If $Z \sim N(0,1)$, then $Z^2 = V \sim \chi^2(1)$

— what we did at the beginning

If $\underbrace{Z_1, Z_2, \dots, Z_n}$ $\sim N(0,1)$,

then $V = \sum_{i=1}^{n} Z_i^2 \sim \chi^2(n)$

! crucially ! these are assumed independent.

$\boxed{\text{Third Question}}$ Sample Variance

$$S^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - x)^2$$

$\boxed{\text{Theorem}}$ Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ and $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$.

Sampling distribution

Then:

1) $\bar{X}$ and S^2 are independent

2) $\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$