# 0  Amendment

$$\text{Response to } \textbf{25\%;} \tag{-1}$$

$$\text{This is my corrective action and (my) letter (to you).} \tag{0}$$

# 1 Overview

## 1.1 Goal

$$\text{Objective: Build / Compare Two Factor-Based L/S Allocation Models} \tag{1}$$

$$\text{Beta } (\beta) \text{ Constraints} \tag{2}$$

$$\text{First Strategy } (S_{\{1\}}) : \text{Target Beta } \beta_T \in [-0.5, 0.5] \tag{3}$$

$$\text{Second Strategy } (S_{\{2\}}) : \text{Target Beta } \beta_T \in [-2, 2] \tag{4}$$

$$S_{\{1\}} \approxeq \text{Value-at-Risk Utility (Robust Optimization)} \tag{5}$$

$$S_{\{2\}} \Leftarrow \text{Information Ratio} \tag{6}$$

$$\text{Post Optimization: Model Comparative Analysis (Estimator Length Sensitivity)}$$

$$[\text{Covariance Matrix } \boldsymbol{\Sigma} \wedge \text{Expected Returns } \boldsymbol{\mu}] : \; R_j \; \forall \, j \in \{1, 2, \ldots, M\} \tag{8}$$

## 1.2 Reallocation

$$\text{Portfolio Allocation } \{P_t\} \Leftarrow \text{`03-01-2007`} \sim \text{`03-31-2024`} \tag{9}$$

$$P_t \quad \forall \, t \in \{t_0, t_1, t_2, \ldots, t_n\} \quad \text{where} \quad t_0 = \text{03-01-2007}, \quad t_n = \text{03-31-2024} \tag{10}$$

$$t_i = t_{i-1} + 7 \text{ days} \quad \text{for} \quad i = 1, 2, \ldots, n \tag{11}$$

$$\text{Investment Universe} \equiv \text{ETFs ('Global World Economy')} \tag{12}$$

$$\text{Fama–French Three-Factor Model (Momentum, Value, Size)} \tag{13}$$

$$\text{Public Data} \tag{14}$$

## 1.3 Performance Evaluation

$$\text{Performance / Risk Profiles} \Leftarrow \text{Sensitive} \Leftarrow \text{Target Beta} : \beta_T \; \wedge \; \text{'Market'} \tag{15}$$

$$\text{Low Beta} \Rightarrow \text{Decorrelation;} \tag{16}$$

$$\text{High Beta} \equiv \text{Antithesis.} \tag{17}$$

$$\text{Portfolio Characteristics:} \tag{18}$$

$$\bullet \qquad \text{Annualized Return}: \mu_a \tag{19}$$

$$\bullet \qquad \text{Historical Vol}: \sigma_h \tag{20}$$

$$\bullet \qquad \text{Skew}: \mathbb{E}\left[\left(\frac{x-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}} \tag{21}$$

$$\bullet \qquad \text{VaR / ES (CVaR)} \tag{22}$$

$$\bullet \qquad \text{Sharpe}: \frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}} \tag{23}$$

## 1.4 Simplification

$$\text{Look-Back } \boldsymbol{\mu} \text{ Estim}\hat{\text{a}}\text{tors}: \tag{24}$$

$$\bullet \qquad \text{Long-Term Estimator (LTE)}: \text{LT} \Rightarrow \text{LB} \in \{180 \text{ Days}\} \tag{25}$$

$$\bullet \qquad \text{Mid-Term Estimator (MTE)}: \text{MT} \Rightarrow \text{LB} \in \{90 \text{ Days}\} \tag{26}$$

$$\bullet \qquad \text{Short-Term Estimator (STE)}: \text{ST} \Rightarrow \text{LB} \in \{40 \text{ Days}, 60 \text{ Days}\} \tag{27}$$

$$\therefore$$

$$\text{Term-Structure}: \text{Covariance } \boldsymbol{\Sigma} \ \wedge \ \text{Expected Return } \boldsymbol{\mu}. \tag{28}$$

