

## 0 Amendment

Response to **25%**; (-1)

This is my corrective action and (my) letter (to you). (0)

## 1 Overview

### 1.1 Goal

Objective: Build / Compare Two Factor-Based L/S Allocation Models (1)

Beta ( $\beta$ ) Constraints (2)

First Strategy ( $S_{\{1\}}$ ) : Target Beta  $\beta_T \in [-0.5, 0.5]$  (3)

Second Strategy ( $S_{\{2\}}$ ) : Target Beta  $\beta_T \in [-2, 2]$  (4)

$S_{\{1\}} \cong$  Value-at-Risk Utility (Robust Optimization) (5)

$S_{\{1\}} \Leftarrow$  Information Ratio (6)

Post optimization, I compare model outcomes while evaluating estimator length se

[covariance matrix  $\Sigma \wedge$  expected returns  $\mu$ ] across market regimes (8)

### 1.2 Reallocation

Portfolio Allocation  $\{P_t\} \Leftarrow$  ‘03-01-2007’  $\sim$  ‘03-31-2024’ (9)

$P_t \quad \forall t \in \{t_0, t_1, t_2, \dots, t_n\}$  where  $t_0 = 03-01-2007, \quad t_n = 03-31-2024$  (10)

$t_i = t_{i-1} + 7 \text{ days}$  for  $i = 1, 2, \dots, n$  (11)

Investment Universe  $\equiv$  ETFs (‘Global World Economy’) (12)

Fama–French Three-Factor Model (Momentum, Value, Size) (13)

Public Data (14)

### 1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market en

Low Beta  $\Rightarrow$  Decorrelation; (16)

High Beta  $\equiv$  Antithesis. (17)

Portfolio Characteristics: (18)

- Return :  $\mu$  (19)

- Volatility (Vol) :  $\sigma$  (20)

- Skewness (Skew) :  $\mathbb{E} \left[ \left( \frac{x - \mu}{\sigma} \right)^3 \right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}}$  (21)

- Value at Risk (VaR) / Expected Shortfall (ES) (22)

- Sharpe Ratio :  $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$  (23)

## 1.4 Simplification

Look-Back  $\mu$  Estimators : (24)

- Long-Term Estimator (LTE) :  $LT \Rightarrow LB \in \{180 \text{ Days}\}$  (25)

- Mid-Term Estimator (MTE) :  $MT \Rightarrow LB \in \{90 \text{ Days}\}$  (26)

- Short-Term Estimator (STE) :  $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$  (27)

Term-Structure for Covariance  $\Sigma \wedge$  Expected Return  $\mu$ . (28)

## 1.5 Synthesis

Optimal portfolio behavior constructed from (29)

covariance and expected return estimators (30)

will vary due to strategic and market differences. (31)

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (32)$$

Objective: (33)

- Evaluate Hypothesis (34)

- Demonstrate Robustness (Or Lack Thereof) (35)

- Market Regime Stratification (36)

## 2 Strategy

Theory \& Math

### 2.1 Strategic Formulation

Consider two strategies :

$$(\text{Strategy I}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (37)$$

and

$$(\text{Strategy II}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (38)$$

- $\Sigma \equiv$  covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$  Beta of security  $S_i$  (CAPM) s.t.  
 $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$  Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$  Tracking Error Volatility;  
 trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (39)$$

### 2.2 Fama–French Three-Factor Model (FF3FM)

Definition: (40)

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (41)$$

$$\mathbb{E}[\epsilon_i] = 0; \therefore$$

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (42)$$

$$\text{Estimated Coefficient Vector:} \quad (43)$$

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\top \Leftarrow y_i = \rho_i - r_f \quad (44)$$

$$\text{Linear Regression:} \quad (45)$$

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \quad (46)$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression / computed directly.} \quad (47)$$

