Household models and bargaining

with a focus on the limited commitment model

Adam Hallengreen Thomas H. Jørgensen Annasofie Markstrøm Olesen

April 3, 2024

Center of Economic Behavior and Inequality (CEBI), Department of Economics, University of Copenhagen

Motivation

- Many people live in couples and make decisions together.
- But many standard economic models do not recognize this.
- Ignoring households can lead to biased and incorrect results.
- We need to understand how living in a household affects economic behavior

Table of contents

1. Introduction to dynamic household models
2. The Limited Commitment Model
3. Bargaining
4. Policy functions
5. Estimation
6. Conclusion

Intro

Bellman equation

$$V_t(\mathcal{S}_t) = \max_{\mathcal{C}_t}\{u(\mathcal{C}_t, \mathcal{S}_t) + \beta\mathbb{E}_t[V_{t+1}(\mathcal{S}_{t+1})]\} \tag{1}$$

$$\mathcal{S}_{t+1} \sim \Gamma(\mathcal{S}_t, \mathcal{C}_t) \tag{2}$$

- Examples are Deaton's model or the Buffer-Stock model.
- Used to study (investment) decisions over individuals' life.
- Can have multiple phases, such as **education**, **working life**, and **retirement**
- Are solved using backwards induction (needs a terminal condition).

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t}\{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t} \{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Unitary: Constant power, $\mu_t = \mu \, \forall t$.

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t} \{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Unitary: Constant power, $\mu_t = \mu \; \forall t$.

Collective: Power determined by exogenous factors, $\mu_t = \mu_t^\star(\mathcal{Z}_t)$.

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t} \{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Unitary: Constant power, $\mu_t = \mu \, \forall t$.

Collective: Power determined by exogenous factors, $\mu_t = \mu_t^\star(\mathcal{Z}_t)$.

Full commitment: Power determined by endogenous factors at initial period, $\mu_t = \mu_t^\star(\mathcal{S}_0)$

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t}\{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Unitary: Constant power, $\mu_t = \mu \, \forall t$.

Collective: Power determined by exogenous factors, $\mu_t = \mu_t^\star(\mathcal{Z}_t)$.

Full commitment: Power determined by endogenous factors at initial period, $\mu_t = \mu_t^\star(\mathcal{S}_0)$

No commitment: Power determined by endogenous factors in all periods, $\mu_t = \mu_t^\star(\mathcal{S}_t)$

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t}\{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1-\mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Unitary: Constant power, $\mu_t = \mu \; \forall t$.

Collective: Power determined by exogenous factors, $\mu_t = \mu_t^\star(\mathcal{Z}_t)$.

Full commitment: Power determined by endogenous factors at initial period, $\mu_t = \mu_t^\star(\mathcal{S}_0)$

No commitment: Power determined by endogenous factors in all periods, $\mu_t = \mu_t^\star(\mathcal{S}_t)$

Limited commitment Power determined by endogenous factors in some periods, $\mu_t = \mu_t^\star(\mathcal{S}_t, \mu_{t-1})$

Households

Household utility as weighted sum of household member's utility

$$U(\mathcal{C}_t, \mathcal{S}_t, \mu_t) = \max_{\mathcal{C}_t} \{\mu_t \cdot u_1(\mathcal{C}_t, \mathcal{S}_t) + (1 - \mu_t) \cdot u_2(\mathcal{C}_t, \mathcal{S}_t)\} \tag{3}$$

Unitary: Constant power, $\mu_t = \mu \; \forall t$.

Collective: Power determined by exogenous factors, $\mu_t = \mu_t^\star(\mathcal{Z}_t)$.

Full commitment: Power determined by endogenous factors at initial period, $\mu_t = \mu_t^\star(\mathcal{S}_0)$

No commitment: Power determined by endogenous factors in all periods, $\mu_t = \mu_t^\star(\mathcal{S}_t)$

Limited commitment Power determined by endogenous factors in some periods, $\mu_t = \mu_t^\star(\mathcal{S}_t, \mu_{t-1})$

Alternatives: Other functional forms than weighted sum; Non-cooperative (Nash equilibrium).

