0 Amendment

$$\text{Response to } \mathbf{25\%;} \tag{-1}$$

$$\text{This is my corrective action and (my) letter (to you).} \tag{0}$$

1 Overview

1.1 Goal

$$\text{Objective: Build / Compare Two Factor-Based L/S Allocation Models} \tag{1}$$

$$\text{Beta } (\beta) \text{ Constraints} \tag{2}$$

$$\text{First Strategy } (S_{\{1\}}) : \text{Target Beta } \beta_T \in [-0.5, 0.5] \tag{3}$$

$$\text{Second Strategy } (S_{\{2\}}) : \text{Target Beta } \beta_T \in [-2, 2] \tag{4}$$

$$S_{\{1\}} \approxeq \text{Value-at-Risk Utility (Robust Optimization)} \tag{5}$$

$$S_{\{1\}} \Leftarrow \text{Information Ratio} \tag{6}$$

Post optimization, I compare model outcomes while evaluating estimator length se

$$[\text{covariance matrix } \mathbf{\Sigma} \wedge \text{expected returns } \boldsymbol{\mu}] \text{ across market regimes} \tag{8}$$

1.2 Reallocation

$$\text{Portfolio Allocation } \{P_t\} \Leftarrow \text{`03-01-2007`} \sim \text{`03-31-2024`} \tag{9}$$

$$P_t \quad \forall\, t \in \{t_0, t_1, t_2, \ldots, t_n\} \quad \text{where} \quad t_0 = \text{03-01-2007}, \quad t_n = \text{03-31-2024} \tag{10}$$

$$t_i = t_{i-1} + 7 \text{ days} \quad \text{for} \quad i = 1, 2, \ldots, n \tag{11}$$

$$\text{Investment Universe} \equiv \text{ETFs ('Global World Economy')} \tag{12}$$

$$\text{Fama–French Three-Factor Model (Momentum, Value, Size)} \tag{13}$$

$$\text{Public Data} \tag{14}$$

1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market en

$$\text{Low Beta} \Rightarrow \text{Decorrelation;} \tag{16}$$

$$\text{High Beta} \equiv \text{Antithesis.} \tag{17}$$

$$\text{Portfolio Characteristics :}$$

- $$\text{Return : } \mu \tag{18}$$
- $$\text{Volatility (Vol) : } \sigma \tag{19}$$
- $$\text{Skewness (Skew) : } \mathbb{E}\left[\left(\frac{x-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}} \tag{20}$$
- $$\text{Value at Risk (VaR) / Expected Shortfall (ES)} \tag{21}$$
- $$\text{Sharpe Ratio : } \frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}} \tag{22}$$

1.4 Simplification

$$\text{Look-Back } \boldsymbol{\mu} \text{ Estimators :}$$

- $$\text{Long-Term Estimator (LTE) : LT} \Rightarrow \text{LB} \in \{\text{180 Days}\} \tag{23}$$
- $$\text{Mid-Term Estimator (MTE) : MT} \Rightarrow \text{LB} \in \{\text{90 Days}\} \tag{24}$$
- $$\text{Short-Term Estimator (STE) : ST} \Rightarrow \text{LB} \in \{\text{40 Days}, \text{60 Days}\} \tag{25}$$

$$\text{Term-Structure for Covariance } \boldsymbol{\Sigma} \ \wedge \ \text{Expected Return } \boldsymbol{\mu}. \tag{26}$$

1.5 Synthesis

$$\text{Optimal portfolio behavior constructed from} \tag{27}$$

$$\text{covariance and expected return estimators} \tag{28}$$

$$\text{will vary due to strategic and market differences.} \tag{29}$$

$$S_{40}^{90} \equiv \hat{\boldsymbol{\Sigma}} \Rightarrow \text{40 Days} \ \wedge \ \hat{\boldsymbol{\mu}} \Rightarrow \text{90 Days} \tag{30}$$

$$\text{Objective :}$$

- $$\text{Evaluate Hypothesis} \tag{31}$$
- $$\text{Demonstrate Robustness (Or Lack Thereof)} \tag{32}$$
- $$\text{Market Regime Stratification} \tag{33}$$

2 Strategy

Theory \& Math

2.1 Strategic Formulation

Consider two strategies :

$$\text{(Strategy I)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{34}$$

and

$$\text{(Strategy II)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{35}$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t.
  $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}}) \equiv$ Tracking Error Volatility;
  trivial derivation (reader exercise) :

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\intercal \Sigma \omega - 2\omega^\intercal \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \tag{36}$$

2.2 Fama–French Three-Factor Model (FF3FM)

$$\text{Definition:} \tag{37}$$

$$r_i = r_f + \beta_i^3 (r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{38}$$

