## AuW-u01-bf

### 1.

$G = (V, E)$, zusammenhängender Graph mit $|V| \geq 3$

### a.

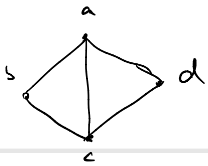
i)
$\forall v \in V,\ \deg(v) \equiv_2 0 \implies G$ ist 2-Kanten-zusammenhängend

No, consider as counter example the complete graph on five vertices, $K_5$. Since $\forall v \in V\ \deg(v) = 5 - 1 \equiv_2 0$, but removing any two edges still leaves the graph connected.

☐ × I think your misconception is that "2-Kanten-zusammenhängend" means there exists a u-v seperator of size 2 but that doesn't need to be the case. Instead it means, that if there exists a seperator it is of at least size 2.

ii)
$G$ ist 2-Kanten-zusammenhängend $\implies \forall v \in V,\ \deg(v) \equiv_2 0$

No, consider the graph on $V = \{a, b, c, d\}$ with $E = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, a\}, \{a, c\}\}$. The graph is 2-edge-connected but there exist some vertices $v \in V$ s.t. $\deg(v) \not\equiv_2 0$, namely (for the counter example provided here) $a$ and $c$.

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### b.

i)
$G$ hat Hamiltonkreis $\implies G$ ist 2-zusammenhängend

Yes, since all vertices $v \in V$ must be in the same equivalence class of the equivalence relation $\sim$ on $E$. Thus they are in the same "block" and there cannot exist another block, since otherwise some vertex (namely the cut vertices) would have to be visited twice.

☐ ✓× The statement is indeed correct, however your argument doesn't work since the equivalence classes are defined on the edges instead of the vertices.

ii)
$G$ ist 2-zusammenhängend $\implies G$ hat Hamiltonkreis

No, consider the $3 \times 3$ grid, which is 2-connected but does not contain a hamiltonian cycle.

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**c.**

Let $G$ be 2-connected. Let $(u, v, w)$ be a path of length 2 in $G$. Show that we can extend this path to a cycle, i.e. that $G$ contains a cycle in which $u$, $v$, and $w$ are adjacent vertices.

By Satz von Menger we know that there exist $k$ internal-vertex-disjunct $u$-$v$-paths. We have as one such path the given $u$-($v$)-$w$-path. There must hence exist one other such $u$-$w$-path that is internally vertex disjunct to the first one. Connecting these two yields the desired cycle, in which $u$, $v$ and $w$ are adjacent.

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1 Point

Good & interesting approaches! Consider reviewing some of the definitions again to avoid unnecessary errors.