FA542 - Homework #5

I pledge my honor that I have abided by the Stevens Honor System.

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Problem 1

The Federal Reserve Bank of St. Louis publishes selected interest rates and U.S. financial data on its website: http://research.stlouisfed.org/fred2/. Consider the monthly 1-year (DGS1) and 10-year Treasury (DGS10) constant maturity rates from January 1962 through December 2021; see the file homework05.csv. The rates are in percentages. Let $c_t = r_t - r_{t-1}$ be the change series of the monthly interest rate r_t.

i. Construct single time series autoregressive models for the single time series c_t^1 and c_t^10.

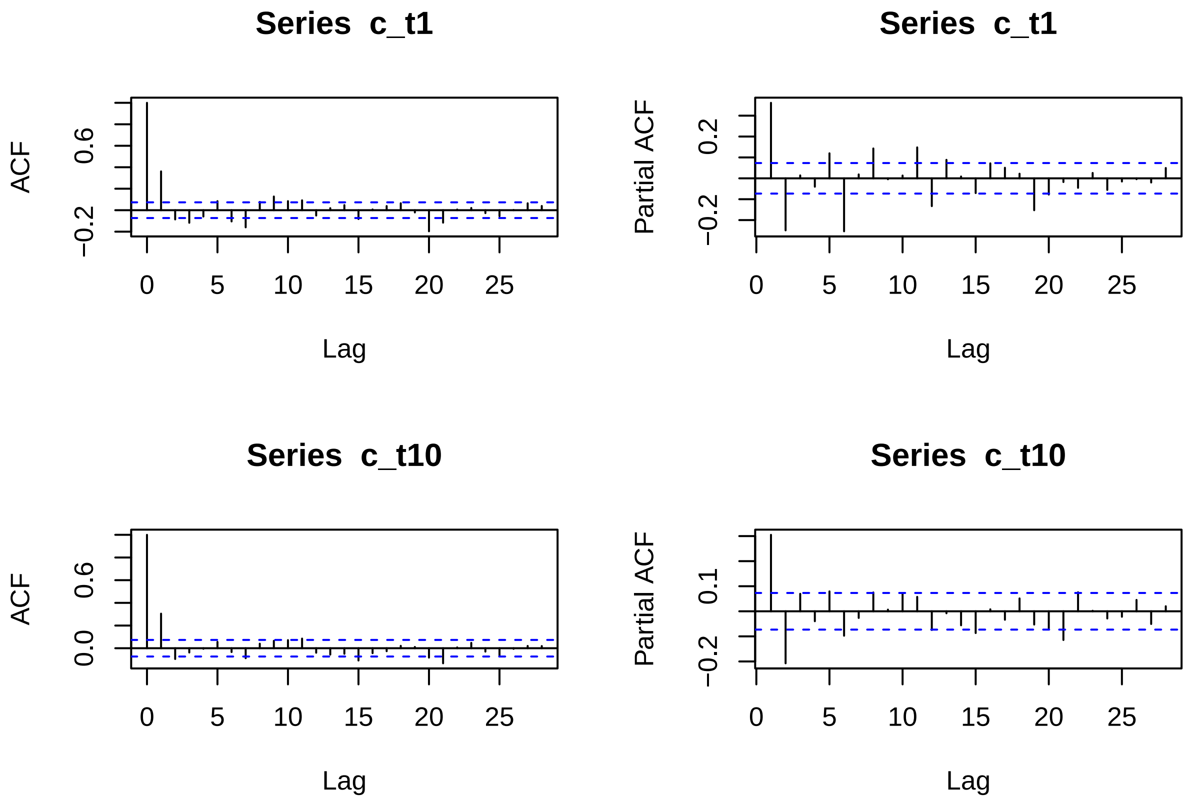
```
# Load required libraries.
library(readr)
library(stats)
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(forecast)

# Load the data from the CSV file.
data <- read.csv("homework05.csv")

head(data)
```

```
##       DATE     DGS1    DGS10
## 1 1/1/1962 3.279545 4.083182
## 2 2/1/1962 3.283889 4.039444
## 3 3/1/1962 3.058182 3.930455
## 4 4/1/1962 2.987500 3.843000
## 5 5/1/1962 3.025909 3.873636
## 6 6/1/1962 3.028095 3.909048
```

```
# Calculate the change series c_t1 and c_t10.
c_t1 <- diff(data$DGS1)
c_t10 <- diff(data$DGS10)

par(mfrow=c(2,2))
acf(c_t1)
pacf(c_t1)
acf(c_t10)
pacf(c_t10)
```




```r
adf.test(c_t1)
```

```
## Warning in adf.test(c_t1): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  c_t1
## Dickey-Fuller = -8.7052, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
```

```r
adf.test(c_t10)
```

```
## Warning in adf.test(c_t10): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  c_t10
## Dickey-Fuller = -8.6373, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
```

```r
auto.arima(c_t1)
```

```
## Series: c_t1
## ARIMA(0,0,2) with zero mean
##
## Coefficients:
##          ma1     ma2
```

```
##       0.4787  -0.100
## s.e.  0.0375   0.043
##
## sigma^2 = 0.1397:  log likelihood = -311.82
## AIC=629.65   AICc=629.68   BIC=643.38
```

```r
auto.arima(c_t10)
```

```
## Series: c_t10
## ARIMA(0,0,2) with zero mean
##
## Coefficients:
##          ma1      ma2
##       0.3908  -0.0999
## s.e.  0.0374   0.0380
##
## sigma^2 = 0.06645:  log likelihood = -44.59
## AIC=95.18   AICc=95.21   BIC=108.91
```

