Only need to show $\bar{X}$ and S^2 are indep.
In order to show that,
we only need to show X_i and S^2 are independent.

Second Step:
Show $\bar{X}$ and $(X_i - \bar{X})$ are independent.

For $X_1, X_2, \dots, X_n$, the joint density function is

$$f(x_1, x_2, \dots, x_n) = \frac{1}{(\sqrt{2\pi})^n \sigma^n} e^{\left[-\frac{1}{2\sigma^2}\sum (x_i - \mu)^2\right]}$$

$$= \frac{1}{(2\pi)^{n/2}(\sigma^2)^{n/2}} e^{\left[-\frac{1}{2\sigma^2}\left(\sum(x_i-\bar{x})^2 + n(\bar{x}-\mu)^2\right)\right]}$$

Transformation:
$y_1 = \bar{x}$, $y_i = x_i - \bar{x}$, $2 \le i \le n$

$$x_1 - \bar{x} = -\sum_{i=2}^{n}(x_i - \bar{x}) = -\sum_{i=2}^{n} y_i$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \underbrace{\left(-\sum_{i=2}^{n} y_i\right)^2}_{(x_1-\bar{x})^2} + \underbrace{\sum_{i=2}^{n} y_i^2}_{\sum_{i=2}^{n}(x_i-\bar{x})^2}$$