# FA542 - Homework #1

I pledge my honor that I have abided by the Stevens Honor System.

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### Problem #1

#### Import Libraries

mean = mean\_val,

```
# Load libraries for skewness, kurtosis, and plotting.
library(moments)
library(fBasics)
##
## Attaching package: 'fBasics'
## The following objects are masked from 'package:moments':
##
##
       kurtosis, skewness
Data Retrieval
# Establish the directory for data.
data_directory <- "C:/Users/sbhatia2/My Drive/University/Academics/Semester V/FA542 - Time Series with
# Load in each dataset.
data_problem_1 <- read.table(paste(data_directory, 'HW1_1.txt', sep=""), header = T)</pre>
data_problem_2 <- read.table(paste(data_directory, 'HW1_2.txt', sep=""), header = T)</pre>
data_problem_3 <- read.table(paste(data_directory, 'HW1_3.txt', sep=""), header = T)</pre>
df_1 <- as.data.frame(data_problem_1)</pre>
# Create function that calculates the sample mean, standard deviation, skewness, kurtosis, minimum, and
compute_statistics <- function(returns)</pre>
  mean_val <- mean(returns)</pre>
  sd_val <- sd(returns)</pre>
  skewness_val <- skewness(returns)</pre>
  kurtosis_val <- kurtosis(returns)</pre>
  min_val <- min(returns)</pre>
  max_val <- max(returns)</pre>
  # Create a list to hold each sample statistic.
  result <- list(
```

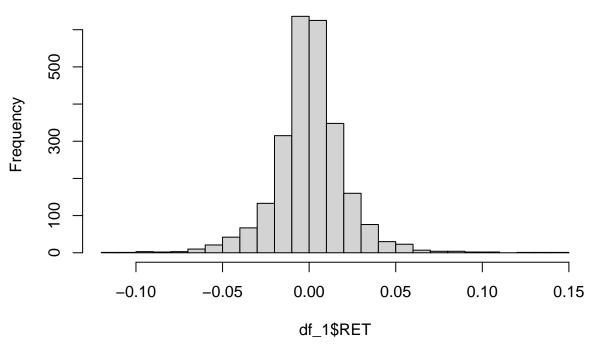
```
standard_deviation = sd_val,
    skewness = skewness_val,
    kurtosis = kurtosis_val,
    minimum = min_val,
    maximum = max_val
  return(result)
}
\# Compute statistics for each simple return series.
CAT_statistics <- compute_statistics(df_1$RET)</pre>
VW_statistics <- compute_statistics(df_1$vwretd)</pre>
EW_statistics <- compute_statistics(df_1$ewretd)</pre>
SP_statistics <- compute_statistics(df_1$sprtrn)</pre>
CAT_statistics
1a)
## $mean
## [1] 0.0004945544
## $standard_deviation
## [1] 0.02092881
##
## $skewness
## [1] 0.2304287
## attr(,"method")
## [1] "moment"
## $kurtosis
## [1] 5.029022
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.11434
##
## $maximum
## [1] 0.147229
VW_statistics
## $mean
## [1] 0.0003372415
## $standard_deviation
## [1] 0.01320677
##
## $skewness
## [1] -0.1879131
## attr(,"method")
## [1] "moment"
##
```

```
## $kurtosis
## [1] 9.14096
## attr(,"method")
## [1] "excess"
## $minimum
## [1] -0.089771
## $maximum
## [1] 0.114887
EW_statistics
## $mean
## [1] 0.0004484198
## $standard_deviation
## [1] 0.01220235
##
## $skewness
## [1] -0.2246611
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 7.847616
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.07824
##
## $maximum
## [1] 0.107422
{\sf SP\_statistics}
## $mean
## [1] 0.0002682593
## $standard_deviation
## [1] 0.01317902
## $skewness
## [1] -0.08946497
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 10.14406
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.09035
##
```

```
## $maximum
## [1] 0.1158
```

```
\# Display the empirical density distribution of simple returns for CAT. hist(df_1$RET, nclass=30)
```

## Histogram of df\_1\$RET



```
1b)

CAT_density_estimate <- density(df_1$RET)

# Obtain density estimate of simple returns for CAT.

CAT_density_estimate
```

```
##
## Call:
  density.default(x = df_1$RET)
## Data: df_1$RET (2518 obs.); Bandwidth 'bw' = 0.002757
##
##
         х
                            у
##
   Min.
         :-0.12261
                      Min.
                            : 0.00068
   1st Qu.:-0.05308
                      1st Qu.: 0.06048
##
  Median : 0.01644
                      Median: 0.22439
  Mean
         : 0.01644
                      Mean
                            : 3.59220
## 3rd Qu.: 0.08597
                      3rd Qu.: 2.89997
## Max. : 0.15550
                      Max.
                            :27.50193
```

```
# Perform a Jarque-Bera test for normality of simple returns.
normalTest(df_1$RET, method = 'jb')
##
## Title:
## Jarque - Bera Normalality Test
```

