THE FIGURES OF ROTATING PLANETS

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Summary

The theory of the figure of the Earth on the hydrostatic hypothesis, developed by Darwin and de Sitter, is applied to models for Jupiter and Saturn proposed by W. H. Ramsey and B. Miles. It is found that the integral equation for the coefficient of the fourth harmonic is more easily solved directly than by conversion into a differential equation. The results show that the ratio D/J^2, which is equal to $35l/12k^2$ in the treatment of H. and G. Struve, is substantially larger than for a homogeneous body in a hydrostatic state, and in satisfactory agreement with observation for Saturn. Comparison with a model given by Darwin indicates that an inequality stated by de Sitter for the coefficient of the fourth harmonic is not general.

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1. Darwin, in a classical paper, developed the hydrostatic theory of a rotating planet to the second order in the ellipticity.* His definitions of the quantities concerned led to considerable algebraic complexity; part of this was removed by de Sitter†, but de Sitter published only a few of the formulae and made several errors, some of which have been corrected previously, especially by Bullard.‡ The theory has been applied hitherto only to the Earth, but can now be usefully applied also to Jupiter and Saturn, since the fourth harmonic in their gravitational potentials has an observable effect on the motions of the nearest satellites. Further, the work of Ramsey and Miles on these planets makes it possible to give a theoretical determination of these terms and compare with observation. Application of the first-order theory to rotating stars is perhaps not altogether out of the question.

The first-order terms in the equation satisfied by the ellipticity lead to Clairaut's differential equation, the numerical treatment of which for the Earth is greatly simplified by Radau's approximation. The second-order terms were included by Darwin and de Sitter, who found that they could be treated by a simple modification of the equation. For Jupiter and Saturn, however, Radau's approximation is seriously wrong and a differential equation has still to be solved numerically; and the equation satisfied by the fourth harmonic has so far been solved only by numerical integration, except in a few special cases.

The equations arise in the first place as integral equations, which are converted into differential equations and a pair of boundary conditions. It will appear in what follows that this is unnecessary, and that for the fourth harmonic at least it is easier to apply numerical methods directly to the integral equation.

In the theory of Darwin and de Sitter it was found convenient to define a modified ellipticity, the relation of which to the original ellipticity appears to

* Sir G. H. Darwin, *M.N.*, **60**, 82–124, 1899; *Sci. Papers*, **3**, 78–118.

† W. de Sitter, *B.A.N.*, **2**, 97–108, 1924.

‡ In addition, in (1), 128/105 should be 32/105; in (11), 32/3 should be 8/3. In the set of equations at the foot of the page, $S_1=\frac{5}{3}J$ should read $S_1=\frac{5}{3}J(1+\frac{3}{7}\epsilon)$. (11) is stated to agree with Darwin, but Darwin's form is correct.