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Performance Analysis of a Simple L1-Adaptive Controller

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***Abstract*— In this note we compare the performance of a simple L1-adaptive controller with standard adaptive control strategies, i.e. Direct Adaptive Control (DAC) and Indirect Adaptive Control (IAC). Performance comparison is carried out with respect to several performance criteria. It is shown that, in this application, L1-controller is significantly outperformed by the IAC algorithm in the cases with and without time delay.**

I. Introduction

In the recent years, a technique referred to as the L1 Adaptive Control (L1-AC) was proposed in the literature (see [1] and references therein), where the authors claim that the approach assures guaranteed transient performance of adaptive systems. The L1 technique is based on adding a fixed compensator to the adaptive controller, and studying the input-output properties of the resulting closed-loop system. The main result is based on the small-gain theorem used to show that the norms of both input and estimation errors depend inversely on the adaptive gain, and that arbitrary large gain and phase margins can be obtained in the system for arbitrarily large values of the adaptive gain.

In this note we carry out a simple simulation study and compare the performance obtained using L1 adaptive control with that obtained using standard adaptive control techniques. It is shown that the L1-AC results in high-frequency oscillation of the parameter estimate, and yields the performance inferior to Indirect Adaptive Control.

II. Problem Statement

Let the plant be described by a first-order model of the form:

$$\dot{x}(t) = -k_m x(t) + b(u(t-\tau) + \theta x(t)), \;\; x(0) = x_o, \tag{1}$$

where $x \in \mathbb{R}$ is the plant state, $u \in \mathbb{R}$ is the control input, $(k_m, b) > 0$ are known, τ is the value of time delay, and $\theta \in [-\bar{\theta}, \bar{\theta}]$ is an uncertain parameter.

Let the reference model dynamics be of the form:

$$\dot{x}_m = -k_m(x_m - r), \tag{2}$$

where $x_m \in \mathbb{R}$ is the reference model state, and r denotes a bounded piece-wise continuous reference input.

In this note we assume that $b = 1, k_m = 1$, and that the initial conditions are as follows: $t_o = 0, x(0) = 0$ and $x_m(0) = 0$.

When the parameter θ is known, the ideal control input that solves the tracking problem with zero error (due to the above choice of initial conditions) is of the form:

$$u^* = -\theta + r.$$

The objective is to implement several adaptive control strategies for controlling the above plant to assure that the state remains bounded, and compare the resulting performance with respect to the following performance indices:

$$J_e = \int_0^T |x(t) - x_m(t)| dt, \quad J_u = \int_0^T |u(t) - u^*(t)| dt,$$

$$J_\theta = \int_0^T |\hat{\theta}(t) - \theta| dt, \quad J_{eF} = |x(T) - x_m(T)|,$$

where $t \in [0, T]$. The above performance indices J_e, J_u and J_θ evaluate the tracking, input and estimation errors over the course of the simulation, while the index J_{eF} evaluates the final tracking performance.

We next discuss the adaptive control algorithms whose performance will be compared.

L1-Adaptive Control (L1-AC): Assuming that $\tau = 0$, this algorithm is implemented as:

$$\begin{aligned} u &= C(s)(-\hat{\theta}x + r) \\ \dot{\hat{x}} &= -\hat{x} + u + \hat{\theta}x, \;\; \hat{x}(0) = 0, \\ \dot{\hat{\theta}} &= \text{Proj}_{[-\bar{\theta},\bar{\theta}]}\{-\gamma_{L1}(\hat{x} - x)x\}, \;\; \hat{\theta}(0) = 0, \end{aligned}$$

where $\gamma_{L1} > 0$, $\text{Proj}_{[\cdot]}\{\cdot\}$ denotes the projection operator, $C(s) = \lambda_F/(s + \lambda_F)$ and $\lambda_F > 0$. In this study we chose $\gamma_{L1} = 10,000$ and $\lambda_F = 4$.

