0 Amendment

$$\text{Response to } \mathbf{25\%}; \tag{-1}$$

$$\text{This is my corrective action and (my) letter (to you).} \tag{0}$$

1 Overview

1.1 Goal

$$\text{Objective: Build / Compare Two Factor-Based L/S Allocation Models} \tag{1}$$

$$\text{Beta } (\beta) \text{ Constraints} \tag{2}$$

$$\text{First Strategy } (S_{\{1\}}) : \text{Target Beta } \beta_T \in [-0.5, 0.5] \tag{3}$$

$$\text{Second Strategy } (S_{\{2\}}) : \text{Target Beta } \beta_T \in [-2, 2] \tag{4}$$

$$S_{\{1\}} \approxeq \text{Value-at-Risk Utility (Robust Optimization)} \tag{5}$$

$$S_{\{2\}} \Leftarrow \text{Information Ratio} \tag{6}$$

$$\text{Post Optimization: Model Comparative Analysis (Estimator Length Sensitivity)}$$

$$[\text{Covariance Matrix } \mathbf{\Sigma} \wedge \text{Expected Returns } \boldsymbol{\mu}] : \ R_j \ \forall \ j \in \{1, 2, \ldots, M\} \tag{8}$$

1.2 Reallocation

$$\text{Portfolio Allocation } \{P_t\} \Leftarrow \texttt{`03-01-2007`} \sim \texttt{`03-31-2024`} \tag{9}$$

$$P_t \quad \forall\, t \in \{t_0, t_1, t_2, \ldots, t_n\} \quad \text{where} \quad t_0 = \texttt{03-01-2007}, \quad t_n = \texttt{03-31-2024} \tag{10}$$

$$t_i = t_{i-1} + 7 \text{ days} \quad \text{for} \quad i = 1, 2, \ldots, n \tag{11}$$

$$\text{Investment Universe} \equiv \text{ETFs ('Global World Economy')} \tag{12}$$

$$\text{Fama–French Three-Factor Model (Momentum, Value, Size)} \tag{13}$$

$$\text{Public Data} \tag{14}$$

1.3 Performance Evaluation

$$\text{Performance / Risk Profiles} \Leftarrow \text{Sensitive} \Leftarrow \text{Target Beta} : \beta_T \ \wedge \ \text{'Market'} \tag{15}$$

$$\text{Low Beta} \Rightarrow \text{ Decorrelation;} \tag{16}$$

$$\text{High Beta} \equiv \text{ Antithesis.} \tag{17}$$

$$\text{Portfolio Characteristics:} \tag{18}$$

- $$\text{Annualized Return : } \mu_a \tag{19}$$
- $$\text{Historical Vol : } \sigma_h \tag{20}$$
- $$\text{Skew : } \mathbb{E}\left[\left(\frac{x-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}} \tag{21}$$
- $$\text{VaR / ES (CVaR)} \tag{22}$$
- $$\text{Sharpe : } \frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}} \tag{23}$$

1.4 Simplification

$$\text{Look-Back } \boldsymbol{\mu} \text{ Estimators : } \tag{24}$$

- $$\text{Long-Term Estimator (LTE) : } \hat{\text{LT}} \Rightarrow \text{LB} \in \{180 \text{ Days}\} \tag{25}$$
- $$\text{Mid-Term Estimator (MTE) : } \hat{\text{MT}} \Rightarrow \text{LB} \in \{90 \text{ Days}\} \tag{26}$$
- $$\text{Short-Term Estimator (STE) : } \hat{\text{ST}} \Rightarrow \text{LB} \in \{40 \text{ Days}, 60 \text{ Days}\} \tag{27}$$

$$\therefore$$

$$\text{Term-Structure : Covariance } \boldsymbol{\Sigma} \ \wedge \ \text{Expected Return } \boldsymbol{\mu}. \tag{28}$$

1.5 Synthesis

$$\text{Optimal Portfolio} \Leftarrow \hat{\boldsymbol{\Sigma}} \mid \hat{\boldsymbol{\mu}} \tag{29}$$

$$\text{Variance} \Leftarrow \text{Strategic } \backslash\& \text{ Market } \boldsymbol{\Delta} \tag{30}$$

$$\Downarrow$$

$$S_{40}^{90} \equiv \hat{\boldsymbol{\Sigma}} \Rightarrow 40 \text{ Days} \ \wedge \ \hat{\boldsymbol{\mu}} \Rightarrow 90 \text{ Days} \tag{31}$$

$$\therefore$$

$$\text{Objective:} \tag{32}$$

- $$\text{Evaluate Hypothesis} \tag{33}$$
- $$\text{Demonstrate Robustness (Or Lack Thereof)} \tag{34}$$
- $$\text{Market Regime Stratification} \tag{35}$$

2 Strategy

Theory \& Math

2.1 Strategic Formulation

Consider two strategies :

$$\textbf{(Strategy I)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{36}$$

and

$$\textbf{(Strategy II)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{37}$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t.
  $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
  (non)trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\intercal \Sigma \omega - 2\omega^\intercal \text{Cov}(r, r_{\text{SPY}}) + \sigma^2_{\text{SPY}}}. \tag{38}$$

2.2 Fama–French Three-Factor Model (FF3FM)

Definition: (39)

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{40}$$

$\mathbb{E}[\epsilon_i] = 0$; $\therefore$

$$\rho_i = r_f + \beta_i^3 (\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{41}$$

$$\text{Estimated Coefficient Vector:} \tag{42}$$

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\intercal \Leftarrow y_i = \rho_i - r_f \tag{43}$$

$$\text{Linear Regression:} \tag{44}$$

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \tag{45}$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression / computed directly.} \tag{46}$$

