AuD_U3_bf

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3.1. b) 2)

Prove or disprove the following statement: $\sum_{i=1}^{n}\sum_{j=1}^{i} j = \Theta(n^3)$.

The equality can be rewritten as:

$$\sum_{i=1}^{n} \frac{i(i+1)}{2} = \Theta(n^3)$$

To prove the equality, we need to show that the left-hand side grows at a rate that is bounded by and bounds n^3.

$$\sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{n(n+1)(n+2)}{6} = \Theta(n^3)$$

Dividing both sides of the inequality by n^3 gives:

$$c_1 \leq \frac{(n+1)(n+2)}{6n^2} \leq c_2$$

Now, let's evaluate the limit of the middle term as n approaches infinity:

$$\lim_{n\to\infty} \frac{(n+1)(n+2)}{6n^2} = \frac{1}{6}$$

Thus, the middle term converges to $\frac{1}{6}$ as n approaches infinity.

We can choose $c_1 = \frac{1}{12}$ and $c_2 = \frac{1}{6}$, which are positive constants that satisfy the inequality for sufficiently large n:

$$\frac{1}{12} \cdot n^3 \leq \frac{n(n+1)(n+2)}{6} \leq \frac{1}{6} \cdot n^3$$

This completes the proof, demonstrating that $\sum_{i=1}^{n}\sum_{j=1}^{i} j$ is indeed in the $\Theta(n^3)$ class.

3.1. b) 3)

To prove or disprove the statement:

$$\log(n^4 + n^3 + n^2) \leq O(\log(n^3 + n^2 + n))$$

Let's break it down step by step:
First, let's analyze the left side:

$$\log(n^4 + n^3 + n^2)$$

This can be simplified using logarithm properties:

$$\log(n^4 + n^3 + n^2) = \log(n^2(n^2 + n + 1))$$

Now, let's analyze the right side:

$$O(\log(n^3 + n^2 + n))$$

We can observe that the largest term in the parentheses on the right side is n^3, so we can write this as:

$$O(\log(n^3(1 + \frac{1}{n} + \frac{1}{n^2}))$$

Now, we can use logarithmic properties to simplify this:

$$O(\log(n^3) + \log(1 + \frac{1}{n} + \frac{1}{n^2}))$$

The logarithmic term on the right can be bounded by $\log(1 + \frac{1}{n} + \frac{1}{n^2}) \leq \log(1 + \frac{1}{n})$.
Now, we have:

$$O(\log(n^3) + \log(1 + \frac{1}{n}))$$

Since $\log(n^3)$ is dominated by $3\log(n)$, and $\log(1 + \frac{1}{n})$ grows very slowly as n becomes large, we can conclude that:

$$\log(n^4 + n^3 + n^2) \leq O(3\log(n))$$

So, the statement is proven:

$$\log(n^4 + n^3 + n^2) \leq O(3\log(n))$$