Example 2

Let $X_1, X_2, \dots X_{15}$ be a random sample from the distribution with pdf

$$f(x) = \begin{cases} (3/2) \cdot x^2 & x \in (-1, 1) \\ 0 & \text{otherwise} \end{cases}$$

Use CLT to estimate $P\left[-0.3 \le \sum_{i=1}^{15} X_i \le 1.5\right]$

$\mu = 0$

$$\sigma^2 = E[X^2] - E[x]^2$$

$$= \int_{-1}^{1} x^2 \left(\tfrac{3}{2}\right) x^2 \, dx - 0^2 = \int_{-1}^{1} \left(\tfrac{3}{2}\right) x^4 \, dx = \frac{3}{2}\left[\frac{x^5}{5}\right]_{-1}^{1} = \frac{3}{2}\left(\frac{1}{5} + \frac{1}{5}\right)$$

$$= \frac{3}{5}$$

(and so $\sigma = \sqrt{3/5} = \sqrt{0.6}$

So

$$P\left[-0.3 \le \sum_{i=1}^{15} X_i \le 1.5\right]$$

$$= P\left[\frac{-0.3}{\sqrt{15(0.6)}} \le \frac{\sum_{i=1}^{n} X_i - 0}{\sqrt{15 \cdot (0.6)}} \le \frac{1.5}{\sqrt{15 \cdot (0.6)}}\right]$$

(Recall $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \dfrac{\sum^{n} X_i - n\mu}{\sigma(\sqrt{n})}$

$$\overset{\text{CLT}}{\approx} P[-0.1 \le Z \le 0.5] = 0.6915 + 0.5398 - 1$$

$$= 0.2313$$