

前提假設：

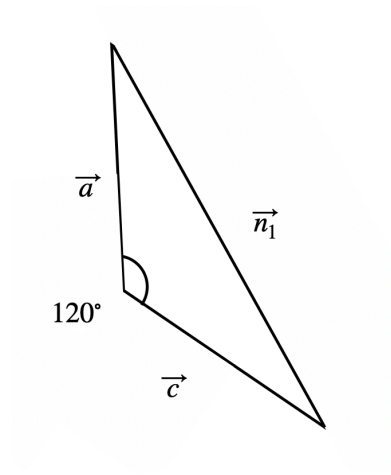
$$\begin{cases} \theta = 60^\circ \\ \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta = 0.5(|\vec{a}| |\vec{b}|) \\ \vec{b} \cdot \vec{c} = |\vec{b}| |\vec{c}| \cos\theta = 0.5(|\vec{b}| |\vec{c}|) \\ \vec{c} \cdot \vec{a} = |\vec{c}| |\vec{a}| \cos\theta = 0.5(|\vec{c}| |\vec{a}|) \end{cases}$$

\vec{n}_1 為目標動量，可控制範圍為 \vec{a} 、 \vec{c} ，即導出：

$$\begin{cases} \theta = 90^\circ \\ \vec{n}_1 \cdot \vec{b} = |\vec{n}_1| |\vec{b}| \cos\theta = 0 \Rightarrow \vec{n}_1 \perp \vec{b} \\ \vec{n}_1 = \vec{a} + \vec{c} \end{cases}$$

如果針對 \vec{n}_1 的向量動量來解釋：

設 n 為方向強度，則

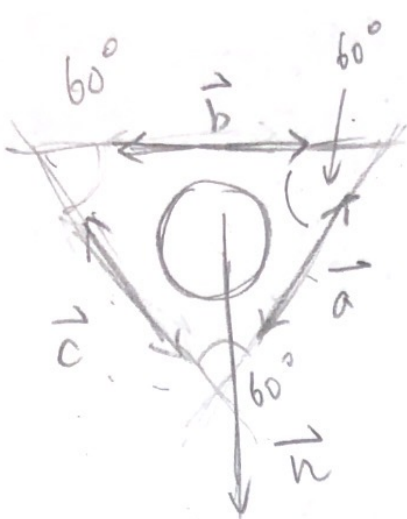


(圖一)

$$\left\{ \begin{array}{l} |\vec{a}| = |\vec{c}| \\ |\vec{c}| < |\vec{n}_1| \\ |\vec{a}| < |\vec{n}_1| \\ (n \cdot |\vec{n}_1|)^2 = (n |\vec{a}|)^2 + (n |\vec{c}|)^2 \end{array} \right.$$

且因為向量 \vec{a} 、 \vec{c} 為等量，且馬達的佈局是以正三角形排列。根據（圖一）所示，我們把向量整合起來，會發現 \vec{a} 與 \vec{c} 的夾角成120度，且其之頂點垂直於 \vec{n}_1 ，另外其他夾角都是 30° 。得知sin的比例特性後利用畢氏定理假設出實值之比例，且利用 n 來修正比例值之強度。

$$\left\{ \begin{array}{l} \sqrt{\sin(30^\circ)^2 + \sin(30^\circ)^2} = \sqrt{\sin(120^\circ)^2} \Rightarrow \\ \sqrt{(\sin(30^\circ) |\vec{a}|)^2 + (\sin(30^\circ) |\vec{c}|)^2} = \sqrt{(\sin(120^\circ) |\vec{n}_1|)^2} = \\ \sqrt{(0.5 |\vec{a}| \cdot 1)^2 + (0.5 |\vec{c}|)^2} \approx \sqrt{0.86602 |\vec{n}_1|} \Rightarrow \\ \sqrt{|\vec{c}|} = \sqrt{|\vec{a}|} = \left(\frac{|\vec{n}_1| \sin(120^\circ)}{2} \right)^2 \\ \left(\frac{|\vec{n}_1| \sin(120^\circ)}{2} \right)^2 = |\vec{a}| = |\vec{c}| \end{array} \right.$$



前提:

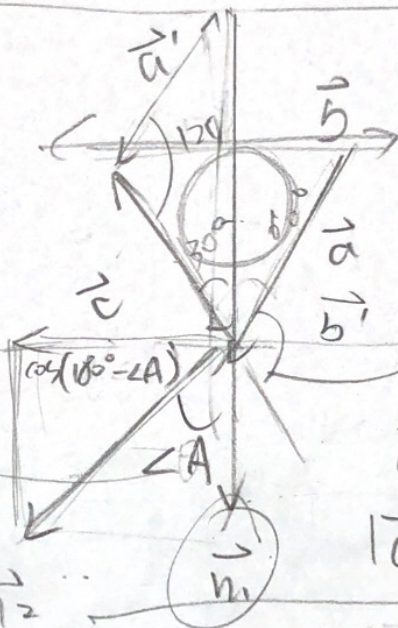
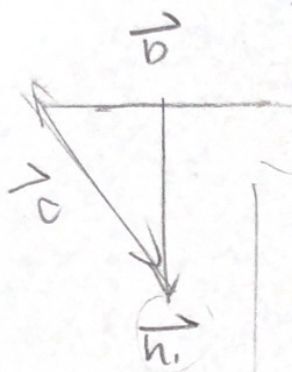
$$\theta = 60^\circ$$

$$\begin{cases} \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = 0.5 |\vec{a}| |\vec{b}| \\ \vec{b} \cdot \vec{c} = |\vec{b}| |\vec{c}| \cos \theta = 0.5 |\vec{b}| |\vec{c}| \\ \vec{c} \cdot \vec{a} = |\vec{c}| |\vec{a}| \cos \theta = 0.5 |\vec{c}| |\vec{a}| \end{cases}$$

