"Which path" decoherence in quantum dot experiments

K. Roszak, P. Machnikowski

Institute of Physics, Wrocław University of Technology, 50-370 Wrocław, Poland

Abstract

We analyze and interpret recent optical experiments with semiconductor quantum dots. We derive a quantitative relation between the amount of information transferred into the environment and the optical polarization that may be observed in a spectroscopy experiment.

Key words: Which way decoherence, pure dephasing, independent boson model, spin-boson model

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1 Introduction

The fundamental difficulty in sustaining the coherence of a quantum system is its interaction with the surrounding world. In the course of joint evolution of the system and its environment, this interaction establishes phase correlations between the former and a macroscopic number of degrees of freedom of the latter. For most states, such correlations perturb and eventually erase the internal phase information of the quantum system turning a non-classical superposition state into a mixture of a small subset of classically allowed states. This effect is known as decoherence or dephasing and seems to be one of the most fundamental aspects of the quantum theory [1,2]. It is also of importance for certain practical tasks, like quantum computing [3], where maintaining system coherence over many control operations is indispensable.

Once the quantum correlations (entanglement) between the system and its environment have been created, a measurement on the environment may, in

Email address: katarzyna.roszak@pwr.wroc.pl (K. Roszak).

principle, yield information on the system. Thus, one may say that a certain amount of information, allowing one to determine the system state, has been transferred to the environment. Since both information transfer and decoherence result from the same underlying physical process of building up correlations, a certain relation between the amount of information transfer and the phase coherence retained in the system should be expected. The most celebrated textbook example of such a relation is the vanishing of interference fringes in a double-slit experiment with single particles whenever one tries to establish through which slit the particle passes. Because of this historical relation to the interference of two spatial paths, the knowledge of the system state is customarily referred to as "which path" (or welcher Weg) information.

The decoherence effect due to the transfer of information to the environment appears always when a quantum system is coupled to the surrounding world. In this paper we revisit one specific example of this effect: the decay of coherent optical polarization in a quantum dot due to the dephasing of confined carrier states, which we will interprete as a result of information leakage from the carrier subsystem to its environment (phonon modes). After introducing the system and defining its model (Sec. 2) we will derive the evolution of the confined carriers coupled to phonons using an algebraic technique based on Weyl operators (Sec. 3). Next, we will discuss a quantitative measure of the distinguishability of quantum states due to the information contained in the environment (Sec. 4). Finally, in Sec. 5, we will use the facts introduced in the preceding sections to show that the pure dephasing effect observed as the decay of the coherent response from an optically excited semiconductor quantum dot (or an ensemble thereof) [4] may be interpreted as a result of which path dephasing. We derive a quantitative complementarity relation between the observed coherent optical response and the amount of information transferred to the environment.

2 The system and the model

In this paper, we consider the simplest version of a time-resolved optical experiment performed on a single quantum dot (for practical reasons, actual experiments are often performed on ensambles of QDs using nonlinear techniques [4]). A very short laser pulse prepares the system in a certain superposition (dependent on the pulse phase and intensity) of the ground state (no exciton, denoted $|0\rangle$) and the single-exciton state (denoted $|1\rangle$). By very short we mean a pulse much shorter than the time scales of phonon dynamics, so that the preparation of the initial state may be considered instantaneous. This corresponds to the actual experimental situation with pulse durations of order of 100 fs [4]. On the other hand, the pulse is long enough to assure a relatively narrow spectrum and prevent the population of higher confined levels. Let us

restrict (for simplicity) to the equal superposition state

$$|\psi_0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}.\tag{1}$$

In such a state, the inter-band component of the electric dipole moment has a non-vanishing average value oscillating at an optical frequency (hence referred to as *optical polarization*) which leads to the emission of coherent electromagnetic radiation with an amplitude proportional to the oscillating dipole moment. In an unperturbed system (e.g., in an atom), the radiation would be emitted over time of the order of the lifetime of the superposition state, i.e., until the system relaxes to the ground state due to radiative energy loss.

