COMP2300-COMP6300-ENGN2219
Computer Organization &
Program Execution

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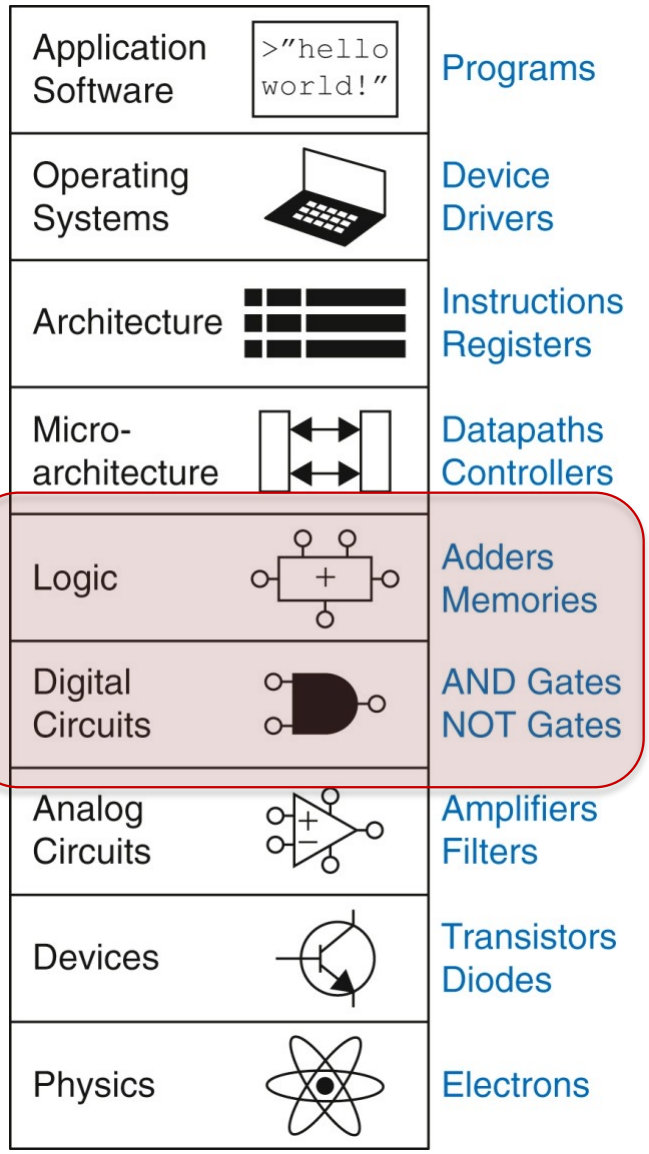


Broadening our horizon "one layer at a time"

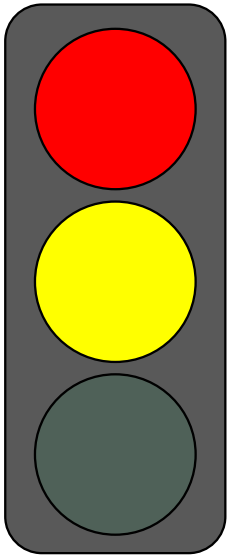
We Covered Combinational Blocks

- Computation
  - Adders
  - ALU
  - Comparator
- Control
  - Multiplexer
  - Decoder
  - Tri-state Buffer

- Standard form (SOP)
- Boolean equation to 2-level implementation

What will we learn this week?

- Circuits that can store information
  - Cross-coupled inverter
  - SR latch
  - D latch
  - D flip-flop
  - Register & Memory
- Synchronous sequential circuits
  - Finite state machines
  - State and Clock
- Synchronous vs. Asynchronous

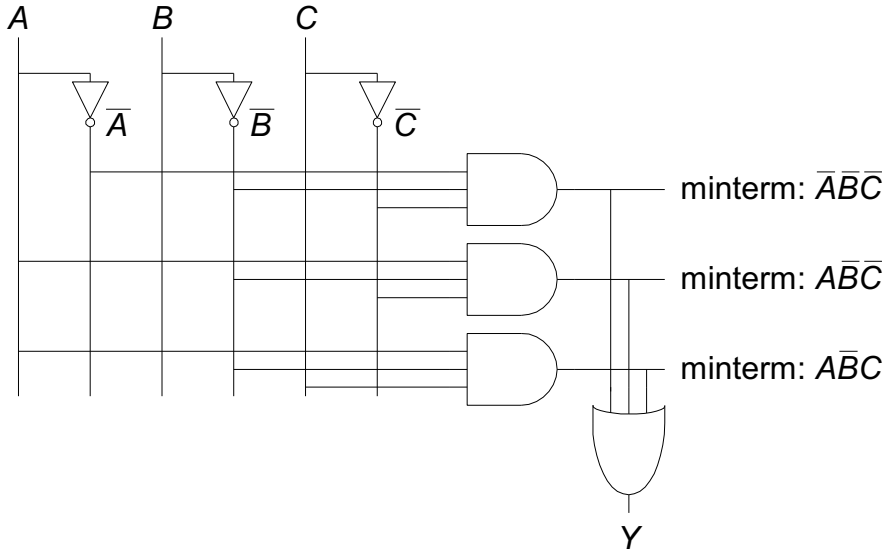


First, We Will Finish

Combinational Logic

Programmable Logic Array (PLA)

- SOP (sum-of-products) leads to two-level logic
- Example: **Y = A'B'C' + AB'C' + AB'C**

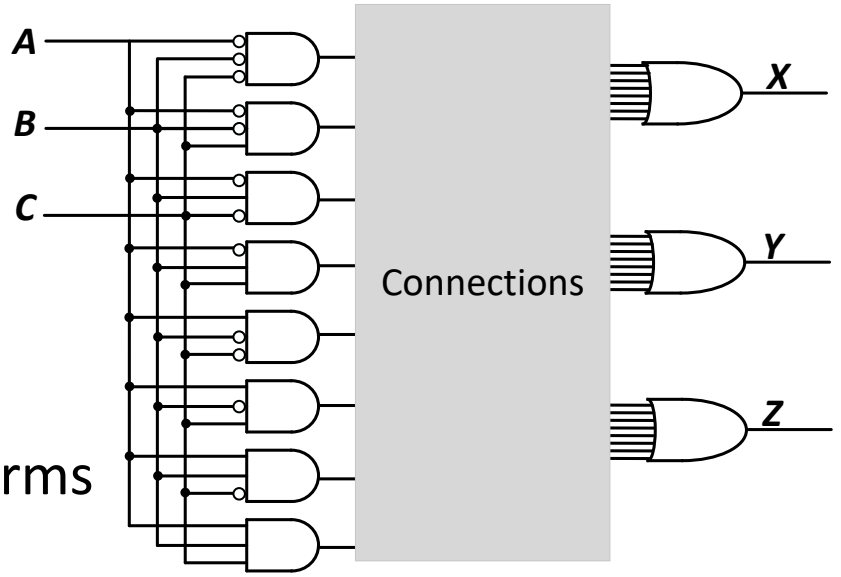


- We can use a PLA to implement **any** N-input M-output function (build many PLA circuits in bulk, and program later in lab to implement any logic function)

Section 5.6.1 of H&H

Programmable Logic Array (PLA)

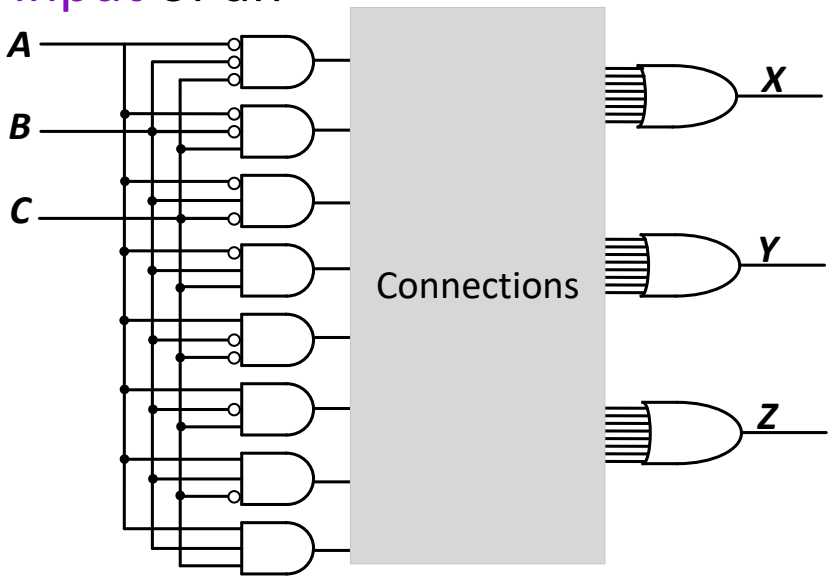
- Common building block for implementing any collection of logic functions
- An **array** of AND gates followed by an **array** of OR gates
- How many AND gates?
  - **Recall SOP:** the number of possible minterms
- How many OR gates?
  - The number of output columns in the truth table



Section 5.6.1 of H&H

Programmable Logic Array (PLA)

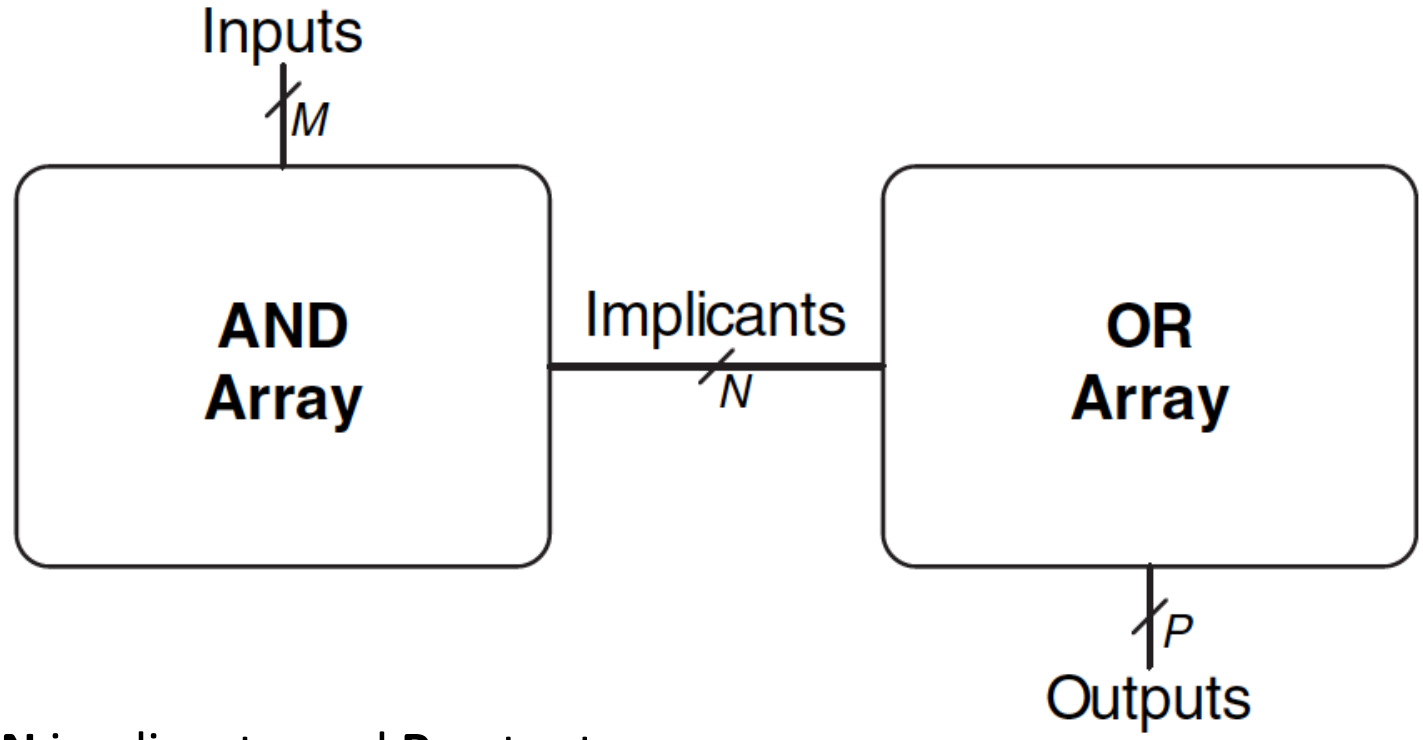
- **How do we implement a logic function?**
  - Connect the output of an AND gate to the input of an OR gate if the corresponding **minterm** is included in the SOP



- **Programming a PLA:** we program the connections from AND gate outputs to OR gate inputs to implement a desired logic function
- Programmable devices we have talked about: CPU/processor, FPGA, PLA

Section 5.6.1 of H&H

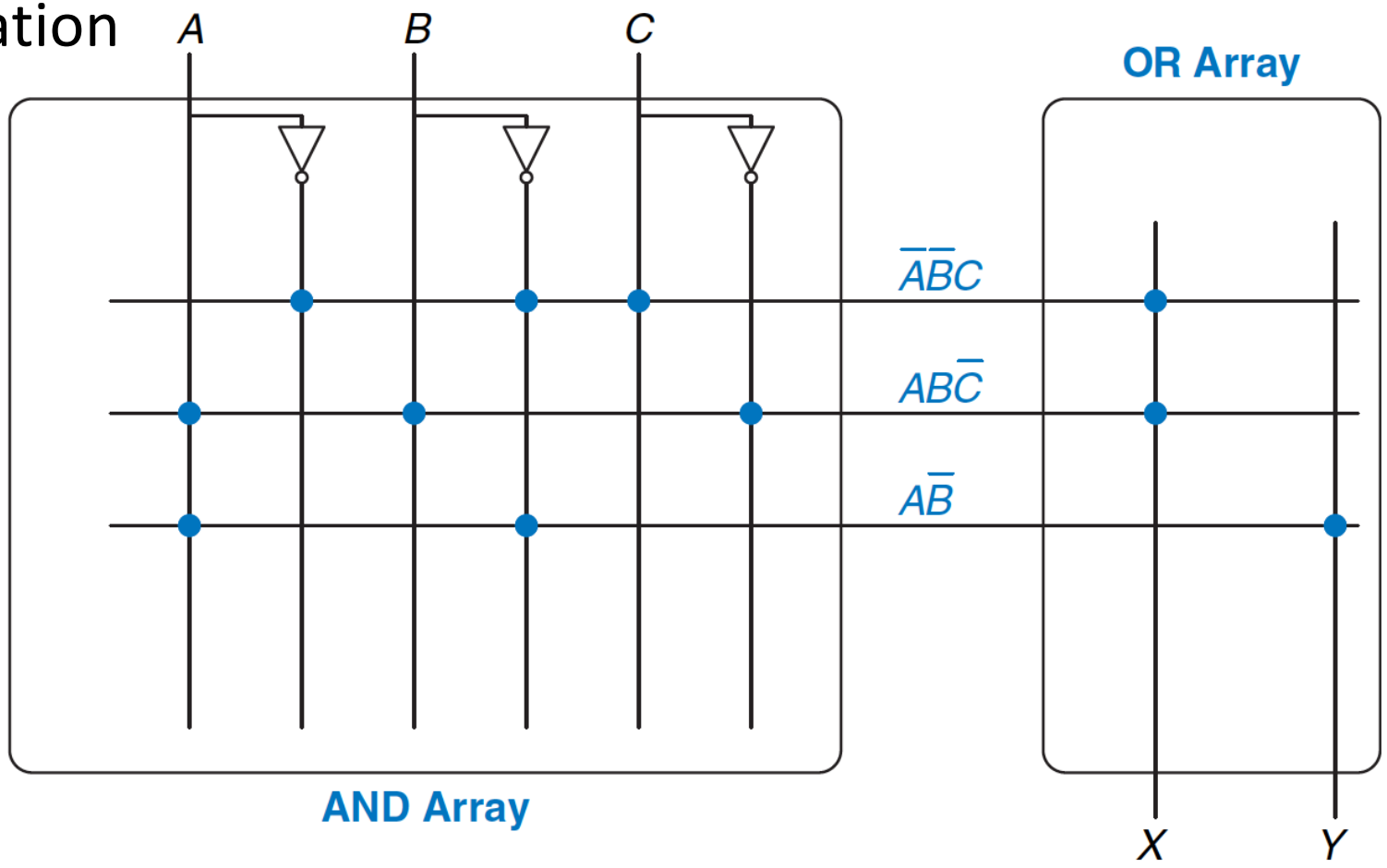
PLA Example (I)



