

M05: Buffer Manager and Storage Hierarchy

1. Importance of Buffer Manager (BM)

- The Buffer Manager (BM) is a crucial component in database systems responsible for caching frequently accessed data pages in DRAM (Dynamic Random-Access Memory)¹².
- Its primary role is to bridge the significant speed gap between faster, volatile main memory (DRAM) and slower, persistent secondary storage (disks/SSDs)¹³.
- DRAM offers much faster access times (50 nanoseconds) and is more expensive per GB than Solid State Disks (SSDs), which have access times around 50,000 nanoseconds but are significantly cheaper¹⁴. Older magnetic disks have even slower access times (5-10 milliseconds)⁵⁶.

2. Buffer Manager Components and Operations (BuzzDB Example)

- **Core Structure** : The `BufferManager` class typically includes a `StorageManager` instance for low-level file I/O, an `std::list (lruList)` to maintain pages in Least Recently Used (LRU) order, and an `std::unordered_map (pageMap)` for quick lookup of pages by their `PageID` ¹⁷.
- ```
**std::list<std::pair<PageID, std::unique_ptr<SlottedPage>>>
lruList** 1 :
```
- This C++ container stores pages that are currently loaded into memory.
- It uses `std::pair` to associate a `PageID` with the actual page data, which is managed by a `std::unique_ptr<SlottedPage>` . The `unique_ptr` ensures proper ownership and automatic memory deallocation when a page is removed from the list or goes out of scope<sup>1</sup>.
- The `std::list` is chosen because its `splice` method allows efficient moving of elements (pages) to the front or back without reallocating other elements, crucial for LRU policy updates<sup>8</sup>.
- ```
**std::unordered_map<PageID, typename PageList::iterator>  
pageMap** 1 :
```
- This map provides $O(1)$ average time complexity for locating a page's position within the `lruList` using its `PageID` ⁷.
- This is essential for quickly updating a page's recency (moving it to the front of `lruList`) when it's accessed (`touch` operation)⁷.
- ```
getPage(int page_id) ** method** 7 :
```
- When a request comes for a `page_id` :
- It first checks `pageMap` to see if the page is already in the cache ( `pageMap.find(page_id)` ).
- If not found (cache miss), it requests the page from the `StorageManager` (which loads

it from disk)7.

- The page is then added to `lruList` and `pageMap`.
- The `touch()` method is called to update the page's position to the most recently used.
- If the cache is full, an eviction policy is triggered (e.g., `evict()` method)9.
- **`touch(PageList::iterator it)` \*\* \*\* method \*\* 8 :**
- This method updates a page's position in the `lruList` to reflect its recent access.
- It uses `lruList.splice(lruList.begin(), lruList, it)` to move the page pointed to by `it` to the very front of the `lruList` 8.
- **`evict()` \*\* \*\* method \*\* 8 :**
- This method removes the least recently used page from the buffer (the page at the back of `lruList`).
- Before removal, if the page has been modified (marked as "dirty"), its changes are flushed to disk using the `storage_manager.flush()` method to ensure data durability8.
- The page is then removed from both `pageMap` and `lruList` 8.
- **Modularity :** BuzzDB is designed so that it interacts primarily with the `BufferManager` for page operations, abstracting away the direct interaction with the `StorageManager` 10.

### 3. Cache Replacement Policies (Policy vs. Mechanism)

- **Policy (What) :** Determines which page to evict from the buffer when space is needed (e.g., LRU, FIFO, TwoQ)1112.
- **Mechanism (How) :** Describes how the eviction is carried out, including flushing modified pages to disk12.
- **`Policy` \*\* \*\* Interface \*\* :** A C++ `Policy` base class with virtual `touch()` and `evict()` methods allows for interchangeable replacement policies, promoting extensibility11.
- **Specific Policies :**
- **FIFO (First-In-First-Out) :** Evicts the page that has been in the cache the longest, regardless of recent access11.
- **Limitation :** Susceptible to "buffer pool flooding" during sequential scans (e.g., iterating through a large table), where frequently accessed "hot" pages might be evicted to make space for less important "cold" pages1314.
- **LRU (Least Recently Used) :** Evicts the page that has not been accessed for the longest period1516.
- **Limitation :** Also prone to buffer pool flooding during sequential scans, as the pages being scanned become the most recently used and push out other, potentially hot,

pages1314.

