Homework

(1) The volatility of an asset is 2% per day. What is the standard deviation of the percentage price change in three days?

(2) Calculate the implied volatility for a European call option with the following parameters:
- Current stock price: $S_0 = \$120$
- Strike price: $K = \$115$
- Time to expiration: $T = 0.5$ years (6 months)
- Risk-free rate: $r = 3\%$
- Market price of the call option: $C_{market} = \$8.75$

(3) The most recent estimate of the daily volatility of an asset is 1.5% and the price of the asset at the close of trading yesterday was \$30.00. The parameter $\lambda$ in the EWMA model is 0.94. Suppose that the price of the asset at the close of trading today is \$30.50. How will this cause the volatility to be updated by the EWMA model?

(4) Suppose that the price of an asset at close of trading yesterday was \$300 and its volatility was estimated as 1.3% per day. The price at the close of trading today is \$298. Update the volatility estimate using

(a) The EWMA model with $\lambda = 0.94$

(b) The GARCH(1,1) model with $\omega = 0.000002$, $\alpha = 0.04$, and $\beta = 0.94$.

(5) Consider GARCH(1,1) model for daily volatility:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

with parameters $\omega = 0.0000013465$, $\alpha = 0.083394$, $\beta = 0.910116$.

(a) Recall that $\omega = \gamma V_L$, where $\gamma + \alpha + \beta = 1$ and $V_L$ is the long-term variance daily rate. Find the long-term volatility.

(b) We assume that the expected value and the variance of $u_n$ are 0 and $\sigma_n^2$, respectively. Suppose $\sigma_0$ is 1.732% per day, find the expected daily volatility after 10-days and 500-days.

(c) Prove that the expected daily volatility converges to the long-term volatility as $n \to \infty$.

(d) For the option of maturity $T$, we denote the predicted volatility per annum as $\sigma(T)$. Find $\sigma(T)$ as 10-days and 500-days, i.e. $T = 10/252, 500/252$.

(e) Suppose there is 1% increase for the current volatility per annum, what is the actual change in $\sigma(T)$ for $T = 10/252, 500/252$?

(f) If we use the approximation

$$\Delta\sigma(T) \approx \frac{1 - e^{-aT}}{aT} \frac{\hat{\sigma}_n}{\sigma(T)} \Delta\hat{\sigma}_n,$$

what is the approximated change in $\sigma(T)$ for $T = 10/252, 500/252$?

Solution

(1) $2\sqrt{3}\%$
(2) 14.12%
(3) $u_0 = .5/30, \sigma_0 = 1.5\%, \lambda = 0.94$, thus $\sigma_1 = \sqrt{(1-\lambda)u_0^2 + \lambda\sigma_0^2} = 1.51\%$
(4) skip
(5) (text chap 10, p219)
  (a) 1.4404% per day.
  (b) 1.72% and 1.45%
  (c) skip
  (d) 27.36 and 24.32
  (e) skip
  (f) 0.97 and 0.33