

Theoretical

Global

Seismology

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and
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Preface

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Part I

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Chapter

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14.1.5 Geographic versus geocentric colatitude

The location of a seismic station situated upon the Earth's surface is conventionally specified by giving its elevation e above the geoid, its longitude ϕ with respect to Greenwich, and its *geographical colatitude* θ' , which is the angle between the normal \hat{n} to the reference ellipsoid and the normal \hat{z} to the equatorial plane. Prior to calculating a synthetic accelerogram or spectrum by mode summation, it is necessary to convert the geographic colatitude θ' to the corresponding *geocentric colatitude* θ , which is the angle between the radius vector \hat{r} and \hat{z} . Correct to first order in the ellipticity, the two colatitudes are related by

$$\tan \theta \approx (1 + 2\varepsilon_a) \tan \theta'. \quad (14.32)$$

Geometrically, the transformation $\theta' \rightarrow \theta$ projects a point on the reference ellipsoid to a point on the unperturbed sphere along a line through the origin. The original and projected points in Figure 14.2 are the feet of the unit normals \hat{n} and \hat{r} , respectively. The hypocentral location of an earthquake source is likewise specified by giving its depth h beneath the geoid, its longitude ϕ_s and its geographical colatitude θ'_s . In calculating the real receiver and source vectors $r_k = \hat{\nu} \cdot s_k(\mathbf{x})$ and $s_k = \mathbf{M} : \varepsilon_k(\mathbf{x}_s)$ we stipulate that

$$\mathbf{x} = (a, \theta, \phi), \quad \mathbf{x}_s = (a - h, \theta_s, \phi_s). \quad (14.33)$$

A similar geographic-to-geocentric coordinate transformation must precede the calculation of a spherical-Earth seismogram or spectrum, using equa-

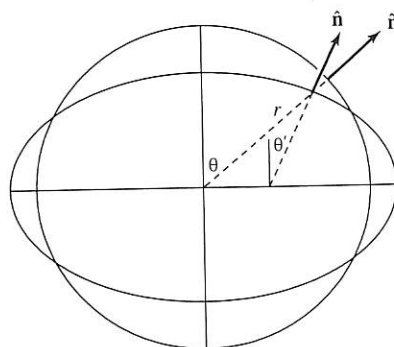


Figure 14.2. Cartoon illustrating the relation between the geographical colatitude θ and the geocentric colatitude θ' . The vectors \hat{n} and \hat{r} are the unit normals to the hydrostatic ellipsoid and the undeformed (equal-mass) sphere, respectively. Note that $\theta \geq \theta'$, with equality prevailing only at the poles and the equator.

tions (10.51)–(10.65). Station elevations e are generally ignored in normal-mode and long-period surface-wave seismology; however, they are routinely accounted for in short-period body-wave travel-time investigations.

14.2. SPLITTING

target multiplet in self-coupling matrix

14.2.1 First-

To begin, we consider the centrifugal splitting. The splitting is the $H^{\text{rot}} = W$. This matrix

$$H^{\text{rot}} = i\chi\Omega \begin{pmatrix} & \\ & \end{pmatrix}$$

The quantity χ is given by Gilbert (1961):

$$\chi = k^{-2} \int_0^a$$

where $k = \sqrt{l(l+1)}$ is the complex-to- of the matrix (D.1

$$Z^H Z = I,$$

where $\Delta = \chi\Omega$ dia

$$Z = \begin{pmatrix} & \ddots & \\ & & \\ & & \ddots & \end{pmatrix}$$

First-order Coriolis quantum energy frequency perturbation

$$\delta\omega_m = m\chi\Omega$$