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Review of Nonlinear Tracking and Setpoint Control Approaches for Autonomous Underactuated Marine Vehicles

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***Abstract*— A review of different approaches to setpoint, trajectory tracking, and path following control for autonomous underactuated surface vessels is presented in this work. The review is focused on rigorous control approaches developed using planar autonomous surface vessel models over approximately the last two decades. The controllers are categorized into setpoint, trajectory tracking, and path following approaches with further classification into discontinuous and smooth time-varying control laws for the setpoint control approaches. An overview of the formulation, advantages, and disadvantages of each approach is presented. Although much progress has been made in autonomous underactuated surface vessel control, there remains a need for the evolution of these approaches to achieve robust, real-time control laws that can be implemented on actual autonomous Unmanned Surface Vessel (USV) platforms in the presence of high sea states and external disturbances in uncertain and unstructured environments.**

I. INTRODUCTION

Control of underactuated marine vehicles is an active area of research that comprises surface, underwater, and hybrid vessel systems. In this review, attention is restricted to rigorous control techniques for autonomous underactuated surface vessels. An autonomous surface vessel is an underactuated system because it possesses either three degrees of freedom (3-DOF) when modeled as a single planar rigid body or 6-DOF when modeled as a three-dimensional rigid body. However, the vast majority of surface vessels have only two actuator inputs. The two actuators are typically two propellers, a propeller and a rudder, or a jet propulsion system that is either steerable or equipped with a rudder.

Setpoint, trajectory tracking, and path following control approaches for autonomous underactuated surface vessels have received increased attention during the last two decades. Most of the research activity in the development of underactuated control laws for these systems has focused on feedback linearization and backstepping methods. However, other control approaches such as passivity-based, direct Lyapunov, sliding mode, and model predictive control methods have also been developed.

For the purposes of this review, the underactuated control laws are divided into the general categories of setpoint control, trajectory tracking, and path following. Setpoint control is important for dynamic positioning of vessels in fixed target operations such as autonomous docking. Trajectory tracking control laws are applied in operations such as docking with a moving vessel, search and rescue, reconnaissance, surveillance, and multi-point or wide-area denial missions. Path following control, which can be applied in similar scenarios, has less stringent control requirements compared to trajectory tracking control because the desired position of the vessel is not specified at all times. Typical path following control applications are way-point navigation, reconnaissance, and surveillance where the vessel is not required to be on a given trajectory at an each instant of time.

This review is limited to planar surface vessel models because full three dimensional models are rarely used for the development of setpoint or trajectory tracking/path following control laws. One exception is Krishamurthy et al. [1] who propose a three dimensional modeling framework for surface vessels that includes hydrodynamic effects, wave, wind, and ocean current disturbances, and actuator models. However, only a proportional-integral control law with backstepping is presented without a stability proof. This work was continued in [2] where the control law was successfully implemented in a hardware-in-the-loop environment for a way-point type experiment although with some error. Another exception is Li et. al. [3] who proposed a backstepping nonlinear robust path following control law using a 4-DOF model that included roll motion. The approach is experimentally implemented on a small model boat where position feedback is provided through GPS. One of the advantages of this method is that it does not rely on (partial) feedback linearization of the system which results in more robust performance. However, the application of the control law is limited because it is only developed for the ruder angle under a given forward motion resulting in a control problem that is not truly underactuated.

The emphasis of this review is also on underactuated control laws that are derived using surface vessel models which include the full surge, sway, and yaw motion dynamics. Therefore, control laws that ignore one or more of these motions, such as the work presented by Encarnacao and Pascoal [4], Greytak and Hover [5]–[7], and many others, are not included in this review although some of this work does include relatively successful experimental results. Because it is costly and not very practical to fully actuate a surface vessel in each DOF in practice, fully actuated surface vessel control approaches such as Khaled and Chalhoub [8], who proposed a sliding mode controller for a 3-DOF fully actuated surface vessel in simulation, are also not considered.

The setpoint control approaches considered in this review are [9]–[25]. Due to the underactuated nature of the system, only discontinuous or smooth time-varying control of such systems is possible if all three coordinates are to be stabilized. Therefore, the setpoint control approaches are divided into discontinuous and smooth time-varying controllers in the review. The discontinuous setpoint control laws have been proposed in [9]–[13]. The smooth time-varying setpoint control laws have been proposed in [14]–[25]. The trajectory tracking control approaches considered in this review are [17], [26]–[42]. Finally, the path following control approaches considered in this review are [43]–[54].

A common thread throughout most of the work considered in this review is the lack of a comprehensive and practical control law that is robust to the uncertainties and disturbances associated with the surface vessel dynamics and sea conditions. With the exception of simple autopilot controls that only attempt to maintain vessel heading and speed, there is also a lack of experimental implementation. To our knowledge, the only experimental work for the underactuated surface vessel control approaches considered in this review is [13], [16], [18], [24] for setpoint stabilization and [33], [37], [42], [44], [47]–[49] for trajectory tracking/path following. This experimental work is carried out using small model vessels rather than actual unmanned surface vessel system platforms. All of this experimental work has also been performed in only four different laboratories.

II. Planar Surface Vessel Dynamic Modeling

In this section, the planar kinematic and dynamic model used to develop the control laws in this review is presented. A detailed discussion of the general two- and three-dimensional kinematic transformations and equations of motion for surface vessels, along with environmental disturbance models and basic autopilot control approaches, can be found in the comprehensive work by Fossen [55]. The 3-DOF planar model of the surface vessel shown in Fig. 1 is considered here. This model considers only surge (v_x), sway (v_y), and yaw ($\omega = \dot{\theta}$) motion. The dynamics associated with roll, heave, and pitch are neglected. The geometrical relationship between the inertial reference frame and the vessel-based body-fixed frame is

$$\begin{aligned} \dot{x} &= v_x \cos\theta - v_y \sin\theta \\ \dot{y} &= v_x \sin\theta + v_y \cos\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

where (x, y) denotes the position of the center of mass, θ is the orientation angle of the vessel in the inertial reference frame, (v_x, v_y) are the surge and sway velocities, and ω is the angular velocity of the vessel.

