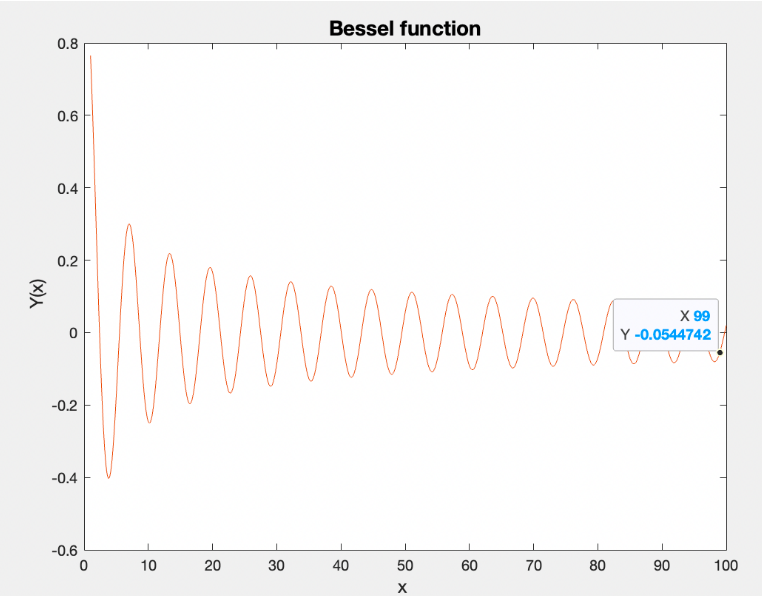


It can be observed from the graph of the Bessel function that the roots occur approximately at the fixed interval of 3.14.

After analysing the graph, I chose the first a to be 0 and b to be 5 since the first root occurs between those two intervals. For successive roots, I first set a to be equal to b and then incremented b by 1 until f(a) * f(b) < 0. Then set a to be equal to b -1. This is efficient and robust because the difference between the endpoints is always 1 therefore we do not miss any roots since we've already observed that the difference between the roots is approximately 3.14.

I chose M to be 10000 since it is a large enough number to give accurate results without taking significant amount of computer memory. Having such a large M would make O(1/M) almost insignificant making $X_M = \alpha$ (M+ b), M -> $\infty$.

I chose my tolerance interval to be 10^-8. A larger tolerance lead to the divergence of the function while a smaller tolerance of 10^-2 lead to inaccurate results. A tolerance of this size ensures convergence and accuracy.

Finding the roots of the Bessel function using the bisection function and then finding the linear approximation of the roots using polyfit gives us the slope as 3.141592633845049 and the y intercept is -0.785259475371024 .

Alpha and beta can be found by using substitution method. First we write an equation in terms of alpha and then substitute that in the second equation which gives us alpha and then substitute it again in the first equation to find beta.

1. $X_{9999}$ -> α(M + β) : 3.141199954905361 * 10^-4= α (9999+ $\beta$)
2. $X_{10000}$ -> α(M + β) 3.141514114172012* 10^-4 = α (10000+ $\beta$)

Which gives us alpha as 3.141592666520413 and beta as -0.2500398897599926.

I believe that the true values of alpha and beta are pi and -0.25 respectively. I believe so because the values are getting closer to pi and -0.25 as M increases.