USV Modeling

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Abstract

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I. Introduction

The goal of this effort is to develop a USV model that

- Makes use of physically measurable quantities (mass and inertia)
- retains fidelity appropriate for
- with predictive capabilities with and without water current
- that can be identified using static tests and manuevering trials with a USV and onboard sensors

II. Background

A. *[Sonnenburg et al., 2010] and [Sonnenburg and Woolsey, 2013]*

[Sonnenburg and Woolsey, 2013] and [Sonnenburg et al., 2010] examine model for USV with steerable outboard motor (vectored thrust) where sideslip is a major concern. Uses notation and maneuvering model from [Fossen, 1994].

- Full vessel model is includes linear and quadratic damping
- All models are then linearized (perturbation dynamics) for the purpose of identification where the coefficients are parameterized by the states (surge, sway and yaw-rate)
- Actuation model thrust as a linear and quadratic with velocity depedence.

Model identification appears to throw out the phyical model and rely on identification of linear models (perturbation models) and a set of discrete speeds.

- Model idetification includes open-loop manuevers to identify steady-state parameters and closed-loop maneuvers to identify dynamic parameters.

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1) Steady-state values are identified by as linear relationships between yaw-rate, side-slip angle and side-sleep speed. The coefficients of these a relationships are determined for a set of discrete, constant speeds. The values of these coefficients change significantly over the range of forward speeds and now functional relationship is offered.
2) Thurster model conflates both the thrust relationship and the inertia of the system. Model is identified by measuring initial acceleration during step changes in engine RPM and identifying the two linear coefficients of a linear model. This single model is constant over the range of speeds. Sparce data at higher speeds.
3) Speed/surge is modeled as first-order model parameterized over speed range.
4) Steering model (first order Nomoto model with sideslip) is indentified by minimizing a quadratic cost function of sideslip angle and yaw-rate over close-loop time histories. Again, the linearized models have coefficients that change with speed. The results again show significant changes over the speed envelope without offering a functional relationship.

B. [Caccia et al., 2008]

Uses the nonlinear model of Blanke [**?**] as reported in [Fossen, 1994]. Speed/surge model:

- Neglects added mass term based on Blanke comments. Blanke suggests that the surge added mass term "will typically be less then 5%" [Fossen, 1994].
- Neglects r^2 terms, as suggested by Blanke, based on assuming reasonabley low yaw-rate (¡ 10 degrees/s).
- Assumes linear+quadratic drag. The Blanke model reported in [Fossen, 1994] includes only the quadratic term.
- Neglects the Coriolis terms based on assuing negligible sway velocity. The experimental evidence from [Sonnenburg and Woolsey, 2013] and [Sonnenburg et al., 2010] indicate that there is a significant sideslip angle, hence a significant sway velocity.

Steering model

- Neglects added mass in surge and yaw.

Sway (sideslip) was not observable in experiments! Used only GPS and heading for identification making making many of the estimated quantities unreliable or unobservable.

Thrust model neglects speed of advance, assumes thrust is independen of vessel speed.

III. MANEUVERING MODELS

A. Nonlinear Maneuvering Model based on Second-order Modulus Functions

This model is uses second-order (linear and quadratic) terms for the dissapative terms. In this section we follow the notation and process detailed in [Fossen, 2011], Chapter 7. The horiozontal-plane maneuvering model captures is formulated using state vector $\boldsymbol{\nu} = [u, v, r]^T$ where the velocities u, v and r are in the surge, sway and yaw directions respectively. The velocities are considered to be relative to an irrotational constant ocean current. The nonlinear maneuvering equations from [Fossen, 2011] are

$$\underbrace{\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{rigid-body forces}} + \underbrace{\boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r}_{\text{hydrodynamic forces}} = \boldsymbol{\tau} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{waves} \tag{1}$$

where $\boldsymbol{\nu}_r$ is the velocity vector relative to an irrotational water current $\boldsymbol{\nu}_c$, i.e., $\boldsymbol{\nu} = \boldsymbol{\nu}_r + \boldsymbol{\nu}_c$. The rigid body kinetics are represented by the rigid body mass $\boldsymbol{M}_{RB}$

