

0 Amendment

Response to **25%**; (-1)

This is my corrective action. (0)

1 Overview

1.1 Goal

Objective: Build / Compare Two Factor-Based L/S Allocation Models (1)

Beta (β) Constraints (2)

First Strategy ($S_{\{1\}}$) : Target Beta $\beta_T \in [-0.5, 0.5]$ (3)

Second Strategy ($S_{\{2\}}$) : Target Beta $\beta_T \in [-2, 2]$ (4)

$S_{\{1\}} \cong$ Value-at-Risk Utility (Robust Optimization) (5)

$S_{\{1\}} \Leftarrow$ Information Ratio (6)

Post optimization, I compare model outcomes while evaluating estimator length se

[covariance matrix $\Sigma \wedge$ expected returns μ] across market regimes (8)

1.2 Reallocation

Portfolio Allocation $\{P_t\} \Leftarrow$ '03-01-2007' \sim '03-31-2024' (9)

$P_t \quad \forall t \in \{t_0, t_1, t_2, \dots, t_n\}$ where $t_0 = 03-01-2007, \quad t_n = 03-31-2024$ (10)

$t_i = t_{i-1} + 7 \text{ days}$ for $i = 1, 2, \dots, n$ (11)

Investment Universe \equiv ETFs ('Global World Economy') (12)

Fama-French Three-Factor Model (Momentum, Value, Size) (13)

Public Data (14)

1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market en

Low Beta \Rightarrow Decorrelation; (16)

High Beta \equiv Antithesis. (17)

Portfolio Characteristics :

- Return : μ (18)

- Volatility (Vol) : σ (19)

- Skewness (Skew) : $\mathbb{E} \left[\left(\frac{x - \mu}{\sigma} \right)^3 \right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}}$ (20)

- Value at Risk (VaR) / Expected Shortfall (ES) (21)

- Sharpe Ratio : $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$ (22)

1.4 Simplification

Look-Back μ Estimators :

- Long-Term Estimator (LTE) : $LT \Rightarrow LB \in \{180 \text{ Days}\}$ (23)

- Mid-Term Estimator (MTE) : $MT \Rightarrow LB \in \{90 \text{ Days}\}$ (24)

- Short-Term Estimator (STE) : $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$ (25)

Term-Structure for Covariance $\Sigma \wedge$ Expected Return μ . (26)

1.5 Synthesis

Optimal portfolio behavior constructed from (27)

covariance and expected return estimators (28)

will vary due to strategic and market differences. (29)

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (30)$$

Objective :

- Evaluate Hypothesis (31)

- Demonstrate Robustness (Or Lack Thereof) (32)

- Market Regime Stratification (33)

2 Strategy

Theory \& Math

2.1 Strategic Formulation

Consider two strategies :

$$(\text{Strategy I}) \quad \left\{ \begin{array}{l} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{array} \right. \quad (34)$$

and

$$(\text{Strategy II}) \quad \left\{ \begin{array}{l} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{array} \right. \quad (35)$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t.
 $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
 trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (36)$$

2.2 Fama–French Three-Factor Model (FF3FM)

Definition: (37)

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (38)$$

$\mathbb{E}[\epsilon_i] = 0; \therefore$

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (39)$$

Estimated Coefficient Vector: (40)

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\top \Leftarrow y_i = \rho_i - r_f \quad (41)$$

Linear Regression: (42)

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \quad (43)$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression / computed directly.} \quad (44)$$

2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

2.3.1 Strategy I Breakdown

1. **Objective:** Maximize returns while considering risk.
2. **Constraints:**
 - The portfolio's beta must be between -0.5 and 0.5 .
 - The sum of the weights assigned to each asset in the portfolio must equal 1.
 - Each individual weight can range from -2 to 2 .

2.3.2 Strategy II Breakdown

1. **Objective:** Maximize returns relative to the portfolio's **tracking error volatility (TEV)**, which measures how much the portfolio's returns deviate from a benchmark.
2. **Constraints:**
 - The portfolio's beta must be between -2 and 2 .
 - The sum of the weights assigned to each asset in the portfolio must equal 1.
 - Each individual weight can range from -2 to 2 .

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

3 Assumptions and (Analysis) Setup

3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.
2. I define three cases:
 - Long-Term Look-Back Period : 120 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario LT $\equiv S_{120}$.
 - Medium-Term Look-Back Period : 90 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario MT $\equiv S_{90}$.
 - Short-Term Look-Back Period : 40 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario ST $\equiv S_{40}$.
3. Consider two possible values for the **Target Beta** (again, **not** the [colloquial slang term](#)) : 0 & 1.
4. Consider two possible values for the λ (the **risk aversion parameter**; i.e., how much are you [putting on black?](#)) : 0.10 & 0.50.