clustering step by step until you have a complete dendrogram.

76. Compare single linkage and complete linkage for a simple case. Which is more conservative? Which produces tighter clusters?

4.5 Level 5: Synthesis and Critical Thinking

77. HRP uses correlation-based distance, which depends only on correlation, not on variances. Could we incorporate variance information into the distance metric? What would be the trade-offs?

78. Hierarchical clustering imposes a tree structure on data that might not be truly hierarchical. When might this be a poor assumption? Can you think of asset relationships that don't fit a tree?

79. The choice of linkage method affects the tree structure, which affects the final portfolio. How sensitive is HRP to this choice? Is this a bug or a feature?

80. Graph theory offers many structures beyond trees: minimum spanning trees, community detection algorithms, etc. Could these be used for portfolio construction? What would be the advantages and challenges?

5 Part V: The HRP Algorithm

5.1 Level 1: Foundational Questions

81. List the three steps of the HRP algorithm.

82. What is quasi-diagonalization? What does it accomplish?

83. What is recursive bisection? Why is it called "recursive"?

84. How do you calculate cluster variance in Step 3?

85. In recursive bisection, which sub-cluster gets more weight: the one with higher or lower variance?

5.2 Level 2: Conceptual Understanding

86. Explain why HRP doesn't require matrix inversion. Where does this fundamentally differ from Markowitz?

87. In quasi-diagonalization, we reorder rows and columns of the covariance matrix. Does this change the matrix's eigenvalues? Its determinant? Explain.

88. Why is the reordered matrix called "quasi-diagonal" rather than "diagonal"? What pattern do we expect to see?

89. Explain the risk parity principle embedded in recursive bisection: "allocate inversely to risk."

90. HRP makes $N - 1$ binary decisions (one at each internal node of the tree). Compare this to Markowitz, which solves a single global optimization. What are the implications for stability?

91. Why does HRP typically give positive weight to all assets, while Markowitz often gives zero weights to many assets?

5.3 Level 3: Mathematical Derivations

92. For a cluster C containing assets with variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2$, derive the inverse-variance weights:

$$w_i^{\text{IV}} = \frac{1/\sigma_i^2}{\sum_{j \in C} 1/\sigma_j^2}$$

Show that these weights minimize the portfolio variance if correlations are ignored (i.e., if $\rho_{ij} = 0$ for all $i \neq j$).

93. Prove that in recursive bisection, the weight allocation formula:

$$W_L = W_P \cdot \frac{V_R}{V_L + V_R}$$

ensures that $W_L + W_R = W_P$ (weights sum to parent weight).

94. Show that for two clusters with variances V_L and V_R receiving weights W_L and W_R as above, the risk contributions are equal:

$$W_L^2 V_L = W_R^2 V_R$$

This is the risk parity condition at each bifurcation.

5.4 Level 4: Applied Problems

95. Complete the HRP algorithm for the following 4-asset example:

Dendrogram structure: $\{(\{A, B\}, \{C, D\})\}$

Volatilities: $\sigma_A = 18\%$, $\sigma_B = 22\%$, $\sigma_C = 12\%$, $\sigma_D = 15\%$

Covariances:

- Within cluster $\{A, B\}$: $\sigma_{AB} = 0.7 \cdot 18\% \cdot 22\%$
- Within cluster $\{C, D\}$: $\sigma_{CD} = 0.6 \cdot 12\% \cdot 15\%$

Compute the final HRP weights step-by-step.

96. Given the dendrogram structure: $\{\{A, B\}, \{\{C, D\}, E\}\}$

(i.e., assets A and B cluster first, C and D cluster next, then $\{C, D\}$ clusters with E, and finally $\{A, B\}$ clusters with $\{C, D, E\}$)

If all assets have equal variance and all pairwise correlations are 0.5, what are the HRP weights?

97. Implement the quasi-diagonalization step: Given a dendrogram and a 4×4 covariance matrix, reorder the matrix according to the tree structure.

5.5 Level 5: Synthesis and Critical Thinking

98. HRP uses inverse-variance weighting within clusters but risk-parity weighting between clusters. Why this hybrid approach? Could we use risk parity everywhere?

99. The dendrogram from Step 1 could be "cut" at different heights to produce different numbers of clusters. HRP implicitly uses all levels of the hierarchy. Could we benefit from explicit multi-level analysis?

100. Quasi-diagonalization doesn't change any numerical values, only the ordering. Yet this step is crucial for the algorithm. Why? What would happen if we skipped it?

101. Compare the computational graph of HRP (a tree with local computations) to Markowitz (a global optimization). How does this relate to concepts in machine learning like local vs. global minima?

