FE630 - Final Project

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Pledge: I pledge my honor that I have abided by the Stevens Honor System.

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1. Overview

1.1 Goal

The goal of this project to build and compare *two factor-based long short allocation models* with constraints on their *betas*. The first strategy considers a **target Beta** in the interval $[-0.5, 0.5]$, while the second has a target Beta in the interval $[-2, +2]$.

The first strategy operates similar to a **Value-at-Risk Utility** corresponding to **Robust Optimization**; the second strategy incorporates an **Information Ratio** term to limit the deviations from a benchmark, provided those deviations yield a 'high return.'

Once the optimization models are built, we want to *compare* the outcomes of the two models while simultaneously evaluating their sensitivity to the *length* of the estimators for the **covariance matrix** in tandem with the **expected returns** under various market regimes/scenarios.

1.2 Reallocation

The portfolios will be *reallocated* or, in other words, 'reoptimized' weekly from the beginning of **March 2007** to the end of **March 2024**. Our *investment universe* encompasses a set of exchange-traded funds (**ETFs**) which is large enough to represent the '**Global World Economy**' (as according to some).

We will utilize the Fama–French Three-Factor Model which incorporates the following factors:

- Momentum
- Value
- Size.

Regarding data accessability, these factors have historical values available for ***free*** from ***Ken French's*** personal website in tandem with Yahoo Finance.

1.3 Performance Evaluation

Naturally, the performance as well as the risk profiles of the aforementioned strategies may be (relatively) sensitive to the *target Beta* and the (current) market environment.

For example, a '**low Beta**' (essentially) means that a strategy is created with the objective or aim to be '**decorrelated**' (no linear relationship between entites) with the 'Global Market,' which, in our case, is represented by the **S&P 500** (i.e., no *systematic relationship*).

A '**high Beta**' is simply the antithesis, or opposite, of what we just discussed. In layman's terms, we have a (higher) appetite for '*risk*' (in this case, let's keep it simple and define our premise as σ or **standard deviation**) and desire to ride or 'scale up' the *market risk* (**systematic risk**).

Moreover, it's imperative that one acknowledges that such a (described) strategy is more probable to be (quite) sensitive to the *estimators* used for the **Risk Model** and the **Alpha Model** (e.g., the length of the *look-back period* utilized); therefore, it is necessary to understand and, most importantly, *comprehend* the impact of said estimators on the **Portfolio's** characteristics:

- (Realized) Return : μ_h
- (Historical) Volatility) : σ_h
- Skewness : $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma_3} = \frac{\kappa_3}{\kappa_2^{3/2}}$
- VaR / Expected Shortfall
- Sharpe Ratio : $S_a = \frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$

1.4 Simplification

To make it easier, we assume that once the **Factor Model** (FM) has been constructed, we will use trend following estimators for the **Expected Returns**. Since the quality of the estimators depend on the **look-back period**, we define three cases:

- ***Long-Term Estimator (LTE)*** : $\mathrm{LT} \Rightarrow \mathrm{LB} \in \{180 \text{ Days}\}$.
- ***Mid-Term Estimator (MTE)*** : $\mathrm{MT} \Rightarrow \mathrm{LB} \in \{90 \text{ Days}\}$.
- ***Short-Term Estimator (STE)*** : $\mathrm{ST} \Rightarrow \mathrm{LB} \in \{40 \text{ Days}, 60 \text{ Days}\}$.

Specifically, we define a **Term-Structure** for the $\text{Covariance } \mathbf{\Sigma}$ and $\text{Expected Return } \boldsymbol{\mu}$.

1.5 Synthesis

To (briefly) summarize, the behavior of a (potential) '*optimal*' portfolio built from a melting pot of *estimators* for **Covariance** and **Expected Return** may vary according to the cadence of the '**Market**' (environment/regime) or an aforementioned strategy.

For example, the (mathematical) notation S_{40}^{90} is just fancy jargon to visually illustrate that we are using **40 days** for the covariance estimation and **90 days** for the expected returns estimations—it's not that deep.

Overall, the goal of this fun, entertaining project is to conceptualize, visualize, understand, analyze, and compare the behavior of our ideas; we want to *see* if we can (actually) make some $\mathbf{\$\$\$}$, especially during momentous, historical (time) periods such as the **Subprime Mortgage Crisis** of *2008*, the horrendous commencement of **Coronavirus SARS-CoV-2 Disease** of *2019*, et cetera.

2. (Investment) Strategy

Alrighty, let's get to the fun, juicy portion; shall we?

