3.2 From Natural Language to a Formula (*)

Consider the universe $U = \mathbb{R}$. Express each of the following statements with a formula in predicate logic, in which the only predicates appearing are less(x, y), equals(x, y), and integer(x). Instead of less(x, y) and equals(x, y) you can write x < y and x = y. You can also use the symbols + and $\cdot$ to denote the addition and multiplication functions, and you can use constants (e.g., 0, 1, . . .). Do not use division. No justification is required.

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i) There exists two positive integers whose sum is negative.

$\exists x,\, y \in \mathbb{Z}^+ \,(x + y) < 0$

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ii) Any real number is not greater than all rational numbers.

$\forall x \in \mathbb{Q}\, \neg \exists y \in \mathbb{R},\, x < y$

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iii) If for every pair of real numbers there exist an integer which is smaller than one of the two and larger than the other, then all real numbers are greater than zero.

$\forall (x,\, y \in \mathbb{R})\, \exists i \in \mathbb{Z},\, (y < i < x) \rightarrow (\forall r \in \mathbb{R},\, 0 < r)$

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iv) All integers whose sum is odd have different parity.

$\forall (x,\, y \in \mathbb{R}),\, a \in \mathbb{R},\, x + y = 2a + 1,\, ((x = 2a + 1) \wedge (y = 2a)) \vee ((x = 2a) \wedge (y = 2a + 1))$

3.7 For each of the following proof patterns, prove or disprove that it is sound.

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a) To prove a statement S, find two appropriate statements T_1 and T_2. Assume that S is false and show (from this assumption) that one between the statements T_1 and T_2 is true. Then show that one statement between T_1 and T_2 is false.

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b) To prove an implication $S \implies T$, find an appropriate statement R. Assume that S is true and T is false, and prove that (from these assumptions) R is true. Then show that R is false.
