(2) Markov Inequality

$u(x) = |X|^r$ and consider $c^r$ in (1)

$$P\left[|X|^r \geq c^r\right] \leq \frac{E[|X|^r]}{c^r}$$

$$P\left[|X| \geq c\right] \leq \frac{E[|X|^r]}{c^r}$$

(3) Chebyshev inequality

$$P\left[|X-\mu| \geq k\sigma\right] \leq \frac{1}{k^2}$$

$$u(x) = (x-\mu)^2, \quad c = k^2\sigma^2$$

$$P\left[(x-\mu)^2 \geq k^2\sigma^2\right] \leq \frac{E[(x-\mu)^2]}{k^2\sigma^2} = \frac{\cancel{\sigma^2}}{k^2\cancel{\sigma^2}}$$

$\hookrightarrow = P\left[|x-\mu| > k\sigma\right]$

Now $P\left[|\bar{x}-\mu| < \varepsilon\right] \geq 1 - \sigma^2/\varepsilon^2 n \longrightarrow 1$ as $n \to \infty$

and $P\left[|X-\mu| < k\sigma\right] \geq 1 - 1/k^2$

Weak LLN

$P\left[|x-\mu| < \varepsilon\right] \geq 1 - \frac{\sigma^2}{k^2}$ so $P\left[|x-\mu| \geq \varepsilon\right] = \frac{\sigma^2}{\varepsilon^2}$