2.1 (Mathematical) Strategic Formulation

Let's make things interesting—spicy, one may say.

Consider two strats [(clipping) of 'strategies,' as embodied in *Morphology*)]:

$$\text{(Strategy I)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \; \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^{n} \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^{n} \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{1}$$

and

$$\text{(Strategy II)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \; \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^{n} \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^{n} \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{2}$$

where we define the hieroglyphics used above:

- Σ is the covariance matrix between the securities returns (as computed from the **FF3FM**);
- $\beta_i^m = \frac{\text{Cov}(r_i, r_M)}{\sigma^2(r_M)}$ is the Beta) (not to be confused with the colloquial slang usage) of some security) S_i as defined by the CAPM Model such that $\beta_P^m = \sum_{i=1}^{n} \beta_i^m \omega_i$ is the **Portfolio Beta**;

- $\text{TEV}(\omega) = \sigma(r_P(\omega) - r_{\text{SPY}})$ is the '**Tracking Error Volatility**', which (if you're *really nerdy*) you can derive it as such:

$$\sigma(r_P(\omega) - r_{\text{SPY}}) = \sqrt{\omega^\intercal \Sigma \omega - 2\omega^\intercal \text{Cov}(r, r_{\text{SPY}}) + \sigma_{\text{SPY}}^2} \tag{3}$$

Oh yeah, I should probably define what '**FF3FM**' means; that would (probably) be helpful.

2.2 Fama–French Three-Factor Model

So, to echo the previous sentiment, we should (almost surely) explain what is this *funky* model we kept referencing:

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\text{SMB}} + b_i^v r_{\text{HML}} + \alpha_i + \epsilon_i \tag{4}$$

Sorry for writing (or, to be *really technical*, typesetting) more hieroglyphics. We gotta keep going for a bit—stay with me!

If we assume our white noise/error terms, on 'average', have a (numerical) value of 0 (i.e., $\mathbb{E}[\epsilon_i] = 0$), we can derive a new goofy equation:

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\text{SMB}} + b_i^v \rho_{\text{HML}} + \alpha_i \tag{5}$$

In the new cursive script defined above, the 3 coefficients β_i^3, b_i^s, and b_i^v are estimated by making a linear regression, or, in 'plain English', drawing a line of best fit of the time series $y_i = \rho_i - r_f$ against the other cool time series $\rho_M - r_f$ (**Momentum Factor**), r_{SMB} (**Size Factor**), and ρ_{HML} (**Value Factor**).

I feel like I'm forgetting something ...

Oh yeah! There's an extra (nerdy) thingy we gotta verify: (generally), $\beta_i^m \neq \beta_i^3$ and needs to be estimated by a separate regression or directly computed.

2.3 'Plain' English Formulation

Whew. Let's a take breather, shall we?

I get it; that was a *mouthful*, to say the least.

But, let's try and *digest* that in a slower, easier fashion.

Overall, we are exploring two *different investment strategies*, each with its own set of rules and objectives; let's dive right into them.

2.3.1 Strategy I Breakdown

1. **Objective**: Maximize returns while considering risk.
2. **Constraints**:

- The portfolio's beta (a measure of its *volatility* relative to the market; i.e., how *silly* and *spread out* it is relative to the 'market') must be between -0.5 and 0.5.
- The sum of the weights assigned to each asset in the portfolio must equal 1 (i.e., ***we gotta put our money to work!*** As such, let's buy a bunch of stuff that can make us money but, also, let's (try) not to violate the Laws of Probability Theory).
- Each individual weight can range from -2 to 2 (i.e., we can be like *certain individuals* from WallStreetBets and put all our eggs in one basket or, like a more prudent investor, do anything *but that*).

2.3.2 Strategy II Breakdown

1. **Objective**: Maximize returns relative to the portfolio's **tracking error volatility** (**TEV**), which measures how much the portfolio's returns deviate from a benchmark (e.g., the S&P 500 or 'big boy stock market').
2. **Constraints**:
   - The portfolio's beta (a measure of its *volatility* relative to the market; i.e., how *wild* and *crazy* it gets compared to the 'market') must be between -2 and 2.
   - The sum of the weights assigned to each asset in the portfolio must equal 1 (i.e., ***we need to make sure all our money is actively working!*** So, let's diversify our investments while still following the Laws of Probability Theory).
   - Each individual weight can range from -2 to 2 (i.e., we can either go *all in* on one asset like *those wild investors* on WallStreetBets, or spread our investments more wisely).