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Predicting goal attainment in process-oriented behavioral interventions using a data-driven system identification approach

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ABSTRACT

Behavioral interventions (such as those developed to increase physical activity, achieve smoking cessation, or weight loss) can be represented as dynamic process systems incorporating a multitude of factors, ranging from cognitive (internal) to environmental (external) influences. This facilitates the application of system identification and control engineering methods to address questions such as: what drives individuals to improve health behaviors (such as engaging in physical activity)? In this paper, the goal is to efficiently estimate personalized, dynamic models which in turn will lead to control systems that can optimize this behavior. This problem is examined in system identification applied to the *Just Walk* study that aimed to increase walking behavior in sedentary adults. The paper presents a Discrete Simultaneous Perturbation Stochastic Approximation (DSPSA)-based modeling of the *Goal Attainment* construct estimated using AutoRegressive with eXogenous inputs (ARX) models. Feature selection of participants and ARX order selection is achieved through the DSPSA algorithm, which efficiently handles computationally expensive calculations. DSPSA can search over large sets of features as well as regressor structures in an informed, principled manner to model behavioral data within reasonable computational time. DSPSA estimation highlights the large individual variability in motivating factors among participants in *Just Walk*, thus emphasizing the importance of a personalized approach for optimized behavioral interventions.

1. Introduction

The prevalence of sedentary, unhealthy lifestyles is a leading cause of chronic diseases among adults. Behaviors such as lack of physical activity, smoking, and unhealthy diet lead to diabetes, cancer, and heart disease [1]. These behaviors collectively contribute to more than 50% of deaths among adults [2] that could be prevented otherwise. Changing behaviors such as averaging 8000 steps/day instead of 4000 steps/day, can decrease the risk of mortality arising from unhealthy behavior by 51% [3,4]. While such sustained physical activities can improve general public health, a lack of personalized interventions that promote and reinforce these behaviors poses a great challenge to our society. This may occur in part due to an improper understanding of behavior change dynamics at an individual level. However, the abundance of mobile and wireless health (mHealth) devices such as *Fitbits* that keep track of user data such as the number of steps taken per day or environmental context (e.g., ambient temperature or whether the day is a weekday or weekend) offer a potential solution [5,6]. They can be utilized to implement data-centric estimation and predictive modeling algorithms aimed at improving personalized behavioral interventions. Principles from behavior change theory and control engineering can be used on the data collected by the devices to implement behavioral interventions through an engineering approach. For this purpose, the *Just Walk* study has been developed [7–9], which shows proof-of-concept for control engineering principles leading to time-varying "just-in-time" interventions for increasing walking activity in sedentary adults. *Just Walk* has been acknowledged in the behavioral medicine community as an innovative approach in implementing smartphone-based behavioral interventions [10]. *Just Walk* is an initial demonstration of the Control Optimization Trial (COT) framework [11,12], which is an experimental paradigm that uses data-centric models to perform system identification and closed-loop control in an integrated manner. The COT framework is based on the widely accepted Social Cognitive Theory [13] which involves modeling physical activity as a control-oriented dynamic model of exogenous and endogenous constructs. Essential concepts of SCT are further discussed in [14] which explores the dynamic modeling of behavior through process models based on a fluid analogy. Other research

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in a similar context with process- and control-oriented models includes gestational weight gain interventions [15,16] and interventions related to smoking cessation [6] and fibromyalgia pain [17].

The focus of prior work in the *Just Walk* study has been on experimental design and estimation of *Behavior*-output dynamic models with black-box prediction-error models involving the interrelationship between various psychological constructs. However, there is a risk of oversimplifying the relationship between step goals and physical activity [18], which calls for a deeper understanding of the optimal goal-setting zone to improve the participant's behavioral performance. Chevance et al. [19] highlight the importance of understanding an individual's ability to attain the step goals to set optimal daily targets.

This study focuses on *Goal Attainment* in the context of data-driven modeling to conceptualize the influence of physiological and psychological factors on a person's capabilities and allows us to generate models with an improved understanding of an individual's physical competence. The examination of *Goal Attainment* has implications in the realm of behavioral medicine and consequently in control system design for the intervention. Generating a model that relates *Behavior* to various influencing factors such as *Temperature*, availability of a *Weekday* or a *Weekend*, or *Predicted Busyness* serves predictive purposes, which is to say, it helps to forecast the number of steps the person might take when subjected to a given set of conditions. However, the overall study extends beyond just the prediction and covers the formulation of prescriptive actions by implementing a closed-loop-control-based behavioral intervention, where it is necessary to interpret to what extent an increase in *Goals* or *Points* can improve activity. Such prescriptive models aim to understand the performance of the person relative to the *Goals* provided and observe the participant's response in scenarios when tasks are difficult to achieve, rewards change, or when they are not motivated enough. For example, it is crucial for an efficient control system design to understand that while higher *Goals* may evoke higher step counts in the immediate future, the associated increase in *Goal* difficulty would reduce long-term commitment, leading to an eventual decline in the desired activity.

In the closed-loop setting control, a behavior-output model may improperly capture such dynamics, leading to suboptimal control actions. This motivates modeling the *Goal Attainment* construct, which provides a broader understanding of the dynamics of the participant's physical activity.

A notable aspect of this study is that models are estimated from the data measured from individual participants without clustering; this ensures that behavioral idiosyncrasies are taken into consideration during modeling for a given participant. Such an approach is termed as *idiographic* or single subject ($N = 1$; [8,20]) and is essential for undertaking decisions that cannot otherwise be addressed in a group, such as understanding specific motivating factors behind an individual's physical activity behavior. However, idiographic approaches face two primary challenges that make time-varying behavioral interventions a non-deterministic problem without a straightforward solution: (i) small datasets containing noise and missingness and (ii) incomplete knowledge of all possible features influencing a person's behavior. The collection of real-time data is further limited by the period of study and the frequency at which the data is obtained. For example, trying to capture data from users too frequently can lead to notification fatigue which can adversely influence the intervention. The limited availability of information demands judicious use of the data while generating estimation/validation datasets for predictive modeling. This is addressed in a unique manner in the *Just Walk* study. The study spans 80 days, comprising five cycles (sub-experiments), combinations of which are used to generate the estimation/validation datasets. The *Just Walk* study also employs a non-conventional method for input signal design where it uses multisine signals for the input variables *Goals* and *Expected Points* to generate an informative dataset. Prior work that has undertaken exhaustive searches through these databases [9] which demanded large computational time. It can be shown that the number of possible models increases exponentially with increases in features and regressor structures, making exhaustive search an impractical approach [21].

This work uses the Discrete Simultaneous Perturbation Stochastic Approximation (DSPSA) algorithm to deal with the aforementioned challenge through a stochastic search routine [22,23]. DSPSA is applied within the context of AutoRegressive with eXogenous Input (ARX) modeling for generating person-specific behavioral models based on the *Just Walk* dataset [21,24]. This allows us to use DSPSA to efficiently search over sets of features as well as regressor structures within an acceptable computational time. A comparative study with exhaustive ARX modeling demonstrates the ability of DSPSA to search over an expanded set of model characteristics with a better sense of contextually relevant constructs. The estimated model is then suitable for "just in time" adaptive interventions using control-oriented formulations such as Nandola and Rivera [25] and Khan et al. [26].

This paper is organized as follows: Section 2 describes modeling behavior as a process system and introduces the *Just Walk* study and its key features. Section 3 elaborates on system identification of the *Just Walk* dataset. It presents a motivating example based on fixed structure ARX-modeling that provides important insights into a participant's behavior dynamics and highlights the challenges of this approach. Section 4 presents the Discrete Simultaneous Perturbation Stochastic Approximation algorithm as a suitable alternative and demonstrates its application to the *Just Walk* dataset. It shows how DSPSA can play a significant role in accomplishing model estimation for behavioral data and in laying the path for personalized interventions. Section 5 briefly mentions the challenges associated with implementing the ARX SPSA models in closed-loop settings for successful behavioral intervention and the associated avenues in this regard. Section 6 sums up some important conclusions and describes directions for future research.

2. Behavior change as a dynamic process model

Adaptive behavioral interventions aim at developing decision frameworks tailored to meet individual requirements [27]. Control systems engineering principles present a comprehensive methodology to design these interventions using a process-based approach [15,28]. System identification methods generate data-centric models based on the information collected from users at suitable intervals. These predictive models are further implemented in closed-loop settings to generate appropriate control sequences that promote and sustain healthy behavior. A fundamental assumption underlying these approaches involves considering behavior as a dynamic system of several internal and external factors interacting to generate an outcome. A prevalent model is the Social Cognitive Theory (SCT) model, which stems from early learning theories such as the Social Learning Theory, the details of which are discussed in [29]. SCT allows us to capture complex individual behavior dynamics as the outcome of internal and external influences and their interactions in a simple yet effective way. SCT can be represented as a fluid analogy [14] that allows depicting physical activity as a process system through the use of inflow–outflow streams and inventories. The current work analyzes the *Just Walk* study which is a behavioral intervention developed to use system identification to characterize and predict person-specific behavior. The aim of the *Just Walk* study is to use an engineering methodology to promote sustained physical activity among sedentary adults. Fig. 1 illustrates a reduced SCT fluid analogy model for the *Just Walk* study. The model lays its foundation on multiple time-varying constructs which are discussed in detail in [14]. The behavioral constructs relevant to this study are noted as follows:

1. *Behavior* (η_5): The number of steps taken by an individual per day, which is the primary measured output; it forms a basis for defining multiple output constructs needed for modeling physical activity.