##
## Test Results:
## STATISTIC:
## X-squared: 2682.5018
## P VALUE:
## Asymptotic p Value: < 2.2e-16</pre>

At  $\alpha = 0.05$  or at the 5% significance level, we reject the null hypothesis:

 $H_0$ : The simple returns of CAT are normally distributed

since the p-value is less than 0.05.

1c) Log returns in relation to simple returns are defined as the following:

$$r_t = \ln(1 + R_t)$$

where  $r_t$  are log returns and  $R_t$  are simple returns.

```
# Create function that transforms simple returns to log returns. log(...) is base e.
simple_to_log <- function(returns) {
    return(log(1 + returns))
}

# Convert simple returns to log using function.
CAT_log_returns <- simple_to_log(df_1$RET)
W_log_returns <- simple_to_log(df_1$vvretd)
EW_log_returns <- simple_to_log(df_1$evretd)
SP_log_returns <- simple_to_log(df_1$sprtrn)

# Use `compute_statistics()` function to calculate all relevant statistics.
CAT_log_statistics <- compute_statistics(CAT_log_returns)
WW_log_statistics <- compute_statistics(EW_log_returns)
EW_log_statistics <- compute_statistics(EW_log_returns)
SP_log_statistics <- compute_statistics(SP_log_returns)
CAT_log_statistics</pre>
```

```
## $mean
## [1] 0.0002760543
##
## $standard_deviation
## [1] 0.02089984
##
## $skewness
## [1] 0.01646851
## attr(,"method")
## [1] "moment"
##
```

```
## $kurtosis
## [1] 4.739097
## attr(,"method")
## [1] "excess"
## $minimum
## [1] -0.1214221
##
## $maximum
## [1] 0.1373495
VW_log_statistics
## $mean
## [1] 0.0002498325
## $standard_deviation
## [1] 0.01323116
##
## $skewness
## [1] -0.4052208
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 9.027875
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.09405906
##
## $maximum
## [1] 0.1087531
EW_log_statistics
## $mean
## [1] 0.0003737711
## $standard_deviation
## [1] 0.0122235
##
## $skewness
## [1] -0.4018266
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 7.747635
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.08147039
##
```

```
## $maximum
## [1] 0.1020348
SP_log_statistics
## $mean
## [1] 0.0001812947
## $standard_deviation
## [1] 0.01319662
##
## $skewness
## [1] -0.3254499
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 9.905808
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.09469537
##
## $maximum
## [1] 0.1095716
```

1d) To test the following the null hypothesis that the mean log returns are zero, we need to conduct a t-test:

 $H_0$ : Mean log returns are zero.

 $H_0$ : Mean log returns do not equal zero.

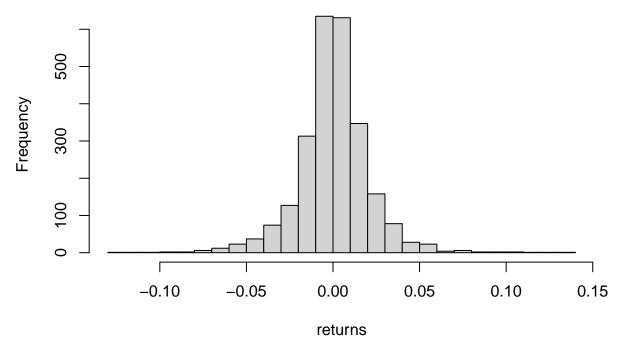
```
t.test(x = CAT_log_returns, alternative = c('two.sided'), mu = 0)
##
##
    One Sample t-test
##
## data: CAT_log_returns
## t = 0.6628, df = 2517, p-value = 0.5075
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## -0.0005406637 0.0010927723
## sample estimates:
##
      mean of x
## 0.0002760543
t.test(x = SP_log_returns, alternative = c('two.sided'), mu = 0)
##
##
    One Sample t-test
## data: SP_log_returns
```

```
## t = 0.68937, df = 2517, p-value = 0.4907
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## -0.0003343990  0.0006969884
## sample estimates:
## mean of x
## 0.0001812947
```

