## Example 3

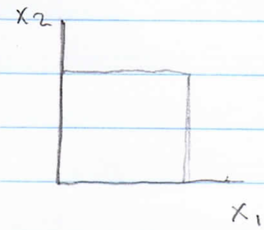
$X_1, X_2 \sim UNIF(0,1)$ and $X_1 \perp X_2$.

Consider $Y = X_1 + X_2$.

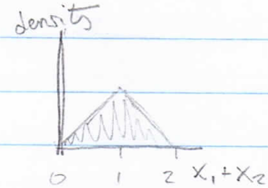
Determine the pdf of $Y$.

$Y \in (0,2)$. For $t \in (0,2)$,

$$P[Y \le t] = P[X_1 + X_2 \le t]$$




pdf of $X_1 + X_2$




imagine what happens as we increase $n$ where we have $\sum_{i=1}^{n} X_i$

## Example 4 Back to $N(\mu, \sigma^2)$

Take a random sample

$$X_1, X_2, \dots, X_n \sim N(\mu, \sigma^2).$$

Then $\bar{X} = \frac{1}{n}\sum X_i \sim N(\quad,\quad)$

$$M_{\bar{x}}(t) = E\left[e^{t\bar{x}}\right] = E\left[e^{(1/n)(t)(x_1)} \cdots e^{(\frac{1}{n})(t)(x_n)}\right]$$

$$= E\left[e^{(1/n)(t)(x_1)}\right] \cdots E\left[e^{(1/n)(t)(x_1)}\right]$$

$$= \left(E\left[e^{(\frac{1}{n})(t)(x_1)}\right]\right)^n$$

$$= \left[e^{\mu(\frac{t}{n}) + \frac{\sigma^2(\frac{t}{n})^2}{2}}\right]^n = e^{\left(\mu t + \frac{\left(\frac{\sigma^2}{n}\cdot t^2\right)}{2}\right)}$$

$$\sim N\left(\mu, \frac{\sigma^2}{n}\right)$$