Continued application of CLT to Analysis

Note:

$$\frac{\ln(n)}{n^r} \xrightarrow[n\to\infty]{} 0$$

Consider $n^n, e^n, n!$. Which grows fastest?

e^n, $n!$ ↑ ↑ These are faster.

So

$$\lim_{n\to\infty} \frac{n^n}{e^n n!} = 0$$

Prove it!

August 29

Recall from last class:

1) $\lim_{n\to\infty} e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!} = \frac{1}{2}$.

by CLT

when $X_1, X_2, \dots, X_n \sim POI(1)$

$Y = \sum X_i \sim POI(n)$

$$P\left[\frac{Y-n}{\sqrt{n}} \le 0\right] = P\left[Y \le n\right] = \sum_{k=0}^{n} P[Y=k]$$

$$\| $$

$$P[Z \le 0] = \frac{1}{2}$$

$$= \sum_{k=0}^{n} \frac{e^{-n} n^k}{k!}$$

So

$$\lim_{n\to\infty} \sum_{k=0}^{\infty} \frac{e^{-n} n^k}{k!} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2}.$$

2) $$\lim_{n\to\infty} \int_0^n \frac{e^{-t} t^{n-1}}{(n-1)!}\, dt = \frac{1}{2}$$