

FE630 - Final Project (Revision)

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Pledge: I pledge my honor that I have abided by the Stevens Honor System.

Professor: Papa Momar Ndiaye

0 Amendment

Response to **25%**.

This is my corrective action.

1 Overview

1.1 Goal

The objective is to build and compare two factor-based long short allocation models with constraints on their betas. The first strategy considers a target Beta in the interval $[-0.5, 0.5]$ while the second has one in the interval $[-2, +2]$.

The first operates similar to a Value-at-Risk Utility (Robust Optimization); the second incorporates an Information Ratio.

Post optimization, I compare the model outcomes while evaluating their sensitivity to estimator length for the covariance matrix and expected returns across market regimes.

1.2 Reallocation

The portfolios are reallocated weekly from **03-01-2007** to **03-31-2024**.

My investment universe \equiv ETFs ('Global World Economy').

I use the Fama–French Three-Factor Model (Momentum, Value, Size).

The data is publicly available.

1.3 Performance Evaluation

The performance / risk profiles are sensitive to the target Beta and the market environment.

A low Beta indicates decorrelation; a high Beta is the antithesis.

Portfolio Characteristics Definition:

- Return : μ
- Vol : σ
- Skew : $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma^3} = \frac{\kappa_3}{\kappa_2^{3/2}}$
- VaR / Expected Shortfall
- Sharpe : $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$

1.4 Simplification

Post Factor Model (FM) construction, I use trend following estimators for μ .

The estimator quality depends on the look-back (LB) period; \therefore

- Long-Term Estimator (LTE) : $LT \Rightarrow LB \in \{180 \text{ Days}\}$.
- Mid-Term Estimator (MTE) : $MT \Rightarrow LB \in \{90 \text{ Days}\}$.
- Short-Term Estimator (STE) : $ST \Rightarrow LB \in \{40 \text{ Days}, 60 \text{ Days}\}$.

I define **Term-Structure** for Covariance $\Sigma \wedge$ Expected Return μ .

1.5 Synthesis

Optimal portfolio behavior constructed from covariance and expected return estimators will vary due to strategic and market differences.

$$S_{40}^{90} \equiv \hat{\Sigma} \Rightarrow 40 \text{ Days} \wedge \hat{\mu} \Rightarrow 90 \text{ Days} \quad (1)$$

Goal :

- Evaluate Hypothesis
- Demonstrate Robustness (Or Lack Thereof)
- Market Regime Stratification

2 (Investment) Strategy

This section delves into the theoretical and mathematical formulations for the investment strategy employed; it is not for the faint of heart.

2.1 (Mathematical) Strategic Formulation

Consider the following two strategies:

$$(\text{Strategy I}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (1)$$

and

$$(\text{Strategy II}) \quad \begin{cases} \max_{\omega \in \mathbb{R}^n} \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \quad (2)$$

where we define the constructions above:

- Σ is the **covariance matrix** between the securities returns (as computed from the **FF3FM**);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)}$ is the **Beta** of some **security** S_i as defined by the **CAPM Model** such that $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i$ is the **Portfolio Beta**;
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}})$ is the '**Tracking Error Volatility**'; the derivation is trivial and left as an exercise to the reader:

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^T \Sigma \omega - 2\omega^T \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \quad (3)$$

2.2 Fama–French Three-Factor Model (FF3FM)

The **Fama-French Three-Factor Model** (FF3FM) is defined as follows:

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \quad (4)$$

Assume $\mathbb{E}[\epsilon_i] = 0$; therefore,

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \quad (5)$$

The 3 coefficients β_i^3 , b_i^s , and b_i^v are estimated by making a linear regression of the time series $y_i = \rho_i - r_f$ against the time series $\rho_M - r_f$ (**Momentum Factor**), r_{SMB} (**Size Factor**), and ρ_{HML} (**Value Factor**).

$\beta_i^m \neq \beta_i^3$ in tandem with the requirement to be estimated via a separate regression or directly computed.

2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

2.3.1 Strategy I Breakdown

1. **Objective:** Maximize returns while considering risk.
2. **Constraints:**
 - The portfolio's beta must be between -0.5 and 0.5 .
 - The sum of the weights assigned to each asset in the portfolio must equal 1.
 - Each individual weight can range from -2 to 2 .

2.3.2 Strategy II Breakdown

1. **Objective:** Maximize returns relative to the portfolio's **tracking error volatility (TEV)**, which measures how much the portfolio's returns deviate from a benchmark.
2. **Constraints:**
 - The portfolio's beta must be between -2 and 2 .
 - The sum of the weights assigned to each asset in the portfolio must equal 1.
 - Each individual weight can range from -2 to 2 .

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

3 Assumptions and (Analysis) Setup

3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.
2. I define three cases:
 - Long-Term Look-Back Period : 120 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario LT $\equiv S_{120}$.
 - Medium-Term Look-Back Period : 90 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario MT $\equiv S_{90}$.
 - Short-Term Look-Back Period : 40 Data Points for estimation of a Sample Covariance & Sample Mean; i.e., Scenario ST $\equiv S_{40}$.

3. Consider two possible values for the **Target Beta** (again, *not* the colloquial slang term) : 0 & 1.
4. Consider two possible values for the λ (the *risk aversion parameter*; i.e., how much are you putting on black?) : 0.10 & 0.50.