- **TwoQ Policy** : A more sophisticated policy designed to mitigate buffer pool flooding14.
- It uses two queues: a FIFO queue for "read-once" pages (like those in a sequential scan) and an LRU queue for frequently accessed "hot" pages14.
- Pages initially enter the FIFO queue. If a page is accessed a second time while in the FIFO queue, it is "promoted" to the LRU queue1718.
- Eviction prioritizes pages from the FIFO queue first, and only if the FIFO queue is empty will it evict from the LRU queue (least recently used)1920. This helps keep valuable hot pages in memory even during sequential scans.
- **LFU (Least Frequently Used)** : Keeps the most frequently used pages in memory21. Limitation: Does not adapt well to changes in access patterns over time21.
- **ARC (Adaptive Replacement Cache)** : Dynamically balances between LRU and LFU, offering good performance across various workloads21. Limitation: More complex implementation21.

#### 4. Umbra's Advanced Buffer Manager Features

- **Variable-Size Pages** : Umbra's buffer manager supports pages of varying sizes, organized into "size classes" (e.g., 64 KiB as the smallest, with subsequent classes doubling in size)2223. This avoids the need for complex mechanisms to handle large objects (like strings or lookup tables) that might not fit into fixed-size pages224.
- **Memory Management (C++ & OS Intricacies)** :
  - Umbra leverages the operating system's virtual memory management ( `mmap` system call) to prevent external fragmentation within the buffer pool2526.
  - It reserves separate, contiguous virtual memory blocks for each size class, which do not immediately consume physical memory26.
  - When data is read from disk ( `pread` ), the OS creates actual physical memory mappings. When a page is evicted, Umbra signals the kernel ( `madvise` with `MADV_DONTNEED` ) to allow immediate reuse of physical memory, ensuring the buffer pool's physical memory consumption stays within limits27.
- **Pointer Swizzling (C++ Intricacies)** :
  - Umbra uses "swips" (a 64-bit integer) for efficient address translation, replacing a global hash table which can be a bottleneck in multi-core systems28.
  - A swip is "swizzled" if it directly contains a virtual memory address (page in memory), and "unswizzled" if it contains a `PageID` and size class (page on disk)29.
  - A tagging bit distinguishes between these states, allowing quick access with a single conditional check30.
  - To simplify eviction, Umbra requires that buffer-managed data structures (like B+ Trees) organize pages in a tree, ensuring each page has exactly one "owning swip" that needs updating upon eviction31. This means B+ Tree leaf nodes cannot have sibling pointers, though efficient scans are still supported31.

- **Versioned Latches (C++ Intricacies) :**
- Umbra adopts optimistic latching from LeanStore to synchronize concurrent page accesses, significantly reducing contention<sup>3233</sup>.
- Each active buffer frame has a versioned latch (a 64-bit integer) that encodes state (locked/unlocked, mode) and a version counter<sup>33</sup>.
- **Exclusive Mode :** For modifications; only one thread at a time. Version counter increments on page modification<sup>34</sup>.
- **Shared Mode :** For reads; multiple threads can acquire if not exclusively locked. Prevents page eviction<sup>35</sup>.
- **Optimistic Mode :** For reads; any number of threads can acquire if not exclusively locked. No contention. Readers remember the version counter, validating it upon release. If the counter changed, the read is restarted<sup>36</sup>. This is legal even if the page is unloaded, as virtual memory regions remain valid and map to a zero page<sup>36</sup>.
- **Buffer-Managed Relations :** Relations are organized in B+ Trees using synthetic 8-byte tuple identifiers<sup>37</sup>. Inner nodes use smallest page size (64 KiB) for high fanout, while leaf nodes adapt to tuple size, often fitting on 64 KiB pages<sup>37</sup>. Tuples within leaf pages use a PAX (Partition Attributes Across) layout, storing fixed-size attributes columnarly and variable-size attributes densely<sup>38</sup>.

## M06: Multi-Threading, Synchronization, and Database Configuration

### 1. Multi-Core CPUs and Multi-Threading

- **Evolution of CPUs :** CPU clock speeds hit a "thermal wall" around mid-2000s, leading to a shift towards multi-core architectures (e.g., AMD Phenom X4, Threadripper) to achieve higher performance through parallelism rather than just faster individual cores<sup>39</sup>.
- **Threads :** A thread is a sequence of instructions. Multi-core CPUs can execute multiple threads concurrently, enabling parallel processing of tasks (e.g., multiple user transactions in a database)<sup>40</sup>.
- **C++ std::thread :** The C++ Standard Library provides `std::thread` for creating and managing threads<sup>41</sup>. You can launch a function in a new thread using `threads.emplace_back(function_name)` and wait for threads to complete with `thread.join()`<sup>42</sup>.

