Proof (in 3 parts)

1) $$\underbrace{\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2}_{\sim\chi^2(n)} = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{\sigma^2} = \frac{\sum(x_i-\bar{x}+\bar{x}-\mu)^2}{\sigma^2}$$

by quadratic formula

$$= \sum\frac{(x_i-\bar{x})^2}{\sigma^2} + \sum\frac{(\bar{x}-\mu)^2}{\sigma^2} + 2\,\frac{\sum(x_i-\bar{x})(\bar{x}-\mu)}{\sigma^2}$$

$$= \sum\frac{(x_i-\bar{x})^2}{\sigma^2} + \frac{n(\bar{x}-\mu)^2}{\sigma^2} + \underbrace{2(\bar{x}-\mu)\frac{\sum(x_i-\bar{x})}{\sigma^2}}_{=0}$$

since $\sum x_i - n\bar{x} = 0$

$$= \underbrace{\frac{\sum(x_i-\bar{x})^2}{\sigma^2}}_{\sim\chi^2(n-1)} + \left(\frac{\bar{x}-\mu}{(\sigma/\sqrt{n})}\right)^2$$

we'll show $\sim\chi^2(n-1)$

Note: Class ended here