Cruise Control - Solution

Dynamics (Engr 3340), Fall2006

Instructions

In this example we will use MATLAB to analyze one example of a first order model - the response of a car with cruise control (figure 2).

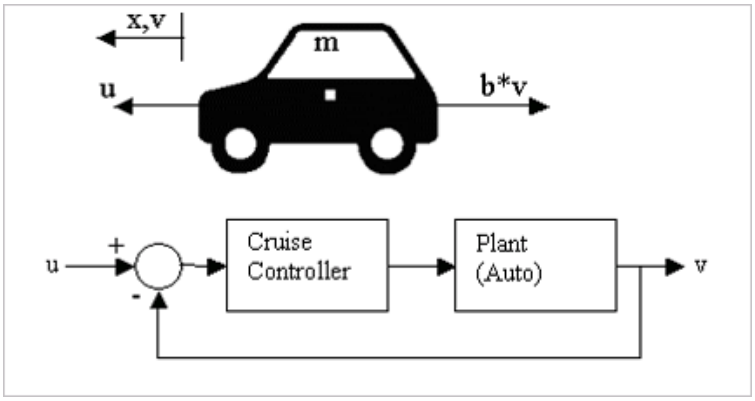

Figure 1: Cruise Control

$$m\dot{v} + bv = u(t)$$

$$G(s) = \frac{V(S)}{U(s)} = \frac{1/m}{s + b/m}$$

1. Write the equations of motion for the speed and forward motion of the car shown in figure 2. Assume that the engine imparts a force $u(t)$ as shown. Drag is modelled as a linear function of velocity. Take the Laplace transform of the resulting differential equation and find the transfer function between the input u and the output v.

2. Use MATLAB to find the response of the velocity of the car for the case in which the input jumps from being $u = 0$ at time $t = 0$ to a constant $u = 500$ N thereafter. Assuming the car's mass is 1000 kg and b = 50 Ns/m.

3. What is the steady-state speed increase for the specified input?

   This can be illustrated with figure 2. You can alternatively do this two other ways. First, you can see that the steady-state value is $v_{ss} = f/b = 500/50$ m/s by considering the differential equation and setting $\dot{v} = 0$. A second method is to use the final value theorem in the Laplace domain. From the transfer function we can see that $V(s) = \frac{F}{s}\frac{1/m}{s+b/m}$. To find the final value we multiply by s and take the limit as $s \to \infty$. This also yields $v_{ss} = f/b$

4. How long does it take for the car to reach this steady-state speed?

   The time constant for this system is $\tau = m/b = 20$ s. The vehicle will never truly reach the steady-state speed, but we might consider it to have reached the steady-state speed when it is 98% of the way there. This happens after three time constants (3τ) or 60 s.

```
% First-Order LTI System
%Cruise Control Example
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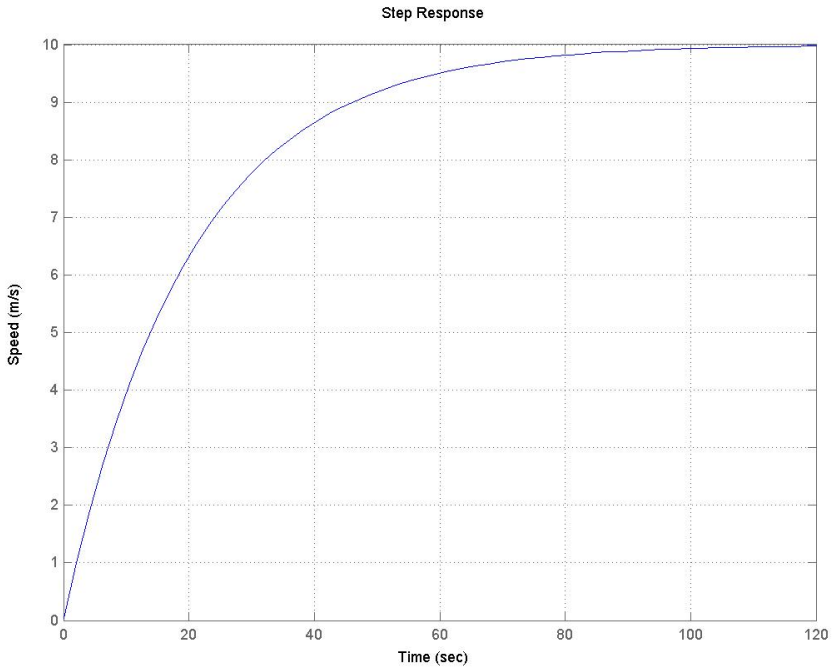

Figure 2: Step response for 500 N input. Notice the steady-state (final) value of 10 m/s and the time-constant of 20 seconds.

```
clear;
close all;

m = 1000; % kg
b = 50; % Ns/m

% Make and LTI system
num = 1/m;
den = [1 b/m];
sys1 = tf(num,den);

% now you can type 'sys1' to see the transfer funtion
sys1

% Look at the time response
figure(1)
step(sys1*500);  % notice that we can multiply the lti system by 500 (because the step input has an ampl

ylabel('Speed (m/s)')
grid on;
```