Pareto Multi Objective Optimization

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Abstract-- **The goal of this chapter is to give fundamental knowledge on solving multi-objective optimization problems. The focus is on the intelligent metaheuristic approaches (evolutionary algorithms or swarm-based techniques). The focus is on techniques for efficient generation of the Pareto frontier. A general formulation of MO optimization is given in this chapter, the Pareto optimality concepts introduced, and solution approaches with examples of MO problems in the power systems field are given.**

Index Terms -- **Particle swarm optimization, Multi-objective optimization, Pareto optimization, trade-off analysis.**

I INTRODUCTION

Optimization is an essential process in many business, management, and engineering applications. In these fields, multiple and often conflicting objectives need to be satisfied. Solving such problems has traditionally consisted of converting all objectives into a single objective (SO) function. The ultimate goal is to find the solution that minimizes or maximizes this single objective while maintaining the physical constraints of the system or process. The optimization solution results in a single value that reflects a compromise between all objectives. The art in this process is to formulate the function to achieve this desired compromise.

Conversion of the multiple objectives into an SO function is usually done by aggregating all objectives in a weighted function, or simply transforming all but one of the objectives into constraints. This approach to solving multi objective (MO) optimization problems has several limitations: 1) it requires a priori knowledge about the relative importance of the objectives, and the limits on the objectives that are converted into constraints 2) the aggregated function leads to only one solution; 3) trade-offs between objectives cannot be easily evaluated; and 4) the solution may not be attainable unless the search space is convex.

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This simple optimization process is no longer acceptable for systems with multiple conflicting objectives. System engineers may desire to know all possible optimization solutions of all objectives simultaneously. In the business world, it is known as a trade-off analysis. In the engineering field, there are several examples of the need to perform trade-off analysis. For example, designing distributed controllers while reducing cost are two conflicting objectives. Similarly, to place more functional blocks on a chip while minimizing that chip area and/or power dissipation are conflicting objectives. To find the vehicle that covers the most distance in a day while requiring the least energy is a multi-objective problem. Minimizing the operating cost of a business while maintaining a stable work force is a conflicting objective optimization problem [[1]-[3]].

In power systems, the operation inherently requires MO optimizations. For instance, in environmental/economic load dispatch, minimizing operation cost, minimizing fossil fuel emissions and minimizing system losses are some of the objectives that can be incorporated together to create a MO problem. Minimizing the fuel cost, emission, and total power loss subject to stability constraints, generation capacity constraints, and security constraints requires MO optimizations techniques [[4]]. Also, the optimization of reactive resources location and sizing of the transmission and distribution system is another example of a MO problem. Some other conflicting objectives in the transmission networks are transmission loss, transmission capacity and voltage stability. Other objectives in distribution networks include the distribution loss, power factor, and voltage stability [[5],[6]].

Compared to SO problems, MO problems are more difficult to solve, because there is no unique solution; rather, there is a set of acceptable trade-off optimal solutions. This set is called Pareto front. MO optimization is in fact considered as the analytical phase of the multi criteria decision making (MCDM) process, and consists of determining all solutions to the MO problem that are optimal in the Pareto sense [[7]]. The preferred solution – the one most desirable to the designer or decision maker (DM) – is selected from the Pareto set.

Generating the Pareto set has several advantages. The Pareto set allows the DM to make an informed decision by seeing a wide range of options since it contains the solutions that are optimum from an "overall" standpoint; unlike SO optimization that may ignore this trade-off viewpoint. From a

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systems engineer's perspective, this feature is useful since it provides better understanding of the system where all the consequences of a decision with respect to all the objectives can be explored [[8]].

The goal of this chapter is to give fundamental knowledge on solving MO problems. The focus will be on the intelligent metaheuristic approaches (evolutionary algorithms or swarm-based techniques). We will motivate the use of these techniques as opposed to the traditional mathematical programming approaches. The former are more appropriate for generic MO problems, particularly when information about preferences or priority of the objectives is not known in advance, and when we seek to present the DM with a wide range of alternatives solutions. Hence, our focus will be on techniques for efficient generation of the Pareto frontier. A general formulation of MO optimization is given in this chapter, the Pareto optimality concepts introduced, and solution approaches with examples of MO problems in the power systems field are given.