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_g \\ 0 & mx_g & I_z \end{bmatrix}, \tag{2}$$

where m is the mass of the vehicle, I_z is the moment of inertia about the body-centered z-axis and x_g is distance, along the x-axis, from the origin of the body-centered frame to the center of gravity of the vessel, and by the rigid body Coriolis-centripetal matrix,

$$\boldsymbol{C}_{RB}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & -m(x_g r + v) \\ 0 & 0 & mu \\ m(x_g r + v) & -mu & 0 \end{bmatrix}. \tag{3}$$

Noting that $\boldsymbol{C}_{RB}(\boldsymbol{\nu})$ is skew-symmetric, i.e., $\boldsymbol{C}_{RB}(\boldsymbol{\nu}) = -\boldsymbol{C}_{RB}^T(\boldsymbol{\nu})$. The hydrodynamic effects are represented by the added mass matrix

$$\boldsymbol{M}_A = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix}. \tag{4}$$

and the Coriolis-centripetal matrix for the added mass

$$\boldsymbol{C}_A(\boldsymbol{\nu}_r) = \begin{bmatrix} 0 & 0 & Y_{\dot{v}} v_r + Y_{\dot{r}} r \\ 0 & 0 & -X_{\dot{u}} u_r \\ -Y_{\dot{v}} v_r - Y_{\dot{r}} r & X_{\dot{u}} u_r & 0 \end{bmatrix}. \tag{5}$$

It is worth noting that $\boldsymbol{C}_A$ includes the nonlinear Munk moment (see [Fossen, 2011] p.121). Following [Fossen, 2011] the SNAME notation for the hydrodynamic derivatives.

The linear and quadratic drag terms

$$\boldsymbol{D}(\boldsymbol{\nu}_r) = \begin{bmatrix} X_u + X_{u|u|}|u| & 0 & 0 \\ 0 & Y_v + Y_{v|v|}|v| & Y_r + Y_{r|r|}|r| \\ 0 & N_v + N_{v|v|}|v| & N_r + N_{r|r|}|r| \end{bmatrix}. \tag{6}$$

Equivalently we can express the same model in non-matrix form, where the speed equation in the surge direction is

$$\underbrace{m\dot{u}}_{\text{RB inertia}} - \underbrace{mx_g r^2}_{\text{RB centripetal}} - \underbrace{mvr}_{\text{RB Coriolis}} = \underbrace{X_{\dot{u}}\dot{u}_r}_{\text{AM inertia}} + \underbrace{Y_{\dot{v}}v_r r}_{\text{AM Coriolis}} + \underbrace{Y_{\dot{r}}r^2}_{\text{AM centripetal}} + \underbrace{X_u u + X_{u|u|}|u|u}_{\text{Drag}} + \underbrace{\tau}_{\text{Thrust}} \tag{7}$$

and the coupled steering questions in the sway direction

$$\underbrace{m\dot{v} + mx_g\dot{r}}_{\text{RB inertia}} + \underbrace{mur}_{\text{RB Coriolis}} \underbrace{-Y_{\dot{v}}\dot{v} - Y_{\dot{r}}\dot{r}}_{\text{AM inertia}} = +\underbrace{Y_v v + Y_{v|v|}|v|v}_{\text{Drag}} + \underbrace{Y_r r + Y_{r|r|}|r|r}_{\text{Coupled Drag}} \tag{8}$$

and yaw directions

$$\underbrace{I_z\dot{r} + mx_g\dot{v}}_{\text{RB inertia}} + \underbrace{mx_g ru}_{\text{RB Coriolis}} \underbrace{-Y_{\dot{r}}\dot{v} - N_{\dot{r}}\dot{r}}_{\text{AM inertia}} - \underbrace{\overbrace{Y_{\dot{v}}v_r u + X_{\dot{u}}u_r v}^{\text{Monk Moment}} - Y_{\dot{r}}ru}_{\text{AM Coriolis}} - \underbrace{N_v v + N_{v|v|}|v|v}_{\text{Coupled Drag}} + \underbrace{N_r r + N_{r|r|}|r|r}_{\text{Drag}} = +\underbrace{\tau}_{\text{Torque}} \tag{9}$$

B. *Nonlinear Model of Blanke*

As reported by [Fossen, 1994], this model "retains the most imoprtant terms for steering and propulsion". Formulating these equations for a rudder-less vessel, the speed equation becomes

$$(m - X_{\dot{u}})\dot{u} = X_{u|u|}|u|u + (m + X_{vr})vr + (mx_g + X_{rr})r^2 + \tau \tag{10}$$

Comparing (7) and (10), noting that $Y_{\dot{v}}$ from (7) is equivalent to X_{vr} in (10), we can see that the only difference between these models is that the full model in (7) includes both linear and quadratic drag terms, while Blanke's model includes only a quadratic term. The linear term is important because it forces the system to coverge to zero surge. Futhermore, results of other experiments suggest that the drag is not purely quadratic.