### 2. Race Conditions

- **Definition :** Occur when multiple threads access and modify shared data concurrently, and the final result depends on the non-deterministic order of operations<sup>4344</sup>.
- **Example (Bank Balance) :** If two threads simultaneously deposit \$10 to a shared `bankBalance` without synchronization, the final balance might be incorrect (e.g., \$1010 instead of \$1020 from an initial \$1000)<sup>4042</sup>.
- **Assembly Level :** This happens because operations like `bankBalance += 10` are not

atomic. They involve multiple low-level instructions: `LOAD` value to register, `ADD` to register, `STORE` back to memory. If threads interleave these steps, one thread's update can overwrite another's without incorporating the intermediate change<sup>43</sup><sup>45</sup>.

### 3. Synchronization Mechanisms (C++ Intricacies)

- **`std::mutex`** (Mutual Exclusion)<sup>46</sup> :
  - Acts like a "door lock" for a "critical section" of code, ensuring that only one thread can execute that section at any given time<sup>45</sup>....
  - **Usage** : `bankMutex.lock()` acquires the lock, and `bankMutex.unlock()` releases it<sup>46</sup>.
  - **Assembly Level** : Locking a mutex typically involves atomic operations like `LOCK` `CMPXCHG` (Compare and Exchange) to ensure that only one thread successfully sets the mutex to a "locked" state<sup>47</sup>.
  - **Limitations** : A single mutex for all shared resources (e.g., all bank accounts) can severely limit concurrency, forcing even non-conflicting operations to wait<sup>48</sup><sup>49</sup>. This is called "coarse-grained locking."
- **`std::lock_guard`** (RAII for Mutexes)<sup>50</sup> :
  - A C++ RAII (Resource Acquisition Is Initialization) wrapper for `std::mutex` <sup>50</sup>.
  - **RAII Principle** : Resources (like mutex locks, memory, file handles) are acquired in an object's constructor and automatically released in its destructor, tying resource management to object lifetime<sup>51</sup><sup>52</sup>.
  - **Benefit** : `std::lock_guard` automatically acquires the mutex lock upon construction and releases it automatically when it goes out of scope (e.g., at the end of a function or block), preventing forgotten unlocks and potential deadlocks<sup>50</sup><sup>53</sup>.
- **Fine-Grained Locking** :
  - To increase concurrency, separate mutexes can be used for independent shared resources (e.g., one mutex per bank account)<sup>54</sup>.
  - **C++ Implementation** : `std::mutex bankAccountMutexes` <sup>55</sup> for an array of mutexes<sup>54</sup>. In data structures like a hash table, this could involve `std::vector<std::unique_ptr<std::mutex>> mutexes;` for per-slot locking. `std::unique_ptr` is necessary because `std::mutex` objects are not copyable or movable<sup>56</sup><sup>57</sup>.
  - **Benefits** : Increased parallelism and reduced contention, as non-conflicting operations can proceed simultaneously<sup>58</sup><sup>59</sup>.
  - **Limitations** : Increased overhead for lock acquisition/release and higher memory consumption due to more mutexes<sup>59</sup>.
- **`std::shared_mutex`** (Reader-Writer Lock)<sup>60</sup> :
  - Addresses the limitation of `std::mutex` where even read-only operations block each other<sup>59</sup>.

- Allows multiple threads to hold a "shared" (read) lock concurrently<sup>60</sup>.
- Ensures exclusive access for "write" operations: if any thread holds a shared lock, no thread can acquire an exclusive lock; if a thread holds an exclusive lock, no other thread can acquire any lock (shared or exclusive)<sup>61</sup><sup>62</sup>.
- **C++ Usage** : `std::shared_lock<std::shared_mutex>` for shared (read) access and `std::unique_lock<std::shared_mutex>` for exclusive (write) access<sup>60</sup><sup>61</sup>. Both are RAII-style wrappers.
- **Benefit** : Significantly improves performance in read-heavy workloads by allowing concurrent reads<sup>60</sup>.

