

# FE630 - Final Project (Revision)

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**Pledge:** I pledge my honor that I have abided by the Stevens Honor System.

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## 0 Disclaimer

This work is in response to the original (numerical) grade I received as a **25%** due to my imprudence and deviation from the requested prompt.

I (profusely) apologize for my incompetence; **This is my corrective action.**

## 1 Overview

### 1.1 Goal

The objective is to build and compare two factor-based long short allocation models with constraints on their betas. The first strategy considers a target Beta in the interval  $[-0.5, 0.5]$  while the second has one in the interval  $[-2, +2]$ .

The first operates similar to a Value-at-Risk Utility (Robust Optimization); the second incorporates an Information Ratio.

Post optimization, I compare the model outcomes while evaluating their sensitivity to estimator length for the covariance matrix and expected returns across market regimes.

### 1.2 Reallocation

The portfolios are reallocated weekly from 03-01-2007 to 03-31-2024 .

My investment universe  $\equiv$  ETFs ('Global World Economy').

I use the Fama–French Three-Factor Model (Momentum, Value, Size).

The data is publicly available.

### 1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market environment.

A low Beta indicates decorrelation; a high Beta is the antithesis.

Portfolio Characteristics Definition:

- Return :  $\mu$
- Vol :  $\sigma$
- Skew :  $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma_3} = \frac{\kappa_3}{\kappa_2^{3/2}}$
- VaR / Expected Shortfall
- Sharpe :  $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$

## 1.4 Simplification

Post Factor Model (FM) construction, I use trend following estimators for  $\mu$ .

The estimator quality depends on the look-back (LB) period;  $\therefore$

- Long-Term Estimator (LTE) :  $LT \Rightarrow LB \in \{180 \text{ Days}\}$ .
- Mid-Term Estimator (MTE) :  $MT \Rightarrow LB \in \{90 \text{ Days}\}$ .
- Short-Term Estimator (STE) :  $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$ .

I define **Term-Structure** for Covariance  $\Sigma \wedge$  Expected Return  $\mu$ .

## 1.5 Synthesis

Optimal portfolio behavior constructed from covariance and expected return estimators will vary due to strategic and market differences.

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (1)$$

Goal :

- Evaluate Hypothesis
- Demonstrate Robustness (Or Lack Thereof)
- Market Regime Stratification

## 2 (Investment) Strategy

This section delves into the theoretical and mathematical formulations for the investment strategy employed; it is not for the faint of heart.

## 2.1 (Mathematical) Strategic Formulation

Consider the following two strategies:

$$(\text{Strategy I}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (1)$$

and

$$(\text{Strategy II}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (2)$$

where we define the constructions above:

- $\Sigma$  is the **covariance matrix** between the securities returns (as computed from the **FF3FM**);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)}$  is the **Beta** of some **security**  $S_i$  as defined by the **CAPM Model** such that  $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i$  is the **Portfolio Beta**;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}})$  is the '**Tracking Error Volatility**'; the derivation is trivial and left as an exercise to the reader:

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (3)$$

## 2.2 Fama–French Three-Factor Model (FF3FM)

The **Fama-French Three-Factor Model** (FF3FM) is defined as follows:

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (4)$$

Assume  $\mathbb{E}[\epsilon_i] = 0$ ; therefore,

$$\rho_i = r_f + \beta_i^3 (\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (5)$$

The 3 coefficients  $\beta_i^3$ ,  $b_i^s$ , and  $b_i^v$  are estimated by making a linear regression of the time series  $y_i = \rho_i - r_f$  against the time series  $\rho_M - r_f$  (**Momentum Factor**),  $r_{\text{SMB}}$  (**Size Factor**), and  $\rho_{\text{HML}}$  (**Value Factor**).

$\beta_i^m \neq \beta_i^3$  in tandem with the requirement to be estimated via a separate regression or directly computed.

## 2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

### 2.3.1 Strategy I Breakdown

1. **Objective:** Maximize returns while considering risk.
2. **Constraints:**
  - The portfolio's beta must be between  $-0.5$  and  $0.5$ .
  - The sum of the weights assigned to each asset in the portfolio must equal 1.
  - Each individual weight can range from  $-2$  to  $2$ .

### 2.3.2 Strategy II Breakdown

1. **Objective:** Maximize returns relative to the portfolio's **tracking error volatility (TEV)**, which measures how much the portfolio's returns deviate from a benchmark.
2. **Constraints:**
  - The portfolio's beta must be between  $-2$  and  $2$ .
  - The sum of the weights assigned to each asset in the portfolio must equal 1.
  - Each individual weight can range from  $-2$  to  $2$ .

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

## 3 Assumptions and (Analysis) Setup

### 3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.
2. I define three cases:
  - Long-Term Look-Back Period : 120 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario LT  $\equiv S_{120}$ .
  - Medium-Term Look-Back Period : 90 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario MT  $\equiv S_{90}$ .
  - Short-Term Look-Back Period : 40 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario ST  $\equiv S_{40}$ .

3. Consider two possible values for the **Target Beta** (again, *not* the colloquial slang term) : 0 & 1.
4. Consider two possible values for the  $\lambda$  (the *risk aversion parameter*; i.e., how much are you putting on black?) : 0.10 & 0.50.