Image Denoising Based on Haar Wavelet Transform

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***Abstract*—Wavelet transform has widely applied in image denoising, and how to select the threshold is the key to wavelet image denoising.The paper put forward the algrithm for image denoising based on harr wavelet transform. Fisrt , The image with noise is decomposed by Haar wavelt transform, then select the soft threshold to clean the image noise. Experiments show that the algorithm can reduce image noise effectively and improve the detail effect of picture and raise the PSNR value of image.**

Keywords- Haar Wavelt Transform, Soft Threshold, Image Denoising

I. INTRODUCTION

How to clean the noise of image, and reserve the detail and edge information in the image as far as possible is the important contents of image processing. Up to the present, Some researcher have already put forward a lot of image denoising method. In the space area ,image denoising method mainly have median filter , mean filter,and Wiener filter ,etc[1].

Supose the value of current pixel is f(i, j), the value of image-denoisied pixel is $\widehat{f}(i,j)$.Then, the mean filter can mean for:

$$\widehat{f}(i,j)=\frac{1}{M}\sum_{(k,j)\in\Omega(i,j)}f(k,l) \tag{1}$$

In the formula, the $\Omega i,j$ means the neighborhood-windows, which takes as the center of f(i, j), the M is the total amount of pixels in the the neighborhood-windows.

The reference expresses that the mean filter will clean the high-frequency and detail information when clean the image denoising,and make the image-edge faint[1].

Median filter can mean for:

$$\widehat{f}(i,j)=median\{f(k,l):(k,l)\in\Omega i,j\} \tag{2}$$

The reference expresses that the median filter can keep the edge and detail information when clean the image denoising[3]. The median filter can clean the impulse noise and salt-pepper noise effectly.

Wiener filter can mean for[1]:

$$\widehat{f}(i,j)=\frac{\partial_y^2}{\partial^2+\partial_y^2}f(i,j) \tag{3}$$

In the formula, ∂_y^2 is the signal variance, ∂^2 is the noise variance.

Wiener filter can obtain higher PSNR after cleaned the image noise,but still remain a little noise in the image[1].

In the frequency area ,the main algrithm of image denoising is wavelet, the image denoising method of wavelet mainly includes three steps[2] .The first step is carry on wavelet transformation to the image with noising;The secand step is carry on a certain processing to the wavelet coefficient, and clean the noise of image. The third step is carry on wavelet negative transformation, get the denoised image.

In this paper,the image with noise is decomposed by Haar wavelet transform, then selected thereshold to clean the image noise ,at last , arry on wavelet negative transformation, get the denoised image..

II. HAAR WAVELET TRANSFORM

Wavelet Analysis has already been applied in the signal processing, automatic control and image analysis realms, etc. The wavelet decomposition can produce a gradually approxmiate expression of image, the low frequency have the function of filter[4,5].This paper selected the harr wavelet to carry on decomposition to the iamge with noise.

Define the harr wavelet group as [7]:

$\{h_i(x)\mid i=0,1,2,\ldots\ldots,2^{j+1}-1\}$,Where :

$$ho(x)=\begin{cases}1,x\in[0,1]\\ o,x\in otherwise\end{cases} \tag{4}$$

$$hi(x)=\begin{cases}1,x\in[k/n,(k+1/2)/n)\\ -1,x\in[(k+1/2)/n,(k+1)/n]\\ 0,otherwise\end{cases} \tag{5}$$

In the formula , n=2*j,j=0,1,2,….. , the j meas the number of wavelet layer,which decide the biggest level of resolution[7]. k=0,1,…..n-1, It is the translation parameter,and i=n+k.

Define the Haar matrix of m rank:

$$Hm=[hm(1/2n),hm(3/2n),...,hm((2m-1)/2n)]m\times m \tag{6}$$

Where , $hm(x)=[ho(x),h1(x),...,hm-1(x)]^T$ ($x_l=(2l-1)/(2n),l=1,2,...,m.$) is the Haar function vector, $m=2^{j+1}, j\in\{0,1,2,....\}$ 。

The Haar scaling function $\phi(x)$ can define as[6] :

$$\phi(x)=\sum_{k\in 2}Pk\phi(2x-k)=\phi(2x)+\phi(2x-1) \tag{7}$$

Haar wavelet function can define as [6]:

$$W_H(x)=\sum_{k\in z}q_k\phi(2x-k)=\phi(2x)-\phi(2x-1) \tag{8}$$

Where, scaling sequence :

$p_k : p_0=p_1=1, p_i=0, i>=2,$

Equally:

$$q_k=\begin{cases}1,k=0,1\\0,k>=2\end{cases} \tag{9}$$

and

$$\phi(x)=\begin{cases}1, for 0<=x<1\\0, otherwise\end{cases} \tag{10}$$

For an image I, which can describe as:

$$I(x,y)=\begin{bmatrix} i_{0,0} & i_{0,1} & \cdots & i_{0,2N-1}\\ i_{1,0} & i_{1,1} & \cdots & i_{1,2N-1}\\ \cdots & \cdots & \cdots & \cdots\\ i_{2N-1,0} & i_{2N-1,1} & \cdots & i_{2N-1,2N-1}\end{bmatrix}_{2N\times 2N} \tag{11}$$