Divorce

Divorce can be **absent**, **exogenous**, or **endogenous**. In general

$$V_t^m(\mathcal{S}_t, \mu_{t-1}) = (1 - D_t^\star)V_t^{m\to m}(\mathcal{S}_t, \mu_t) + D_t^\star V_t^{m\to s}(\mathcal{S}_t) \tag{4}$$

Divorce can be **absent**, **exogenous**, or **endogenous**. In general

$$V_t^m(\mathcal{S}_t, \mu_{t-1}) = (1 - D_t^\star) V_t^{m \to m}(\mathcal{S}_t, \mu_t) + D_t^\star V_t^{m \to s}(\mathcal{S}_t) \tag{4}$$

Can be **absent**, $D_t^\star = 0 \quad \forall t$

Divorce

Divorce can be **absent**, **exogenous**, or **endogenous**. In general

$$V_t^m(\mathcal{S}_t, \mu_{t-1}) = (1 - D_t^\star)V_t^{m\to m}(\mathcal{S}_t, \mu_t) + D_t^\star V_t^{m\to s}(\mathcal{S}_t) \tag{4}$$

Can be **absent**, $D_t^\star = 0 \quad \forall t$

Can be introduced with **exogenous** divorce probability in **all specifications**, $\mathbb{P}(D_t^\star = 1) = \delta_t(Z_t)$.

Divorce

Divorce can be **absent**, **exogenous**, or **endogenous**. In general

$$V_t^m(\mathcal{S}_t, \mu_{t-1}) = (1 - D_t^\star) V_t^{m\to m}(\mathcal{S}_t, \mu_t) + D_t^\star V_t^{m\to s}(\mathcal{S}_t) \tag{4}$$

Can be **absent**, $D_t^\star = 0 \quad \forall t$

Can be introduced with **exogenous** divorce probability in **all specifications**, $\mathbb{P}(D_t^\star = 1) = \delta_t(Z_t)$.

Can be introduced as **endogenous** decision in **no commitment** and **limited commitment**.

$$D_t^\star = 1 \quad \text{iff} \quad S_{j,t} \equiv V_{j,t}^{m\to m}(\mathcal{S}_t, \mu) - V_{j,t}^{m\to s}(\mathcal{S}_t) < 0, \forall \mu, \quad \exists j \in [1,2] \tag{5}$$

The Limited Commitment Model

Examples

The model has recently gained attraction in the field of household dynamics and bargaining

- The role of divorce laws on couples' choices [Voena, 2015]
- Education choices of women [Bronson, 2019]
- Taxation system's effect on couples decisions [Bronson et al., 2023]
- The effect of time limits on women's welfare and decisions [Low et al., 2018]

Setup

We follow agents for T periods and in period t=T, they die with certainty.

Setup

We follow agents for T periods and in period t=T, they die with certainty.

In each period, t>0, they make **choices** on **private consumption** ($c_{1,t}, c_{2,t}$); **public consumption** (c_t); and **divorce** ($D_t^\star$).

Setup

We follow agents for T periods and in period t=T, they die with certainty.

In each period, t>0, they make **choices** on **private consumption** $(c_{1,t}, c_{2,t})$; **public consumption** (c_t); and **divorce** $(D_t^\star)$.

Choices are based on their current **states** of **match quality** (ψ_t), **asset level** (A_{t-1}); and **bargaining power** (μ_{t-1}).

We follow agents for T periods and in period t=T, they die with certainty.

In each period, t>0, they make **choices** on **private consumption** ($c_{1,t}, c_{2,t}$); **public consumption** (c_t); and **divorce** ($D_t^\star$).

Choices are based on their current **states** of **match quality** (ψ_t), **asset level** (A_{t-1}); and **bargaining power** (μ_{t-1}).

