Assignment 3: Due Friday 2018-09-19

Main Assignment

Chapter 14.4:

17. Suppose $X_j = Z_j - Z_{j-1}$, where $j = 1, 2, ..., n$ and $Z_0, Z_1, ..., Z_n$ are independent and identically distributed with common variance σ^2. What is the variance of the random variable $\frac{1}{n}\sum_{j=1}^{n} X_j$?

19. Let $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_n$ be two random sample from the independent normal distributions with $Var[X_i] = \sigma^2$ and $Var[Y_i] = 2\sigma^2$, for $i = 1, 2, ..., n$ and $\sigma^2 > 0$. If $U = \sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ and $V = \sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)^2$, then what is the sampling distribution of the statistic $\frac{2U+V}{2\sigma^2}$?

Chapter 15.4:

2. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with a probability density function

$$f(x;\theta) = \begin{cases} (\theta+1)\,x^{-\theta-2} & \text{if} \quad 1 < x < \infty \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta$ is a parameter. Using the moment method find an estimator for the parameter θ

5. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with a probability density function

$$f(x;\theta) = \begin{cases} (\theta+1)\,x^{-\theta-2} & \text{if} \quad 1 < x < \infty \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta$ is a parameter. Using the maximum likelihood method find an estimator for the parameter θ.

6. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with a probability density function

$$f(x;\theta) = \begin{cases} \theta^2\,x\,e^{-\theta\,x} & \text{if} \quad 0 < x < \infty \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta$ is a parameter. Using the maximum likelihood method find an estimator for the parameter θ.

Extra Credit:

Chapter 14.4:

16. Let X and Y be joint normal random variables with common mean 0, common variance 1, and covariance $\frac{1}{2}$. What is the probability of the event $\left(X + Y \leq \sqrt{3}\right)$, that is $P\left(X + Y \leq \sqrt{3}\right)$?

20. Suppose $X_1, X_2, ..., X_6$ and $Y_1, Y_2, ..., Y_9$ are independent, identically distributed normal random variables, each with mean zero and variance $\sigma^2 > 0$. What is the 95^{th} percentile of the statistics $W = \left[\sum_{i=1}^{6} X_i^2\right] / \left[\sum_{j=1}^{9} Y_j^2\right]$?

Chapter 15.4:

3. Let $X_1, X_2, ..., X_n$ be a random sample of size n from a distribution with a probability density function

$$f(x;\theta) = \begin{cases} \theta^2\, x\, e^{-\theta\, x} & \text{if} \quad 0 < x < \infty \\ 0 & \text{otherwise}, \end{cases}$$

where $0 < \theta$ is a parameter. Using the moment method find an estimator for the parameter θ.