The two-dimensional haar wavelet transform can realize used the following Mallat calculate,the calculation formula is[4,5,6]:

$$LL_{x,y}=\frac{1}{4}\sum_{k_1,k_2=0}^{l}p_{k_1}p_{k_2}i_{k_1+2x,k_2+2y}$$
$$=\frac{1}{4}(i_{2x,2y}+i_{2x,2y+1}+i_{2x+1,y}+i_{2x+1,2y+1}) \tag{12}$$

$$LH_{x,y}=\frac{1}{4}\sum_{k_1,k_2=0}^{l}p_{k_1}q_{k_2}i_{k_1+2x,k_2+2y}$$
$$=\frac{1}{4}(i_{2x,2y}-i_{2x,2y+1}+i_{2x+1,y}-i_{2x+1,2y+1}) \tag{13}$$

$$HL_{x,y}=\frac{1}{4}\sum_{k_1,k_2=0}^{l}q_{k_1}p_{k_2}i_{k_1+2x,k_2+2y}$$
$$=\frac{1}{4}(i_{2x,2y}+i_{2x,2y+1}-i_{2x+1,y}-i_{2x+1,2y+1}) \tag{14}$$

$$HH_{x,y}=\frac{1}{4}\sum_{k_1,k_2=0}^{l}q_{k_1}q_{k_2}i_{k_1+2x,k_2+2y}$$
$$=\frac{1}{4}(i_{2x,2y}-i_{2x,2y+1}-i_{2x+1,y}+i_{2x+1,2y+1}) \tag{15}$$

III. IMAGE DENOISING

The key to the image denoising of wavelet is how to carry on correction to the coefficient of transformation. The main calculate way has the threshold method, correlation method and modulus maximum method, the threshold method is applied most extensive[8]. The threshold selection count for much. If the value is bigger, will cause the useful signal throw to lose;If the value is small, then can not ability clean the image noise. At present the threshold method of image denoising have a hard threshold method and the soft threshold method.

The hard threshold method reserved the bigger value than the wavelet coefficient, and set the smaller value to zero than the wavelet coefficient . The hard threshold method can clean the image noise effectily ,and remain the edge and detail information. The soft threshold method set the smaller value to zero than the wavelet coefficient, and use the absolute value of the bigger value than the wavelet coefficient to sub the threshold to clean the image noise. The soft threshold method can clean the image noise ,but will misty the image[1].

This paper used the soft threshold method to clean the image noise. In the conferrence [8], Mallet build the generalized gaussian distribution (GGD) model with the image sub-band confficient, the generalized gaussian distribution (GGD) also to be called generalized Laplace distribution (GLD)[8]:

$$f_Y(y)=\frac{v}{2s\tau(\frac{1}{v})}\exp(-|y/s|^v), s,v>0 \tag{16}$$

Where v is shape parameter, mean that the peak descends velocity.

$$\tau(t)=\int_j^{\infty}e^{-w}u^{j-1}du$$

The s and v parameter can estimate according to the columnar section of sub-band confficient.

Establish the image detail columnar section of sub-band confficient is h(u), the second moment of h(u) is:

$$m_2=\int_{-\infty}^{+\infty}u^2h(u)du \tag{17}$$

Take the generalized Laplace distribution into the calculus formula, can get:

$$m_2=s^2\frac{\tau(3/v)}{\tau(1/v)} \tag{18}$$

The second moment m_2 is the variance ∂_y^2 of image wavelet cofficient.

This paper used the bayesian threshold to obtain the threshold of image denoising. In the sub-bandle b,the threshold λ_b can define[9]:

$$\lambda_b = \frac{\hat{\partial}^2}{\hat{\partial}_\gamma} \tag{19}$$

Where, $\hat{\partial}^2$ is the estimated value of noise variance, $\hat{\partial}_\gamma$ is the estimated value of standard deviation.

IV. EXPERIMENT AND ANALYSIS

This paper select the the lotus leaf picture of windows as the test image,and join the gaussian white noise. This paper adopt the peak signal noise rate (PSNR) as the evaluation index,the PSNR define as[10] :

$$PSNR = 10\lg \frac{(I^{\max}_{i,j})^2}{\frac{1}{MN}\sum_{(i,j)}(I_{i,j} - Z_{i,j})^2} \tag{20}$$

Where, $0 \le i \le M-1, 0 \le j \le N-1, I^{\max}_{i,j}$ is 255.