- Match quality follows random walk: $\psi_{t+1} = \psi_t + \varepsilon_t,\ \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$

Setup

We follow agents for T periods and in period t=T, they die with certainty.

In each period, t>0, they make **choices** on **private consumption** ($c_{1,t}, c_{2,t}$); **public consumption** (c_t); and **divorce** ($D_t^\star$).

Choices are based on their current **states** of **match quality** (ψ_t), **asset level** (A_{t-1}); and **bargaining power** (μ_{t-1}).

- Match quality follows random walk: $\psi_{t+1} = \psi_t + \varepsilon_t,\ \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$
- Budget constraint is: $A_t + c_t + c_{1,t} + c_{2,t} = RA_{t-1} + Y_{1,t} + Y_{2,t}$.

Setup

We follow agents for T periods and in period t=T, they die with certainty.

In each period, t>0, they make **choices** on **private consumption** $(c_{1,t}, c_{2,t})$; **public consumption** (c_t); and **divorce** $(D_t^\star)$.

Choices are based on their current **states** of **match quality** (ψ_t), **asset level** (A_{t-1}); and **bargaining power** (μ_{t-1}).

- Match quality follows random walk: $\psi_{t+1} = \psi_t + \varepsilon_t,\ \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$
- Budget constraint is: $A_t + c_t + c_{1,t} + c_{2,t} = RA_{t-1} + Y_{1,t} + Y_{2,t}$.
- Bargaining power is updated according to the bargaining rule (later).

Approach

Goal: Calculate all value and policy functions.

The value of starting as a couple:

$$V^{m}_{j,t}(\psi_t, A_{t-1}, \mu_{t-1}) = (1 - D^{\star}_t)V^{m\to m}_{j,t}(\psi_t, A_{t-1}, \mu_t) + D^{\star}_t V^{m\to s}_{j,t}(A_{t-1}) \quad (6)$$

To get this, we need to calculate:

- The value of transitioning to single, $V^{m\to s}_{j,t}(A_{t-1})$;
- The value of remaining a couple, $V^{m\to m}_{j,t}(\psi_t, A_{t-1}, \mu)$;
- If it is optimal to update bargaining power, μ_t, or divorce, $D^{\star}_t$.

Value of transitioning to single

Can be solved as stand-alone DP problem with backwards induction (and EGM).

$$V_{j,t}^{m\to s}(A_{t-1}) = \max_{c_{j,t},c_t}\{u_{j,t}(c_{j,t},c_t) + \beta\mathbb{E}_t[V_{j,t}^{s}(A_t)]\} \tag{7}$$

$$A_{j,t} + c_{j,t} + c_t = RA_{j,t-1} + Y_{j,t} \tag{8}$$

For simplicity, assume that singlehood is an absorbing state, such that $V_{j,t}^{s} = V_{j,t}^{m\to s}$.

Value of remaining a couple

In a couple, you can't single-handedly decide on consumption levels - they depend on bargaining power.

$$V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) = u_t(\tilde{c}_{j,t}, \tilde{c}_t) + \psi_t + \beta \mathbb{E}_t[V_{j,t}^{m}(\psi_{t+1}, A_t, \mu)] \tag{9}$$

where

$$\begin{aligned}\tilde{c}_{w,t}(\mu), \tilde{c}_{m,t}(\mu), \tilde{c}_t(\mu) = \arg\max_{c_{w,t}, c_{m,t}, c_t} & \ \mu v_{w,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu) \\ & + (1-\mu) v_{m,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu)\end{aligned} \tag{10}$$

Both are subject to

$$\begin{aligned} A_t &= RA_{t-1} + Y_{w,t} + Y_{m,t} - (c_t + c_{w,t} + c_{m,t}) \\ \psi_{t+1} &= \psi_t + \varepsilon_{t+1},\ \varepsilon_t \sim iid\mathcal{N}(0, \sigma_\psi^2) \end{aligned}$$

