Homework

(1) Consider a discrete random variable X with the following probability mass function (pmf):

$$P(X = x) = \begin{cases} 0.2 & \text{if } x = 1, \\ 0.5 & \text{if } x = 2, \\ 0.3 & \text{if } x = 3, \\ 0 & \text{otherwise.} \end{cases}$$

   a. Find the cumulative distribution function (CDF) of X.
   b. Compute the expected value $\mathbb{E}[X]$.
   c. Compute the variance $\mathrm{Var}(X)$.

(2) The loss from a portfolio during six months is normally distributed with mean 2 million and standard deviation 10 million. What is the probability that the loss is larger than 12 million?

(3) The loss from a portfolio during six months follows a t-distribution with 5 degrees of freedom, mean 2, scale 10. What is the probability that the loss is larger than 12 million?

(4) Let $X \sim T(6, 0, 100)$. Find $\mathbb{P}(F(X) > 0.5)$, where F is the CDF of X.

(5) Let X be a random variable with finite second moment, i.e., $E[X^2] < \infty$.

   1. Show that $E[|X|] < \infty$.
   2. Show that $Var(X) = E[X^2] - (E[X])^2$.
   3. Show that $\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$ for any constants a, b.
   4. Show that $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$ for any constants a, b.