Example 1 Consider $X_1, X_2, X_3, X_4 \sim N(2,1)$
$Z_1, Z_2, Z_3, Z_4 \sim N(0,1)$
(Independent)

Identify the sampling distribution of the following

1) $$\frac{\sqrt{2}(Z_1+Z_2)}{\sqrt{(X_1-X_2)^2+(Z_3+Z_4)^2}}$$

Note:
$Z_1+Z_2 \sim N(0,2)$
$Z_3+Z_4 \sim N(0,2)$
$X_1-X_2 \sim N(0,2)$
(These 3 variables are independent)

$\frac{Z_1+Z_2}{\sqrt{2}} \sim N(0,1)$ $\quad$ $\left(\frac{Z_3+Z_4}{\sqrt{2}}\right)^2 \sim \chi^2(1)$

$\frac{Z_3+Z_4}{\sqrt{2}} \sim N(0,1)$ $\quad$ so $\quad$ $\left(\frac{X_1-X_2}{\sqrt{2}}\right)^2 \sim \chi^2(1)$

$\frac{X_1-X_2}{\sqrt{2}} \sim N(0,1)$

and

$$\left(\frac{X_1-X_2}{\sqrt{2}}\right)^2 + \left(\frac{Z_3+Z_4}{\sqrt{2}}\right)^2 \sim \chi^2(2)$$