6 Part VI: Performance Analysis and Comparisons

6.1 Level 1: Foundational Questions

102. What three methods are compared in the paper: CLA, IVP, and HRP? Briefly describe each.

103. In the Monte Carlo simulations, which method performed best out-of-sample? Which performed worst?

104. What is meant by "out-of-sample" performance? Why is it more important than in-sample performance?

105. How much variance reduction did HRP achieve compared to Markowitz (CLA)?

106. What is the time complexity of HRP? How does it compare to Markowitz?

6.2 Level 2: Conceptual Understanding

107. Explain the paradox: Markowitz is "optimal" in-sample but performs worst out-of-sample. What does this teach us about optimization?

108. IVP (inverse variance portfolio) ignores correlations entirely. Why does it outperform Markowitz despite using less information?

109. HRP strikes a balance between CLA (too aggressive) and IVP (too naive). Explain what this means in terms of bias-variance tradeoff.

110. The paper shows HRP is more data-efficient than Markowitz. Explain why this is important for practical implementation.

111. Why does portfolio concentration (putting most weight in one or two assets) increase out-of-sample variance?

6.3 Level 3: Mathematical Analysis

112. Given out-of-sample variances:

    - $\sigma^2_{\text{CLA}} = 0.1157$
    - $\sigma^2_{\text{HRP}} = 0.0671$

    Assuming equal expected returns, calculate the Sharpe ratio improvement of HRP over CLA.

113. If a portfolio has variance $\sigma^2 = 0.09$ and we can reduce it to 0.06 through better methodology, what is the percentage reduction in standard deviation? Why is this more meaningful than percentage reduction in variance?

114. The paper uses $N = 50$ assets and $T = 260$ observations. Calculate the ratio T/N. Is this sufficient for reliable covariance estimation according to the rule $T \geq 10N$?

115. HRP has complexity $O(N^2 \log N)$ while Markowitz has $O(N^3)$. For $N = 1000$, approximately how much faster is HRP?

6.4 Level 4: Applied Problems

116. You manage a portfolio and are deciding between three approaches:

    - Equal weighting (1/N)
    - Markowitz optimization
    - HRP

    You have 100 assets and 3 years of daily data (750 observations). Which method would you choose? Justify based on data requirements and stability.

117. Implement a simple comparison: For 5 assets with a given covariance matrix, compute:

    (a) Minimum variance (Markowitz) weights

    (b) Inverse variance weights

    (c) HRP weights

    Compare the resulting portfolio variances.

118. A portfolio manager shows you backtest results: Markowitz outperforms HRP by 2% per year in-sample. How would you interpret this result? What additional analysis would you request?

6.5 Level 5: Synthesis and Critical Thinking

119. The paper tests performance using Monte Carlo simulation, not just historical backtests. Why is this methodologically stronger? What can simulations tell us that backtests cannot?

120. Transaction costs favor less concentrated portfolios (since you're not trading extreme positions). How would including transaction costs affect the relative performance of CLA, IVP, and HRP?

121. Could we create an ensemble that combines HRP, Markowitz, and equal weighting? How would you determine the optimal combination? Would this be better than any single method?

122. The performance comparison assumes we're only trying to minimize variance (risk). How might results change if we also considered expected returns? Which method would be easier to extend to include return forecasts?

7 Part VII: The Bigger Picture

7.1 Level 1: Foundational Questions

123. According to López de Prado, what is the "least interesting" application of ML in finance? Why?

124. Name three valuable applications of ML in finance (beyond price prediction).

125. What is an ensemble method? Give an example.

126. What is meant by "the representation problem"?

127. List three possible extensions of HRP mentioned in the document.

7.2 Level 2: Conceptual Understanding

128. Explain why price prediction is particularly difficult in financial markets. What makes financial data different from, say, image recognition?

129. HRP succeeds not by better optimization, but by better representation (tree vs. complete graph). Give another example from machine learning where representation choice matters more than optimization algorithm.

130. What is "structural break detection" and why is it important for portfolio management?

131. Explain the concept of "bet sizing" in the context of portfolio management. How does this differ from traditional position sizing?

132. What does it mean to "diversify the methods" rather than just diversifying assets?

7.3 Level 3: Synthesis and Critical Thinking

133. The document argues that HRP's success comes from matching the mathematical representation to the structure of financial data (hierarchical clusters). Can you think of other domains where hierarchical structure exists but standard methods use flat representations?

134. Machine learning is often criticized for being a "black box." Is HRP more or less interpretable than Markowitz optimization? Defend your position.

135. Many quantitative strategies fail when they become widely adopted (the "capacity problem"). Do you think HRP would suffer from this? Why or why not?

136. The paper was published in 2016. Research recent developments: Has HRP been adopted in practice? What limitations have emerged? What improvements have been proposed?

