Homework

(1) Suppose i/2-year zero rates (semi-annual) are quoted as

$$r_1 = 5\%, r_2 = 5.8\%, r_3 = 6.4\%, r_4 = 6.8\%, r_5 = 7\%, r_6 = 7.1\%.$$

Consider 3-year bond, principal = 100, semi-annual coupon rate = 5%
  (a) Find the bond price.
  (b) Find the YTM.

(2) Semi-annual Coupon bonds with a maturity $T = 10$ years, face value $F = \$10,000$, and annual coupon rate $q = 4\%$ are currently trading at \$9,060 per bond. Coupon bonds with maturity $T = 10$ years, face value \$5,000 and annual coupon rate $q = 8\%$ are currently trading at \$6,146.50 per bond. Determine the value of the following securities:
  (a) A zero coupon bond with face value \$20,000 and maturity 10 years.
  (b) An annuity that will make payments of \$500 twice per year for the next 10 years.
  (c) A coupon bond with maturity 10 years, face value \$7,500 and annual coupon rate $q = 6\%$.

(3) Assume that the spot curve is flat at 6%. A portfolio consists of three zero-coupon bonds each having a face value of \$1,000,000. The maturities of the bonds are 2, 5, and 10 years.
  (a) Find the DV01 and duration of the portfolio of bonds.
  (b) Suppose that you are asked to purchase a single semiannual par-coupon bond having maturity of 7 years such that the DV01 of the par-coupon bond will match the DV01 of portfolio of zero-coupon bonds. What should the face of the par-coupon bond be?
  (c) Suppose that the 2 year spot rate increases by 50 basis points (bp), the 5 year rate increases by 42 bp and the 10 year rate increases by 38 bp. Compute the exact price change of the portfolio of the zero coupon bonds.
  (d) Now assume a *parallel* shift in the (flat) spot curve from parts $(a), (b)$: i.e. all the spot rates move by the same amount. What parallel shift would explain the price change in part (c)? Try to find a shift which exactly explains the price change. Is this possible? What if you use the first order approximation implied by the portfolio's duration? What shift does this yield? Compare the results.

(4) Consider a zero-coupon bond with face value \$10,000 and maturity 10 years. Assume that semiannual spot rate $\hat{r}(10) = 4.8736\%$.
  (a) Compute the convexity of the bond.
  (b) Compute the exact price change in the bond corresponding to a 35 bp increase in $\hat{r}(10)$.
  (c) Compute the first-order approximation to the price change in the bond for a 35 bp increase in $\hat{r}(10)$.
  (d) Compute the second-order approximation to the price change in the bond for a 35 bp increase in $\hat{r}(10)$.

(c) Under the given rate changes we have that $\hat{r}(2) = .065$, $\hat{r}(5) = .0642$ and $\hat{r}(10) = 0.0638$. Using these values (which are still the respective yields since we have zeros) we obtain

$$\mathrm{Px}(\mathbf{B}_1) = 1,000,000 \left(1 + \frac{0.065}{2}\right)^{-4} = 879,913;$$
$$\mathrm{Px}(\mathbf{B}_2) = 1,000,000 \left(1 + \frac{0.0642}{2}\right)^{-10} = 729,092;$$
$$\mathrm{Px}(\mathbf{B}_3) = 1,000,000 \left(1 + \frac{0.0638}{2}\right)^{-20} = 533,639.$$

The portfolio value is thus

$$\mathrm{Px}(X) = \mathrm{Px}(\mathbf{B}_1) + \mathrm{Px}(\mathbf{B}_2) + \mathrm{Px}(\mathbf{B}_3) = 2,142,644.$$

The price change is thus

$$\Delta\mathrm{Px}(X) = 2,142,644 - 2,286,256 = -43,612.$$

(d) Now, assume a parallel shift in the spot curve so all the spot rates have moved from 0.06 to some level y. We are thus trying to find y so the portfolio price $\mathrm{Px}(X) = 2,142,644$. Using the formula for the zero price above we want y so that

$$\begin{aligned} 2,142,644 &= \mathrm{Px}(\mathbf{B}_1) + \mathrm{Px}(\mathbf{B}_2) + \mathrm{Px}(\mathbf{B}_3), \\ &= 1,000,000 \left((1+y/2)^{-4} + (1+y/2)^{-10} + (1+y/2)^{-20}\right). \end{aligned}$$

Solving this for y we see that $y = 0.0641327$. Thus an upward parallel shift of 41.3 basis points will yield the same price. Using the first order approximation based on the portfolios duration, we are looking for Δy so that

$$\Delta\mathrm{Px}(X) = -D(X)\mathrm{Px}(X)\Delta y.$$

Thus, using the original $\mathrm{Px}(X) = 2,286,256$, duration $D(X) = 4.90007$ and $\Delta\mathrm{Px}(X) = -43,612$ we see that

$$\Delta y = .004071 \Longrightarrow y = 0.0641007.$$

Thus, the first order approximation does well in suggesting that an 40.7 bp upwards parallel shift explains the actual price change, when the exact parallel shift was an upwards shift of 41.3 bp.

(4) (a) For the convexity of a zero-coupon bond we have

$$C = \frac{T^2 + T/2}{(1+y/2)^2} = 100.06386939324206.$$

(b) For exact price change we have original bond price

$$P = 10,000(1 + 0.048736/2)^{-20} = 6178.455689688011.$$

and new bond price

$$P_{new} = 10,000(1 + (0.048736 + 0.0035)/2)^{-20} = 5971.093612806781.$$

The difference is

$$\Delta P = P_{new} - P = 5971.093612806781 - 6178.455689688011 = -207.362076881229.$$

(c) First order approximation gives

$$dP = P \cdot (-D) \cdot (y_{new} - y) = -211.10181998957464,$$

where $D = T/(1 + y/2) = 9.76$.