

Sampling from  $N(\mu, \sigma^2)$

Question:  
What if  $\sigma^2$  is unknown?

1.  $\bar{X} \sim N(\mu, \sigma^2)$   $\rightarrow \left( \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \right) \sim N(0, 1)$

(More generally,  $L = \sum_{i=1}^n a_i X_i$   
 $L \sim N(\sum a_i \mu_i, \sum a_i^2 \sigma_i^2)$ )

2. If  $V = Z^2$ ,

then  $V \sim \chi^2(1)$ .

If  $V = \sum \left( \frac{x_i - \mu_i}{\sigma_i} \right)^2$  then  $V \sim \chi^2(n)$   
(because of independence).

3.  $S^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$

$E[S^2] = \sigma^2$  (ie,  $S^2$  is an unbiased estimator of  $\sigma^2$ )

$$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum (x_i - \bar{x})^2}{\sigma^2} \sim \chi^2(n-1)$$