Direct Adaptive Control (DAC): This algorithm is implemented as:

$$\begin{aligned} u &= r - \hat{\theta}x \\ \dot{\hat{\theta}} &= \text{Proj}_{[-\bar{\theta},\bar{\theta}]}\{\gamma_{DAC}(x - x_m)x\}, \;\; \hat{\theta}(0) = 0, \end{aligned}$$

where we chose $\gamma_{DAC} = 4$.

Indirect Adaptive Control (IAC): This algorithm is implemented as a two degrees-of-freedom indirect adaptive controller:

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$$
\begin{aligned}
u &= r - \hat{\theta}x \\
\dot{\hat{x}} &= -\lambda(\hat{x} - x) - x + u + \hat{\theta}x, \;\; \hat{x}(0) = 0, \\
\dot{\hat{\theta}} &= \text{Proj}_{[-\bar{\theta},\bar{\theta}]}\{-\gamma(\hat{x} - x)x\}, \;\; \hat{\theta}(0) = 0,
\end{aligned}
$$

where the degrees of freedom for tuning are $\lambda > 0$ and $\gamma > 0$. The following values are chosen in this study: $\lambda = 12$ and $\gamma = 30$.

We note that DAC and IAC are, in the case of plant (1), identical when in (3) $\lambda = 1$. However, unlike DAC that has a single tuning parameter, in IAC we can tune γ and λ separately without affecting the overall system stability [2]. The IAC in which we tune γ and λ separately is referred to as the IAC with two-degrees-of-freedom. L1-AC is also a two-degree-of-freedom controller since we can tune λ_F and γ_{L1} separately. However, we note that in [1] it was shown that λ_F needs to satisfy the L1-AC stability condition. Hence its values need to be selected with care.

III. Simulations

In all simulations step size is chosen to be 0.01 sec. To integrate differential equations we used simple Euler integration. Simulation duration is 10 seconds. Also, the bound on θ is chosen as $\bar{\theta} = 3$. The scenario chosen starts with $\theta = 0$. Then, at $t = 2$ seconds, θ switches to $\theta = 2$, making the open-loop plant unstable.

Summary of simulation cases is given in Table I.

Cases	**Reference Input**	**Time Delay**
Case 1	$r(t) = 1, \;\forall t \geq 0$	$\tau = 0$
Case 2	$r(t) = 1, \;\forall t \geq 0$	$\tau = 0.3$ seconds
Case 3	$r(t) = 1, \;\forall t \geq 0$	$\tau = 0.6$ second
Case 4	$r(t) = sin(t)$	$\tau = 0$
Case 5	$r(t) = sin(t)$	$\tau = 0.3$ seconds
Case 6	$r(t) = sin(t)$	$\tau = 0.6$ second

TABLE I: A List of Simulation Cases

Simulation results are given in Figures 1-6 for the case when $\theta = 2$. We also ran Monte-Carlo simulations whereby the parameter θ was chosen randomly for each simulation from the interval $[-3, 3]$. A total of 1,000 simulations was ran for each controller. The average values of the corresponding performance indices are given in Tables II-VII.