Where Ii,j means the gray value of image in the palce (i,j) of the image(I), Zi,j means the gray value of the de-noised image in the palce (i,j) of the image(I)。

Experiment select the model of Haar wavelet is:

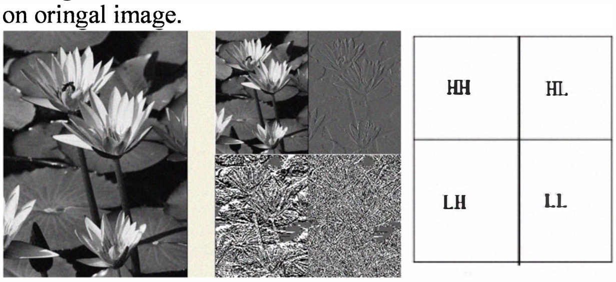
$$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 1 & 1 \\ \hline\end{array} \quad \begin{array}{|c|c|}\hline 1 & -1 \\ \hline 1 & -1 \\ \hline\end{array} \quad \begin{array}{|c|c|}\hline 1 & 1 \\ \hline -1 & -1 \\ \hline\end{array} \quad \begin{array}{|c|c|}\hline 1 & -1 \\ \hline -1 & 1 \\ \hline\end{array}$$

(a) (b) (c) (d)

Figure 1. the model of Haar wavelet

In the Figer 1, (a) is the low frequency;(b) is the horizontal component; (c) is the vertical component; （d） is the diagonal component.

Figer 2 is the test result of Haar wavelet transformation on oringal image.



(a) (b) (c)

Figure 2. The test result of Haar wavelet transformation on oringal image.

In the Figer 2, (a) is the oringal image ; (b) is the result of Haar wavelt transformation; (c) is the model of component.

In experiment, one orignal image(300*400) has been added 100 points of noise unreguely, another image (300*400) has been added regular noise as 1:64. The image with noised as the figer 3.

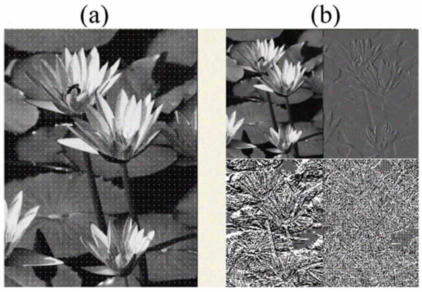
(a) (b)

Figure 3. the image with noise

In the figer 3, (a) is the image with 100 points noised ;(b) is the image with regular noise.

Figer 4 is the test result of Haar wavelet transformation on image with noise.

(a) (b)



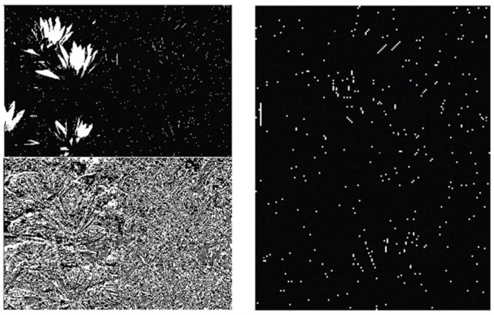
(c) (d)

Figure 4. The result of wavelet transformation to the noised image.

In the figer 4,(a) is the image with unregularly noise ;(b) is the result of wavelet transformation to image with unregularly noise;(c) is image with regularly noise;(d) is the result of wavelet transformation to image with regularly noise。

Figer 5 is the result of binaryzation to the image with wavelet transformation .In the result of binaryzation,the noise of image has been showed in the HL area.

In the figer 5, (a) is the test result of binaryzation to the image with wavelet transformation ; (b) is the noise showed of wavelet transformation.



(a) (b)

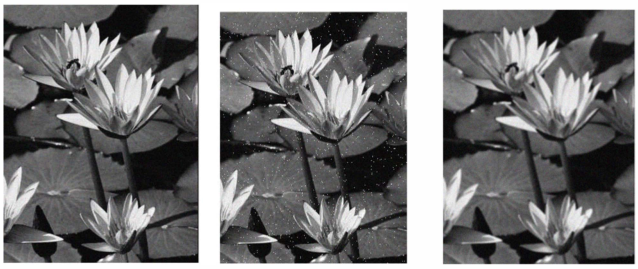
Figure 5. the test result of binaryzation to the image with wavelet transformation.

Table 1 is the image de-noising test value of PNSR ,after join the gaussian white noise.

TABLE I. THE PSNR OF THE TEST VALUE

σ	σ =10	σ =20	σ =30	σ =40
PSNR	27.8	23.6	22.5	20.3

Figure 6 is the result of image de-noising test where σ=10.



(a) Original image (b)Noisy image (c) De-noised image

Figure 6. The result of image de-noising test where σ=10.

The experiment show that the algorithm can reduce noise effectively and improve the detail effect of picture and raise the PSNR value of image.

V. CONCLUSION

The paper put forward the algrithm for image denoising based on Haar wavelet transform.The algrithm can effectively clean the noise of image and reserve the detail and veins of image, and obtain a good visual of image. The experiment show that the algorithm can reduce noise effectively , and and raise the PSNR value of iamge. The algrithm has practical value.

ACKNOWLEDGMENT

This work was supported by the "Youth fundation of Yibin University (No. 2010 Q36) ".

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