$$= 1 - \left(\sum_{k=0}^{n} \frac{e^{-n} n^k}{k!} \right) + \frac{e^{-n} n^n}{n!}$$

Take this limit:

Note:

$$\frac{1}{2} = 1 - \frac{1}{2} + \lim_{n \to \infty} \frac{e^{-n} n^n}{n!}$$

$$= \frac{1}{2} + \lim_{n \to \infty} \frac{e^{-n} n^n}{n!}$$

So

$$0 = \lim_{n \to \infty} \frac{e^{-n} n^n}{n!}$$

▽▽▽
△△△