Let $X_1, X_2, \ldots, X_n \sim EXP(1)$

$$Y = \sum_{i=1}^{n} X_i \sim GAM(\underset{\alpha}{n}, \underset{\theta}{1})$$

$$f(t) = \frac{1}{\Gamma(n)\, 1^n} t^{n-1} e^{-t}$$

$$= \frac{1}{(n-1)!} t^{n-1} e^{-t}$$

So $P\left[\frac{Y-n}{\sqrt{n}} \le 0\right] = \lim_{n\to\infty} P[Y \le n] = \int_0^n \frac{e^{-t} t^{n-1}}{(n-1)!} dt$

$= P[Z \le 0]$

$= 1/2$

Taking the limit:

$$\lim_{n\to\infty} \int_0^n \frac{e^{-t} t^{n-1}}{(n-1)!} dt = \lim_{n\to\infty} 1/2 = 1/2$$

So we put 1) and 2) together

using IBP:

$u = \frac{t^{n-1}}{(n-1)!}$ $\qquad dv = e^{-t} dt$

$du = \frac{t^{n-2}}{(n-2)!}$ $\qquad -e^{-t}$

$$\int_0^n \frac{e^{-t} t^{n-1}}{(n-1)!} dt$$

$$= -e^{-t}\left(\frac{t^{n-1}}{(n-1)!}\right)\Big|_0^n + \int_0^n e^{-t} \frac{t^{n-2}}{(n-2)!} dt$$

$$= 1 + \int_0^n e^{-t}\left(\frac{t^{n-2}}{(n-2)!}\right) dt$$

$$= 1 - \sum_{k=0}^{n-1} \frac{e^{-n} n^k}{k!}$$