0 Amendment

Response to **25%;**

This is my corrective action.

1 Overview

1.1 Goal

The objective is to build and compare two factor-based long short allocation models with constraints on their betas.

The first strategy considers a target Beta in the interval $[-0.5, 0.5]$
while the second has one in the interval $[-2, +2]$.

The first operates similar to a Value-at-Risk Utility (Robust Optimization);

the second incorporates an Information Ratio.

Post optimization, I compare the model outcomes while evaluating their sensitivity to estimator length for the covariance matrix

and expected returns across market regimes.

1.2 Reallocation

The portfolios are reallocated weekly from ‘`03-01-2007`‘ to `03-31-2024`.

My investment universe ≡ ETFs (’Global World Economy’).

I use the Fama–French Three-Factor Model (Momentum, Value, Size).

The data is publicly available.

1.3 Performance Evaluation

The performance / risk profiles are sensitive

to the target Beta and the market environment.

A low Beta indicates decorrelation;

a high Beta is the antithesis.

Portfolio Characteristics Definition:

- Return : μ
- Vol : σ
- Skew : $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma_3} = \frac{\kappa_3}{\kappa_2^{3/2}}$
- VaR / Expected Shortfall
- Sharpe : $\frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$

1.4 Simplification

Post Factor Model (FM) construction,

I use trend following estimators for $\boldsymbol{\mu}$.

The estimator quality depends on the look-back (LB) period; $\therefore$

- Long-Term Estimator (LTE) : LT $\Rightarrow$ LB $\in \{180 \text{ Days}\}$.
- Mid-Term Estimator (MTE) : MT $\Rightarrow$ LB $\in \{90 \text{ Days}\}$.
- Short-Term Estimator (STE) : ST $\Rightarrow$ LB $\in \{40 \text{ Days}, 60 \text{ Days}\}$.

I define Term-Structure for Covariance $\boldsymbol{\Sigma}$ $\wedge$ Expected Return $\boldsymbol{\mu}$.

1.5 Synthesis

Optimal portfolio behavior constructed from covariance and expected return estimators will vary due to strategic and market differences.

$$S_{40}^{90} \equiv \hat{\boldsymbol{\Sigma}} \Rightarrow 40 \text{ Days} \wedge \hat{\boldsymbol{\mu}} \Rightarrow 90 \text{ Days} \tag{1}$$

Goal :

- Evaluate Hypothesis
- Demonstrate Robustness (Or Lack Thereof)
- Market Regime Stratification

2 (Investment) Strategy

Theory \& Math

2.1 (Mathematical) Strategic Formulation

Consider two strategies:

$$
\text{(Strategy I)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \rho^T\omega - \lambda\sqrt{\omega^T\Sigma\omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{1}
$$

and

$$
\text{(Strategy II)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \frac{\rho^T\omega}{\text{TEV}(\omega)} - \lambda\sqrt{\omega^T\Sigma\omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{2}
$$

- $\Sigma \equiv$ covariance matrix between security returns (FF3FM).
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)} \equiv$ Beta of security S_i (CAPM) s.t. $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i \equiv$ Porfolio Beta
- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}})$ is the '**Tracking Error Volatility**'; the derivation is trivial and left as an exercise to the reader:

$$
\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\intercal\Sigma\omega - 2\omega^\intercal \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2}. \tag{3}
$$

2.2 Fama–French Three-Factor Model (FF3FM)

The Fama-French Three-Factor Model (FF3FM) is defined as follows:

$$
r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{4}
$$

Assume $\mathbb{E}[\epsilon_i] = 0$; therefore,

$$
\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{5}
$$

The 3 coefficients β_i^3, b_i^s, and b_i^v are estimated by making a linear regression of the time series $y_i = \rho_i - r_f$ against the time series $\rho_M - r_f$ (**Momentum Factor**), r_{SMB} (**Size Factor**), and ρ_{HML} (**Value Factor**).

$\beta_i^m \neq \beta_i^3$ in tandem with the requirement to be estimated via a separate regression or directly computed.

2.3 Executive Summary Formulation

This section elaborates on the mathematical formulation established in Sections 2.1 but for executives (innumerate):

2.3.1 Strategy I Breakdown

1. **Objective**: Maximize returns while considering risk.

2. **Constraints**:

   - The portfolio's beta must be between -0.5 and 0.5.
   - The sum of the weights assigned to each asset in the portfolio must equal 1.
   - Each individual weight can range from -2 to 2.

2.3.2 Strategy II Breakdown

1. **Objective**: Maximize returns relative to the portfolio's **tracking error volatility** (**TEV**), which measures how much the portfolio's returns deviate from a benchmark.

2. **Constraints**:

   - The portfolio's beta must be between -2 and 2.
   - The sum of the weights assigned to each asset in the portfolio must equal 1.
   - Each individual weight can range from -2 to 2.

The next section establishes the necessary assumptions considered for strategic formulation and implementation.

3 Assumptions and (Analysis) Setup

3.1 Setup

To simplify, we will make the following assumptions for this experiment:

1. The portfolios will be reallocated weekly from the beginning of **March 2007** to the end of **March 2024**.

2. I define three cases:

   - $\text{Long-Term Look-Back Period}: 120\,\text{Data Points}$ for estimation of a $\text{Sample Covariance \& Sample Mean}$; i.e., $\text{Scenario LT} \equiv S_{120}$.
   - $\text{Medium-Term Look-Back Period}: 90\,\text{Data Points}$ for estimation of a $\text{Sample Covariance \& Sample Mean}$; i.e., $\text{Scenario MT} \equiv S_{90}$.
   - $\text{Short-Term Look-Back Period}: 40\,\text{Data Points}$ for estimation of a $\text{Sample Covariance \& Sample Mean}$; i.e., $\text{Scenario ST} \equiv S_{40}$.
3. Consider two possible values for the **Target Beta** (again, ***not*** the colloquial slang term) $: 0 \ \& \ 1$.

4. Consider two possible values for the $\boldsymbol{\lambda}$ (the ***risk aversion parameter***; i.e., how much are you putting on black?) : 0.10 & 0.50.