Homework #1 - Due 29 September 2023

FE-570

October 2, 2023

Problem 1.1
Assume that the limit order book is as shown in the table below.

1) What is the bid-ask spread in cents?

2) If you buy 100 shares with a market order, an immediately sell them also with a market order, what is your P&L (profit/loss)?

3) A limit order to buy 250 shares at $25.50 arrives in the market. What transactions occur, and what is the resulting bid-ask spread in cents?

4) A limit order buy order for 2000 shares at $25.54 arrives. List all transactions, and determine the bid-ask spread after the order is executed.

Orders	Shares	Price
offer	2000	25.56
offer	1500	25.55
offer	1200	25.53
offer	400	25.52
offer	200	25.51
bid	1000	25.48
bid	1100	25.47
bid	1400	25.45
bid	1500	25.44
bid	2200	25.42

Solution.

1) The bid-ask spread is 3 cents.

2) The P&L is - 100 $\times 0.03$ = - \$3. We realize a loss.

3) The buy order is not executed and is entered as a limit buy order. No transactions occur. The new bid-ask spread is 1 cent.

4) The following transactions take place: 200 shares are purchased at 25.51, 400 shares are purchased at 25.52 and 1200 shares are purchased at 25.53. The remaining 200 shares are entered as a limit buy order at 25.54.

The new best ask is 25.55, and the best bid is 25.54. The new bid-ask spread is 1 cent.

Problem 1.2
Assume that the buy and sell orders shown in the table below have been submitted prior to an auction.

A single-price auction is conducted at a price p_*, chosen such that the number of exchanged shares is maximal.

1) What is the auction price p_*, and how many shares exchange hands?

2) What is the resulting Limit Order Book after the auction?

Buyers		Order Price	Sellers	
Order	Size		Order	Size
		50.3	S1	500
B1	80	50.2	S2	350
B2	110	50.1	S3	300
B3	120	50.0		
B4	250	49.9	S4	100
B5	500	49.8	S5	50
B6	750	49.7		

Figure 1: Orders submitted for an auction.

Solution.

1) Compute the aggregate supply and demand, as in the upper panel of Figure 2. The number of shares traded is maximized at $p_* = 50.1$ and is equal to 190.

2) The Limit Order Book after the auction is shown in Figure 2.

Aggregate demand	Buyers		Order Price	Sellers		Aggregate supply	Shares traded
	Order	Size		Order	Size		
0			50.3	S1	500	1300	0
80	B1	80	50.2	S2	350	800	80
190	B2	110	50.1	S3	300	450	190
310	B3	120	50.0			150	150
560	B4	250	49.9	S4	100	150	150
1060	B5	500	49.8	S5	50	50	50
1810	B6	750	49.7			0	0

after

Buyers		Order Price	Sellers	
Order	Size		Order	Size
		50.3	S1	500
		50.2	S2	350
		50.1	S3	260
B3	**120**	50		
B4	250	49.9		
B5	250	49.8		
B6	200	49.7		

Figure 2: Solution for problem 2. Upper figure: the computation of the auction price p_*. Lower figure: the Limit Order Book after the auction.