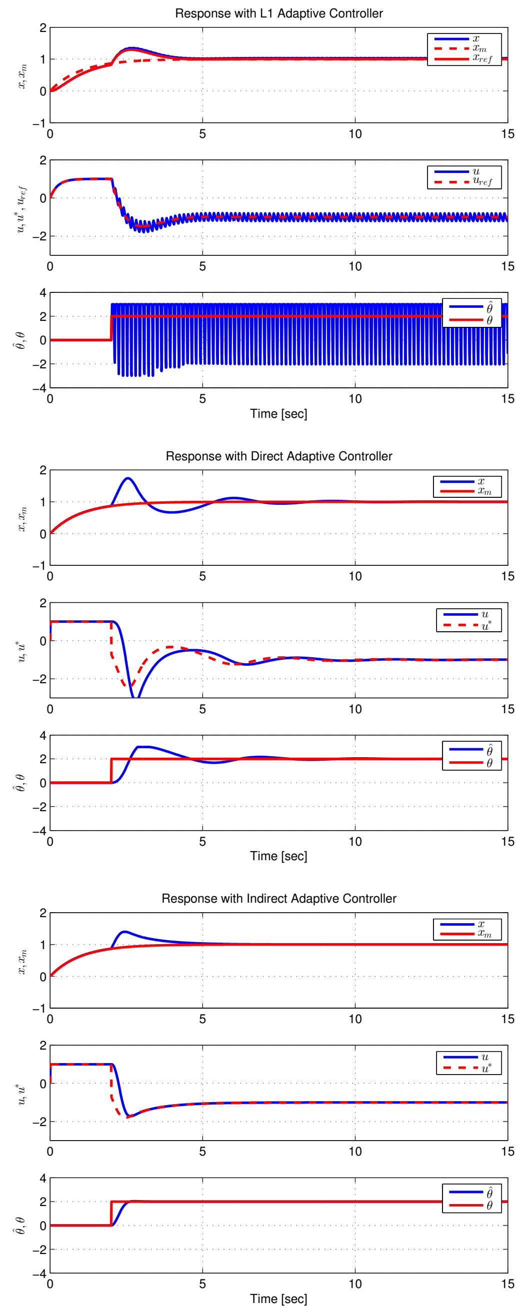

Fig. 1: System response: Case 1 ($r(t) = 1, \theta = 2, \tau = 0$ sec)

From Figure 1 we can already notice the main features of system response obtained using L1 adaptive control: after θ switching to $\theta = 2$ and a transient, the output slowly converges to the reference input value (i.e. $r(t) = 1$); control input is oscillatory, while the parameter estimate exhibits high frequency oscillations between the saturation bounds. Oscillation in u (but not in $\hat{\theta}$) can be somewhat decreased by lowering λ_F, but lowering it too much causes closed-loop system instability. Hence $\lambda_F = 4$ is chosen for all cases.

Controller	J_e	J_u	J_θ	J_{eF}
L1 Adaptive Controller	0.812	2.022	20.933	0.016
Direct Adaptive Controller	0.986	1.765	2.184	0.000
Indirect Adaptive Controller	0.755	0.768	1.120	0.000

TABLE II: Average values of performance indices in the Case 1 ($r(t) = 1, \tau = 0$) after Monte-Carlo simulations

From the same figure we can see that DAC tracking response is somewhat oscillatory, however, the resulting performance in the input and parametric errors is still superior to that achieved using L1, Table II. It is also seen that the IAC achieves the best overall performance.

Figures 2 and 3 show the response in the case of time delay of 0.3 and 0.6 seconds, respectively. We note that the control inputs are implemented only after the delay resulting in reduced tracking performance. In both cases the DAC algorithm achieves poor tracking performance, see Tables III and IV. In the case of $\tau = 0.3$ seconds, IAC and L1 have similar tracking performance, while IAC achieves input and parameter error performance far superior to that with L1. In the case of $\tau = 0.6$ seconds, *the closed-loop system with L1 became unstable in several occasions during Monte-Carlo simulations*, while the IAC achieved acceptable performance.

Similar conclusions can be made in the case of reference input $r(t) = sin(t)$. System response in this case is shown in Figures 4-6 and corresponding values of the performance indices are listed in Tables V-VII. In the last case, i.e. with $\tau = 0.6$ seconds, the closed-loop with L1 again becomes unstable.

IV. Discussion

In this study we compare the performance of an uncertain first-order plant obtained using standard adaptive control algorithms, namely Direct Adaptive Control (DAC) and Indirect Adaptive Control (IAC), with the performance obtained using the L1 Adaptive Control (L1-AC). The following was found through the simulation study:

1. L1-AC results in a highly oscillatory behavior of the parameter estimate. This is due to the use of large adaptation gains and parameter projection algorithm which is used to keep the parameter estimates within their bounds. We note that the use of large adaptation gains has actually been recommended by the authors of L1-AC [1]. The oscillations of the parameter estimate also cause oscillations of the control input and the system state, which is a highly undesirable feature in practical applications.