II MULTIOBJECTIVE OPTIMIZATION

The objective of MO optimization is to find the set of acceptable solutions and present them to the DM, which will then choose among them. Additional constraints or criteria specified either before or after the search by the DM can help guide, refine or narrow the search, but we will look at the generic case where there is no *a priori* information from the DM.

II.1 Simple Example

Consider the problem of determining the most efficient transportation mode. Assume two criteria are used to determine this efficiency: (a) distance covered in a day, and (b) energy used in the process. The following transportation modes are considered: walking, bicycling, riding a cow, car, motorcycle, horse, airplane, rocket, balloon, boat and scooter.

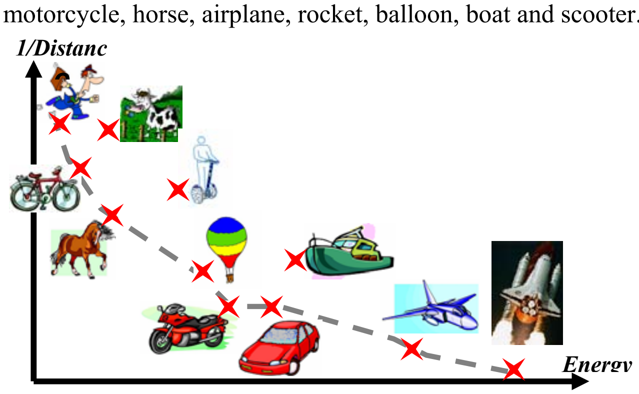

Fig 1. Transportation mode example.

Common sense can be used to obtain all potential solutions. For example, the average car will need more fuel than the average motorcycle, but the car can travel longer distance. Similarly, given the same amount of food, it would be generally expected that a cow would cover a smaller distance than a horse. Using this reasoning, we can generate Fig 1

II.2 Generic Formulation of MO Optimization

The general MO problem requiring the optimization of N objectives may be formulated as follows:

Minimize $\vec{y} = \vec{F}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), ..., f_N(\vec{x})]^T$,

Subject to $g_j(\vec{x}) \le 0$, $j = 1, 2, \ldots, M$ **(1)**

Where $\vec{x} = [x_1, x_2, \ldots, x_P]^T \in \Omega$

$\vec{y}$ is the objective vector, the g_j's represent the constraints and $\vec{x}$ is a P-dimensional vector representing the decision variables within a parameter space Ω. The space spanned by the objective vectors is called the objective space. The subspace of the objective vectors that satisfies the constraints is called the feasible space.

The *utopian* solution is the solution that is optimal for all objectives.

$$\vec{x}_0^* \in \Omega : \forall \vec{x} \in \Omega, f_i(\vec{x}_0^*) \le f_i(\vec{x}), \text{ for } i \in \{1, 2, \ldots, N\} \quad \textbf{(2)}$$

For N=1, the MO problem is reduced to an SO problem. In that case, the utopian solution is simply the global optimum. It always exists, even if it cannot be found.

For the more general case where N>1 however, the utopian solution does not generally exists since the individual objective functions $\{f_i\}$ are typically conflicting. Rather there is a possibly uncountable set of solutions, the so-called non-dominated solutions (for which an objective can not be improved without degrading at least another one) that represent different compromises or trade-offs between the objectives.

II.3 Pareto optimality Concepts

To compare candidate solutions to the MO problems, the concepts of Pareto dominance and Pareto optimality are commonly used. These concepts were originally introduced by Francis Ysidro, and then generalized by Vilfredo Pareto [[3]]. A solution belongs to the Pareto set if there is no other solution that can improve at least one of the objectives without degradation any other objective.

