

FE630 - Homework #2

Author: Sid Bhatia

Date: May 7th, 2023

Pledge: I pledge my honor that I have abided by the Stevens Honor System.

Professor: Papa Momar Ndiaye

Topics

- Algebra & Optimization;
- Geometry of Efficient Frontiers;
- Applications of One-Fund & Two-Fund Theorems.

P1 - Optimization w/Equality Constraints (40 pts)

Consider the optimization problem **Max Expected Return w/Target Risk:**

$$\begin{cases} \max_{\omega_1, \omega_2} & R_P(\omega_1, \omega_2) = \mu_1\omega_1 + \mu_2\omega_2 \\ \text{s.t.} & \sqrt{\sigma_1^2\omega_1^2 + 2\rho_{1,2}\sigma_1\sigma_2\omega_1\omega_2 + \sigma_2^2\omega_2^2} = \sigma_T \\ & \omega_1 + \omega_2 = 1 \end{cases} \quad (1)$$

where we have two securities with **Expected Returns** μ_1 and μ_2 for the column vector $(\mu_1, \mu_2)^\top \in \mathbb{R}^{2 \times 1}$, **volatilities** $(\sigma_1, \sigma_2) \in \mathbb{R}^+$, and **Pearson correlation coefficient** $\rho_{1,2} \in [-1, 1]$. Additionally, $\sigma_T \in \mathbb{R}^+$ denotes the **target risk/vol.**