In the body-fixed frame, the nonlinear equations of motion have the following general form

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{D}(\mathbf{v})\mathbf{v} = \mathbf{f}(\mathbf{v}) \tag{2}$$

where $\mathbf{v} = [v_x, v_y, \omega]^T$ is the velocity vector, $\mathbf{M}$ and $\mathbf{D}$ are inertia and hydrodynamic drag matrices, and $\mathbf{f}(\mathbf{v})$ includes the actuator and environmental forces acting on the vessel.

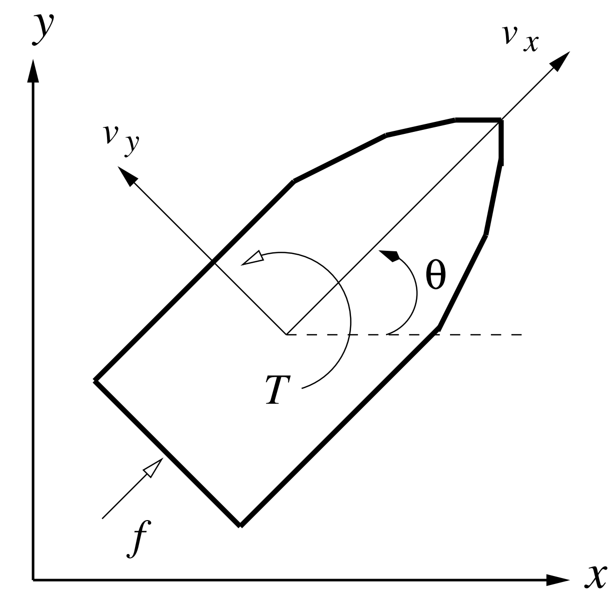

Fig. 1. Planar surface vessel schematic.

A. Control-Oriented Planar Surface Vessel Models

If the environmental forces are neglected and coupling in the drag and inertia matrices are restricted such that

$$\begin{aligned} \mathbf{D}(\mathbf{v}) &= \begin{bmatrix} d_{11}(v_x) & 0 & 0 \\ 0 & d_{22}(v_y) & d_{23} \\ 0 & d_{32} & d_{33}(\omega) \end{bmatrix} \\ \mathbf{M} &= \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} \end{aligned} \tag{3}$$

the resulting two-dimensional planar model of the surface vessel dynamics becomes

$$\begin{aligned} m_{11}\dot{v}_x - m_{22}v_y\omega + d_{11}(v_x)v_x &= f \\ m_{22}\dot{v}_y + m_{23}\dot{\omega} + m_{11}v_x\omega + d_{22}(v_y)v_y - d_{23}\omega &= 0 \\ m_{32}\dot{v}_y + m_{33}\dot{\omega} + m_d v_x v_y + d_{33}(\omega)\omega - d_{32}v_y &= T \end{aligned} \tag{4}$$

where m_{ii} are the mass and inertia model terms with $m_d = m_{22} - m_{11} > 0$. The m_{ii} parameters include added mass contributions that represent hydraulic pressure forces and torque due to forced harmonic motion of the vessel that are proportional to acceleration. The model terms $d_{ii}(\cdot)$ represent the hydrodynamic damping forces that are functions of the corresponding velocities in general. The surge force f and yaw moment T control inputs in the model are provided through the two actuators on the vessel.

A further simplification to the model in Eq. 4 can be made by eliminating the off-diagonal terms in the inertia and drag matrices and only considering drag forces that are linear functions of velocity. The equations of motion for the underactuated surface vessel then become

$$\begin{aligned} m_{11}\dot{v}_x - m_{22}v_y\omega + d_1 v_x &= f \\ m_{22}\dot{v}_y + m_{11}v_x\omega + d_2 v_y &= 0 \\ m_{33}\dot{\omega} + m_d v_x v_y + d_3\omega &= T \end{aligned} \tag{5}$$

where the model terms d_i denote the linear hydrodynamic damping coefficients. The vast majority of the controllers for both setpoint control and trajectory tracking/path following discussed in this review are derived using this model of the vessel dynamics.

B. Partial Feedback Linearization

Many of the setpoint and trajectory tracking control algorithms presented in this review are based on partial feedback linearization. This approach also facilitates a discussion of the controllability and stabilizability properties of the system. As a first step in developing the nonlinear control system model based on partial feedback linearization, the two control inputs of the system model in Eq. 5, the surge force f and yaw moment T, are redefined to be the angular acceleration, $\dot{\omega} = \ddot{\theta}$, and the surge acceleration, $\dot{v}_x$, by the following transformation

$$\begin{aligned} u_1 &= \dot{\omega} = \big(T - (m_{22} - m_{11})v_x v_y - d_{33}\omega\big)/m_{33} \\ u_2 &= \dot{v}_x = (f + m_{22} v_y \omega - d_{11} v_x)/m_{11} \end{aligned} \tag{6}$$

where u_1 is the angular acceleration and u_2 is surge acceleration. The first three states of the model are then defined as the vessel orientation angle and the body-fixed coordinates of the vessel center of mass

$$\begin{aligned} x_1 &= \theta - \theta_d \\ x_2 &= (x - x_d)\cos\theta + (y - y_d)\sin\theta \\ x_3 &= -(x - x_d)\sin\theta + (y - y_d)\cos\theta \end{aligned} \tag{7}$$

where (x_d, y_d, θ_d) represent the desired position and orientation setpoint or trajectory of the vessel. Three additional states are defined as the sway velocity, angular velocity, and surge velocity of the vessel.

$$\begin{aligned} x_4 &= v_y \\ x_5 &= \dot{\theta} \equiv \omega \\ x_6 &= v_x \end{aligned} \tag{8}$$

The state equations of the partial feedback linearized system can now be represented as

$$\begin{aligned} \dot{x}_1 &= x_5 \\ \dot{x}_2 &= x_6 + x_3 x_5 \\ \dot{x}_3 &= x_4 - x_2 x_5 \\ \dot{x}_4 &= -\alpha x_4 - \beta x_5 x_6 \\ \dot{x}_5 &= u_1 \\ \dot{x}_6 &= u_2 \end{aligned} \tag{9}$$

where $\alpha = \frac{d_{22}}{m_{22}}$ and $\beta = \frac{m_{11}}{m_{22}}$. The underactuated system model given in Eq. 9 is not asymptotically stabilizable to an equilibrium point using time-invariant continuous feedback because it does not satisfy Brockett's necessary condition [56]. However, it can be shown that such systems are strongly accessible at all points and small-time locally controllable [14], [57].