Formally, a decision vector $\vec{u} = [u_1, u_2, \ldots, u_P]^T$ is said to *Pareto-dominate* the decision vector $\vec{v} = [v_1, v_2, \ldots, v_P]^T$, in a minimization context, if and only if:

$$\forall i \in \{1, ..., N\}, f_i(\vec{u}) \le f_i(\vec{v}), \text{ and } \exists j \in \{1, ..., N\} : f_j(\vec{u}) < f_j(\vec{v}) \quad \textbf{(3)}$$

In the context of MO optimization, Pareto dominance is used to compare and rank decision vectors: $\vec{u}$ dominates $\vec{v}$ in the Pareto sense means that $\vec{F}(\vec{u})$ is better than $\vec{F}(\vec{v})$ for all objectives, and there is at least one objective function for which $\vec{F}(\vec{u})$ is strictly better than $\vec{F}(\vec{v})$.

A solution $\vec{a}$ is said to be Pareto optimal if and only if there does not exist another solution that dominates it. In other words, solution $\vec{a}$ cannot be improved in one of the objectives without adversely affecting at least one other objective. The corresponding objective vector $\vec{F}(\vec{a})$ is called a Pareto dominant vector, or non-inferior or non-dominated vector. The set of all Pareto optimal solutions is called the Pareto optimal set. The corresponding objective vectors are said to be on the Pareto front. It is generally impossible to come up with an analytical expression of the Pareto front.

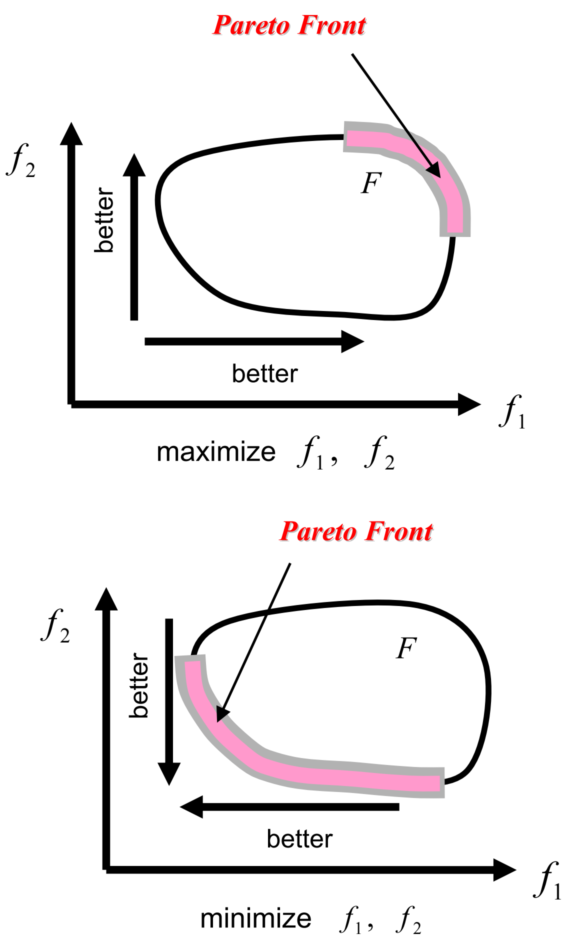

Fig 2. Illustration of Pareto front for a biobjective **optimization problem.**

Fig 2 depicts a Pareto set for a two-objective minimization problem. Potential solutions that optimize f_1 and f_2 are shown on the graph.

III SOLUTION APPROACHES

There are several methods to solve MO problems. Classical approaches consist of converting the MO problem into an SO problem, which then can be solved using traditional scalar optimization techniques. Because most of these approaches assume *a priori* information from the DM (either ranking the objectives in order of importance, or indication of target optimal values), these techniques are geared toward finding a unique solution, the one that best satisfies the criteria and additional information (preferences) provided by the DM.

The second class of techniques, even though proposed early, only took rise in recent years thanks to the advances in computational power and the development of population-based metaheuristic algorithms. These techniques are geared towards direct determination of the Pareto frontier by optimizing all the objectives separately. Some of these techniques make explicit use of Pareto ranking. These techniques are advantageous for real-life problems, particularly those appearing in the context of power systems since they present to the DM all possible, or at least a wide range of trade-offs between objectives. This allows them to make an informed decision.