7.4 Level 4: Integration Across Topics

137. Trace the full intellectual journey from Markowitz (1952) to HRP (2016):
    - What problem did Markowitz solve?
    - What problems did his solution create?
    - How did decades of research try to fix these problems?
    - What was López de Prado's key insight?
138. Create a comparison table with axes:
    - Rows: Equal Weight, IVP, Markowitz, HRP
    - Columns: Complexity, Data Requirements, Stability, Diversification, Performance

    Fill in each cell and discuss trade-offs.
139. Design a research project: "HRP for Cryptocurrencies"
    - What challenges would you face?
    - What modifications to HRP might be needed?
    - How would you evaluate success?
140. Philosophical question: HRP shows that simpler, more robust methods can outperform complex "optimal" methods. What does this teach us about the nature of optimization under uncertainty? Can you draw parallels to other fields (medicine, engineering, policy-making)?

8 Comprehensive Challenge Problems

These problems require integrating knowledge from multiple sections:

141. **The Complete Pipeline**: You are given monthly returns for 20 stocks over 10 years.
    (a) Estimate the correlation matrix
    (b) Compute the condition number - is it well-conditioned?
    (c) Convert to distance matrix
    (d) Perform hierarchical clustering
    (e) Implement quasi-diagonalization
    (f) Execute recursive bisection
    (g) Calculate the final HRP portfolio
    (h) Compare to equal-weighted and minimum-variance portfolios
    (i) Backtest all three out-of-sample

142. **Sensitivity Analysis**:
    (a) Take your correlation matrix and perturb each entry by adding random noise $\epsilon \sim N(0, 0.01)$
    (b) Recompute Markowitz and HRP portfolios
    (c) Measure how much the weights changed (e.g., using $\|\mathbf{w}_{\text{new}} - \mathbf{w}_{\text{old}}\|$)
    (d) Repeat 100 times
    (e) Compare the stability of Markowitz vs. HRP

143. **Theoretical Analysis**: Prove that the HRP portfolio can be written as:

$$\mathbf{w}_{\text{HRP}} = f(\mathcal{T}, \mathbf{\Sigma})$$

where $\mathcal{T}$ is the tree structure and f is a function that doesn't require matrix inversion. What are the properties of f? Is it continuous? Differentiable?

144. **Extension: Dynamic HRP**: Design a dynamic version of HRP where:
    - The tree structure can change over time
    - You detect when the hierarchical structure has shifted significantly
    - You decide when to rebalance based on both weights and tree structure

    How would you implement this? What metrics would you use?

145. **Critique and Improve**: Write a critical review of HRP identifying:
    (a) Three key assumptions that might not hold in practice
    (b) Scenarios where HRP would likely fail
    (c) Three concrete improvements you would propose

(d) How you would test whether your improvements work

146. **Real-World Implementation**: You're implementing HRP for a $100M portfolio. Address:

    (a) How often would you recompute the tree?

    (b) How would you handle transaction costs?

    (c) What do you do when the tree structure changes drastically?

    (d) How would you explain this to clients unfamiliar with graph theory?

    (e) What risk management overlays would you add?

9 Self-Assessment Rubric

Rate your understanding for each section (1-5 scale):

Topic	**Level 1**	**Level 2**	**Level 3**	**Level 4**	**Level 5**
Part I: Foundations					
Part II: Markowitz					
Part III: Problems					
Part IV: Prerequisites					
Part V: HRP Algorithm					
Part VI: Performance					
Part VII: Big Picture					

Scoring Guide

For each section and level:

- **1**: Cannot answer questions; need to review material
- **2**: Can answer with notes; basic understanding
- **3**: Can answer most without notes; solid understanding
- **4**: Can answer all confidently; strong understanding
- **5**: Can answer and extend; mastery level

Target Benchmarks

- **Minimum competency**: All Level 1 questions at score 4+, Level 2 at score 3+
- **Strong understanding**: All Level 1-2 at score 5, Level 3-4 at score 4+
- **Mastery**: All levels at score 4+, Level 5 at score 3+

Study Recommendations Based on Gaps

- **If struggling with Level 1-2**: Review the corresponding section in the main document. Focus on definitions and intuition before mathematics.
- **If struggling with Level 3**: Work through the derivations in the document step-by-step. Fill in missing steps. Derive related results.
- **If struggling with Level 4**: Practice more problems. Implement algorithms in code. Test with simulated data.
- **If struggling with Level 5**: Read the original paper by López de Prado. Read related research. Try to extend the ideas to new domains.

Conclusion

These questions are designed to test deep understanding, not just memorization. If you can work through Level 3-5 questions for most sections, you have a strong grasp of HRP and its foundations.

Remember: Understanding comes from doing. Attempt to answer questions without immediately looking at the document. Struggle is part of learning.

Good luck with your review!