Assignment 4: Due Friday 2018-09-28

Main Assignment

Chapter 15.4

7. Let X_1, X_2, X_3, X_4 be a random sample from a distribution with density function

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{\frac{-(x-4)}{\beta}} & \text{for} \quad x > 4 \\ 0 & \text{otherwise,} \end{cases}$$

where $\beta > 0$. If the data from this random sample are 8.2, 9.1, 10.6 and 4.9, respectively, what is the maximum likelihood estimate of β?

13. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with probability density

$$f(x) = \begin{cases} \frac{1}{\theta} & \text{if } 2\theta \leq x \leq 3\theta \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$. What is the maximum likelihood estimator of θ?

21. A sample of size n is drawn from a gamma distribution

$$f(x;\beta) = \begin{cases} \frac{x^3 e^{-\frac{x}{\beta}}}{6\beta^4} & \text{if } 0 < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

What is the maximum likelihood estimator of β ?

22. The probability density function of the random variable X is defined by

$$f(x;\lambda) = \begin{cases} 1 - \frac{2}{3}\lambda + \lambda\sqrt{x} & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the maximum likelihood estimate of the parameter λ based on two independent observations $x_1 = \frac{1}{4}$ and $x_2 = \frac{9}{16}$?

28. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a population distribution with the probability density function

$$f(x;\theta) = \begin{cases} \theta^2\, x\, e^{-\theta\, x} & \text{if} \quad 0 < x < \infty \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta$ is a parameter. Find the Fisher information in the sample about the parameter θ.

Extra Credit

14. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with probability density

$$f(x) = \begin{cases} 1 - \theta^2 & \text{if } 0 \leq x \leq \frac{1}{1-\theta^2} \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$. What is the maximum likelihood estimator of θ?

23. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with density function $f(x;\sigma) = \frac{\sigma}{2}\, e^{-\sigma|x-\mu|}$. What is the maximum likelihood estimator of σ ?