- **M** inputs, **N** implicants, and **P** outputs
- Chips are manufactured in bulk with the same layout (**low cost**)
- Programmed **once** to implement the required function by **programming connections**

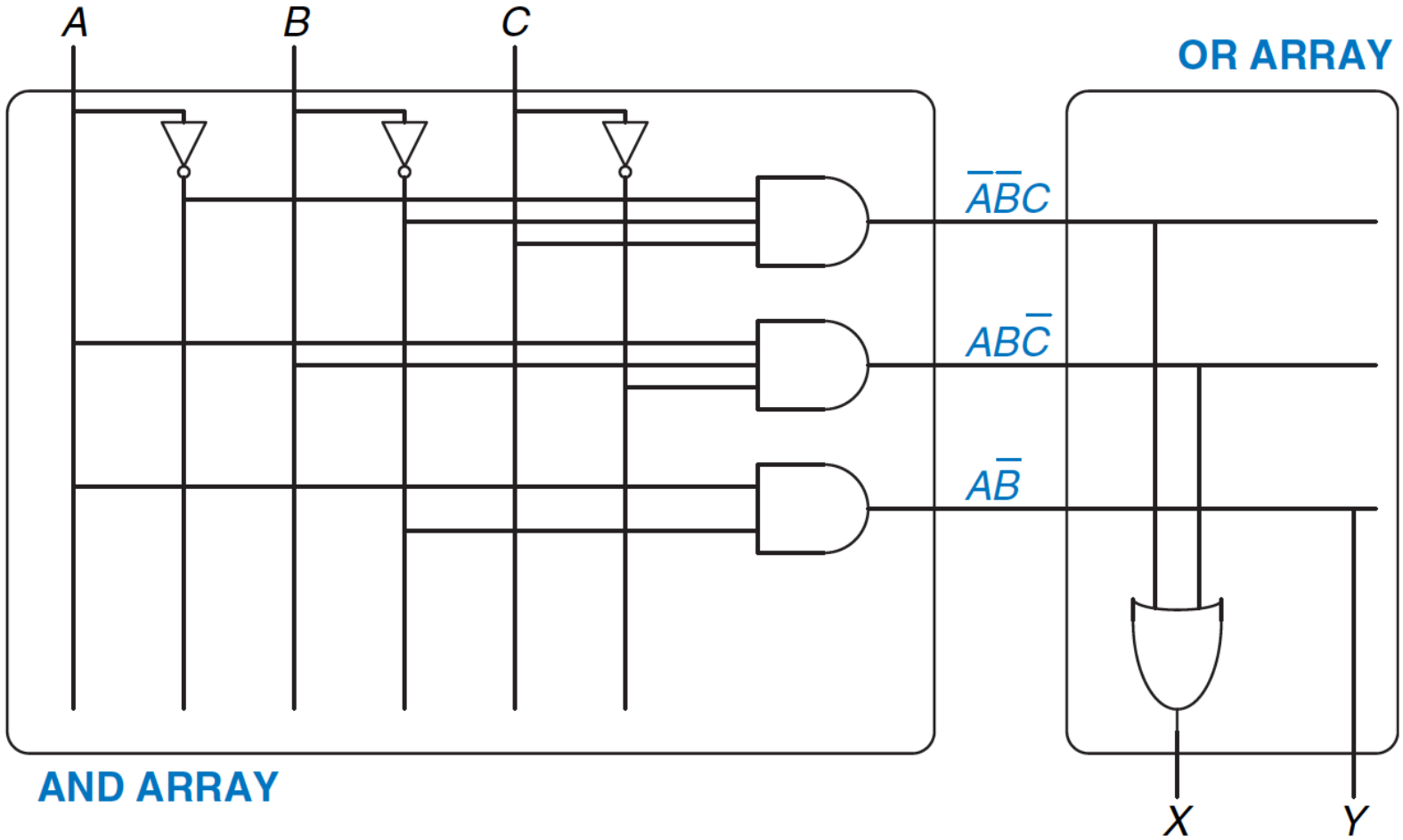
PLA Example (II)

Dot Notation



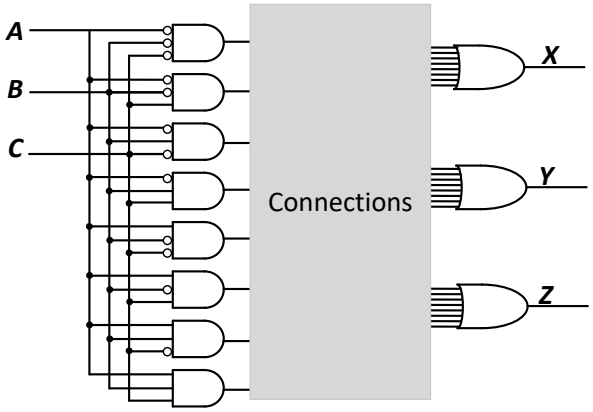
Section 5.6.1 of H&H

PLA Example (III)



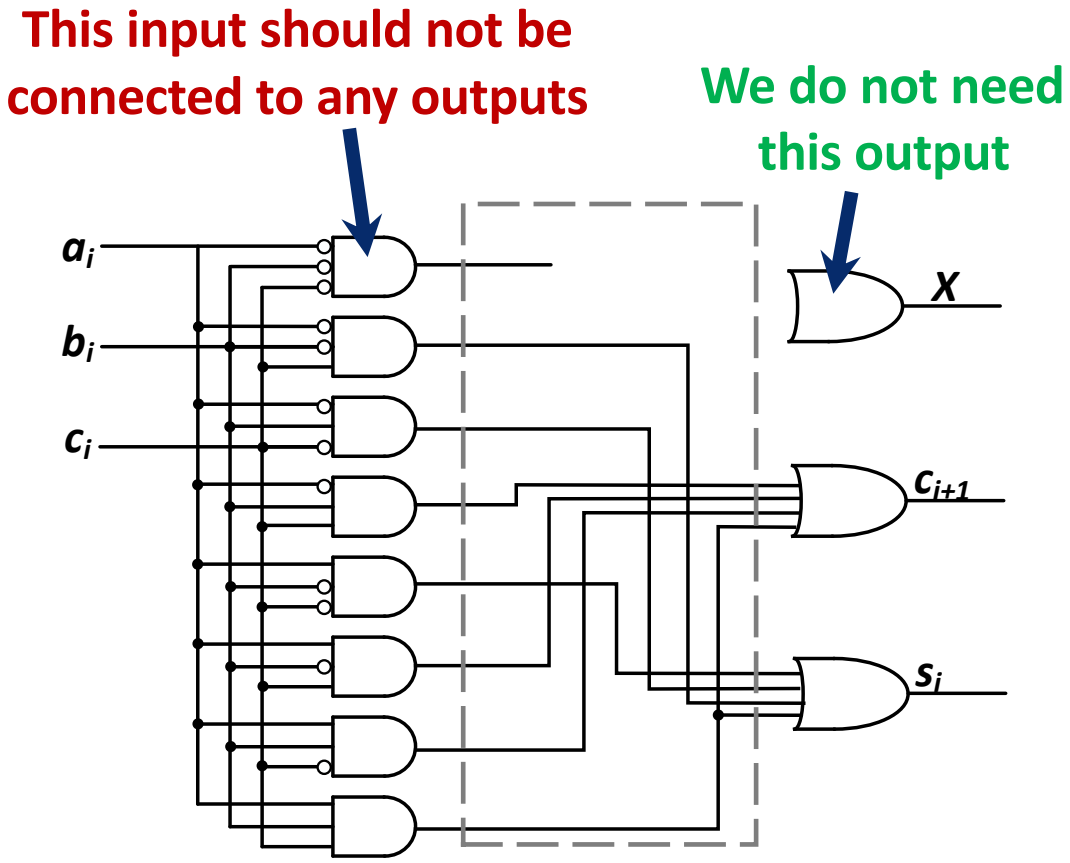
Implementation: Pick the **literals** & **implicants** by programming connections

Full Adder Implementation w/t PLA



Truth table of a full adder

a_i	b_i	$carry_i$	$carry_{i+1}$	S_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Implementation: Pick the **implicants** by programming connections

Lessons from PLA

- **Programmability:** Programmable devices incur a **cost**
  - Some logic in PLA is redundant if a subset of **minterms** are needed
  - On the other hand, PLAs can be **programmed** after **bulk** manufacturing which is their key **programmability advantage**

Implementing Combinational Logic (Recall this Slide)

- Steps in implementing combinational Logic
  - Initial specification (e.g., in English)
  - Construct the truth table
  - Derive the Boolean equation

  Functional specification

  - *Simplify the Boolean equation (use Boolean algebra)*
  - Implement the equation using logic gates

We Study Boolean Algebra for

Logic Minimization

Because we care about minimizing area, cost, logic complexity, energy, footprint, ...

Boolean Algebra (Logic Minimization)

- The **sum-of-products (SOP)** canonical form does not lead to the simplest logic gate implementation
- We can eliminate some **minterms** → Less # logic gates
- We can reduce the **# literals** in minterms → Smaller gates
- We use Boolean algebra to simplify Boolean equations
  - Similar in spirit to simplification in ordinary algebra except we are dealing with 0 and 1 **(much easier)**

Section 2.2 of H&H

Boolean Algebra

- Boolean algebra consists of
  - Axioms (**correct by definition**)
  - Theorems of one variable
  - Theorems of several variables

- Any theorem can be proved via the axioms
  - An axiom is the ground truth and cannot be proven wrong

- The **Principle of Duality**
  - If the symbols **0** and **1** and the operators **AND** and **OR** are interchanged, the statement will still be correct

Boolean Axioms

Number	Axiom	Dual	Name
A1	B = 0 if B ≠ 1	B = 1 if B ≠ 0	Binary Field
A2	$\overline{0} = 1$	$\overline{1} = 0$	NOT
A3	0 • 0 = 0	1 + 1 = 1	AND/OR
A4	1 • 1 = 1	0 + 0 = 0	AND/OR
A5	0 • 1 = 1 • 0 = 0	1 + 0 = 0 + 1 = 1	AND/OR

Dual: Replace: • with +

0 with 1

Boolean Theorems of One Variable

Number	Theorem	Dual	Name
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	$B + \overline{B} = 1$	Complements

Dual: Replace: • with +
0 with 1

Theorems: Several Variable

#	Theorem	Dual	Name
T6	$B \bullet C = C \bullet B$	$B+C = C+B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)\,(B+D)$	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

Warning: T8' (dual of T8) differs from traditional algebra: OR (+) distributes over AND (•)

Proving Theorems

- **Method 1: Perfect induction**
  - Proof by exhaustion: Check all possible input combinations
  - Two expressions are equal if they produce the same value for every possible input combination

- **Method 2: Use other theorems/axioms to simplify equations**
  - As in ordinary algebra, make one side of the equation look like the other side of the equation

Example: Perfect Induction

Number	Theorem	Name
T6	B•C = C•B	Commutativity

B	*C*	*BC*	*CB*
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

Example: Perfect Induction

Number	Theorem	Name
T9	B• (B+C) = B	Covering

B	*C*	*(B+C)*	*B(B+C)*
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Method 2: T9 (Covering)

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 2: Prove true using other axioms and theorems.

B•(B+C)	= B•B + B•C	T8: Distributivity
	= **B** + B•C	T3: Idempotency
	= B•(1 + C)	T8: Distributivity
	= B•(**1**)	T2: Null element
	= B	T1: Identity

Method 2: T10 (Combining)

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combining

Prove true using other axioms and theorems:

$B \bullet C + B \bullet \overline{C} = B \bullet (C + \overline{C})$ T8: Distributivity

$= B \bullet (\mathbf{1})$ T5': Complements

$= B$ T1: Identity

Simplifying Boolean Equations

- A basic principle for simplifying sum-of-product equations
  - **P**A + **P**A' = **P**
  - **P** is any implicant
  - Y = A'B + AB = B(A'+A) = B(1) = B

- **An equation is minimized if**
  - it uses the fewest number of implicants
  - if there are multiple equations with the same number of implicants, then the one with the fewest literals

Section 2.2 of H&H

Simplification Example – 1

$Y = AB + AB'$

Y = A T10: Combining

or

= $A(B + B')$ T8: Distributivity

= $A(1)$ T5': Complements

= A T1: Identity

Simplification Example – 2

$Y = A(AB + ABC)$

$= A(AB(1 + C))$	T8: Distributivity
$= A(AB(1))$	T2': Null Element
$= A(AB)$	T1: Identity
$= (AA)B$	T7: Associativity
$= AB$	T3: Idempotency

Simplification Example – 3A

Y = AB'C + ABC + A'BC

= AC(B + B') + A'BC T8: Distributivity

= AC(1) + A'BC T5: Complements

= AC + A'BC T1: Identity

- The two implicants AC and BC share the minterm ABC
- Are we stuck with simplifying only one of the minterm pairs?

Simplification Example – 3B

Y = AB'C + ABC + A'BC

= AB'C + **ABC + ABC** + A'BC	T3': Idempotency
= (AB'C+ABC) + (ABC+A'BC)	T7': Associativity
= AC + BC	T10: Combining

- The two implicants AC and BC are called prime implicants
- They cannot be combined with any other implicants in the equation to get a new implicant with fewer literals

Simplification Example – 4

Y = A’B’C’ + AB’C’ + AB’C

De Morgan's Theorem

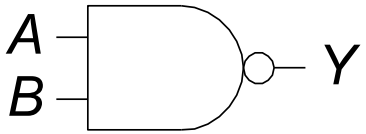
#	Theorem	Dual	Name
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} = \overline{B_0} + \overline{B_1} + \overline{B_2} \ldots$	$\overline{B_0 + B_1 + B_2 \ldots} = \overline{B_0} \bullet \overline{B_1} \bullet \overline{B_2} \ldots$	DeMorgan's Theorem

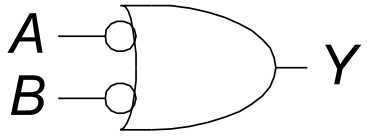
- The complement of the product is the sum of the complements
- **Dual:** The complement of the sum is the product of the complements

Section 2.2 of H&H

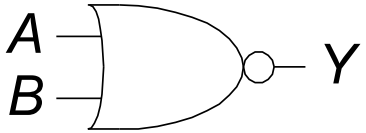
De Morgan's Theorem

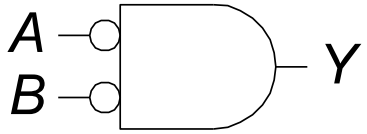
- $Y = \overline{AB} = \overline{A} + \overline{B}$





- $Y = \overline{A + B} = \overline{A} \bullet \overline{B}$



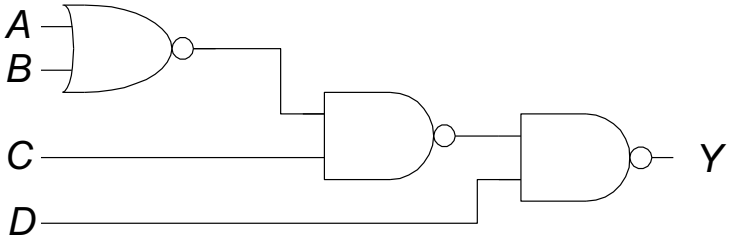


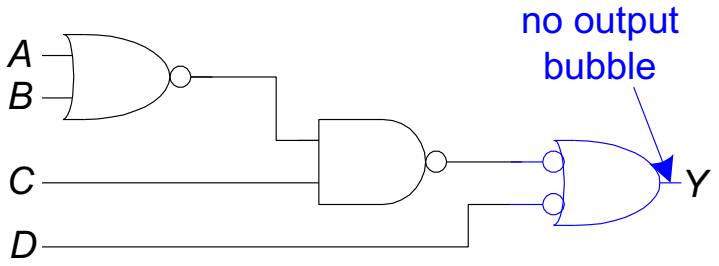
Bubble Pushing Rules

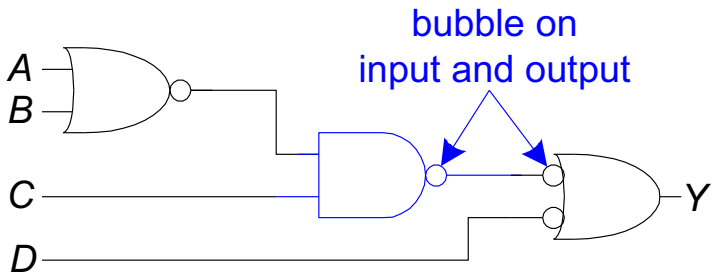
- Pushing bubbles backward/forward *changes the body of the gate* from **AND/OR** to **OR/AND**
- Pushing a bubble *from output back to inputs* put bubbles on all gate inputs
- Pushing *bubbles on all gate inputs forward* towards the output puts a bubble on the output

