

Let's Standardize!

$$W_n = \frac{\bar{X} - \mu}{(\sigma/\sqrt{n})} \sim N(0, 1)$$

$$\text{"lim"}(W_n) = Z \sim N(0, 1)$$

this is shorthand for i.i.d.

The CLT:

Observe a random sample  $X_1, X_2, \dots, X_n$   
from a population with mean  $\mu$   
and variance  $\sigma^2 < \infty$ .

$$\text{Let } W_n = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{(\sum X_i) - (n\mu)}{(\sqrt{n})(\sigma)}$$

Then  $W_n \xrightarrow{d} Z \sim N(0, 1)$  as  $n \rightarrow \infty$ .

Note

1) " $\xrightarrow{d}$ " converges in dist. means that  
 $\lim_{n \rightarrow \infty} G_n(y) = G(y)$  for all  $y$

(i.e. the sequence of functions converges pointwise)

2) IF  $M_n(t) \rightarrow M(t)$  on  $(-\delta, \delta)$ ,  
then  $W_n \xrightarrow{d} W$

★ Q1 What's "lim"?

Q2 What if  $X_i$  are not normal?