#### 4. Database Configuration

- Database systems have numerous parameters ( `PAGE_SIZE` , `MAX_SLOTS` per page, `MAX_PAGES_IN_MEMORY` for the buffer pool) that can be tuned to optimize performance for specific workloads and hardware<sup>63</sup><sup>64</sup>.
- **Parameter Types** : Some parameters are immutable (fixed once set), while others are mutable (can be changed dynamically)<sup>65</sup>.
- **Challenges** : Finding the "sweet spot" for parameters is complex, as values can interact and have non-linear effects on throughput<sup>64</sup>. This often involves balancing performance (e.g., transactions per second) against cost<sup>65</sup>.
- **Application Types** : Database configurations should consider the application type:
- **OLTP (Online Transaction Processing)** : Characterized by many small, short transactions; often CPU- or I/O-bound. Database size can be slightly larger than DRAM<sup>66</sup>.
- **OLAP (Online Analytical Processing)** : Characterized by large, complex reporting queries; often I/O- or DRAM-bound. Database size typically much larger than DRAM<sup>66</sup>.
- **Web Application** : Often CPU-bound with many simple queries. Database size can be much smaller than DRAM<sup>66</sup>.
- **Automated Tuning** : Reinforcement learning can be used to automatically recommend configurations based on current state (parameters, workload), actions (changing parameters), and rewards (improved throughput/price-performance)<sup>67</sup>.

#### 5. Debugging Database Systems

- Debugging is crucial for identifying and fixing issues like data corruption, performance degradation, and system crashes<sup>68</sup>.
- **Tools** :
- **GDB (GNU Debugger)** : A powerful command-line debugger for C++<sup>68</sup>.
- **Commands** : `g++ -g` compiles with debug info; `gdb ./executable` starts debugger<sup>68</sup>.
- `break` (set breakpoint), `run` (start execution), `next` (step to next line), `print` (display variable value), `info locals` (display local variables), `continue` (resume

execution), `backtrace` (show call stack)69....

- **Print Statements** ( `**std::cout**` ) : Simple and effective for tracing execution flow and variable changes71. Overloading `operator<<` for custom classes can improve readability7172.
- **Debugging Principles** :
- **Stress Testing** : Design tests that push the system to its limits (e.g., using a very small buffer pool `MAX_PAGES_IN_MEMORY` to trigger evictions frequently) to expose edge cases and concurrency issues earlier7273.
- **Modular Design** : Design components with clear interfaces and separation of concerns (e.g., separating buffer metadata management from data storage). This makes it easier to isolate bugs to specific modules7475.
- **Literate Programming** : Emphasize clear documentation within the code. Well-documented code (comments, purpose, inputs, outputs, exceptions) is easier to understand and debug by humans74....
- **Robustness** : Build systems that can gracefully handle failures (e.g., disk I/O errors). Implement error handling and recovery mechanisms ( `try-catch` blocks) to prevent crashes and data inconsistencies73....

## M07: Hash Indexing

### 1. Introduction to Hash Index

- **Purpose** : Hash indexes are used for efficient "point queries" (finding a single value based on a key)79. They aim for  $O(1)$  average time complexity80.
- **Limitation** : They are generally inefficient for "range queries" (finding all values within a range), as the data is not stored in sorted order, often requiring a full table scan ( $O(N)$  complexity)81.
- **C++ Standard Library Options** :
- `**std::unordered_map**` : Implements a hash table. Provides average  $O(1)$  time complexity for insertion and retrieval, but worst-case can degrade to  $O(N)$  in case of many collisions. Data is not sorted8082.
- `**std::map**` : Implements a balanced binary search tree (usually a Red-Black Tree). Provides  $O(\log N)$  time complexity for all operations (insert, find, delete) and keeps data sorted by key8082.

### 2. Custom Hash Table Implementation

- **Concept** : A hash table can be thought of as an array (the "cabinet of drawers") where a "hash function" maps a key to a specific index (slot) in the array83.
- `**hashFunction(int key)**` : A function that takes a key and returns an index in the hash table. A simple example is `key % TABLE_SIZE` 84.
- `**HashEntry**` `** Struct (C++ Intricacies)**` : Represents a key-value pair and often includes additional metadata.

