

a)

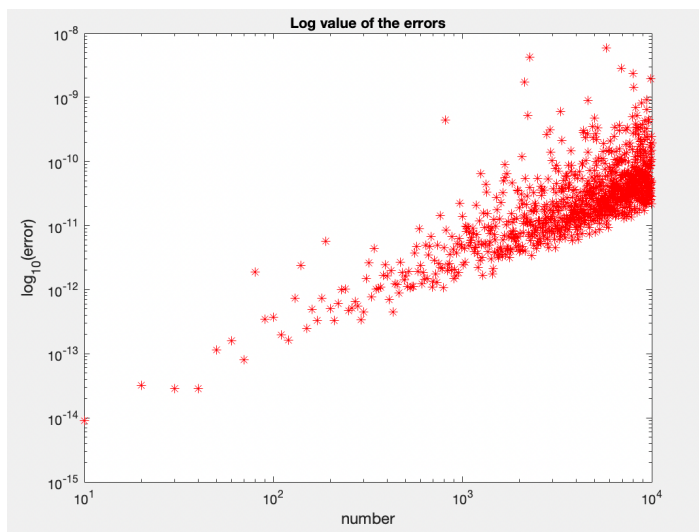


Figure 1

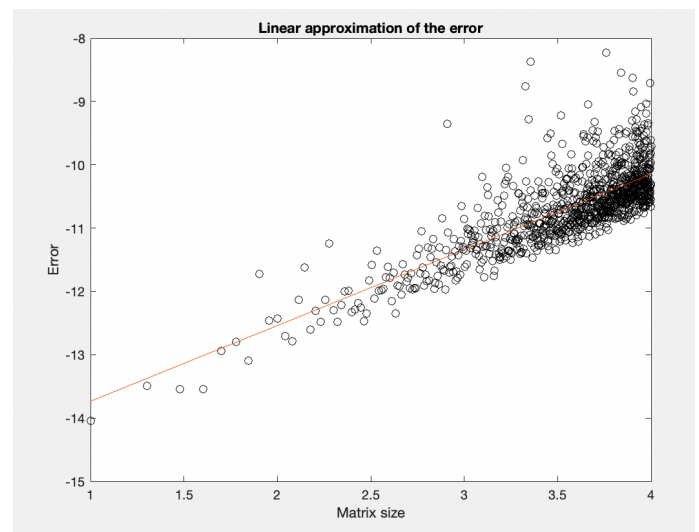


Figure 2

b) I chose  $N$  to be values from 10 to 10000 with an increment of 10 so that we have enough data points to have a more accurate linear approximation of the error. It also allowed me to have a big enough end value without making the computation extremely time consuming. The large number of points made it easy to observe the linear pattern in the error. I chose  $M$  to be 100 so that we have sufficient trials to improve the accuracy of the linear estimation. I also tried  $N$  values that went until  $10^5$  but the matrix size was too big for Matlab. A lower  $M$  (10) value gave me a weaker approximation of the slope and the y-intercept.

c) For different  $n$  values from 10 to 10000, random tridiagonal matrices are created and gaussian elimination is used by the `backslash` command to come up with the solution vector,  $M$  times for each  $n$ . The max error in each trial is stored and then later used to find the mean error for that  $n$  value. Log of the Mean errors of all the  $n$  values are then plotted against the respective  $n$  value. It can be observed from Figure 1 that the the log error follows a linear pattern. Then, `polyfit` is used to estimate the slope and the y-intercept of the line. The returned  $p_1$  (1.19563301207961) and  $p_2$  (-14.9308145029054) are used in the equation:  $1 = \log N^* * p_1 + p_2$  ; to find the value of  $N^*$  where the error in the solution is 1.

D) Polyfit returns 1.19563301207961 and -14.9308145029054 as the slope and the y-intercept. Using,

$$\begin{aligned}
 y &= m \log x + c \\
 1 &= \log N^* * p_1 + p_2 \\
 1 &= 1.19563301207961 \log N^* - 14.9308145029054 \\
 1 &= \log N^* * (1.19563301207961) - 14.9308145029054 \\
 \log N^* &= 13.32419.. \\
 N^* &= 10^{13.32419..} \\
 N^* &= 2.1095..e13
 \end{aligned}$$