ii. Build a **bivariate** autoregressive model for the two change series.

```r
library(vars)
```

```
## Loading required package: MASS

## Loading required package: strucchange

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: sandwich

## Loading required package: urca

## Loading required package: lmtest
```

```r
# Combine the two change series into a data frame.
change_data <- data.frame(c_t1, c_t10)

selected_order <- VARselect(change_data, lag.max = 10)

selected_order
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      8      6      2      8
##
## $criteria
##                  1            2          3            4            5
## AIC(n) -5.416709816 -5.495223367 -5.5053182 -5.516289960 -5.517522764
## HQ(n)  -5.401788888 -5.470355154 -5.4705027 -5.471527176 -5.462812695
## SC(n)  -5.378087625 -5.430853049 -5.4151998 -5.400423388 -5.375908065
## FPE(n)  0.004441737  0.004106341  0.0040651  0.004020748  0.004015804
##                  6            7            8            9           10
```

```
## AIC(n) -5.576481976 -5.569119916 -5.581046043 -5.574695798 -5.570573185
## HQ(n)  -5.511824622 -5.494515276 -5.496494118 -5.480196588 -5.466126689
## SC(n)  -5.409119150 -5.376008962 -5.362186963 -5.330088591 -5.300217850
## FPE(n)  0.003785892  0.003813884  0.003768691  0.003792726  0.003808429
```

