The Application of Wavelet Transform in Digital Image Processing

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***Abstract*—The edge is the most important high-frequency information of a digital image. The traditional filter eliminates the noise effectively. But it will make the image blurry. So we should protect the image of the edge when reduce the noise of the image. The wavelet analysis method is a time-frequency analysis method which selects the appropriate frequency band adaptively based on the characteristics of the signal. Then the frequency band matches the spectrum which improves the time- frequency resolution. The wavelet analysis method has an obvious effect on the removal of noise in the signal. The paper outlines the principles of wavelet analysis. According to the features of the multi-scale edge of the wavelet, we analyze the de-nosing method of the orthogonal wavelet transform which based on soft and hard threshold. A de-noising method is put forward based on the wavelet transform to address this dilemma of the noise reduction and protection the image edge, and realizes the de-noising of two-dimensional image signal based on MATLAB。**

Keywords-digital image; wavelet transform; multi-scale edge detection; de-nosing

I. INTRODUCTION

During the processing of the collection, transmission and conversion, the image is often degraded by the interference of imaging equipment and external environmental noise. Image de-noising is a common technology in the image processing. Essentially, the classic image de-noising method is a low-pass filter. The low-pass filter eliminates the noise effectively. But, the image will also make the image blurry. Therefore, all the image de-noising methods are addressing this dilemma of the de-noising and the protection of the image edge [1].

The wavelet transform method is a mathematical method developing in the 1980s which can decompose the signal with finite energy in the spatial domain into a set of function as a standard in the modular spatial domain of orthogonal. Then we analyze the characteristics of the signal in the modular spatial domain. Compared with the traditional Fourier analysis, the wavelet transform can analyze the function in the modular spatial domain and timing domain which has better local capacity of the frequency and time. It is the development and sublimation of Fourier transform, which has a lot of advantages. The Fourier analysis method and other traditional methods of analysis of the frequency domain don't have these advantages. So it has a unique effect on image de-noising based on the wavelet transform. The study of image de-noising has important significance based on the wavelet transform [2].

The conventional de-noising method will make the edge of image blurry. In order to retain and enhance the edge of the image, it would affect the effects of de-noising. In the domain of the wavelet transform a de-nosing algorithm was found which can balance the de-noising, the reservation of the edge of the image and other reservations. At present, image de-noising method based on wavelet threshold is researched more popular than others. Based on the wavelet transform of de-nosing, many scholars have extensive research field on select of threshold and function of the threshold [3, 4].

II. TRADITIONAL DIGITAL IMAGE PROCESSING

There are many noise sources such as grating scanning, ppaper film, mechanical components and transmission channels, and different kinds of noise are so complex such as noise additive noise, multiplicative noise and quantization noise that smoothing methods are different too. The smoothing method can not only be carried out in the spatial domain, but also in the frequency domain. The traditional de-nosing method achieves only in the spatial domain [5]. Image de-noising algorithm in the spatial domain can be divided into linear and nonlinear methods. Linear method has a more complete theoretical basis whose typical representative is the mean filter which is suitable for zero mean of Gaussian noise.

Some noise of the images can be removed in a linear filter, such as the neighborhood average of local average filter which is very applicable to remove the ppaper noise by scanning images [6]. The thought of the mean filter is as follows: for a given pixel (x, y) of the image f (x, y), we select the neighborhood w. If w contains M pixels, the average of noise is used as a department of gray of the post-image pixel (x, y). In other words, a pixel with the neighborhood average gray pixels replaces the original gray pixels. If the window size of the template is the 3x3, the pixel gray value R of the image at the point (x, y) is calculated according to formula 1x.

$$f(x,y)=\frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1}f(x+i,y+i) \qquad (1)$$

If the noise n is additive noise, irrelevant points in the space, the expectation for 0, the variance for σ^2 , and g is not a contaminated image, F (x, y) containing noise is averaged in the neighborhood domain as formula 2.

$$f(x,y)=\frac{1}{M}\sum f(i,j)=\frac{1}{M}\sum g(i,j)+\frac{1}{M}\sum n(i,j) \qquad (2)$$

The formula (2) shows that the mean of the noise is unchanged after the neighborhood average. Noise variance becomes smaller which means the weakness of the intensity of the noise and noise reduction achieved. Linear smoothing filter is easy to implement and has a good performance in the signal and noise frequency spectrum. However, we often get signals with the steep edge in the

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actual image processing. We use a linear smoothing filter which will blur the edge of images. On the other hand, linear filters can not remove impulse noise completely. Therefore, in many cases, we need to improve the filtering method to overcome these problems.