In a semiconductor structure an additional effect, related to carrier-phonon coupling, appears on a time scale much shorter than the lifetime of the state. Due to the interactions between confined carriers and lattice ions, the ground state of the lattice in the presence of a charge distribution is different than in its absence. As a result, after the creation of a confined exciton the lattice relaxes to a new equilibrium, which is accompanied by the emission of phonon wave packets [5,6] that form a trace in the macroscopic crystal distinguishing the exciton state from an empty dot. As we will discuss below, this information broadcast via emitted phonons leads to a decay of the coherence of the superposition state although the average occupations of the system states remain unaffected (hence the process is referred to as pure dephasing). Since coherent dipole radiation requires well-defined phase relations between the components of a quantum superposition, the amplitude of this radiation, measured in the experiment, gives access to the coherence properties of the quantum state of confined carriers itself. The dephasing of the quantum superposition is therefore directly translated into the decay of coherent optical radiation from the system.

For the carrier-phonon system in a semiconductor quantum dot, a microscopic model of the dephasing effect exists (involving the interaction with acoustic phonons), which reproduces experimental data very well [7,8] and may serve as a reliable starting point to describe the evolution of the combined system of confined carriers and lattice modes.

The Hamiltonian of the system is

$$H = \epsilon |1\rangle\langle 1| + H_{\rm ph} + |1\rangle\langle 1| \sum_{\mathbf{k}} (f_{\mathbf{k}}^* b_{\mathbf{k}} + f_{\mathbf{k}} b_{\mathbf{k}}^{\dagger}), \tag{2}$$

where the first term describes the energy of the confined exciton (ϵ is the energy difference between the states without phonon corrections), $H_{\rm ph} = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}$ is the Hamiltonian of the phonon subsystem and the third term describes the

interaction. Carrier-phonon interaction constants in (2) are given by

$$f_{\mathbf{k}} = (\sigma_{\rm e} - \sigma_{\rm h}) \sqrt{\frac{\hbar k}{2\varrho V_{\rm N} c}} \int_{-\infty}^{\infty} d^3 \mathbf{r} \psi^*(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}} \psi(\mathbf{r}), \tag{3}$$

and describe the deformation potential coupling between the carriers and the lattice modes, which is the dominating mechanism under the assumed optical driving conditions [9]. Here ϱ is the crystal density, $V_{\rm N}$ is the normalization volume of the phonon system, $\omega_{\bf k}=ck$ is the frequency of the phonon mode with the wave vector $\bf k$ (c is the speed of longitudinal sound), and $b_{\bf k}^{\dagger}$, $b_{\bf k}$ are phonon creation and annihilation operators. The exciton wave function is modelled as a product of two identical single-particle wave functions $\psi(r_{\rm e})$ and $\psi(r_{\rm h})$, corresponding to the electron and hole, respectively.

In our calculations we use typical parameters for a self-assembled InAs/GaAs structure: single particle wave functions $\psi(\mathbf{r})$ modelled by Gaussians with 4 nm width in the xy plane and 1 nm along z, the deformation potential difference $\sigma_{\rm e} - \sigma_{\rm h} = 9.5$ eV, crystal density $\varrho = 5300$ kg/m³, and the speed of longitudinal sound c = 5150 m/s.

3 Phonon-induced pure dephasing of optical polarization

In this Section we describe the dephasing of a confined exciton state within the exactly solvable model of interaction with the environment presented above. After describing the time evolution of the system we find the degree of coherence remaining in the system as manifested by the amplitude of the experimentally measurable coherent dipole radiation. In this way we reproduce the recent theoretical description [9] using a simple algebraic method which provides the complete density matrix of the carrier subsystem, necessary for the discussion to be presented in Sec. 5.

The carrier-phonon interaction term in Eq. (2) is linear in phonon operators and describes a shift of the lattice equilibrium induced by the presence of a charge distribution in the dot. The stationary state of the system corresponds to the exciton and the surrounding coherent cloud of phonons representing the lattice distortion to the new equilibrium. The transformation that creates the coherent cloud is the shift $wb_{\mathbf{k}}w^{\dagger} = b_{\mathbf{k}} - f_{\mathbf{k}}/(\hbar\omega_{\mathbf{k}})$, generated by the Weyl operator (see Appendix)

$$w = \exp\left[\sum_{\mathbf{k}} \left(\frac{f_{\mathbf{k}}}{\hbar \omega_{\mathbf{k}}} b_{\mathbf{k}}^{\dagger} - \frac{f_{\mathbf{k}}^{*}}{\hbar \omega_{\mathbf{k}}} b_{\mathbf{k}}\right)\right]. \tag{4}$$

A straightforward calculation shows that the Hamiltonian (2) is diagonalized by the unitary transformation $W = |0\rangle\langle 0| \otimes \mathbb{I} + |1\rangle\langle 1| \otimes w$, where \mathbb{I} is the

identity operator and the tensor product refers to the carrier subsystem (first component) and its phonon environment (second component). As a result one gets

$$\widetilde{H} = WHW^{\dagger} = E|1\rangle\langle 1| + H_{\rm ph},$$

where $E = \epsilon - \sum_{\mathbf{k}} |f_{\mathbf{k}}|^2 / (\hbar \omega_{\mathbf{k}})$.