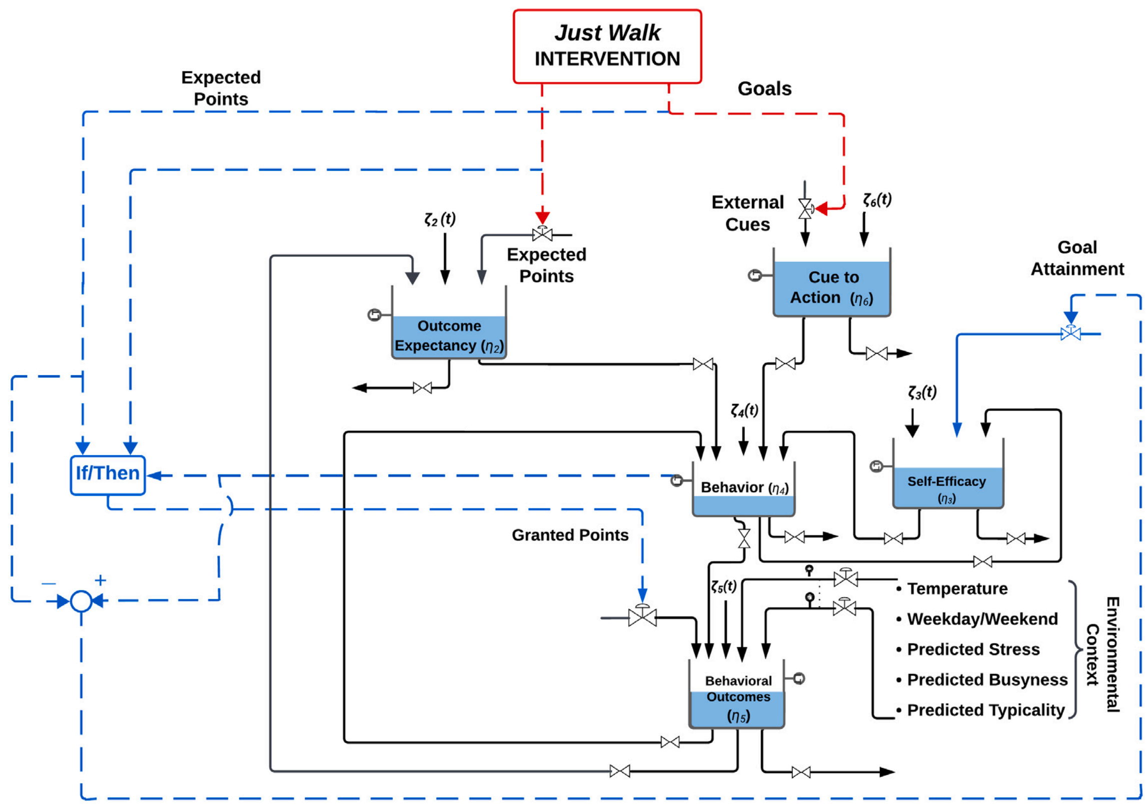

Fig. 1. Schematic illustrating the SCT Fluid Analogy Model for *Just Walk* depicting Physical Activity as a process system through the use of inflows/outflows and inventories. *Source:* Adapted from Martín et al. [14].

2. *Goals* (u_8): The daily step target provided to the participant as a manipulated input signal through their wearable devices. These are unique to each participant and are based on *a priori* knowledge.
3. *Expected Points* (ξ_9): Along with *Goals* (u_8), this is a second manipulated input signal provided to the participant. It defines the anticipated number of points that can be granted on a daily basis.
4. *Granted Points* (ξ_{10}): This is a dependent input signal that describes the actual number of points issued to a participant once they achieve or exceed their daily step *Goals*.
5. *Goal Attainment* (ξ_{11}): The difference (*Behavior-Goals*) between the actual behavior and the goals set for a given day.
6. *Goal Achievement*: The percentage of daily step goals walked by a person.
7. *Self-Efficacy (η_3)*: This construct serves to assess the perceived competence of an individual in undertaking a particular action. This factor largely influences a participant's ability to accomplish the behavior of interest.
8. *Outcome Expectancies (η_2)*: This construct denotes the likelihood of a particular behavior resulting in a given outcome.
9. *Behavioral Outcomes (η_5)*: These are positive or negative reinforcements (such as fitness or fatigue) arising from the participant's behavior.

From this list of signals, *Behavior*, *Goal Attainment*, and *Goal Achievement* are considered to be the outputs of interest. Keeping in mind that the study aims to facilitate interventions, the *External Cues* signals such as *Goals, Expected Points, Granted Points* are categorized as manipulated input variables, and *Environmental Contexts* such as *Predicted Stress, Predicted Busyness, Predicted Typicality, Weekday/Weekend*, and *Temperature* are considered as measured disturbances.

The constructs *Behavioral Outcomes*, *Self-Efficacy*, and *Cue to Action*, though present challenges in measurement, are conceptualized to interplay with a participant through a process-system orientation, making it pertinent to model their interdependence through inventories and inflow/outflow streams. These inventories also include unmeasured disturbances (ζ) and purge streams entering and exiting them respectively. Such modeling allows us to take a fluid analogy standpoint in understanding their dynamics clearly. For example, the measured disturbance signals can be considered to affect the participant's *Behavioral Outcomes* with non-zero gains. The impact of such signals on *Behavior* can be further examined as inputs to different *Behavioral Outcomes* components (e.g., fatigue, fitness) if such components can eventually be measured separately [14,30]. Consequently, higher-order dynamics for the system can be modeled with this approach.

The *Goal Attainment* signal forms a crucial feature of the SCT model as it captures the essence of interdependence among various behavioral constructs. It is formulated as a dependent variable that is a direct outcome of the participant's success or failure in pursuing their daily step targets, but can also be theorized to influence the participant's *Behavior* indirectly. An increase in *Goal Attainment*, for example, causes an increase in *Self-Efficacy*, which further improves the individual's *Behavior*. Appropriately, it becomes important to study such constructs as dynamic models to set ambitious but doable *Goals* that promote and sustain healthy behavior. This paper undertakes a data-driven modeling approach to elucidate the dynamics of these constructs and also validates the use of such an approach through the results demonstrated in the latter sections using multiple criteria. It is observed that the models exhibit transient characteristics such as overdamped responses that agree well with the physical intuition of *Behavior* and *Goal Attainment*. As will be noted in the ensuing section, validation results on the estimated models, particularly step responses, fit percentages, and the applicability of the ARX-based estimation, support that the process model perspective explored in this paper is highly relevant to this analysis.

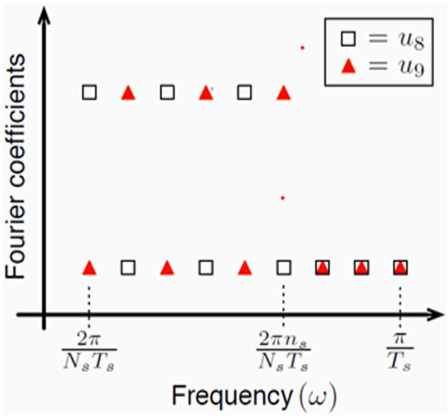

Fig. 2. Schematic illustrating the concept of "zippered" spectra design for two designed multisine inputs (*Goals* and *Expected Points*).
Source: Adapted from [33].

3. System identification of *Just Walk* data

Just Walk (NSF IIS-1449751) is a one-of-a-kind study that enables undertaking an engineering-based approach to designing behavioral interventions. It demonstrates how system identification can play an integral part in developing process-based interventions through a novel input signal design approach followed by a structured model estimation and validation methodology.

3.1. Experimental design

As a part of the system-identification-based experimental design, the manipulated variables are defined using multisine signals with different amplitudes and fundamental frequencies [31]. These signals are designed to be "pseudo-random" in nature; which means although they appear random to participants, they are deterministic by design. As a result, they are expected to align with the properties of randomness that are helpful for causal inference [32]. The spectra of these signals can be specified directly by the designer and a single period of such a signal can be described by

$$u_n(k) = \lambda_n \sum_{j=1}^{N_s/2} \sqrt{2\alpha_{n,j}}\, cos(\omega_j k T_s + \phi_{n,j}) \tag{1}$$

where λ_n is the amplitude scaling factor, T_s denotes the sampling period and N_s gives the number of samples in one period. For the jth harmonic of the signal, the factor $\alpha_{n,j}$ specifies the relative power at the frequency ω_j which is given by $\omega_j = 2\pi j/N_s T_s$. $\phi_{n,j}$ is the phase for each harmonic. $n_s \leq N_s/2$ symbolizes the total number of sinusoids excited, taking into account all inputs. The overall input signal is obtained by repeating a unit period for M cycles, resulting in the total duration of the experiment being $N_s M$ days. Furthermore, the signals are "zippered" in the frequency domain to ensure independent estimates of transfer functions, accomplished by choosing $\alpha_{n,j}$ such that both the input signals are orthogonal in the frequency domain. Mathematically, this is achieved when a non-zero-valued Fourier coefficient in a signal for a given frequency is complemented by a zero-valued coefficient in the other signal. An illustration of the concept is depicted in Fig. 2. The time period of these signals N_s and the overall length $N_s M$ are judiciously decided to strike a balance between the capacity to successfully capture model dynamics and the ability to maintain an optimal resolution of the process.

Based on these considerations, the *Just Walk* study was conducted for 80 days comprising five cycles of 16 days each. For the manipulated inputs, *Goals*, and *Expected Points* were defined using zippered multisine signals, while the *Granted Points* signal was dependent on these. The signals *Predicted Stress*, *Predicted Busyness*, and *Predicted Typicality* were generated through scale-based responses from the users, while the *Weekday/Weekend* was defined as a binary signal where 1 denotes a weekend and 0 denotes a weekday. A plot depicting an intervention database involving input/output signals for a representative participant of the *Just Walk* study is shown in Fig. 3 where the five cycles (sub-experiments) are expressed through the individually-colored sections.

3.2. Model structure selection and estimation

A personalized behavioral intervention demands an idiographic (i.e. single-subject) modeling approach where individual participants are characterized by unique sets of inputs and their corresponding model structures. The *Just Walk* data is modeled using an AutoRegressive with eXogenous input (ARX) [34] based estimation algorithm involving order selection. Mathematically, the ARX model can be described as,

$$y_k + \sum_{l=1}^{n_a} a_l y_{k-l} = \sum_{j=1}^{n_u} \sum_{i=0}^{n_{b_j}-1} b_{(i+1)(j)} u_{j,k-n_{k_j}-i} + e_k \tag{2}$$

where y_k, u_k, and e_k denote output, input, and error variables, respectively. Keeping in mind the computational cost for performing exhaustive ARX modeling, the ranges for the polynomial orders and the delays are chosen to be $[n_a n_b n_k]$ = [1:3 1:3 0:2] for order selection with the modeling being performed using a fixed sets of inputs. An essential part of the estimation procedure entails pre-processing the data. This involves dealing with noise, besides data imputation for missingness and data detrending procedures. Noise in behavioral data is typically non-stationary and is influenced by daily routines and seasonal factors (holidays, or summers/winters). Considering this, combinations of experiments are used to form the estimation and the validation datasets, as opposed to the conventional method wherein the data is directly split into an initial set of estimation data followed by the validation data. This removes any underlying trend originating from the changing noise characteristics in behavioral data. Given that the *Just Walk* data comprises five cycles, a total of 20 such combinations can be generated with at least two sub-experiments forming an estimation dataset. Once the overall data is properly organized, an exhaustive search through these combinations is performed along with order selection for a representative *Just Walk* participant to generate a suitable predictive model. The merits of this procedure is observed in the case studies discussed in the ensuing sections.