As seen above, at the  $\alpha = 0.05$  or the 5% significance level, we fail to reject the null hypothesis that the mean log returns for both CAT and S&P are zero since the p-values are well above 0.05.

```
# Create function to retrieve empirical density distribution function and plot it.
density_plot <- function(returns)
{
   hist(returns, nclass=30)
   return(density(returns))
}
density_plot(CAT_log_returns)</pre>
```

## **Histogram of returns**

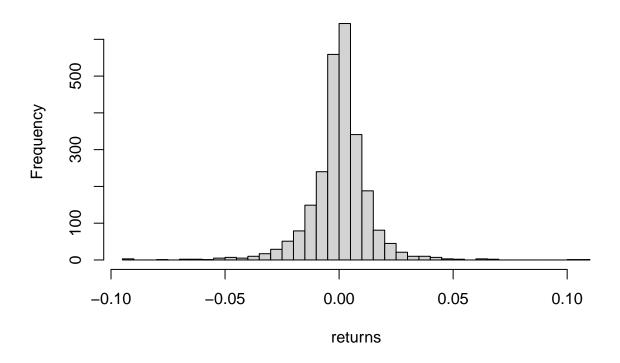


```
le)
##
## Call:
## density.default(x = returns)
##
## Data: returns (2518 obs.); Bandwidth 'bw' = 0.002755
##
```

```
##
           :-0.129687
                                : 0.000663
##
                        Min.
                        1st Qu.: 0.058853
    1st Qu.:-0.060862
   Median : 0.007964
                        Median : 0.255782
##
    Mean
          : 0.007964
                        Mean
                                : 3.628847
    3rd Qu.: 0.076789
                         3rd Qu.: 2.927840
   Max.
           : 0.145615
                        Max.
                                :27.510088
```

density\_plot(SP\_log\_returns)

# **Histogram of returns**



```
##
## Call:
    density.default(x = returns)
##
##
## Data: returns (2518 obs.); Bandwidth 'bw' = 0.001465
##
##
          х
                              : 0.00000
           :-0.099090
   1st Qu.:-0.045826
                        1st Qu.: 0.03419
   Median : 0.007438
                        Median: 0.22369
   Mean
           : 0.007438
                               : 4.68935
                        Mean
    3rd Qu.: 0.060702
                        3rd Qu.: 2.07732
          : 0.113966
   Max.
                        Max.
                               :55.73129
```

### Problem #2

```
df_2 <- as.data.frame(data_problem_2)</pre>
compute_statistics(df_2$RET)
1a)
## $mean
## [1] 0.01034235
## $standard_deviation
## [1] 0.05538326
##
## $skewness
## [1] -0.2969337
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 3.220276
## attr(,"method")
## [1] "excess"
## $minimum
## [1] -0.357041
##
## $maximum
## [1] 0.250931
compute_statistics(df_2$vwretd)
## $mean
## [1] 0.00887971
##
## $standard_deviation
## [1] 0.04403532
## $skewness
## [1] -0.522434
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 1.984566
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.225363
##
## $maximum
## [1] 0.165585
compute_statistics(df_2$ewretd)
```

## \$mean

```
## [1] 0.01136855
##
## $standard_deviation
## [1] 0.05554414
## $skewness
## [1] -0.1806713
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 2.919352
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.272248
##
## $maximum
## [1] 0.29926
compute_statistics(df_2$sprtrn)
## $mean
## [1] 0.006359545
## $standard_deviation
## [1] 0.04251519
##
## $skewness
## [1] -0.4228902
## attr(,"method")
## [1] "moment"
## $kurtosis
## [1] 1.813676
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.21763
## $maximum
## [1] 0.163047
# Display the empirical density distribution of simple returns for PG.
hist(df_2$RET, nclass=30)
```

## Histogram of df\_2\$RET

```
Production of the second of th
```

```
2b)
PG_density_estimate <- density(df_2$RET)
# Obtain density estimate of simple returns for CAT.
PG_density_estimate
##
## Call:
    density.default(x = df_2$RET)
##
## Data: df_2$RET (672 obs.);
                                Bandwidth 'bw' = 0.01197
##
##
          Х
          :-0.39296
                             :0.000000
                       Min.
    1st Qu.:-0.22301
                       1st Qu.:0.004268
##
##
   Median :-0.05305
                       Median :0.079307
##
   Mean
          :-0.05305
                       Mean
                              :1.469559
    3rd Qu.: 0.11690
                       3rd Qu.:1.647373
   Max.
           : 0.28685
                       Max.
                              :8.486108
# Perform a Jarque-Bera test for normality of simple returns.
normalTest(df_2$RET, method = 'jb')
##
```