C. System Identification and Parameter Estimation

A uniform requirement for the application of a control-oriented model for controller development and implementation is the determination of the model parameter values. These parameters are vessel dependent and must be typically determined by system identification experiments. A very brief overview of various system identification approaches presented in the literature is provided in this section.

The USV model in Eq. 5 contains six parameters, three mass parameters and three hydrodynamic damping coefficients, that are usually determined from experimental tests, ship maneuvering data, or some combination of experimental tests and data. Formulas for the mass parameters are presented in [55] that can be used to provide an initial estimate of these values as follows

$$\begin{aligned} m_{11} &\approx m(1+\kappa) \\ m_{22} &\approx m + 0.5(\rho\pi D^2 L) \\ m_{33} &\approx \frac{m(L^2 + W^2) + \frac{1}{2}(0.1mB^2 + \rho\pi D^2 L^3)}{12} \end{aligned} \tag{10}$$

where m is the actual mass of the vessel, $\kappa \in [0.05, 0.01]$, L is the effective vessel length, W is the width, D is the mean submerged depth, and ρ is the density of water. These equations, however, only provide an approximate value that must be further refined. There can be significant error between the values obtained from these relationships and those obtained from system identification as shown in [58] for an experimental model USV. More advanced calculations for the inertia matrix parameters are mentioned in [44]. The hydrodynamic damping coefficients for the surge and sway dynamics are typically identified using drag tests. Turning experiments or free running maneuvering tests are required to determine the damping coefficients for the yaw motion.

Various techniques to identify the model parameter values from experimental or sea trial data, such as the extended Kalman filter [59], [60], simulated annealing [61], maximum likelihood [58], [62], and adaptive estimation [44], [63], [64] have been proposed. A discussion of maneuvering experiments for larger sea-going vessels is presented in [59], [65]. The use of towing tests and maneuvering experiments for small experimental vessels is presented in [44], [58].

III. Setpoint Control

The underactuated surface vessel setpoint control approaches are presented as either discontinuous or continuous time-varying control laws in this section. The smooth time-varying control literature is much richer due to the more practical nature of the controllers in terms of implementation.

A. Discontinuous Setpoint Control

Reyhanoglu [10] introduced a hybrid, time-varying, discontinuous control law that exponentially stabilizes underactuated surface vessels. The proposed method was a less restrictive extension of his earlier work in [9]. In this approach, the sixth order system in Eq. 9 is first reduced to a fourth order one using a discontinuous rational coordinate transformation by applying the σ-process borrowed from geometrical methods in ordinary differential equations [66]. An exponentially stable feedback control law is then introduced for the reduced system which is valid as long as singularity is avoided. A finite-time stable control law is introduced to move the system away from the singular state. The control law is derived based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. The method does not consider any uncertainties or environmental disturbances, does not consider actuator saturation, and does not distinguish between the vessel forward and backward motion dynamics.

Laiou and Astolfi [11] proposed a discontinuous control approach for nonlinear systems that have a generalized high-order form applicable to higher nonholonomic systems such as underactuated vessels. This method also uses rational coordinate transformation, however, it is more general. It can be applied to other nonholonomic systems such as mobile robots and the discontinuity is much less restrictive. The control is only discontinuous along a line in phase space while the method in [10] is discontinuous on an area that can only be reduced by increasing a control gain. The control law is derived based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. The method does not consider any uncertainties or environmental disturbances, does not consider actuator saturation, and does not distinguish between the vessel forward and backward motion dynamics.

Fantoni et al. [12] proposed two discontinuous control laws for an underactuated hovercraft. The first control law stabilizes the surge, sway, and yaw velocities considering the surge force and yaw moment as inputs. The second control law globally asymptotically stabilizes the position and sway velocity using the surge and the angular velocities as inputs. Their method, however, does not stabilize the orientation of the hovercraft. It also produces a discontinuous control effort due to use of the *sign* function. The control law is derived based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. The method does not consider any uncertainties or environmental disturbances, does not consider actuator saturation, and does not distinguish between the vessel forward and backward motion dynamics.

McNinch et al. [13] proposed a sliding mode control approach to setpoint control of underactuated surface vessels based on the finite-time control concepts presented in [67], [68]. The method is implemented experimentally on a small model surface vessel with two propellers where an overhead camera is used for position and orientation feedback. The approach showed some promise although actuator saturation, which is not included in the control derivation, presented a challenge in obtaining precise convergence to the setpoint in the experimental application. The control law is derived based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. The method does not distinguish between the vessel forward and backward motion dynamics. While uncertainties and environmental disturbances are not considered in this work, they can be incorporated into the controller formulation using the standard sliding mode control approach.

B. Smooth Time-Varying Setpoint Control

Pettersen and Egeland [14] proposed a continuous periodic time-varying feedback control law for underactuated surface vessels. However, the controller results in a oscillatory path due to the periodic nature of the input. Furthermore, the control law is only locally stable and the domain of attraction is not known. The authors extended this work in [15] to include thruster dynamics and some parameter uncertainties. Pettersen and Fossen [16] later proposed integral action to improve the control law and applied it to an experimental model ship. The integral action was introduced by adding the integral gain terms to the original control law. The integral action did reduce the stabilization errors but the vessel continued to exhibit oscillation around the setpoint. The approach was slightly modified and applied to a small experimental model ship by Pettersen and Fossen [18]. The experimental results demonstrated that the control law is able to guide the vessel to the neighborhood of the desired equilibrium point with oscillations. Pettersen and Nijmeijer [17] introduced a more practical time-varying feedback control law for global stabilization (and tracking) through a combined averaging and integrator backstepping method where exponential convergence to an arbitrarily small neighborhood of the origin (or reference trajectory) is guaranteed. The control laws derived in these works are all based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. They do not consider any uncertainties or environmental disturbances, do not consider actuator saturation, and do not distinguish between the vessel forward and backward motion dynamics.

Mazenc et al. [22] modified the backstepping control approach introduced in the Pettersen et al. earlier work [14], [16]–[18]. They determined explicit expressions for the smooth time-varying periodic state feedback that render the origin globally uniformly asymptotically stable. The construction of strict Lyapunov functions for time-varying systems as outlined in [69] was used to prove controller stability. The resulting controller design approach is a more flexible and robust control law for surface vessels than the original work. The method was later implemented experimentally using a small surface vessel by Pettersen et al. [24] where three cameras were used to provide position and orientation feedback. The theoretical results for uniformly global asymptotic convergence were verified since the vessel always converged to the neighborhood of the equilibrium point and oscillated about it. In particular, the control law was shown to be robust when the vessel was subjected to perturbations such as unmodeled dynamics, waves, currents, and measurement noise. The experimental work also suggested that unmodeled ship dynamics and environmental disturbances should be taken into account in the controller design for practical implementation.