We assume that at the beginning (t = 0) the state of the whole system is $\rho_0 = (|\psi_0\rangle\langle\psi_0|) \otimes \rho_E$, where ρ_E is the density matrix of the phonon subsystem (environment) at thermal equilibrium and $|\psi_0\rangle$ is the equal superposition state (1) prepared by a properly chosen ultrashort pulse.

The evolution operator $U(t) = e^{-iHt/\hbar}$ may be written as

$$U(t) = W^\dagger W U(t) W^\dagger W = W^\dagger \tilde{U}(t) W = W^\dagger \tilde{U}(t) W \tilde{U}^\dagger(t) \tilde{U}(t) = W^\dagger W(t) \tilde{U}(t),$$

where $\widetilde{U}(t) = e^{-i\widetilde{H}t/\hbar}$ and $W(t) = \widetilde{U}(t)W\widetilde{U}^{\dagger}(t)$. Since $\widetilde{U}(t)$ is diagonal the explicit form of W(t) may easily be found and one gets

$$U(t) = \left[|0\rangle\langle 0| \otimes \mathbb{I} + |1\rangle\langle 1| \otimes w^{\dagger}w(t) \right] \widetilde{U}(t), \tag{5}$$

where $w(t) = e^{-iH_{\rm ph}t/\hbar}we^{iH_{\rm ph}t/\hbar}$.

Using the evolution operator in the form (5) the system state at a time t may be written as

$$\rho(t) = \frac{1}{2} \begin{pmatrix} \rho_{\rm E} & e^{iEt/\hbar} \rho_{\rm E} w^{\dagger}(t) w \\ e^{-iEt/\hbar} w^{\dagger} w(t) \rho_{\rm E} & w^{\dagger} w(t) \rho_{\rm E} w^{\dagger}(t) w \end{pmatrix}, \tag{6}$$

where we used the tensor product notation in which an operator A is expanded as $A = \sum_{m,n} |m\rangle\langle n| \otimes A_{mn}$ with a set of operators A_{mn} acting on the second subsystem, and written in the matrix form with respect to the first subsystem. The density matrix for the carrier subsystem is obtained by tracing out the phonon degrees of freedom, i.e., $\rho_{\rm S} = \text{Tr}_{\rm E} \rho$. Hence,

$$\rho_{S}(t) = \frac{1}{2} \begin{pmatrix} 1 & e^{iEt/\hbar} \langle w^{\dagger}(t)w \rangle \\ e^{-iEt/\hbar} \langle w^{\dagger}w(t) \rangle & 1 \end{pmatrix}, \tag{7}$$

where the average may be calculated by first using the multiplication rule for Weyl operators [Eq. (A.3)] to combine w^{\dagger} and w(t) and then Eq. (A.5). The result is

$$\langle w^{\dagger}(t)w\rangle = \exp\left\{-\sum_{\mathbf{k}} \left|\frac{f_{\mathbf{k}}}{\hbar\omega_{\mathbf{k}}}\right|^{2} \left[i\sin\omega_{\mathbf{k}}t + (1-\cos\omega_{\mathbf{k}}t)(2n_{\mathbf{k}}+1)\right]\right\},$$

where $n_{\mathbf{k}}$ are bosonic equilibrium occupation numbers.

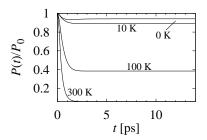


Fig. 1. Decay of the coherent radiation from a confined exciton at various temperatures, as shown.