3.3. Model validation

For model validation, no single measure is used to determine model adequacy; rather, multiple criteria are involved. A standard approach is to quantify the model fits using the Normalized Mean Square Error (NRMSE) fit index. NRMSE fit percentage is calculated for each sub-experiment belonging to estimation and validation datasets and is mathematically defined as:

$$NRSME\ fit\,(\%) = 100\ x \left(1 - \frac{\|y(k) - \hat{y}(k)\|_2}{\|y(k) - \bar{y}\|_2}\right) \tag{3}$$

where $y(k)$ is the measured output, $\hat{y}(k)$ denotes the estimated output, $\bar{y}$ gives the mean of the measured values $y(k)$, and $\|.\|_2$ is the vector 2-norm. Measures like average NRMSE validation fits (F_v), evaluated by averaging over NRMSE fits of the validation cycles, are used to assess the predictive ability of the models. Furthermore, fit over the unpartitioned overall data (F_o), as well as weighted averages F_w of F_v and F_o given by,

$$F_w = W_v F_v + W_o F_o \tag{4}$$

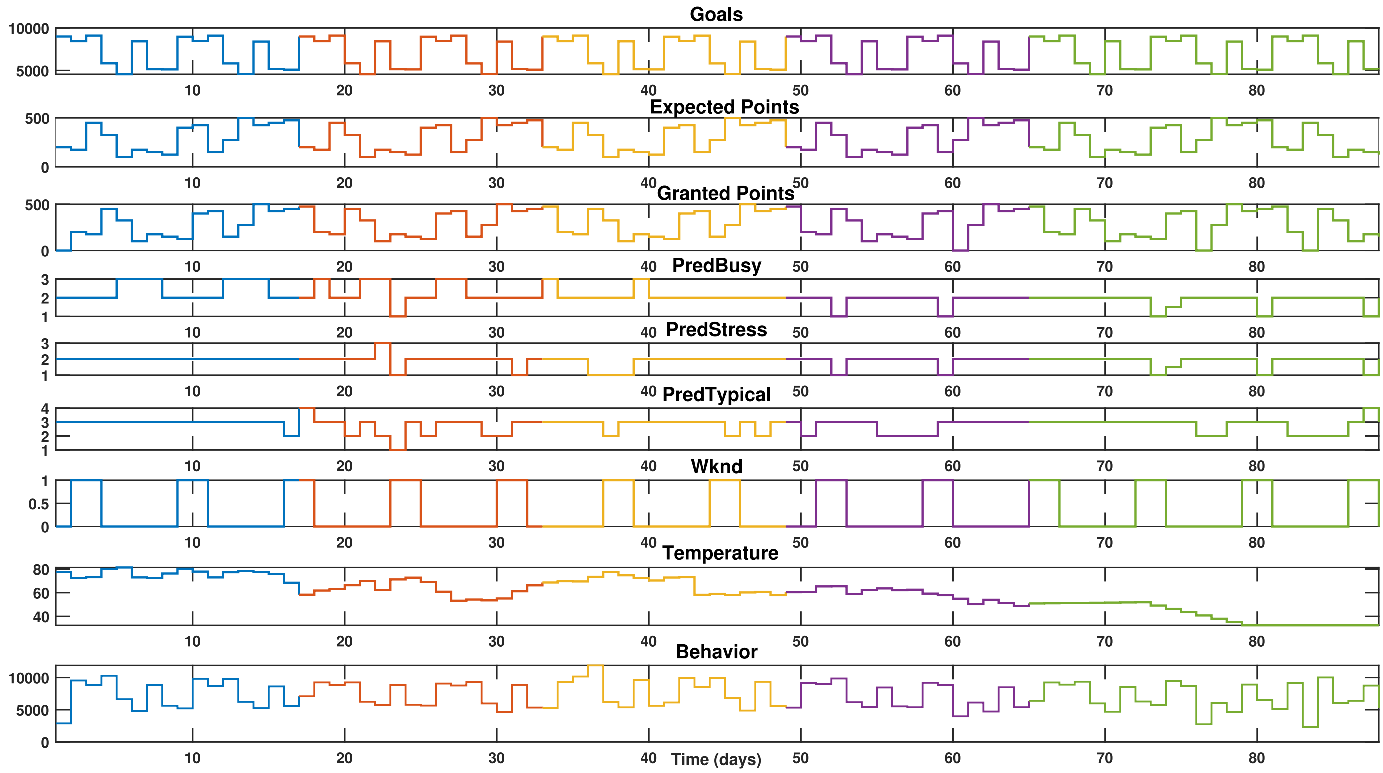

Fig. 3. Illustrative *Just Walk* time series data for the measured output *Behavior* and the eight inputs for Participant 230. The five sub-experiments are denoted through the multi-colored cycles. The first two subplots correspond to the zippered multisine input signals *Goals* and *Expected Points*.

Table 1
Fixed-ARX estimation for Participant 230 for 20 combinations (arising from 5 cycles/sub-experiments) of estimation (marked in green) and validation datasets (marked in yellow), for the 3 inputs- *Goals*, *Granted Points*, and *Temperature*. E^* and V^* denote the cycle numbers corresponding to the estimation and the validation sets, respectively. Fits on the estimation, validation, and overall data are provided along with the weighted fits (the desired measure for model selection). The best weighted/validation model is highlighted in orange.

	NRMSE Fit (%)											ARX Regressor Structure (3-Input)
	E*	V*	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Est	Val	Overall	Weighted	$[n_a, n_{b1}, n_{b2}, n_{b3}, n_{k1}, n_{k2}, n_{k3}]$
1	[1,2]	[3,4,5]	70.88346	71.32452	56.61905	35.55963	39.38238	71.10399	43.85369	36.79913	41.03186	[1,1,3,1,0,2,1]
2	[1,3]	[2,4,5]	67.86368	66.94928	73.10395	44.06539	43.28824	70.48381	51.43431	50.94318	51.23785	[1,1,1,3,0,0,2]
3	[1,4]	[2,3,5]	81.20374	69.16919	70.75834	59.8921	46.88025	70.54792	62.26926	57.85541	60.50372	[2,2,3,3,0,0,2]
4	[1,5]	[2,3,4]	76.33457	77.06865	60.73132	46.31562	44.47462	60.4046	61.37186	44.04209	54.43995	[3,2,3,2,0,2,1]
5	[2,3]	[1,4,5]	88.84943	84.37902	86.74809	36.65733	24.1599	85.56355	49.88889	49.20277	49.61444	[3,3,3,2,0,2,1]
6	[2,4]	[1,3,5]	79.718	74.83333	67.13297	59.2334	45.28369	67.03336	64.04489	57.83194	61.55971	[2,2,3,3,0,0,2]
7	[2,5]	[1,3,4]	72.54821	64.64476	61.58703	59.49978	51.57746	58.11111	64.545	53.14805	59.98622	[2,2,2,3,0,0,2]
8	[3,4]	[1,2,5]	79.47838	61.58703	77.41834	55.57721	46.19864	66.49777	64.99517	58.2405	62.2933	[2,3,1,3,0,0,2]
9	[3,5]	[1,2,4]	80.3568	69.30849	68.26859	46.9439	42.39172	55.33016	69.44824	50.2425	61.76595	[1,2,3,1,0,2,1]
10	[4,5]	[1,2,3]	61.35909	81.04404	48.19456	57.26977	50.10493	53.68735	54.08872	49.73618	52.34771	[2,2,2,1,0,0,0]
11	[3,4,5]	[1,2]	76.96015	52.71251	65.22202	49.58696	44.22889	53.01262	73.9806	49.61776	64.23547	[1,2,3,2,0,2,0]
12	[2,4,5]	[1,3]	70.11791	60.85917	59.40722	62.53389	52.56299	58.65202	64.76257	52.45538	59.83969	[2,2,2,3,0,0,2]
13	[2,3,5]	[1,4]	78.67846	68.65165	69.93324	56.38005	48.98764	62.52418	67.52925	57.06258	63.34259	[3,2,2,2,0,0,2]
14	[2,3,4]	[1,5]	85.82697	77.77283	79.10688	54.90718	43.18856	70.59563	64.50777	59.47658	62.49529	[2,3,2,3,0,0,2]
15	[1,4,5]	[2,3]	75.23178	77.44973	60.71236	49.5119	39.72869	54.82412	69.08104	55.05024	63.46872	[1,3,1,1,0,1,0]
16	[1,3,5]	[2,4]	81.88792	78.44545	70.93062	44.48977	41.39477	64.73777	61.46761	51.81332	57.60589	[3,3,2,3,0,2,1]
17	[1,3,4]	[2,5]	69.60866	64.95014	68.63931	50.04684	46.27613	62.76494	55.61313	49.52313	53.17713	[1,1,3,3,0,0,2]
18	[1,2,5]	[3,4]	77.13327	69.17987	68.1487	58.22799	50.91971	65.74428	63.18835	56.29418	60.43068	[2,3,2,3,0,0,2]
19	[1,2,4]	[3,5]	78.12169	72.86467	64.43789	53.84628	44.80584	68.27755	54.62186	42.26888	49.68067	[3,1,3,3,0,2,2]
20	[1,2,3]	[4,5]	65.42678	61.9551	55.16956	37.05392	36.68754	60.85048	36.87073	48.04867	41.3419	[1,1,2,1,0,1,0]

$$W_v = 1 - W_o \tag{5}$$

are also evaluated to make an informed decision while determining suitable regressor structures, where W_v and W_o are weights on F_v and F_o respectively. In this study, W_v is set to 0.6 and W_o to 0.4 to emphasize predictive ability over the previously unseen validation dataset while maintaining the accuracy of the model.

