Computing Assignment 6

We were tasked with computing the integral I using several methods. At first I used the Simpsons rule which finds the area underneath a polynomial interpolant, n taking values from 100 to 1000 with a step size of 100. It returns converges to -0.4758 which is an inaccurate approximation on the integrand.

Further, I found the zeros of the integrand. These zeros were used to define subintervals in the range [0,1]. Given the subintervals, integral was computed and summed to find the corresponding Q_n values. Q_n appears to slowly converge to -0.46. While using n from 100 to 1000 with step size of 100 an accuracy of only 4 decimal places was observed. Increasing the n to 10,000 gives us an accuracy of 6 decimal places in reasonable computing time.

N = 100	200	300	400	500
-0.461967715156174	-0.460687377158077	-0.460253704737583	-0.460034802499441	-0.459902584904608

N = 600	700	800	900	1000
-0.459813991842412	-0.459750453197773	-0.459702638008753	-0.459665341330723	-0.459635429447389

In order to speed up the convergence of Aitken's Δ^2 Method is used as an extrapolation algorithm which defines a new sequence. At n = 10,000 and a step size of 100 an accuracy of 6 decimal places is observed. This method converges to -0.459375. I chose this level of accuracy as at Qhat9999 = -0.459375461758346 and at n = 10000 Qhat =-0.459375318076043 since the first 6 digits match the accuracy is upto 6 decimal places. Similarly, if we choose n = 1000 with a step size of 1 $Q^{\wedge}_{9999}$ = -0.459360912739237 and $Q^{\wedge}_{10000}$ = -0.459360912874103 giving an accuracy of 9 digits. This is achieved in a reasonable computing time. It can be observed from the figure on the bottom right that this converges faster than the summation method used previously.

For the bonus question, when n = 2000 and step size is 1 $Q^{\wedge}$ converges to approximately 0.2795060966 with an accuracy of 10 digits. In order to get the output I made the following changes to my code:

1. The function and b values to find out the subintervals were found out using `x∗ exp(2∗x)−i ∗ pi+(pi/2),0`. The addition of pi/2 is made to account for the zeros of the cosine function.