## 1.5 Synthesis

$$\text{Optimal portfolio behavior constructed from} \tag{29}$$

$$\text{covariance and expected return estimators} \tag{30}$$

$$\text{will vary due to strategic and market differences.} \tag{31}$$

$$S_{40}^{90} \equiv \hat{\boldsymbol{\Sigma}} \Rightarrow 40 \text{ Days} \ \wedge \ \hat{\boldsymbol{\mu}} \Rightarrow 90 \text{ Days} \tag{32}$$

$$\text{Objective:} \tag{33}$$

$$\bullet \qquad \text{Evaluate Hypothesis} \tag{34}$$

$$\bullet \qquad \text{Demonstrate Robustness (Or Lack Thereof)} \tag{35}$$

$$\bullet \qquad \text{Market Regime Stratification} \tag{36}$$

# 2  Strategy

Theory \& Math

## 2.1 Strategic Formulation

Consider two strategies :

$$\text{(Strategy I)} \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \ \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\[2mm] -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\[2mm] \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{37}$$

and

$$\text{(Strategy II)} \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \ \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\[2mm] -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\[2mm] \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{38}$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);

- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security $S_i$ (CAPM) s.t.
  $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta;

- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
  trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\mathsf{T} \Sigma \omega - 2\omega^\mathsf{T} \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \tag{39}$$

## 2.2 Fama–French Three-Factor Model (FF3FM)

$$\text{Definition:} \tag{40}$$

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{41}$$

$\mathbb{E}[\epsilon_i] = 0; \therefore$

$$\rho_i = r_f + \beta_i^3 (\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{42}$$

$$\text{Estimated Coefficient Vector:} \tag{43}$$

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\mathsf{T} \Leftarrow y_i = \rho_i - r_f \tag{44}$$

$$\text{Linear Regression:} \tag{45}$$

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \tag{46}$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression / computed directly.} \tag{47}$$

## 2.3 Executive Summary Formulation

$$\text{Innumerate:} \tag{48}$$

$$\text{Strategy I} \tag{49}$$

$$\text{1. Objective} \equiv \text{Maximize Returns w/Risk.} \tag{50}$$

$$\text{2. Constraints :} \tag{51}$$

- $$\text{The portfolio's beta must be between } -0.5 \text{ and } 0.5. \tag{52}$$

- The sum of the weights assigned to each asset in the portfolio must equal 1.

- $$\text{Each individual weight can range from } -2 \text{ to } 2. \tag{54}$$

$$\text{Strategy II} \tag{55}$$

$$\text{1. Objective} \equiv \text{Maximize Returns Relative to Tracking Error Volatility (TEV).}$$

$$\text{2. Constraints :} \tag{57}$$

- $$\text{The portfolio's beta must be between } -2 \text{ and } 2. \tag{58}$$

- The sum of the weights assigned to each asset in the portfolio must equal 1.

- $$\text{Each individual weight can range from } -2 \text{ to } 2. \tag{60}$$

# 3  Assumptions

## 3.1 Setup

$$\text{1. Reallocation : `03-01-2007` } \sim \text{ `03-31-2024`} \tag{61}$$

$$\text{2. Input Construction :} \tag{62}$$

$$\bullet \qquad \text{LT LB Period} : n_{\text{LT}} = 120 \ \mid \ \Sigma_s \ \wedge \ \mu_s \ \mid \ \text{LT} \equiv S_{120} \tag{63}$$

$$\bullet \qquad \text{MT LB Period} : n_{\text{LT}} = 90 \ \mid \ \Sigma_s \ \wedge \ \mu_s \ \mid \ \text{MT} \equiv S_{90} \tag{64}$$

$$\bullet \qquad \text{ST LB Period} : n_{\text{LT}} = 40 \ \mid \ \Sigma_s \ \wedge \ \mu_s \ \mid \ \text{MT} \equiv S_{40} \tag{65}$$

$$\text{3. } \boldsymbol{\beta}_T \in \{0, 1\} \tag{66}$$

$$\text{4. } \boldsymbol{\lambda} \in \{0.10, 0.50\} \tag{67}$$

## 3.2 Period Analysis

$$\text{Period Stratification:} \tag{68}$$

$$\bullet \qquad \text{Period 1} \ \equiv \ \text{Pre-Subprime} \tag{69}$$

$$\bullet \qquad \text{Period 2} \ \equiv \ \text{Subprime} \tag{70}$$

$$\bullet \qquad \text{Period 3} \ \equiv \ \text{Post-Subprime} \tag{71}$$

$$\bullet \qquad \text{Period 4} \ \equiv \ \text{COVID} \tag{72}$$

$$\bullet \qquad \text{Period 5} \ \equiv \ \text{Post-Covid} \tag{73}$$

## 3.3 BackTesting

$$\text{Definition: Historical Data} \Rightarrow \text{Performance} \tag{74}$$

$$\text{Logistical Considerations:} \tag{75}$$

$$\bullet \qquad \text{BackTest} \neq \text{Forecasts} \Rightarrow \textbf{Snooping Bias / P-Hacking} \tag{76}$$