The emitted coherent dipole radiation is proportional to the non-diagonal element of the density matrix $\rho_s(t)$ and its amplitude is

$$P(t) = P_0 |\langle w^{\dagger}(t)w \rangle|. \tag{8}$$

In Fig. 1 we show the normalized polarization amplitude $P(t)/P_0$ (first derived in Ref. [9]). The interaction with the macroscopic crystal environment leads to a reduction of coherent radiation due to pure dephasing of the exciton state, reflected by the reduced value of the non-diagonal element of the density matrix $\rho_{\rm S}$. At t=0 one has $\langle w^{\dagger}(t)w\rangle=1$, while at large values of t, $\cos \omega_{\bf k} t$ oscillates very quickly as a function of $\bf k$ and averages to 0 (see also Ref. [9]). Thus, for long times, the polarization amplitude tends to a temperature-dependent finite value

$$P(t) \to P_0 \exp \left[-\sum_{\mathbf{k}} \left| \frac{f_{\mathbf{k}}}{\hbar \omega_{\mathbf{k}}} \right|^2 (2n_{\mathbf{k}} + 1) \right] < P_0.$$

This partial decay of coherence is a characteristic feature of short-time dephasing for carrier-phonon couplings encountered in real systems [4].

On the other hand, it is clear from the diagonal elements in Eq. (6) that the two carrier states $|0\rangle$ and $|1\rangle$ are accompanied by different states of the phonon environment so that correlations between the two subsystems are present. These correlations may be related to the phonon wave packets that are emitted into the bulk of the crystal after ultrafast optical excitation [6,5]. These wave packets traveling away from the QD region carry the information about the system state into the environment. We will see that dephasing may be quantitatively related to the amount of these correlations, i.e., to the information on the exciton state extracted by its phonon environment. First, however, we need a quantitative measure for this information.

4 Distinguishability of quantum states

For completeness, in this Section we review the definition of a measure for the quantity of *which path* information on a quantum state of the system contained in its environment [10].

Qualitatively, the definition is as follows. Let us consider a two-level system S with the Hilbert space spanned by the states $|0\rangle$, $|1\rangle$, interacting with its environment E. A which path measurement of the system S, e.g., a measurement of the observable $|0\rangle\langle 0|$, determines the system state. Given the state of the total system S+E, how well can we predict the outcome of this measurement based on a previous measurement on the environment? The measure of distinguishability is based on the probability p of a correct guess for the best possible choice of the measurement on the environment. If the systems are completely uncorrelated then any measurement on the environment is of no help and the probability of correct prediction remains equal to 1/2. If the systems are in a maximally entangled state like $|0\rangle\langle E_0\rangle + |1\rangle\langle E_1\rangle$ with $\langle E_0|E_1\rangle = 0$ then the result of the measurement of the environment observable $|E_0\rangle\langle E_0|$ completely determines the state of the system S, so that p = 1. The distinguishability measure is defined as

$$\mathcal{D} = 2\left(p - \frac{1}{2}\right) \tag{9}$$

and changes correspondingly from 0 to 1.

In general, for a given state of the total system ρ and a result e of the environment measurement, the optimal prediction procedure [11] is the usual statistical inference rule: one calculates the probabilities

$$p(i,e) = \text{Tr}\left[(|i\rangle\langle i| \otimes |e\rangle\langle e|)\rho \right] \tag{10}$$

of finding the system S in the state $|i\rangle$ simultaneously with the environment in the state $|e\rangle$ associated with the measurement outcome e. Then, given the outcome e, one opts for the state i for which this probability is larger. The prediction is correct if the total system is indeed in this more probable state, whichever it is. Allowing for all possible outcomes e, the total likelihood of guessing right is equal to the probability that the system is in any of the "more probable states", i.e,

$$p = \sum_{e} \max [p(0, e), p(1, e)] = \frac{1}{2} \sum_{e} [p(0, e) + p(1, e)] + \frac{1}{2} \sum_{e} |p(0, e) - p(1, e)|.$$

The first term contains the sum of probabilities for any state and is therefore equal to 1/2. Writing the full density matrix (for equal probabilities of the two system states) in the form $\rho = (1/2) \sum_{ij} |i\rangle\langle j| \otimes \rho_{ij}$, where ρ_{00} and ρ_{11} (but neither ρ_{01} nor ρ_{10}) are density matrices, and using Eq. (10) we can write

$$p = \frac{1}{2} + \frac{1}{4} \sum_{e} |\langle e|\rho_{00} - \rho_{11}|e\rangle|.$$

This formula gives the probability of correct prediction for a fixed measurement on the environment, i.e. for a specific basis of environment eigenstates