2. The L1-AC, in this case, is a two-degree-of-freedom controller since the adaptation gain γ_{L1} and the filter parameter λ_F can be tuned independently. However, the adaptation gain γ_{L1} is commonly chosen to be very large (e.g. of the order of $10^4 - 10^6$), so that the only tuning parameter is λ_F. In this study we found that large λ_F results in a highly oscillatory response. For smaller values of λ_F, the L1-AC stability condition can be violated. We note that the L1-AC stability condition, in this case, reduces to the condition that the sum of λ_F and the gain k_m (in our case $k_m = 1$) is larger than the upper bound on the parametric uncertainty.

3. Besides resulting in inferior performance compared to IAC, L1-AC is also shown to have lower time delay margin compared to both DAC and IAC. In this example and in the case of time delay of $\tau_D = 0.6$ seconds, the response with L1-AC would became unbounded in several occasions during Monte-Carlo simulations, which was not the case with either DAC or IAC.

V. Conclusions

In this note we show through a simple simulation example that performance obtained using L1 adaptive control spans the range from “comparable” to “much worse” when compared to performance obtained using standard adaptive control techniques. Even though this conclusion is based on a limited amount of simulations, it indicates that: (a) There is no “silver bullet” in adaptive control, i.e. a single technique that achieves performance superior to that obtained using other approaches in all cases; and (ii) A feasible approach to adaptive control design and implementation is to compare the performance obtained using different adaptive control algorithms before choosing one of them for a specific application.

References

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[2] J. D. Bošković and N. Knoebel, “Adaptive Control using Reduced-Order Observers”, in Proceedings of the *2008 American Control Conference*, Seattle, WA, June 11–13, 2008.

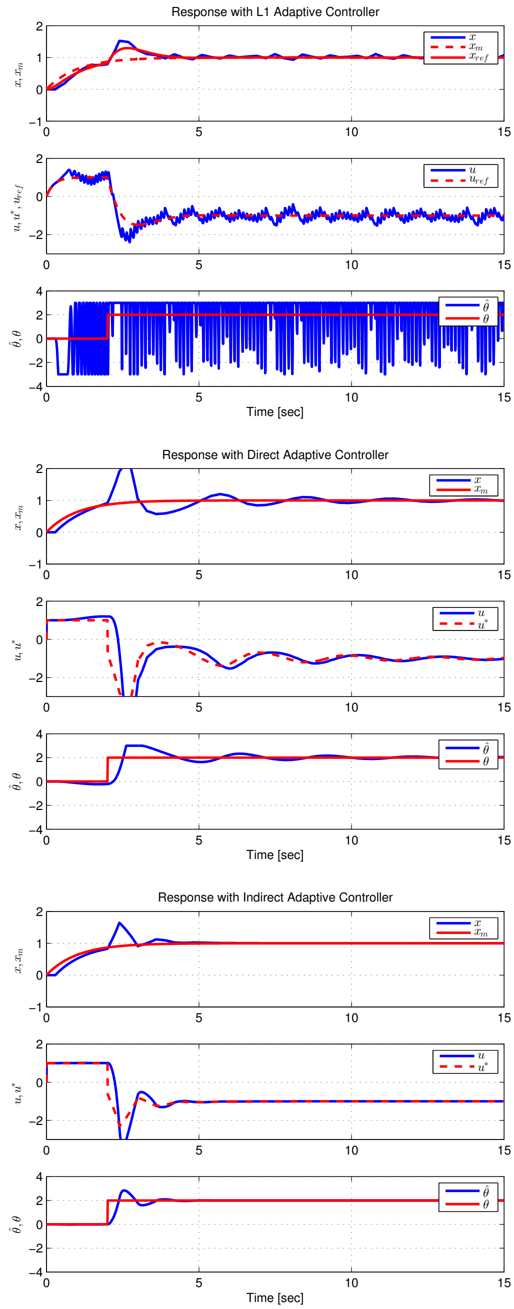

Fig. 2: System response: Case 2 ($r(t) = 1, \theta = 2, \tau = 0.3s$)