## 2.3 Executive Summary Formulation

$$\text{Innumerate:} \quad (48)$$

$$\text{Strategy I} \quad (49)$$

$$1. \text{ Objective} \equiv \text{Maximize Returns w/Risk.} \quad (50)$$

$$2. \text{ Constraints :} \quad (51)$$

- The portfolio's beta must be between  $-0.5$  and  $0.5$ . (52)
- The sum of the weights assigned to each asset in the portfolio must equal 1.
- Each individual weight can range from  $-2$  to  $2$ . (54)

$$\text{Strategy II} \quad (55)$$

$$1. \text{ Objective} \equiv \text{Maximize Returns Relative to Tracking Error Volatility (TEV).}$$

$$2. \text{ Constraints :} \quad (57)$$

- The portfolio's beta must be between  $-2$  and  $2$ . (58)
- The sum of the weights assigned to each asset in the portfolio must equal 1.
- Each individual weight can range from  $-2$  to  $2$ . (60)

## 3 Assumptions

### 3.1 Setup

$$1. \text{ Reallocation : '03-01-2007' } \sim \text{'03-31-2024'} \quad (61)$$

$$2. \text{ Input Construction :} \quad (62)$$

$$\bullet \quad \text{LT LB Period : } n_{\text{LT}} = 120 \mid \Sigma_s \wedge \mu_s \mid \text{LT} \equiv S_{120} \quad (63)$$

$$\bullet \quad \text{MT LB Period : } n_{\text{LT}} = 90 \mid \Sigma_s \wedge \mu_s \mid \text{MT} \equiv S_{90} \quad (64)$$

$$\bullet \quad \text{ST LB Period : } n_{\text{LT}} = 40 \mid \Sigma_s \wedge \mu_s \mid \text{MT} \equiv S_{40} \quad (65)$$

$$3. \beta_T \in \{0, 1\} \quad (66)$$

$$4. \lambda \in \{0.10, 0.50\} \quad (67)$$

### 3.2 Period Analysis

$$\text{Period Stratification:} \quad (68)$$

$$\bullet \quad \text{Period 1} \equiv \text{Pre-Subprime} \quad (69)$$

$$\bullet \quad \text{Period 2} \equiv \text{Subprime} \quad (70)$$

$$\bullet \quad \text{Period 3} \equiv \text{Post-Subprime} \quad (71)$$

$$\bullet \quad \text{Period 4} \equiv \text{COVID} \quad (72)$$

$$\bullet \quad \text{Period 5} \equiv \text{Post-Covid} \quad (73)$$

### 3.3 BackTesting

$$\text{Definition: Historical Data} \Rightarrow \text{Performance} \quad (74)$$

$$\text{Logistical Considerations:} \quad (75)$$

$$\bullet \quad \text{BackTest} \neq \text{Forecasts} \Rightarrow \textbf{Snooping Bias / P-Hacking} \quad (76)$$

$$\bullet \quad \text{Weekly Rebalance} \quad (77)$$

$$\bullet \quad \{t_i\}_{i=1}^n : \quad (78)$$

{ For the initial date  $t_1$ , use the prior 60 days of historical data to estimate input  
 Store the portfolio weights:  $\omega_{t_1}$ .  
 { For each subsequent date  $t_{i+1}$ , roll the historical data window by 5 days, re-est  
 Store the new portfolio weights:  $\omega_{t_{i+1}}$ .  
 { Repeat this process until the target date  $t_n$  is reached.

## 4 ToolKit|Arsenal

Strat I  $\Rightarrow$  CVXPY | Strat II  $\Rightarrow$  Nonlinear Optimizer (80)

Data (ETFs) : yfinance (81)

• 1. FXE (82)

• 2. EWJ (83)

• 3. GLD (84)

• 4. QQQ (85)

• 5. SPY (86)

• 6. SHV (87)

• 7. DBA (88)

• 8. USO (89)

• 9. XBI (90)

• 10. ILF (91)

• 11. EPP (92)

• 12. FEZ (93)

To Do: (94)

• Task 1 : 'download\\_data(start\\_date, end\\_date)', 'compute\\_dail

• Task 2 : 'factor\\_model(...)' (96)

• Task 3 : 'optimize\\_model(...)' (97)

- Task 4 : 'backtest(...)' (98)
- Task 5 : 'analyze(...)' (99)
- Task 6 : 'summarize(...)' (100)