 $\{|e\rangle\}$. In order to measure the amount of information inherently contained in the environment, independently of the possibly poor choice of the measurement basis, one maximizes this quantity with respect to all possible observables that can be measured, i.e. to all possible complete eigensystems $\{|e\rangle\}$. To find the optimal (upper bound) value for p, we split the hermitian operator $\Delta \rho = \rho_{00} - \rho_{11}$ into a positive and negative part. Let $\Delta \rho_l$ and $|l\rangle$ be the eigenvalues and the corresponding eigenstates of $\Delta \rho$. Let L_+, L_- be the sets of quantum numbers l for which $\Delta \rho_l$ is nonnegative and negative, respectively. Then $\Delta \rho = \Delta \rho^{(+)} - \Delta \rho^{(-)}$, with the positive operators $\Delta \rho^{(\pm)} = \sum_{l \in L_{\pm}} \Delta \rho_l |l\rangle\langle l|$. Since these operators are positive, one has

$$\left| \langle e | \Delta \rho^{(+)} | e \rangle - \langle e | \Delta \rho^{(-)} | e \rangle \right| \le \langle e | \Delta \rho^{(+)} | e \rangle + \langle e | \Delta \rho^{(-)} | e \rangle = \langle e | | \Delta \rho | | e \rangle,$$

where $|\Delta \rho|$ is the modulus of the operator $\Delta \rho$, defined as

$$|\Delta \rho| = \sum_{l} |\Delta \rho_l| |l\rangle\langle l| = \Delta \rho^{(+)} + \Delta \rho^{(-)}.$$

Hence,

$$\frac{1}{4} \sum_{e} |\langle e | \Delta \rho | e \rangle| \le \frac{1}{4} \operatorname{Tr} |\Delta \rho| = \frac{1}{2} D(\rho_{00}, \rho_{11}),$$

where $D(\rho, \rho')$ is known as the trace distance [3] between the density matrices ρ and ρ' . On the other hand, the equality is clearly attained if the eigensystem $\{|e\rangle\}$ coincides with the eigenstates of $\Delta\rho$, i.e., when the observable measured on the environment is $\Delta\rho$ itself. Thus, using the definition (9), we finally get

$$\mathcal{D} = D(\rho_{00}, \rho_{11}). \tag{11}$$

5 The complementarity relation

In this Section we derive a quantitative complementarity relation between the degree of coherence in the quantum dot system, as manifesteed by the amplitude of coherent radiation described in Sec. 3, and the amount of *which* path information transferred to the environment, as defined in Sec. 4. This relation is analogous to the visibility-distinguishability relation in the doubleslit setup [10,12].

For the carrier-phonon state of Eq. (6) the distinguishability of carrier states due to the correlations with the phonon environment is, from Eq. (11),

$$\mathcal{D}(t) = D(\rho_E, w^{\dagger}w(t)\rho_E w^{\dagger}(t)w).$$

One has, in general [3], $D^2(\rho, \rho') \leq 1 - F^2(\rho, \rho')$, where $F(\rho, \rho') = \text{Tr} \sqrt{\rho^{1/2} \rho' \rho^{1/2}}$

is the fidelity measure of the distance between the states ρ and ρ' . Hence

$$\mathcal{D}^{2}(t) \leq 1 - \left[\operatorname{Tr} \sqrt{\rho_{E}^{1/2} w^{\dagger} w(t) \rho_{E} w^{\dagger}(t) w \rho_{E}^{1/2}} \right]^{2} = 1 - \left[\operatorname{Tr} \left| \rho_{E}^{1/2} w^{\dagger} w(t) \rho_{E}^{1/2} \right| \right]^{2}.$$
(12)

Since $\operatorname{Tr} |A| \ge |\operatorname{Tr} A|$ we may write

$$\operatorname{Tr} |\rho_E^{1/2} w^{\dagger} w(t) \rho_E^{1/2}| \ge |\operatorname{Tr} (\rho_E w^{\dagger} w(t))| = |\langle w^{\dagger} w(t) \rangle| = P(t)/P_0, \tag{13}$$

where we used Eq. (8). Combining Eqs. (12) and (13) leads to the relation

$$\left[\frac{P(t)}{P_0}\right]^2 + \mathcal{D}^2(t) \le 1,\tag{14}$$

which shows that the relative decay of polarization is related to the *which* path information transfer to the environment. Note that for pure states $\rho_{\rm E}$, equality holds in Eqs. (12) and (13), so that the relation (14) also turns into an equality.