III.1 Classical Methods

The classical methods consist of converting the MO problem into an SO problem by either aggregating the objective functions or optimizing one objective and treating the other as constraints. The SO problem can then be solved using traditional scalar-valued optimization techniques. These techniques are geared towards finding a single solution and are ideal for cases where preferential information about the objectives is known in advance. It is also possible, by modifying the aggregation parameters and solving the newly created SO problem, to approximate the non-dominated front.

a. Weighted Aggregation

The weighted aggregation method is a special case of the utility function method, which converts the MO problem into an SO problem by applying a function operator to the objective vector. This function is designed by the DM to capture his preferences [[2]]. A simple and popular utility function is a linear combination of the objectives.

$$\text{Minimize } Z = \sum_{j=1}^{N} w_j f_j(\vec{x}) \text{ with } w_j \geq 0 \text{ and } \sum_{j=1}^{N} w_j = 1 \qquad \textbf{(4)}$$

where the weights (w_j's) can, for example, indicate the relative importance the DM attaches to objective j and must be specified for each of the k objectives *a priori*. The optimum values of w_j's cannot be determined within the SO optimization process. The solution of this SO problem yields a single result that is as good as the selection of the weights.

Without prior information, choosing the weights can be problematic. In a variant of this method called dynamic weighted aggregation (DWA) [[9]], the weights are incrementally changed. For each new combination of weights, the problem is solved, thus generating a new compromise solution. While simple to implement, this method does not generally yield the non-dominating front, and also misses concave portion of the frontier. Furthermore, it is difficult to control diversity along the Pareto front [[2],[10]].

b. Goal Programming

A variation of the above technique is the goal programming (or goal attainment) which seeks to minimize deviation from prespecified goals. (5) is a common formulation.

$$\text{Minimize } Z = \sum_{j=1}^{N} w_j \left| f_j(\vec{x}) - T_j \right| \qquad \textbf{(5)}$$

where T_j represents the target or goal set by the DM for the j^{th} objective function, and the w_j's now capture the priorities [[2]]. As in the weighted aggregation approach, the main drawback is the need for a priori information (priorities and targets).

c. ε-Constraint

This is a method designed to discover Pareto optimal solutions based on optimization of one objective while treating the other objectives as constraints bound by some allowable range ε_i. The problem is repeatedly solved for different values of ε_i to generate the entire Pareto set.

Minimize $f_k(\vec{x})$, $\vec{x} \in \Omega$, Subject to $f_i(\vec{x}) \le \varepsilon_i$ and $g_j(\vec{x}) \le 0$. $i = 1, 2, \ldots, N; i \neq k$

$j = 1, 2, \ldots, M$

Repeat (1) for different values of ε_i.

This is a relatively simple technique, yet it is computationally intensive. Furthermore the solutions found are not necessarily globally non-dominated [[6],[10]].

d. Discussion on Classical Methods

Most of the traditional methods attempt to ease the decision-making process by incorporate preferential information from the DM, and are geared towards finding the single best solution representing the best compromise given the information from the DM. Such techniques can approximate the Pareto front by essentially repeating the solution process after modifying the aggregation parameters (weights or target levels). The obtained front while locally non-dominated is not necessarily globally non-dominated. Furthermore, the distribution of solutions along the front depends on efficiency of SO optimization solver [[11]]. While the techniques are relatively simple to implement, they mostly are inefficient, and sometimes sensitive to the shape of the Pareto front.

III.2 Intelligent Techniques

In contrast to the aggregation-based techniques, some intelligent techniques are geared towards direct generation of the Pareto front by simultaneously optimizing the individual objectives. Computational advances and the development of population-based metaheuristic algorithms contributed to the rise of these methods in recent years. Population-based algorithms have the advantage of evaluating multiple potential solutions in a single iteration (**Fig 3**). In addition, they offer greater flexibility for the decision-maker, mainly in cases where no *a priori* information is available as is the case for most real-life MO problems. However, the challenge is how to guide the search towards the Pareto-optimal set, and how to maintain a diverse population in order to prevent premature convergence [[12]].

