

4th International Conference on Recent Trends in Computer Science & Engineering Comparative Analysis of Wavelet Thresholding Techniques with Wavelet-Wiener Filter on ECG Signal

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Abstract

The electrocardiogram (ECG) records the electrical activity of the heart muscle and displays this data as a trace on a screen or on paper. This data is then interpreted for identification of a particular malfunctioning of the heart. Before the identification of a particular disease the ECG signal is first de-noised, as the raw ECG signal is contaminated with various other signals called artefacts. This is the crucial step as the signal has to be extracted from the noisy signal, without losing much of the valid information. In this paper, wavelet transform technique is considered for de-noising the ECG signal. The ECG signal is de-noised using different threshold techniques like hard, soft, SURE shrink, hybrid shrink and compared with the wavelet based wiener filter. The performance of these techniques is analysed. It is observed that there is a trade-off between the bias and variance in case of hard, soft, sure shrink, hybrid shrink whereas, in case of wavelet-wiener filter, bias and variance reduces simultaneously and gives a minimum MSE.

Keywords: MSE, bias, variance, Savitzky-Golay filter, wiener filter, shrinkage.

1. Introduction

The ECG which is recorded is a non-stationary signal obscured with various noises like Power-line interference, Base-line wander, Electrode contact noise, Motion artefacts, EMG etc. The frequency spectrum of the ECG signal ranges from 0.05 to 100 Hz. And the frequency spectrum of the EMG which is overlapped is in the range from 20 to 200 Hz. It is of primary interest to recover the true signal from the noisy ECG signal. The ECG signal should be clean and clear as possible so that accurate decisions can be taken by the physicians. To de-noise the ECG signal various kinds of time domain and frequency domain filters are implemented. But when compared to other filters like adaptive (LMS and RLS), Savitzky-Golay, wavelet transform techniques can better de-noise the ECG signal [1]. Wavelet transform has been an efficient tool for a variety of applications including estimation, classification and compression. Here the wavelet transform is used for estimating the true signal from the corrupted signal. The noisy signal is decomposed into different levels and the signal energy is concentrated in a small number of large wavelet coefficients and the noisy components are concentrated in a large number of small wavelet coefficients for which

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threshold is applied. The noisy wavelet coefficients are discarded / reduced and the true signal is estimated by reconstructing the signal from the remaining wavelet coefficients. The signal to be considered is modelled as a vector in R^N space. Let x, s, n denote $N \times 1$ column vectors. The noisy signal can be written as

$$x(i) = s\left(\frac{i}{N}\right) + n(i) \quad i = 1, 2, \dots, N. \quad (1)$$

With $x(i)$ is the i^{th} sample of the noisy signal, $s\left(\frac{i}{N}\right)$ is the i^{th} sample of the true signal, $n(i)$ a zero mean white Gaussian noise of variance σ .

Let W denote the $N \times N$ orthogonal wavelet transform matrix. Applying wavelet transform to Eq. (1) becomes

$$y = \theta + z \quad (2)$$

Where, $y = Wx$, $\theta = Ws$, $z = Wn$

θ is the true signal wavelet coefficients, y is given noisy observations. The true signal s is compactly represented into a small number of large wavelet coefficients in θ . Similarly, the wavelet transform map n noise terms to z noise wavelet coefficients.

Now the wavelet transform estimates the signal by shrinking the small entries of y (no signal) while retaining the large entries of y (where the signal is). On the other hand, ideal Wiener filter is the optimal filter to obtain the minimum MSE. The new algorithm given by [3] is the wavelet domain based wiener filter. In this algorithm, the wavelet shrinkage estimate is used as a means to design wavelet-wiener filter. The unwanted EMG signal components are best suppressed using DWT technique [2].

There are various de-noising algorithms which are used to de-noise the signal and to obtain an uncorrupted signal. In this paper, noisy ECG signal is de-noised using hard, soft, sure shrink, hybrid shrink and wavelet wiener filter. The de-noised signal is tested for MSE, bias squared, variance. Better results are achieved using wavelet-wiener filter because both bias-square and variance decreases simultaneously and maintain the minimum MSE.

