

Einzelprüfung „Theoretische Informatik / Algorithmen (vertieft)“

Einzelprüfungsnummer 66115 / 2019 / Herbst

Thema 1 / Aufgabe 4

(SAT DOPPELSAT)

Stichwörter: Polynomialzeitreduktion

Betrachten Sie die folgenden Probleme:

SAT

Gegeben: Aussagenlogische Formel F in KNF

Frage: Gibt es mindestens eine erfüllende Belegung für F ?

DOPPELSAT

Gegeben: Aussagenlogische Formel F' in KNF

Frage: Gibt es mindestens eine erfüllende Belegung für F , in der mindestens zwei Literale pro Klausel wahr sind?

(a) Führen Sie eine polynomielle Reduktion von SAT auf DOPPELSAT durch.

Lösungsvorschlag

<https://courses.cs.washington.edu/courses/csep531/09wi/handouts/sol4.pdf>

DOUBLE-SAT is in NP. The polynomial size certificate consists of two assignments f_1 and f_2 . First, the verifier verifies if $f_1 \neq f_2$. Then, it verifies if both assignments satisfy ϕ by substituting the values for the variables and evaluate the clauses of ϕ . Both checks can be done in linear time. DOUBLE-SAT is NP-hard. We give a reduction from SAT. Given an instance ϕ of SAT which is a CNF formula of n variables x_1, x_2, \dots, x_n , we construct a new variable x_{n+1} and let $\psi = \phi \wedge (x_{n+1} \vee \neg x_{n+1})$ be the corresponding instance of DOUBLE-SAT. We claim that ϕ has a satisfying assignment iff ψ has at least two satisfying assignments. On one hand, if ϕ has a satisfying assignment f , we can obtain two distinct satisfying assignments of ψ by extending f with $x_{n+1} = T$ and $x_{n+1} = F$ respectively. On the other hand, if ψ has at least two satisfying assignments then the restriction of any of them to the set x_1, x_2, \dots, x_n is a satisfying assignment for ϕ . Thus, DOUBLE-SAT is NP-complete.

<https://cs.stackexchange.com/questions/6371/proving-double-sat-is-np-complete>

Here is one solution:

Clearly Double-SAT belongs to NP, since a NTM can decide Double-SAT as follows: On a Boolean input formula $\phi(x_1, \dots, x_n)$, nondeterministically guess 2 assignments and verify whether both satisfy ϕ .

To show that Double-SAT is NP-Complete, we give a reduction from SAT to Double-SAT, as follows:

On input $\phi(x_1, \dots, x_n)$:

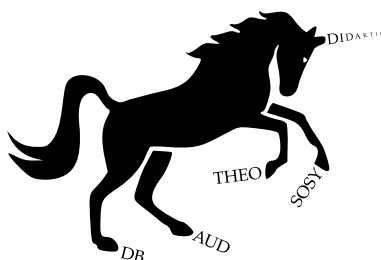
1. Introduce a new variable y . 2. Output formula $\phi'(x_1, \dots, x_n, y) = \phi(x_1, \dots, x_n) \wedge (y \vee \bar{y})$.

If $\phi(x_1, \dots, x_n)$ belongs to SAT, then ϕ has at least 1 satisfying assignment, and therefore $\phi'(x_1, \dots, x_n, y)$ has at least 2 satisfying assignments as we can satisfy the new clause $(y \vee \bar{y})$ by assigning either $y = 1$ or $y = 0$ to the new variable y , so $\phi'(x_1, \dots, x_n, y) \in \text{Double-SAT}$.

On the other hand, if $\phi(x_1, \dots, x_n) \notin \text{SAT}$, then clearly $\phi'(x_1, \dots, x_n, y) = \phi(x_1, \dots, x_n) \wedge (y \vee \bar{y})$ has no satisfying assignment either, so $\phi'(x_1, \dots, x_n, y) \notin \text{Double-SAT}$.

Therefore, $\text{SAT} \leq_p \text{Double-SAT}$, and hence Double-SAT is NP-Complete.

(b) Zeigen Sie, dass DOPPELSAT NP-vollständig ist.



Die Bschlangaul-Sammlung

Hermine Bschlangaul and Friends

Eine freie Aufgabensammlung mit Lösungen von Studierenden für Studierende zur Vorbereitung auf die 1. Staatsexamensprüfungen des Lehramts Informatik in Bayern.



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