

## Test 2 - Schlüsselthemen

### Aufgabe 1 (Recap Basics)

- 1) A  $7 \times 5$  matrix has  rows.
- 2) Let  $A \in \mathbb{R}^{2 \times 4}$ , then  $A \cdot B$  is defined, if  $B$  is a   $\times 4$  matrix.
- 3) Let  $A$  be a  $n \times n$  matrix.  $A$  is invertible, iff  $\text{rank} A = n -$  .
- 4) Let  $A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & 3 \\ 2 & 4 & 4 \end{pmatrix}$ .  $A$  has  free variable(s) and  pivot(s).
- 5) For any  $A$  and  $x$ , the homogeneous linear systems of equations ( $Ax = 0$ ) always has at least  solution(s).
- 6) Let  $x = \begin{pmatrix} 2 & 4 & 6 & 8 \end{pmatrix}^T \in \mathbb{R}^4$ . Then  $\min_{i \in \{1, \dots, 4\}} \|x \cdot e_i\| =$   and  $\arg \min_{i \in \{1, \dots, 4\}} \|x \cdot e_i\| =$  .
- 7) Let  $A = CR$  where  $C$  contains the  $r$  linearly independent columns of  $A$ . We know that  $R$  has 2 rows, then  $r =$  .

Summe: \_\_\_\_\_

### Aufgabe 2 (Schlüssel)

- 1) Let  $u, v, w \in \mathbb{R}^2$  and  $u, v$  be orthonormal. Then  $\dim(\text{span}\{u, v, w\}) =$  .
- 2) Let  $V = \mathbb{R}^{3 \times 2}$  be a vector space. A basis of  $V$  has exactly  vectors.
- 3) Given a spanning set  $S$  of a subspace  $U$  of  $\mathbb{R}^{10}$ . We know  $|S| = 4$ , then  $\dim U \leq$   and  $\dim U \geq$  .
- 4) Given a set  $M$  of vectors in a vector space  $V$  with  $\dim V = 2$ . We know  $\exists x \in M$  with  $\text{span}(M - \{x\}) = V$ .  $M$  has at least  vector(s).

- 5) Given any basis. For all vectors there exists  linear combination(s) from the basis vectors.
- 6) Let  $U \subseteq \mathbb{R}^3$ .  $U$  is a subspace of  $\mathbb{R}^3$ , if  $U = \{ (a \ b \ c)^T \in \mathbb{R}^3 \mid a + b + c = \text{$  } \}.
- 7) Let  $F : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation and  $A$  the transformation matrix.  $A$  has  rows und  columns.
- 8) Given  $Ax = b$ ,  $\text{ref}(A) = \begin{pmatrix} 2 & -5 & 0 & 1 & 0 \\ 0 & 0 & 1 & 5 & 1 \end{pmatrix}$ . Then we know any  $b$  has  dimension(s) of solutions  $x$ .
- 9) Given  $Ax = b$ ,  $\text{ref}(A) = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ . Then any basis of  $C(A)$  has  vector(s) and any basis of  $N(A)$  has  vector(s).
- 10) Let  $F : \mathbb{R}^5 \rightarrow \mathbb{R}^6$  be a linear transformation and  $A$  the transformation matrix with  $\dim C(A) = 3$ . Then  $\dim N(A) = \text{$

**Summe:** \_\_\_\_\_

**Summe Insgesamt:** \_\_\_\_\_