Problem 1.1:

From the question we have, for every $a \in A$, $b \in B$ and $c \in C$

a) $a \subseteq \gamma_1(b)$ iff $\alpha_1(a) \leq b$
b) $b \subseteq \gamma_2(c)$ iff $\alpha_2(b) \leq c$

We have to prove, $a \subseteq \gamma_1\gamma_2(c)$ iff $\alpha_2\alpha_1(a) \leq c$

From the contractive property we get,
$\alpha_2\gamma_2(c) \leq c$
$\alpha_2(b) \leq c$ [As $b \subseteq \gamma_2(c)$]
$\alpha_2\alpha_1(a) \leq c$ [As $\alpha_1(a) \leq b$]

And from the expansive property we get,
$a \subseteq \gamma_1\alpha_1(a)$
$a \subseteq \gamma_1(b)$ [As $\alpha_1(a) \leq b$]
$a \subseteq \gamma_1\gamma_2(c)$ [As $b \subseteq \gamma_2(c)$]

So, $\alpha_2\alpha_1$ and $\gamma_1\gamma_2$ form a Galois connection

Problem 1.2:

From Galois connection property we know, for every $a \in A$ and every $c \in C$
$c \subseteq \gamma(a)$ iff $\alpha(c) \leq a$

To prove α is monotone, we need to prove that for any c1, c2 if $c1 \subseteq c2$ then $\alpha(c1) \leq \alpha(c2)$.
From the expansive property we get,
$c2 \subseteq \gamma(\alpha(c2))$
So, $c1 \subseteq \gamma(\alpha(c2))$ [As $c1 \subseteq c2$]
So, $\alpha(c1) \leq \alpha(c2)$ [From Galois connection property]

Problem 1.3:

From Galois connection property we know, for every $a \in A$ and every $c \in C$

$c \subseteq \gamma(a)$ iff $\alpha(c) \leq a$

To prove γ is monotone, we need to prove that for any a1, a2 if $a1 \leq a2$ then $\gamma(a1) \subseteq \gamma(a2)$.

From the contractive property we get,

$\alpha(\gamma(a1)) \leq a1$

So, $\alpha(\gamma(a1)) \leq a2$ [As $a1 \leq a2$]

So, $\gamma(a1) \subseteq \gamma(a2)$ [From Galois connection property]

Problem 2.2:

$(\lambda x.\lambda y.x)(\lambda z.(\lambda x.\lambda y.x)\ z\ ((\lambda x.z\ x)(\lambda x.z\ x)))$

Normal Order:

$\rightarrow \lambda y.(\lambda z.(\lambda x.\lambda y.x)\ z\ ((\lambda x.z\ x)(\lambda x.z\ x))$

$\rightarrow \lambda y.((\lambda x.\lambda y.x)\ ((\lambda x.z\ x)(\lambda x.z\ x)))$

Apllicative Order:

$\rightarrow (\lambda x.\lambda y.x)((\lambda x.\lambda y.x)\ \ ((\lambda x.z\ x)(\lambda x.z\ x)))$

$\rightarrow (\lambda x.\lambda y.x)((\lambda x.\lambda y.x)\ \ (z\ (\lambda x.z\ x)))$

Problem 2.3:

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interpret(x) = x

interpret(λx.E1) = let E2 = interpret(E1)
                   In case E2 of
                   x = λx.E2
                    _ = interpret(E2)

interpret(E1 E2) = let E3 = interpret(E1)
                    In case E3 of
                    λx.E4 = interpret(E4[E2/x])
                    x = E3 interpret(E2)
                    _ = interpret(E3 E2)
```