Term project 2: Modelling a Stellar Core

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Abstract

In this report, this, that and more of this is discussed.

I Introduction

When trying to understand the mechanics of a star the most logical place to start is with what you can observe. Which properties of the star can we measure rather easily?

Well, we can look at how much radiation energy hits one particular spot on the earth, and by assuming that the star radiates equally in all directions we can calculate the total amount of energy sent out by the star, the so-called luminosity (denoted as L).

In addition, we can find the surface temperature (denoted T) by looking at the color of the light emitted from the sun and using Wien's displacements law.

Lastly, we might be able to look at the orbits of all the planets and do some number crunching and figure out the total mass of the star (denoted M).

But that's about it for what we can figure out by just looking at the outside effects. There are many other properties we would like to know about, e.g. the radius (R), density (ρ) and pressure (P). In particular, we would like to have all of these properties as a function of the radius, so that we can get an idea of what the cross section of the star looks like. In order to achieve this, we need to deploy many different branches of physics as well as a good number of assumptions to simplify things for us.

II The Governing Equations

In this project, we will only be looking at the radiative core of the star. This means that we will not look at the outer convection layer, and assume that all energy transport is done through photon radiation. For reference, this corresponds to the inner $\sim 70\,\%$ of the sun. Furthermore, we will not consider time evolution; we look at one particular moment in time.

The following four differential equations govern the internal structure of radiative core of our star[1]:

[1] Derivations of the equations are not given, as they are shown in the lecture notes [1]. A qualitative description of their meaning is given instead.

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \tag{1}$$
$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \tag{2}$$
$$\frac{\partial L}{\partial m} = \epsilon \tag{3}$$
$$\frac{\partial T}{\partial m} = -\frac{3\kappa L}{256\pi^2 \sigma r^4 T^3} \tag{4}$$

The first thing to notice about these equations are that they are differentials with respect to mass, and not radius. Remember that what we want in the end is all the properties of the star as a function of radius, so it would make sense to write the equations with respect to r instead[2]. It turns out that it is more numerically stable to use dm compared to dr, so we are going to treat the radius as $r = r(m)$. When we plot the data later, however, we will return to using r as the variable. Eq. (1) is simply stating the relation between taking an infinitesimal step in r, compared to an infinitesimal step in m.

Eq. (2) is the assumption of hydrostatic equilibrium, and states that if the gas in the star is to be at rest, then the outward pressure must exactly balance the force of gravity acting on the gas.

The ϵ in Eq. (3) represents the amount of energy produced by nuclear fusion per time and mass. This quantity will be treated more in a later section (ref??).

Eq. (4) is the temperature gradient need to transport all the produced energy out of the sun. I should be noted that this is only the correct temperature gradient when all the energy is transported by radiation.

References

[1] B. V. Gudiksen. *AST3310: Astrophysical plasma and stellar interiors.* 2016.

[2] I will use lower case r and m when referring to the radius and mass as variables, and upper case R and M for the total radius and mass.