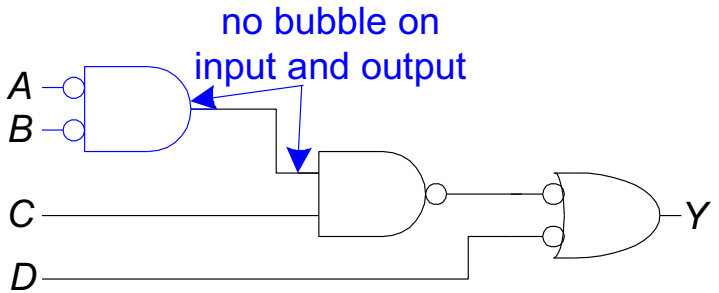
Section 2.5.1 and 2.5.2 of H&H

Bubble Pushing Example







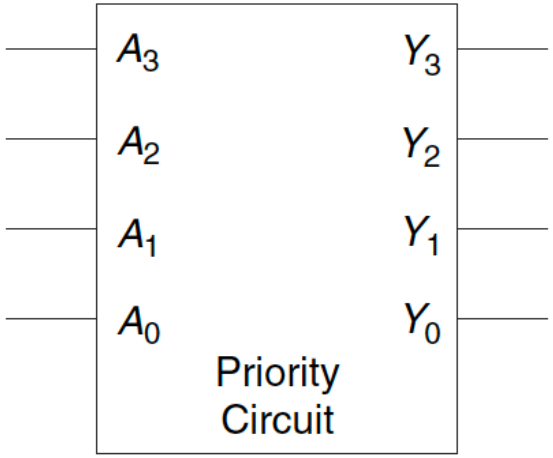


$Y = \overline{A}\overline{B}C + \overline{D}$

Priority Circuit

Example 2.7 of H&H

- **Priority circuit**
  - Inputs: “Requestors” with priority levels
  - Outputs: “Grant” signal for each requestor
  - **Example:** 4-bit priority circuit
- Room reservation system with priority levels
- Computer bus demanded by four CPUs



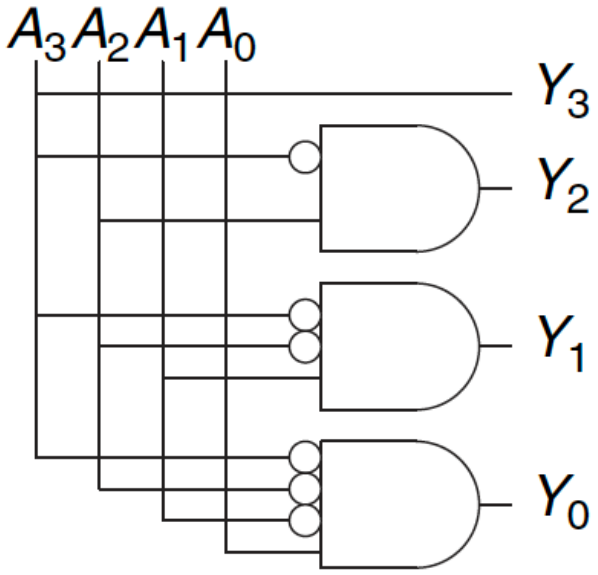
Requestors				Grant Signals			
A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

Priority Circuit

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

Figure 2.29 Priority circuit truth table with don't cares (X's)



X (Don't Care) means *We don't care what the value of this input is*

$Y_3 = A_3$

$Y_2 = A_3'A_2$

$Y_1 = A_3'A_2'A_1$

$Y_0 = A_3'A_2'A_1'A_0$

Optional Self-Study

- Product of Sums (POS)
  - Interesting but not entirely needed if you understand SOP well
  - Follows from Demorgan

Section 2.2.3 of H&H

Alternative Canonical Form: POS

- Product of Sums (**POS**)
- Find all the input combinations (maxterms) for which the output of the function is FALSE
- The function evaluates to FALSE (i.e., the output is 0) if any of the Sums (**maxterms**) causes the output to be 0
- Think: DeMorgan of SOP of $\overline{F}$

Alternative Canonical Form: POS

Product of Sums (POS)

product

$$F = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)$$

sums

This input

Each sum term represents one of the "zeros" of the function

$$F = \underbrace{(A + B + C)}_{0\ 0\ 0} \underbrace{(A + B + \overline{C})}_{0\ 0\ 1} \underbrace{(A + \overline{B} + C)}_{0\ 1\ 0}$$

Activates this term

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

For the given input, only the shaded sum term will equal 0

$$A + \overline{B} + C = 0 + \overline{1} + 0$$

Anything ANDed with 0 is 0; Output F will be 0

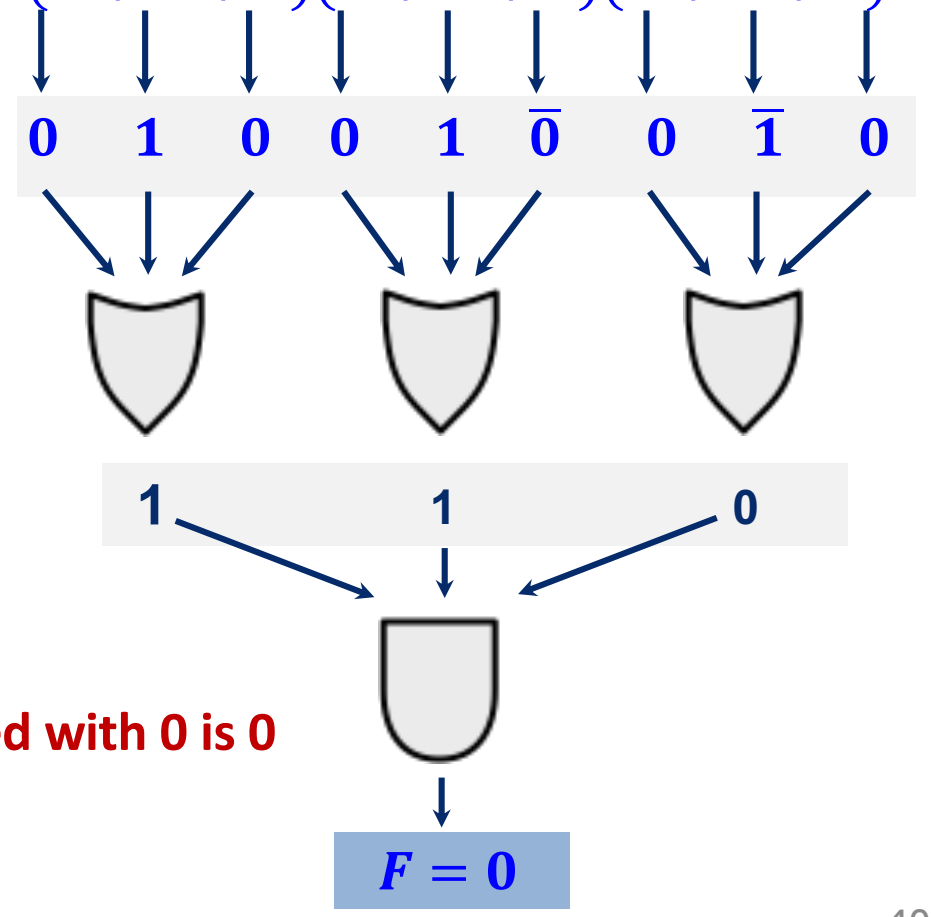
Consider A=0, B=1, C=0

0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Input

0 1 0 →

$$F = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)$$



Only one of the products will be 0, anything ANDed with 0 is 0

Therefore, the output is F = 0

Optional Self-Study

- More combinational circuits
  - Shifters
  - Rotators
  - Multiplication
  - Division
  - FPGAs

Section 5.2.5, 5.2.6, 5.2.7, 5.6.2 of H&H

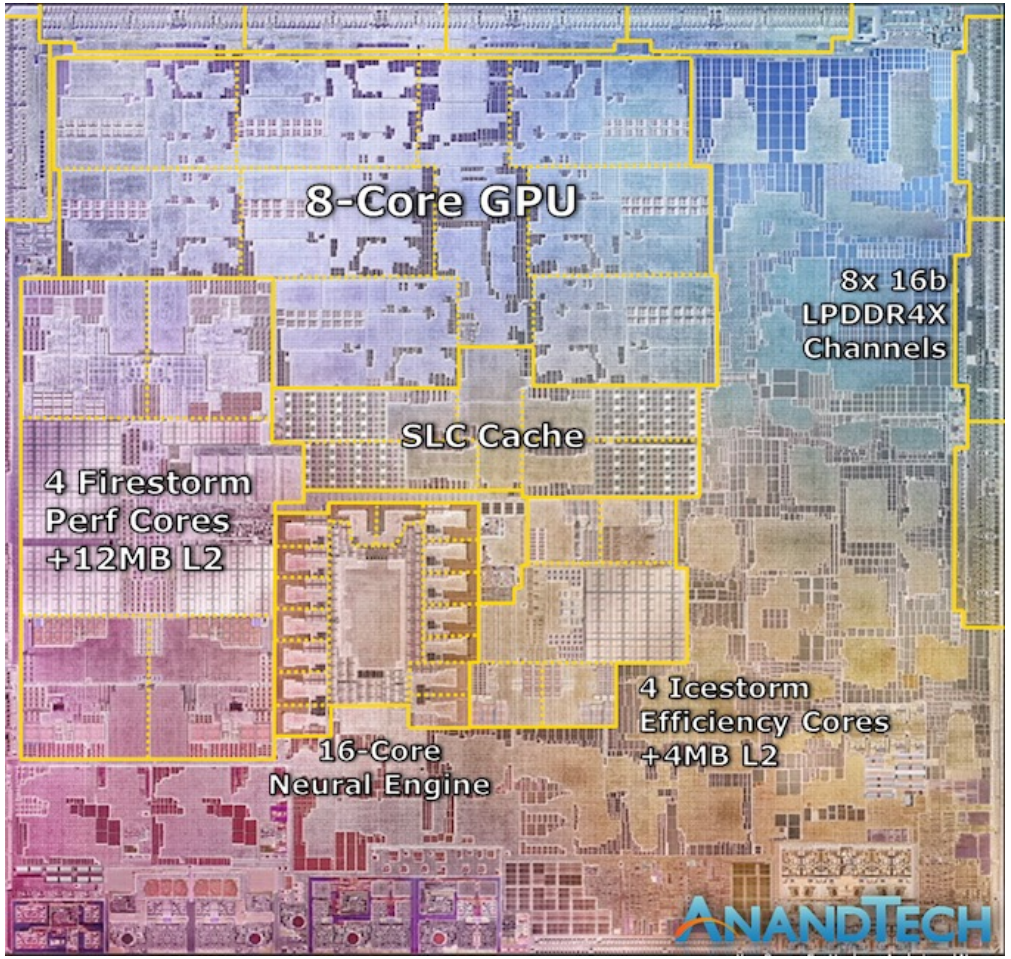
What We Have Done So Far

- Building blocks of modern computers
  - Transistors
  - Logic gates
- Combinational logic fundamentals
- Boolean algebra
- Using Boolean algebra to implement combinational circuits
- Basic combinational logic blocks
- Simplifying combinational logic circuits