3.4. Initial case study using fixed ARX modeling

As an initial case study, ARX estimation of *Goal Attainment* for a representative *Just Walk* participant (Participant 230) is performed as a function of the fixed inputs *Goals, Granted Points*, and *Temperature*. An exhaustive search was conducted on over 20 combinations of estimation-validation datasets, and order selection was performed

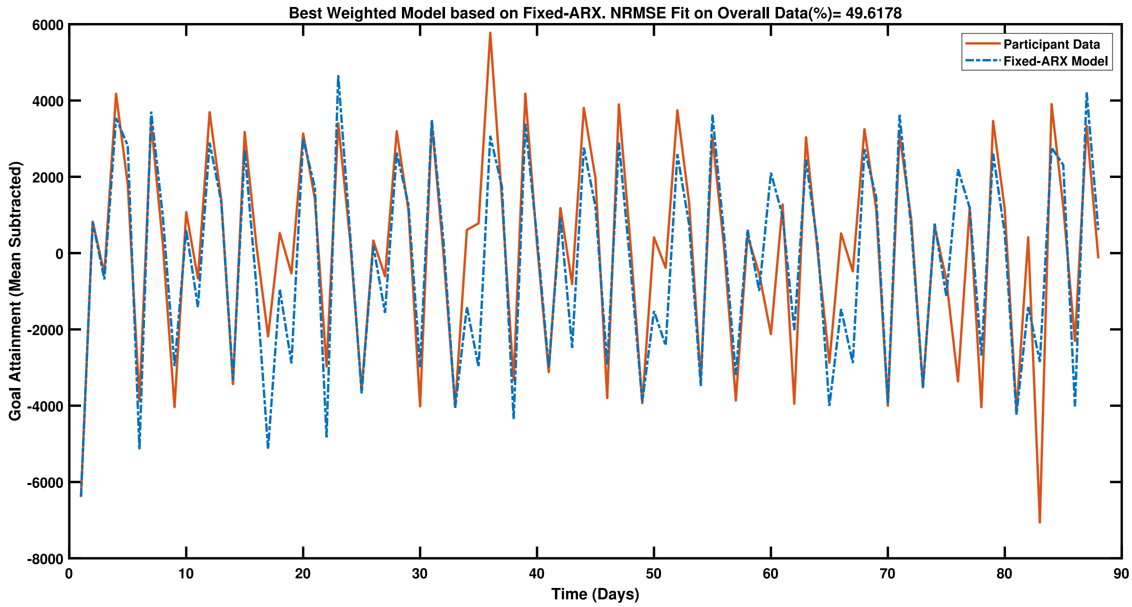

Fig. 4. Plot illustrating the simulation result of the best-weighted model with fixed inputs (*Goals*, *Granted Points*, and *Temperature*) fitted across the overall dataset for Participant 230. NRMSE Fit = 49.62% (results are mean-subtracted).

Table 2
Steady-state gains for the *Goal Attainment* model for Participant 230.

Model output	*Goals*	*Granted Points*	*Temperature*
Goal Attainment	−0.516	−6.63	−2.6

for each combination. Here, the ARX regressor orders $[n_a\ n_b\ n_k]$ are chosen to be in the range [1:3, 1:3, 0:2]. Once the dataset is available, as seen in Table 1, the model with the highest value of the weighted fit percentage is considered the most suitable model. As evident from the table, the best-weighted *Goal Attainment* model is characterized by $n_a = 1$, $n_b = [2\ \ 3\ \ 2]$, and $n_k = [0\ \ 2\ \ 0]$ structure and it corresponds to Set 11 on Table 1 with experiments {3,4,5} chosen for estimation, and {1,2} for validation. Coincidentally, this also corresponds to the model with the best fit on the validation database. When tested on the overall dataset, the model generates an NRMSE fit percentage of 49.62%, as shown in Fig. 4, which can be considered reasonable in the context of behavioral data.

The transient (step) response plots of the estimated *Goal Attainment* model are illustrated in Fig. 5. The model demonstrates a negative association between *Goals* and *Goal Attainment*, or in other words, a *Goal-to-Behavior* gain of less than one. As noted in Table 2, the steady-state gain for the *Goal-to-Goal Attainment* model is −0.516, implying that the participant fails to attain more than 50% of their daily step targets. This is indicative of high task complexity or low commitment towards attaining the goals provided. The steady-state gain constitutes a crucial measure in setting optimal targets, as the corrective actions needed for a person failing to attain 50% of their *Goals* would be very different from those needed for someone missing 10% of their *Goals*. Additionally, the underdamped dynamics displayed by the model, settling over a time period of multiple days, can further be interpreted as improved responsiveness of the individual over time, thus providing additional insights into one's adherence to the step goals. This systematic approach to analyzing behavioral data supports the process model view described earlier, and it can be highly instrumental in designing personalized adaptive interventions.

Different sets of inputs can be considered to determine the responsiveness of an individual toward different influencing factors and suitable models can be generated to explain the associations of these factors with the behavior-related outcome of the participant. However, a primary challenge with this exhaustive ARX modeling approach is the substantial computational time this may consume. Among the eight possible inputs, only three could be used to perform the calculations within a reasonable time frame, considering that order selection is achieved through exhaustive iteration. The number of possible cases increases exponentially with the features and regressor orders, and therefore, an exhaustive search through them becomes impractical. N measured input variables correspond to N possible model features. Additionally, for each set of features, the exhaustive approach demands iteration through ranges of $[n_a\ n_b\ n_k]$ for order selection, resulting in $(2^N - 1)(n_a)(n_b)^N(n_k + 1)^N$ total model combinations. For $N = 8$ and $[n_a\ n_b\ n_k] = [1{:}3,\ 1{:}3,\ 0{:}2]$, the total number of possible combinations equals $(2^8 - 1)(3)(3)^8(3)^8 \simeq 3.2 * 10^{10}$, making it impractical to traverse through all of them. Furthermore, considering that the exhaustive approach can search over a limited number of inputs due to high computational time, features need to be chosen manually. This can often result in the computation of models that fail to capture the behavior dynamics of an individual properly. This motivates us to explore the DSPSA-based stochastic search routine to identify model features and regressor structures for ARX modeling of the participant's behavior, as demonstrated in the ensuing section.

4. DSPSA: Discrete Simultaneous Perturbation Stochastic Approximation

Simultaneous Perturbation Stochastic Approximation (SPSA) is a simulation-based stochastic optimization algorithm that approximates gradient descent. It is useful for optimization problems for which there are no closed-form objective functions (or it is difficult to obtain one) and when measurements of the objective function may be noisy, both of which are the case for behavioral interventions [22]. Such problems are usually solved using finite-difference-method-based stochastic approximation (SA) algorithms of the Kiefer-Wolfowitz/Blum type [35]. However, for an objective function $\mathcal{L}(\theta) : \mathcal{R}^p \to \mathcal{R}^1$, this conventional approach requires $2p$ noisy measurements of the objective function to calculate the gradient at each iteration. The DSPSA algorithm, due to the simultaneous perturbation, requires only 2 per iteration; that is to say, it needs $1/p$ times fewer overall measurements while providing similar statistical accuracy as the conventional SA for a specified number of iterations [36]. This makes it suitable to analyze behavioral data at an accelerated pace. This paper specifically uses a discrete

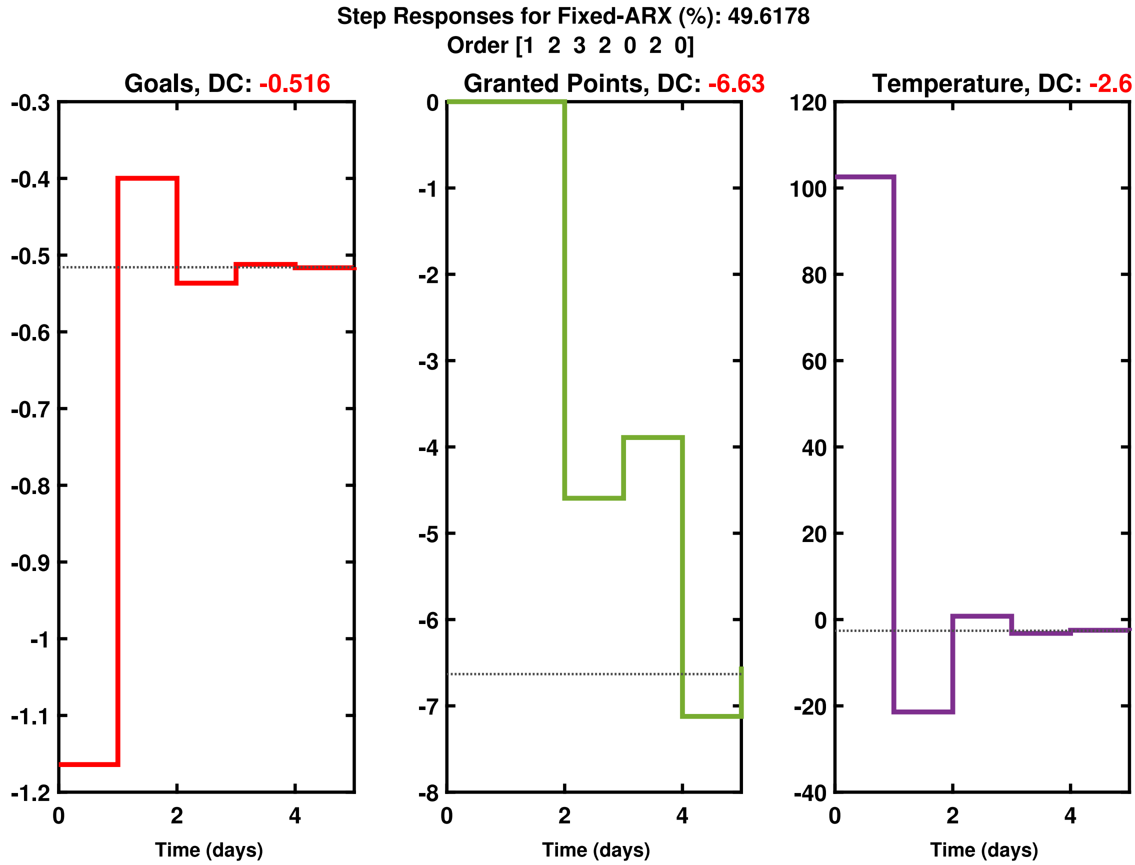

Fig. 5. Unit step responses corresponding to the inputs *Goals, Granted Points, Temperature* (left to right) of the best-weighted ARX model with fixed inputs for Participant 230 along with their corresponding steady-state gains and the regressor structures.

application of SPSA (DSPSA) since feature selection is formulated as a binary problem (in which 1 indicates that a feature is used and 0 indicates that a feature is not used in a model), and the ARX model parameters subjected to search $[n_a,\ n_b,\ n_k]$ take on integer values.

4.1. DSPSA algorithm

SPSA is initialized by first defining the variables of the search in an input vector (or multiple vectors), $\hat{\theta}$, starting with an initial guess of their values. To estimate a gradient, the input vectors are subjected to random, two-sided simultaneous perturbations, which are then used to evaluate the objective function, $J(\hat{\theta})$. These two evaluations are then used to approximate the gradient, which is subsequently used to update the values of the input vector(s). This is repeated for a user-specified number of iterations K_{iter}.

The objective function chosen for the optimization problem often takes on the form of a loss function, $L(\hat{\theta})$. However, an explicit closed-form expression of this loss function may be unavailable or difficult to obtain, such that users cannot directly compute the gradient. Instead, a noisy approximation of the objective function is used, $J(\hat{\theta}) = L(\hat{\theta})+\epsilon(\hat{\theta})$. SPSA then minimizes the approximated loss function through a process that resembles gradient descent, iterating and updating $\hat{\theta}$. SPSA can also be re-formulated as a maximization problem, as was done in the work here. For the discrete application of SPSA (DSPSA), the variables of the search ($\hat{\theta}$) take on discrete values and are rounded (and bounded if there are limitations as to what values $\hat{\theta}$ can take on) before being used to evaluate $J(\hat{\theta})$.