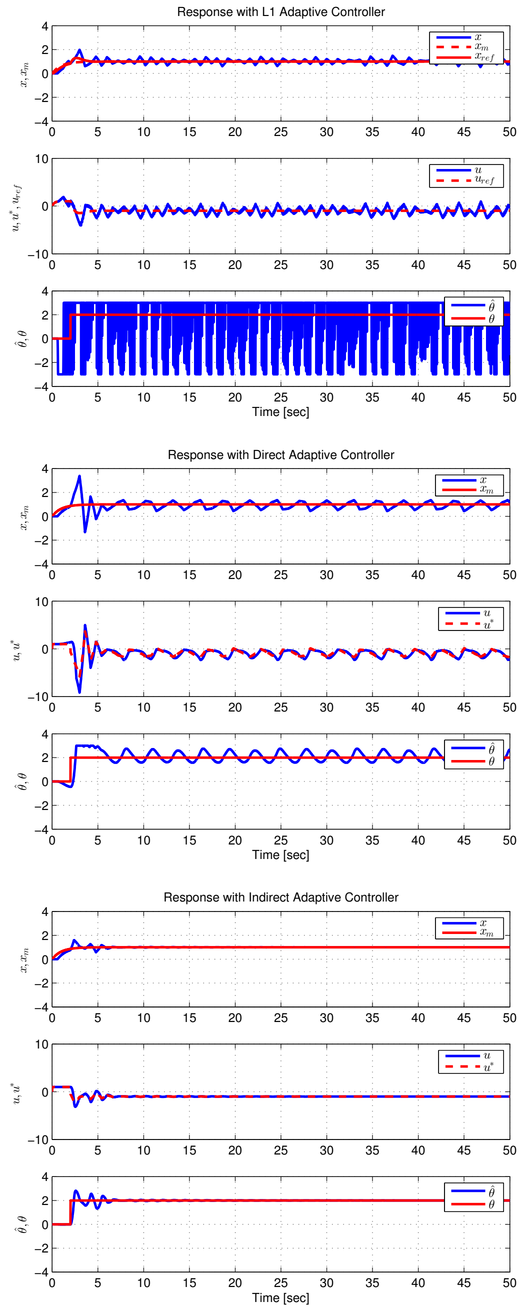

Fig. 3: System response: Case 3 ($r(t) = 1, \theta = 2, \tau = 0.6$ s)

Controller	J_e	J_u	J_θ	J_{eF}
L1 Adaptive Controller	1.057	2.616	25.144	0.025
Direct Adaptive Controller	1.758	2.897	3.315	0.020
Indirect Adaptive Controller	1.041	0.873	1.232	0.000

TABLE III: Average values of performance indices in the Case 2 ($r(t) = 1, \tau = 0.3$ sec) after Monte-Carlo simulations

Controller	J_e	J_u	J_θ	J_{eF}
L1 Adaptive Controller	∞	∞	84.478	∞
Direct Adaptive Controller	8.358	14.077	14.856	0.111
Indirect Adaptive Controller	4.515	8.328	8.667	0.028

TABLE IV: Average values of performance indices in the Case 3 ($r(t) = 1, \tau = 0.6$ s) after Monte-Carlo simulations

