

Note: For any distribution, $E[\bar{x}] = \mu$, and $\text{Var}(\bar{x}) = \frac{\sigma^2}{n}$
where \bar{x} is the mean of sample size n .

Hence $E[W_n] = 0$, and $\text{Var}(W_n) = 1$

" \xrightarrow{d} "

1) $\lim_{n \rightarrow \infty} G_n(x) = \Phi(x)$ for all x .

2) If $\lim_{n \rightarrow \infty} M_n(t) = M(t)$ on $(-\delta, \delta)$, $\leftarrow (\text{for some } \delta > 0)$

then $W_n \xrightarrow{d} W$.

(this is
sufficient,
but not
necessary)

3) When n is large,
 $W_n \approx Z$

$$\bar{X}_n \approx N\left(\mu, \frac{\sigma^2}{n}\right)$$

(Since $W_n \xrightarrow{d} Z \sim N(0, 1)$)