

0 Amendment

Response to **25%**; (-1)

This is my corrective action and (my) letter (to you). (0)

1 Overview

1.1 Goal

Objective: Build / Compare Two Factor-Based L/S Allocation Models (1)

Beta (β) Constraints (2)

First Strategy ($S_{\{1\}}$) : Target Beta $\beta_T \in [-0.5, 0.5]$ (3)

Second Strategy ($S_{\{2\}}$) : Target Beta $\beta_T \in [-2, 2]$ (4)

$S_{\{1\}} \cong$ Value-at-Risk Utility (Robust Optimization) (5)

$S_{\{2\}} \Leftarrow$ Information Ratio (6)

Post Optimization: Model Comparative Analysis (Estimator Length Sensitivity)

[Covariance Matrix $\Sigma \wedge$ Expected Returns μ] : $R_j \forall j \in \{1, 2, \dots, M\}$ (8)

1.2 Reallocation

Portfolio Allocation $\{P_t\} \Leftarrow$ '03-01-2007' \sim '03-31-2024' (9)

$P_t \quad \forall t \in \{t_0, t_1, t_2, \dots, t_n\}$ where $t_0 = 03-01-2007, \quad t_n = 03-31-2024$ (10)

$t_i = t_{i-1} + 7 \text{ days}$ for $i = 1, 2, \dots, n$ (11)

Investment Universe \equiv ETFs ('Global World Economy') (12)

Fama-French Three-Factor Model (Momentum, Value, Size) (13)

Public Data (14)

1.3 Performance Evaluation

Performance / Risk Profiles \Leftarrow Sensitive \Leftarrow Target Beta : $\beta_T \wedge$ 'Market' (15)

Low Beta \Rightarrow Decorrelation; (16)

High Beta \equiv Antithesis. (17)

Portfolio Characteristics: (18)

- Return : μ (19)

- Volatility (Vol) : σ (20)

- Skewness (Skew) : $\mathbb{E} \left[\left(\frac{x - \mu}{\sigma} \right)^3 \right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}}$ (21)

- Value at Risk (VaR) / Expected Shortfall (ES) (22)

- Sharpe Ratio : $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$ (23)

1.4 Simplification

Look-Back μ Estimators : (24)

- Long-Term Estimator (LTE) : $LT \Rightarrow LB \in \{180 \text{ Days}\}$ (25)

- Mid-Term Estimator (MTE) : $MT \Rightarrow LB \in \{90 \text{ Days}\}$ (26)

- Short-Term Estimator (STE) : $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$ (27)

Term-Structure for Covariance $\Sigma \wedge$ Expected Return μ . (28)

1.5 Synthesis

Optimal portfolio behavior constructed from (29)

covariance and expected return estimators (30)

will vary due to strategic and market differences. (31)

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (32)$$

Objective: (33)

- Evaluate Hypothesis (34)

- Demonstrate Robustness (Or Lack Thereof) (35)

- Market Regime Stratification (36)

2 Strategy

Theory \& Math

2.1 Strategic Formulation

Consider two strategies :

$$(\text{Strategy I}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (37)$$

and

$$(\text{Strategy II}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (38)$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t.
 $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
 trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (39)$$

2.2 Fama–French Three-Factor Model (FF3FM)

Definition: (40)

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (41)$$

$$\mathbb{E}[\epsilon_i] = 0; \therefore$$

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (42)$$

$$\text{Estimated Coefficient Vector:} \quad (43)$$

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\top \Leftarrow y_i = \rho_i - r_f \quad (44)$$

$$\text{Linear Regression:} \quad (45)$$

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \quad (46)$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression} / \text{computed directly.} \quad (47)$$

2.3 Executive Summary Formulation

$$\text{Innumerate:} \quad (48)$$

$$\text{Strategy I} \quad (49)$$

$$1. \text{ Objective} \equiv \text{Maximize Returns w/Risk.} \quad (50)$$

$$2. \text{ Constraints :} \quad (51)$$

- The portfolio's beta must be between -0.5 and 0.5 . (52)
- The sum of the weights assigned to each asset in the portfolio must equal 1.
- Each individual weight can range from -2 to 2 . (54)

$$\text{Strategy II} \quad (55)$$

$$1. \text{ Objective} \equiv \text{Maximize Returns Relative to Tracking Error Volatility (TEV).}$$

$$2. \text{ Constraints :} \quad (57)$$

- The portfolio's beta must be between -2 and 2 . (58)
- The sum of the weights assigned to each asset in the portfolio must equal 1.
- Each individual weight can range from -2 to 2 . (60)