The following summarizes the DSPSA process for $k \in \{1, 2, 3, \dots, K_{iter}\}$ iterations, as described in [23,37]:

1. *Initialize the Input Vector and Gain Sequences*. Specify an initial p-dimensional input vector, $\hat{\theta}$, in which p corresponds to the number of features or parameters subject to stochastic search. The gain sequences a_k and c_k define the step size of each iteration and perturbation, respectively.
2. *Generate the Perturbation Vector*. Generate a perturbation vector (Δ_k) of dimension p using a Bernoulli ± 1 distribution with probability $1/2$.
3. *Create Two Input Vectors for Gradient Approximation*. From the input vector, create a new vector, $\pi(\hat{\theta}_k) = \lfloor \hat{\theta}_k \rfloor + \mathbf{1}_p/2$, in which $\lfloor \cdot \rfloor$ is the floor operator, which rounds down the values of $\hat{\theta}_k$, and $\mathbf{1}_p$ is a p-dimensional vector of ones. From $\pi(\hat{\theta}_k)$, create two input vectors for gradient approximation, $\hat{\theta}_k^+ = \pi(\hat{\theta}_k) + c_k \Delta_k$ and $\hat{\theta}_k^- = \pi(\hat{\theta}_k) - c_k \Delta_k$. Apply bounds to limit between values and round $\hat{\theta}_k^+$ and $\hat{\theta}_k^-$.
4. *Approximate the Gradient*. Evaluate the objective function $J(\cdot)$ at the bounded and rounded input vectors, $\hat{\theta}_k^+$ and $\hat{\theta}_k^-$. Use these two evaluations to approximate the gradient using a finite difference approximation:
$$\hat{g_k} = \frac{J(\hat{\theta}_k^+) - J(\hat{\theta}_k^-)}{2 c_k \Delta_k} \tag{6}$$
5. *Update the Input Vector*. Using the gradient approximation, update the input vector:
$$\hat{\theta}_{k+1} = \hat{\theta}_k - a_k \hat{g_k} \tag{7}$$
Apply bounds to limit between discrete values and round the new input vector.
6. *Report the Best Solution Vector*. Once the SPSA search has reached its final iteration, report the best solution.

For the binary application of DSPSA, the input vector $\hat{\theta}_k$ is bounded and rounded between 0 and 1. For the gain sequences, c_k is kept constant, while a_k is updated between each iteration according to Eq. (8).

Table 3
DSPSA-based ARX estimation of the *Goal Attainment* model for Participant 230. A full search is conducted over all available model inputs, and the regressor range $[n_a\ n_b\ n_k]$ = [1:3 1:3 0:1]. The top-performing model is marked in green, and the other two of the top three are marked in yellow. For the features: 1- Included in the model. 0- Not included. The objective function is the weighted average fit percentage.

	Datasets		NRMSE Fit Percentages(%)				Possible Inputs for the Model (1- Included, 0- Not Included)								Total	Regressor Structure		
Set #	Estimation	Validation	Est_avg	Val_avg	Overall	Objective	Goals	Exp. Pts.	Gr. Pts.	PredBusy	PredStress	PredTyp	Wknd	Temp	Inputs	n_a	n_b	n_k
1	{1,2}	{3,4,5}	91.81	48.81	54.28	51.00	1	0	1	0	0	1	1	1	5	2	[2,2,2,2,3]	[0,0,0,0,0]
2	{1,3}	{2,4,5}	89.25	79.59	80.70	80.03	1	1	1	0	1	0	0	0	4	3	[3,1,3,1]	[0,1,0,1]
3	{1,4}	{2,3,5}	66.86	64.74	56.91	61.60	1	0	0	0	0	0	0	1	2	2	[2,1]	[0,1]
4	{1,5}	{2,3,4}	88.59	80.67	81.08	80.84	1	1	1	0	1	0	0	0	4	3	[3,3,3,3]	[0,0,0,1]
5	{2,3}	{1,4,5}	84.64	53.73	54.24	53.93	1	0	1	1	1	1	1	0	6	3	[3,2,1,1,2,3]	[0,0,1,1,1,1]
6	{2,4}	{1,3,5}	92.35	79.53	78.85	79.26	1	1	1	1	0	0	0	1	5	2	[2,2,3,1,3]	[0,1,0,1,1]
7	{2.5}	{1,3,4}	87.13	83.11	81.89	82.62	1	1	1	0	0	1	0	0	4	3	[3,1,1,1]	[0,1,0,1]
8	{3,4}	{1,2,5}	86.86	82.79	81.55	82.29	1	1	1	0	0	0	0	0	3	3	[3,3,1]	[0,1,0]
9	{3,5}	{1,2,4}	53.66	65.78	57.82	62.60	1	0	1	0	0	0	1	0	3	1	[2,1,2]	[0,0,1]
10	{4,5}	{1,2,3}	86.40	84.06	82.26	83.34	1	1	1	0	0	0	0	0	3	1	[2,2,2]	[0,1,0]
11	{3,4,5}	{1,2}	83.95	85.78	81.71	84.16	1	1	1	0	0	0	0	1	4	2	[2,1,1,3]	[0,1,0,0]
12	{2,4,5}	{1,3}	57.15	67.14	57.74	63.38	1	0	1	0	0	1	1	1	6	2	[2,1,3,3,3]	[0,1,0,1,1]
13	{2,3,5}	{1,4}	62.11	68.66	59.55	65.02	1	1	0	0	1	0	0	1	4	1	[3,3,3,1]	[0,0,0,0]
14	{2,3,4}	{1,5}	68.37	55.71	55.11	55.47	1	1	1	0	1	0	1	0	5	2	[2,1,2,1,2]	[0,1,1,0,1]
15	{1,4,5}	{2,3}	55.34	71.42	56.14	65.31	1	0	0	0	0	0	0	1	2	1	[2,2]	[0,1]
16	{1,3,5}	{2,4}	84.77	83.15	82.37	82.84	1	1	1	0	0	0	0	1	4	1	[3,1,3,1]	[0,1,0,1]
17	{1,3,4}	{2.5}	85.96	80.37	80.94	80.60	1	1	1	0	0	0	0	1	4	1	[3,3,1,1]	[0,0,0,0]
18	{1,2,5}	{3,4}	87.40	82.43	82.85	82.60	1	1	1	1	0	1	0	0	5	2	[2,3,1,1,2]	[0,1,0,1,0]
19	{1,2,4}	{3,5}	74.90	56.94	59.20	57.84	1	1	0	1	0	0	0	0	3	2	[3,3,3]	[0,0,1]
20	{1,2,3}	{4,5}	87.68	38.88	54.56	45.15	1	0	1	1	0	1	0	0	4	1	[3,1,1,3]	[0,1,0,1]

$$a_k = \frac{a}{(k + A + 1)^\alpha} \tag{8}$$

Each of the parameters a, A, and α are determined as per [38]. The parameter A is set to $0.1K_{iter}$, in which K_{iter} is the total number of iterations. α is set to 0.501, and a is determined by choosing a value such that a_k results in initial steps of a desired magnitude, depending on the range of values being searched. For example, since n_a and n_b have a range of 1 to 3, an initial step size ($a_k g_k$) of around 0.5 was needed, so a was then chosen such that $a_k g_k$ was approximately 0.5 for the first few iterations.

4.2. DSPSA applied to Just Walk data

When applied to the *Just Walk* dataset, the DSPSA searches over a range of regressor structures and model features for a fixed number of iterations to choose a suitable model to define the participant dynamics. To demonstrate the effectiveness of DSPSA, a "full search" is conducted while choosing the features, which includes all possible combinations of all the model inputs. An exhaustive approach for such a search would be tremendously time-consuming and would be infeasible for all practical purposes. However, owing to the stochastic nature of DSPSA, the searches take place within a comparatively much shorter run-time. While a full, exhaustive search may take up to 7 h to complete, the DSPSA-based search cuts down the required time to roughly 8 min for 20 iterations over the entire search space [21]. This is achieved by taking a binary approach while assigning importance to features. The vector corresponding to the model inputs is initialized with a set of values, and all the elements are perturbed simultaneously to determine the direction of the decrease in the approximate gradient of the objective function in the regressor space. The range of variation is set to 0 and 1, where 1 denotes that the feature is needed to characterize the particular participant and 0 implies that the feature is left out of the model. On the other hand, the search over [n_a n_b n_k] is rather multi-level where each whole number within the bounds of the regressor values is considered a valid state. For the full-search, four input vectors (θ^w, θ^{n_a}, θ^{n_b}, θ^{n_k}) are initialized, where $\theta^w \in \mathbb{Z}^8$ corresponding to eight inputs to the model. The gain sequences for θ^w are specified separately from those of θ^{n_a}, θ^{n_b}, θ^{n_k} which are kept the same.

4.3. DSPSA-based ARX modeling of illustrative Just Walk participants

For a representative *Just Walk* participant (Participant 230), 20 combinations of estimation-validation datasets were generated. For each such combination, the regressor structure [n_a n_b n_k] was varied in the range [1:3 1:3 0:1]. The [θ^w, θ^{n_a}, θ^{n_b}, θ^{n_k}] vectors were initialized at [**0.5**, 2, 2, 1] where $\mathbf{0.5} \in \mathbb{Z}^8$. All eight inputs to the model were considered. For each of the 20 estimation-validation combinations, the number of iterations performed was K_{iter} = 20, and it took about five minutes to evaluate these 20 X 20 = 400 models. The gain sequences are noted in Table 4. The best models (according to the weighted objective function) corresponding to each estimation-validation combination are documented in Table 3. Once these were generated, further refinement was done by selecting the model with the highest weighted fit among all the combinations (set 11 in Table 3, marked in green). The corresponding estimation-validation cycles comprised the experiments {3,4,5} for estimation and {1,2} for validation. The model was characterized by four inputs: *Goals, Expected Points, Granted Points, and Temperature* and the polynomial order corresponding to these inputs were given by n_a = 2, n_b = [2 1 1 3], n_k = [0 1 0 0]. This model was considered to possess the highest representative ability, and it generated a weighted average NRMSE fit of 84.15%. It was observed that the same model when tested on the overall dataset, generated a fit percentage of 81.7%, as illustrated in Table 3 (also see Fig. 8), which is substantially higher than the model attained through fixed-ARX modeling with a 49.62% fit. Interestingly, this model was also the one with the highest predictive ability, with a fit of 85.78% over the validation dataset. This is an important measure to gauge the chances of overparameterization, as a high value of the average validation fit percentage provides confidence that the best-weighted model does not run the risk of being overdetermined. The superior nature of the models generated through this process can be ascribed to the fact that DSPSA allows us to search over an expanded set of features and regressor structures, thus enabling us to pick up high-performing models within reasonable amounts of computational time. To gain a deeper understanding of the DSPSA-based selection process, the search over 20 iterations for the particular dataset that resulted in the best fit model (obtained from the estimation-validation combination {3,4,5}–{1,2}) is discussed below.

