

## 0 Amendment

Response to **25%**;

This is my corrective action.

## 1 Overview

### 1.1 Goal

Objective: Build / Compare Two Factor-Based L/S Allocation Models

Beta ( $\beta$ ) Constraints

$$\text{First Strategy } (S_{\{1\}}) : \text{Target Beta } \beta_T \in [-0.5, 0.5] \quad (1)$$

$$\text{Second Strategy } (S_{\{2\}}) : \text{Target Beta } \beta_T \in [-2, 2] \quad (2)$$

$$S_{\{1\}} \cong \text{Value-at-Risk Utility (Robust Optimization)} \quad (3)$$

$$S_{\{1\}} \Leftarrow \text{Information Ratio} \quad (4)$$

Post optimization, I compare model outcomes while evaluating estimator length se

$$[\text{covariance matrix } \Sigma \wedge \text{expected returns } \mu] \text{ across market regimes} \quad (6)$$

### 1.2 Reallocation

$$\text{Portfolio Allocation } \{P_t\} \Leftarrow \text{'03-01-2007'} \sim \text{'03-31-2024'} \quad (7)$$

$$P_t \quad \forall t \in \{t_0, t_1, t_2, \dots, t_n\} \quad \text{where} \quad t_0 = \text{'03-01-2007'}, \quad t_n = \text{'03-31-2024'} \quad (8)$$

$$t_i = t_{i-1} + 7 \text{ days} \quad \text{for} \quad i = 1, 2, \dots, n \quad (9)$$

$$\text{Investment Universe} \equiv \text{ETFs ('Global World Economy')} \quad (10)$$

$$\text{Fama-French Three-Factor Model (Momentum, Value, Size)} \quad (11)$$

$$\text{Public Data} \quad (12)$$

### 1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market en

$$\text{Low Beta} \Rightarrow \text{Decorrelation}; \quad (14)$$

$$\text{High Beta} \equiv \text{Antithesis}. \quad (15)$$

## Portfolio Characteristics :

- Return :  $\mu$  (16)

- Volatility (Vol) :  $\sigma$  (17)

- Skewness (Skew) :  $\mathbb{E} \left[ \left( \frac{x - \mu}{\sigma} \right)^3 \right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}}$  (18)

- Value at Risk (VaR) / Expected Shortfall (ES) (19)

- Sharpe Ratio :  $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$  (20)

## 1.4 Simplification

Post Factor Model (FM) construction,

I use trend following estimators for  $\mu$ .

The estimator quality depends on the look-back (LB) period;  $\therefore$

- Long-Term Estimator (LTE) :  $LT \Rightarrow LB \in \{180 \text{ Days}\}$ . (1)

- Mid-Term Estimator (MTE) :  $MT \Rightarrow LB \in \{90 \text{ Days}\}$ . (2)

- Short-Term Estimator (STE) :  $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$ . (3)

Term-Structure for Covariance  $\Sigma \wedge$  Expected Return  $\mu$ .

## 1.5 Synthesis

Optimal portfolio behavior constructed from

covariance and expected return estimators

will vary due to strategic and market differences.

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (4)$$

Goal :

- Evaluate Hypothesis
- Demonstrate Robustness (Or Lack Thereof)

- Market Regime Stratification

## 2 Strategy

Theory \& Math

### 2.1 Strategic Formulation

Consider two strategies :

$$(\text{Strategy I}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (5)$$

and

$$(\text{Strategy II}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (6)$$

- $\Sigma \equiv$  covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$  Beta of security  $S_i$  (CAPM) s.t.  
 $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$  Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$  Tracking Error Volatility;  
 trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (7)$$

### 2.2 Fama–French Three-Factor Model (FF3FM)

Definition :

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (8)$$

$$\mathbb{E}[\epsilon_i] = 0; \therefore$$

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (9)$$

Estimated Coefficient Vector :

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\top \Leftarrow y_i = \rho_i - r_f \quad (10)$$

Linear Regression :

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \quad (11)$$

$\beta_i^m \neq \beta_i^3$  | estimated via separate regression / computed directly.

## 2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

### 2.3.1 Strategy I Breakdown

1. **Objective:** Maximize returns while considering risk.
2. **Constraints:**
  - The portfolio's beta must be between  $-0.5$  and  $0.5$ .
  - The sum of the weights assigned to each asset in the portfolio must equal 1.
  - Each individual weight can range from  $-2$  to  $2$ .

### 2.3.2 Strategy II Breakdown

1. **Objective:** Maximize returns relative to the portfolio's **tracking error volatility (TEV)**, which measures how much the portfolio's returns deviate from a benchmark.
2. **Constraints:**
  - The portfolio's beta must be between  $-2$  and  $2$ .
  - The sum of the weights assigned to each asset in the portfolio must equal 1.
  - Each individual weight can range from  $-2$  to  $2$ .

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

## 3 Assumptions and (Analysis) Setup

### 3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.
2. I define three cases:
  - Long-Term Look-Back Period : 120 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario LT  $\equiv S_{120}$ .
  - Medium-Term Look-Back Period : 90 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario MT  $\equiv S_{90}$ .
  - Short-Term Look-Back Period : 40 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario ST  $\equiv S_{40}$ .
3. Consider two possible values for the **Target Beta** (again, **not** the colloquial slang term) : 0 & 1.
4. Consider two possible values for the  $\lambda$  (the **risk aversion parameter**; i.e., how much are you putting on black?) : 0.10 & 0.50.