Circuits that

Store Information

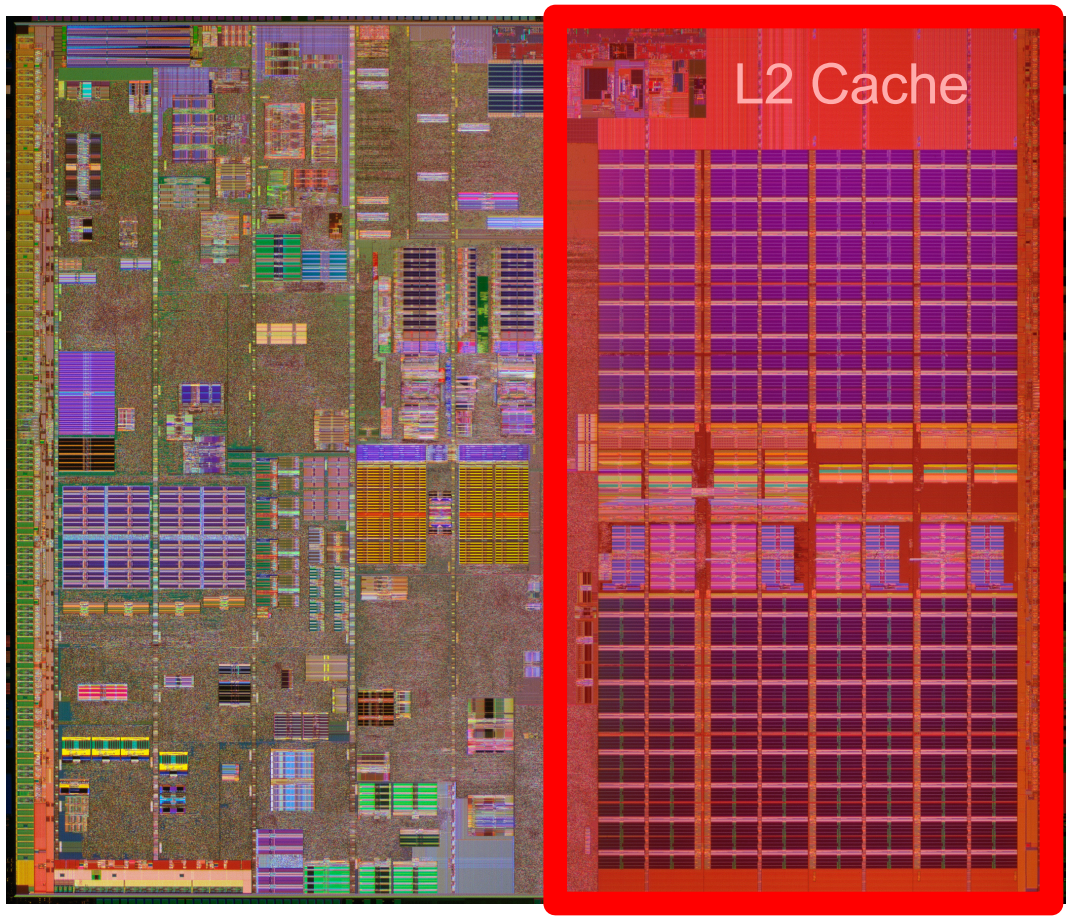
All Computers Need Memory to Work



Apple M1, 2021

Source: https://www.anandtech.com/show/16252/mac-mini-apple-m1-tested

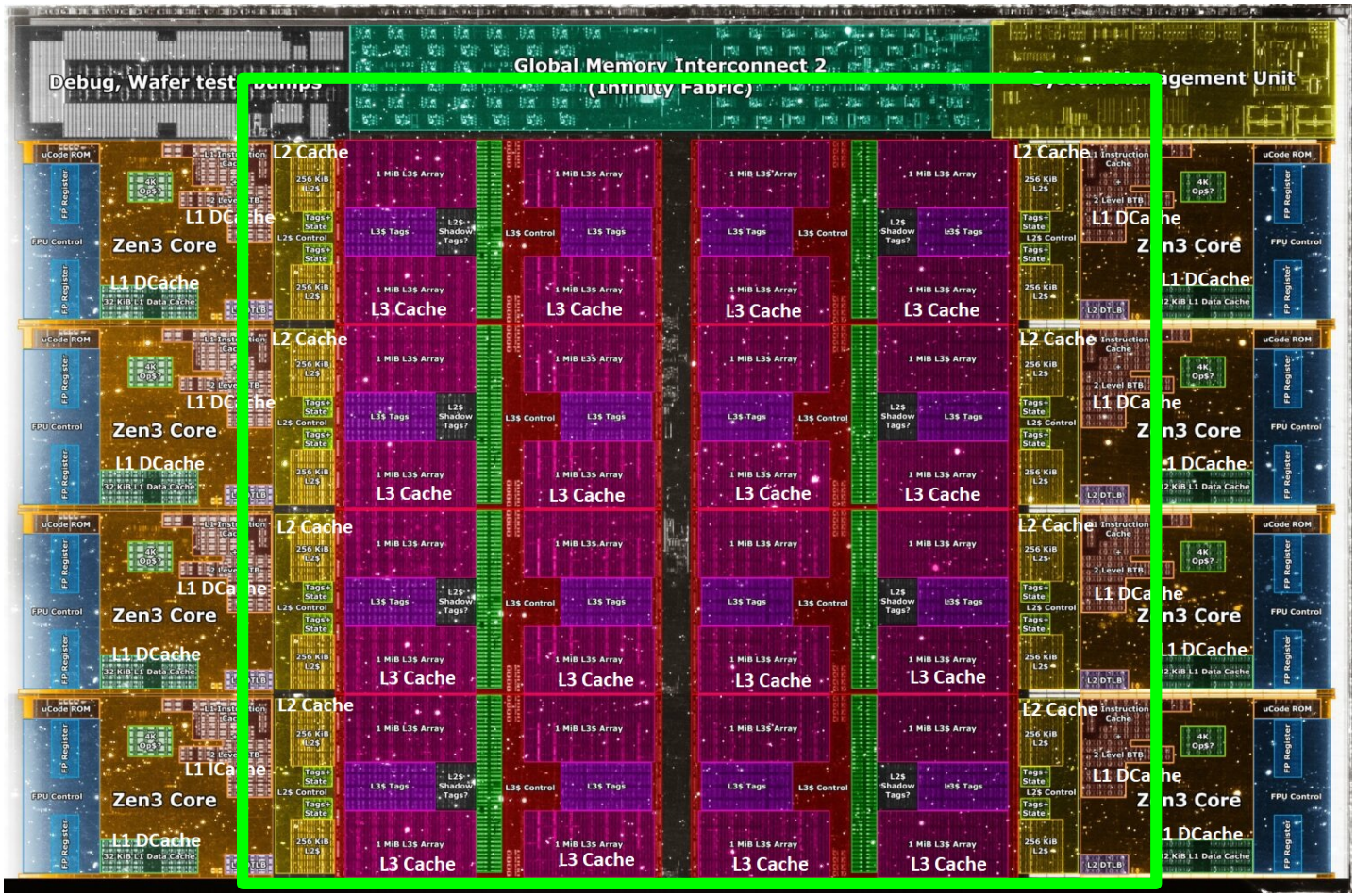
Large Portion of a System is Memory



https://download.intel.com/newsroom/kits/40thanniversary/gallery/images/Pentium_4_6xx-die.jpg

Intel Pentium 4, 2000

Large Portion of a System is Memory



Core Count:

8 cores/16 threads

L1 Caches:

32 KB per core

L2 Caches:

512 KB per core

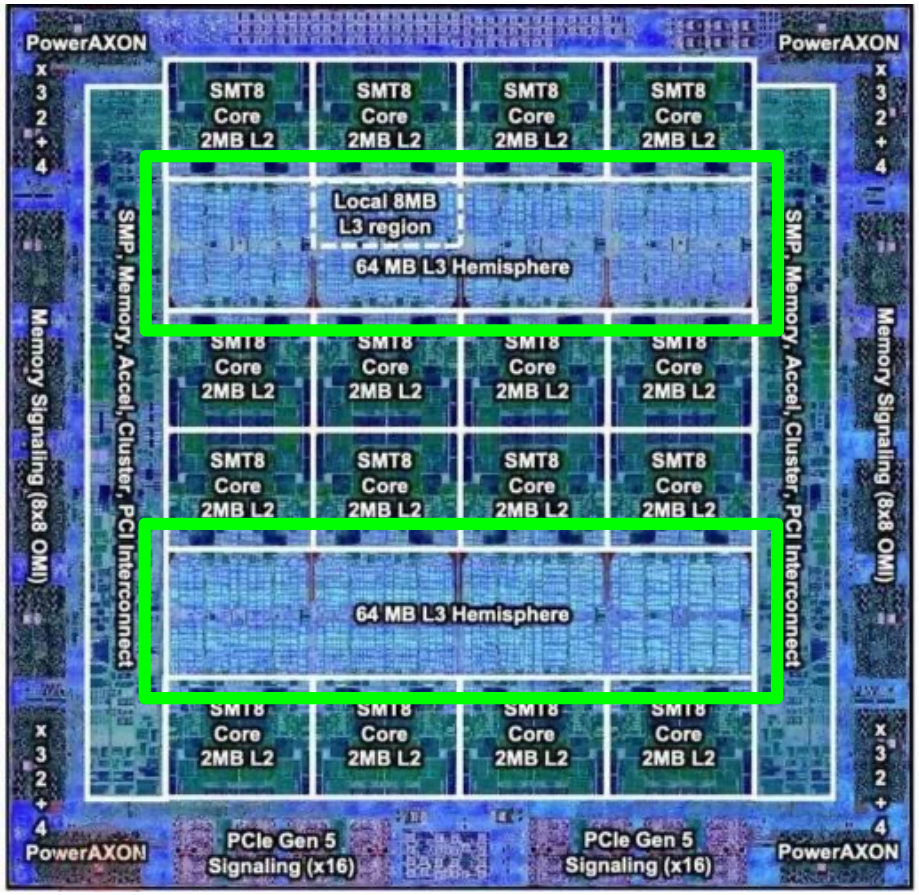
L3 Cache:

32 MB shared

AMD Ryzen 5000, 2020

https://wccftech.com/amd-ryzen-5000-zen-3-vermeer-undressed-high-res-die-shots-close-ups-pictured-detailed/

Large Portion of a System is Memory



IBM POWER10, 2020

Cores:

15-16 cores,
8 threads/core

L2 Caches:

2 MB per core

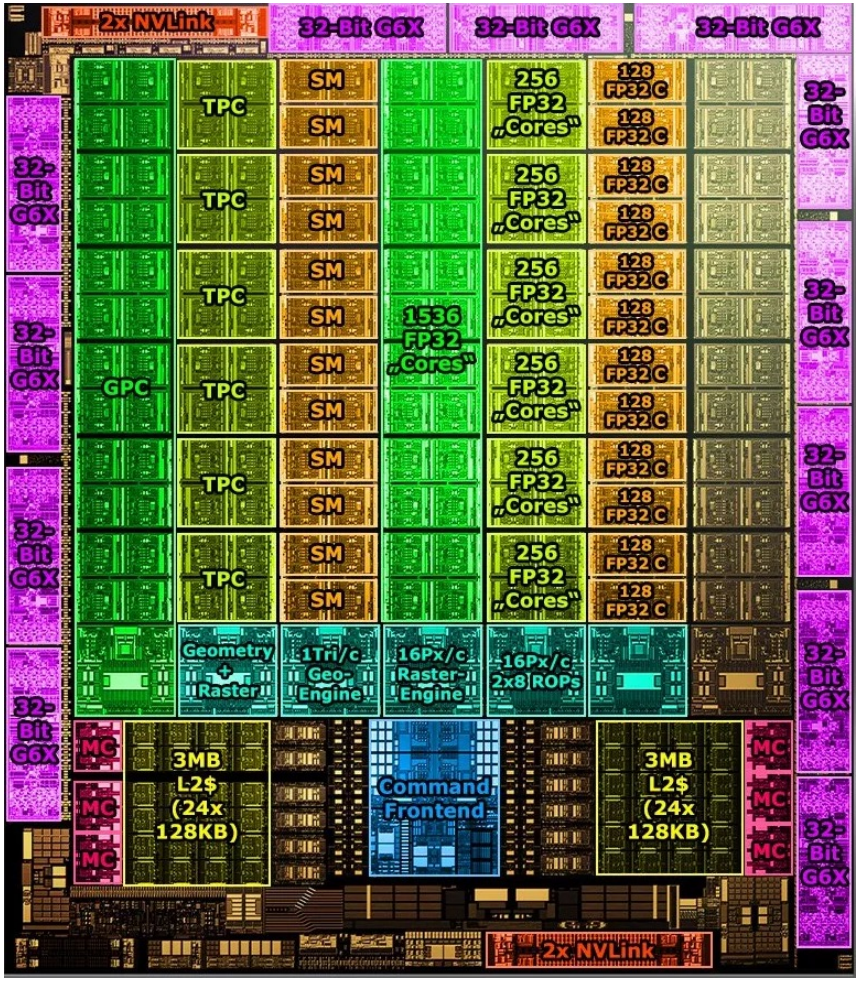
L3 Cache:

120 MB shared

https://www.it-techblog.de/ibm-power10-prozessor-mehr-speicher-mehr-tempo-mehr-sicherheit/09/2020/

Large Portion of a System is Memory

Nvidia Ampere, 2020



Cores:
128 Streaming Multiprocessors

L1 Cache or Scratchpad:
192KB per SM
Can be used as L1 Cache and/or Scratchpad

L2 Cache:
40 MB shared

https://www.tomshardware.com/news/infrared-photographer-photos-nvidia-ga102-ampere-silicon

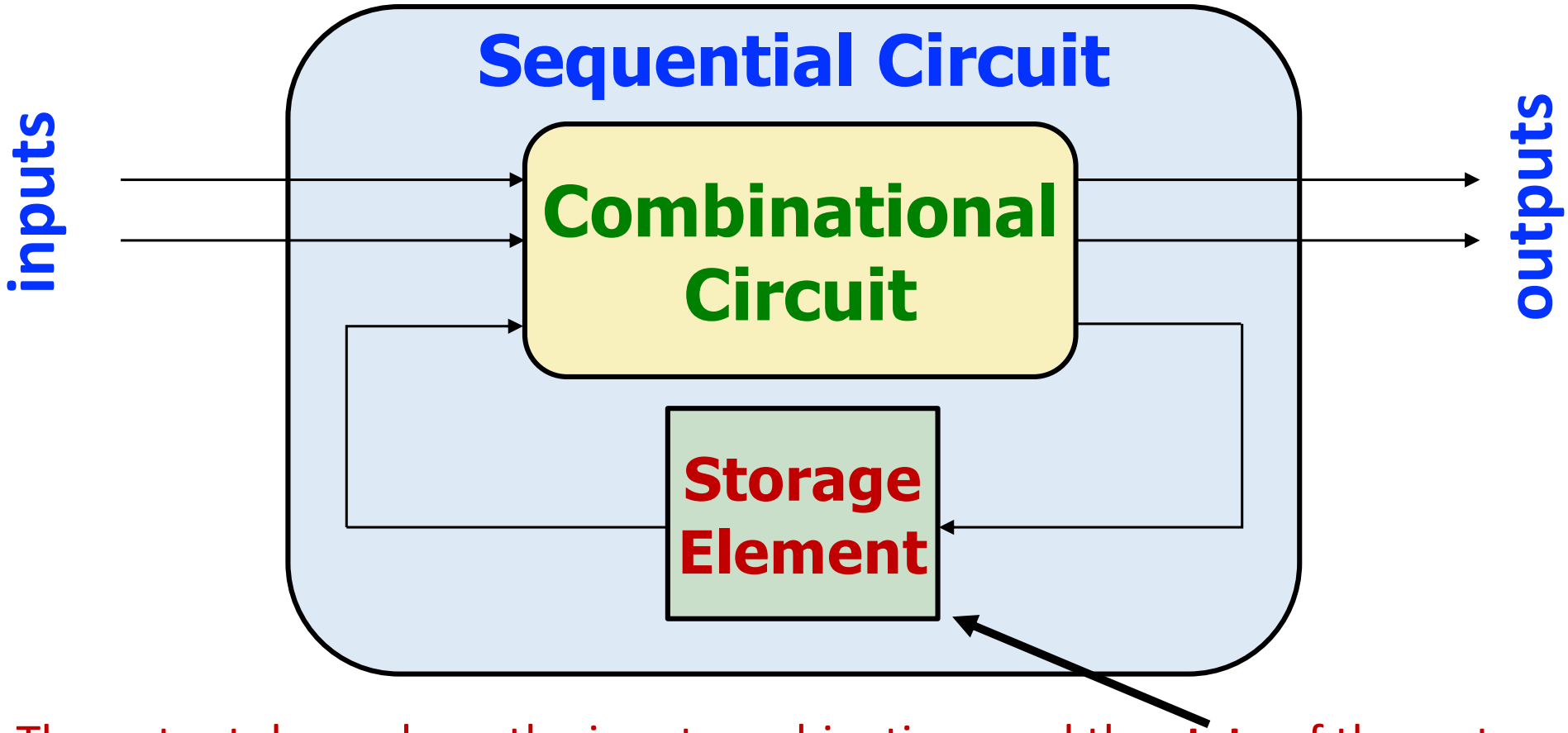
Motivation

- **Combination**al circuit output depends **only** on current input
  - Output only depends on the input **combination**s

- We want circuits that produce output depending on **current** and **past** input values – circuits with **memory**

- **How can we design a circuit that stores information?**

Sequential Circuit



The output depends on the input combinations and the **state** of the system

What is State?

- What is **state**?
  - A set of bits that summarize the past behavior of the system
  - The set of bits are called state variables

- The state of a system *informs us everything about the past we need to know to predict the future*

- The output of sequential logic depends on both the current input values and the *state* of the system

Example of State

- Consider a **controller** for a traffic light

- To set the next state, the controller needs to remember the current state

- The past input values (e.g., signals from traffic sensors) are summarized by the "current" state stored as a **2-bit** value

Another Example

- We remember the meaning of the words in time **t** since
  - we remembered them in time **t – 1**
    - all the way back to the point of time
      - when we first committed them to memory

Capturing State

- To remember or to store the **state** of the system (e.g., current state of traffic light), we need a logic element with memory

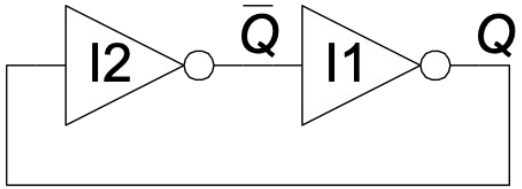
- **How can we capture data and persist it until the next input?**

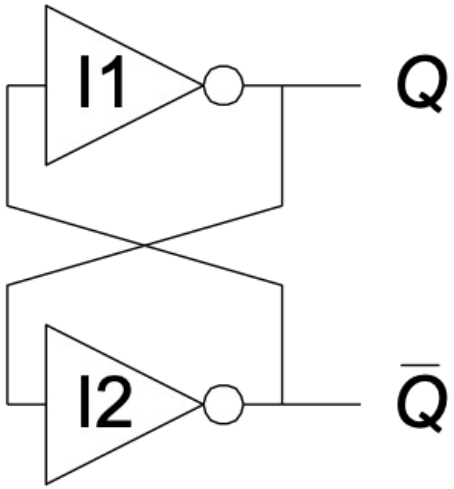
Capturing and Storing One Bit

- One bit of information represents two possible states
- To store one bit, we need
  - An element with two stable states (**bistable** element)
  - The ability to change the state
- First, we will find the bistable element
  - And then focus on the *ability to change state*

Bistable element

- **The fundamental building block of memory is a bistable element with two stable states**