3 Assumptions

3.1 Setup

$$1. \text{ Reallocation : '03-01-2007' } \sim \text{ '03-31-2024' } \quad (61)$$

$$2. \text{ Input Construction : } \quad (62)$$

$$\bullet \quad \text{LT LB Period : } n_{\text{LT}} = 120 \mid \Sigma_s \wedge \mu_s \mid \text{LT} \equiv S_{120} \quad (63)$$

$$\bullet \quad \text{MT LB Period : } n_{\text{LT}} = 90 \mid \Sigma_s \wedge \mu_s \mid \text{MT} \equiv S_{90} \quad (64)$$

$$\bullet \quad \text{ST LB Period : } n_{\text{LT}} = 40 \mid \Sigma_s \wedge \mu_s \mid \text{MT} \equiv S_{40} \quad (65)$$

$$3. \beta_T \in \{0, 1\} \quad (66)$$

$$4. \lambda \in \{0.10, 0.50\} \quad (67)$$

3.2 Period Analysis

$$\text{Period Stratification:} \quad (68)$$

$$\bullet \quad \text{Period 1} \equiv \text{Pre-Subprime} \quad (69)$$

$$\bullet \quad \text{Period 2} \equiv \text{Subprime} \quad (70)$$

$$\bullet \quad \text{Period 3} \equiv \text{Post-Subprime} \quad (71)$$

$$\bullet \quad \text{Period 4} \equiv \text{COVID} \quad (72)$$

$$\bullet \quad \text{Period 5} \equiv \text{Post-Covid} \quad (73)$$

3.3 BackTesting

$$\text{Definition: Historical Data} \Rightarrow \text{Performance} \quad (74)$$

$$\text{Logistical Considerations:} \quad (75)$$

$$\bullet \quad \text{BackTest} \neq \text{Forecasts} \Rightarrow \textbf{Snooping Bias / P-Hacking} \quad (76)$$

$$\bullet \quad \text{Weekly Rebalance} \quad (77)$$

$$\bullet \quad \{t_i\}_{i=1}^n : \quad (78)$$

- For the initial date t_1 , use the prior 60 days of historical data to estimate input
- Store the portfolio weights: ω_{t_1} .
- For each subsequent date t_{i+1} , roll the historical data window by 5 days, re-est
- Store the new portfolio weights: $\omega_{t_{i+1}}$.
- Repeat this process until the target date t_n is reached.

4 ToolKit|Arsenal

Strat I \Rightarrow CVXPY | Strat II \Rightarrow Nonlinear Optimizer (80)

Data (ETFs) : yfinance (81)

• 1. FXE (82)

• 2. EWJ (83)

• 3. GLD (84)

• 4. QQQ (85)

• 5. SPY (86)

• 6. SHV (87)

• 7. DBA (88)

• 8. USO (89)

• 9. XBI (90)

• 10. ILF (91)

• 11. EPP (92)

• 12. FEZ (93)

To Do: (94)

• Task 1 : 'download_data(start_date, end_date)', 'compute_dail

• Task 2 : 'factor_model(...)' (96)

• Task 3 : 'optimize_model(...)' (97)

- Task 4 : ‘backtest(...)’ (98)
- Task 5 : ‘analyze(...)’ (99)
- Task 6 : ‘summarize(...)’ (100)

5 Performance + Risk Reporting 4 Strats

KPIs: (101)

- Cumulative PnL / Return (102)
- Average Daily Arithmetic / Geometric Return| Daily Min Return (103)
- 10 Day Max Drawdown| Sharpe (104)
- Vol, Skew, (Excess) Kurt, (Modified) VaR, Expected Shortfall (CVar) (105)

Tabular Formulation: (106)

	$S_{40}(\beta_T = 0)$	$S_{90}(\beta_T = 1)$	$S_{120}(\beta_T = 0)$	SPY
Mean Return			12	
:			:	
Max DD			8	

Furthermore: (107)

1. Evolution Plot : Cumulative Daily PnL | $P_0 = \$100, SPY_0 = \100 . (108)
2. Plot + Analyze (Daily) Return Distribution. (109)



6 Deliverables

- 1 ≡ Report : Findings, Conclusions, Estimator Impact on Strats (When? Why?)
Crisis Periods (Subprime, COVID);
- 2 ≡ Axes : Estimator Term-Structure Sensitivity (ST, MT, LT) for $\Sigma \wedge \mu$ | Ta
- 3 ≡ Notation, Strats, Graphs, \& Tables Description; (112)

4 ≡ Code .

(113)