$$\bullet \qquad \text{Weekly Rebalance} \tag{77}$$

$$\bullet \qquad \{t_i\}_{i=1}^n : \tag{78}$$

$$\begin{cases} \text{For the initial date } t_1, \text{ use the prior 60 days of historical data to estimate input} \\[2mm] \text{Store the portfolio weights: } \omega_{t_1}. \\[2mm] \text{For each subsequent date } t_{i+1}, \text{ roll the historical data window by 5 days, re-est} \\[2mm] \text{Store the new portfolio weights: } \omega_{t_{i+1}}. \\[2mm] \text{Repeat this process until the target date } t_n \text{ is reached.} \end{cases}$$

# 4  ToolKit|Arsenal

$$\text{Strat I} \Rightarrow \texttt{CVXPY} \mid \text{Strat II} \Rightarrow \texttt{Nonlinear Optimizer} \tag{80}$$

$$\text{Data (ETFs) : } \texttt{yfinance} \tag{81}$$

- $$\text{1. FXE} \tag{82}$$
- $$\text{2. EWJ} \tag{83}$$
- $$\text{3. GLD} \tag{84}$$
- $$\text{4. QQQ} \tag{85}$$
- $$\text{5. SPY} \tag{86}$$
- $$\text{6. SHV} \tag{87}$$
- $$\text{7. DBA} \tag{88}$$
- $$\text{8. USO} \tag{89}$$
- $$\text{9. XBI} \tag{90}$$
- $$\text{10. ILF} \tag{91}$$
- $$\text{11. EPP} \tag{92}$$
- $$\text{12. FEZ} \tag{93}$$

$$\text{To Do:} \tag{94}$$

- $$\text{Task 1 : `download\_data(start\_date, end\_date)`, `compute\_dail}$$
- $$\text{Task 2 : `factor\_model(...)`.} \tag{96}$$
- $$\text{Task 3 : `optimize\_model(...)`} \tag{97}$$
- $$\text{Task 4 : `backtest(...)`} \tag{98}$$
- $$\text{Task 5 : `analyze(...)`} \tag{99}$$
- $$\text{Task 6 : `summarize(...)`} \tag{100}$$

# 5  Performance + Risk Reporting 4 Strats

$$\text{KPIs:} \tag{101}$$

- $$\text{Cumulative PnL / Return} \tag{102}$$

- $$\text{Average Daily Arithmetic / Geometric Return| Daily Min Return} \tag{103}$$

- $$\text{10 Day Max Drawdown| Sharpe} \tag{104}$$

- $$\text{Vol, Skew, (Excess) Kurt, (Modified) VaR, Expected Shortfall (CVar)} \tag{105}$$

$$\text{Tabular Formulation:} \tag{106}$$

|  | $S_{40}(\beta_T = 0)$ | $S_{90}(\beta_T = 1)$ | $S_{120}(\beta_T = 0)$ | **SPY** |
|---|---|---|---|---|
| **Mean Return** |  |  | 12 |  |
| $\vdots$ |  |  | $\vdots$ |  |
| **Max DD** |  |  | 8 |  |

$$\text{Furthermore:} \tag{107}$$

1. $$\text{Evolution Plot : Cumulative Daily PnL } \mid P_0 = \$100, \text{SPY}_0 = \$100. \tag{108}$$

2. $$\text{Plot + Analyze (Daily) Return Distribution.} \tag{109}$$

# 6  Deliverables

$$1 \equiv \text{Report : Findings, Conclusions, Estimator Impact on Strats (When? Why?)}$$
$$\text{Crisis Periods (Subprime, COVID);}$$

$$2 \equiv \text{Axes : Estimator Term-Structure Sensitivity (ST, MT, LT) for } \mathbf{\Sigma} \wedge \boldsymbol{\mu} \mid \text{Ta}$$

$$3 \equiv \text{Notation, Strats, Graphs, \\& Tables Description;} \tag{112}$$

$$4 \equiv \texttt{Code.} \tag{113}$$