Sep. 4

meaning: $\cap$ — ω denotes the compound event

$$P[X \le x, Y \le y] \quad — \quad P[\omega \mid X(\omega) \le x, Y(\omega) \le y]$$

i.e.

$$P[(X \le x) \cap (Y \le y)]$$

$$X_1, X_2, \ldots, X_{100} \sim POI(\lambda)$$

$$\bar{X} = \frac{1}{100} \sum_{i=1}^{100} x_i$$

Recall $\sum_{i=1}^{100} x_i \sim POI(100\lambda)$

Interesting note: It's very hard to identify the sampling distribution of $\bar{x}$ for a random sample from a uniform dist. The CLT is useful in this case.

Rule of thumb
When you have a random sample of 100, you should see 100 somewhere in your solution

$$P[\bar{X} \ge 1] = 1 - P[\bar{X} < 1]$$

(since Poisson RV is in $\{0\} \cup \{\mathbb{N}\}$)

$$= 1 - P[\bar{X} = 0]$$

$$= 1 - P\left[\tfrac{1}{100} \sum x_i = 0\right]$$

☆ !

$$= 1 - P[\sum x_i = 0]$$

$$= 1 - e^{-100\lambda}$$

Important!
It's only kosher to multiply by 100 here because the RHS is equal to zero. Doesn't generally work for any real number.

Note: For HW2, we need to note that independence is a crucial condition