Jarque - Bera Normalality Test

## Test Results:

```
##
     STATISTIC:
##
       X-squared: 303.6398
##
     P VALUE:
       Asymptotic p Value: < 2.2e-16
##
At \alpha = 0.05 or at the 5% significance level, we reject the null hypothesis:
                       H_0: The simple returns of PG are normally distributed
since the p-value is less than 0.05.
compute_statistics(simple_to_log(df_2$RET))
2c)
## $mean
## [1] 0.008756106
##
## $standard_deviation
## [1] 0.05580647
##
## $skewness
## [1] -0.8330476
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 6.351822
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.4416743
##
## $maximum
## [1] 0.2238881
compute_statistics(simple_to_log(df_2$vwretd))
## $mean
## [1] 0.007869959
##
## $standard_deviation
## [1] 0.04434013
##
## $skewness
## [1] -0.7880992
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 2.859168
## attr(,"method")
## [1] "excess"
##
## $minimum
```

```
## [1] -0.2553607
##
## $maximum
## [1] 0.1532231
compute_statistics(simple_to_log(df_2$ewretd))
## $mean
## [1] 0.009774514
## $standard_deviation
## [1] 0.05564156
##
## $skewness
## [1] -0.5980258
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 3.53031
## attr(,"method")
## [1] "excess"
## $minimum
## [1] -0.3177949
##
## $maximum
## [1] 0.2617949
compute_statistics(simple_to_log(df_2$sprtrn))
## $mean
## [1] 0.005433599
##
## $standard_deviation
## [1] 0.0427925
##
## $skewness
## [1] -0.6717913
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 2.538153
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.2454275
## $maximum
## [1] 0.1510433
```

**2d)** To test the following the null hypothesis that the mean log returns are zero, we need to conduct a t-test:

 $H_0$ : Mean log returns are zero.

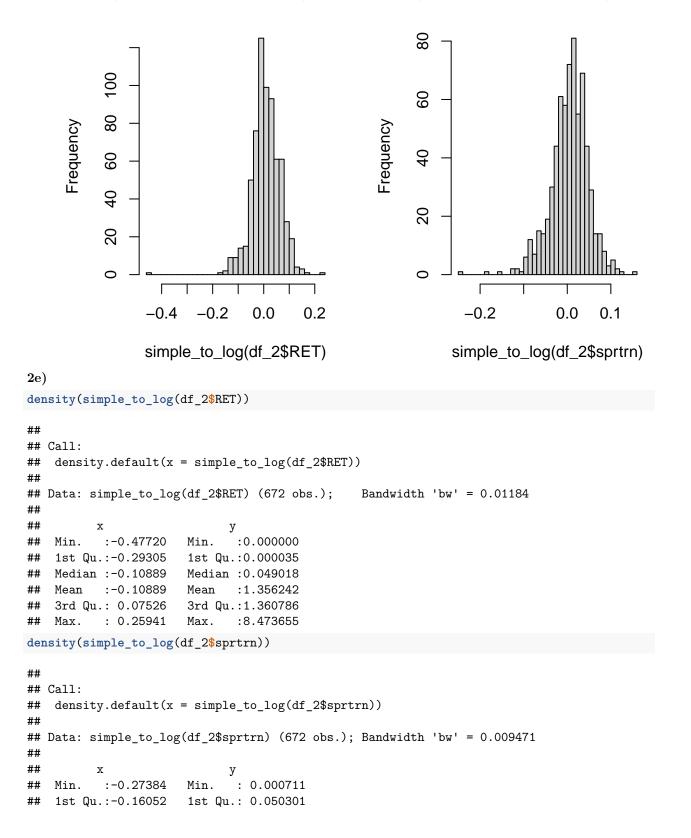
 $H_0$ : Mean log returns do not equal zero.

```
t.test(x = simple_to_log(df_2$RET), alternative = c('two.sided'), mu = 0)
    One Sample t-test
##
##
## data: simple_to_log(df_2$RET)
## t = 4.0673, df = 671, p-value = 5.32e-05
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## 0.004529107 0.012983105
## sample estimates:
     mean of x
##
## 0.008756106
t.test(x = simple_to_log(df_2$sprtrn), alternative = c('two.sided'), mu = 0)
##
    One Sample t-test
##
## data: simple_to_log(df_2$sprtrn)
## t = 3.2916, df = 671, p-value = 0.001048
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## 0.002192330 0.008674869
## sample estimates:
     mean of x
## 0.005433599
```

