Assignment 5: Due Friday October 5, 2018

Main Assignment

8. Given θ, the random variable X has a binomial distribution with $n = 2$ and probability of success θ. If the prior density of θ is

$$h(\theta) = \begin{cases} k & \text{if } \frac{1}{2} < \theta < 1 \\ 0 & \text{otherwise,} \end{cases}$$

what is the Bayes' estimate of θ for a squared error loss if the sample consists of $x_1 = 1$ and $x_2 = 2$.

9. Suppose two observations were taken of a random variable X which yielded the values 2 and 3. The density function for X is

$$f(x/\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise,} \end{cases}$$

and prior distribution for the parameter θ is

$$h(\theta) = \begin{cases} 3\,\theta^{-4} & \text{if } \theta > 1 \\ 0 & \text{otherwise.} \end{cases}$$

If the loss function is quadratic, then what is the Bayes' estimate for θ?

12. Suppose one observation was taken of a random variable X which yielded the value 2. The density function for X is

$$f(x/\mu) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(x-\mu)^2} \qquad -\infty < x < \infty,$$

and prior distribution of μ is

$$h(\mu) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\mu^2} \qquad -\infty < \mu < \infty.$$

If the loss function is quadratic, then what is the Bayes' estimate for μ?

15. Given θ, the random variable X has a binomial distribution with $n = 3$ and probability of success θ. If the prior density of θ is

$$h(\theta) = \begin{cases} k & \text{if } \frac{1}{2} < \theta < 1 \\ \\ 0 & \text{otherwise,} \end{cases}$$

what is the Bayes' estimate of θ for a *absolute difference error loss* if the sample consists of one observation $x = 1$?

26. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a population with a probability density function

$$f(x; \alpha, \lambda) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} \, x^{\alpha-1} e^{-\lambda x} & \text{if} \quad 0 < x < \infty \\ \\ 0 & \text{otherwise,} \end{cases}$$

where α and λ are parameters. Using the moment method find the estimators for the parameters α and λ.

Extra Credit

16. Suppose the random variable X has the cumulative density function $F(x)$. Show that the expected value of the random variable $(X - c)^2$ is minimum if c equals the expected value of X.

17. Suppose the continuous random variable X has the cumulative density function $F(x)$. Show that the expected value of the random variable $|X - c|$ is minimum if c equals the median of X (that is, $F(c) = 0.5$).