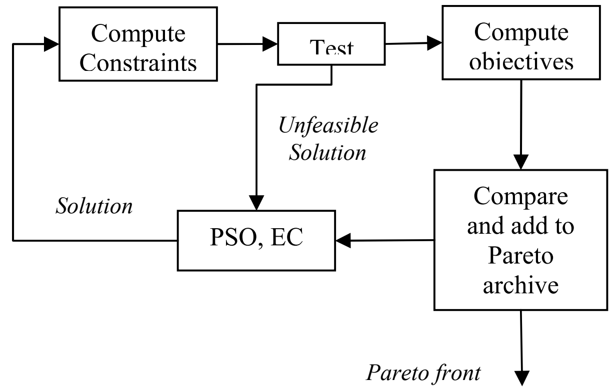

Fig 3. Pareto front generation using population-based techniques

Evolutionary computing emulates the biological evolution process. A population of individuals representing different solutions is evolving to find the optimal solutions. The fittest individuals are chosen, mutation and crossover operations applied, thus yielding a new generation (offspring). These methods include genetic algorithms (GA), evolutionary algorithms (EA) and evolutionary strategies (ES) which only differ in the way the fitness selection, mutation and crossover operations are performed. Evolutionary techniques have been successfully applied to all sort of SO optimization, especially those where the objective functions is not well-behaved (not differentiable, discontinuous, and/or no analytical formulation) [[3],[12]-[14]]. They often appear for the embedded optimization step of the traditional techniques presented in the previous section. Confusion should be avoided between these hybrid evolutionary algorithm-based techniques, and the ones geared toward determining the Pareto efficient solutions presented in this section.

a. Non-Pareto-Based Approach: Vector Evaluated Genetic Algorithm (VEGA)

Schaffer's vector-evaluated genetic algorithm (VEGA)[[15]] is a non-Pareto-based technique that differs from the conventional genetic algorithm only in the way in which the selection step is performed. At each generation, the population is divided into as many equal-size subgroups as there are objectives, and the fittest individuals for each objective functions are selected (**Fig 4**). Regular mutation and crossover operations are then performed to obtain the next generation.

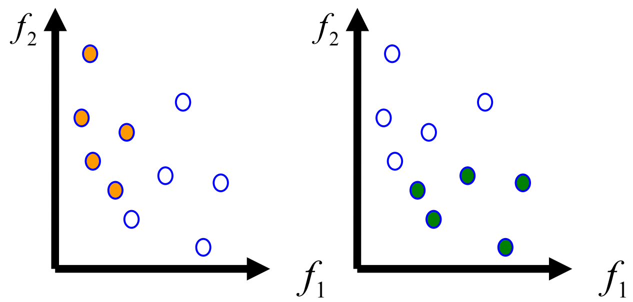

Fig 4. Illustration of VEGA approach for a biobjective minimization problem. Each of the figures shows the selected individuals according to each objectives. The original population size is 10, and 5 individuals are selected for each of the objectives.

The VEGA algorithm is easy to implement; however, it suffers from the speciation problem (evolution of species that excel in one of the objectives). This causes the algorithm to fail to generate compromise solutions (those that are not necessarily the best in one objective, but are optimal in the Pareto sense). In addition, the algorithm is susceptible to the shape of the Pareto front.

b. Pareto-Based Approaches

EAs in this category explicitly use Pareto-ranking in order to determine the probability of replication of an individual. The basic idea is to find the set of non-dominated individuals in the population. These are assigned the highest rank and eliminated from further contention. The process is then repeated with the remaining individuals until the entire population is ranked and assigned a fitness value. In conjunction with Pareto-based fitness assignment, a niching mechanism is used to prevent the algorithm from converging to a single region of the Pareto front [[10]]. A popular niching technique called sharing consists of regulating the density of solutions in the hyperspace spanned by either the objective vector or the decision vector. The schemes presented below essentially differ in the way the fitness value of an individual is determined prior to the selection step of the EA. Sharing is often used in the computation of the fitness value. Mutation and crossover operations are then performed to get the next generation of individuals.