- `struct HashEntry { int key; int value; bool exists; }; 85.`
- `std::vector<std::optional<HashEntry>> hashTable;` is a C++ way to represent the hash table array, where `std::optional` indicates if a slot is occupied or empty<sup>86</sup>.
- **Collision Handling** : When two different keys hash to the same slot, a strategy is needed to find an alternative slot.
- **Linear Probing** : If the target slot is occupied, it checks the next slot, and so on, linearly, until an empty slot or the key is found. The formula for probing is  $(\text{HASH}(\text{KEY}) + K) \% \text{TABLE\_SIZE}$ , where K is the probe count<sup>84</sup><sup>87</sup>.
- **Limitation** : Can lead to "clustering," where many consecutive slots become occupied, increasing search times<sup>88</sup><sup>89</sup>.
- **Quadratic Probing** : Addresses clustering by using a quadratic formula for the probe sequence:  $(\text{HASH}(\text{KEY}) + K^2) \% \text{TABLE\_SIZE}$  <sup>90</sup>. It spreads out keys more effectively than linear probing<sup>91</sup>.
- **Limitation** : Can still suffer from "secondary clustering," where keys that initially hash to the same spot follow the same quadratic probe sequence<sup>91</sup>.
- **Double Hashing** : Uses a second hash function ( $\text{HASH2}(\text{KEY})$ ) to determine the step size for probing:  $(\text{HASH1}(\text{KEY}) + K * \text{HASH2}(\text{KEY})) \% \text{TABLE\_SIZE}$  <sup>91</sup>. This generates unique probe sequences for keys, even if they initially hash to the same spot, further reducing clustering<sup>92</sup><sup>93</sup>.
- **Deletion Handling** :
- Directly removing an entry can break the probe chain for other keys that relied on that entry's slot being occupied.
- **Lazy Deletion** : Instead of physically removing the entry, it's marked as "deleted" (e.g., by setting an `exists` flag to `false` in `HashEntry`)<sup>85</sup>. This maintains the probe chain. Insert operations can reuse marked-deleted slots, while find operations skip them but continue probing<sup>85</sup>.
- **Casting in C++ (Intricacies)** :
- `**static_cast**` : Used for converting between compatible types (e.g., `float` to `int` or base class to derived class pointer)<sup>94</sup><sup>95</sup>.
- `**reinterpret_cast**` : A powerful, low-level cast that converts any pointer type to any other pointer type (e.g., `char*` to `Slot*` for raw memory buffer access). Use with caution as it bypasses type safety checks<sup>95</sup><sup>96</sup>.
- `**std::istringstream**` : A C++ library for treating strings as input streams, useful for deserializing data from a string format back into structured objects (e.g., `Tuple` objects)<sup>94</sup><sup>97</sup>.

## M08: Thread-Safe Hash Index

### 1. Parallel Index Construction and Challenges



- **Benefit** : Building a hash index can be parallelized by dividing the workload (e.g., assigning ranges of pages to different threads). This significantly boosts performance on multi-core CPUs<sup>98</sup>.
- **Challenge** : When multiple threads concurrently insert or update entries in a shared hash index, it creates "thread safety" issues.
- **Race Conditions** : As explained in M06, concurrent modifications without synchronization lead to non-deterministic outcomes and potential data corruption<sup>44</sup>. For a hash table, this means threads might read stale data, overwrite each other's updates, or misplace entries during probing<sup>44</sup>.

## 2. Concurrency Control Protocols (C++ Intricacies)

- **Global Mutex** ( `**std::mutex**` ) :
- **Approach** : A single `std::mutex` protects the entire hash table<sup>53</sup>.
- **Benefit** : Simplest to implement, guarantees thread safety<sup>53</sup>.
- **Limitation** : Severely limits parallelism. Only one thread can perform any operation (insert, find, update) on the hash table at any given time, regardless of whether operations conflict<sup>49</sup>. This "serializes" access.
- **C++ Implementation** : `std::mutex indexMutex;` and `std::lock_guard<std::mutex> guard(indexMutex);` within critical methods<sup>53</sup>.
- **Fine-Grained Locking (Mutex Per Slot)** :
- **Approach** : Each slot (or bucket) in the hash table has its own dedicated `std::mutex` <sup>56</sup>.
- **Benefit** : Increases concurrency significantly. Threads can operate in parallel on different slots without contention. Only operations targeting the same slot will contend for that specific slot's mutex<sup>56</sup>.
- **C++ Implementation** : `std::vector<std::unique_ptr<std::mutex>> mutexes;` is commonly used to store mutexes for each slot because `std::mutex` objects are not movable or copyable, but `std::unique_ptr`s are<sup>56</sup><sup>57</sup>.
- **Limitations** :
- **Increased Overhead** : More mutexes mean more memory consumption and higher overhead for acquiring and releasing individual locks<sup>59</sup>.
- **Probing Challenges** : If a probing sequence spans multiple slots, a thread might need to acquire and release multiple locks, or acquire multiple locks simultaneously (which can lead to deadlocks if not handled carefully)<sup>57</sup>.
- **Fine-Grained Locking with `**std::shared_mutex**` (Shared Mutex Per Slot)** :
- **Approach** : Uses `std::shared_mutex` for each slot<sup>60</sup>.
- **Benefit** : This is the most flexible and often highest-performing approach for mixed

read/write workloads.