Blanke's steering model in sway, without rudder terms, is

$$(m - Y_{\dot{v}})\dot{v} + (mx_g - Y_{\dot{r}})\dot{r} = -(m - Y_{ur})ur + Y_{uv}uv + Y_{v|v|}|v|v + Y_{|v|r}|v|r. \tag{11}$$

Comparing (8) and (11) we note the following differences

- The "Coupled Drag" term in (8) is neglected in the Blanke model. What is the affect of neglecting these coupled drag terms? Appears faily commonly in simplified models.

- The linear component of cross-flow drag (8) is neglected in the Blanke model
- The following second-order cross-coupling terms are present in Blanke that are not represented in (8)
  - $Y_{ur}ur$ - which appears as an added mass Coriolis term
  - $Y_{uv}uv$
  - $Y_{|v|r}|v|r$
  - What is the consequence of neglecting these? Ditto for the yaw eqn.

The steering equation for yaw is

$$(mx_g - N_{\dot{v}})\dot{v} + (I_z - N_{\dot{r}})\dot{r} = -(mx_g - N_{ur})ur + N_{uv}uv + N_{v|v|}|v|v + N_{|v|r}|v|r \tag{12}$$

What/Why is the quadratic yaw drag expressed as $N_{v|v|}|v|v$? Seems like it would be $N_{r|r|}|r|r$.
Comparing (9) and (12), we note that the Monk moment has been collapsed into a single term $N_{uv}uv$ in (12). We note the following differences

- The "Coupled Drag" term in (9) is neglected in the Blanke model
- The linear component of yaw drag (8) is neglected in the Blanke model
- The second-order cross-coupling term $N_{|v|r}|v|r$ is present in Blanke model and not represented in (9).

C. Simplified Model of Blanke from [Caccia et al., 2008]

In [Caccia et al., 2008] the Blanke model is used, with the addition of linear drag terms, and the following assumptions are made:

- Speed: Surge
  - Added mass in surge ($X_{\dot{u}}$) is negligible - less than 5% of vessel mass. This seem reasoable, but is not really necessary since the mass and added mass are lumped together in the identification.
  - The Coriolis terms $(m + X_{vr})vr$, both rigid body and added mass contributions, are negligible because the sway speed is negligible. This would seem to limit the sideslip. The experiments in [Sonnenburg et al., 2010]suggest the sideslip would be an important component for this type of vehicle.
  - The centripetal terms $(mx_g + X_{rr})r^2$ are neglected on the assumption of a low turn rate. This is probably reasonable?
- Steering: Sway and Yaw
  - Added mass in both directions is neglected. Again, probably not necessary since the identification estimates the combination of rigid body and added mass.

- Although the author's don't make it explicit(!), they neglect the rigid body and added mass coupling terms I believe this is equivalent to assuming that the principle axes of inertia for the vessel (the body-centered coordinate frame) are co-located with the center of mass of the vessel.
  * $(mx_g - Y_{\dot{r}})\dot{r}$ in the sway equation
  * $(mx_g - N_{\dot{v}})\dot{v}$ in the yaw equation
- Again, this is not explicit, but the authors omit the Coriolis terms, both rigid body and added mass, $-(mx_g - N_{ur})ur$. The reason for omission is not provided in the text.
- The "coupled drag terms are neglected because v and r are typically small". We can only assume they are referring to the following terms - Are these correctly identified as "coupling drag terms"? Are they negligible?
  * $Y_{uv}uv$ and $Y_{|v|r}|v|r$ are negligible in sway
  * $N_{uv}uv$ and $N_{|v|r}|v|r$ are negligible in yaw
- The quadratic term $N_{|v|v}|v|v$ in the Blanke model is substituted with a linear and quadratic term as a function of yaw, $N_r r + N_{|r|r}|r|r$. As indicated previously, I'm not clear whey this term is related to the sway velocity in the first place.

