# Math 562 (Math. Stats)

Example 1:

$\int \frac{1}{\theta} e^{-x/\theta}$

$\frac{1}{\theta} \int e^{-x/\theta}$

1) $X \sim Exp(\theta)$

$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0$

and $Y = \ln X$.

Want to find pdf of Y.

Start by identifying domain: $Dom(Y) = (-\infty, \infty)$

$\ln(x) \in (-\infty, \infty), \ \forall x > 0$

Then for any $y \in (-\infty, \infty)$

$$P[Y \le y] = P[\ln X \le y]$$
$$= P[X \le e^y]$$
$$= 1 - e^{(-1/\theta)(e^y)}$$

$$f_Y(y) = -e^{-1/\theta \, e^y}\left(-\frac{1}{\theta} e^y\right)$$
$$= \left(\frac{1}{\theta}\right) e^{-1/\theta \, e^y}\left(e^y\right)$$

Example 2:

If we have $X_1, \ldots, X_n \sim Exp(\theta)$, iid.

and $Y = \sum_{i=1}^{n} X_i$.

What is the distribution of Y?

(Rule of thumb: with sum of ind. variables, easiest to work with mgf)

$$M_Y(t) = E[e^{tY}] = E[e^{t\sum x_i}] = E[e^{tx_1 + tx_2 + \cdots}]$$
$$= E[e^{tx_1} e^{tx_2} \cdots e^{tx_n}] = E[e^{tx_1}]E[e^{tx_2}]\cdots E[e^{tx_n}]$$
$$= (E[e^{tx_1}])^n \quad \text{(because iid)}$$
$$= \frac{1}{(1-\theta t)^n} \sim GAM(\overset{\alpha}{\theta}, \overset{\theta}{n})$$

$n \quad \theta$

(See ☆2)