6 Conclusion

In this paper, we have interpreted the dephasing effects observed in recent optical experiments on semiconductor quantum dots in terms of "which path" information transfer to the environment. Coherence properties of carriers confined in a QD may be tested by detecting coherent radiation emitted from the system as a result of optical polarization induced by optical excitation. Due to the coupling between the system and its phonon environment, in the course of quantum evolution lattice modes get excited, which leads to the dephasing of the carrier state manifested by a decrease of the amplitude of this radiation. We have derived an inequality between the remaining amplitude of emitted radiation and the amount of information carried to the bulk of the macroscopic crystal by lattice excitations. From this point of view the dephasing of optical polarization is of the same nature as the well-known disappearance of interference fringes in a double-slit (Young) experiment due to the which path knowledge.

The complementarity relation (14) is analogous to similar relations between the distinguishability of paths and the visibility of interference fringes in the double-slit setup [10,12]. In fact, an interference experiment analogous to the double-slit experiment has been performed using optically excited states of semiconductor quantum dots [13], where the two states (analogous to paths) are the absence or presence of a single, optically created exciton, confined by the binding potential of the QD. It may be shown that the visibility of interference fringes in these time-domain interference experiments is governed by the same non-diagonal element of the reduced density matrix as the coherent polarization in our discussion.

Our discussion confirms that incompatibility of the quantum behavior and which path information is a general feature of the quantum world. We have shown that a manifestation of this general property may be identified in an experimental situation that is extremely different from the historical concept of two spatial paths in a double-slit experiment.

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Appendix

A Rules for Weyl operators

Applying the Weyl operator (4) to the phonon annihilation operator b_k and expanding the result we get [14]

$$wb_{\mathbf{k}}w^{\dagger} = b_{\mathbf{k}} + [S, b_{\mathbf{k}}] + \frac{1}{2}[S, [S, b_{\mathbf{k}}]] + \dots,$$

where $S = \sum_{\mathbf{k}} (g_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} - g_{\mathbf{k}}^{*} b_{\mathbf{k}})$, $g_{\mathbf{k}} = f_{\mathbf{k}} / (\hbar \omega_{\mathbf{k}})$. Since $[S, b_{\mathbf{k}}] = -g_{\mathbf{k}}$ and $[S, [S, b_{\mathbf{k}}]] = 0$ (as all the higher commutators), we obtain the shift effect of the Weyl operator, $w b_{\mathbf{k}} w^{\dagger} = b_{\mathbf{k}} - g_{\mathbf{k}}$.

Multiplying Weyl operators is done using the Baker-Hausdorff formula for separating operator exponents [14]

$$e^{A+B} = e^A e^B e^{-[A,B]/2},$$
 (A.1)

where A and B are some operators and [A, [A, B]] = [B, [A, B]] = 0. Some simple algebra using Eq. (A.1) shows that the product of two Weyl operators,

$$w_i = \exp\left[\sum_{\mathbf{k}} \left(g_{\mathbf{k}}^{(i)} b_{\mathbf{k}}^{\dagger} - g_{\mathbf{k}}^{(i)*} b_{\mathbf{k}}\right)\right],\tag{A.2}$$

i=1,2, is equal to

$$w_1 w_2 = w_3 \exp\left[-\frac{1}{2} \sum_{\mathbf{k}} \left(g_{\mathbf{k}}^{(1)*} g_{\mathbf{k}}^{(2)} - g_{\mathbf{k}}^{(1)} g_{\mathbf{k}}^{(2)*}\right)\right], \tag{A.3}$$

where w_3 is given by (A.2) with $g_{\mathbf{k}}^{(3)} = g_{\mathbf{k}}^{(1)} + g_{\mathbf{k}}^{(2)}$.

Now it is possible to calculate the equilibrium average of a Weyl operator (following Ref. [14]), $\langle w \rangle = \text{Tr}(\rho_{\rm E} w)$, where $\rho_{\rm E}$ is the density matrix of the phonon reservoir at thermal equilibrium. Since $\rho_{\rm E} = e^{-\beta H_{\rm ph}}/\text{Tr}\,e^{-\beta H_{\rm ph}}$, $\beta = 1/(k_{\rm B}T)$, and $H_{\rm ph} = \sum_{\bf k} \hbar \omega_{\bf k} b_{\bf k}^{\dagger} b_{\bf k}$, we may write