Table 4
Gain sequence values for Participant 230.

Input vector	a	c_k	A	α
θ^w	0.003	0.1	2	0.501
θ^{n_a}, θ^{n_b}, θ^{n_k}	0.1	1	2	0.501

Fig. 6 illustrates the values of [n_a, n_b, n_k] regressor terms and the feature choices examined by the algorithm at each iteration. As depicted in the subplot for feature selection (top left, Fig. 6), for each iterate, the DSPSA evaluates behavioral models using a subset of features, by either considering (value = 1) or discarding (value =

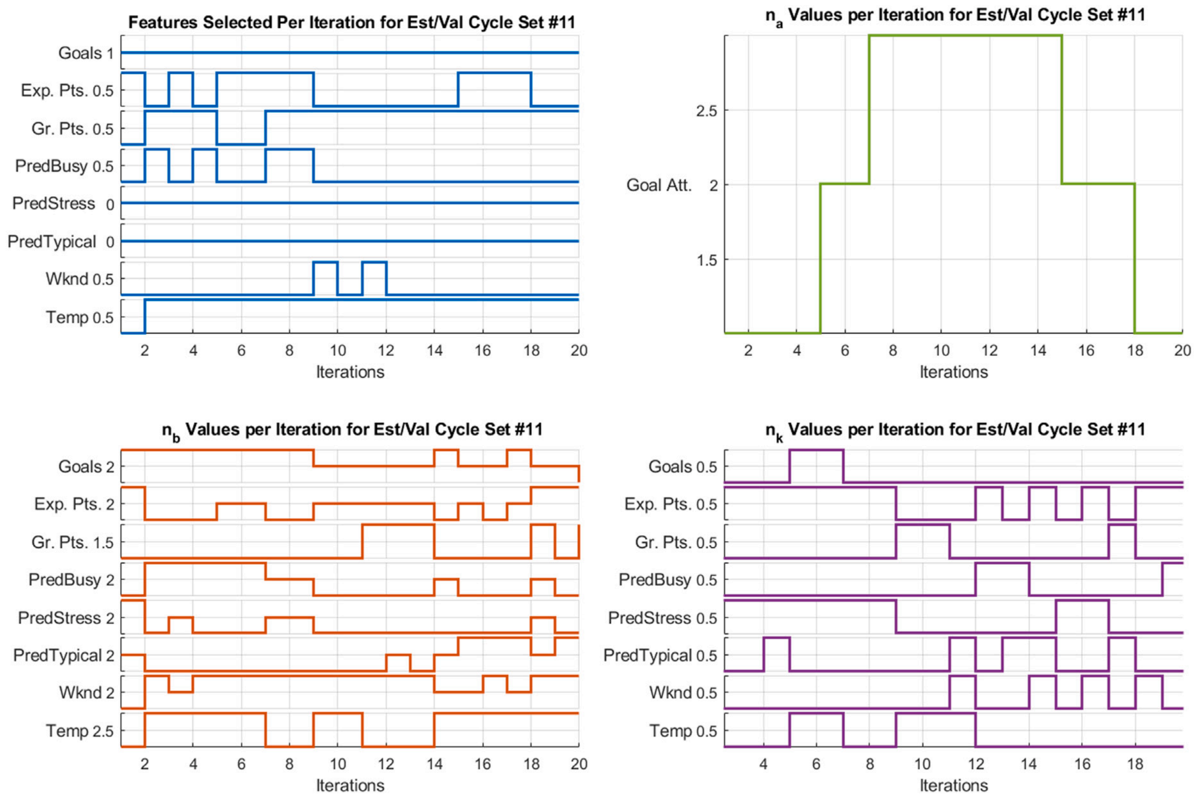

Fig. 6. Plots illustrating the iterations over DSPSA inputs for Participant 230. These inputs refer to the vector (θ^{w}, θ^{n_a}, θ^{n_b}, θ^{n_k}) corresponding to the features and the regressors $[n_a, n_b, n_k]$. The set of values for these inputs that generate the maximum value for the objective function over the 20 iterations is used to characterize the participant.

0) a given input. Simultaneous operations are performed on n_a (top right, Fig. 6), n_b (bottom left, Fig. 6), and n_k (bottom right, Fig. 6) by considering their values from the prespecified set of choices. 20 such models, each fully characterized by their [θ^{w}, θ^{n_a}, θ^{n_b}, θ^{n_k}] vector, are generated and validated using various measures for each iteration. The corresponding NRMSE fit percentages are plotted in Fig. 7 for visualization, and it is noted that the best representing model described earlier in Table 3, corresponds to iteration 16 of this search (see also Fig. 8).

One must note that the model obtained from this process is not necessarily the optimal one. The best-weighted model is based on the number of iterations specified by the user, 20 for this case, and is greatly influenced by the stochastic nature of the algorithm, and the non-convexity of the objective function. However, it can be observed from Table 3, that the other two models (marked in yellow) among the three top-performing ones are characterized by similar sets of model inputs, with slight variations in regressor structure, thus giving us a fair idea about the most influential features for understanding the given individual's *Goal Attainment*.

Participant 230 can be considered to be an "operant learner", in other words, one who follows the inputs provided to them in an adherent manner and achieves their step goals daily, as observed from Fig. 3. Such participants are considered to be highly engaged and thus viewed as desirable for the intervention. Nonetheless, to establish a broader appeal of the modeling approach beyond these ideal settings, we analyze a less adherent participant, namely, Participant 180. This individual fails to exceed their daily step goals as observed from Fig. 9. A DSPSA-based search over the defining features and regressor structure for Participant 180 reveals the inputs *Goals, Expected Points, Granted Points, Predicted Busyness*, and *Weekday/Weekend* to be the instrumental constructs for characterizing this person's *Goal Attainment*. The best-fit model comprises a regressor structure of $n_a = 1$, $n_b = [2\ 2\ 3\ 2\ 1]$, and $n_k = [0\ 1\ 0\ 1\ 1]$ with an NRMSE fit percentage of 58.3% on the overall data, and it utilizes experiments {1,2,3} for estimation and {4,5} for validation. While the fit on the overall data is comparatively lesser than that of Participant 230, the simulation results illustrated in Fig. 10 demonstrate that the model sufficiently predicts the person's *Goal Attainment* using the requisite features.

Table 5
Steady-state gains for top three models for Participant 230.

Set	*Goals*	*Expected Points*	*Granted Points*	*Temperature*
11 (Best weighted model)	0.089	−43.8	43.4	4.97
10	0.065	−40.9	40.4	–
16	0.098	−46.5	46.6	1.59

To better understand the nature of the influence of various inputs on the output for different participants, we study the magnitude and direction of the step responses of the models generated through the DSPSA-based ARX modeling for both participants. Fig. 11 illustrates the response of *Goal Attainment* to unit steps in the four inputs corresponding to the best-weighted model for Participant 230. The step responses of *Goals* and *Granted Points* are characterized by positive steady-state gains as anticipated from a behavioral standpoint. This indicates that the increase in these inputs would motivate a person to attain their daily step goals. Additionally, the plot for *Goals* shows second-order behavior with an inverse response. From the given step-response, one can infer that a unit increase in *Goals* is marked by a comparatively slower increase in the participant's *Behavior* which eventually dominates in magnitude, thus generating the inverse response with a positive gain.

Furthermore, contrary to our intuitive understandings, the negative and positive gains in response to unit increases in *Expected Points* and *Temperature* respectively strongly indicate the need for idiosyncratic models for describing individual behaviors. Table 5 lists the steady-state gains corresponding to the inputs for the top three best-weighted models. It can be seen that the steady-state gains are very similar in magnitude and direction for these models, which demonstrates consistency in modeling participant-specific dynamics.

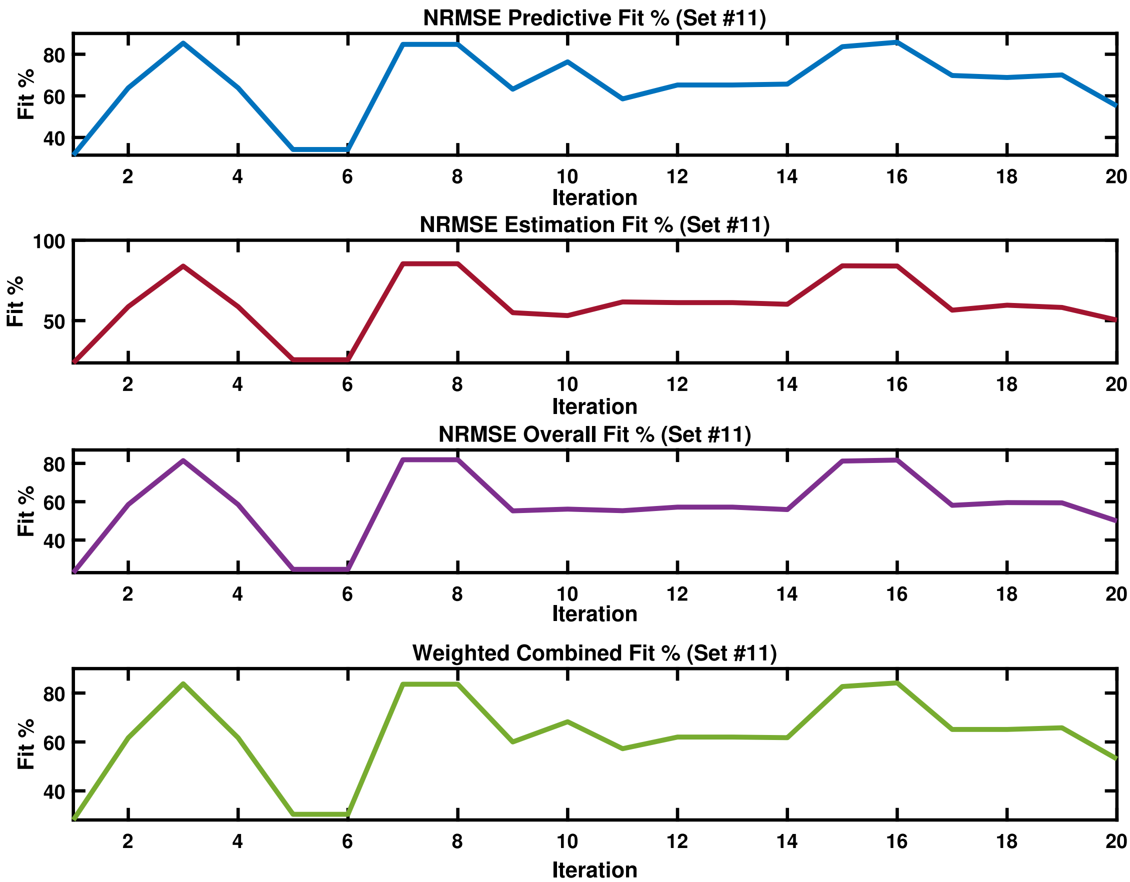

Fig. 7. Plot illustrating the NRMSE model fits over the validation dataset, estimation dataset, overall dataset, and the objective function (top to bottom in that order) for 20 iterations for the*Goal Attainment* model of Participant 230 for the estimation-validation combination {3,4,5}–{1,2}. The objective function is the weighted average of the NRMSE fits over the prediction ($W_v = 0.6$) and the overall data ($W_o = 0.4$). 20 iterations were performed and the DSPSA objective was maximized at iteration 16 with a weighted NRMSE fit of 84.15%.

