

2) Weak Law of Large Numbers

Let X_1, X_2, \dots, X_n be a random sample from $f(x)$ with μ and $\sigma^2 < \infty$

Then $\bar{X} \xrightarrow{P} \mu$ (conv. in probability)

$$\left(\text{i.e. } \lim_{n \rightarrow \infty} P[|\bar{X} - \mu| > \varepsilon] \leq \lim_{n \rightarrow \infty} \frac{\sigma^2}{n\varepsilon^2} = 0 \right)$$

3) Strong Law of Large Numbers

If μ exists,

$$P[\omega \mid \lim_{n \rightarrow \infty} \bar{X}(\omega) = \mu] = 1$$

i.e. $\bar{X} \rightarrow \mu$ almost surely

We will prove the weak LLN,
but the strong LLN cannot
be proven in this course
(it's very difficult)