Dynamic Programming Exercise Class 5

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Case study: Bus engine replacement model

Problem: When to replace bus engine to minimize costs.

$$V(x_t, \epsilon_t) = \max_{d_t \in \{0,1\}} u(x_t, d_t) + \beta E_t[V(x_{t+1})]$$

$$u(x_t, d_t) = \begin{cases} -c(x_t, \theta_1) + \epsilon_0 & \text{if } d_t = 0 \\ -RC - c(0, \theta_1) + \epsilon_1 & \text{if } d_t = 1 \end{cases}$$

$$c(x_t, \theta_1) = x_t \theta_1$$

$$\epsilon_0, \epsilon_1 \sim q(\theta_2) \text{ where } q \text{ has mean zero}$$

$$x_{t+1} = x_t(1 - d_t) + \nu_t$$

$$\nu_t \sim f(x_t(1 - d_t), \theta_3)$$

- Structural parameters: β, θ_1, θ_2, θ_3, RC
- **NFXP** will estimate θ_1, RC, θ_3.

Nested Fixed Point Algorithm (NFXP)

- Maximum likelihood estimation: Evaluate log-likelihood to see how far model is from data
- Log-likelihood will be function of choice-probability. We must solve the model for a given set of parameters to get choice probability
- $\Rightarrow$ We must solve model for each parameter guess
- Model solution time becomes very important.

Execise 1

- You dont have to do much yourself
- Mostly about understanding code we give you.
- Look at slides, look at code make sure you can see the similarity between math and code.

Successive approximations (SA)

- So far: Successive approximations to find fixed point on the bellman operator.
- Last time the **bellman operator** $\bar{\Gamma}$ looked like this:

$$\bar{\Gamma}(\bar{V}) = (\log\left(\exp\{v^{replace}(\bar{V})\} + \exp\{v^{keep}(\bar{V})\}\right) \tag{1}$$

$$v^{replace}(\bar{V}) = u^{replace} + \beta P(replace)\bar{V} \tag{2}$$

$$v^{keep}(\bar{V}) = u^{keep} + \beta P(keep)\bar{V} \tag{3}$$

  - You keep iterating on this until $|\bar{V} - \bar{\Gamma}(\bar{V})|$ is sufficiently small.
  - Slow for large β

Newton approach (NK)

- Today we will combine successive approximations with a newton approach.
- Newtons method in general solves $f(x) = 0$.
- First order taylor approx. around x_0:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) = 0 \iff x = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{4}$$

- Fixed point problem as equation to be solved:

$$\bar{V} - \bar{\Gamma}(\bar{V}) = 0 \tag{5}$$

- Rust combines this with successive approximations: Calls it a **poly-algorithm**.