$$\langle w \rangle = \frac{\text{Tr}\left(e^{-\beta \sum_{\boldsymbol{k}} \hbar \omega_{\boldsymbol{k}} b_{\boldsymbol{k}}^{\dagger} b_{\boldsymbol{k}}} e^{\sum_{\boldsymbol{k}} (g_{\boldsymbol{k}} b_{\boldsymbol{k}}^{\dagger} - g_{\boldsymbol{k}}^* b_{\boldsymbol{k}})}\right)}{\text{Tr}(e^{-\beta \sum_{\boldsymbol{k}} \hbar \omega_{\boldsymbol{k}} b_{\boldsymbol{k}}^{\dagger} b_{\boldsymbol{k}}})},$$

and separate the thermal equilibrium into

$$\langle w \rangle = \prod_{\mathbf{k}} \langle w \rangle_{\mathbf{k}},\tag{A.4}$$

where

$$\langle w \rangle_{\mathbf{k}} = \frac{\operatorname{Tr}\left(e^{-\beta\hbar\omega_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}b_{\mathbf{k}}}e^{(g_{\mathbf{k}}b_{\mathbf{k}}^{\dagger} - g_{\mathbf{k}}^{*}b_{\mathbf{k}})}\right)}{\operatorname{Tr}(e^{-\beta\hbar\omega_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}b_{\mathbf{k}}})}.$$
(A.5)

Using Eq. (A.1) and keeping in mind that $\text{Tr}(e^{-\beta\hbar\omega_k b_k^{\dagger}b_k}) = (1 - e^{-\beta\hbar\omega_k})^{-1}$, we can transform Eq. (A.5) into

$$\langle w \rangle_{\mathbf{k}} = e^{\frac{1}{2}|g_{\mathbf{k}}|^2} (1 - e^{-\beta\hbar\omega_{\mathbf{k}}}) \operatorname{Tr}(e^{-\beta\hbar\omega_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}b_{\mathbf{k}}} e^{g_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}} e^{-g_{\mathbf{k}}^*b_{\mathbf{k}}}). \tag{A.6}$$

For the trace one has

$$\operatorname{Tr}(e^{-\beta\hbar\omega_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}b_{\mathbf{k}}}e^{g_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}}e^{-g_{\mathbf{k}}^{*}b_{\mathbf{k}}}) = \sum_{m=0}^{\infty} e^{-\beta\hbar\omega_{\mathbf{k}}m} \langle m|e^{g_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}}e^{-g_{\mathbf{k}}^{*}b_{\mathbf{k}}}|m\rangle, \tag{A.7}$$

where $|m\rangle$ denotes a state with m excitations. The exponents are expanded in a power series,

$$e^{-g_{\mathbf{k}}^* b_{\mathbf{k}}} |m\rangle = \sum_{l=0}^{\infty} \frac{(-g_{\mathbf{k}}^*)^l}{l!} b_{\mathbf{k}} |m\rangle. \tag{A.8}$$

Since $b_{\mathbf{k}}^l|m\rangle = [m!/(m-l)!]^{1/2}|m-l\rangle$ for $l \leq m$ and $b_{\mathbf{k}}^l|m\rangle = 0$ for l > m, Eq. (A.8) transforms into

$$e^{-g_{\mathbf{k}}^* b_{\mathbf{k}}} |m\rangle = \sum_{l=0}^m \frac{(-g_{\mathbf{k}}^*)^l}{l!} \left[\frac{m!}{(m-l)!} \right]^{1/2} |m-l\rangle.$$
 (A.9)

Treating the other exponent in the same manner we end up with

$$\langle m|e^{g_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}}e^{-g_{\mathbf{k}}^{*}b_{\mathbf{k}}}|m\rangle = \sum_{l=0}^{m} \frac{(-|g_{\mathbf{k}}|^{2})^{l}}{(l!)^{2}} \frac{m!}{(m-l)!},$$
 (A.10)

that is, a Laguerre polynomial of order m, $L_m(|g_k|^2)$. Using the fact that

$$\sum_{m=0}^{\infty} L_m(|g_{\mathbf{k}}|^2) z^m = (1-z)^{-1} e^{|g_{\mathbf{k}}|^2 \frac{z}{z-1}},$$

with a specific $z = e^{-\beta\hbar\omega_k}$ [so that $n_k = z/(1-z)$], and inserting the result into Eq. (A.6), and then into Eq. (A.4), we get the thermal average of the Weyl operator in the form

$$\langle w \rangle = e^{-\frac{1}{2} \sum_{k} |g_{k}|^{2} (2n_{k} + 1)}.$$
 (A.11)

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