Cross-coupled inverters: A pair of inverters connected in a loop

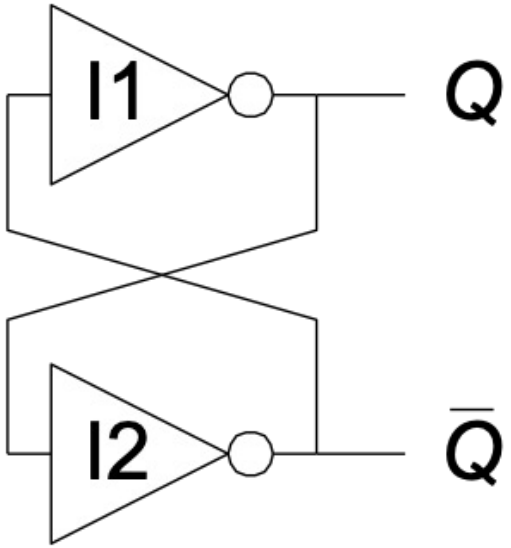
Section 3.2 of H&H

Analysis of Bistable Element

- **How should we analyze circuits with cyclic paths?**
  - When the circuit is switched on, **Q** is either 0 or 1 (**scenarios**)
  - **Show:** Output is stable (consequence – A)
  - **Show: Q** and **Q'** are complements of each other (consequence – B)

- Scenario # 1: **Q** = 0 (FALSE)
  - I2 receives 0, **Q'** = 1: **B** is satisfied
  - **Q'** = 1, **Q** = 0, **A** is satisfied
  - Consistent with original assumption

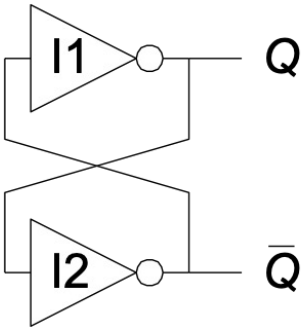
- Scenario # 2: **Q** = 1 (TRUE)
  - I2 receives 1, **Q'** = 0: **B** is satisfied
  - **Q'** = 0, **Q** = 1, **A** is satisfied



Section 3.2 of H&H

Bistable Element: Observations

- **Bistable element has two stable states**
  - Stores one bit of information



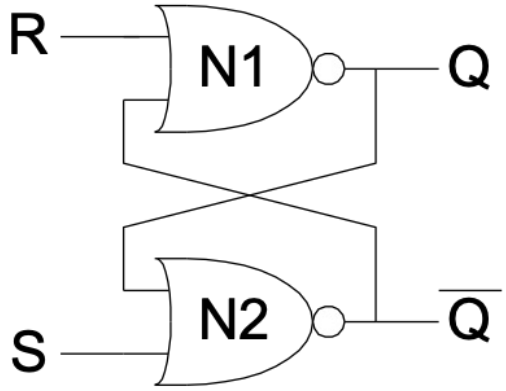
- **Q** reveals about past, necessary to explain the future
  - If **Q** = 0, it will remain 0 forever
  - If **Q** = 1, it will remain 1 forever

- The (initial) state of the bistable element is unpredictable (powered on)
  - Bistable element is *not practical* because the user lacks inputs to control its state
  - **We need something else!**

Section 3.2 of H&H

SR Latch

- Two cross-coupled NOR gates
- The state can be *controlled* with S/R inputs
  - S = Set to 1
  - R = Reset to 0



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

- NOR gate revision
  - When both inputs are 0, the output is 1
  - If any of the inputs is 1, the output is 0

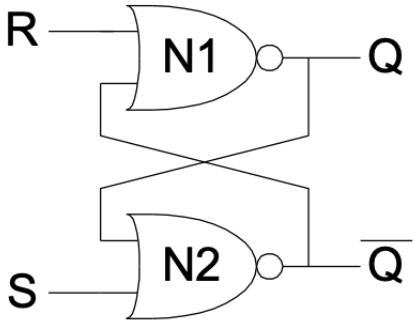
SR Latch: Analysis

- SR latch has inputs (unlike the *cross-coupled* inverter)
- Four scenarios in the truth table (Sim = Simultaneous)

Scenario	S	R	Q	Q’
Sim-0	0	0		
Reset	0	1		
Set	1	0		
Sim-1	1	1		

Section 3.2.1 of H&H

Whiteboard: SR Latch



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

S	R	Q
0	0	
0	1	
1	0	
1	1	

Section 3.2.1 of H&H

SR Latch: Analysis

- Scenario # 1 (**Reset**): **R** = 1, **S** = 0
  - N1 sees at least one TRUE (1) input
    - **Q** = FALSE (0)
  - N2 sees both Q and S FALSE
    - **Q'** = TRUE (1)

- Scenario # 2 (**Set**): **R** = 0, **S** = 1
  - N1 sees 0 and **Q'**
    - What is **Q'**?
  - N2 sees at least one TRUE input
    - **Q'** = FALSE (0)
  - Revisit N1 (**R** = 0 and **Q'** = 0)
    - **Q** = TRUE (1)

Section 3.2.1 of H&H

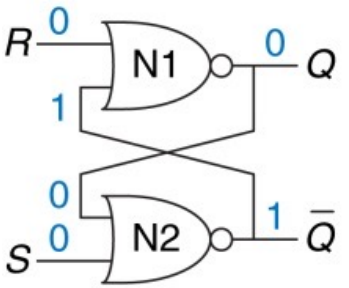
SR Latch: Analysis

- Scenario # 3 (**Sim-1**): **R** = 1, **S** = 1
  - N1 sees at least one TRUE (1) input
    - **Q** = FALSE (0)
  - N2 sees at least one TRUE (1) input
    - **Q'** = FALSE (0)

- Scenario # 4 (**Sim-0**): **R** = 0, **S** = 0
  - N1 sees 0 and **Q'**
    - What is **Q'**?
  - N2 sees 0 and **Q**
    - What is **Q?**
  - **We are stuck!**
    - Wait: remember the cross-coupled inverter? **Q** can be 0 or 1 ☺

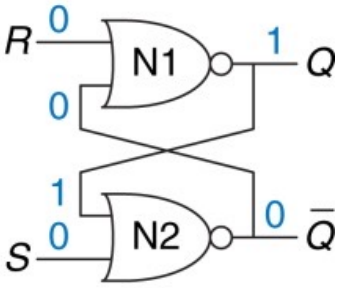
SR Latch: Analysis

- Scenario # 4-A (**Sim-0**): **R** = 0, **S** = 0, **Q** = 0
  - N2 sees **S** = 0 and **Q** = 0
    - **Q'** = 1
  - N1 sees one TRUE (1) input
    - **Q** = 0 (hindsight: **Q** is indeed 0 as assumed)



(a)

- Scenario # 4-B (**Sim-0**): **R** = 0, **S** = 0, **Q** = 1
  - N2 sees **Q** = 1
    - **Q'** = FALSE
  - N1 receives two FALSE inputs
    - **Q** = 1 (hindsight: **Q** is indeed 1 as assumed)



(b)

SR Latch: Analysis Summary

- SR latch has inputs (unlike the cross-coupled inverters)
- Four scenarios in the truth table (Sim = Simultaneous)

Scenario	S	R	Q	Q’
Sim-0	0	0	Q_{prev}	Q'_{prev}
Reset	0	1	0	1
Set	1	0	1	0
Sim-1	1	1	0	0

Section 3.2.1 of H&H

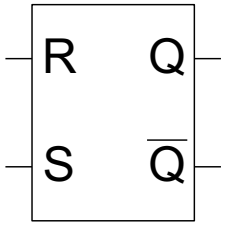
SR Latch: Observations

- **SR latch is a bistable element but it's state can be controlled**
  - To set a bit means to make it TRUE. To reset is to make it FALSE

- **Q** accounts for the entire history of past inputs
  - All *prior set/reset patterns* are irrelevant
  - The *most recent set/reset* event predicts the future behavior of the **SR** latch

- Asserting both **set/reset** to 1 does not make sense
  - Neither intuitively nor physically
  - The circuit outputs 0 on **Q** and **Q'** which is inconsistent
  - **We need something else!**

SR Latch Symbol



D Latch

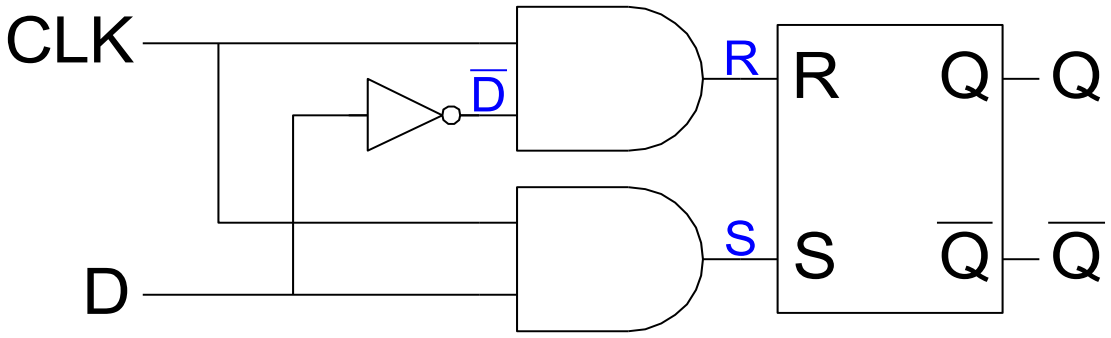
- **Two drawbacks of SR latch**
  - Strange behavior when **S** = 1 and **R** = 1
  - **S** and **R** inputs serve two roles: *what* the state is and *when* the state changes

- Designing sequential circuits is **easier** when we have control over *when* the state changes

- The D latch **overcomes** the two drawbacks
  - A data input (**D**) ***controls*** what the next state should be
  - The clock input (**CLK**) controls *when* the state should change

Section 3.2.2 of H&H

D Latch

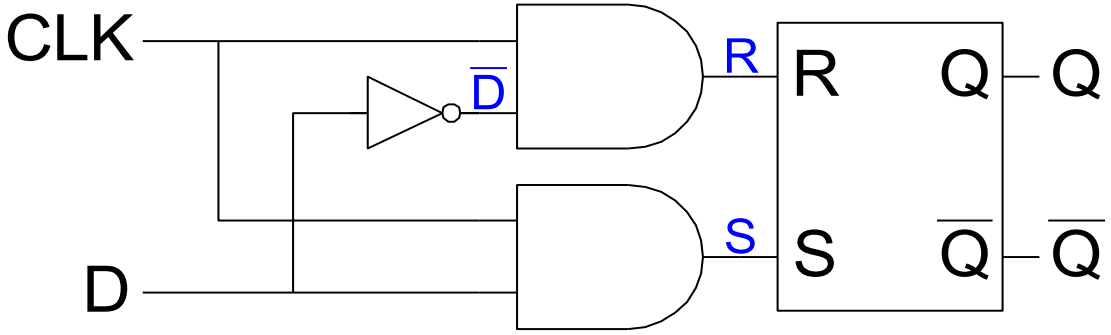


- Scenario # 1: **CLK** = 0, **S** = 0, **R** = 0, **D** = X
  - The value of **D** is irrelevant
  - **Q** = $\mathbf{Q_{prev}}$ (remember the old value)
- Scenario # 2: **CLK** = 1, **D** = 0, **S** = 0, **R** = 1
  - The latch is **reset**
- Scenario # 3: **CLK** = 1, **D** = 1, **S** = 1, **R** = 0
  - The latch is **set**

Latch is opaque (blocks new data from flowing to Q)

Latch is transparent (acts like a buffer)

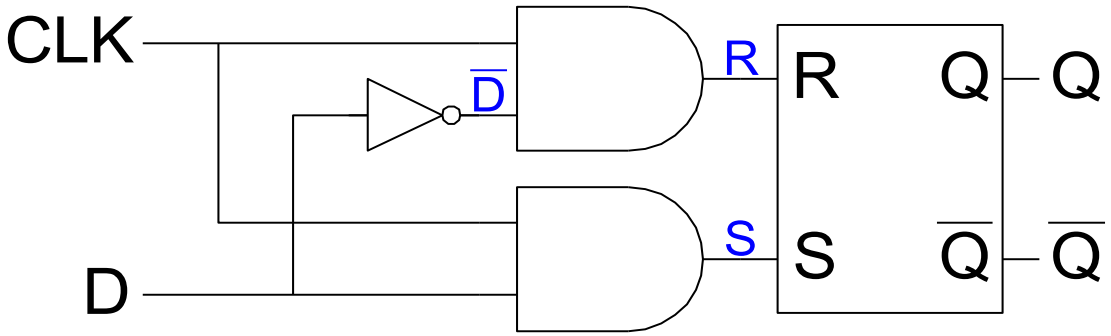
Whiteboard: D Latch



S	R	Q
0	0	Q_{prev}
0	1	0
1	0	1
1	1	0

Section 3.2.2 of H&H

D Latch: Truth Table

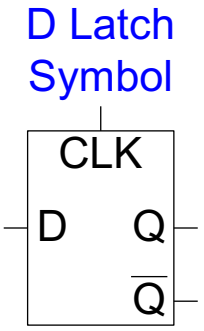


Scenario	CLK	D	Q	Q'
Opaque	0	X	Q_{prev}	Q'_{prev}
Transparent/0	1	0	0	1
Transparent/1	1	1	1	0

Section 3.2.2 of H&H

D Latch: Observations

- **D latch is a level-triggered or a level-sensitive circuit**
  - Reacts to the level (0 or 1) of the CLK input
- D latch avoids the **awkward** case of both **S/R** asserted
- D latch **changes** its state **continuously** when **CLK** = 1
- Designing **correct & efficient** sequential circuits is easier when
  - the state changes only at a **specific instant** in time instead of **changing continuously**
  - **We need something else!**



Summary

- **Cross-coupled inverter**
  - Two stable states *but* no inputs to control the state
- **SR Latch**
  - Two control inputs: **S** for **S**et (1) and **R** for **R**eset (0)
  - Awkward when **S** = 1 and **R** = 1
  - The state changes instantly (*need greater control over when it changes*)
- **D Latch**
  - One **D**ata input (**D**) and one control input (**CLK**)
  - Level-sensitive: **CLK** = 1 (**Transparent**), **CLK** = 0 (**Opaque**)
  - Output (**Q**) changes continuously when **CLK** = 1

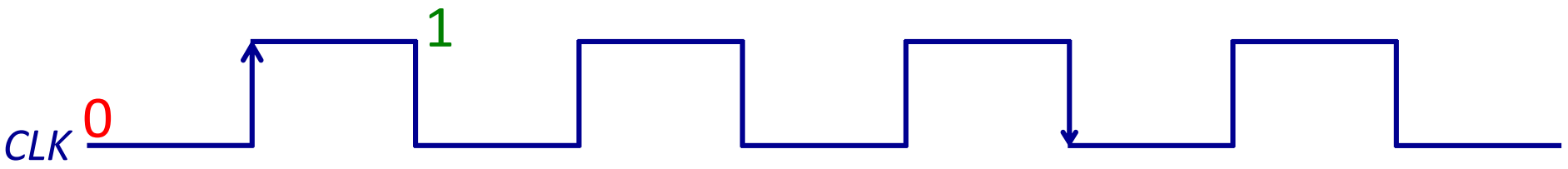
Next: Flip-Flop

First, We Will Learn to Handle the Progression of Time

Representing Time

- Physicist and philosopher's view
  - Relentless arrow continuously progressing forward
  - Changes in the world can be infinitesimally small

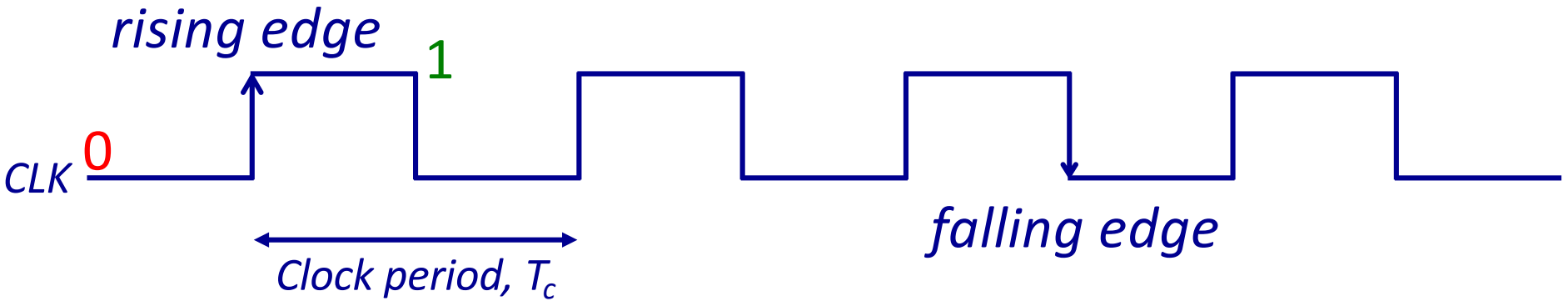
- **Too mysterious and deep for computer scientists**
  - We prefer a discrete representation
  - Break time into fixed-length intervals called cycles
  - cycle 1, cycle 2, cycle 3, and so on .... **Cycles are indivisible and atomic**



