Celebration of Learning #3: A chance to show me what you know!

- You may use a calculator, graphing calculator, and/or Desmos on this exam. You may not use any outside help (from other people, the internet, AI, textbooks, etc.).
- You do not have to complete the exam in one sitting.
- All of your work should be on lined paper.
- Number each problem and clearly indicate your answer (for instance, box-in your answers). Cross out any work you don't want graded.
- You should not talk to others about these problems. This work should be solely yours. Any evidence otherwise (i.e., the work is found on the internet, it is discovered you worked with someone, etc) will result in a zero score and the reporting of the academic integrity violation.
- Due under my door by noon on Monday, December 9th.

Good Luck!!!

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1. (*10 points*) Calculate the determinant of the following matrix by expanding upon the 2nd row of the matrix:

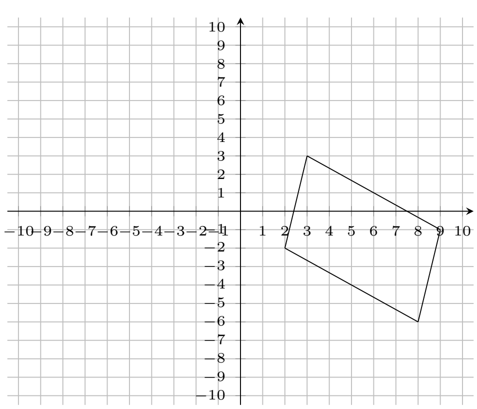
$$A = \begin{bmatrix} 1 & -4 & 2 & 3 \\ 3 & 0 & -2 & 0 \\ 1 & 2 & 3 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}$$

2. (*10 points*) If A is a matrix with $\det(A) = 4$, determine (if possible) $\det((A^{\intercal})^{-1})$. Make sure to justify your answer.

3. (*10 points*) Let A be a 3×3 matrix with $|A| = -2$. Determine $|B|$ if B is formed from A by performing the following sequence of row operations:

   - R1 $\leftrightarrow$ R3
   - -3R1+R2 $\rightarrow$ R2
   - $\frac{1}{4}$R3
   - 2R3+R1 $\rightarrow$ R1
   - 3R3+R2 $\rightarrow$ R2

4. (*10 points*) Use a determinant to find the area of the following parallelogram:



5. (*15 points*) Use Cramer's rule to find the value of z in a solution to

$$\begin{array}{rcrcrcrcr} -x & + & 3y & & & + & 4w & = & -2 \\ & & -2y & + & z & + & 2w & = & -1 \\ & & 4y & + & z & - & w & = & -1 \\ x & + & 2y & & & - & 7w & = & 1 \end{array}$$

6. (*15 points* each) For each matrix, find the real eigenvalues, along with a basis for each corresponding eigenspace.

   (a) $A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

   (b) $B = \begin{bmatrix} -5 & -5 & -9 \\ 8 & 9 & 18 \\ -2 & -3 & -7 \end{bmatrix}$

7. (*15 points*) Determine a matrix A that has eigenvalues $\lambda = 1$ and $\lambda = -2$, where a basis for the eigenspace when $\lambda = 1$ is $\left\{ \left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right] \right\}$ and a basis for the eigenspace when $\lambda = -2$ is $\left\{ \left[\begin{smallmatrix} 0 \\ 3 \end{smallmatrix}\right] \right\}$.