2. Wavelet thresholding techniques

In wavelet shrinkage, the shrink/retain operation is to reduce the small amplitude (noise) wavelet coefficients to zero. A suitable threshold value is chosen such that pure noise coefficients may pass the threshold and discarded. And the signal is estimated from the remaining wavelet coefficients [4]. If the chosen threshold value is small then some of the noisy wavelet coefficients cannot pass the threshold and thus the estimated signal may still contain noise terms. On the other hand, if the chosen threshold is large then the signal is over smoothed, which may lead to the loss of valid signal information. Thus the optimal threshold value selection is an important criteria to get minimum MSE [5].

The wavelet shrinkage function determines how the thresholds are applied to the data. There are two basic wavelet shrinkage functions.

2.1 Hard threshold

This method was proposed by Dohono. This is a linear function in which, it discards coefficients below a threshold value τ , that is determined by the noise variance.

The hard threshold filter coefficients are given from [6],

$$h_h(i) = \begin{cases} 1, & \text{if } |y(i)| > \tau; \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

It is basically a “keep or kill” procedure. In this scheme, spurious ‘blips’ and discontinuous appear in the output when noise terms does not pass the threshold [5].

2.2 Soft threshold

This method was also proposed by Dohono. This is a non-linear function which is similar to hard threshold but it shrinks the large magnitude wavelet coefficients above the threshold. This causes the output to be more smoothed and continuous when compared to the hard threshold. The soft threshold filter coefficients are given from [6]

$$h_s(i) = \begin{cases} \text{sgn}[y(i)][y(i) - \tau], & \text{if } |y(i)| > \tau; \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

In case of hard and soft thresholds universal threshold value is considered and is given by

$\tau = \sqrt{\sigma^2 \log(N)}$, N is the length of the input signal, σ^2 is the variance estimated from the finest level of detail coefficients.

2.3 Sure shrink:

Sure shrink method is proposed by Dohono and Johnstone to select the optimal threshold value [7]. This method is based on the principle of Stein's unbiased estimator for risk (SURE). This method estimates the loss $\|(\hat{\mu} - \mu)^2\|$ in an unbiased fashion. Using the SURE (quadratic loss) function, estimate of risk for a particular threshold value (' τ ') is obtained. Now a threshold value is selected by minimizing the risks in ' τ '. This method gives a better MSE properties for large sample of data. The stein's unbiased estimate of risk is given by

$$R_s(\tau) = N + E\{\|g(y)\|^2 + 2\nabla \cdot g(y)\} \quad (5)$$

Now the threshold value is selected by considering the minimum risk value among the set $\{y_0, y_1, \dots, y_{N-1}\}$.

$$\text{surethreshold}(\tau) = \arg \min R_s(\tau) \quad \text{where } \tau = \{y_0, y_1, \dots, y_{N-1}\} \quad (6)$$

2.4 Hybrid threshold

According to Donoho's algorithm, hard-threshold function causes discontinuity and the soft-threshold function has bigger bias. To reduce the drawbacks of thresholding function in the Donoho's algorithm and the adjustment of more parameters in the thresholding function, a hybrid threshold function is used [2]. The hybrid threshold function is constructed as

$$h_y(i) = \begin{cases} y(i) - \frac{\tau^2}{y(i)}, & |y(i)| > \tau; \\ 0, & |y(i)| \leq \tau \end{cases} \quad (7)$$

where, $\tau = \sqrt{3\sigma^2}$, σ^2 is the estimated variance of the finest level of the detail coefficients.