2.3 Executive Summary Formulation

$$\textit{(In)numerate}: \tag{47}$$

$$\textbf{Strategy I} \tag{48}$$

$$\text{1. Objective} \equiv \text{Maximize Returns w/Risk.} \tag{49}$$

$$\text{2. Constraints :} \tag{50}$$

- $$\text{The portfolio's beta must be between } -0.5 \text{ and } 0.5. \tag{51}$$
- $$\text{The sum of the weights assigned to each asset in the portfolio must equal 1.}$$
- $$\text{Each individual weight can range from } -2 \text{ to } 2. \tag{53}$$

$$\textbf{Strategy II} \tag{54}$$

$$\text{1. Objective} \equiv \text{Maximize Returns Relative to Tracking Error Volatility (TEV).}$$

$$\text{2. Constraints :} \tag{56}$$

- $$\text{The portfolio's beta must be between } -2 \text{ and } 2. \tag{57}$$
- $$\text{The sum of the weights assigned to each asset in the portfolio must equal 1.}$$
- $$\text{Each individual weight can range from } -2 \text{ to } 2. \tag{59}$$

3 Assumptions

3.1 Setup

$$\text{1. Reallocation : } \texttt{`03-01-2007`} \sim \texttt{`03-31-2024`} \tag{60}$$

2. Input Construction : (61)

- LT LB Period : $n_{\text{LT}} = 120 \mid \hat{\Sigma}_s \wedge \hat{\mu}_s \mid \text{LT} \equiv S_{120}$ (62)
- MT LB Period : $n_{\text{LT}} = 90 \mid \hat{\Sigma}_s \wedge \hat{\mu}_s \mid \text{MT} \equiv S_{90}$ (63)
- ST LB Period : $n_{\text{LT}} = 40 \mid \hat{\Sigma}_s \wedge \hat{\mu}_s \mid \text{MT} \equiv S_{40}$ (64)

3. $\boldsymbol{\beta}_T \in \{0, 1\}$ (65)

4. $\boldsymbol{\lambda} \in \{0.10, 0.50\}$ (66)

3.2 Period Analysis

Period Stratification: (67)

- Period 1 $\equiv$ Pre-Subprime (68)
- Period 2 $\equiv$ Subprime (69)
- Period 3 $\equiv$ Post-Subprime (70)
- Period 4 $\equiv$ COVID (71)
- Period 5 $\equiv$ Post-Covid (72)

3.3 BackTesting

Definition: Historical Data$\rightarrow$ Performance (73)

Logistical Considerations: (74)

- BackTest $\neq$ ForeCast $\Rightarrow$ **Snooping Bias / P-Hacking** (75)
- Non-Adapted Filtrations :

  $\{\mathcal{F}_t\}_{t=1}^{T} \equiv$ Information Available (Up to Time) t
- Weekly Rebalance (76)
- $\{t_i\}_{i=1}^{n}$: (77)

For the initial date t_1, use the prior 60 days of historical data to estimate input

Store the portfolio weights: ω_{t_1}.

For each subsequent date t_{i+1}, roll the historical data window by 5 days, re-est

Store the new portfolio weights: $\omega_{t_{i+1}}$.

Repeat this process until the target date t_n is reached.

4 ToolKit|Arsenal

Strat I $\Rightarrow$ `CVXPY` | Strat II $\Rightarrow$ `Nonlinear Optimizer` (79)

Data (ETFs) : `yfinance` (80)

- 1. FXE (81)
- 2. EWJ (82)
- 3. GLD (83)
- 4. QQQ (84)
- 5. SPY (85)
- 6. SHV (86)
- 7. DBA (87)
- 8. USO (88)
- 9. XBI (89)
- 10. ILF (90)
- 11. EPP (91)
- 12. FEZ (92)

To Do: (93)

- Task 1 : ‘`download\_data(start\_date, end\_date)`‘, ‘`compute\_dail`
- Task 2 : ‘`factor\_model(...)`‘. (95)
- Task 3 : ‘`optimize\_model(...)`‘ (96)

- Task 4 : ‘`backtest(...)`‘ (97)
- Task 5 : ‘`analyze(...)`‘ (98)
- Task 6 : ‘`summarize(...)`‘ (99)

5 Performance + Risk Reporting 4 Strats

KPIs: (100)

- Cumulative PnL / Return (101)
- Average Daily Arithmetic / Geometric Return| Daily Min Return (102)
- 10 Day Max Drawdown| Sharpe (103)
- Vol, Skew, (Excess) Kurt, (Modified) VaR, Expected Shortfall (CVar) (104)

Tabular Formulation: (105)

	$S_{40}(\beta_T = 0)$	$S_{90}(\beta_T = 1)$	$S_{120}(\beta_T = 0)$	**SPY**
Mean Return			12	
⋮			⋮	
Max DD			8	

Furthermore: (106)

1. Evolution Plot : Cumulative Daily PnL | $P_0 = \$100,\ \text{SPY}_0 = \$100.$ (107)
2. Plot + Analyze (Daily) Return Distribution. (108)

6 Deliverables

$1 \equiv$ Report : Findings, Conclusions, Estimator Impact on Strats (When? Why?) Crisis Periods (Subprime, COVID, et cetera.);

$2 \equiv$ Axes : Estimator Term-Structure Sensitivity (ST, MT, LT) for $\mathbf{\Sigma} \wedge \boldsymbol{\mu}$ | T

$3 \equiv$ Notation, Strats, Graphs, \& Tables Description; (111)

$$4 \equiv \texttt{Code.} \tag{112}$$