AuW-pg01-bf

1.

a)

We use Prim's algorithm to find the MST of G. Starting it on the node A results in the nodes being visited in the following order, with the associated weights.

1. $D, w = 2$
2. $B, w = 3$
3. $C, w = 4$
4. $G, w = 3$
5. $F, w = 5$
6. $E, w = 4$

Thus T contains the following edges:

$$E_T = \{\{A, D\}, \{A, B\}, \{B, C\}, \{C, G\}, \{G, F\}, \{F, E\}\}$$

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b)

We construct Z as follows:

$$E_Z = \{\{A, D\}, \{D, E\}, \{E, F\}, \{F, G\}, \{G, C\}, \{C, B\}, \{B, A\}\},\ C_Z = 21.$$

In particular, we add one edge, $e_Z = \{D, E\}, w(e_Z) = 5$. Since the rest of the edges are the ones already in the MST T, and $w(e_Z) < C_Z$, thus follows:

$$C_Z + w(e_Z) < 2C_Z$$

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c)

The MST T is, per definition, the walk of minimum length to visit every node. In order for any closed walk Z on G that visits all nodes to be closed, we must add an edge e_Z to T to "close" the walk. Since $w(e_Z) \geq \min(w(e_T)\ \forall e_T \in E_T$, $\sum_{e \in E_Z} w(e) > C_T$.

$\square$