AuW-u01-bf

1.

$G = (V, E)$, zusammenhängender Graph mit $|V| \geq 3$

a.

i)
$\forall v \in V,\ \deg(v) \equiv_2 0 \implies G$ ist 2-Kanten-zusammenhängend

No, consider as counter example the complete graph on five vertices, K_5. Since $\forall v \in V \deg(v) = 5 - 1 \equiv_2 0$, but removing any two edges still leaves the graph connected.
□

ii)
G ist 2-Kanten-zusammenhängend $\implies \forall v \in V,\ \deg(v) \equiv_2 0$

No, consider the graph on $V = \{a, b, c, d\}$ with $E = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, c\}\}$. The graph is 2-edge-connected but there exist some vertices $v \in V$ s.t. $\deg(v) \not\equiv_2 0$, namely (for the counter example provided here) a and c.
□

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b.

i)
G hat Hamiltonkreis $\implies G$ ist 2-zusammenhängend

Yes, since all vertices $v \in V$ must be in the same equivalence class of the equivalence relation $\sim$ on E. Thus they are in the same "block" and there cannot exist another block, since otherwise some vertex (namely the cut vertices) would have to be visited twice.
□

ii)
G ist 2-zusammenhängend $\implies G$ hat Hamiltonkreis

No, consider the 3×3 grid, which is 2-connected but does not contain a hamiltonian cycle.
□

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c.

Let G be 2-connected. Let (u, v, w) be a path of length 2 in G. Show that we can extend this path to a cycle, i.e. that G contains a cycle in which u, v, and w are adjacent vertices.

By Satz von Menger we know that there exist k internal-vertex-disjunct u-v-paths. We have as one such path the given u-(v)-w-path. There must hence exist one other such u-w-path that is internally vertex disjunct to the first one. Connecting these two yields the desired cycle, in which u, v and w are adjacent.
□