2.5 Wavelet wiener filter

Wiener filter is the optimal in the sense of minimizing the MSE. The noisy signal wavelet coefficients are applied to the wiener filter coefficients and the output of the filter is inverse transformed for the reconstruction of the estimate signal. The diagonal wiener filter coefficients is given by [8]

$$H_w = \frac{\theta^2}{\theta^2 + \sigma^2} \quad i = 1, 2, \dots, N_s \quad (8)$$

To design the wiener filter first θ and variance (σ^2) are to be estimated. σ^2 is the estimated variance of the finest scale of the wavelet detail coefficients. $\theta(i)$ is estimated by considering the upper part as shown in fig(1). At first the noisy ECG signal $x(i)$ is decomposed into suitable levels using wavelet transform (WT1). Then the obtained hard threshold coefficients $\hat{\theta}_1$, collects all the high SNR trustworthy (N_t) coefficients and discards all the low SNR dubious (N_d) coefficients in y . Now the N_t coefficients are inverse transformed to obtain the estimate of the signal \hat{s}_1 , which is mathematically represented as

$$\hat{s}_1 = W_{T1}^{-1} \hat{\theta}_1 W_{T1} x \quad (9)$$

This estimate signal \hat{s}_1 is used as signal model for the wiener filter design. The aim of the wiener filter is to predict the (N_d) coefficients which have low SNR in y_1 and discarded. For this approach a slightly different wavelet transform (WT2) is chosen which is used to increase the low SNR (N_d) coefficients in y to high SNR values. In the second part, the noisy ECG signal is again decomposed into different levels using another wavelet transform (WT2) and these coefficients θ_2 are further modified by the wiener filter coefficients. The wiener filter coefficients

are designed from the estimated signal \hat{s}_1 from the upper part of the fig (1).

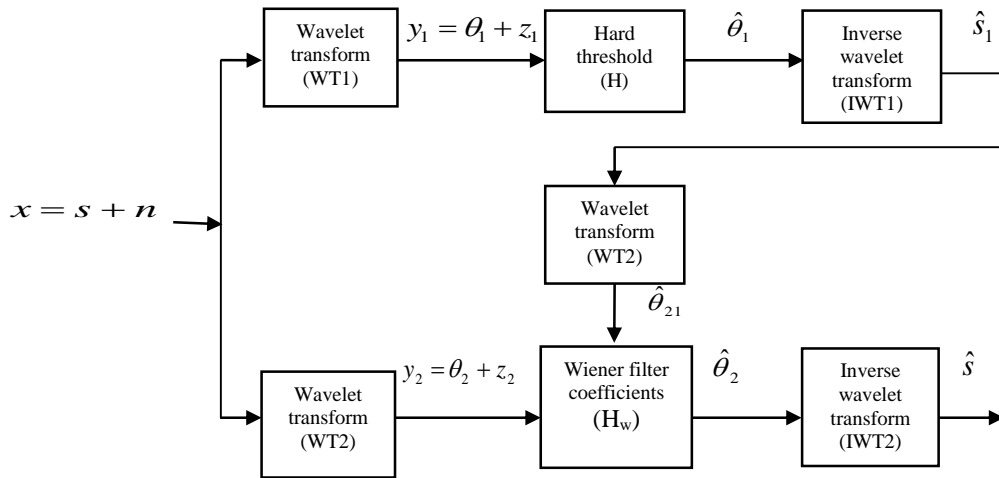


Fig.1. Block diagram of wavelet based wiener filter.

The \hat{s}_1 is further decomposed using WT2 and these coefficients are used to modify the noisy wavelet coefficients $\hat{\theta}_{21}$. Now the estimates $\hat{\theta}_1$ and $\hat{\theta}_{21}$ are related as

$$\hat{\theta}_{21} = W_{T2} W_{T2}^{-1} \hat{\theta}_1 \quad (10)$$

The estimate $\hat{\theta}_{21}$ of the signal coefficients is then used to design the wiener filter coefficients in W_{T2} domain using Eq.(9). The wiener filtered wavelet coefficients are inverse transformed (IWT2) to obtain the de-noised signal. The main idea of the wavelet wiener filter is to stretch the wavelet coefficients to bring the (N_d) coefficients and thus to implicitly obtain the entire $(N_s = N_t + N_d)$ wavelet coefficients.