- **Multiple Readers** : Allows multiple threads to read from the same slot concurrently by acquiring a "shared lock"<sup>6199</sup>.
- **Exclusive Writer** : Only one thread can write to a slot at a time by acquiring an "exclusive lock," which blocks all other readers and writers for that slot<sup>6061</sup>.
- **C++ Implementation** :
  - For write operations (e.g., `insertOrUpdate`): use `std::unique_lock<std::shared_mutex> lock(mutexes[index]);` for exclusive access<sup>60</sup>.
  - For read operations (e.g., `getValue`): use `std::shared_lock<std::shared_mutex> lock(mutexes[index]);` for shared access<sup>61</sup>.
- **Overall** : Provides increased parallelism, especially for read-heavy workloads, by reducing contention<sup>99</sup>.

### 3. Simulation Framework for Protocol Comparison

- **Purpose** : Real-world multi-threading behavior can be non-deterministic and hard to analyze directly<sup>99</sup>. A simulation framework provides a controlled and deterministic environment to compare the efficiency of different concurrency control protocols (Global Mutex, Mutex Per Slot, Shared Mutex Per Slot)<sup>99100</sup>.
- **How it works** :
  - Simulated threads perform operations (`try_acquire`, `find`, `insert`, `release`, `unlock_and_relock`) on a simulated hash table<sup>101102</sup>.
  - Each thread takes one step per "logical time step"<sup>102</sup>.
  - The total number of logical time steps required to complete a given set of operations indicates the efficiency of the protocol<sup>103</sup>.
- **Typical Results** : Shared Mutex Per Slot usually shows the lowest logical time steps (highest efficiency) for mixed or read-heavy workloads, followed by Mutex Per Slot, and then Global Mutex<sup>103</sup>....

## M09: B + Tree Indexing and Range Queries

### 1. Ordered Index: B + Tree for Range Queries

- **Necessity** : While hash tables excel at point queries, they are inefficient for range queries because their data is unordered<sup>7981</sup>. An "ordered index" like a B + Tree is required for efficient range queries.
- **B + Tree Advantage** : B + Trees store data in a sorted manner and have linked "leaf nodes," which allows for efficient retrieval of all data within a specified range<sup>106107</sup>. Once the starting point of the range is found, a linear scan across the linked leaf nodes can retrieve all relevant data efficiently<sup>107</sup>.

## 2. B+ Tree Structure

- **Nodes** : A B+ Tree consists of two types of nodes:
- **Inner Nodes** : Store keys and pointers (or file offsets) to child nodes/pages. These keys define ranges for navigating down the tree<sup>108109</sup>.
- **Leaf Nodes** : Store the actual keys and their associated values (or tuple IDs). All leaf nodes are at the same depth (uniform height) and are linked together in a sorted sequence using "sibling pointers" (or `next` pointers in C++ implementation)<sup>107....</sup>
- **Height and Fan-Out** :
  - B+ Trees are "short and wide" due to their high "fan-out" (the number of children an inner node can point to)<sup>112</sup>.
  - A high fan-out means fewer levels (shorter height) in the tree. This is critical for disk-based systems because each level typically corresponds to a disk page, and thus, a disk I/O operation<sup>113114</sup>. Minimizing height reduces disk reads for searches.
- **Keys and Values in Nodes** :
  - Inner nodes: Typically store  $K-1$  keys and  $K$  pointers to child nodes<sup>108</sup>.
  - Leaf nodes: Store  $K$  keys and  $K$  associated values<sup>108</sup>.
  - **`**Entry**` Struct (C++ Intricacies)** : A common C++ representation for a key-value pair within a node, potentially also holding child pointers ( `NodePtr next` ) for inner nodes<sup>115</sup>.
  - **`**Node**` Struct (C++ Intricacies)** : Contains a boolean `isLeaf` flag, a `std::vector<Entry>` to hold its entries, and a `std::unique_ptr<Node> next` for sibling pointers in leaf nodes<sup>111</sup>. In Umbra, separate `std::vector<Key>` keys and `std::vector<Value>` values are used in the Node struct for columnar storage within a page<sup>116</sup>.