Can also be solved with iEGM [Hallengreen et al., 2] iEGM

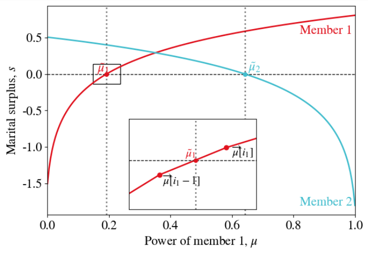
Bargaining

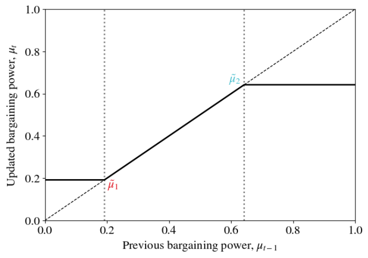
Calculate marital surplus

$$S_{j,t}(\mathcal{S}_t, \mu) \equiv V_{j,t}^{m \to m}(\mathcal{S}_t, \mu) - V_{j,t}^{m \to s}(\mathcal{S}_t) \tag{11}$$

Check if marital surplus is positive for both household members

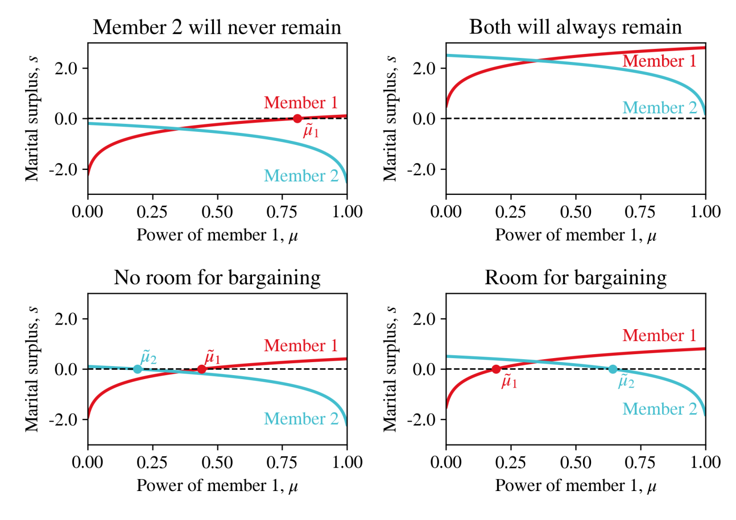
Update bargaining power





Source: [Hallengreen et al.,]

Bargaining cases



Source: [Hallengreen et al.,]

Approach - completed

Goal: Calculate all value and policy functions.

The value of starting as a couple:

$$V^{m}_{j,t}(\psi_t, A_{t-1}, \mu_{t-1}) = (1 - D^{\star}_t)V^{m\to m}_{j,t}(\psi_t, A_{t-1}, \mu_t) + D^{\star}_t V^{m\to s}_{j,t}(A_{t-1})$$

To get this, we need to calculate:

- The value of transitioning to single, $V^{m\to s}_{j,t}(A_{t-1})$;
- The value of remaining a couple, $V^{m\to m}_{j,t}(\psi_t, A_{t-1}, \mu)$;
- If it is optimal to update bargaining power, μ_t, or divorce, $D^{\star}_t$.

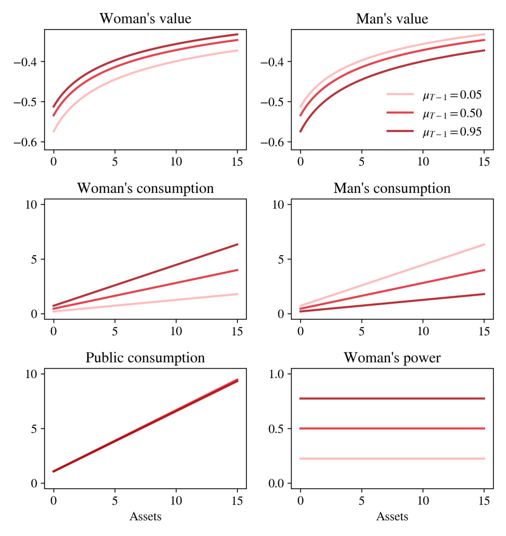
Policy functions

Table 1: Parameter Values.