For our vessel this would be equivalent to the following speed and steering equations

$$m\dot{u} = X_u u X_{|u|u}|u|u + \tau \tag{13}$$

$$(m)\dot{v} = -(m - Y_{ur})ur + Y_v v + Y_{v|v|}|v|v. \tag{14}$$

$$I_z\dot{r} = N_r r + N_{r|r|}|r|r + \tau \tag{15}$$

This model is greatly simplified. Us such a high degree of simplification justified?

D. [Muske et al., 2008]

The model in [Muske et al., 2008] supposedly comes from [Fossen, 1994] (although I haven't found it yet). This model includes a power-law drag model in each direction, combined rigid body and added mass terms and (unlike the simplified Blanke model from [Caccia et al., 2008]) includes the Coriolis terms. The equations of motion are

$$m_{11}\dot{u} - m_{22}vr + d_1 u^{\alpha_1} = \tau_{Thrust} \tag{16}$$

$$m_{22}\dot{v} + m_{11}ur + d_2 u^{\alpha_2} = 0 \tag{17}$$

$$m_{33}\dot{r} + (m_{22} - m_{11})uv + d_3 u^{\alpha_3} = \tau_{Torque} \tag{18}$$

$$\tag{19}$$

Comparing this model to the nonlinear maneuvering model in (**??**) we see that

- Rigid body mass and added mass are combined in all terms
- They assume the body-axis and center of mass are collocated
- All centripetal terms are neglected
- Drag modeled as a power law, which may be more general (with the same number of parameters) as the linear+quadratic model.
- There are some additional assumptions here to make this work out - i.e., deriving this from the original nonlinear model.

E. Thrust Model

Two options:

Assume thrust is independent of speed (as done in the Caccia papers).

Or assume and unknown, linear decrease in thrust with speed.

$$T = T_o(1 - au)$$

where a is the linear speed reduction

F. Speed model

Consider the surge state of the model above where

$$\underbrace{m\dot{u}}_{\text{RB inertia}} - \underbrace{mx_g r^2}_{\text{RB centripetal}} - \underbrace{mvu}_{\text{RB Coriolis}} = \underbrace{X_{\dot{u}}\dot{u}}_{\text{AM inertia}} + \underbrace{Y_{\dot{v}}v_r r}_{\text{AM Coriolis}} + \underbrace{Y_{\dot{r}}r^2}_{\text{AM centripetal}} + \underbrace{X_u u + X_{u|u|}|u|u}_{\text{Drag}} + \underbrace{T}_{\text{Thrust}} \tag{20}$$

Following [Caccia et al., 2008], based on [Fossen, 1994], we neglect the second-order centripetal terms

$$m\dot{u} - mvu = X_{\dot{u}}\dot{u} + Y_{\dot{v}}v_r r + X_u u + X_{u|u|}|u|u + T \tag{21}$$

For steady state forward motion ($\dot{u} = v = r = 0$) in stationary water ($v_r = 0$)

$$0 = +X_u u + X_{u|u|}|u|u + T \tag{22}$$

We can estimate X_u and $X_{u|u}$ from steady state forward motion trials with known thrust input by testing at a series of known forward speeds and measuring u.

Considering forward-only acceleration

$$m\dot{u} = X_{\dot{u}}\dot{u} + X_u u + X_{u|u|}|u|u + T \tag{23}$$

we can identify the added mass ($X_{\dot{u}}$) by either estimating the initial acceleration (see [Sonnenburg et al., 2010]) or by examing the 'time constant' for such tests.

This leaves the coefficient $Y_{\dot{v}}$, related to the added-mass Coriolis force, as the single unknown.

G. Steering Model

IV. Model Identification Tests

A. Physical Measurements

- Measure the mass (m) directly.
- Measure the moment of inertia (I_z) using a bifilar pendulum.

B. Thrust Characterization

Bollard tests in the tank to measure thrust force (at zero velocity) as a function of motor command.

C. Steady-State Tests

- Surge: Measure the steady-state speed at a variety of thrust inputs to identify the drag terms.
- Yaw: Measure the steady-state yaw rate at variety of torque inputs to identify the yaw drag terms.

D. Open-Loop Dynamic Tests

- Surge: Measure step response to forward thruste (with heading control?) to estimate added surge mass.
- Yaw: Measure step response to torque to estimate added mass/ineriat in yaw.

E. Closed-Loop Dynamic Tests

References

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