- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}})$ is the '**Tracking Error Volatility**', which (if you're *really nerdy*) you can derive it as such:

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\intercal \Sigma \omega - 2\omega^\intercal \text{Cov}(r, r_{\text{SPY}}) + \sigma^2_{\text{SPY}}} \tag{3}$$

Oh yeah, I should probably define what '**FF3FM**' means; that would (probably) be helpful.

2.2 Fama–French Three-Factor Model

So, to echo the previous sentiment, we should (almost surely) explain what is this *funky* model we kept referencing:

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{4}$$

Sorry for writing (or, to be *really technical*, typesetting) more hieroglyphics. We gotta keep going for a bit—stay with me!

If we assume our white noise/error terms, on 'average', have a (numerical) value of 0 (i.e., $\mathbb{E}[\epsilon_i] = 0$), we can derive a new goofy equation:

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{5}$$

In the new cursive script defined above, the 3 coefficients β_i^3, b_i^s, and b_i^v are estimated by making a linear regression, or, in 'plain English', drawing a line of best fit of the time series $y_i = \rho_i - r_f$ against the other cool time series $\rho_M - r_f$ (**Momentum Factor**), r_{SMB} (**Size Factor**), and ρ_{HML} (**Value Factor**).

I feel like I'm forgetting something . . .

Oh yeah! There's an extra (nerdy) thingy we gotta verify: (generally), $\beta_i^m \neq \beta_i^3$ and needs to be estimated by a separate regression or directly computed.

2.3 'Plain' English Formulation

Whew. Let's a take breather, shall we?

I get it; that was a *mouthful*, to say the least.

But, let's try and *digest* that in a slower, easier fashion.

Overall, we are exploring two *different investment strategies*, each with its own set of rules and objectives; let's dive right into them.

2.3.1 Strategy I Breakdown

1. **Objective**: Maximize returns while considering risk.
2. **Constraints**:

- The portfolio's beta (a measure of its *volatility* relative to the market; i.e., how *silly* and *spread out* it is relative to the 'market') must be between -0.5 and 0.5.
- The sum of the weights assigned to each asset in the portfolio must equal 1 (i.e., ***we gotta put our money to work!*** As such, let's buy a bunch of stuff that can make us money but, also, let's (try) not to violate the Laws of Probability Theory).
- Each individual weight can range from -2 to 2 (i.e., we can be like *certain individuals* from WallStreetBets and put all our eggs in one basket or, like a more prudent investor, do anything *but that*).

2.3.2 Strategy II Breakdown

1. **Objective**: Maximize returns relative to the portfolio's **tracking error volatility** (**TEV**), which measures how much the portfolio's returns deviate from a benchmark (e.g., the S&P 500 or 'big boy stock market').
2. **Constraints**:
   - The portfolio's beta (a measure of its *volatility* relative to the market; i.e., how *wild* and *crazy* it gets compared to the 'market') must be between -2 and 2.
   - The sum of the weights assigned to each asset in the portfolio must equal 1 (i.e., ***we need to make sure all our money is actively working!*** So, let's diversify our investments while still following the Laws of Probability Theory).
   - Each individual weight can range from -2 to 2 (i.e., we can either go *all in* on one asset like *those wild investors* on WallStreetBets, or spread our investments more wisely).