Path Integrals in QFT

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Introduction

Quantum field theory (QFT) is a powerful framework for describing the behaviour and interaction of particles in the quantum regime, combining classical field theory, special relativity and quantum mechanics. **A particular mathematical formalism of QFT is the path integral formalism.**

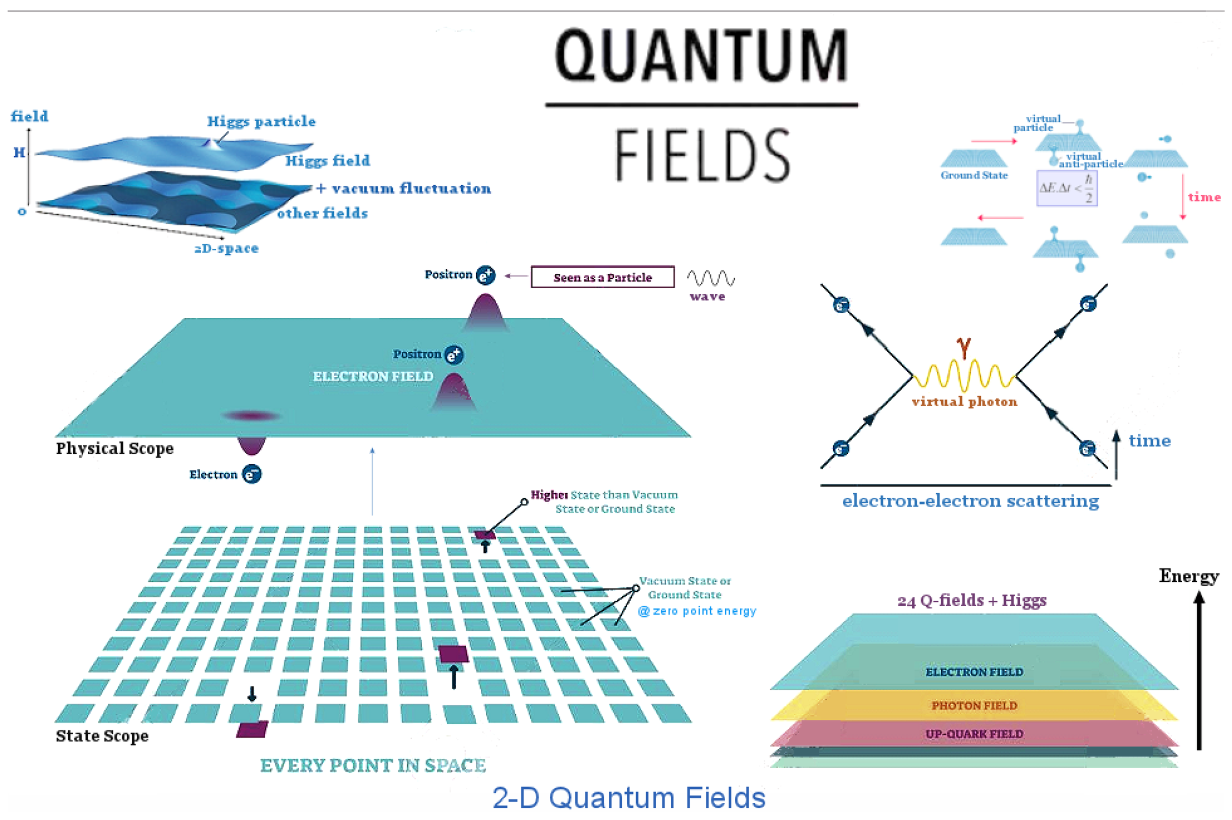

Fig. 1: Visualisation of quantum fields.[1]

Path Integral formalism of QM

Path integrals provide an alternative formalism to tackle the problems of quantum mechanics, equation (1) replaces the Schrodinger equation in describing the evolution of a wavefunction where the final state, $\Psi(\mathbf{x}'', t'')$ is described by an integral involving the initial state, $\Psi(\mathbf{x}', t')$:

$$\Psi(\mathbf{x}'', t'') = \int d\mathbf{x}' K(\mathbf{x}'', \mathbf{x}'; t'', t')\Psi(\mathbf{x}', t') \qquad (1)$$

$$K(\mathbf{x}'', \mathbf{x}'; T) = \int_{\mathbf{x}(0)=\mathbf{x}'}^{\mathbf{x}(T)=\mathbf{x}''} \mathcal{D}\mathbf{x}(t) e^{\frac{i\mathbf{R}[\mathbf{x}(t)]}{\hbar}} \qquad (2)$$

