

0 Amendment

Response to **25%**;

This is my corrective action.

(-1)

(0)

1 Overview

1.1 Goal

Objective: Build / Compare Two Factor-Based L/S Allocation Models

Beta (β) Constraints

- First Strategy ($S_{\{1\}}$) : Target Beta $\beta_T \in [-0.5, 0.5]$
- Second Strategy ($S_{\{2\}}$) : Target Beta $\beta_T \in [-2, 2]$
- $S_{\{1\}} \cong$ Value-at-Risk Utility (Robust Optimization)
- $S_{\{1\}} \Leftarrow$ Information Ratio
- (1)
- (2)
- (3)
- (4)

Post optimization, I compare model outcomes while evaluating estimator length se
[covariance matrix $\Sigma \wedge$ expected returns μ] across market regimes

(6)

1.2 Reallocation

- Portfolio Allocation $\{P_t\} \Leftarrow$ ‘03-01-2007‘ \sim ‘03-31-2024‘
- $P_t \quad \forall t \in \{t_0, t_1, t_2, \dots, t_n\}$ where $t_0 =$ 03-01-2007, $t_n =$ 03-31-2024
- $t_i = t_{i-1} + 7$ days for $i = 1, 2, \dots, n$
- Investment Universe \equiv ETFs (‘Global World Economy’)
- Fama–French Three-Factor Model (Momentum, Value, Size)
- Public Data
- (7)
- (8)
- (9)
- (10)
- (11)
- (12)

1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market en

- Low Beta \Rightarrow Decorrelation;
- High Beta \equiv Antithesis.
- (14)
- (15)

Portfolio Characteristics :

- Return : μ (16)

- Volatility (Vol) : σ (17)

- Skewness (Skew) : $\mathbb{E} \left[\left(\frac{x - \mu}{\sigma} \right)^3 \right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}}$ (18)

- Value at Risk (VaR) / Expected Shortfall (ES) (19)

- Sharpe Ratio : $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$ (20)

1.4 Simplification

Post Factor Model (FM) construction,

I use trend following estimators for μ .

The estimator quality depends on the look-back (LB) period; \therefore

- Long-Term Estimator (LTE) : $LT \Rightarrow LB \in \{180 \text{ Days}\}$. (1)

- Mid-Term Estimator (MTE) : $MT \Rightarrow LB \in \{90 \text{ Days}\}$. (2)

- Short-Term Estimator (STE) : $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$. (3)

Term-Structure for Covariance $\Sigma \wedge$ Expected Return μ .

1.5 Synthesis

Optimal portfolio behavior constructed from

covariance and expected return estimators

will vary due to strategic and market differences.

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (4)$$

Goal :

- Evaluate Hypothesis
- Demonstrate Robustness (Or Lack Thereof)

- Market Regime Stratification

2 Strategy

Theory \& Math

2.1 Strategic Formulation

Consider two strategies :

$$(\text{Strategy I}) \quad \left\{ \begin{array}{l} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{array} \right. \quad (5)$$

and

$$(\text{Strategy II}) \quad \left\{ \begin{array}{l} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{array} \right. \quad (6)$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t.
 $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
 trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (7)$$

2.2 Fama–French Three-Factor Model (FF3FM)

Definition :

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (8)$$

$$\mathbb{E}[\epsilon_i] = 0; \therefore$$

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (9)$$

Estimated Coefficient Vector :

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\top \Leftarrow y_i = \rho_i - r_f \quad (10)$$

Linear Regression :

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \quad (11)$$

$\beta_i^m \neq \beta_i^3$ | estimated via separate regression / computed directly.

2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

2.3.1 Strategy I Breakdown

1. **Objective:** Maximize returns while considering risk.
2. **Constraints:**
 - The portfolio's beta must be between -0.5 and 0.5 .
 - The sum of the weights assigned to each asset in the portfolio must equal 1.
 - Each individual weight can range from -2 to 2 .

2.3.2 Strategy II Breakdown

1. **Objective:** Maximize returns relative to the portfolio's **tracking error volatility (TEV)**, which measures how much the portfolio's returns deviate from a benchmark.
2. **Constraints:**
 - The portfolio's beta must be between -2 and 2 .
 - The sum of the weights assigned to each asset in the portfolio must equal 1.
 - Each individual weight can range from -2 to 2 .

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

3 Assumptions and (Analysis) Setup

3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.
2. I define three cases:
 - Long-Term Look-Back Period : 120 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario LT $\equiv S_{120}$.
 - Medium-Term Look-Back Period : 90 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario MT $\equiv S_{90}$.
 - Short-Term Look-Back Period : 40 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario ST $\equiv S_{40}$.
3. Consider two possible values for the **Target Beta** (again, **not** the colloquial slang term) : 0 & 1.
4. Consider two possible values for the λ (the **risk aversion parameter**; i.e., how much are you putting on black?) : 0.10 & 0.50.