Problem #2 (Basket Options & Correlation)

Basket Call Options Overview

Basket call options are financial derivatives that derive their value from the performance of a *basket* of underlying assets. These options allow investors to bet on the **average performance** of several assets, rather than just one, providing a way to *diversify risk* across multiple securities.

Payoff Formula

The payoff of a basket call option at maturity can be described by the formula:

$$\left(\frac{1}{d}\sum_{i=1}^{d} S_T^i - K\right)^+, \tag{5}$$

where:

- S_T^i is the price at maturity T of the i-th asset in the basket.
- d is the total number of assets in the basket.
- K is the strike price of the option.

This payoff formula calculates the *average* of the final prices of the d assets, subtracts the strike price, and applies a **positive part function**, which ensures that the payoff is **non-negative**.

Asset Dynamics

$$\frac{dS_t^i}{S_t^i} = rdt + \sigma^i dW_t^i, \tag{6}$$

where $\{W_t^1, \ldots, W_t^d\}_{t\geq 0}$ is a d-dimensional Brownian motion with the covariance matrix

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{pmatrix}. \tag{7}$$

In other words,

$$\{W_t^1, \ldots, W_t^d\} \sim \mathcal{N}(\mu = 0, \sigma^2 = t \cdot \mathbf{\Sigma}),\ \forall t \in \mathbb{R}^+ \tag{8}$$