Assignment #6: Due October 15, 2018

Main Exercises

2. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with density function

$$f(x;\theta) = \frac{1}{2\theta}\, e^{-\frac{|x|}{\theta}} \qquad -\infty < x < \infty,$$

where $0 < \theta$ is a parameter. What is the expected value of the maximum likelihood estimator of θ ? Is this estimator unbiased?

3. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with density function

$$f(x;\theta) = \frac{1}{2\theta}\, e^{-\frac{|x|}{\theta}} \qquad -\infty < x < \infty,$$

where $0 < \theta$ is a parameter. Is the maximum likelihood estimator an efficient estimator of θ?

5. Suppose X and Y are independent random variables each with density function

$$f(x) = \begin{cases} 2\,x\,\theta^2 & \text{for } 0 < x < \frac{1}{\theta} \\ 0 & \text{otherwise.} \end{cases}$$

If $k\,(X + 2Y)$ is an unbiased estimator of θ^{-1}, then what is the value of k?

10. Let $X_1, X_2, ..., X_n$ be a random sample from a population with probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise ,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. If $\overline{X}$ denotes the sample mean, then what should be value of the constant k such that $k\overline{X}$ is an unbiased estimator of θ ?

20. Let $X_1, X_2, ..., X_n$ be a random sample from a population X with density function

$$f(x;\theta) = \begin{cases} \binom{m}{x}\theta^x(1-\theta)^{m-x} & \text{for } x = 0, 1, 2, ..., m \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta < 1$ is parameter. Show that $\frac{\overline{X}}{m}$ is a uniform minimum variance unbiased estimator of θ for a fixed m.

21. Let $X_1, X_2, ..., X_n$ be a random sample from a population X with density function

$$f(x;\theta) = \begin{cases} \theta\, x^{\theta-1} & \text{for } 0 < x < 1 \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 1$ is parameter. Show that $-\frac{1}{n}\sum_{i=1}^{n} \ln(X_i)$ is a uniform minimum variance unbiased estimator of $\frac{1}{\theta}$.

Extra Credit

9. Let X_1, X_2 be a random sample of size 2 from population with probability density

$$f(x;\theta) = \begin{cases} \frac{1}{\theta}\, e^{-\frac{x}{\theta}} & \text{if } 0 < x < \infty \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. If $Y = \sqrt{X_1 X_2}$, then what should be the value of the constant k such that kY is an unbiased estimator of the parameter θ ?

11. Let $X_1, X_2, ..., X_n$ be a random sample from a population with probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ \\ 0 & \text{otherwise ,} \end{cases}$$

where $\theta > 0$ is an unknown parameter. If X_{med} denotes the sample median, then what should be value of the constant k such that kX_{med} is an unbiased estimator of θ ?