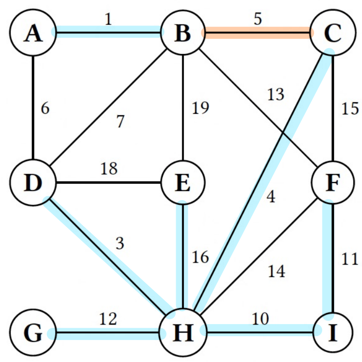
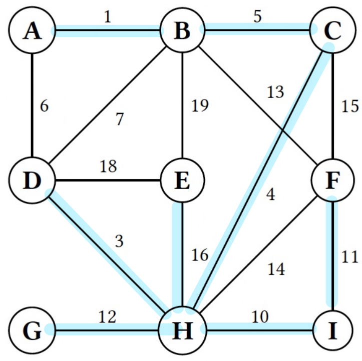
Serie 12

Exercise 12.1

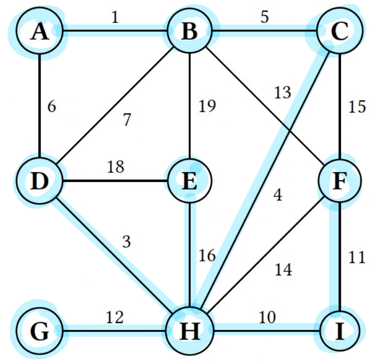
a) Boruvka's algorithm



b) Kruskal's algorithm

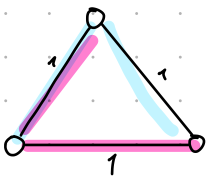


c) Prim's algorithm



Exercise 12.2

a)



b) T_1 und T_2 $\qquad e \in T_1 \setminus T_2$

$T_2 \cup \{e\}$ $\qquad |V| = n \quad |E| = n-1$

$\hookrightarrow |E| = n > n-1$

$T_2 \cup \{e\}$ kein Baum ist ~ zusammenhängend

=> enthält Zyklus

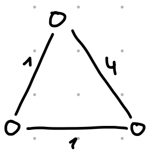
$f \in T_2 \setminus T_1$ $\qquad w(f) > w(e)$

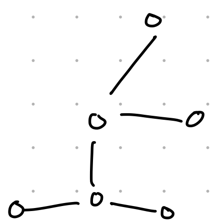
T_1 hat keine Kreise => $\exists f$ im Kreis C, der nicht in T_1 ist.

- $f \in T_2 \cup \{e\}$ $\quad e \in T_1$ $\quad f \neq e$ $\quad \Rightarrow f \in T_2$

$\Rightarrow f_2 \in T_2 \setminus T_1 \Rightarrow w(e) \leq w(f) \Rightarrow w(e) < w(f)$

(d)





Exercise 12.4

$tst(G)$

a) $M \subseteq E$ $\qquad w(M) \leq tst(G)$

mst

P $\qquad E(P) \leftarrow$ spannen V auf

$G' = (V, E(P))$ connected

T $\qquad w(P)$

$\Downarrow$

auch Spannbaum für G $\qquad w(M) \leq w(T) \leq w(P)$

c)

$2 \cdot mst(G)$

Exercise 12.3 *Constructing a Fiber Optic Network.*

The government of Atlantis put you in charge of installing a fiber optic network that connects all its n cities. There are two technologies of fibre optic that you can use:

- Fibre 1.0: It is a good reliable technology that is relatively cheap. There is a list of pairs of cities between which it is possible to install a direct Fibre 1.0 link. Furthermore, for each such pair, there is a corresponding positive integer cost.
- Fibre 2.0: It is an emerging technology that it extremely good and can directly connect any two cities. However, its cost is too high and the government cannot afford a single Fibre 2.0 link.

Note that all direct links are two-directional. The installed network should connect all the cities of Atlantis: Between any two cities, there should be a connected path of direct links in the network that connects them.

A philanthropist volunteered to donate the cost of exactly $k < n$ direct Fibre 2.0 links, and you can use them to connect any k pairs of cities. Your goal is to minimize the cost that is paid by the government for the Fibre 1.0 links that are needed to construct a connected network. Describe an algorithm that finds the network that costs the government the minimum amount of money.

Note that it is possible to construct a network connecting all the cities of Atlantis using only Fibre 1.0 links, but we would like to benefit from the k Fibre 2.0 links that were donated by the philantropist in order to minimize the cost that is paid by the government.

Hint: *Modify Kruskal's algorithm.*

Vorlesung Recap

$\{1, 2, \dots, n\}$

Floyd-Warshall

d^i_{uv} kürzester Pfad $u \rightsquigarrow v$ $\{1, \dots, i\}$

for $u \in V$: $d^0_{uu} \leftarrow 0$

for $u \in V, v \in V \setminus \{u\}$: if $(u,v) \in E$ then $d^0_{uv} \leftarrow c(u,v)$; else $d^0_{uv} \leftarrow \infty$

for $i = 1 \dots n$

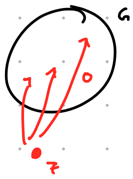
for $u = 1 \dots n$

for $v = 1 \dots n$

$d^i_{uv} \leftarrow \min\{d^{i-1}_{uv}, d^{i-1}_{ui} + d^{i-1}_{iv}\}$

return d^n $\quad \boxed{}^n$ $\quad O(n^3)$ $\quad d^n_{vv} < 0$

Johnson



$h(u)$ = Länge d. kürz. wegs $z \rightsquigarrow u$

$\hat{c}(u,v) = c(u,v) + h(u) - h(v) \geq 0$

$h(v) \leq h(u) + c(u,v)$

$\underbrace{c(u,v) + h(u) + h(v)}_{= \hat{c}(u,v)} \geq 0$

$O(n+m)$

$O(m \cdot n)$

$O(n \cdot (m+n) \cdot \log(n))$

$\Rightarrow O(n \cdot (m+n) \cdot \log(n))$

A_G $\quad (A_G)^k_{uv}$ ist die Anz. der u-v wege der länge k.