Where $K(\mathbf{x}'', \mathbf{x}'; T)$ is the **Green's function of the time-dependent Schrodinger equation**, analogous to the integral kernel of the quantum mechanical time evolution operator. Here the function $\mathbf{R}[\mathbf{x}(t)] = \int_{t'}^{t''} \mathcal{L}(x(t), \dot{x})dt$ defines the classical action of the particles motion. [2]

Path integral formalism of QFT

The path integral formalism of QFT combines the path integral approach to QM with classical field theories:

- Instead of summing over all possible paths the particle can take, you sum over all possible histories of the quantum fields. This allows us to describe the behaviour of particles by excitations in the electron or Electromagnetic (EM) fields.
- An electron is viewed as an excitation that propagates as a wave through the electron field.
- Similarly, photons are viewed as excitations propagating through the EM field.

Feynman Diagrams

Feynman Diagrams were developed as pictorial devices to represent the complex mathematics behind QFT. Using the diagram shown as an example:

- The photons (γ) represent the quantized EM field excitation of the photon.
- Electrons and positrons (e^+ *and* e^-) represent quantized excitations in the electron field.
- These 'lines' (straight and sinusoidal) are a representation of the particles Feynman propagator.
- Vertices represent the interaction of the electrons and is mathematically described by the coupling constant.

Each Feynman diagram represents a particular term in the path integral, thus, the path integral can be thought of as summing over all possible Feynman diagrams.

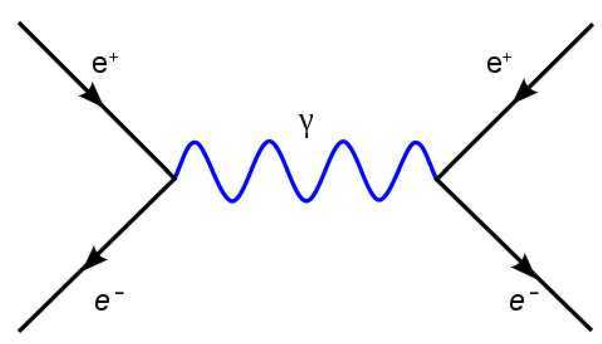

Fig. 2: Feynman diagram of electron-positron annihilation and creation.[5]

Current research

Current research in the energy levels of Positronium by G.S. Adkins, D.B. Cassidy and J. Perez Rios aims to provide experimental evidence of bound-state QED theory beyond the standard model. [3]

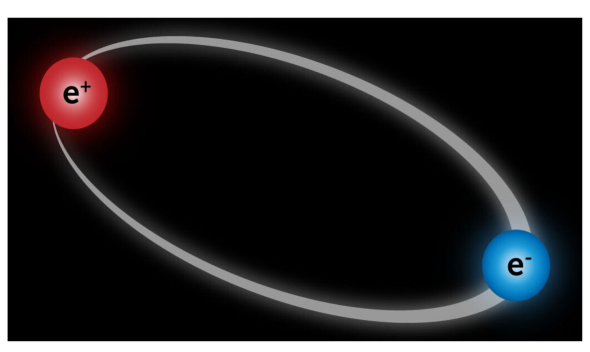

Fig. 3: Positronium atom.[4]

References

[1] Cartoon Dealer. Quantum field theory. https://cartoondealer.com/illustrations/pg1/theory.html, 2018.

[2] C. Grosche F.Steiner. *Hankbook of Feynman Path Integrals*. Springer, 1998.

[3] J. Pérez-Ríos G.S. Adkins, D.B. Cassidy. https://www.sciencedirect.com/science/article/pii/S0370157322001508, 2022.

[4] Science News. Positronium. https://www.sciencenews.org/article/positronium-energy-levels-exotic-atom-physics, 2020.

[5] Wikemedia. Electron positron annihilation. https://commons.wikimedia.org/wiki/File:ElectronPositronAnnihilation.svg, 2020.

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