$$\text{Var}(S^2) = \text{Var}\left(\frac{\sigma^2}{n-1}\,\frac{(n-1)}{\sigma^2}S^2\right)$$

$$= \frac{\sigma^4}{(n-1)^2}\,\text{Var}\left(\frac{n-1}{\sigma^2}S^2\right) = \frac{\sigma^4}{(n-1)^2}\cdot 2(n-1)$$

$$= \frac{2\sigma^4}{(n-1)}$$

So far, we have from our theorem that

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \longrightarrow \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim Z \sim N(0,1)$$

$$S^2 \longrightarrow \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

Two problems:

1) If σ^2 is unknown, we have to use S^2 to replace σ^2

What will be

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim ?$$

2) To compare σ_1^2 and σ_2^2 by using S_1^2 and S_2^2,

we do

$$\frac{\frac{S_1^2}{\sigma_1^2}}{\frac{S_2^2}{\sigma_2^2}} \sim ?$$