Dynamic Programming Exercise Class 4

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Part 2 case study: Bus engine replacement model

Problem: When to replace bus engine to minimize costs.

$$V(x_t) = \max_{d_t \in \{0,1\}} u(x_t, d_t) + \beta E_t[V(x_{t+1})]$$

$$u(x_t, d_t) = \begin{cases} -c(x_t, \theta_1) + \epsilon_0 & \text{if } d_t = 0 \\ -RC - c(0, \theta_1) + \epsilon_1 & \text{if } d_t = 1 \end{cases}$$

$$c(x_t, \theta_1) = x_t \theta_1$$

$$\epsilon_0, \epsilon_1 \sim q(\theta_2) \text{ where } q \text{ has mean zero}$$

$$x_{t+1} = x_t(1 - d_t) + \nu_t$$

$$\nu_t \sim f(x_t(1 - d_t), \theta_3)$$

- Structural parameters: β, θ_1, θ_2, θ_3, RC
- **NFXP** will be used to solve and estimate some of these parameters.

Nested Fixed Point Algorithm (NFXP)

- Goal of NFXP: Estimate structural parameters.
- Why the name?
  - When we solve infinite horizon models we are trying to find the fixed point of the bellman equation:

$$V(x_t) = \max_{d_t \in \{0,1\}} u(x_t, d_t) + \beta E_t[V(x_{t+1})]$$

  - Estimation: Maximum likelihood
  - Evaluation of likelihood function requires knowing "policy" function. We must solve model again for each parameter guess: **The fixed point solution method is nested inside estimation procedure**
- **Today**: We focus on the solution part. Estimation next time.

Execise 0

1. Solve model without any uncertainty
2. Add stochastic state transition
3. Add Extreme value taste shocks - Monte carlo integration
4. Add Extreme value taste shocks - Analytic integration
5. Add gaussian taste shocks - Monte carlo integration
6. (Optional) Vectorize code

Some final notes

- With taste shocks choices are stochastic: Policy function is choice probability.
- Lectures: Bertel talked about solving in expected value function space vs integrated value function space.
    - See slides 13 and 14 in lecture in part 2
    - In the exercises we use integrated value function space.
    - I encourage you to try to understand the difference.