設控制 $\vec{n}, \vec{a}, \vec{c}$

$$\theta = 90^\circ$$

$$\begin{cases} \vec{n}_1 \cdot \vec{b} = |\vec{n}_1| |\vec{b}| \cos \theta = 0 \Rightarrow \vec{n}_1 \perp \vec{b} \\ \vec{n}_1 = \vec{a} + \vec{c} \end{cases}$$



$$\angle A = 180^\circ - \angle A'$$

$$\sin(180^\circ - \angle A)$$

$$\cos(180^\circ - \angle A)$$

$$\vec{a} = \vec{a}'$$

$$|\vec{c}| = |\vec{a}| < |\vec{n}_1|$$

$$\vec{n}_2 = \begin{pmatrix} \cos(\angle A') \\ \sin(\angle A') \end{pmatrix}$$

$$\begin{cases} \vec{n}_1 + \vec{a} + \vec{c} \\ \vec{n}_1 \cdot \vec{b} = 0 \\ \Rightarrow \vec{n}_1 \perp \vec{b} \end{cases}$$

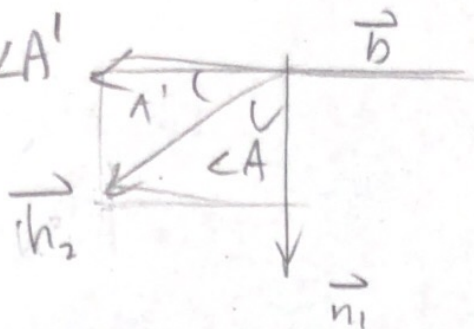
$$\begin{pmatrix} n_x \\ n_y \end{pmatrix} = \begin{pmatrix} \vec{c}_x + \vec{a}_x \\ \vec{c}_y + \vec{a}_y \end{pmatrix}$$

$$\begin{aligned} \vec{n}_2 &= \vec{n}_1 + \vec{b} \\ \Rightarrow \vec{n}_1 &= \vec{n}_2 - \vec{b} \end{aligned}$$

$$\vec{n}_2 = \vec{n}_1 + \vec{b}$$

$$\begin{aligned} \vec{n}_2 &= (\vec{a} + \vec{c}) + \vec{b} \\ &= \vec{a} + \vec{b} + \vec{c} \end{aligned}$$

• $\angle A = 180^\circ - \angle A'$

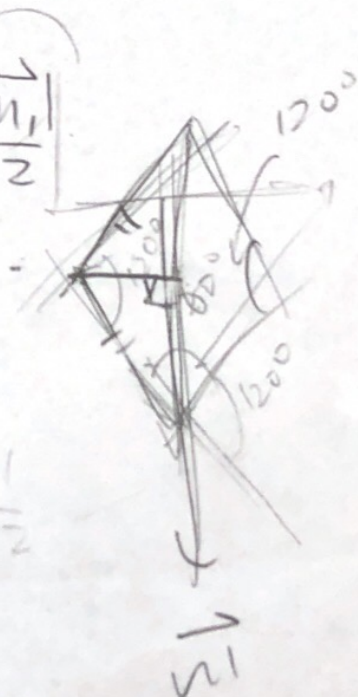


$$\vec{h}_2 = \begin{pmatrix} \cos(\angle A) \\ \sin(\angle A) \end{pmatrix} = \vec{h}_1 + \vec{b} = \begin{pmatrix} 0 \\ \sin(\angle A) \end{pmatrix} + \begin{pmatrix} \cos(\angle A) \\ 0 \end{pmatrix}$$

$$\vec{h}_1 = \begin{pmatrix} 0 \\ \sin(\angle A) \end{pmatrix} \quad \angle A \text{ 為 傾斜角}$$

$$\vec{b} = \begin{pmatrix} \cos(\angle A) \\ 0 \end{pmatrix}$$

$$\vec{h}_1 = \begin{pmatrix} 0 \\ \sin(\angle A) \end{pmatrix} = \vec{a} + \vec{c} \quad |\vec{a}| = |\vec{c}| = \frac{|\vec{h}_1|}{2}$$



$$n\vec{h}_1 = n\vec{a} + n\vec{c}$$

$$= n(\vec{a} + \vec{c}) \quad \frac{|\vec{h}_1|}{2} = \frac{|\vec{a}|}{2} + \frac{|\vec{c}|}{2}$$

$$= n(2i \frac{|\vec{h}_1|}{2}) \quad |\vec{a}| = |\vec{c}|$$

$$\vec{a} = \frac{|\vec{h}_1|}{2} \times \frac{1}{n}$$

$$\vec{h}_1 = \frac{n \left(\frac{|\vec{h}_1|}{2} \right)}{\vec{a}} + \frac{n \left(\frac{|\vec{h}_1|}{2} \right)}{\vec{c}}$$