A simple and efficient method is Multi Objective Genetic Algorithm (MOGA) [16]. The fitness value of an individual is proportional to the number of other individuals it dominates (**Fig 5**). Niching can be performed either in the objective space or the decision space.

Another version is the *Non-dominated Sorting Genetic Algorithm* (NSGA), which uses a layered classification technique [[17]]. All non-dominated individuals are assigned the same fitness value and sharing is applied in the decision variable space. The process is repeated for the remainder of the population with a progressively lower fitness value assigned to the non-dominated individuals.

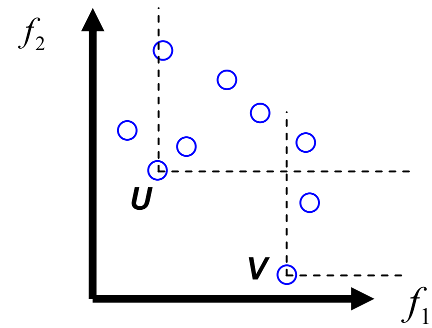

Fig 5. Illustration of fitness computation for MOGA in a biobjective minimization problem. Fitness of a given individual is proportional to the number of individual it dominates. For example, *U* which dominates 5 other individuals, has a higher fitness, and hence probability of selection than *V* which only dominates 2 individuals.

In the *Niched Pareto Genetic Algorithm* (NPGA) [[18]], instead of bilateral direct comparison, two individuals are compared with respect to a comparison set (usually 10% of the entire population). When one candidate is dominated by the set while the other is not, the latter is selected. If neither or both the candidates are dominated, fitness sharing is used to decide selection. NPGA introduces a new variable (size of the comparison set), but is computationally faster than the previous techniques, since the selection step is applied only to a subset of the population.

Strength Pareto Evolutionary Algorithm (SPEA) [[19]] uses an external archive to maintain the non-dominated solutions found during the evolution. Candidate solutions are compared to the archive. A MOGA-style fitness assignment is applied: fitness of each member of the current population is computed according to the strengths of all external non-dominated solutions that dominate it. A clustering technique is applied to maintain diversity.

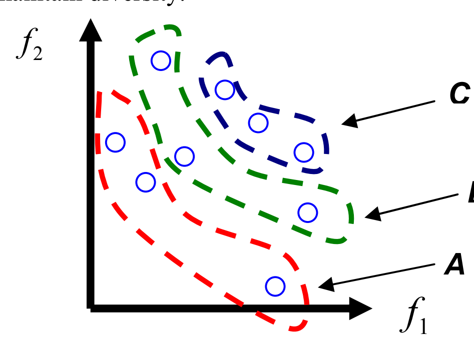

Fig 6. Illustration of fitness computation for NSGA in a biobjective minimization problem. A layered classification technique is used whereby the population is incrementally sorted using Pareto dominance. Individuals in set A have the same fitness value, which is higher than the fitness of individuals in set B, which in turn are superior to individuals in set C.

More recently, swarm intelligence approaches have been developed for MO problems [[20]-[22]]. In particular, in *Multi Objective Particle Swarm Optimization* (MOPSO), the global best (towards which the particles flock while exploring the search space) changed after a specified number of PSO epochs to a heuristically selected point of the emerging non-dominated front. The selection method is designed to emphasize regions of low density, thus at the same time maintaining diversity. The algorithm also features a mutation operator and a dynamic grid-based Pareto front management mechanism [[22]].

c. Discussion on Modern Methods

The metaheuristic techniques successfully address the limitations of the classical approaches when generating the Pareto front. Because they allow concurrent exploration of different points of the Pareto front, they can generate multiple solutions in a single run. The optimization can be performed without a priori information about objectives relative importance. These techniques can handle ill-posed problems (with incommensurable or mixed-type objectives). They are not susceptible to the shape of the Pareto front. Their main drawback is performance degradation as the number of objectives increases, since there does not exist computationally efficient methods to perform Pareto ranking. Furthermore, they require additional parameters such as sharing factor, or number of Pareto samples which need to be tweaked

IV Applications

The power systems field has always been ripe with problems of MO nature: Economic dispatch, maintenance scheduling, reactive resource allocation, etc. These problems are complex because they are often large scale problems for which the parameter space explodes as the size of the system increases. In addition, the objectives and constraints are typically incommensurable and of mixed-type.

