FE630 - Homework #2

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Pledge: I pledge my honor that I have abided by the Stevens Honor System.

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Topics

- Algebra & Optimization;
- Geometry of Efficient Frontiers;
- Applications of One-Fund & Two-Fund Theorems.

P1 - Optimization w/Equality Constraints (40 pts)

Consider the optimization problem **Max Expected Return w/Target Risk**:

$$\begin{cases} \max\limits_{\omega_1,\omega_2} & R_p(\omega_1,\omega_2) = \mu_1\omega_1 + \mu_2\omega_2 \\ \text{s.t.} & \sqrt{\sigma_1^2\omega_1^2 + 2\rho_{1,2}\sigma_1\sigma_2\omega_1\omega_2 + \sigma_2^2\omega_2^2} = \sigma_T \\ & \omega_1 + \omega_2 = 1 \end{cases} \tag{1}$$

where we have two securities with **Expected Returns** μ_1 and μ_2 for the column vector $(\mu_1, \mu_2)^\intercal \in \mathbb{R}^{2\times 1}$, **volatilities** $(\sigma_1, \sigma_2) \in \mathbb{R}^+$, and **Pearson correlation coefficient** $\rho_{1,2} \in [-1, 1]$. Additionally, $\sigma_T \in \mathbb{R}^+$ denotes the **target risk/vol**.

1. Solve the *problem (3)* using a **Lagrangian approach**. You will denote the solution (the ***optimal solution***) by $\omega^*(\sigma_T)$ and the ***optimal value*** of the problem by $R_p(\omega_1^*(\sigma_T), \omega_2^*(\sigma_T))$ by $R_p(\sigma_T)$.

2. Assume that $\mu_1 = 5\%$, $\mu_2 = 10\%$, $\sigma_1 = 10\%$, $\sigma_2 = 20\%$, and $\rho_{1,2} = -0.5$ (moderate negative correlation).

- Consider a sequence of successive values of σ_T in the range $[2\%, 30\%]$ by step of 0.5%
- Plot the efficient frontier: namely, the graph from the *mapping* $\sigma_T \mapsto R_p(\sigma_T)$.

The (aforementioned) graph maps the sequence of values of σ_T from the x-axis into the sequence of values $R_p(\sigma_T)$ on the y-axis.