III. WAVELET TRANSFORM

The basic method of the wavelet transform is selecting a function whose integral is zero in time-domain as the basic wavelet. By the expansion and translation of the basic wavelet, we can get a family function which may constitute a framework for the function space. We decompose the signal by projecting the analysis signals on the framework. The signal in original time domain can get a time-scale expression by several scaling in the wavelet transform domain. Then we are able to achieve the most effective signal processing purpose transform domain [7].

The essence of wavelet de-noising is searching for the best mapping of signals from of the actual space to wavelet function space in order to get the best restoration of the original signal. From the view of the signal processing, the wavelet de-noising is a signal filtering. The wavelet de-noising is able to retain the characteristics of the image successfully. Actually, it is comprehensive with feature extraction and low-pass filtering [8].

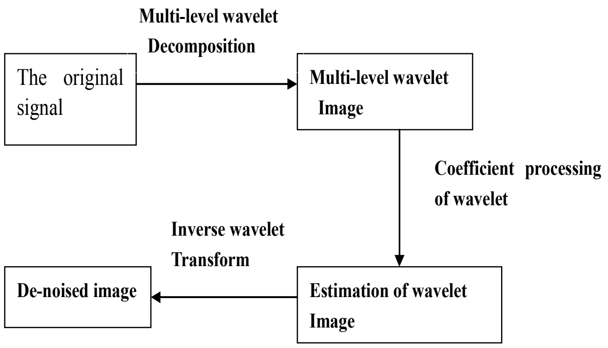


Figure 1 the de-noising process based on wavelet transform

The meaning of tensor product space is given at first for realization of this process [9, 10]. F and G is two limited dimensional or countable infinite-dimensional linear space. The basement of F and G are ..., f-1, f0, f-1, ... and ..., g-1, g0, g-1, ... Form shows as fi and gi; (i=0，±1...;J=0，±1,…) is the basement of elements space H, it is known as the tensor space of F and G. It shows as follows:

$$H = F \otimes G \tag{3}$$

If the F, G are the functional space respectively, suppose independent variable of F function being x, independent variable of G being y, the element of the tensor space H is called as the dual tensor function or tensor surface.

Two-dimensional multi-scale analysis is made from the relevant tensor product by one-dimensional multi-scale analysis which is made up of four elements, which defines as two functions and the corresponding two-scale space. Suppose $\{V_j, j \in Z\}$ is generated by the scaling function $\phi(t)$ and scale of space $\{W_j, j \in Z\}$ is by the wavelet $\Psi(t)$, Wj is orthogonal complement of Vj in Vj +1. Wj and Vj are orthogonal complements. According to the tensor product definition, there can be two-dimensional subspace:

$$V_j^2 = V_j \otimes V_j \tag{4}$$

$$\phi(x, y) = \phi(x)\phi(y) \tag{5}$$

$\phi(x, y)$ is the scaling function of $\{V^2{}_j, j \in Z\}$, a set of orthogonal basis of V_j^2 two-dimensional space which shows as the coefficients of the corresponding space of the standard orthogonal basis are recorded as $\{c_{j,m,n}\}\{d^1{}_{j,m,n}\}\{d^2{}_{j,m,n}\}\{d^3{}_{j,m,n}\}$ .Suppose V0 is the result of the N-decomposition, we can reconstruc as follows:

$$f(x,y) = \sum_{j=-N}^{-1}\sum_m\sum_n d^1_{j,m,n}\phi_{j,m}(x) + \sum_{j=-N}^{-1}\sum_m\sum_n d^2_{j,m,n}\psi_{j,m}(x)\phi_{j,n}(x) \quad (6)$$
$$+ \sum_{j=-N}^{-1}\sum_m\sum_n d^1_{j,m,n}\psi_{j,m}(x)\phi_{j,n}(x) + \sum_m\sum_n c_{-N,m,n}\phi_{m,n}(x)\phi_{m,n}(x)$$

In (6), $d^k_{j,m,n}$ is the approximate coefficient of the image f (x, y) in the j-scale, and $d^k_{j,m,n}$ (k = 1,2,3) is the coefficient of the image f (x, y) in horizontal, vertical and diagonal direction. Multi-resolution decomposition process of two-dimensional signal can be described as one after another one-dimensional decomposition to row-column.

There are three ways to de-noising in the processing of the wavelet coefficient: 1) maximum de-noising based on the wavelet transform; 2) correlation de-nosing based on the adjacent scale wavelet coefficient; 3) correlation de-nosing based on the threshold of wavelet transform domain.