Changes in the world occur only during cycle transitions; within cycles, the world stands still

Time in Digital Systems

- Periodic **square wave** (**clock**) generated by a special circuit
- Used to *synchronize* state updates everywhere in the system



Frequency = $1/T_c$

Changes in the world occur only during cycle transitions; within cycles, the world stands still

D Flip-Flop

- **Problem with D Latch:** The output changes *continuously* when **CLK** = 1
  - This behavior leads to timing bugs and circuits that are difficult to analyze

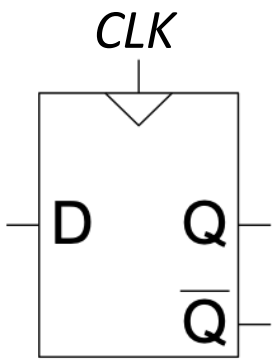
- D **flip-flop** is an edge-triggered circuit
  - Sets its state to the data input when **CLK** changes from 0 to 1
  - At all other times, the D flip-flop simply remembers its state

Flip-flop samples the input at the rising edge of the clock

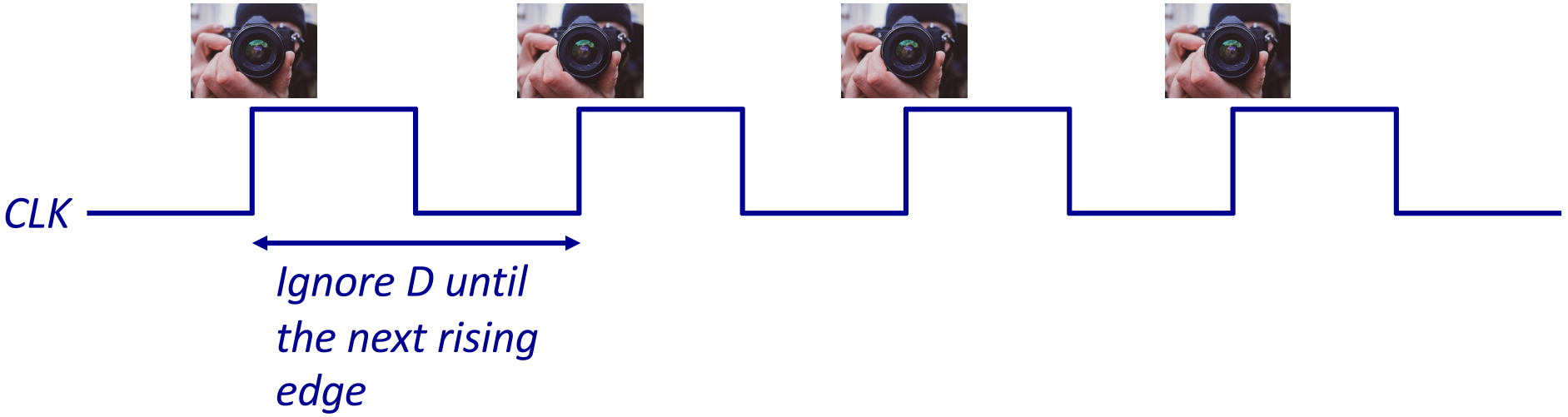
Section 3.2.3 of H&H

Analogy: Photography

- Input: **D**, Output/State: **Q**
- **CLK** = Clock (periodic square wave)

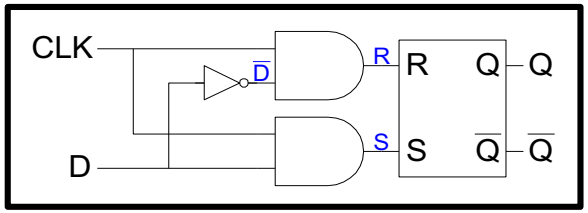


Sample the input just at the rising edge (copy D to Q)

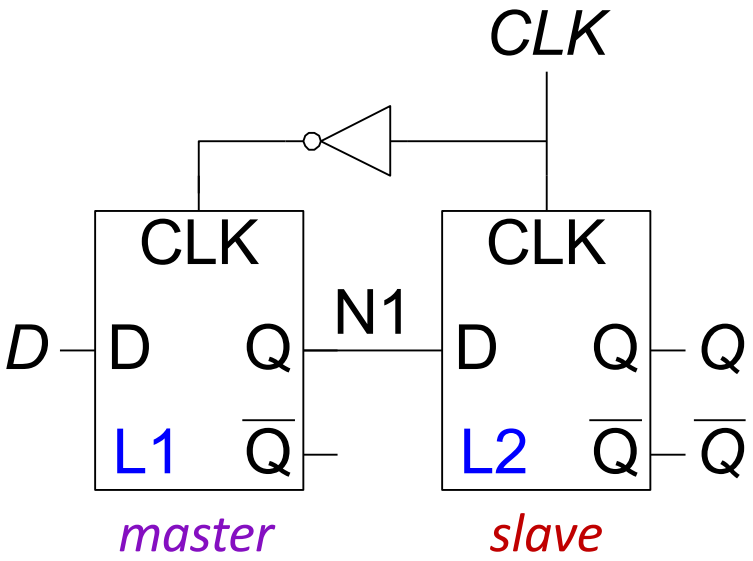


D Flip-Flop

- Two back-to-back D latches controlled by complementary clocks
- The L1 latch is the master and L2 is the slave latch
  - Remember that, when **CLK** = 0, D latch is *opaque*
  - And, when **CLK** = 1, D latch is *transparent*

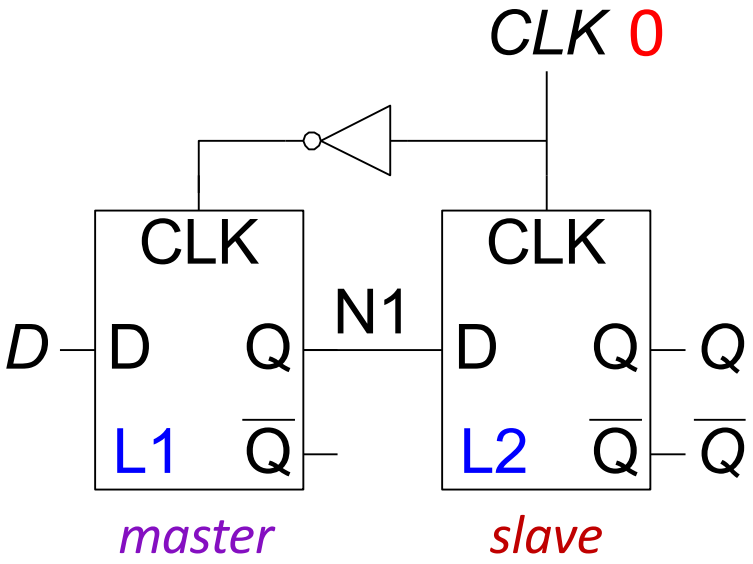


D Latch



D Flip-Flop: Analysis

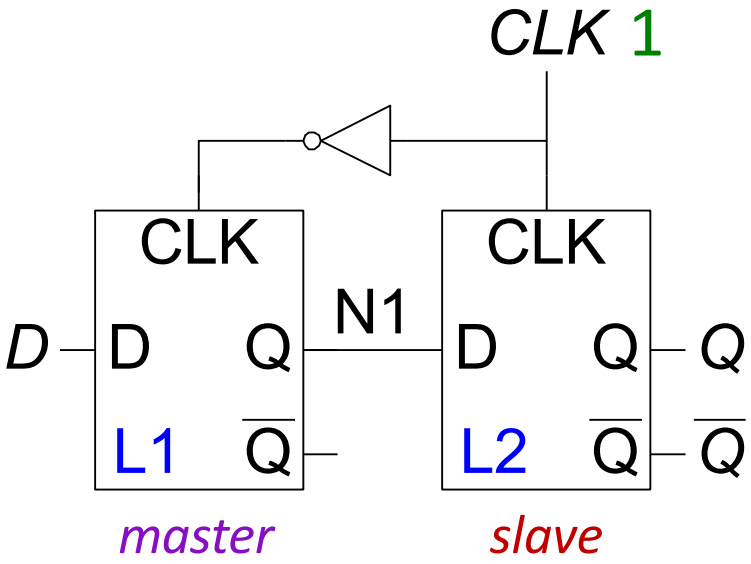
- **CLK** = 0
  - master: transparent
  - slave: opaque



The value at D propagates through to N1, but Q is cut off from N1

D Flip-Flop: Analysis

- **CLK** = 1
  - master: opaque
  - slave: transparent



The value at N1 propagates through to Q, but N1 is cut off from D

D Flip-Flop: Observations

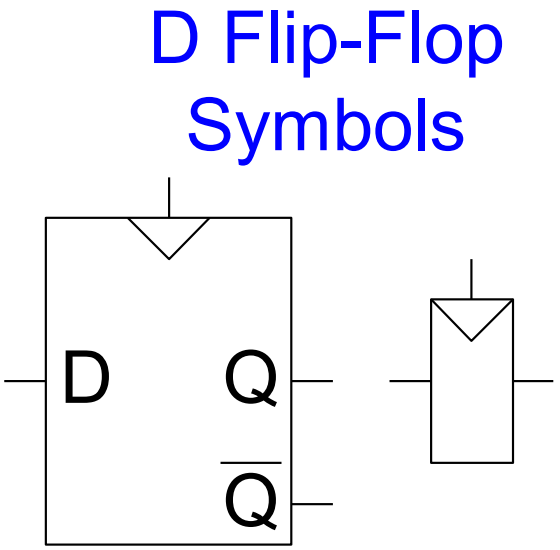
- The value at **D** immediately before the **CLK** rises from 0 to 1 get copied to **Q** immediately after **CLK** rises

- At all other times, **Q** retains its old value
  - There is an *opaque* latch blocking **D** from flowing to **Q**

A **D** flip-flop copies **D** to **Q** on the *rising edge* of the **CLK**, and remembers its state at all other times

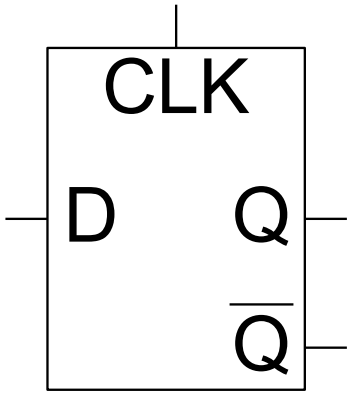
Other Names

- All names means the same
  - Master-slave flip-flop
  - Edge-triggered flip-flop
  - Positive edge-triggered flip-flop

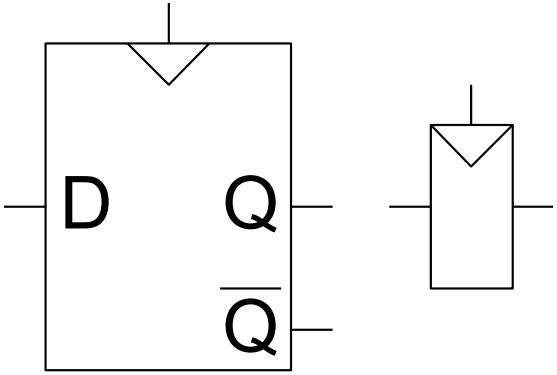


Remember the Symbols

D Latch Symbol



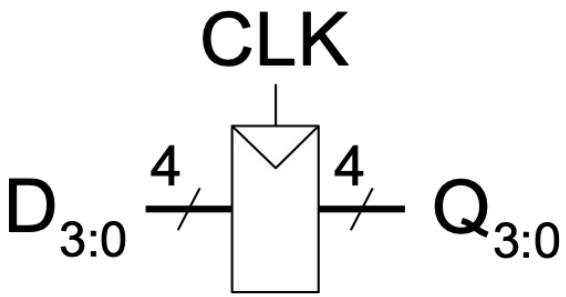
D Flip-Flop Symbols



Register

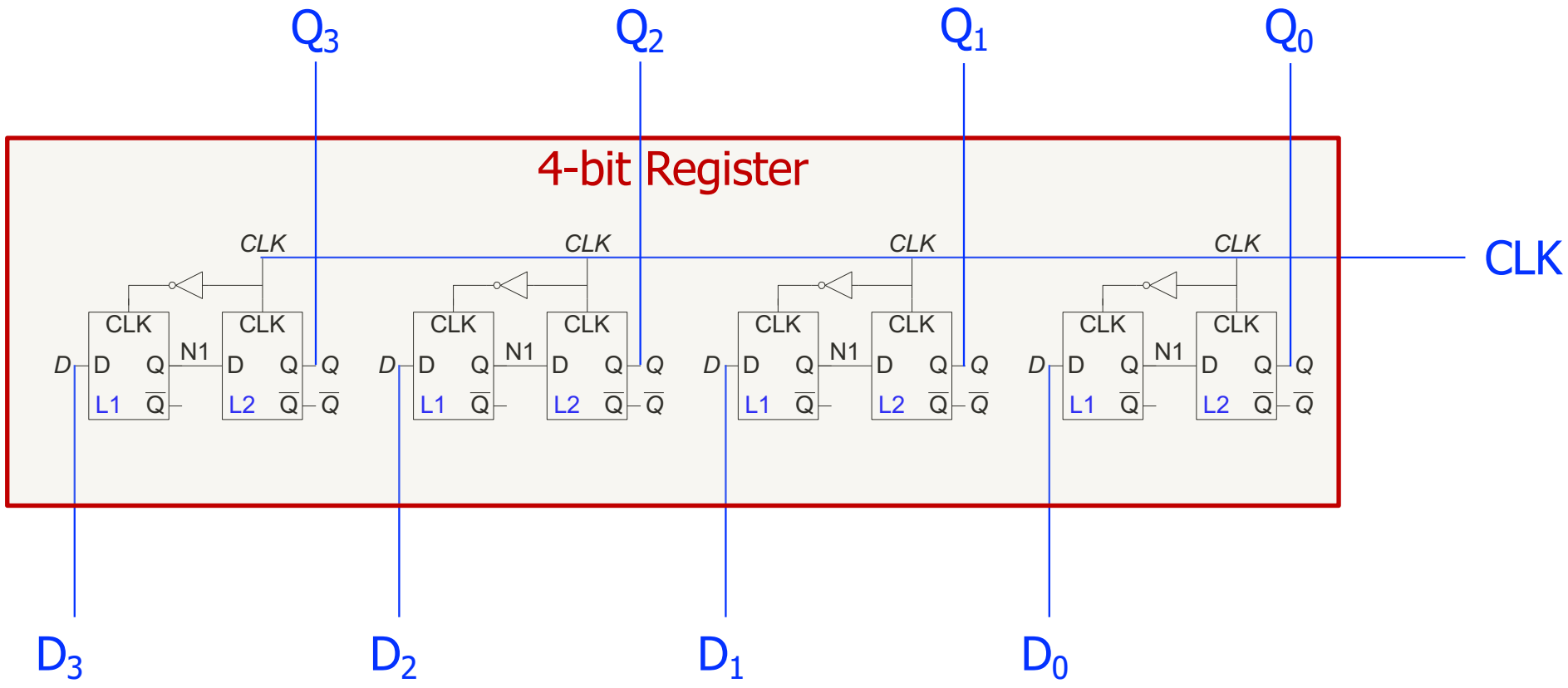
Register

- **How can we use flipflops to store more than one bit?**
  - **Principle of modularity:** Use more flipflops!
  - A single CLK to simultaneously write to all flipflops