Tian and Li [19], [20] introduced an approach for exponential stabilization of nonholonomic systems by smooth time-varying control and applied it to underactuated surface vessels among other examples. They initially designed a controller based on only the kinematic model in [19], but later extended it to the dynamic model with the surge force and yaw moment as control inputs. In this method, the nonlinear system is transformed into a linear time-varying control system by introducing an assistant state variable and a time-varying state transformation based on the concept of minimal dilation degree. The concept allows for formulation of a stabilizable time-invariant system leading to a smooth time-varying exponentially stable control law. The approach results in smooth control inputs, however, parameter uncertainties and disturbances are not discussed since the proof is dependent upon a nominal system system model. The control laws derived in this work are all based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. This work does not consider actuator saturation and does not distinguish between the vessel forward and backward motion dynamics.

Kim, et al. [21] proposed a logic-based regulation controller for asymptotic stabilization of underactuated surface vessels that can be viewed as a time-varying controller. This controller is based on a sequence of maneuvers to stabilize the orientation followed by the elimination of error in the lateral direction and achievement of a linear trajectory that can be easily followed. The controller is very effective at planning a path in very structured environment in order to achieve a desired equilibrium point. Asymptotic stability of the process is proved through Lyapunov analysis. The control law is derived based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. The method does not consider any uncertainties or environmental disturbances, does not consider actuator saturation, and does not distinguish between the vessel forward and backward motion dynamics.

Soro and Lozano [23] propose an asymptotically stable time-varying control law for semi-global stabilization of underactuated surface vessels. The method is based on a combination of nested saturation and backstepping techniques and the stability analysis is provided using averaging theory. In this work, all of the dynamic terms in the model, including drag, added mass effects, and centrifugal and Coriolis terms are ignored. Furthermore, the method does not consider any uncertainties or environmental disturbances, does not consider actuator saturation, and does not distinguish between the vessel forward and backward motion dynamics.

Dong and Guo [25] proposed a control approach which results in asymptotic stabilization of underactuated surface vessels and then proceed to develop a globally exponentially convergent control law where the rate of convergence can be arbitrarily selected. The method reduces the six-state problem into a four-state one and introduces a virtual time-varying control input to determine the actual control inputs. The approach is then modified through the introduction of an exponential time-varying term in the yaw moment control law leading to an exponentially convergent control law. Finally, they redefine the state transformation and revise the partial feedback linearization process to achieve a more convenient formulation of their exponentially convergent control law that allows the rate of exponential convergence to be easily changed. However, the stability of the method depends on the accuracy of the nominal model because uncertainties and disturbances are not considered in the formulation. Further, the method does not consider actuator saturation and does not distinguish between the vessel forward and backward motion dynamics.

IV. Trajectory Tracking Control

Godhavn [26] proposed a backstepping position tracking control law for underactuated surface vessels that is valid as long as input saturations are avoided and surge velocity remains nonzero. The method is proved to be exponentially convergent to the desired trajectory using the standard model given in Eq. 5. However, the angular velocity is only shown to be BIBO (Bounded Input Bounded Output) stable. A series of line segments and arcs are used to define trajectories that can actually be followed given the saturation constraints. The control law derived in this work is based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. This work does not consider any uncertainties or environmental disturbances and does not consider actuator saturation.

Petterson and Nijmeijer [27] introduced a semi-global exponential stabilization of the tracking error for any reasonable desired trajectory using an integrator backstepping approach. They essentially apply the methods developed for chained driftless systems [70] to tracking control of an underactuated surface vessel model with a drift vector. A coordinate transformation similar to their setpoint control approach is performed using a much simplified vessel dynamic model where all nonlinear terms and even the linear damping terms in the surge and yaw equations are ignored. Furthermore, only a simple linear drag model is used in the second-order nonholonomic sway equation. Using this approach, however, they are able stabilize not only the position tracking errors but also the orientation errors. They later combined integrator backstepping with averaging approach for trajectory tracking and global stabilization [17] as mentioned in the previous section.

Berge, et. al. [28] proposed a nonlinear tracking controller with integral action for ships based on state feedback linearization. Lyapunov stability theory is used to prove the exponential convergence of the body-fixed position and velocity errors while the heading is only stabilized indirectly. The authors define a virtual reference point such that the damping in yaw is increased and the zero dynamics are stabilized. The heading angle is claimed to be less sensitive to wind, currents and waves in this work. Simulation results with wind forces are presented but other forms of disturbances and uncertainties are not included. The control laws derived in this work is based on the diagonal inertia and drag matrices and linear drag model in Eq. 5 and actuator saturation limits are not considered.

Indiveri et al. [29] designed a nonlinear globally asymptotically stable control law for underactuated surface vessels based on both kinematic and dynamic models. The controller is smooth and time invariant but has limited application because it only applies to vessels on a linear course. Furthermore, despite the authors' claim, the example presented reveals a discontinuous and largely varying yaw control moment. The control law derived in this work is based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. This work does not consider any uncertainties or environmental disturbances and does not consider actuator saturation.

Toussaint et al. [30] extended the work of Gohavn [26] to include more general models with generalized forces using output redefinition. However, the method still has the drawbacks of the original work including the limited types of trajectories that can be followed. Toussaint et al. [31], [32] have also developed H_∞ techniques for motion planning and control of underactuated vehicles when disturbances are present and imperfect state measurements are available for feedback. The control laws derived in this work are based on the diagonal inertia and drag matrices and linear drag model in Eq. 5 and actuator saturation is not considered.

