Dynamic Programming Exercise Class 9

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Plan

- Exercise 3:
  - Use EGM to solve the Buffer-stock model
  - Exercise will mainly be about understanding the buffer-stock model.
- Exercise 4: Structural estimation of buffer-stock model
  - Maximum-likelihood estimation (MLE)
  - Method of simulated moments (MSM)

Buffer-stock model

Bellman equation:

$$V_t(M_t, P_t) = \max_{C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta E_t[V_{t+1}(M_{t+1}, P_{t+1})]$$
$$M_{t+1} = R(M_t - C_t) + \xi_{t+1} P_{t+1}$$
$$P_{t+1} = G L_t P_t \psi_{t+1}$$
$$+ \text{ More stuff}$$

- Model has two state-variables: M_t, P_t.

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- Model has two state-variables: M_t, P_t.
- Turns out: Because state-transiton in P_{t+1} is a random walk $\Longrightarrow$ We can eliminate P_t as a state variable

Normalizing Bellman equation with P_t

- Utility function is homogenous with degree $1-\rho$:

$$\lambda^{1-\rho} = (\lambda C_t)^{1-\rho}/(1-\rho)$$

- Value function will inherit this from the utility function.
- If we multiply both sides of the bellman equation with $\frac{1}{P_t^{1-\rho}}$ we use this to get the following bellman equation:

$$V_t(M_t/P_t, 1) = \max_{C_t/P_t} \frac{(C_t/P_t)^{1-\rho}}{1-\rho} + \beta E_t[(GL_t\psi_{t+1})^{1-\rho} V_{t+1}(M_{t+1}, 1)]$$

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- By defining variables $x_t = X_t/P_t$ and $v_t(m_t) = V_t(m_t, 1)$ we get:

$$v_t(m_t) = \max_{c_t} \frac{(c_t)^{1-\rho}}{1-\rho} + \beta E_t[(GL_t\psi_{t+1})^{1-\rho} v_{t+1}(m_{t+1})]$$

Normalizing state transitions with P_t

- We can do the same with transitions function. We divide with P_t and get:

$$m_{t+1} = \frac{1}{GL_t\psi_{t+1}}R(m_t - c_t) + \xi_{t+1}$$

- Everything is now normalized by P_t and no equation depend on this state-variable $\implies$ we have eliminated a state from the model!

Other stuff in buffer-stock model

- **Shocks**: 3 shocks - One permanent and 2 transitory income shocks.
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- **Retirement**: After retirement receive fixed income.
- **Borrowing constraint**: Allow for some borrowing. Two borrowing constraints:
    1. Maximum repayable debt if receiving lowest possible income and consuming nothing Ω_t
    2. You cannot borrow below some level λ
    3. We take the most limiting constraint as binding $a_t \geq \max(-\Omega_t, \lambda_t)$

Estimation with maximum likelihood

- Assume that log-consumption is measured with normally distributed i.i.d measurement error $\implies$ difference between observed and optimal consumption follows a log-normal distributon
- We use distributional assumption to estimate with maximum-likelihood
- Should be covered in Bertel's slides.

Estimation with MSM

- Assume that we observe average savings in data.
- **Idea**: Solve model and simulate to get average savings in the model $\Longrightarrow$ Set parameters to minimize distance between moments in model and data
- Bertel's Slides also cover this quite comprehensively.