ME2801, HW #7, Root Locus, Compensation Design

This assignment is two design problems via root locus for a PD and PI compensator respectively. The exercises are verbose and intended to walk you through each step of the root locus design process. To perform the analysis, most of the work will be done in MATLAB. You can document the designs by following the steps and submiting the results as a MATLAB script, live script, written results or a combination.

1 Designing compensation (P and PD) with root locus

Consider the Plant transfer function

$$L(s) = \frac{s+6}{(s+2)(s+3)(s+5)}$$

1.1 Proportional-Only Controller Design

Consider the proportional-only, unity feedback design as shown in Figure 1.

- Plot the root locus for the system. Use the `sgrid()` command to add a line for $\zeta = 0.707$ (%OS=4.3). Submit this plot.
- Find a value for the proportional gain, K, so that the *dominant closed-loop second order poles* have a damping ratio of approximately $\zeta = 0.707$. Report this value of gain, K.
- Using the proportional gain you found, create a closed-loop unit step response for the design and report the settling time, damping ratio and overshoot.

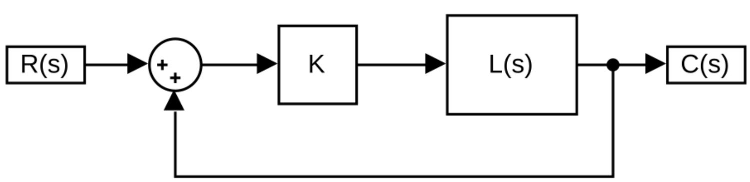

Figure 1: Proportional Control.

1.2 Proportional-Derivative Design

An ideal PD controller can be equivalently represented in the following forms:

$$\begin{aligned} D(s) &= K_p + K_d s && \text{(Standard form)} \\ &= K_d\left(s + K_p/K_d\right) && \text{(Factored form)} \\ &= K_{pd}(s + z_{pd}) && \text{(Root locus PD form)} \end{aligned}$$

where K_p is the proportional gain and K_d is the derivative gain in the standard form; K_{pd} is the root locus gain and z_{pd} is the location of the PD zero in the root locus form. The PD controller is used to

improve the transient response of the system. The PD controller adds a zero to the system, which can be used to change the system dynamics. The PD controller does not change the steady-state error of the system, but it can improve the transient response by increasing the damping ratio and reducing the overshoot.
Consider the PD control design shown in Figure 2. The design goal is to achieve the following closed-loop step response metrics:

- $T_s \approx 0.9\,\text{s}$
- $\zeta \approx 0.707$

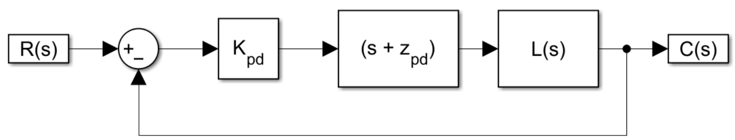

Figure 2: Proportional-Derivative Control.

The PD compensator design has two design parameters. For root locus design we consider these two parameters as the location of the PD zero, $z = K_p/K_d$, and the gain value K_d.

- Find the target closed-loop pole locations that correspond to the performance metric goals, T_s and ζ. That is, based on the provided metrics, what is the closed-loop, underdamped, second-order pole values (real and imaginary) to satisfy the design?
- Find a location for the PD controller zero, the value of $z = K_p/K_d$ in Figure 2, so that the root locus passes through the target pole locations. Try the following values for the zero location: $z = [-4, -7, -10]$. For each value of z, plot the root locus and use the `sgrid` command to add a line for $\zeta = 0.707$ (%OS=4.3). You can plot the root locus for each design as a separate window using the `rlocus` command or plot them all on the same axes using the `rlocusplot` command. Which zero location enables satisfying the design constraints? Choose one of the values for the zero location.
- Find the value of K_d that will place the dominate second order poles near the design target. Use the interactive tool for the root locus figure window (Data tips) to choose a gain value (K_d) for which the closed loop poles will be within the design constraints. (If you used the `rlocusplot` command previously, you may need to generate the RL for just this system to be able to use the tool.)
- Using the zero location (z) and gain (K_d) you found, create a closed-loop step response for the PD design and report the settling time and overshoot.
- Report the compensator design in the form $D(s) = K_d(s + K_p/K_d)$.
- Plot both closed-loop step response for both the P and PD compensator designs on the same axes and annotate your figure with the compensator values (P: K, PD: K_d and zero location K_p/K_d) and the performance metrics (T_s and %OS).

2 Design PI compensation with Root Locus

Consider the plant transfer function

$$G(s) = \frac{1}{(s+5)^2(s+8)}.$$

We might notice that this plant has *Type 0* dynamics, so we anticipate it will have poor steady state error performance. We want to design a proportional-integral (PI) compensator to improve the steady-state error. We can write the PI controller in this form

$$D(s) = K_p + \frac{K_i}{s} = K_i\left(\frac{s + K_p/K_i}{s}\right) = K\left(\frac{s+z}{s}\right),$$

which illustrates that the controller adds a pole at zero and a zero at $-K_p/K_i$.
The design contraints for the problem are:

- $T_r \le 0.9\,\mathrm{s}$
- $\%OS \le 2\%$
- $e_{\text{step}}(t \to \infty) = 0$

For an underdamped second-order system, the rise time can be approximated as

$$T_r \approx \frac{1.8}{\omega_n}.$$

for $\zeta < 0.7$.

1. Translate the design constraints into s-plane parameters. Based on T_r and %OS, find the ω_n and ζ bounds for the desired pole locations to meet the design specification.
2. Plot the root locus for three different choices of PI zero location: $z = K_p/K_i = [-1, -4, -7]$. Use the `sgrid` command to provide gridlines to illustrate the design constraints. You can plot the root locus for each design as a separate window using the `rlocus` command or plot them all on the same axes using the `rlocusplot` command.
3. Which of the design choices for the PI zero locations enables satisfying the design constraints? Choose one of the values for the zero location.
4. Using the zero location chosen in the previous step. From the root locus plot, use the interactive tool (Data tips) to choose a gain value (K) for which the closed loop poles will be within the design constraints. (If you used the `rlocusplot` command previously, you may need to generate the RL for just this system to be able to use the tool.)
5. With your chosen values of K and z, find the closed-loop transfer function for your design.
6. Verify that the closed-loop poles are in the location predicted by the root locus for your chosen value of K.

7. Plot the unit step response, and print closed-loop pole locations (see `pzmap` and `damp` commands) and step response metrics (`stepinfo`) to verify that the closed-loop design meets the design specification and has zero steady-state error. Hint: Because the closed-loop system is more complex than a single second-order system with no zeros, even if your closed-loop poles are meet the s-plane constraints, the total system response metric goals. A design that "gets close" is sufficient.

8. Report your compensator design in the form

$$D(s) = K\frac{s+z}{s}.$$