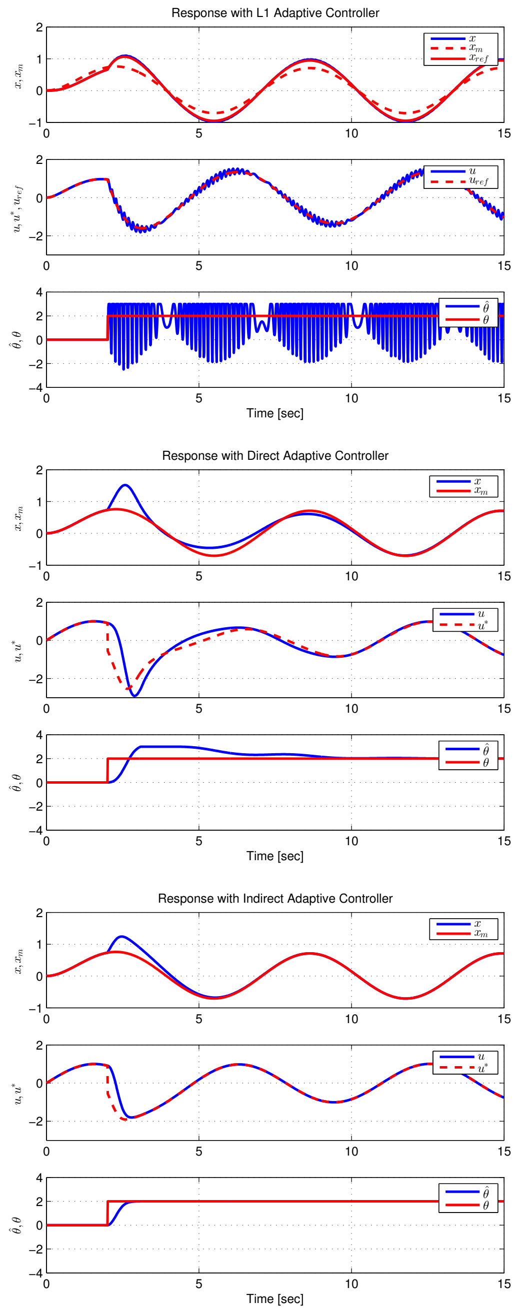

Fig. 4: System response: Case 4 ($r(t) = sin(t), \theta = 2, \tau = 0$)

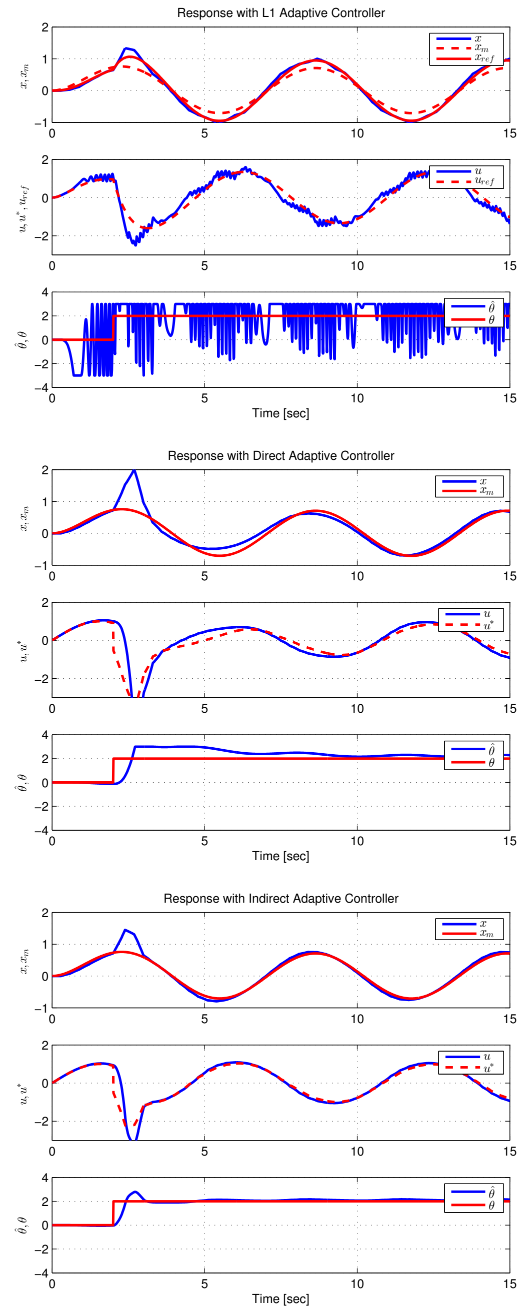

Fig. 5: System response: Case 5 ($r(t) = sin(t), \theta = 2, \tau = 0.3$ sec)

