0 Amendment

Response to **25%;**

This is my corrective action.

1 Overview

1.1 Goal

Objective : Build / Compare Two Factor-Based L/S Allocation Models;

Beta (β) Constraints.

First Strategy ($S_{\{1\}}$) : Target Beta $\beta_T \in [-0.5, 0.5]$;
Second \ldots $\in\ [-2, +2]$.

$S_{\{1\}} \cong$ Value-at-Risk Utility (Robust Optimization);

$S_{\{1\}} \Leftarrow$ Information Ratio.

Post optimization, I compare the model outcomes while evaluating
estimator length sensitivity for covariance matrix $\boldsymbol{\Sigma}$

and expected returns $\boldsymbol{\mu}$ across market regimes.

1.2 Reallocation

The portfolios are reallocated weekly from `03-01-2007` to `03-31-2024`;

Investment Universe $\equiv$ ETFs ('Global World Economy');

Fama–French Three-Factor Model (Momentum, Value, Size);

Public Data.

1.3 Performance Evaluation

The performance / risk profiles are sensitive

to the target Beta and the market environment.

A low Beta indicates decorrelation;

a high Beta is the antithesis.

Portfolio Characteristics :

- Return : μ

- Vol : σ

- Skew : $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma_3} = \frac{\kappa_3}{\kappa_2^{3/2}}$

- VaR / ES

- Sharpe : $\frac{\mathbb{E}[R_a-R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a-R_b]}{\sqrt{\mathbb{V}(R_a-R_b)}}$

1.4 Simplification

Post Factor Model (FM) construction,

I use trend following estimators for μ.

The estimator quality depends on the look-back (LB) period; $\therefore$

$$\text{Long-Term Estimator (LTE) : LT} \Rightarrow \text{LB} \in \{180 \text{ Days}\}. \tag{1}$$

$$\text{Mid-Term Estimator (MTE) : MT} \Rightarrow \text{LB} \in \{90 \text{ Days}\}. \tag{2}$$

$$\text{Short-Term Estimator (STE) : ST} \Rightarrow \text{LB} \in \{40 \text{ Days}, 60 \text{ Days}\}. \tag{3}$$

Term-Structure for Covariance $\boldsymbol{\Sigma}$ $\wedge$ Expected Return $\boldsymbol{\mu}$.

1.5 Synthesis

Optimal portfolio behavior constructed from

covariance and expected return estimators

will vary due to strategic and market differences.

$$S_{40}^{90} \equiv \hat{\boldsymbol{\Sigma}} \Rightarrow 40 \text{ Days} \wedge \hat{\boldsymbol{\mu}} \Rightarrow 90 \text{ Days} \tag{4}$$

Goal :

- Evaluate Hypothesis
- Demonstrate Robustness (Or Lack Thereof)
- Market Regime Stratification

2 Strategy

Theory \& Math

2.1 Strategic Formulation

Consider two strategies :

$$
\text{(Strategy I)} \quad
\begin{cases}
\max\limits_{\omega\in\mathbb{R}^n}\ \rho^T\omega - \lambda\sqrt{\omega^T\Sigma\omega} \\[2mm]
-0.5 \le \sum_{i=1}^{n}\beta_i^m\omega_i \le 0.5 \\[2mm]
\sum_{i=1}^{n}\omega_i = 1, \quad -2 \le \omega_i \le 2,
\end{cases}
\tag{5}
$$

and

$$
\text{(Strategy II)} \quad
\begin{cases}
\max\limits_{\omega\in\mathbb{R}^n}\ \dfrac{\rho^T\omega}{\text{TEV}(\omega)} - \lambda\sqrt{\omega^T\Sigma\omega} \\[2mm]
-2 \le \sum_{i=1}^{n}\beta_i^m\omega_i \le 2 \\[2mm]
\sum_{i=1}^{n}\omega_i = 1, \quad -2 \le \omega_i \le 2,
\end{cases}
\tag{6}
$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);

- $\beta_i^m = \frac{\text{Cov}(r_i,r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t.
  $\beta_P^m = \sum_{i=1}^{n}\beta_i^m\omega_i \equiv$ Porfolio Beta;

- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
  trivial derivation (reader exercise) :

$$
\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\mathsf{T}\Sigma\omega - 2\omega^\mathsf{T}\text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}.
\tag{7}
$$

2.2 Fama–French Three-Factor Model (FF3FM)

Definition :

$$
r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i
\tag{8}
$$

$\mathbb{E}[\epsilon_i] = 0; \therefore$

$$
\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i
\tag{9}
$$

$$\text{Estimated Coefficient Vector :}$$

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\mathsf{T} \Leftarrow y_i = \rho_i - r_f \tag{10}$$

$$\text{Linear Regression :}$$

$$= \hat{\beta}_i^3 (\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \tag{11}$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression / computed directly.}$$

2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

2.3.1 Strategy I Breakdown

1. **Objective**: Maximize returns while considering risk.

2. **Constraints**:

   - The portfolio's beta must be between -0.5 and 0.5.
   - The sum of the weights assigned to each asset in the portfolio must equal 1.
   - Each individual weight can range from -2 to 2.

2.3.2 Strategy II Breakdown

1. **Objective**: Maximize returns relative to the portfolio's **tracking error volatility** (**TEV**), which measures how much the portfolio's returns deviate from a benchmark.

2. **Constraints**:

   - The portfolio's beta must be between -2 and 2.
   - The sum of the weights assigned to each asset in the portfolio must equal 1.
   - Each individual weight can range from -2 to 2.

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

3 Assumptions and (Analysis) Setup

3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.

2. I define three cases:

   - Long-Term Look-Back Period : 120 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario $\mathrm{LT} \equiv S_{120}$.
   - Medium-Term Look-Back Period : 90 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario $\mathrm{MT} \equiv S_{90}$.
   - Short-Term Look-Back Period : 40 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario $\mathrm{ST} \equiv S_{40}$.

3. Consider two possible values for the **Target Beta** (again, ***not*** the colloquial slang term) : 0 & 1.

4. Consider two possible values for the **λ** (the ***risk aversion parameter***; i.e., how much are you putting on black?) : 0.10 & 0.50.