HOMEWORK

(1) Let $F(x,y) = (xy)^2$ for $x, y \in [0,1]$. Find the marginals $F_1(x), F_2(y)$ and the copula $C(u,v)$ such that $F(x,y) = C(F_1(x), F_2(y))$.

(2) Consider (X,Y) with joint cdf $F(x,y) = xy$ for $x, y \in [0,1]$. Find the copula of $(e^X, \ln(1+Y))$.

(3) Let $b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. Show that, if Y is a 2-dimensional random vector, $X = (D^2 + I)(Y - b)$ and Y has the same copula, where I is the identity matrix.

(4) Let (X,Y) follow a bivariate normal distribution $\mathcal{N}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}\right)$. Find its correlation matrix.

(5) Consider one-factor model:

$$Z_i = \sqrt{\rho} Y + \sqrt{1-\rho}\,\epsilon_i, i = 1, 2, 3,$$

where $Y, \epsilon_i \sim N(0,1)$ are independent, and $\rho \in (0,1)$. Find the correlation matrix R and covariance matrix Σ of (Z_1, Z_2, Z_3).