The paper chooses the threshold of the wavelet coefficients. The basic theory is supposing that there are numbers of wavelet coefficients which are polluted seriously very small or near to 0. So we can give the threshold to remove the polluted points in order to remove the noise.

This threshold divided mainly in the form of two categories: hard and soft threshold. The soft-threshold makes the model which is smaller than the threshold th of wavelet coefficients replaced by 0. The formula shows as follows:

$$\mathrm{th}s(W) = \begin{cases} \mathrm{sgn}(W)(|W| - th), |W| \ge th \\ 0, else \end{cases} \tag{7}$$

The hard threshold retains the model whose value is greater than the threshold of the wavelet coefficients, and makes the model whose value is smaller than the threshold 0. The formula shows as follows:

$$th_h(W) = \begin{cases} W, |W| \ge th \\ 0, \text{else} \end{cases} \tag{8}$$

The key is to use the threshold to select the value th. If you choose greater, the noise can't be removed mostly which makes the result worse. If smaller, it makes the detail of the image lost which has the deviation. We should consider the selection of the threshold th at each layer because the wavelet transform is a multi-resolution analysis.

IV. EXPERIMENT

In the digital image processing, the choice of the basic wavelet is very important. Haar wavelet is unique symmetry wavelet in the whole orthogonal wavelet. Haar wavelet's support is very short which can be high-pass and low-pass filter, what's more, it can save the computational complexity. So this paper chooses Haar wavelet as the basis function for digital image analysis. The expression of Harr wavelet and its scaling function follows as follows:

$$\psi_h(x)=\begin{cases}1 & 0<x\le 0.5\\ -1 & 0.5\le x<1\\ 0 & \text{else}\end{cases} \qquad (9)$$

$$\phi_h(x)=\begin{cases}1, & 0\le x\le 1\\ 0, & \text{else}\end{cases} \qquad (10)$$

The corresponding function graphs are shown in Figure 2:

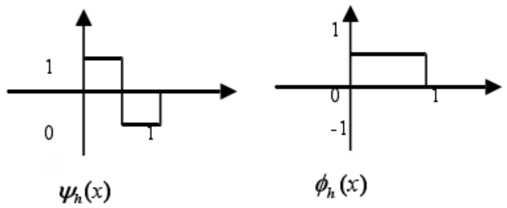


Figure 2 Haar wavelet function and its scale function

The experiment has three steps by using wavelet analysis to deal with image noise.

Step 1: Wavelet decomposition of two-dimensional image.

We should select the appropriate wavelet and determine the level of N to make the de-noised image f(x) wavelet transform. Then we can get a set of wavelet decomposition coefficient $\omega_{j,k}$.

Step 2: Quantifying the high-frequency coefficients after the decomposition.

We can process the wavelet coefficients $\omega_{j,k}$ which are decomposed by the threshold in the wavelet transform domain to get the new wavelet coefficients $\omega'_{j,k}$. It can make the $\left\|\omega_{j,k}-u_{j,k}\right\|$ as small as possible, where $u_{j,k}$ is the wavelet coefficient of non-polluted image signal, and we select the minimum of mean square error always.

Step 3: Reconstruction image signal of two-dimensional wavelet.

By making the processed wavelet coefficients inversing and reconstruction, we can get the energy concentrating on a small number of wavelet coefficients. But white noise on the orthogonal transformation is still white noise and has the same amplitude. In comparison, the wavelet coefficients of the image are greater than wavelet coefficients which have scattered energy and smaller amplitude. We can successfully remove the noise and retain the useful signal by choosing the appropriate threshold to process the wavelet coefficients. The key of the Threshold is the determination of the threshold. Because if the threshold is too large, this will make in useful high-frequency information (such as the edge of the information) lost and make the image blurred; and if the threshold is too small, it will be retained too much noise and make the de-nosing effect not obvious. As a result, we must choose a suitable threshold which is the only way to remove noise at the same time, and without fear of the damage to the image edge features.

Figure 3 shows an original image. The corresponding wavelet decomposition picture is showed in Figure 5. In Figure 5 (a), we can see that the overall perspective of the son image is close to the original image, but the edge become blurred. The image 5 (b), 5 (c) and 5 (d) reflect the details of the image and the brink of three directions. Figure 4 is a polluted image by the noise, and we can see that the image quality has seriously affected. The decomposition result is show as Figure 6. In Figure 6 (a) we can be see that overall perspective of the son image is close to the original image, but the whole has become blurred. The image 6 (b), 6 (c) and 6 (d) reflect the edge of details in three directions, but the edge is impacted by noise which makes the image which decomposed by wavelet transform doesn't reflect the details of the image. That may make the segmentation and identification worse. So it is necessary to de-noise before image processing. Figure 8 shows the noise mainly shows as the high-frequency information of the image. Therefore research on the relationship between noise and the multi-scale wavelet transform is important.

