FE630 - Final Project (Revision)

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Pledge: I pledge my honor that I have abided by the Stevens Honor System.

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0 Disclaimer

This work is in response to the original (numerical) grade received as a $\mathbf{25\%}$ due to my imprudence and deviation from the requested prompt.

I (profusely) apologize for my incompetence; I will take corrective action.

1 Overview

1.1 Goal

The goal of this project to build and compare *two factor-based long short allocation models* with constraints on their *betas*. The first strategy considers a **target Beta** in the interval $[-0.5, 0.5]$, while the second has a target Beta in the interval $[-2, +2]$.

The first strategy operates similar to a **Value-at-Risk Utility** corresponding to **Robust Optimization**; the second strategy incorporates an **Information Ratio** term to limit the deviations from a benchmark, provided those deviations yield a 'high return.'

Once the optimization models are built, we want to *compare* the outcomes of the two models while simultaneously evaluating their sensitivity to the *length* of the estimators for the **covariance matrix** in tandem with the **expected returns** under various market regimes/scenarios.

1.2 Reallocation

The portfolios will be *reallocated* or, in other words, 'reoptimized' weekly from the beginning of **March 2007** to the end of **March 2024**. Our *investment universe* encompasses a set of exchange-traded funds (**ETFs**) which is large enough to represent the '**Global World Economy**.'

We will utilize the Fama–French Three-Factor Model which incorporates the following factors:

- Momentum
- Value
- Size.

Regarding data accessability, these factors have historical values available for ***free*** from ***Ken French's*** personal website in tandem with Yahoo Finance.

1.3 Performance Evaluation

Naturally, the performance as well as the risk profiles of the aforementioned strategies may be (relatively) sensitive to the *target Beta* and the (current) market environment.

For example, a '**low Beta**' (essentially) means that a strategy is created with the objective or aim to be '**decorrelated**' (no linear relationship between entites) with the 'Global Market,' which, in our case, is represented by the **S&P 500** (i.e., no *systematic relationship*).

A '**high Beta**' is simply the antithesis, or opposite, of what we just discussed. In layman's terms, we have a (higher) appetite for '*risk*' (in this case, let's keep it simple and define our premise as σ or **standard deviation**) and desire to ride or 'scale up' the *market risk* (**systematic risk**).

Moreover, it's imperative that one acknowledges that such a (described) strategy is more probable to be (quite) sensitive to the *estimators* used for the **Risk Model** and the **Alpha Model** (e.g., the length of the *look-back period* utilized); therefore, it is necessary to understand and, most importantly, *comprehend* the impact of said estimators on the **Portfolio's** characteristics:

- (Realized) Return : μ_h
- (Historical) Volatility) : σ_h
- Skewness : $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma_3} = \frac{\kappa_3}{\kappa_2^{3/2}}$
- VaR / Expected Shortfall
- Sharpe Ratio : $S_a = \frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$

1.4 Simplification

To make it easier, we assume that once the **Factor Model** (FM) has been constructed, we will use trend following estimators for the **Expected Returns**. Since the quality of the estimators depend on the **look-back period**, we define three cases:

- ***Long-Term Estimator (LTE)*** : $\text{LT} \Rightarrow \text{LB} \in \{180 \text{ Days}\}$.
- ***Mid-Term Estimator (MTE)*** : $\text{MT} \Rightarrow \text{LB} \in \{90 \text{ Days}\}$.
- ***Short-Term Estimator (STE)*** : $\text{ST} \Rightarrow \text{LB} \in \{40 \text{ Days}, 60 \text{ Days}\}$.

Specifically, we define a **Term-Structure** for the $\text{Covariance } \mathbf{\Sigma}$ and $\text{Expected Return } \boldsymbol{\mu}$.

1.5 Synthesis

To (briefly) summarize, the behavior of a (potential) '*optimal*' portfolio built from a melting pot of *estimators* for **Covariance** and **Expected Return** may vary according to the cadence of the '**Market**' (environment/regime) or an aforementioned strategy.

