Homework

(1) Suppose that the change in the value of a portfolio over a 10-day time horizon is normally distributed with a mean of zero and a standard deviation of 20 million. Compute the 10-day 99% VaR and ES for the portfolio.

(2) Consider two independent random variables L_1 and L_2 with the following distributions:
- $P(L_1 = 0) = 0.98$, $P(L_1 = 10) = 0.02$
- $P(L_2 = 0) = 0.98$, $P(L_2 = 10) = 0.02$

1. Prove that $\mathrm{VaR}_{0.98}$ is not subadditive.
2. Determine whether $\mathrm{VaR}_{0.99}$ is subadditive.
3. Find $\mathrm{ES}_{0.98}$ for $L_1, L_2, L_1 + L_2$, respectively.

(3) A bank has two 10 million one-year loans. The probabilities of default are as follows:
- $\mathbb{P}$(Neither loan defaults) = 0.975
- $\mathbb{P}$(Loan 1 defaults, Loan 2 does not default) = 1.25%
- $\mathbb{P}$(Loan 2 defaults, Loan 1 does not default) = 1.25%
- $\mathbb{P}$(Both loans default) = 0%

If a default occurs, all losses between 0% and 100% of the principal are equally likely. If the loan does not default, a profit of 0.2 million is made.

1. Compute one-year 99% VaR for the Loan 1.
2. Compute one-year 99% VaR for the bank loan portfolio.
3. Compute the one-year 99% expected shortfall (ES) for the Loan 1 and bank loan portfolio, respectively.

(4) VaR monotonicity says that If $L_1 \leq L_2$, then $\mathrm{VaR}_\alpha(L_1) \leq \mathrm{VaR}_\alpha(L_2)$. Justify the following statement:

If $L_1 < L_2$ almost surely, then $\mathrm{VaR}_\alpha(L_1) < \mathrm{VaR}_\alpha(L_2)$ for any $\alpha < 1$.

(5) Prove that if f is an increasing function and $X = f(Z)$ for some random variable Z, then $\mathrm{VaR}_\alpha(X) = f(\mathrm{VaR}_\alpha(Z))$.

(6) Prove the monotonicity of Expected Shortfall (ES).