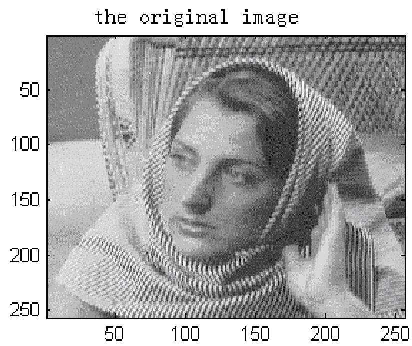


Figure 3 the original image

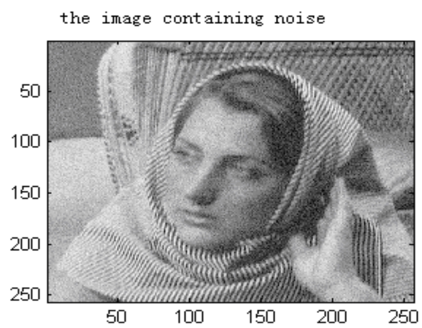
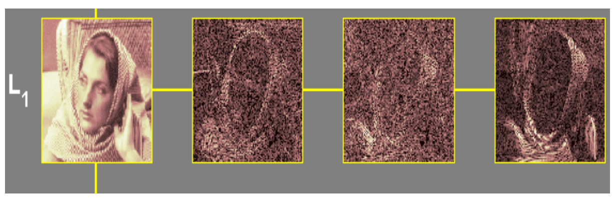
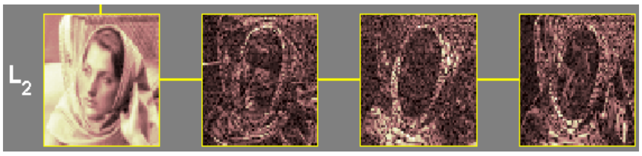


Figure 4 the image containing noise



a) Approximations b) Horizontal Details c) Vertical Details d) Diagonal Details

Figure 5 the coefficient of the first order decomposition of the original image



a) Approximations b) Horizontal Details c) Vertical Details d) Diagonal Details

Figure 6 the coefficient of the first order decomposition of the image with nosie

At the same time, the experiment shows that when wavelet analysis is used to deal with image noise based on the threshold of wavelet noise in the process, as the use of quantitative disposals of the threshold, the threshold is the key to de-noise processing. The forced de-noise method is relatively simple and easy processing way, but it is less effective to dispose the picture which has little high-frequency noise. So the range of application is quite narrow; although independent threshold noise has a good and reliable result. However, the threshold selection is more difficult. The key factor of selection wavelet threshold is the specific estimate of the threshold. If the threshold is too small, the image is still having noise after de-noising; in contrary, if the threshold is too large and important, characteristics of the images will be filtered. Visually, the choice of thresholds is a hot research field. Soft threshold de-noising wavelet decomposition uses different threshold in different layers and reflects the different propagation of signal coefficient and noise coefficient in the processing of the wavelet decomposition.

The effects of de-nosing and the degree of details retaining are influenced by several factors. We should analyze specific issues in practical applications.

V. CONCLUSIONS

The paper provides the image de-noising based on wavelet transform. At first, we decide the edge point of the image before the threshold de-nosing. Then we start to de-noise. The experiment shows that it is very important to choose the threshold during the image de-nosing. In the threshold selection, according to the wavelet coefficients of the two-dimensional Gaussian noise are still subject to Gaussian distribution, the paper analyzes the two-dimensional wavelet transform noise characteristics. Under certain conditions, using wavelet multi-scale features, the wavelet coefficients corresponding to the noise and the distribution of the wavelet coefficients corresponding to signal are opposite. By setting specific thresholds for wavelet coefficients, we can finish de-noising procedure. The method is simple, small amount of calculation, and has a wide range of applications in practice. However, the experiment also finds that a certain shortcoming still exist on the threshold section. Although it is giving the best universal threshold in Donoho theory, its practical applications have not been very effective [11]. Therefore, the improved algorithm of the selection of threshold and the estimation of wavelet coefficients are still the main direction of the research in future.

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