Order of 8 has the lowest AIC.

```r
# Fit the VAR model with the selected order.
var_model <- VAR(change_data, p = 8, type = "none")

# Print model summary.
summary(var_model)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: c_t1, c_t10
## Deterministic variables: none
## Sample size: 711
## Log Likelihood: 2.29
## Roots of the characteristic polynomial:
## 0.8683 0.8683 0.8394 0.8394 0.8093 0.8093 0.742 0.742 0.7172 0.7172 0.6996 0.6996 0.675 0.675 0.6598
## Call:
## VAR(y = change_data, p = 8, type = "none")
##
##
## Estimation results for equation c_t1:
## =====================================
## c_t1 = c_t1.l1 + c_t10.l1 + c_t1.l2 + c_t10.l2 + c_t1.l3 + c_t10.l3 + c_t1.l4 + c_t10.l4 + c_t1.l5 +
##
##           Estimate Std. Error t value Pr(>|t|)
## c_t1.l1   0.368309   0.056691   6.497 1.56e-10 ***
## c_t10.l1  0.252896   0.081121   3.118  0.00190 **
## c_t1.l2  -0.162847   0.060437  -2.694  0.00722 **
## c_t10.l2 -0.167711   0.084899  -1.975  0.04862 *
## c_t1.l3  -0.018533   0.061194  -0.303  0.76209
## c_t10.l3  0.090477   0.086444   1.047  0.29562
## c_t1.l4  -0.119791   0.061470  -1.949  0.05172 .
## c_t10.l4  0.005487   0.087057   0.063  0.94977
## c_t1.l5   0.245731   0.061552   3.992 7.24e-05 ***
## c_t10.l5 -0.065648   0.086904  -0.755  0.45026
## c_t1.l6  -0.284161   0.062237  -4.566 5.89e-06 ***
## c_t10.l6  0.110200   0.086387   1.276  0.20250
## c_t1.l7  -0.009973   0.061780  -0.161  0.87180
## c_t10.l7 -0.044708   0.085067  -0.526  0.59936
## c_t1.l8   0.171361   0.056515   3.032  0.00252 **
## c_t10.l8 -0.075528   0.081258  -0.929  0.35296
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.3576 on 695 degrees of freedom
## Multiple R-Squared: 0.2809,  Adjusted R-squared: 0.2644
## F-statistic: 16.97 on 16 and 695 DF,  p-value: < 2.2e-16
##
```

```
##
## Estimation results for equation c_t10:
## ======================================
## c_t10 = c_t1.l1 + c_t10.l1 + c_t1.l2 + c_t10.l2 + c_t1.l3 + c_t10.l3 + c_t1.l4 + c_t10.l4 + c_t1.l5 +
##
##            Estimate Std. Error t value Pr(>|t|)
## c_t1.l1  -0.0007239  0.0398195  -0.018 0.985501
## c_t10.l1  0.4171323  0.0569787   7.321 6.83e-13 ***
## c_t1.l2   0.0462710  0.0424502   1.090 0.276088
## c_t10.l2 -0.2725643  0.0596319  -4.571 5.75e-06 ***
## c_t1.l3  -0.1186313  0.0429821  -2.760 0.005932 **
## c_t10.l3  0.1902492  0.0607175   3.133 0.001801 **
## c_t1.l4   0.0184265  0.0431761   0.427 0.669675
## c_t10.l4 -0.0627060  0.0611481  -1.025 0.305495
## c_t1.l5   0.1667285  0.0432335   3.856 0.000126 ***
## c_t10.l5 -0.0916959  0.0610408  -1.502 0.133498
## c_t1.l6  -0.1823120  0.0437149  -4.170 3.42e-05 ***
## c_t10.l6  0.0983381  0.0606773   1.621 0.105542
## c_t1.l7   0.0023941  0.0433933   0.055 0.956017
## c_t10.l7 -0.0206966  0.0597502  -0.346 0.729158
## c_t1.l8   0.0509165  0.0396956   1.283 0.200034
## c_t10.l8  0.0250031  0.0570746   0.438 0.661466
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.2512 on 695 degrees of freedom
## Multiple R-Squared: 0.209,   Adjusted R-squared: 0.1908
## F-statistic: 11.48 on 16 and 695 DF,  p-value: < 2.2e-16
##
##
##
## Covariance matrix of residuals:
##          c_t1   c_t10
## c_t1  0.12791 0.06713
## c_t10 0.06713 0.06310
##
## Correlation matrix of residuals:
##         c_t1  c_t10
## c_t1  1.0000 0.7472
## c_t10 0.7472 1.0000
```

iii. Transform the constructed bivariate model into a structural form.

```r
structural_var <- BQ(var_model)

summary(structural_var)
```

```
##
## SVAR Estimation Results:
## ========================
##
## Call:
## BQ(x = var_model)
##
```

```
## Type: Blanchard-Quah
## Sample size: 711
## Log Likelihood: -13.893
##
## Estimated contemporaneous impact matrix:
##         c_t1    c_t10
## c_t1  0.3569 -0.02275
## c_t10 0.1980  0.15466
##
## Estimated identified long run impact matrix:
##         c_t1  c_t10
## c_t1  0.4752 0.0000
## c_t10 0.2649 0.2157
##
## Covariance matrix of reduced form residuals (*100):
##         c_t1 c_t10
## c_t1  12.791 6.714
## c_t10  6.714 6.311
```

iv. Briefly discuss the implications of the vector autoregressive model and compare with the single time series models.

Vector Autoregressive (VAR) models simultaneously model multiple variables and their dynamic interactions, ideal for datasets with interdependencies. They capture how variables depend on their past values and those of others, revealing how changes in one variable affect the entire system.

In contrast to single time series models, VAR models are more complex and require a larger dataset for accurate estimation due to multiple parameters. Single time series models, like ARIMA, are simpler, focusing on one variable, making them easier to interpret.

The choice between VAR and single time series models depends on research goals. VAR suits interconnected systems, while single time series models offer simplicity for analyzing individual variables in isolation.