DMath_U2_bf

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2.3 Simplifying a Formula (*)

Consider the propositional formula

$$((\neg A \lor \neg B) \rightarrow (A \land \neg B)) \land (C \lor A)$$

Give a formula G that is equivalent to F, but in which each atomic formula A, B, and C appears at most once. Prove that $F \equiv G$ by providing a sequence of equivalence transformations with at most 9 steps.

Solution

Let's prove that $G \equiv F$ by a sequence of equivalence transformations.

$$
\begin{aligned}
G &\equiv ((\neg A \lor \neg B) \rightarrow (A \land \neg B)) \land (C \lor A) & \\
&\equiv ((\neg(A \land B) \rightarrow (A \land \neg B)) \land (C \lor A)) & \text{1st Step (de Morgan's rule)} \\
&\equiv ((\neg\neg(A \land B) \lor (A \land \neg B)) \land (C \lor A)) & \text{2nd Step } (A \rightarrow B \equiv \neg A \lor B) \\
&\equiv ((A \land B) \lor (A \land \neg B)) \land (C \lor A) & \text{3rd Step (double negation)} \\
&\equiv (A \lor (B \land \neg B)) \land (C \lor A)) & \text{4th Step (first distributive law)} \\
&\equiv (A \lor \perp) \land (C \lor A)) & \text{5th Step } (B \land \neg B \equiv \perp) \\
&\equiv (A \lor \perp) \land (A \lor C)) & \text{6th Step (associativity)} \\
&\equiv (A \lor (\perp \land C)) & \text{7th Step (second distributive law)} \\
&\equiv (A \lor \perp) & \text{8th Step } (\perp \land C \equiv \perp) \\
&\equiv A & \text{9th Step } (A \land \perp \equiv A)
\end{aligned}
$$

Thus the formula $G \models A$.

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