Petterson and Nijmeijer [33] improved the control law in [27] and provided experimental results with a small model ship where three cameras were used to provide position and orientation feedback. The experimental results were relatively successful with up to 20% tracking error for a circular reference trajectory. The authors attribute the experimental error to modeling errors, measurement noise, thruster limitations, waves, currents, hydrodynamic effects from the pool walls, and errors in the feedback due to observer estimation errors. The approach in [33] is further improved to a globally exponentially convergent control law by Lefeber et al. [37]. However, in this case, the control problem is divided into a cascade of a linear time-invariant subsystem for the yaw control and a linear time-varying subsystem for the surge control. The method was tested with the same vessel and on the same trajectory as the one presented in [33]. The parameters of the control law were tuned using an optimal control procedure applied to two linear time-invariant subsystems resulting from the cascade analysis.

Behal et al. [34], [35] proposed a continuous, time-varying tracking control law for underactuated surface vessels based on the transformation of open-loop tracking error dynamics into a skew-symmetric form. This method has the advantage of using a non-diagonal drag matrix although the drag model is still linear. The control law is shown to globally exponentially force the tracking errors to converge to a neighborhood of zero that can be made arbitrary small. In addition, the control law can be applied to setpoint problems as a simplified case of the trajectory tracking problem. However, the method is limited to linear drag models and robustness analysis is not provided. Disturbances, modeling uncertainties and actuator saturation are not included in the formulation.

Aquiar and Hespanha [39] proposed a position tracking control law for underactuated autonomous vehicles with both planar and spatial models and applied it to a hovercraft. The authors recursively introduce Lyapunov functions based on a backstepping approach to develop the control law assuming a drag model linear in angular velocity. The control law is claimed to be globally stable with exponential convergence to the neighborhood of the origin that can be made arbitrarily small. However, stability is not strictly proved and only presented as a practical matter. In addition, the controller seems to have complicated terms that may be difficult to calculate. However, the method has been applied to steer a simple model of a hovercraft in a circular motion in simulation. The control law derived in this work is based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. This work does not consider any uncertainties or environmental disturbances and does not consider actuator saturation.

Jiang [36] proposed two global tracking control laws for underactuated vessels based on Lyapunov's direct method. Both methods use the model presented in Eq. 5 with linear drag terms. The first method is a passivity-based design approach. Under a sufficient condition for persistent excitation, the control law is shown to be time-varying Lipschitz continuous asymptotically convergent. The second control law is a combined cascade-backstepping approach resulting in a time-varying and continuous exponentially convergent tracking controller. The method is shown to be robust to unmeasured thruster dynamics, but does not consider other parameter uncertainties or environmental disturbances and does not consider actuator saturation.

Do et al. [38] extended Jiang's work and developed a control law robust to environmental disturbances such as wave, wind, and ocean currents for ships on a linear course. In this work, the authors formulate the kinematic and dynamic model of the vessel assuming that it is moving on a linear course. Hence, the main task is to determine the yaw moment control law in order to reject disturbances. This assumption makes the development of the control law much simpler than the general trajectory tracking problem. Initially, disturbances are included in developing a full state feedback control law based on the backstepping approach where nonlinear quadratic damping terms are also included. An output feedback control law is then developed using a nonlinear observer that estimates sway and yaw velocities from measured sway displacement and yaw angle. Simulation results are presented using a ship model where the control law is able to reject a combination of wind, wave, and ocean current disturbances. The control law derived in this work is based on diagonal inertia and drag matrices and does not consider actuator saturation. Do and Pan [40] continued the work in [36], [38] by including nonzero off-diagonal terms to the system inertia and drag matrices. However, only linear drag is considered. The method essentially uses a coordinate transformation to convert the ship dynamics into a diagonal form and then proceeds with a similar backstepping technique used in the previous work. They also address constant environmental disturbances by modifying the control law.

Cao and Tian [41] introduce a smooth time-varying cascade design for trajectory tracking control of a general class of nonholonomic systems that include surface vessels. The idea is borrowed from the earlier work in Tian and

Li [20] for setpoint control where the nonlinear system is transformed into a linear time-varying control system by introducing a time-varying state transformation based on the concept of minimal dilation degree but with no assistant states. The control law, however, depends on the structure of the reference signal which must be exponentially decaying. Furthermore, it is based on the diagonal inertia and drag matrices and linear drag model in Eq. 5. This work does not consider any uncertainties or environmental disturbances and does not consider actuator saturation.

Ashrafiuon et al. [42] presented an exponentially stable tracking control law for underactuated surface vessel following any desired trajectory based on the sliding mode control approach. The method uses the second-order nonholonomic sway equation in defining a second-order sliding surface in order to determine the yaw control moment. The control law is shown to be exponentially convergent for tracking a position while the angular velocity is only BIBO stable as long as the vessel is in motion (i.e. surge velocity is not zero). The method used diagonal inertia and drag matrices but with a more general power law drag model that was obtained experimentally [58]. The approach was experimentally verified using a small model boat following straight line and circular trajectories where a single camera was used to provide position and orientation feedback. Control input saturation and environmental disturbances were not considered and the method required the vessel to start on the trajectory. Furthermore, yaw motion is not directly controlled in this method. The sliding mode control law in [42] was later extended to the case where the vessel could start from any initial condition and follow any desired trajectory at a practical desired orientation by Soltan et al. [71]. The method includes setpoint control as a special case of trajectory tracking control. The optimal determination of the sliding mode control tuning parameters that achieve a desired performance objective was presented by McNinch et al. [72]. The controller was further extended to coordinated control [73] and obstacle avoidance [74] by the same authors.

V. Path Following Control

Skjetne and Fossen [43] proposed a three-step backstepping method to develop a path-following control law for maneuvering ships. They avoid the problem of 3-DOF zero dynamics by first decoupling the surge motion from the sway and yaw motions and derived an independent control law to keep a nearly constant surge speed. They considered coupling and non-zero off-diagonal terms in the inertia and linear drag submatrices used for sway and yaw to determine a 2-DOF zero dynamics. Skjetne et. al. [44] modeled, identified, and designed a basic controller for maneuvering experiments with a small ship in a marine control laboratory. This work considered inertia and drag matrices with non-zero off diagonal terms, nonlinear quadratic and cubic drag terms, and actuator dynamics. The control law was based on a more general robust output maneuvering technique proposed in [75]. Ihle et. al. [45] continued this work by developing an output feedback controller using an observer backstepping approach that applied damping terms to counteract disturbances to the controller and applied it to a problem where only position feedback was available. The authors transform the problem into an output-feedback form where nonlinearities are only present in the output. The control law derived in this work is based on a linear drag model and does not consider actuator saturation. However, the formulation includes linear wave spectra and slowly varying environmental disturbances.