	Model 1	Model 2
Income		
R	1.03	
Y_w	1.0	
Y_m	1.0	
Preferences		
β	$1/R$	
ρ_w	2.0	
ρ_m	2.0	
$\alpha_{1,w}$	1.0	
$\alpha_{1,m}$	1.0	
$\alpha_{2,w}$	1.0	
$\alpha_{2,m}$	1.0	
ϕ_w	0.2	
ϕ_m	0.2	
Household bargaining		
κ_w	0.5	0.23
κ_m	0.5	0.77
σ_ψ	0.1	
χ	0.0	

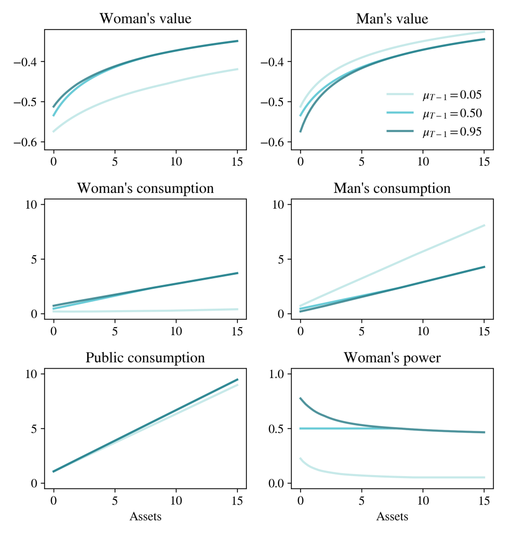
Source: [Hallengreen et al.,]

Base model



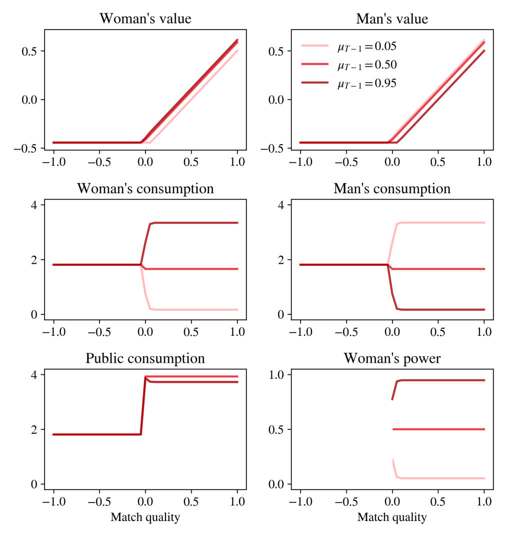
Source: [Hallengreen et al.,]

Model with unequal asset split upon divorce



Source: [Hallengreen et al.,]

Effect of match quality



Source: [Hallengreen et al.,]

Estimation

Estimation

We have a problem when estimating:
- we do not observe match quality or bargaining power.

So we cannot estimate with maximum likelihood.

Instead, we can estimate with Simulated Minimum Distance.

Conclusion

Conclusion

- Many people live in couples and make decisions together within the household.
- Household models are essential for studying decisions made in the household.
- The dynamics of household power is not static, so we need bargaining.
- The limited commitment model accounts for bargaining and endogenous divorce.
- Good choice when studying household behavior.

References i

Bronson, M. A. (2019).
Degrees Are Forever: Marriage, Educational Investment, and Lifecycle Labor Decisions of Men and Women.

Bronson, M. A., Haanwinckel, D., and Mazzocco, M. (2023).
Taxation and Household Decisions: An Intertemporal Analysis.

Hallengreen, A., Jørgensen, T. H., and Olesen, A. M.
Household Bargaining with Limited Commitment: A Practitioners Guide.