$\mathbb{E}[\epsilon_i] = 0; \therefore$

$$\rho_i = r_f + \beta_i^3 (\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{39}$$

$$\text{Estimated Coefficient Vector:} \tag{40}$$

$$(\hat{\beta}_i^3, \hat{b}_i^s, \hat{b}_i^v)^\intercal \Leftarrow y_i = \rho_i - r_f \tag{41}$$

$$\text{Linear Regression:} \tag{42}$$

$$= \hat{\beta}_i^3(\rho_M - r_f) + \hat{\beta}_i^s r_{\text{SMB}} + \hat{b}_i^v \rho_{\text{HML}} + \epsilon_i \tag{43}$$

$$\beta_i^m \neq \beta_i^3 \mid \text{estimated via separate regression / computed directly.} \tag{44}$$

2.3 Executive Summary Formulation

$$\text{Innumerate:} \tag{45}$$

$$\text{Strategy I} \tag{46}$$

$$\text{1. Objective} \equiv \text{Maximize Returns w/Risk.} \tag{47}$$

2. Constraints :

- The portfolio's beta must be between -0.5 and 0.5. (48)
- The sum of the weights assigned to each asset in the portfolio must equal 1.
- Each individual weight can range from -2 to 2. (50)

$$\text{Strategy II} \tag{51}$$

1. Objective $\equiv$ Maximize Returns Relative to Tracking Error Volatility (TEV).

2. Constraints :

- The portfolio's beta must be between -2 and 2. (53)
- The sum of the weights assigned to each asset in the portfolio must equal 1.
- Each individual weight can range from -2 to 2. (55)

3 Assumptions

3.1 Setup

1. Reallocation : `‘03-01-2007‘` $\sim$ `‘03-31-2024‘` (57)

$$2.\ \text{Input Construction :} \tag{58}$$

- $$\text{LT LB Period : } n_{\text{LT}} = 120 \ | \ \Sigma_s \ \wedge \ \mu_s \ | \ \text{LT} \equiv S_{120} \tag{59}$$
- $$\text{MT LB Period : } n_{\text{LT}} = 90 \ | \ \Sigma_s \ \wedge \ \mu_s \ | \ \text{MT} \equiv S_{90} \tag{60}$$
- $$\text{ST LB Period : } n_{\text{LT}} = 40 \ | \ \Sigma_s \ \wedge \ \mu_s \ | \ \text{MT} \equiv S_{40} \tag{61}$$

$$3.\ \boldsymbol{\beta}_T \in \{0, 1\} \tag{62}$$

$$4.\ \boldsymbol{\lambda} \in \{0.10, 0.50\} \tag{63}$$

3.2 Period Analysis

$$\text{Period Stratification:} \tag{64}$$

- $$\text{Period 1} \ \equiv \ \text{Pre-Subprime} \tag{65}$$
- $$\text{Period 2} \ \equiv \ \text{Subprime} \tag{66}$$
- $$\text{Period 3} \ \equiv \ \text{Post-Subprime} \tag{67}$$
- $$\text{Period 4} \ \equiv \ \text{COVID} \tag{68}$$
- $$\text{Period 5} \ \equiv \ \text{Post-Covid} \tag{69}$$

3.3 BackTesting

$$\text{Definition: Historical Data} \Rightarrow \text{Performance} \tag{70}$$

Logistical Considerations:

- $$\text{BackTest} \neq \ \text{Forecasts} \Rightarrow \textbf{Snooping Bias / P-Hacking} \tag{71}$$
- $$\text{Weekly Rebalance} \tag{72}$$
- $$\{t_i\}_{i=1}^{n} : \tag{73}$$

$$\begin{cases} \text{For the initial date } t_1, \text{ use the prior 60 days of historical data to estimate input} \\ \text{Store the portfolio weights: } \omega_{t_1}. \\ \text{For each subsequent date } t_{i+1}, \text{ roll the historical data window by 5 days, re-est} \\ \text{Store the new portfolio weights: } \omega_{t_{i+1}}. \\ \text{Repeat this process until the target date } t_n \text{ is reached.} \end{cases}$$

4 ToolKit|Arsenal

$$\text{Strat I} \Rightarrow \texttt{CVXPY} \mid \text{Strat II} \Rightarrow \texttt{Nonlinear Optimizer} \tag{75}$$

$$\text{Data (ETFs) } : \texttt{yfinance} \tag{76}$$

- $$\text{1. FXE} \tag{77}$$
- $$\text{2. EWJ} \tag{78}$$
- $$\text{3. GLD} \tag{79}$$
- $$\text{4. QQQ} \tag{80}$$
- $$\text{5. SPY} \tag{81}$$
- $$\text{6. SHV} \tag{82}$$
- $$\text{7. DBA} \tag{83}$$
- $$\text{8. USO} \tag{84}$$
- $$\text{9. XBI} \tag{85}$$
- $$\text{10. ILF} \tag{86}$$
- $$\text{11. EPP} \tag{87}$$
- $$\text{12. FEZ} \tag{88}$$

- $$\text{To Do}$$

1. **Task 1** : `download_data(start_date, end_date)`, `compute_daily_returns(...)`, `annualize_data(...)`.
2. **Task 2** : `factor_model(...)`.

3. **Task 3** : `optimize_model(...)` .

4. **Task 4** : `backtest(...)`

5. **Task 5** : `analyze(...)`

6. **Task 6** : `summarize(...)`