Homework

(1) In Merton's model, show that the default probability is decreasing as the equity value increases. In particular, if the equity value goes to infinity, prove that the default probability goes to zero.

(2) Show that, under the Merton's model, the credit spread on a T-year zero bond (with face value 1) is given by:

$$\text{Credit Spread} = -\frac{\log\left(V_0 e^{rT}\Phi(-d_1) + \Phi(d_2)\right)}{T}.$$

where

$$d_1 = \frac{\log(V_0) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

(3) Consider a firm with the following parameters:
- Current asset value: $V_0 = \$120$ million
- Debt obligation: $B = \$90$ million
- Asset volatility: $\sigma = 0.25$ (25% per year)
- Drift rate: $\mu = 0.06$ (6% per year)
- Risk-free rate: $r = 0.04$ (4% per year)
- Time to maturity: $T = 2$ years

Answer all questions.
1. Compute the default probability $P(V_T \leq B)$ using the Merton model framework.
2. Compute the equity value S_0 using the risk-neutral valuation approach.
3. Compute the debt value B_0 using the risk-neutral valuation approach.
4. Compute the credit spread using the risk-neutral valuation approach.
5. Compute the default probability under the risk-neutral measure.
6. Compute the rate of the expected loss on the debt.
7. Compute the Delta of the equity.
8. Compute the equity volatility.
9. Compute the recovery rate on the debt.

(4) Consider a firm with the following parameters:
- The value of a company's equity is 3 million and the volatility of the equity is 80%.
- The debt that will have to be paid in one year is 10 million.
- The risk-free rate is 5% per annum.

Answer all questions.
1. Find the current asset value V_0 and the asset volatility σ_V.
   Hint: S_0 and σ_S are given instead of V_0 and σ_V. One may solve these two equations:

   $$S_0 = V_0\Phi(d_1) - Be^{-rT}\Phi(d_2)$$

   $$S_0\sigma_S = \Phi(d_1)V_0\sigma_V$$

   This is a system of two nonlinear equations in two unknowns and can be solved using numerical methods.
2. Compute the default probability under risk-neutral measure.
3. Compute the recovery rate on the debt.