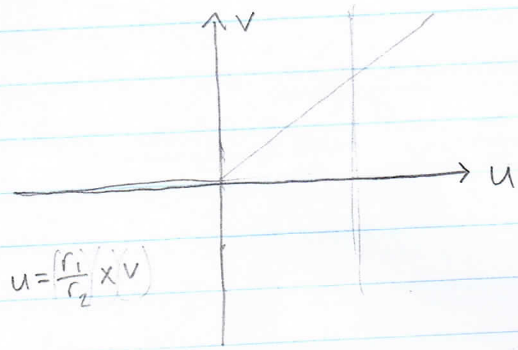
Given U and V, the joint pdf is

$$g(u,v) = \frac{u^{(\frac{r_1}{2}-1)} e^{-\frac{u}{2}}}{\Gamma(\frac{r_1}{2})\, 2^{(r_1/2)}} \; \frac{v^{(\frac{r_2}{2}-1)} e^{-\frac{v}{2}}}{\Gamma(\frac{r_2}{2})\, 2^{(r_2/2)}}$$

For $x > 0$,

$$P[W \le x] = P\left[\frac{U/r_1}{(V/r_2)} \le x\right] = P\left[\frac{u}{r_1} \le x\left(\frac{v}{r_2}\right)\right]$$



$$= \int_0^\infty \int_0^{(r_1/r_2)vx} g(u,v)\, du\, dv$$

$$= \frac{1}{\Gamma(\frac{r_1}{2})\Gamma(\frac{r_2}{2})} \int_0^\infty \left[\int_0^{(\frac{r_1}{r_2}vx)} u^{\frac{r_1}{2}-1} e^{-\frac{u}{2}}\, du\right] \left(\frac{1}{2^{\frac{r_1+r_2}{2}}}\right) v^{(\frac{r_2}{2}-1)} e^{(-\frac{v}{2})}\, dv$$

$$f(x) = \frac{1}{\Gamma(\frac{r_1}{2})\Gamma(\frac{r_2}{2})} \int_0^\infty \left(\frac{r_1}{r_2} v x\right) e^{\left(\frac{-\frac{r_1}{r_2} v x}{2}\right)} \left(\frac{r_1}{r_2} v\right) \left(\frac{1}{2^{\frac{r_1+r_2}{2}}}\right) v^{(\frac{r_2}{2}-1)} e^{(-\frac{v}{2})}\, dv$$

See lecture notes on Blackboard

$$= \ldots$$

$$= \frac{\Gamma(\frac{r_1+r_2}{2})}{\Gamma(\frac{r_1}{2})\Gamma(\frac{r_2}{2})} \left(\frac{r_1}{r_2}\right)^{r_1/2} x^{(\frac{r_1}{2}-1)} \left(1 + \frac{r_1}{r_2} x\right)^{-\left(\frac{r_1+r_2}{2}\right)}$$