HOMEWORK

(1) Let $Y = X^{2n}$ for some positive integer n, where $X \sim \mathcal{N}(0, 1)$. Prove that
  (a) X and Y are uncorrelated, i.e. $\mathbb{E}[XY] = 0$.
  (b) X and Y are not independent.

(2) Let X and Y be random variables with joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{if } 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

  (a) Find the marginal densities $f_X(x)$ and $f_Y(y)$.
  (b) Are X and Y independent? Justify your answer.
  (c) Compute $\text{Cov}(X, Y)$ and $\text{Corr}(X, Y)$.

(3) Consider two random variables X and Y such that:

$$X \sim \text{Uniform}(-1, 1)$$

$$Y = \begin{cases} 1 & \text{if } |X| < 0.5 \\ -1 & \text{if } |X| \geq 0.5 \end{cases}$$

  (a) Show that $\mathbb{E}[X] = 0$ and $\mathbb{E}[Y] = 0$.
  (b) Compute $\text{Cov}(X, Y)$ and show that X and Y are uncorrelated.
  (c) Are X and Y independent? Explain why or why not.

(4) Suppose that the current daily volatilities of asset A and asset B are 1.6% and 2.5%, respectively. The prices of the assets at close of trading yesterday were \$20 and \$40 and the estimate of the coefficient of correlation between the returns on the two assets made at that time was 0.25. The parameter λ used in the EWMA model is 0.95.
  (a) Calculate the current estimate of the covariance between the assets.
  (b) On the assumption that the prices of the assets at close of trading today are \$20.50 and \$40.50, update the correlation estimate.

(5) Let (X, Y) follow a bivariate normal distribution with parameters $\mu_X = 0$, $\mu_Y = 0$, $\sigma_X^2 = 1$, $\sigma_Y^2 = 4$, and correlation coefficient $\rho = 0.6$. Find $\mathbb{P}(Y > 2X + 1)$.