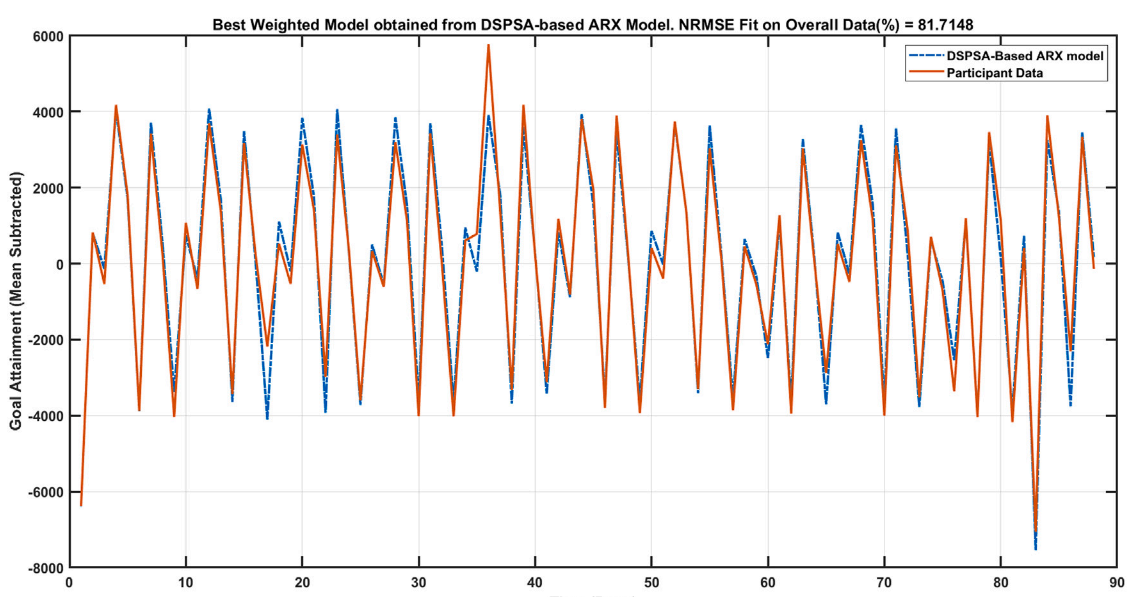

Fig. 8. Plot illustrating the simulation result of the DSPSA-based best-weighted ARX model for *Goal Attainment*, fitted across the overall dataset for Participant 230. NRMSE Fit = 81.71% (results are mean-subtracted).

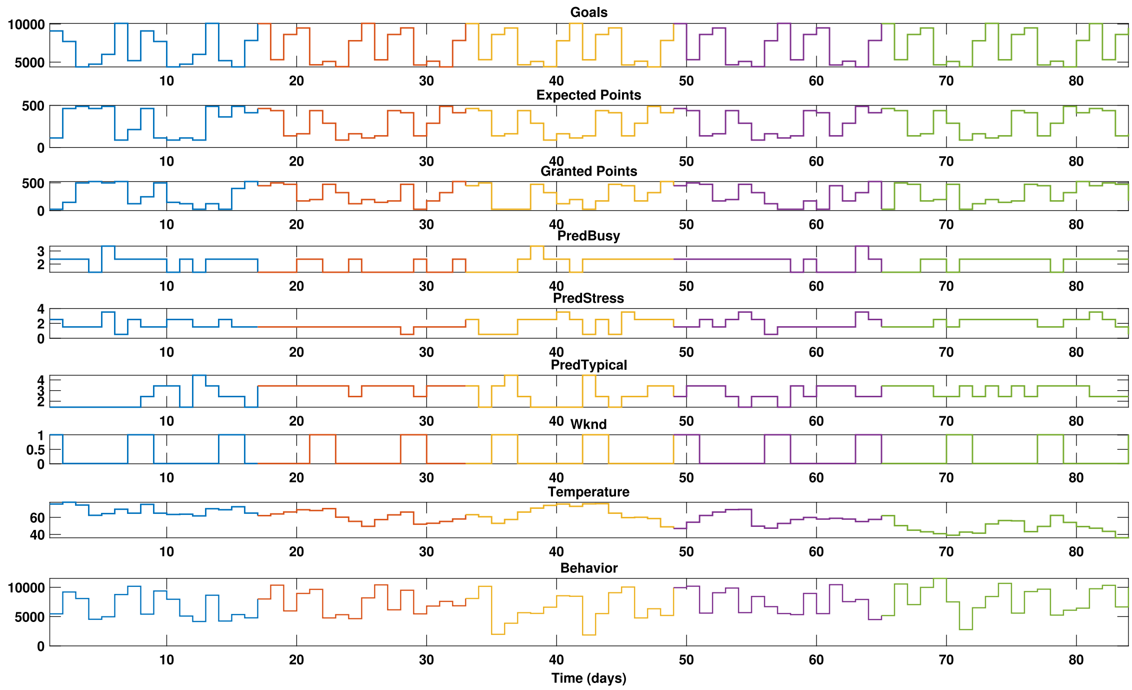

Fig. 9. *Just Walk* time series data for the measured output *Behavior* and the eight inputs for Participant 180.

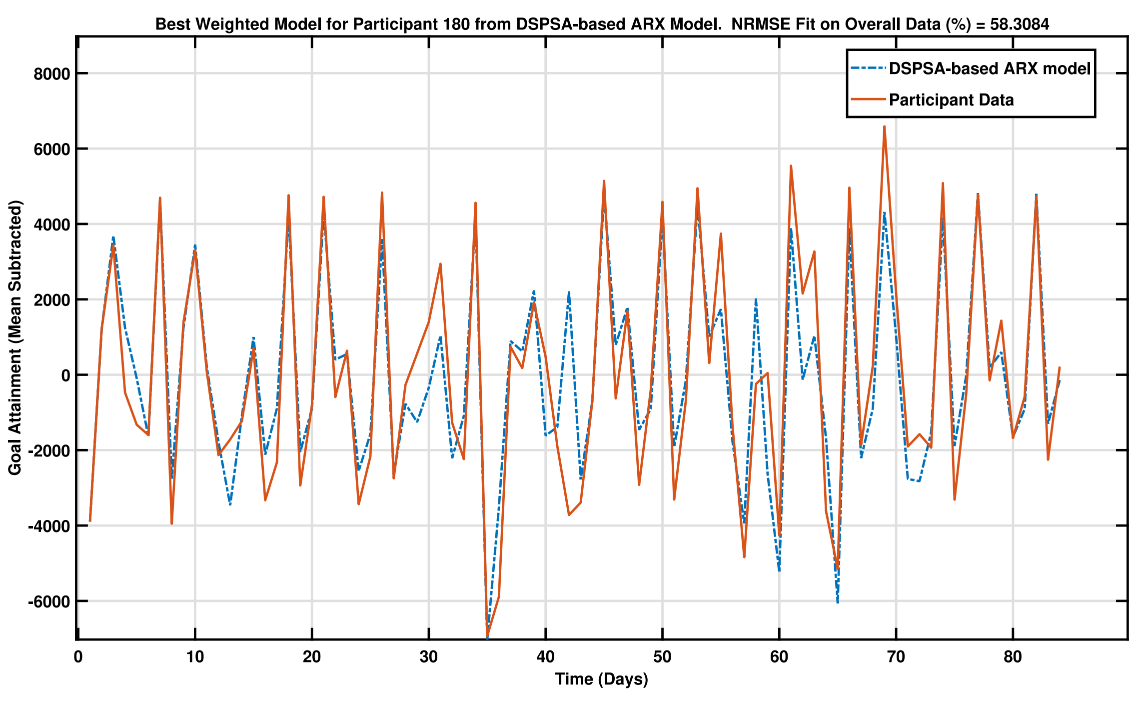

Fig. 10. Simulation results for the *Goal Attainment* model of Participant 180. NRMSE fit on mean-subtracted overall data = 58.3%.

Similarly, for Participant 180, a study of the step responses of the best-fit model (see Fig. 12) shows the association of the *Goals* and the *Goal Attainment* to be negative (steady-state gain = −0.136). This implies that the step goals do not sufficiently motivate the person to attain them on a daily basis. This is in contrast to what was observed for Participant 230, who demonstrated a positive *Goal*-to-*Goal Attainment* gain. However, the response to *Expected Points* with a negative gain of −10.3 for this participant is observed to be similar to the best-fit model of Participant 230. Furthermore, it is observed that an increase in *Granted Points* incentivizes the person to take more steps (steady-state gain = 12.3). Additionally, while *Busyness* influences the person to walk more with a significant increase in daily step count (steady-state gain = 753), the presence of *Weekend* reduces the number of steps taken by the person (steady-state gain = −622). It can be, therefore, inferred that the participant's physical activity is greatly shaped by their daily schedule, and the models involving these relevant constructs can suitably predict their behavior.