3. Result

The ECG signal is added with various kinds of noises like power-line interference, EMG interference etc. Power-line interference noise is generated by adding a sinusoidal wave of 50 Hz signal to the ECG signal. The experiment was conducted based on two slightly different wavelet transforms.

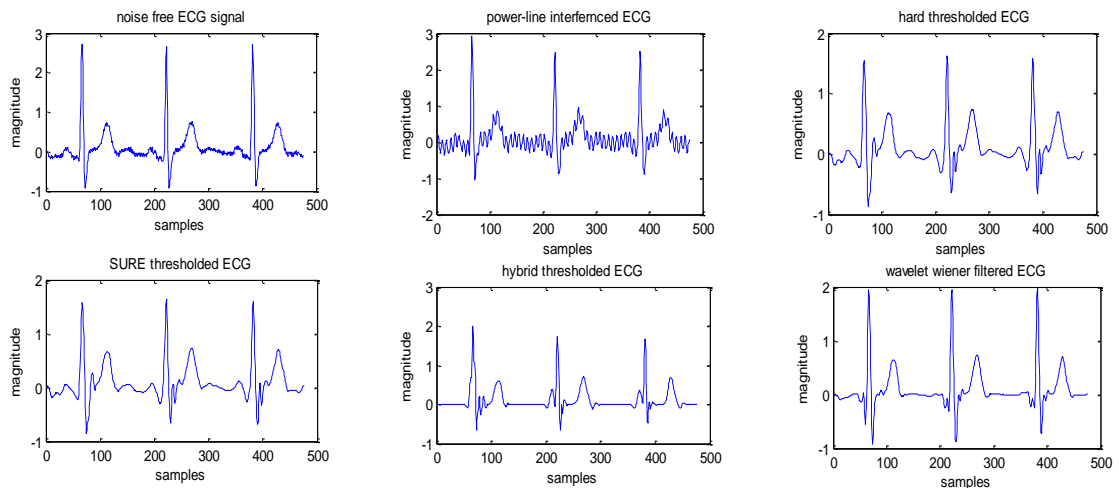


Fig.2. output waveforms of ECG noise free, power-line interference and de-noised waveforms.

Daubechies (db5) and symlet (sym6) wavelets compactly represent the signal. Filter bank of sym6 is slightly larger than the db5, so that N_d coefficients can be estimated. The noisy ECG signal is decomposed into three levels and de-noised using various wavelet de-noising techniques like hard, soft, universal, SURE shrink, hybrid shrink, and wavelet domain wiener filter as shown in fig (2). Similarly EMG interference noise is generated by adding random noise to a frequency range of about 20 to 200 Hz, then it is added to the ECG signal. This noisy ECG signal is decomposed into three levels and de-noised using various wavelet de-noising techniques like hard, soft, universal, SURE shrink, hybrid shrink, and wavelet domain wiener filter. Their denoised signals are shown in fig (3)

