

Last Part : F distribution

Motivation: We wanted to compare σ_1^2 and σ_2^2
by comparing S_1^2 and S_2^2

$$N(\underline{\mu}_1, \sigma^2) \text{ and } N(\underline{\mu}_2, \sigma^2)$$

$$\sigma_1^2 = \sigma_2^2$$

$$W = \frac{\left(\frac{S_1^2}{\sigma_1^2} \right)}{\left(\frac{S_2^2}{\sigma_2^2} \right)} = \frac{\frac{(n_1-1)S_1^2}{\sigma_1^2} / (n_1-1)}{\frac{(n_2-1)S_2^2}{\sigma_2^2} / (n_2-1)}$$

$\swarrow \chi^2(n_1-1)$
 $\swarrow \chi^2(n_2-1)$

In defining the F-dist, we start with two χ^2 -distributions.

Def. Let $U \sim \chi^2(r_1)$, $V \sim \chi^2(r_2)$

$$U \perp V. \quad \text{Define } W = \frac{(U/r_1)}{(V/r_2)}$$

W is called $F(r_1, r_2)$ "F-distribution with r_1 and r_2 degrees of freedom"