FE630 - Final Project (Revision)

Author: Sid Bhatia

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Pledge: I pledge my honor that I have abided by the Stevens Honor System.

Professor: Papa Momar Ndiaye

0 Disclaimer

This work is in response to the original (numerical) grade received as a $\mathbf{25\%}$ due to my imprudence and deviation from the requested prompt.

I (profusely) apologize for my incompetence; I will take corrective action.

1 Overview

1.1 Goal

The goal of this project to build and compare *two factor-based long short allocation models* with constraints on their *betas*. The first strategy considers a **target Beta** in the interval $[-0.5, 0.5]$, while the second has a target Beta in the interval $[-2, +2]$.

The first strategy operates similar to a **Value-at-Risk Utility** corresponding to **Robust Optimization**; the second strategy incorporates an **Information Ratio** term to limit the deviations from a benchmark, provided those deviations yield a 'high return.'

Once the optimization models are built, we want to *compare* the outcomes of the two models while simultaneously evaluating their sensitivity to the *length* of the estimators for the **covariance matrix** in tandem with the **expected returns** under various market regimes/scenarios.

1.2 Reallocation

The portfolios will be *reallocated* or, in other words, 'reoptimized' weekly from the beginning of **March 2007** to the end of **March 2024**. Our *investment universe* encompasses a set of exchange-traded funds (**ETFs**) which is large enough to represent the '**Global World Economy**.'

We will utilize the Fama–French Three-Factor Model which incorporates the following factors:

- Momentum
- Value
- Size.

Regarding data accessability, these factors have historical values available for ***free*** from ***Ken French's*** personal website in tandem with Yahoo Finance.

1.3 Performance Evaluation

Naturally, the performance as well as the risk profiles of the aforementioned strategies may be (relatively) sensitive to the *target Beta* and the (current) market environment.

For example, a '**low Beta**' (essentially) means that a strategy is created with the objective or aim to be '**decorrelated**' (no linear relationship between entites) with the 'Global Market,' which, in our case, is represented by the **S&P 500** (i.e., no *systematic relationship*).

A '**high Beta**' is simply the antithesis, or opposite, of what we just discussed. In layman's terms, we have a (higher) appetite for '*risk*' (in this case, let's keep it simple and define our premise as σ or **standard deviation**) and desire to ride or 'scale up' the *market risk* (**systematic risk**).

Moreover, it's imperative that one acknowledges that such a (described) strategy is more probable to be (quite) sensitive to the *estimators* used for the **Risk Model** and the **Alpha Model** (e.g., the length of the *look-back period* utilized); therefore, it is necessary to understand and, most importantly, *comprehend* the impact of said estimators on the **Portfolio's** characteristics:

- (Realized) Return : μ_h
- (Historical) Volatility) : σ_h
- Skewness : $(\mathbb{E}[(\frac{x-\mu}{\sigma})^3]) = \frac{\mu_3}{\sigma_3} = \frac{\kappa_3}{\kappa_2^{3/2}}$
- VaR / Expected Shortfall
- Sharpe Ratio : $S_a = \frac{\mathbb{E}[R_a - R_b]}{\sigma_a} = \frac{\mathbb{E}[R_a - R_b]}{\sqrt{\mathbb{V}(R_a - R_b)}}$

1.4 Simplification

To make it easier, we assume that once the **Factor Model** (FM) has been constructed, we will use trend following estimators for the **Expected Returns**. Since the quality of the estimators depend on the **look-back period**, we define three cases:

- ***Long-Term Estimator (LTE)*** : $\mathrm{LT} \Rightarrow \mathrm{LB} \in \{180 \text{ Days}\}$.
- ***Mid-Term Estimator (MTE)*** : $\mathrm{MT} \Rightarrow \mathrm{LB} \in \{90 \text{ Days}\}$.
- ***Short-Term Estimator (STE)*** : $\mathrm{ST} \Rightarrow \mathrm{LB} \in \{40 \text{ Days}, 60 \text{ Days}\}$.

Specifically, we define a **Term-Structure** for the $\text{Covariance } \mathbf{\Sigma}$ and $\text{Expected Return } \boldsymbol{\mu}$.

1.5 Synthesis

To (briefly) summarize, the behavior of a (potential) '*optimal*' portfolio built from a melting pot of *estimators* for **Covariance** and **Expected Return** may vary according to the cadence of the '**Market**' (environment/regime) or an aforementioned strategy.

For example, the (mathematical) notation S_{40}^{90} is implemented to illustrate that we are using **40 days** for the covariance estimation and **90 days** for the expected returns estimations.

Overall, the goal of this experiment is to conceptualize, visualize, understand, analyze, and compare the behavior of our hypotheses; we want to see if we can deliver robust risk-adjusted performance, especially during momentous, historical periods such as the **Subprime Mortgage Crisis** of *2008*, the horrendous commencement of **Coronavirus SARS-CoV-2 Disease** of *2019*, et cetera.

2 (Investment) Strategy

This section delves into the theoretical and mathematical formulations for the investment strategy employed; it is not for the faint of heart.

2.1 (Mathematical) Strategic Formulation

Consider the following two strategies:

$$\text{(Strategy I)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \rho^T \omega - \lambda \sqrt{\omega^T \Sigma \omega} \\ -0.5 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 0.5 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{1}$$

and

$$\text{(Strategy II)} \quad \begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \ \frac{\rho^T \omega}{\text{TEV}(\omega)} - \lambda \sqrt{\omega^T \Sigma \omega} \\ -2 \leq \sum_{i=1}^n \beta_i^m \omega_i \leq 2 \\ \sum_{i=1}^n \omega_i = 1, \quad -2 \leq \omega_i \leq 2, \end{cases} \tag{2}$$

where we define the constructions above:

- Σ is the covariance matrix between the securities returns (as computed from the **FF3FM**);

- $\beta_i^m = \frac{\mathrm{Cov}(r_i, r_M)}{\sigma^2(r_M)}$ is the Beta) of some security) S_i as defined by the CAPM Model such that $\beta_P^m = \sum_{i=1}^n \beta_i^m \omega_i$ is the **Portfolio Beta**;
- $\mathrm{TEV}(\omega) = \sigma(r_P(\omega) - r_{\mathrm{SPY}})$ is the '**Tracking Error Volatility**'; the derivation is trivial and left as an exercise to the reader:

$$\sigma(r_P(\omega) - r_{\mathrm{SPY}}) = \sqrt{\omega^\intercal \Sigma \omega - 2\omega^\intercal \mathrm{Cov}(r, r_{\mathrm{SPY}}) + \sigma_{\mathrm{SPY}}^2}. \tag{3}$$

2.2 Fama–French Three-Factor Model (FF3FM)

The Fama-French Three-Factor Model (FF3FM) is defined as follows:

$$r_i = r_f + \beta_i^3(r_M - r_f) + b_i^s r_{\mathrm{SMB}} + b_i^v r_{\mathrm{HML}} + \alpha_i + \epsilon_i \tag{4}$$

Assume $\mathbb{E}[\epsilon_i] = 0$; therefore,

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{\mathrm{SMB}} + b_i^v \rho_{\mathrm{HML}} + \alpha_i \tag{5}$$

The 3 coefficients β_i^3, b_i^s, and b_i^v are estimated by making a linear regression of the time series $y_i = \rho_i - r_f$ against the time series $\rho_M - r_f$ (**Momentum Factor**), r_{SMB} (**Size Factor**), and ρ_{HML} (**Value Factor**).

$\beta_i^m \neq \beta_i^3$ in tandem with the requirement to be estimated via a separate regression or directly computed.