Hallengreen, A., Jørgensen, T. H., and Olesen, A. M. (2).
The Endogenous Grid Method without Analytical Inverse Marginal Utility.

Low, H., Meghir, C., Pistaferri, L., and Voena, A. (2018).
Marriage, Labor Supply and the Dynamics of the Social Safety Net.

Voena, A. (2015).
Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?
American Economic Review, 105(8):2295–2332.

iEGM

Okay, let's look at that complex value function

In a couple, you can't single-handedly decide on consumption levels - they depend on bargaining power.

$$V_{j,t}^{m\to m}(\psi_t, A_{t-1}, \mu) = u_t(\tilde{c}_{j,t}, \tilde{c}_t) + \psi_t + \beta \mathbb{E}_t[V_{j,t}^{m}(\psi_{t+1}, A_t, \mu)]$$

where

$$\begin{aligned}\tilde{c}_{w,t}(\mu), \tilde{c}_{m,t}(\mu), \tilde{c}_t(\mu) = \arg \max_{c_{w,t}, c_{m,t}, c_t} & \mu v_{w,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu) \\ & + (1-\mu) v_{m,t}(\psi_t, A_{t-1}, c_{w,t}, c_{m,t}, c_t, \mu)\end{aligned}$$

Both are subject to

$$\begin{aligned} A_t &= RA_{t-1} + Y_{w,t} + Y_{m,t} - (c_t + c_{w,t} + c_{m,t}) \\ \psi_{t+1} &= \psi_t + \varepsilon_{t+1},\ \varepsilon_t \sim iid\mathcal{N}(0, \sigma_\psi^2) \end{aligned}$$

back

Simplify problem

Realize that if total consumption is known, consumption allocation can be inferred. We refer to this as the **intra-period** problem.

$$c_w(\mu, C), c_m(\mu, C), c(\mu, C) = \arg\max_{c_j, c_m, c} \mu U_w(c_w, c) + (1-\mu) U_m(c_m, c)$$
$$st. \quad C = c_w + c_m + c$$

The **inter-period** problem is then to find the C that maximizes the value

$$V_t^{m\to m}(\psi_t, A_{t-1}, \mu) = \max_{C_t}\{U_t(C_t) + \psi_t + \beta \mathbb{E}_t[V_t^m(\psi_{t+1}, A_t, \mu)]\}$$

EGM

FOC

$$U'(C_t) = \beta R \mathbb{E}_t \left[\frac{\partial V_{t+1}(\psi_{t+1}, A_t, \mu)}{\partial A_t} \right] \equiv W_t \tag{12}$$

Consumption

$$C_t = U'^{-1}(W_t) \tag{13}$$

Endogenous grid

$$M_t = A_t + C_t \tag{14}$$

iEGM

The problem is that U is not analytically invertible.

We can use iEGM [Hallengreen et al., 2] to circumvent this issue.

Idea: Use FOC to go from grid of C to grid of W.

$$\overrightarrow{W} = U'(\overrightarrow{C}) \tag{15}$$

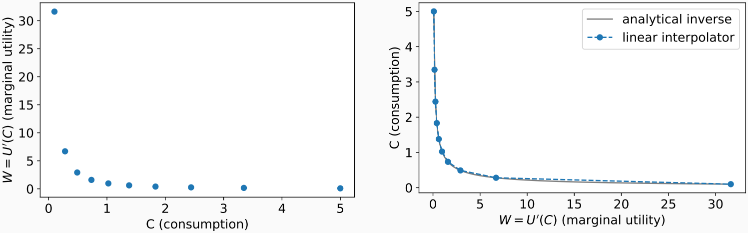
If the function is invertible, we can also go from W to C.

$$\overrightarrow{C} = U'^{-1}(\overrightarrow{W}) \tag{16}$$

So, we create an interpolator of C over W.

$$C_t^{\star} = \check{C}(W_t). \tag{17}$$

Flip the axis



Source: [Hallengreen et al., 2]

back