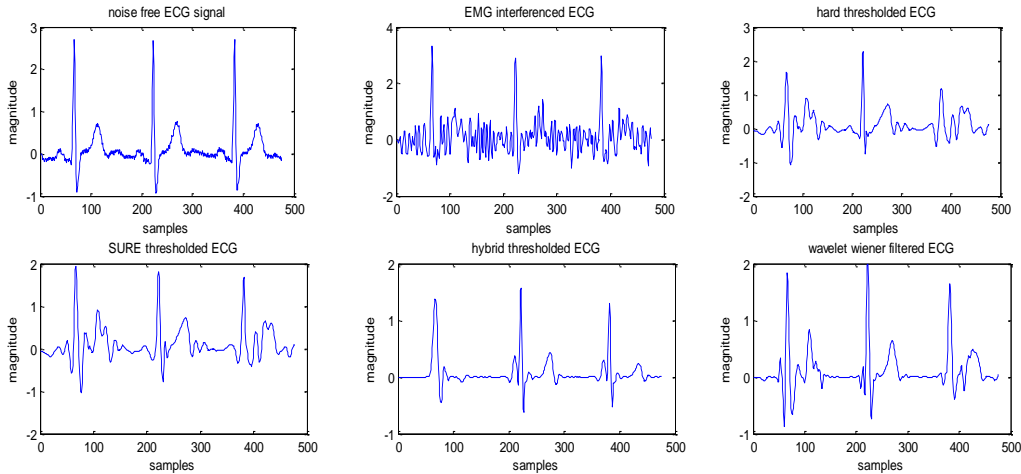


Fig.3. Output waveforms of ECG noise-free, EMG interference and de-noised waveforms.

To measure the performance of these wavelet thresholding techniques, the following are considered.

MSE: One of the most commonly used measure. It is an average of the squares of the difference between the actual observations and those predicted. MSE is a risk function, corresponding to the expected value of the squared error loss or quadratic loss.

$$MSE = \left(\frac{1}{N} \right) \sum_{i=1}^N (\hat{y}(i) - y(i))^2 \quad (11)$$

\hat{y} is the estimated signal using wavelet threshold techniques, y is the observed noisy signal.

Variance: It is the expected value of the squared deviation from the mean. It gives the variation of predicted value from its mean.

$$\text{variance}(\sigma^2) = \left(\frac{1}{N-1} \right) \sum_{i=1}^N (\hat{y}(i) - \mu)^2 \quad (12)$$

\hat{y} is the estimated signal, μ is the mean of the estimated signal. Variance must be of a small value.

Bias-squared: It is the difference of variance from the MSE. The lower the value the better is the performance. To obtain minimum MSE (risk function) there is a trade-off between the variance and bias-squared.

For ECG corrupted with power-line interference signal, the above thresholding techniques are implemented and MSE, variance and bias square values are calculated as shown in table 1.

Table 1. Tabular form for ECG added with power-line interference

Threshold method	MSE	Variance	Bias-square
Hard	0.0362	6.2810×10^{-05}	0.0362
Soft	0.0572	6.2810×10^{-05}	0.0572
SURE shrink	0.0558	6.2810×10^{-05}	0.0558
Hybrid shrink	0.0901	2.5078×10^{-05}	0.0901
Wavelet wiener filter (WT1≠WT2)	0.0291	1.3997×10^{-05}	0.0291

For ECG corrupted with EMG interference signal, the above thresholding techniques are implemented and MSE, variance and bias square values are calculated as shown in table2.

Table 2. Tabular form for ECG added with EMG interference

Threshold method	MSE	Variance	Bias-square
Hard	0.1792	9.8173×10^{-08}	0.1792
Soft	0.1613	9.8173×10^{-08}	0.1613
SURE shrink	0.1500	9.8173×10^{-08}	0.1500
Hybrid shrink	0.2378	7.5046×10^{-06}	0.2378
Wavelet wiener filter (WT1≠WT2)	0.1440	6.6706×10^{-06}	0.1440

4. Conclusions

In this paper, various thresholding techniques are experimented on the noisy ECG signal. The hard threshold method smoothens the signal, but spurious blips are observed in the output. Soft threshold method overcomes this disadvantage. Sure shrink and hybrid shrink methods better smoothens the signal but does not give minimize the MSE. In case of ECG interference with power-line noise, it is observed that in hard, soft, SURE shrink methods the variance is large and MSE increases, whereas in wavelet-wiener filter both variance and bias square value decreases and so the MSE. In case of ECG interference with EMG signal noise, it is observed that in hard, soft, SURE shrink methods the variance and bias square values increases or decreases simultaneously, and the MSE increases, whereas in wavelet-wiener filter both variance and bias square value decreases and thus the MSE is small.

It is observed that when the two wavelet functions have slightly different filter bank a better performance is obtained, as it extracts all the low SNR coefficients. When the two wavelet functions have different filter banks with large variation then the signal spreads out and does not give a better performance.

5. References

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