AW_T3_DH

1

(a)

We assume that $"\text{Anzahl Kanten über den Schnitt } (S, V \setminus S)"$ means the number of edges with an incident vertex in each of these sets.

Thus we define an indicator variable for every *edge* e.

$Y_e = \begin{cases} 1, & e \text{ is an edge over } (S, V \setminus S) \\ 0, & \text{otherwise} \end{cases}$

For every edge e the chance that the vertices are in different subsets $(S, V \setminus S)$ is the chance that the second vertex is in a different vertex than the first one. Which is $\frac{1}{2}$ as all subsets are equally likely.

$\forall e \in E : \mathbb{E}[Y_e] = \frac{1}{2}$

$\mathbb{E}[X] \overset{\text{linearity of } \mathbb{E}}{=} \sum_{e \in E} \mathbb{E}[Y_e] = m \cdot \frac{1}{2} = \frac{m}{2}$

(b)

For the weighted average to be $\frac{m}{2}$ there has to exist at least one value with magnitude at least $\frac{m}{2}$. (→ Lecture)

2

$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = Pr[A]$

(i)

To show that $\bar{A}$ and B are *independent* it suffices to show that $Pr[\bar{A}|B] = Pr[\bar{A}]$.

$$
\begin{aligned}
Pr[\bar{A}|B] &= \frac{Pr[\bar{A} \cap B]}{Pr[B]} && \text{(def. conditional prob.)} \\
&= \frac{Pr[B] - Pr[A \cap B]}{Pr[B]} && \text{(law of total prob. over } B) \\
&= 1 - \frac{Pr[A \cap B]}{Pr[B]} \\
&= 1 - Pr[A|B] && \text{(def. conditional prob.)} \\
&= 1 - Pr[A] && \text{(independence A and B)} \\
&= Pr[\bar{A}] && \text{(def. of complement)}
\end{aligned}
$$

This proves the *independence* of $\bar{A}$ and B

(ii)

Simply by swapping A and B we can show this *independence* analogous to (i).

(iii)