As seen above, at the  $\alpha = 0.05$  or the 5% significance level, we reject the null hypothesis that the mean log returns for both CAT and S&P are zero since the p-values below 0.05.

```
par(mfrow = c(1, 2))
hist(simple_to_log(df_2$RET), nclass=30)
hist(simple_to_log(df_2$sprtrn), nclass=30)
```

# Histogram of simple\_to\_log(df\_2\$Histogram of simple\_to\_log(df\_2\$sr



```
## Median :-0.04719 Median : 0.352586

## Mean :-0.04719 Mean : 2.203906

## 3rd Qu.: 0.06613 3rd Qu.: 2.879510

## Max. : 0.17946 Max. :10.731430
```

#### Problem 3

```
# Create function to calculate the confidence interval for a t-test.
t_confidence_interval <- function(alpha, n, mean, sd)
{
  # Define degrees of freedom as n-1.
  df <- n - 1
  t_{score} \leftarrow qt(p = alpha/2, df)
  lower_bound <- mean - abs(t_score) * (sd / sqrt(n))</pre>
  upper_bound <- mean + abs(t_score) * (sd / sqrt(n))
  return(c(lower_bound, upper_bound))
}
# Define significance level alpha = 0.05 or 95% CI.
alpha <- 0.05
n <- length(CAT_log_returns)</pre>
CAT_mean <- CAT_log_statistics$mean</pre>
CAT_sd <- CAT_log_statistics$standard_deviation
t_confidence_interval(alpha, n, CAT_mean, CAT_sd)
3a)
## [1] -0.0005406637 0.0010927723
t.test(CAT_log_returns)$conf.int
## [1] -0.0005406637 0.0010927723
## attr(,"conf.level")
```

**3b)** We are testing the following:

## [1] 0.95

$$H_0: S(r) = 0$$

$$H_a: S(r) \neq 0$$

Where S(r) is the skewness of the log returns.

The t-statistic for sample skewness is the following:

$$t = \frac{\hat{S}(r)}{\sqrt{6/n}}$$

```
# Construct function to test whether the population skewness of returns is equal to 0 or not.
skewness_test <- function(returns)
{
    t_skewness <- skewness(returns) / sqrt(6 / length(returns))

    p_skewness <- 2 * (1 - pnorm(abs(t_skewness)))

    return(p_skewness)
}
skewness_test(CAT_log_returns)

## [1] 0.7358379
## attr(,"method")
## [1] "moment"</pre>
```

Since the p-value is greater than 0.05, we fail to reject the null hypothesis at the 5% significance level.

**3c)** We are testing the following:

$$H_0: K = 3$$
$$H_a: K \neq 3$$

where K denotes the kurtosis of the returns (excess kurtosis = 0).

The t-statistic for kurtosis of the log returns is the following:

$$t = \frac{\hat{K}(r) - 3}{\sqrt{24/n}}$$

```
# Construct function to test whether the population kurtosis of returns is equal to 3 or not (excess ku
kurtosis_test <- function(returns)
{
    # Since the kurtosis function already computes excess log returns, we do not have to subtract by thre
    t_kurtosis <- (kurtosis(returns)) / sqrt(24 / length(returns))

    p_kurtosis <- 2 * (1 - pnorm(abs(t_kurtosis)))

    return(p_kurtosis)
}
kurtosis_test(CAT_log_returns)

## [1] 0</pre>
```

Since the p-value is less than 0.05, we reject the null hypothesis at the 5% significance level.

#### Problem 4

## attr(,"method")
## [1] "excess"

4a) We are testing the following:

$$H_0: S(r) = 0$$

$$H_a: S(r) \neq 0$$

Where S(r) is the skewness of the daily log returns for S&P from 01/03/2007 to 12/31/2016.

```
skewness_test(SP_log_returns)
```

```
## [1] 2.609224e-11
## attr(,"method")
## [1] "moment"
```

Since the p-value is less than 0.05, we reject the null hypothesis at the 5% significance level that the log returns are symmetric with respect to its mean.

**4b)** We are testing the following:

$$H_0: K = 3$$

$$H_a: K \neq 3$$

where K denotes the kurtosis of the returns (excess kurtosis = 0).

```
kurtosis_test(SP_log_returns)
```

```
## [1] 0
## attr(,"method")
## [1] "excess"
```

Since the p-value is less than 0.05, we reject the null hypothesis at the 5% significance level that the excess kurtosis of the log returns is zero.