Do et al. [46] develop a path-following control law that can reject disturbances for a ship following any predefined path at a desired speed. The backstepping technique, based on Lyaounov's direct method, was adopted to develop a stable control law for the system with non-vanishing uncertainties. The main application of this work is claimed to be parking and point-to-point navigation. The authors address yaw dynamics discontinuities that can cause difficulties for the backstepping technique. Nonlinear quadratic damping is included in the model. Diagonal inertia and drag matrices are used in the development of the control law and control input saturation is not discussed. Do and Pan [47] combined their earlier work to develop a more comprehensive control law that includes non-diagonal inertia and drag matrices, nonlinear quadratic drag terms, and environmental disturbances. They experimentally implemented this control law on a small model ship. The experiments were performed in a river where feedback was provided by a differential global positioning system. A sinusoidal path and a straight line followed by a circular path were used to verify the controller performance. The experimental results demonstrated consistent chattering in the surge and yaw motion. Do and Pan [48] also proposed a global robust and adaptive path following control law for underactuated ships based on a the same model as [47] with very similar performance.

Fredriksen and Pettersen [49] proposed a global exponential path-following control law for way-point maneuvering of underactuated vessels. In this work, the authors use a line-of-sight approach to define the desired yaw angle and parameterize the path and derive a stabilizing speed control law for surge motion. One of the advantages of this method is that non-zero off-diagonal elements are included in the inertia and linear drag matrices. Relatively successful experimental results with the setup used in [24] are presented. The controller development is based on a linear damping model and control input saturation is not included in the formulation which presented some difficulties in the controller performance.

Moreira et. al. [50] proposed a path following control approach through a way-point guidance scheme based on line-of-sight projection. The approach is based on calculation of a dynamic line-of-sight vector norm to improve the speed of convergence to the desired path. The speed controller is developed based on feedback linearization. The control law derived in this work was based on inertia and drag matrices with non-zero off-diagonal terms and a linear drag model. This work considers water current and wind disturbances, but not actuator saturation.

Burger et al. [51] have recently proposed a control law for straight line path following of formations of underactuated surface vessels under the influence of ocean currents and have presented successful simulation results. The control law derived in this work is based on non-diagonal inertia and drag matrices with a linear drag model. However, it does not consider actuator saturation.

Li et al. [52] proposed a path following controller based on linear model predictive control. A linearized 4-DOF model is used in the model predictive control formulation where actuator and roll constraints are imposed in simulation. However, the surge speed is assumed to be constant and surge dynamics are neglected. Sorensen et al. [53] proposed a linear quadratic feedback controller for station keeping and tracking where a reference model calculated the reference trajectory and wave and wind disturbances are considered. In this case, experimental ship data is presented to demonstrate the controller performance. A nonlinear model predictive control approach is presented by McNinch et al. [54] in which the nonlinear model presented in Eq. 5 with power law drag terms is used in an autonomous recovery scenario simulation. Actuator and position constraints are imposed to achieve the desired performance. Input blocking techniques are also used to reduce the computational requirements. The method does not distinguish between the vessel forward and backward motion dynamics. Uncertainties and environmental disturbances are also not considered in this work.

VI. CONCLUSIONS

A summary of setpoint, trajectory tracking, and path following control laws for underactuated autonomous surface vessels is presented in this review. The controllers considered are based on a two-dimensional planar model of the system that include surge, sway, and yaw dynamics. The review reveals the significant progress made during the last two decades in the development of setpoint, trajectory tracking, and path following control algorithms and methodologies. However, the review also reveals a weakness in regard to real-time experimental implementation of these methods. There is still much work to be done in the development of real-time, robust control laws that can be implemented on autonomous surface vessels with uncertain parameters, limited actuation, and environmental disturbances particularly in high sea state conditions.

REFERENCES

[1] P. Krishnamurthy, F. Khorrami, and S. Fujikawa. A modeling framework for six-degree-of-freedom control of unmanned sea surface vehicles. In *Proceedings of the 44th IEEE Conference on Decision and Control*, pages 2676–2681, 2005.

[2] P. Krishnamurthy, F. Khorrami, and T.L. Ng. Control design for unmanned sea surface vehicles: hardware-in-the-loop simulator and experimental results. In *Proceedings of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3660–3665, 2007.

[3] Z. Li, J. Sun, and S. Oh. Design, analysis and experimental validation of a robust nonlinear path following controller for marine surface vessels. *Automatica*, 45(7):1649–1658, 2009.

[4] P. Encarnacao and A. Pascoal. Combined trajectory tracking and path following: An application to the coordinated control of autonomous marine craft. In *Proceedings of the 40th IEEE Conference on Decision and Control*, pages 964–969, 2001.

[5] M.B. Greytak and F.S. Hover. Robust motion planning for marine vehicle navigation. In *Proceedings of the 18th International Offshore and Polar Engineering Conference*, pages 399–406, 2008.

[6] M.B. Greytak and F.S. Hover. Underactuated point stabilization using predictive models with application to marine vehicles. In *Proceedings of the 2008 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3756–3761, 2008.

[7] M.B. Greytak and F.S. Hover. Analytic error variance predictions for planar vehicles. In *Proceedings of the 2009 IEEE International Conference on Robotics and Automation*, pages 471–476, 2009.

[8] N. Khaled and N.G. Chaloub. A dynamic model and a robust controller for a fully actuated marine surface vessel. In R.A. Ibrahim, V.I. Babitsky, and M. Okuma, editors, *Vibro-Impact Dynamics of Ocean Systems and Related Problems*, pages 135–148. Birkhauser–Verlag, 2009.

[9] M. Reyhanoglu. Control and stabilization of an underactuated surface vessel. In *Proceedings of the 35th IEEE Conference on Decision and Control*, pages 2371–2376, 1996.

[10] M. Reyhanoglu. Exponential stabilization of an underactuated autonomous surface vessel. *Automatica*, 33(12):2249–2254, 1997.

[11] M.-C. Laiou and A. Astolfi. Discontinuous control of a high-order generalized chained system. *Systems & Control Letters*, 37:309–322, 1999.

[12] J. Fantoni, R. Lozano, F. Mazenc, and K.Y. Pettersen. Stabilization of a nonlinear underactuated hovercraft. *International Journal of Robust and Nonlinear Control*, 10(8):645–654, 2000.

