

## FE621 - Homework #4

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**Pledge:** I pledge my honor that I have abided by the Stevens Honor System.

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### Problem #1 (Barrier Options)

The price of an **up-and-out put option/knock-out (UOP)** with strike price  $K$  and barrier  $H$  is given by:

$$P = e^{-rT} \mathbb{E}[(K - S_T)_+ \mathbf{1}_{\{\tau > T\}}] \quad (1)$$

where  $\tau$  is the *stopping time* of the asset price process  $(S_t)_{t \geq 0}$  to the barrier  $H$ :

$$\tau = \inf \{t > 0 : S_t \geq H\} \quad (2)$$

The payoff is the **same** as that of a *vanilla put option*, unless the stock price goes above  $H$  during the life of the option, in which case the payoff is **zero**. Assume the process  $(S_t)_{t \geq 0}$  to follow a GBM.

*a. Is an UOP option cheaper or more expensive than a vanilla put option? Explain.*

An **up-and-out put option (UOP)** is generally cheaper than a vanilla put option. This difference in pricing comes from the **additional condition** involved in the UOP, where the option becomes worthless if the stock price exceeds the barrier  $H$  before expiration. In a vanilla put option, the holder has the right to sell the stock at the strike price  $K$  **regardless of how high the stock price has climbed during the option's life**.

This restriction in the UOP **reduces the probability** of a payout compared to a vanilla put option, where there is no upper limit on the stock price affecting the payoff.

Therefore, the UOP has a **lower premium** due to its *reduced likelihood of exercising profitably*. Essentially, the risk of the option knocking out (i.e., becoming worthless if the stock price exceeds the barrier  $H$ ) reduces its cost.