4c) We are constructing a 95% CI for the expected daily log return (mean of log return) of the S&P.

We will use our previously coded function:

```
t_confidence_interval(0.05, length(SP_log_returns), mean(SP_log_returns), sd(SP_log_returns))
## [1] -0.0003343990  0.0006969884

t.test(SP_log_returns)$conf.int

## [1] -0.0003343990  0.0006969884
## attr(,"conf.level")
## [1] 0.95
```

#### Problem 5

**5a)** Log returns are defined as the following:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln\left(S_t\right) - \ln\left(S_{t-1}\right)$$

```
head(data_problem_3)
```

```
## year mon day euro
## 1 2005 1 3 1.3476
## 2 2005 1 4 1.3295
## 3 2005 1 5 1.3292
```

```
## 4 2005    1    6 1.3187
## 5 2005    1    7 1.3062
## 6 2005    1    10 1.3109
# Compute log returns of the Dollar-Euro exchange rate.
euro_log_returns <- diff(log(data_problem_3$euro))
head(euro_log_returns)
## [1] -0.0135223008 -0.0002256742 -0.0079308547 -0.0095242443    0.0035917657
## [6]    0.0039588936</pre>
```

**5b)** We will compute the sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum of the log returns of the exchange rate using the previously coded function <code>density\_plot</code>.

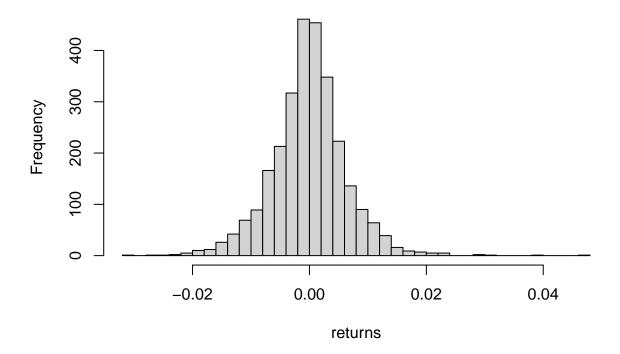
```
compute_statistics(euro_log_returns)
```

```
## $mean
## [1] -6.278971e-05
##
## $standard_deviation
## [1] 0.006362766
##
## $skewness
## [1] 0.2061161
## attr(,"method")
## [1] "moment"
##
## $kurtosis
## [1] 2.890412
## attr(,"method")
## [1] "excess"
##
## $minimum
## [1] -0.03003101
## $maximum
## [1] 0.04620792
```

5c) We will obtain the density plot using the previously coded function, density\_plot.

```
density_plot(euro_log_returns)
```

# **Histogram of returns**



```
##
## Call:
    density.default(x = returns)
## Data: returns (2816 obs.);
                                 Bandwidth 'bw' = 0.0009258
##
##
                               У
                                : 0.00002
           :-0.032808
##
    Min.
                         Min.
    1st Qu.:-0.012360
                         1st Qu.: 0.11177
##
##
    Median: 0.008088
                         Median : 1.12271
           : 0.008088
                         Mean
                                :12.21392
    3rd Qu.: 0.028537
                         3rd Qu.:12.52048
##
    Max.
           : 0.048985
                                :85.42477
```

**5d)** We will test the following:

$$H_0: \mu = 0$$

$$H_a: \mu \neq 0$$

Where  $\mu$  is the mean of the daily log return of the Dollar-Euro exchange rate.

```
t.test(euro_log_returns)$p.value
```

## [1] 0.6005483

Since the p-value is greater than 0.05, we fail to reject the null hypothesis that the mean of the daily log return of the Dollar-Euro exchange rate is zero at the 5% significance level.

**5e)** We will perform the Jarque-Bera test for normality which states the following:

$$H_0: S(r) = 0 = K(r)$$

Where S(r) denotes the skewness of the distribution and K(r) denotes the excess kurtosis.

```
normalTest(euro_log_returns, method = 'jb')
```

```
##
## Title:
## Jarque - Bera Normalality Test
##
## Test Results:
## STATISTIC:
## X-squared: 1003.0606
## P VALUE:
## Asymptotic p Value: < 2.2e-16</pre>
```

Since the p-value is less than 0.05, we reject the null hypothesis that the skewness and excess kurtosis are zero at the 5% significance level. As such, the log returns are most likely not normally distributed since samples from a normal distribution have an expected skewness of 0 and an expected excess kurtosis of 0.