Controller	J_e	J_u	J_θ	J_{eF}
L1 Adaptive Controller	2.266	3.364	18.334	0.1734
Direct Adaptive Controller	1.185	1.745	4.597	0.012
Indirect Adaptive Controller	0.810	1.035	2.740	0.002

TABLE V: Average values of performance indices in the Case 4 ($r(t) = sin(t), \tau = 0$ seconds) after Monte-Carlo simulations

Controller	J_e	J_u	J_θ	J_{eF}
L1 Adaptive Controller	2.653	2.581	22.330	0.180
Direct Adaptive Controller	1.829	2.272	5.423	0.049
Indirect Adaptive Controller	1.368	1.702	4.574	0.073

TABLE VI: Average values of performance indices in the Case 5 ($r(t) = sin(t), \tau = 0.3$ sec) after Monte-Carlo simulations

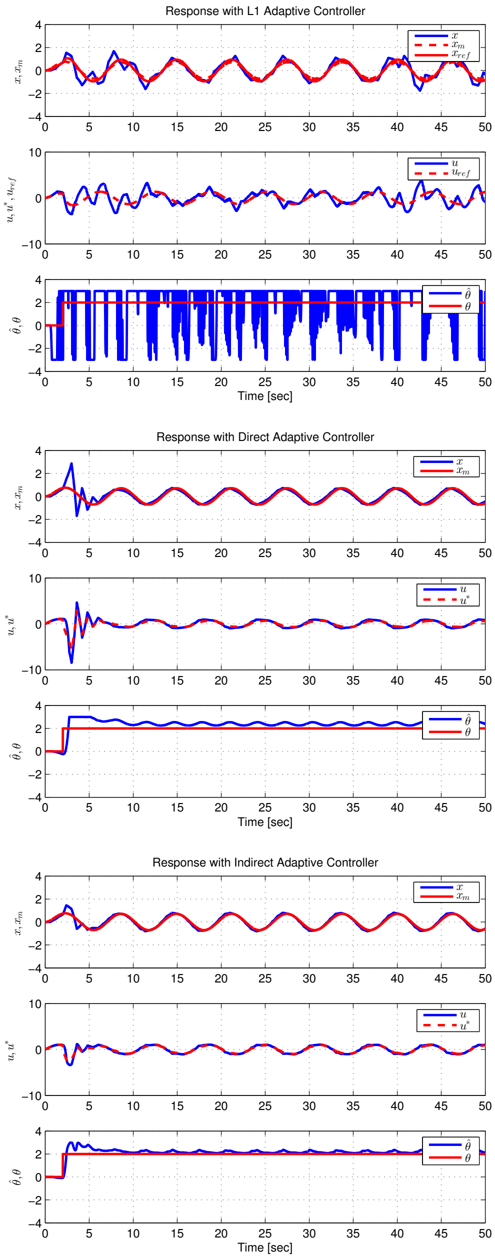

Fig. 6: System response: Case 6 ($r(t) = sin(t), \theta = 2, \tau = 0.6$ sec)

Controller	J_e	J_u	J_θ	J_{eF}
L1 Adaptive Controller	∞	∞	83.165	∞
Direct Adaptive Controller	9.272	7.277	16.437	0.091
Indirect Adaptive Controller	6.374	6.225	17.624	0.084

TABLE VII: Average values of performance indices in the Case 6 ($r(t) = sin(t), \tau = 0.6$ sec) after Monte-Carlo simulations