For example, the (mathematical) notation S_{40}^{90} is implemented to illustrate that we are using **40 days** for the covariance estimation and **90 days** for the expected returns estimations.

Overall, the goal of this experiment is to conceptualize, visualize, understand, analyze, and compare the behavior of our hypotheses; we want to see if we can deliver robust risk-adjusted performance, especially during momentous, historical periods such as the **Subprime Mortgage Crisis** of *2008*, the horrendous commencement of **Coronavirus SARS-CoV-2 Disease** of *2019*, et cetera.

2 (Investment) Strategy

This section delves into the theoretical and mathematical formulations for the investment strategy employed; it is not for the faint of heart.

2.1 (Mathematical) Strategic Formulation

Consider the following two strategies:

$$(\text{Strategy I}) \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \; \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{1}$$

and

$$(\text{Strategy II}) \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \; \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{2}$$

where we define the constructions above:

- Σ is the covariance matrix between the securities returns (as computed from the **FF3FM**);

- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)}$ is the Beta) of some security) S_i as defined by the CAPM Model such that $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i$ is the **Portfolio Beta**;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}})$ is the '**Tracking Error Volatility**'; the derivation is trivial and left as an exercise to the reader:

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\intercal \Sigma \omega - 2\omega^\intercal \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \tag{3}$$

2.2 Fama–French Three-Factor Model (FF3FM)

The Fama-French Three-Factor Model (FF3FM) is defined as follows:

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{4}$$

Assume $\mathbb{E}[\epsilon_i] = 0$; therefore,

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{5}$$

The 3 coefficients β_i^3, b_i^s, and b_i^v are estimated by making a linear regression of the time series $y_i = \rho_i - r_f$ against the time series $\rho_M - r_f$ (**Momentum Factor**), r_{SMB} (**Size Factor**), and ρ_{HML} (**Value Factor**).

$\beta_i^m \neq \beta_i^3$ in tandem with the requirement to be estimated via a separate regression or directly computed.

2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

2.3.1 Strategy I Breakdown

1. **Objective**: Maximize returns while considering risk (i.e., make as much

   $ as humanly possible without it being (bi)polar)
2. **Constraints**:
   - The portfolio's beta (a measure of its *volatility* relative to the market; i.e., how *silly* and *spread out* it is relative to the 'market') must be between -0.5 and 0.5.
   - The sum of the weights assigned to each asset in the portfolio must equal 1 (i.e., ***we gotta put our money to work!*** As such, let's buy a bunch of stuff that can make us money but, also, let's (try) not to violate the Laws of Probability Theory).
   - Each individual weight can range from -2 to 2 (i.e., we can be like *certain individuals* from WallStreetBets and put all our eggs in one basket or, like a more prudent investor, do anything *but that*).

2.3.2 Strategy II Breakdown

1. **Objective**: Maximize returns relative to the portfolio's **tracking error volatility** (**TEV**), which measures how much the portfolio's returns deviate from a benchmark (e.g., the S&P 500 or 'big boy stock market').
2. **Constraints**:
   - The portfolio's beta (a measure of its *volatility* relative to the market; i.e., how *wild* and *crazy* it gets compared to the 'market') must be between -2 and 2.
   - The sum of the weights assigned to each asset in the portfolio must equal 1 (i.e., ***we need to make sure all our money is actively working!*** So, let's diversify our investments while still following the Laws of Probability Theory).
   - Each individual weight can range from -2 to 2 (i.e., we can either go *all in* on one asset like *those wild investors* on WallStreetBets, or spread our investments more wisely).

Don't worry about all the fancy schmancy 'math'(matics); math is for nerds (yours truly, included). All math is, is it's another language. The more you practice it, the better you get.

Anyways, that's enough of my rambling and yapping. Let's explore the setup (in da next section *insert cool kid emoji*)!