## 3. B+ Tree Insertion (C++ Intricacies)

- **Recursive Descent** : Insertion starts at the root and recursively descends through inner nodes to find the appropriate leaf node where the new key-value pair should be inserted<sup>111117</sup>.
- **Insertion in Leaf Node** : Once the correct leaf node is found, the new entry is inserted at the correct sorted position. If the key already exists, its value might be updated<sup>118119</sup>. `std::lower_bound` with a lambda comparator helps find the insertion point while maintaining sorted order<sup>118120</sup>.
- **Node Split** : If inserting a new entry causes a node (leaf or inner) to exceed its maximum capacity ( `maxKeys` ), the node must "split"<sup>121</sup>.
- The node is divided into two new nodes, and a "median key" is promoted to the parent node<sup>119122</sup>.
- **`std::move`** and **`std::back_inserter`** : Used to efficiently transfer elements from the splitting node to the new node without unnecessary copying<sup>119122</sup>.

- **Path Tracking and Backtracking** : As the insertion recurses down the tree, a `std::vector<std::shared_ptr<Node>>` `path` keeps track of the parent nodes visited<sup>123</sup>. This "path" allows the algorithm to "backtrack" upwards if a child node splits, enabling the median key to be inserted into the parent. If the root node splits, a new root is created, increasing the tree's height<sup>124</sup><sup>125</sup>.

#### 4. On-Disk B+ Tree Adaptation

- **Scalability and Durability** : B+ Trees are designed for large tables that reside on disk, offering scalability and ensuring the index persists across system restarts<sup>126</sup>.
- **Node Representation for Disk** :
  - Instead of C++ smart pointers for child nodes, `uint32_t` file offsets are used to point to child nodes/pages on disk<sup>126</sup>.
  - Node structure size is aligned with disk page size (e.g., 4KB or 16KB) to maximize fan-out and minimize I/O<sup>127</sup>.
- **Serialization and Deserialization (C++ Intricacies)** :
  - **Serialization** ( `**serializeNodeToBytes**` ) : Converts an in-memory `DiskNode` structure into a raw byte stream ( `std::vector<uint8_t>` ) that can be written to disk<sup>128</sup>. This involves `reinterpret_cast` to treat data as byte arrays.
  - **Deserialization** ( `**deserializeNodeFromBytes**` ) : Reads a byte stream from disk and reconstructs the in-memory `DiskNode` structure<sup>129</sup>. This often involves `std::memcpy` for efficient byte-level copying.
- **Minimizing Disk I/O** :
  - **Caching** : B+ Tree nodes, especially higher-level inner nodes, are cached in the buffer pool (DRAM) to reduce the number of disk I/O operations for traversing the tree<sup>141</sup><sup>129</sup>.
  - **Uniform Height** : All paths from the root to any leaf are of the same length, ensuring a consistent number of disk reads/writes for any point query<sup>114</sup>.
  - **Linked Leaves** : Efficient range queries are performed by traversing horizontally across linked leaf nodes after finding the starting leaf<sup>130</sup>.

#### 5. C++ Templates (Intricacies)

- **Generic Programming** : C++ templates allow functions and classes to operate with generic types, enabling code reusability<sup>131</sup><sup>132</sup>.
- **B+ Tree Template** : A template `<typename Key, typename Value>` class `BPlusTree` can handle various key-value types (e.g., `BPlusTree<int, int>` for employee ID to salary, `BPlusTree<std::string, int>` for product name to price, or `BPlusTree<TicketKey, std::string>` for custom composite keys)<sup>132</sup>....
- **User-Defined Types** : For custom key types (like `TicketKey` ), comparison operators ( `operator<` ) must be overloaded to define how instances of that type are ordered within the B+ Tree<sup>134</sup><sup>135</sup>.
- **Smart Pointers in B+ Tree** : `std::unique_ptr` and `std::shared_ptr` are widely

used within B+ Tree implementations ( `NodePtr` , `children` , `root` ) to manage memory for nodes and ensure proper ownership and deallocation, preventing memory leaks<sup>111</sup>.... `std::shared_ptr` is useful for child pointers in inner nodes, where multiple paths might implicitly share ownership, or for the root node.