These complex problems were traditionally approached using aggregation-based techniques with simplified assumptions to reduce the complexity of the problem. Because of the non-linear nature of the search space, stochastic search techniques such as GA, PSO or simulated annealing, which are robust enough to deal with these complexities, have been used successfully for the underlying optimization step.

The problem of thermal power dispatch with cost vs. emission level trade-off is considered in [[23]]. The approach taken is interactive optimization by using the *ε-constraint* method to generate non-dominated solutions. The *goal attainment* method in conjunction with a GA-based optimizer is used in [[24]] for tackling a stochastic version of economic dispatch where cost is minimized along with expected values of deviations from target power and heat generation levels.

Another common MO problem in power systems is the reactive power or VAr planning: this is the problem of locating and sizing capacitive resources in the most economical manner so as to power losses and enhance the voltage throughout the network. Early approaches for solving this problem used mathematical programming and were geared to finding a global optimum. In [[25]] the authors use *goal attainment* in combination with simulated annealing for the embedded optimization step to solve the problem. Different evolutionary algorithms were investigated in [[26]] for the reactive power planning problems, but the cost function used is the aggregated sum of operation and investment costs.

As mentioned earlier, the drawbacks of aggregation-based and other traditional MO techniques are the fact that they require good knowledge of the systems in order to appropriately determine how to weight the different objectives. Also, only one solution is generated, which is not always optimal in the Pareto sense. More importantly, for the system engineer's perspective, these methods provide limited flexibility, and are not readily portable in case the operating conditions or requirements change.

From the system-wide perspective, liberalization of energy markets induced tremendous changes in power systems planning and operation [[8]]. The Independent System Operator (ISO) in particular, whose role is to ensure security of the power systems must make quick and informed decisions after taking into account a multiplicity of competing objectives. Because it is necessary to consider all the consequences of a decision, a tool that generates all possible compromises allows performing trade-off analysis and gaining better understanding of the system.

At first, traditional techniques were used to attempt to generate Pareto front. For example, in [[8]], the *ε-constraint* method is applied to problems an ISO would face in a modern energy market (minimizing generation cost and real power losses with or without reserve constraints; maximizing the vulnerable operation, while minimizing real power reserve cost and minimize area import). The recent-years development of metaheuristic techniques and their application to concurrent solving of competing objectives addressed the shortcomings of traditional technique. For example, the economic dispatch with environmental constraints which has become the environmental/economic dispatch (EED) problem since the rise in environmental awareness allows proactive treatment of emission minimization as one of the objectives when performing dispatch. In [[11],[27]], it is proposed to use variants of NSGA to solve the EED problem. In [[4]] a comparative study of popular population-based MO evolutionary algorithms (NPGA, NSGA and SPEA) is applied to EED problems and compared to traditional optimization techniques. The research in [[28]] tackles the generation scheduling problem, determining the operation schedule of generating units in order to meet a time-varying load with a variety of constraints (reserve constraints, security constraints, etc.) while minimizing emission levels. In [[29]], capacitor location in radial distribution networks using a Pareto-front geared population-based technique in combination with Tabu search is proposed. In [[30]], a GA-based Pareto-based approach similar to NPGA is applied to the full Reactive-power Compensation Planning (RCP) problem.

V CONCLUSION

In this Chapter, fundamentals of MO optimization are presented. Solution approaches were categorized into two groups: classical techniques and intelligent techniques. The former consist in converting the MO problem into an SO problem. While simpler to implement, they require *a priori* information about the objectives, information which may not be available at the outset. Furthermore, the solution obtained is not always optimal. More importantly, they are not suitable for trade-off analysis.

The complexity of modern systems, particularly in the power systems requires the generation of a set of acceptable solutions (instead of a single solution) that would allow the operator to choose from. The intelligent MO techniques, which are population-based metaheuristics (evolutionary or swarm-based algorithms) allow the concurrent solving of the different objectives, and generation of the Pareto front.

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