The use of DSPSA for ARX model estimation on the two *Just Walk* previously studied participants gives confidence that this method represents a general-purpose computational tool to examine other participants, as summarized in Table 6. It can be seen that the inputs *Goals*, *Expected Points*, and *Granted Points* are common among all participants,

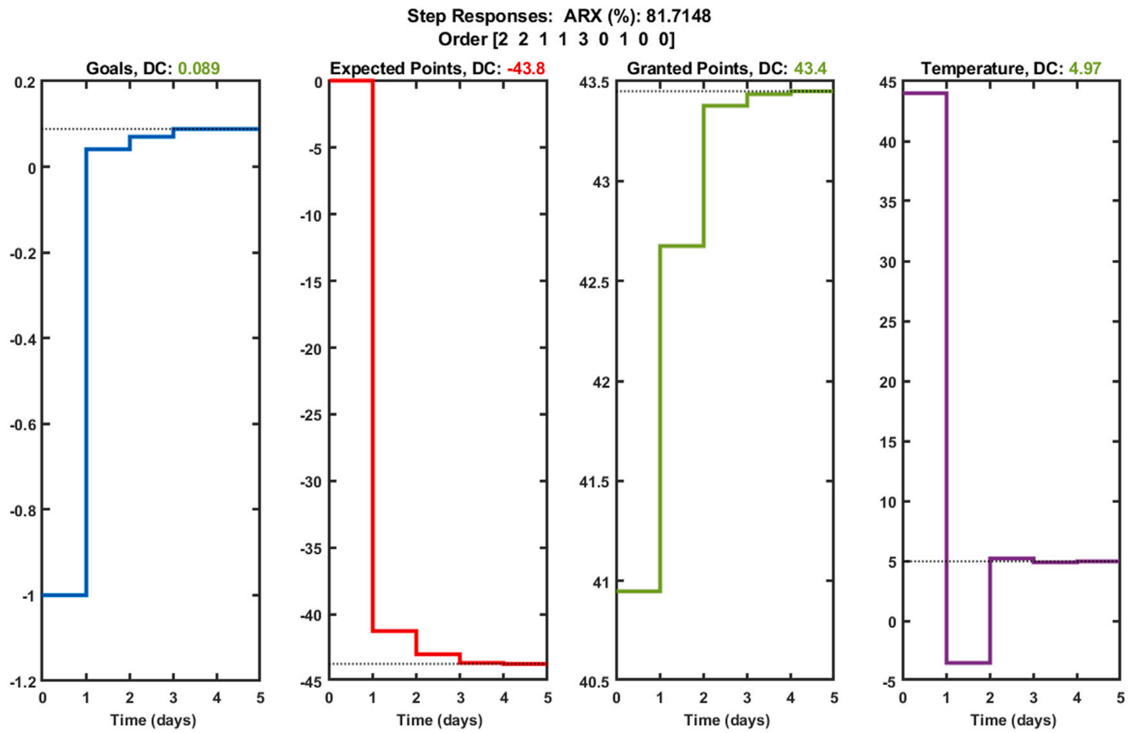

Fig. 11. Figure illustrating unit step responses of the *Goal Attainment* model for Participant 230 corresponding to the inputs *Goals, Expected Points, Granted Points, Temperature* (left to right) of the DSPSA-based best-weighted ARX model along with their corresponding steady-state gains and the regressor structures.

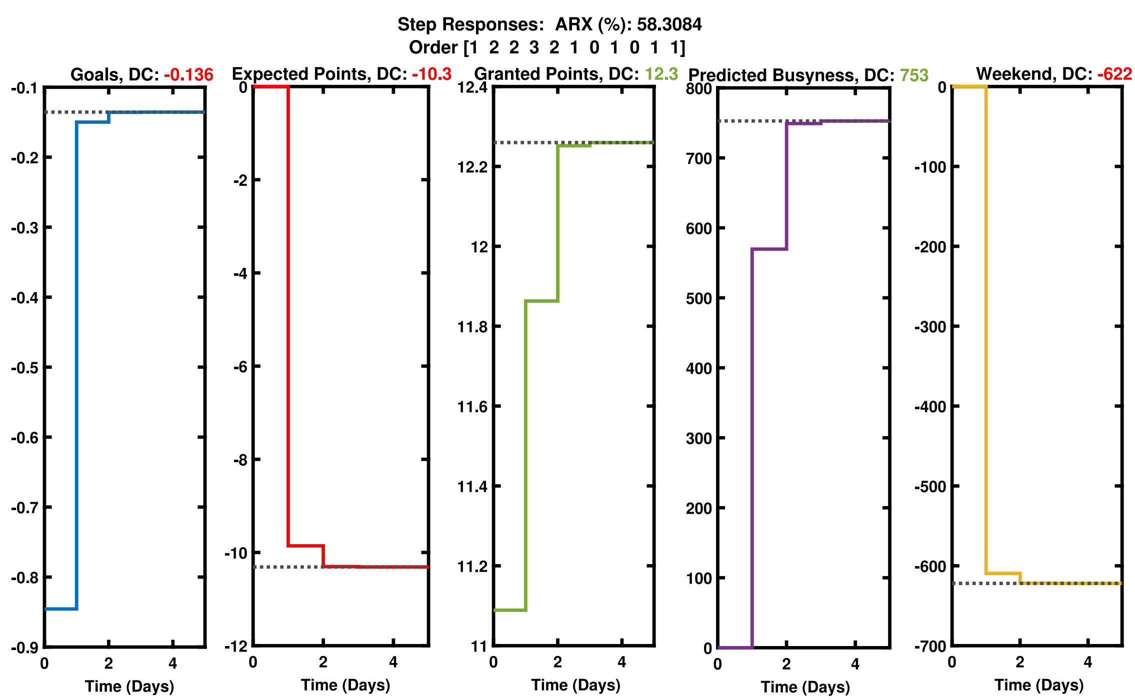

Fig. 12. Figure illustrating unit step responses of the *Goal Attainment* model for Participant 180 corresponding to the inputs *Goals, Expected Points, Granted Points, Temperature* (left to right) of the DSPSA-based best-weighted ARX model along with their corresponding steady-state gains and the regressor structures.

there are significant individual differences in terms of other influencing factors that determine a person's behavior. For example, the walking behavior of Participant 180 is influenced by the availability of the weekend and is not affected by the environmental context *Temperature*. Similarly, the behavior of Participant 008 is dictated by their day-to-day schedule. Their physical activity can be induced by stress, or a busy schedule needing daily commutes, and evaluating models based on these constructs can help predict their behavior greatly. Interestingly, Participant 222 can be seen to be influenced by similar sets of inputs as Participant 230, and yet they need different regressor structures and estimation-validation cycles to best characterize them; such differences highlight the need for personalized behavioral interventions. The high

Table 6
Table summarizing additional participant examples using DSPSA-based ARX modeling. The varied model features and regressor structures for different participants highlight the utility of DSPSA in the idiographic modeling of physical activity. For the dataset: 1- Estimation cycle, 0- Validation cycle.

Part.	*Goals*	*Expected Points*	*Granted Points*	*Pred Busy*	*Pred Stress*	*Pred Typ*	$Wknd$	$Temp$	Weighted fit (%)	n_a	n_b	n_k	Est/Val cycle
230	✓	✓	✓					✓	81.78	2	[2 1 1 3]	[0 1 0 0]	[0 0 1 1 1]
180	✓	✓	✓	✓			✓		58.30	1	[2 2 3 2 1]	[0 1 0 1 1]	[1 1 1 0 0]
198	✓		✓						44.05	2	[2 2]	[0 0]	[0 0 1 1 1]
008	✓	✓	✓	✓	✓	✓			53.54	2	[3 2 1 1 1 1]	[1 0 1 0 1 0]	[1 1 0 0 1]
222	✓	✓	✓					✓	60.49	1	[3 3 1 3]	[0 1 0 0]	[1 0 1 1 0]

values for the weighted fits indicate the effectiveness of the *Just Walk* study methodology and the importance of a data-driven, process system approach to generating personalized behavioral interventions.

5. DSPSA-ARX in a closed-loop intervention setting

An important consideration relating to building models with behavioral data is the reliability of estimated linear ARX models in the face of possibly highly uncertain and time-varying human behavior. It is well understood that any estimated model comes with the caveat of the inability to account for all possible influencing factors, simply due to their multitude and inability to measure. However, understanding the interplay between dynamic modeling and control design provides some answers when the intended purpose of the model is closed-loop control [39].

The answer lies in the fact that while the scope of this work is on dynamic modeling and feature selection, the overarching aim of this study concerns not just the predictive modeling of participant behavior subject to various influencing factors but also the formation of prescriptive actions that guide and improve their physical activities using "ambitious but doable" goals throughout the study. To this end, the models generated using the DSPSA-ARX approach are deployed in a closed-loop setting in synergism with a model-based control framework that generates corrective actions in real time. This falls under the umbrella of the ongoing Control Optimization Trial (COT) based *YourMove* intervention [40], where a 3-degree-of-freedom hybrid model predictive control scheme (3DoF HMPC) [12,25,26] is being successfully validated experimentally for disseminating *Goals, Expected Points*, and *Granted Points* to the participant on a daily basis. The 3DoF architecture uses two intuitively formulated Kalman Filters to decouple the estimated effects of measured and unmeasured disturbances on the actual participant behavior and allows the user to independently adjust the controller parameters relating to the mitigation of these disturbances and the desired speed of response for tracking the reference trajectory. Such formulation can be readily tuned for robustness against model uncertainty/plant-model mismatch. In other words, the integrated modeling and control approach shifts the engineering efforts needed for mitigating plant-model mismatch from the identification phase to the closed-loop control phase where such issues are better managed in a real-time setting. The utility of the DSPSA-ARX models in such scenarios lies in their ability to preserve the interpretability and ease of use of linear modeling techniques that can be seamlessly implemented in closed-loop control. This is critical to personalized behavioral interventions where transparency of the control actions imposed on real-life participants is of significant importance.

6. Summary and conclusions

This paper describes how a process-oriented, control engineering-based approach using ARX estimation and stochastic search algorithms like DSPSA can be used to handle computationally demanding, person-specific behavioral data by expanding the search space significantly compared to conventional exhaustive estimation. The paper demonstrates the ability of DSPSA to manage behavioral data efficiently and to search over multiple features and model orders in limited time, thus allowing the user to gain insights into the dynamics of an individual participant's behavior as an outcome of various internal and external factors. In particular, the integration of behavior change theory and control engineering principles allows us to consider behavioral intervention as a process system problem. Keeping this in mind, *Just Walk* is currently being extended in the form of the *YourMove* intervention [40] which implements a full Control Optimization Trial (COT; [11]) to promote healthy behavior. For future work, the algorithm can be implemented to consider objective functions beyond the traditional NRMSE fit criterion to evaluate the performance of behavioral models within idiosyncratic settings. Furthermore, data-driven modeling algorithms such as Model-on-Demand (MoD) can be used in synergism with DSPSA to address nonlinearity in a more systematic manner [41,42]. Such formulation allows the generation of models with time-varying parameters to reckon the nonlinearities in the behavioral dynamics, thereby generating more efficient forecasts. In summary, DSPSA can consistently be used to design personalized behavioral interventions within a significantly short amount of time frame. The methodology presented is also broadly applicable to similar control-relevant system identification problems that can be found in the process industries.

CRediT authorship contribution statement

Sarasij Banerjee: Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Rachael T. Kha:** Writing – review & editing, Methodology, Investigation, Conceptualization. **Daniel E. Rivera:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Investigation, Funding acquisition, Conceptualization. **Eric Hekler:** Writing – review & editing, Supervision, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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