[13] L. C. McNinch, H. Ashrafiuon, and K. R. Muske. Sliding mode setpoint control of an underactuated surface vessel: simulation and experiment. In *Proceedings of the 2010 American Control Conference*, in press, 2010.

[14] K.Y. Pettersen and O. Egelan. Exponential stabilization of an underactuated surface vessel. In *Proceedings of the 35th IEEE Conference on Decision and Control*, pages 967–971, 1996.

[15] K.Y. Pettersen and O. Egelan. Robust control of an underactuated surface vessel with thruster dynamics. In *Proceedings of the 2007 American Control Conference*, pages 3411–3415, 1997.

[16] K.Y. Pettersen and T.I. Fossen. Underactuated ship stabilization using integral control: experimental results with CyberShip I. In *Proceedings of the 4th IFAC Nonlinear Control Systems Design Symposium, NOLCOS '98*, pages 127–132, 1998.

[17] K.Y. Pettersen and H. Nijmeijer. Global practical stabilization and tracking for an underactuated ship - a combined averaging and backstepping approach. *Modeling, Identification and Control*, 20(4):189–199, 1999.

[18] K.Y. Pettersen and T.I. Fossen. Underactuated dynamic positioning of a ship - experimental results. *IEEE Transactions on Control Systems Technology*, 8(5):891–863, 2000.

[19] Y.-P. Tian and S. Li. Smooth exponential stabilization of nonholonomic systems via time-varying feedback. In *Proceedings of the 39th IEEE Conference on Decision and Control*, pages 1912–1917, 2000.

[20] Y.-P. Tian and S. Li. Exponential stabilization of nonholonomic dynamic systems by smooth time-varying control. *Automatica*, 38:1139–1146, 2002.

[21] T. Kim, T. Basar, and I. Ha. Asymptotic stabilization of an underactuated surface vessel via logic-based control. In *Proceedings of the 2002 American Control Conference*, pages 4678–4683, 2002.

[22] F. Mazenc, K.Y. Pettersen, and H. Nijmeijer. Global uniform asymptotic stabilization of an underactuated surface vessel. *IEEE Transactions on Automatic Control*, 47(10):1759–1762, 2002.

[23] D. Soro and R. Lozano. Semi-global practical stabilization of an underactuated surface vessel via nested saturation controller. In *Proceedings of the 2003 American Control Conference*, pages 2006–2011, 2003.

[24] K.Y. Pettersen, F. Mazenc, and H. Nijmeijer. Global uniform asymptotic stabilization of an underactuated surface vessel: experimental results. *IEEE Transactions on Control Systems Technology*, 12(6):891–903, 2004.

[25] W. Dong and Y. Guo. Global time-varying stabilization of underactuated surface vessel. *IEEE Transactions on Automatic Control*, 50(6):859–864, 2005.

[26] J. Godhavn. Nonlinear tracking of underactuated surface vessels. In *Proceedings of the 35th IEEE Conference on Decision and Control*, pages 975–980, 1996.

[27] K.Y. Pettersen and H. Nijmeijer. Tracking control of an underactuated

surface vessel. In *Proceedings of the 37th IEEE Conference on Decision and Control*, pages 4561–4566, 1998.

[28] S.P. Berge, K. Ohtsu, and T.I. Fossen. Nonlinear control of ships minimizing the position tracking error. *Modeling, Indentification and Control*, 20(3):177–187, 1999.

[29] G. Indiveri, M. Aicardi, and G. Casalino. Nonlinear time-invariant feedback control of an underactuated marine vehicle along a straight course. In *Proceedings of the 2000 Conference on Maneuvering and Control of Marine Craft*, pages 221–226, 2000.

[30] G. Toussaint, T. Basar, and F. Bullo. Tracking for nonlinear underactuated surface vessels with generalized forces. In *Proceedings of the 39th IEEE Conference on Control Applications*, pages 355–360, 2000.

[31] G. Toussaint, T. Basar, and F. Bullo. h_∞-optimal tracking control techniques for nonlinear underactuated systems. In *Proceedings of the 39th IEEE Conference on Decision and Control*, pages 2078–2083, 2000.

[32] G. Toussaint, T. Basar, and F. Bullo. Motion planning for nonlinear underactuated vehicles using h_∞ techniques. In *Proceedings of the 2001 American Control Conference*, pages 4097–4012, 2001.

[33] K.Y. Pettersen and H. Nijmeijer. Underactuated ship control: theory and experiments. *International Journal of Control*, 74(14):1435–1446, 2001.

[34] A. Behal, D.M. Dawson, B. Xian, and P. Seltur. Adaptive tracking control of underactuated surface vessels. In *Proceedings of the 2001 IEEE International Conference on Control Applications*, pages 645–650, 2001.

[35] A. Behal, D.M. Dawson, W.E. Dixon, and Y. Fang. Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics. *IEEE Transactions on Automatic Control*, 47(3):495–500, 2002.

[36] Z.-P. Jiang. Global tracking control of underactuated ships by Lyaounov's direct method. *Automatica*, 38:301–309, 2002.

[37] E. Lefeber, K.Y. Pettersen, and H. Nijmeijer. Tracking control of an underactuated ship. *IEEE Transactions on Control Systems Technology*, 11(1):52–61, 2003.

[38] K.D. Do, Z.-P. Jiang, and J. Pan. Robust global stabilization of underactuated ships on a linear course: state and output feedback. *International Journal of Control*, 76(1):1–17, 2003.

[39] A. Aguiar and J. Hespanha. Position tracking of underactuated vehicles. In *Proceedings of the 2003 American Control Conference*, pages 1988–1993, 2003.

[40] K.D. Do and J. Pan. Global tracking control of underactuated ships with nonzero off-diagonal terms in their system matrices. *Automatica*, 41:87–95, 2005.

[41] K.-C. Cao and Y.-P. Tian. A time-varying cascaded design for trajectory tracking control of non-holonomic systems. *International Journal of Control*, 80(3):416–429, 2007.

[42] H. Ashrafiuon, K.R. Muske, L.C. McNinch, and R.A. Soltan. Sliding-mode tracking control of surface vessels. *IEEE Transactions on Industrial Electronics*, 55(11):4004–4012, 2008.

[43] R. Skjetne and T.I. Fossen. Nonlinear maneuvering and control of ships. In *Proceedings of MTS/IEEE OCEANS 2001*, pages 1808–1815, 2001.

