DMath_U2_bf

Basil Feitknecht

October 4, 2023

2.3 Simplifying a Formula (*)

Consider the propositional formula

$$((\neg A \lor \neg B) \to (A \land \neg B)) \land (C \lor A)$$

Give a formula G that is equivalent to F, but in which each atomic formula A, B, and C appears at most once. Prove that $F \equiv G$ by providing a sequence of equivalence transformations with at most 9 steps.

Solution

Let's prove that $G \equiv F$ by a sequence of equivalence transformations.

This should be F

$\cancel{G} \equiv ((\neg A \lor \neg B) \to (A \land \neg B)) \land (C \lor A)$	
$\equiv ((\neg(A \land B) \to (A \land \neg B)) \land (C \lor A))$	1st Step (de Morgan's rule) ✓
$\equiv ((\neg\neg(A \land B) \lor (A \land \neg B)) \land (C \lor A))$	2nd Step ($A \to B \equiv \neg A \lor B$) ✓
$\equiv ((A \land B) \lor (A \land \neg B)) \land (C \lor A)$	3rd Step (double negation) ✓ This would give you
$\equiv (A \lor (B \land \neg B)) \land (C \lor A))$	4th Step (first distributive law) $\equiv A \land (B \lor \neg B)$ -3
$\equiv (A \lor \bot) \land (C \lor A))$	5th Step ($B \land \neg B \equiv \bot$) ✓
$\equiv (A \lor \bot) \land (A \lor C))$	6th Step (associativity) Here you have applied Commutativity -1
$\equiv (A \lor (\bot \land C))$	7th Step (second distributive law) ✓
$\equiv (A \lor \bot)$	8th Step ($\bot \land C \equiv \bot$) ✓
$\equiv A$	9th Step ($A \cancel{\land} \bot \equiv A$)

Thus the formula $G \cancel{\models} A$.
∎

this should be $G \equiv A$.
And you have shown that $F \equiv G$.

4/8