[44] R. Skjetne, N. Smogeli, and T.I. Fossen. A nonlinear ship manoeuvering model: Identification and adaptive control with experiments for a model ship. *Modeling, Identification and Control*, 25(1):3–27, 2004.

[45] I.-A. F. Ihle, R. Skjetne, and T.I. Fossen. Output feedback control for maneuvering systems using observer backstepping. In *Proceedings of the 2005 Mediterranean Conference on Control and Automation*, pages 1512–1517, 2005.

[46] K.D. Do, Z.-P. Jiang, and J. Pan. Robust adaptive path-following of underactuated ships. *Automatica*, 40:929–944, 2004.

[47] K.D. Do and J. Pan. Robust path-following of the underactuated ships: Theory and experiments on a model ship. *Ocean Engineering*, 33:1354–1372, 2006.

[48] K.D. Do, Z.-P. Jiang, and J. Pan. Global robust adaptive path-following of underactuated ships. *Automatica*, 42(3):1713–1722, 2006.

[49] E. Fredriksen and K.Y. Pettersen. Global exponential way-point maneuvering of ships: Theory and experiments. *Automatica*, 42(4):677–687, 2006.

[50] L. Moreira, T.I. Fossen, and C.G. Soares. Path following control system for a tanker ship model. *Ocean Engineering*, 34:2074–2085, 2007.

[51] M. Burger, A. Pavlov, E. Borhaug, and K.Y. Pettersen. Straight line path following for formations of underactuated surface vessels under influence of constant ocean currents. In *Proceedings of the 2009 American Control Conference*, pages 3065–3070, 2009.

[52] Z. Li, J. Sun, and S. Oh. Path following for marine surface vessels with rudder and roll constraints: an MPC approach. In *Proceedings of the 2009 American Control Conference*, pages 3611–3619, 2009.

[53] A.J. Sorensen, S.I. Sagatun, and T.I. Fossen. Design of a dynamic positioning system using model-based control. *Control Engineering Practice*, 4(3):359–368, 1996.

[54] L. C. McNinch, K. R. Muske, and H. Ashrafiuon. Model-based predictive control of an unmanned surface vessel. In *Proceedings of the 11th IASTED International Conference on Intelligent Systems and Control*, pages 385–390, 2008.

[55] T.I. Fossen. *Guidance and Control of Ocean Vehicles*. Wiley, New York, 1994.

[56] R.W. Brockett. Asymptotic stability and feedback stabilization. In R. W. Brockett, R. S. Millman, and H. J. Sussmann, editors, *Differential Geometric Control Theory*, pages 181–191. Birkhauser–Verlag, 1983.

[57] M. Reyhanoglu, A. van der Schaft, and N. McClamroch. Dynamics and control of a class of underactuated mechanical systems. *IEEE Transactions on Automatic Control*, 44:1663–1671, 1999.

[58] K.R. Muske, H. Ashrafiuon, G. Haas, G. McCloseky, and T. Flynn. Identification of a control oriented nonlinear dynamic usv model. In *Proceedings of the 2008 American Control Conference*, pages 562–567, 2008.

[59] T.I. Fossen, S. Sagatun, and A. Sorensen. Identification of dymanically positioned ships. *Control Engineering Practice*, 4(3):369–376, 1996.

[60] H. Yoon and K. Rhee. Identification of hydrodynamic coefficients in ship maneuvering. *Ocean Engineering*, 30(18):2379–2404, 2003.

[61] A. Tiano and M. Blanke. Multivariable identification of ship steering and roll motions. *Trans. Inst. Meas. Contr.*, 19(2):63–77, 1997.

[62] S. Dudnik. Identification by maximum likelihood method in modeling of ship movement. *Engineering Simulation*, 12(4):658–652, 1995.

[63] S.K. Bhattacharyya and M.R. Haddara. Parametric identification for nonlinear ship maneuvering. *Journal of Ship Research*, 50(3):197–207, 2006.

[64] M. Casado, R. Ferreiro, and F. Velasco. Identification of nonlinear ship model parameters based on the turning circle test. *Journal of Ship Research*, 51(2):174–181, 2007.

[65] M. Blanke and M. Knudsen. A sensitivity approach to identification of ship dynamics from sea trial data. In *Proceedings of the 1998 IFAC Conference on Control Applications in Marine Systems (CAMS'98)*, pages 241–249, 1998.

[66] V.I. Arnold. *Geometrical Methods in the Theory of Ordinary Differential Eqautions*. Springer, New York, 1983.

[67] V.T. Haimo. Finite time controllers. *SIAM Journal on Control and Optimization*, 24(4):760–770, 1986.

[68] S.P. Bhat and D.S. Bernstein. Continuous finite-time stabilization of the translational and rotational double integrators. *IEEE Transactions on Automatic Control*, 43(5):678–682, 1998.

[69] F. Mazenc. Strict Lyapunov functions for time-varying systems. *Automatica*, 39(2):349–353, 2003.

[70] Z.-P. Jiang and H. Nijmeijer. A recursive technique for tracking control of nonholonomic systems in chain form. *IEEE Transactions on Automatic Control*, 44(10):265–279, 1999.

[71] R.A. Soltan, H. Ashrafiuon, and K.R. Muske. State-dependent trajectory planning and tracking control of unmanned surface vessels. In *Proceedings of the 2009 American Control Conference*, pages 3597–3602, 2009.

[72] L.C. McNinch, K.R. Muske, and H. Ashrafiuon. Optimal specification of sliding mode control parameters for unmanned surface vessel systems. In *Proceedings of the 2009 American Control Conference*, pages 2350–2355, 2009.

[73] R. Soltan, H. Ashrafiuon, and K. R. Muske. Coordinated trajectory planning and tracking control. In *Proceedings of the 2009 ASME IDETC/CIE Conference*, Paper DETC2009–86201, 2009.

[74] R. Soltan, H. Ashrafiuon, and K. R. Muske. Real-time obstacle avoidance for underactuated unmanned surface vessels. In *Proceedings of the 2009 ASME IDETC/CIE Conference*, Paper DETC2009–86978, 2009.

[75] R. Skjetne, T.I. Fossen, and P.V. Kokotovic. Robust output maneuvering for a class of nonlinear systems. *Automatica*, 40(3):373–383, 2004.