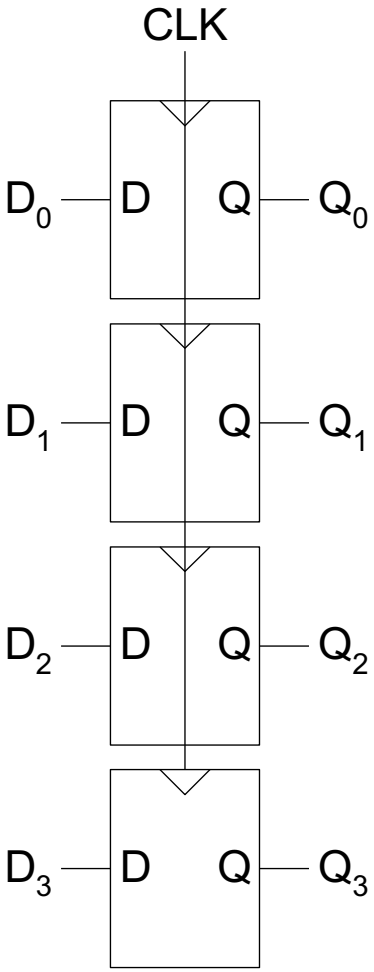
- **Register:** A structure that stores more than **one bit** of information and can be read from and written to
- This **register** holds 4 bits, and its data is referenced as Q[3:0]

4-bit Register

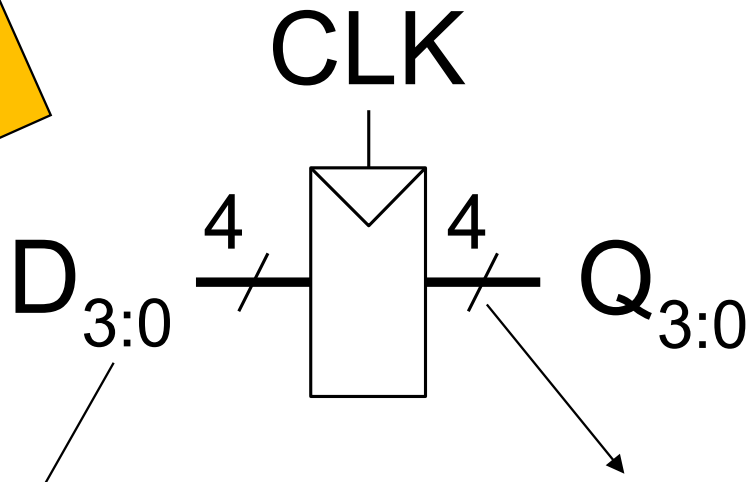


To build an **N-bit** register, use a bank of **N** flipflops with a shared **CLK**

4-bit Register



Condensed



This line represents 4 wires

This register stores 4 bits

Register Width

- Register width is the number of flipflops in a register
  - Typical register widths: 8-bit, 16-bit, 32-bit, 64-bit, ..., 512+ bits today
  - Register width is an important parameter of any computer

- **Why do we need wide registers with large widths?**
  - Solving **large/important** problems require manipulating large numbers
  - Large numbers need large number of bits, hence more flipflops
  - Floating point numbers (section 5.3) with decimal point require extra bits

- **Question:** Do we need 512-bit registers in: microwave controller, automobile engine control unit (ECU), computer used in financial tech, cockpit domain controller in cars?

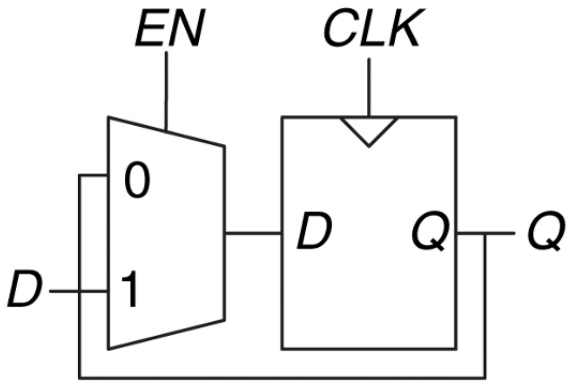
Now, We will See

Different Types of Flipflops

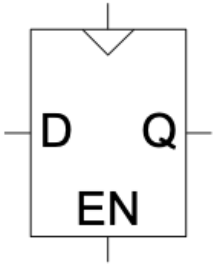
Enabled Flip-Flops

- **Loads a new value on a specific clock edge**
- **Inputs: CLK, D, EN**
  - The enable input (**EN**) controls when new data (**D**) is stored
- **Function:**
  - **EN = 1:** D passes through to Q on clock edge
  - **EN = 0:** the flip-flop retains its previous state

Internal Circuit

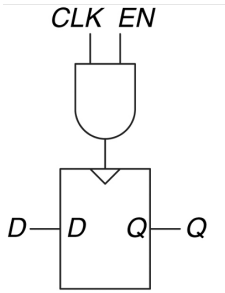


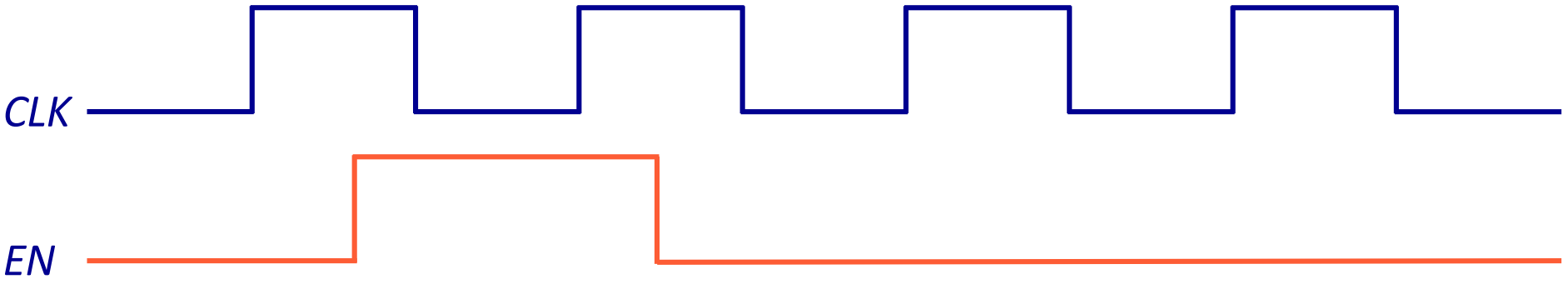
Symbol



Clock Gating

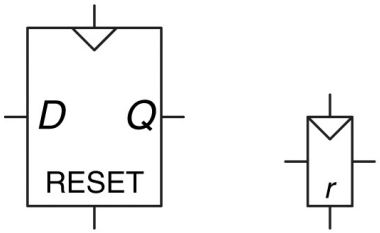
- Doing an **AND** of **CLK** with **EN** is called clock gating
- **Caution:** Performing logic on clock is best avoided
  - AND gate delays the clock
  - If **EN** changes while **CLK** is 1, the flipflop sees a glitch (switch at an incorrect time)



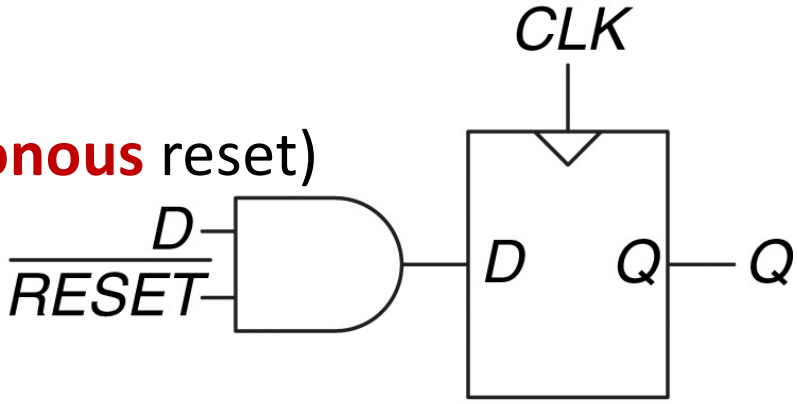


Resettable Flip-Flop

- Add another input (**RESET**)
- **Function:**
  - **RESET** = 1 (ignore **D** and reset output to 0)
  - **RESET** = 0 (**ordinary** D flip-flop)



- **Two types**
  - Reset on clock edge only (**synchronous** reset)

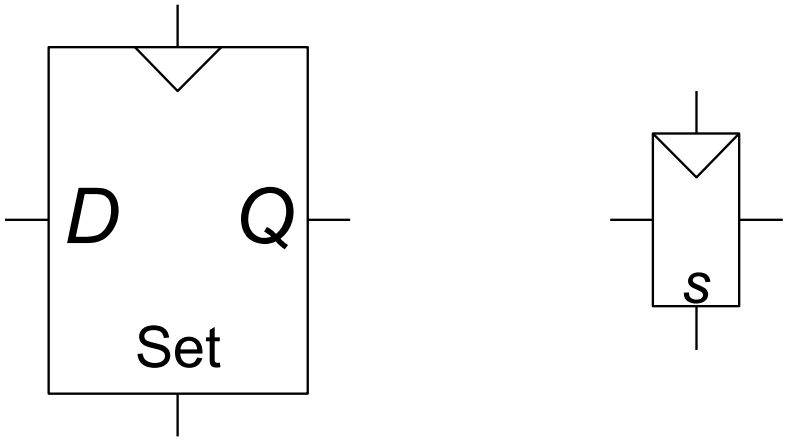


  - Reset immediately (**asynchronous** reset) Homework exercise!

Settable Flip-Flop

- Add another input (**SET**)
- **Function:**
  - **SET** = 1 (**Q** is set to 1)
  - **SET** = 0 (**ordinary** D flip-flop)

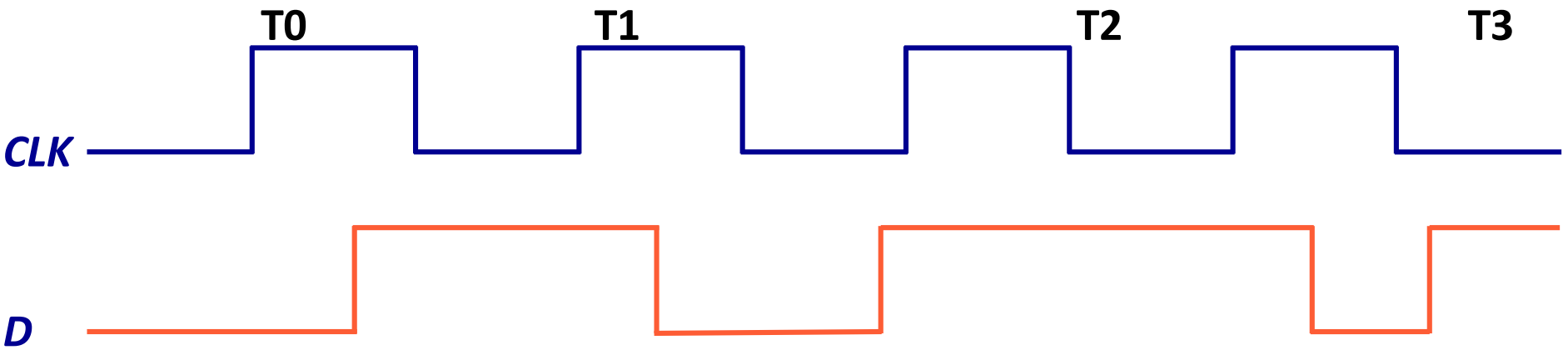
Symbols



Examples:

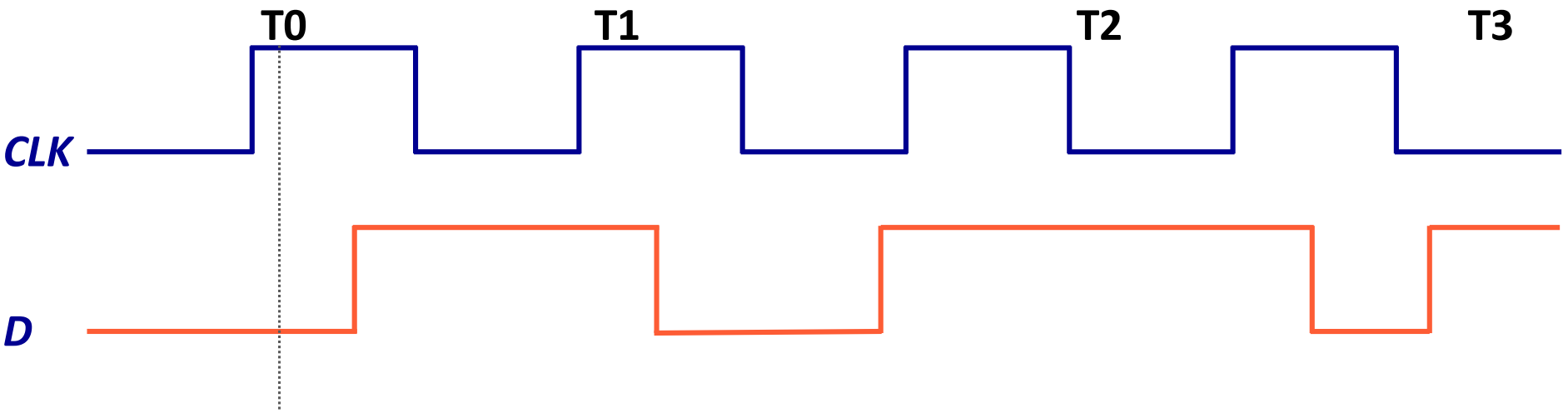
Latch vs. Flipflop

Example - I



	Q (Latch)	Q (Flipflop)
T0		
T1		
T2		
T3		

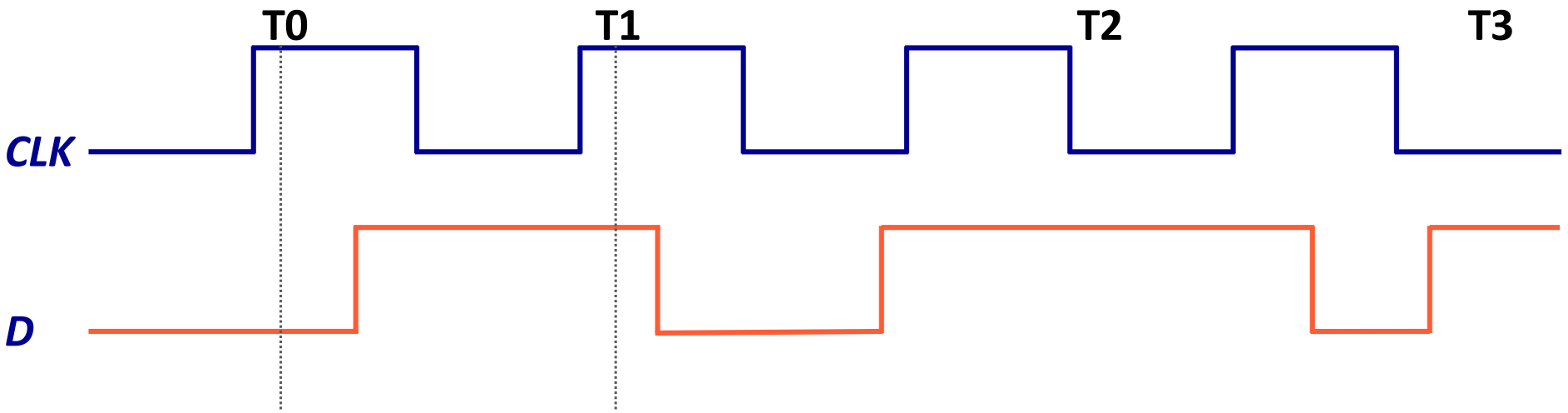
Example - I



	Q (Latch)
T0	0
T1	
T2	
T3	

	Q (Flipflop)
T0	0
T1	
T2	
T3	

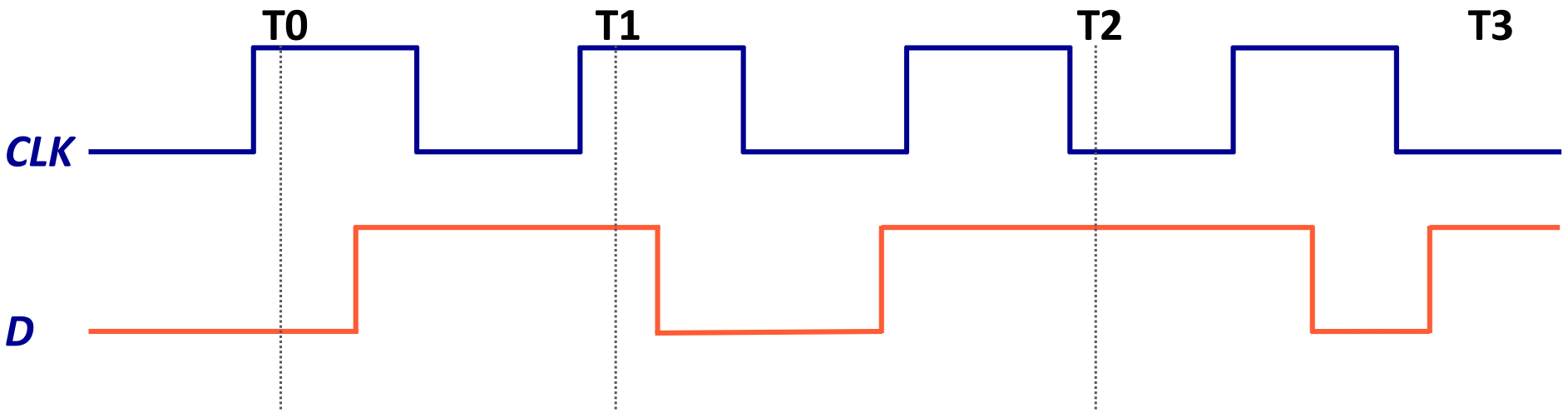
Example - I



	Q (Latch)
T0	0
T1	1
T2	
T3	

	Q (Flipflop)
T0	0
T1	1
T2	
T3	

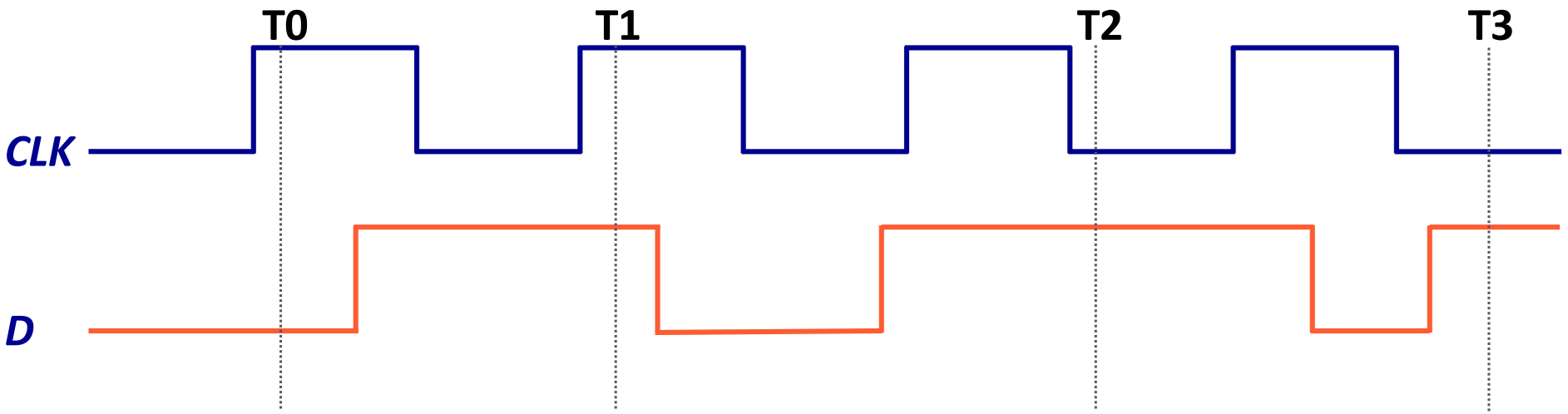
Example - I



	Q (Latch)
T0	0
T1	1
T2	1
T3	

	Q (Flipflop)
T0	0
T1	1
T2	1
T3	

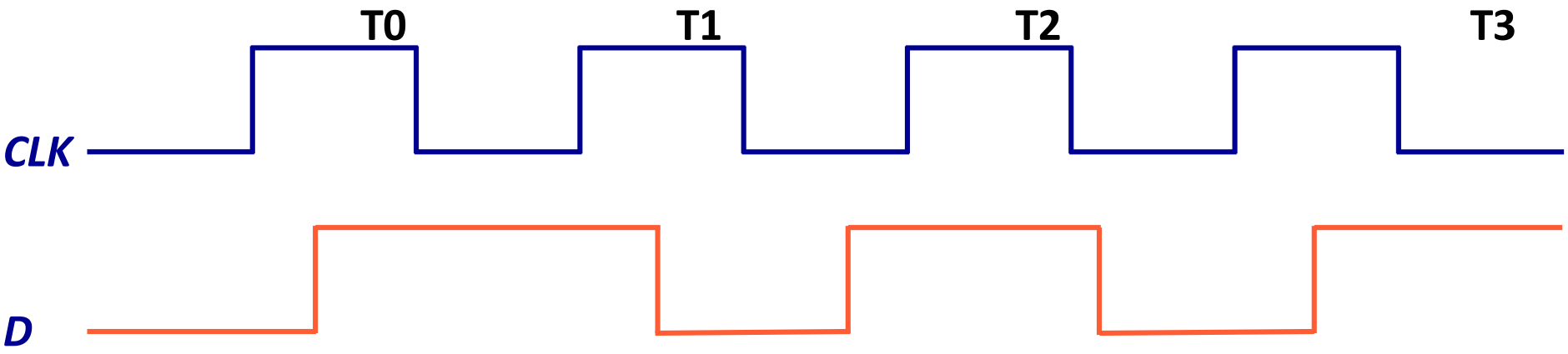
Example - I



	Q (Latch)
T0	0
T1	1
T2	1
T3	0

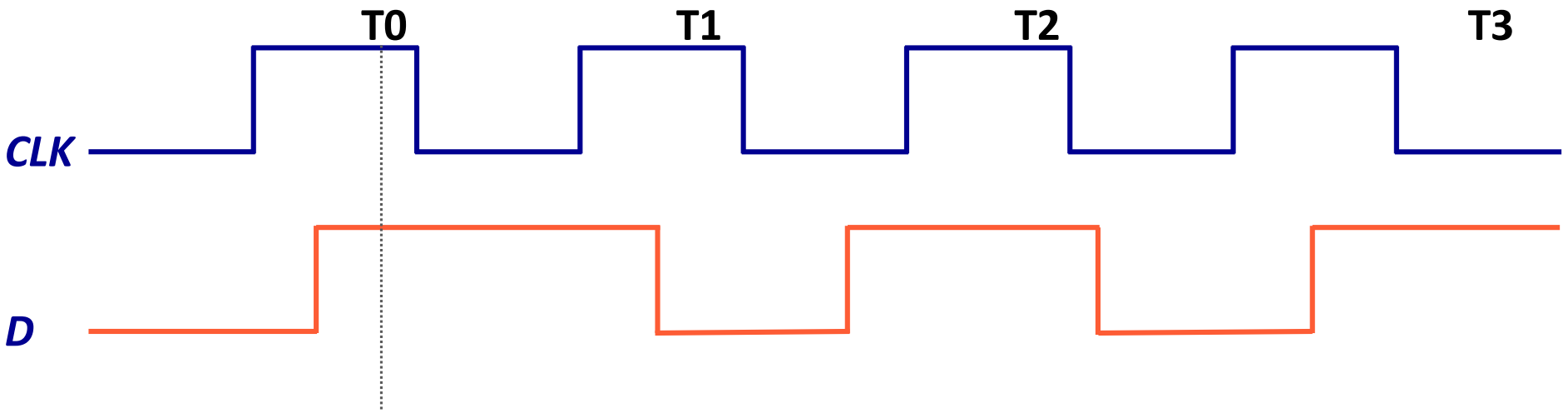
	Q (Flipflop)
T0	0
T1	1
T2	1
T3	1

Example - II



Q (Latch)	Q (Flipflop)
T0	T0
T1	T1
T2	T2
T3	T3

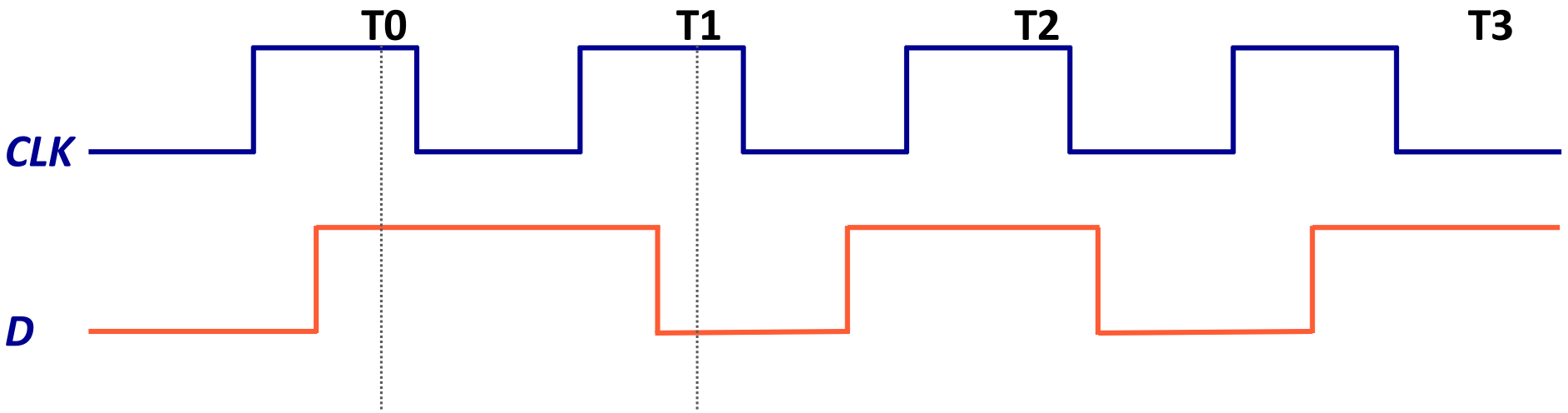
Example - II



	Q (Latch)
T0	1
T1	
T2	
T3	

	Q (Flipflop)
T0	0
T1	
T2	
T3	

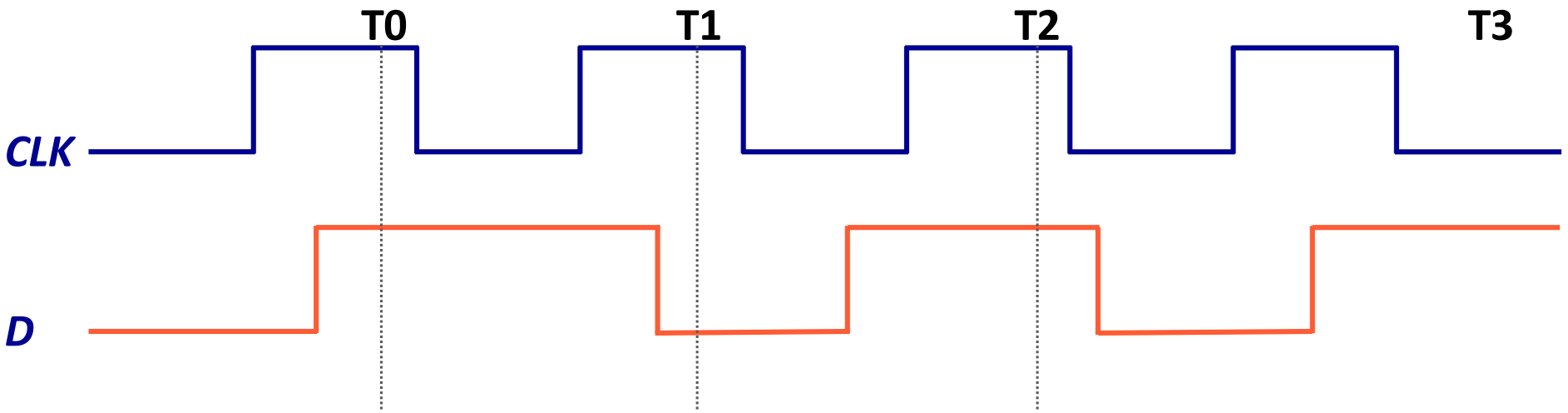
Example - II



	Q (Latch)
T0	1
T1	0
T2	
T3	

	Q (Flipflop)
T0	0
T1	1
T2	
T3	

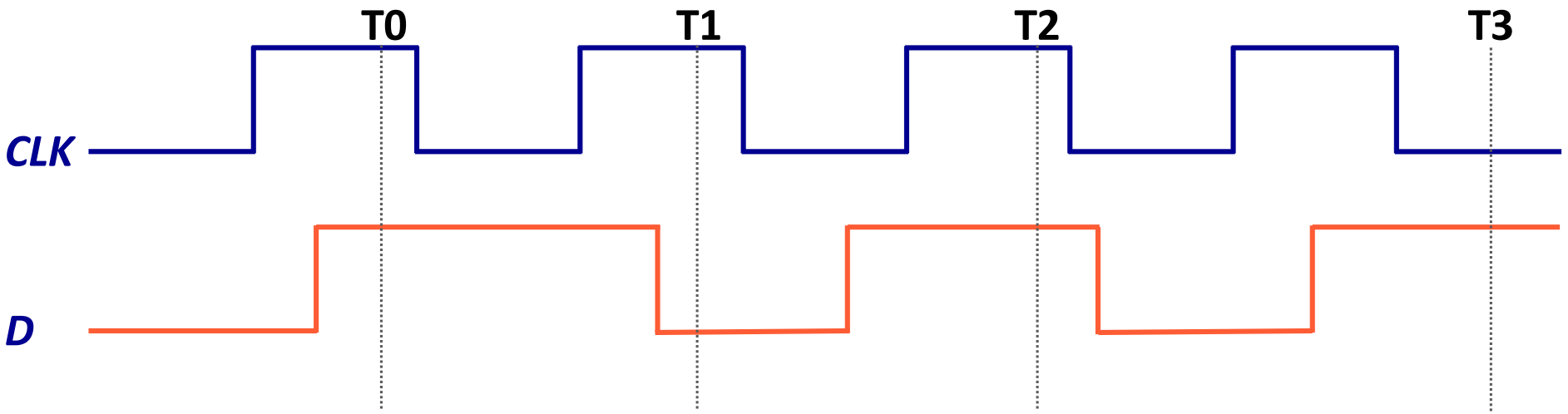
Example - II



	Q (Latch)
T0	1
T1	0
T2	1
T3	

	Q (Flipflop)
T0	0
T1	1
T2	1
T3	

Example - II



	Q (Latch)
T0	1
T1	0
T2	1
T3	1

	Q (Flipflop)
T0	0
T1	1
T2	1
T3	0

Memory

Memory

- **Memory** is made up of locations that can be written to or read from. An example memory array with 4 locations:

Addr(00):	0100 1001	**Addr**(01):	0100 1011
Addr(10):	0010 0010	**Addr**(11):	1100 1001

- Each unique memory location is indexed with a unique **address**. 4 locations require 2 address bits (log[#locations])
- **Addressability:** the number of **bits** of information stored **in each location**. This example: addressability is 8 bits.
- The entire set of **unique locations** in memory is referred to as the **address space.**

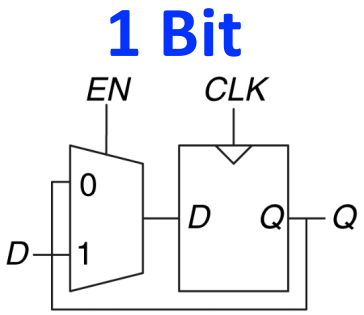
Section 5.5 of H&H

Memory

- **Memory** in almost all computers today is *byte addressable* due to historical reasons (later)
- Typical memory is **MUCH** larger (e.g., billions of locations)
- Two billion locations and **8-bit (1 byte)** addressability
  - Address space is **2 GB** or **2 billion 1-byte** locations

Addressing Memory

- **Let's implement a simple memory array with:**
  - 3-bit addressability & address space of 2 (total of 6 bits)

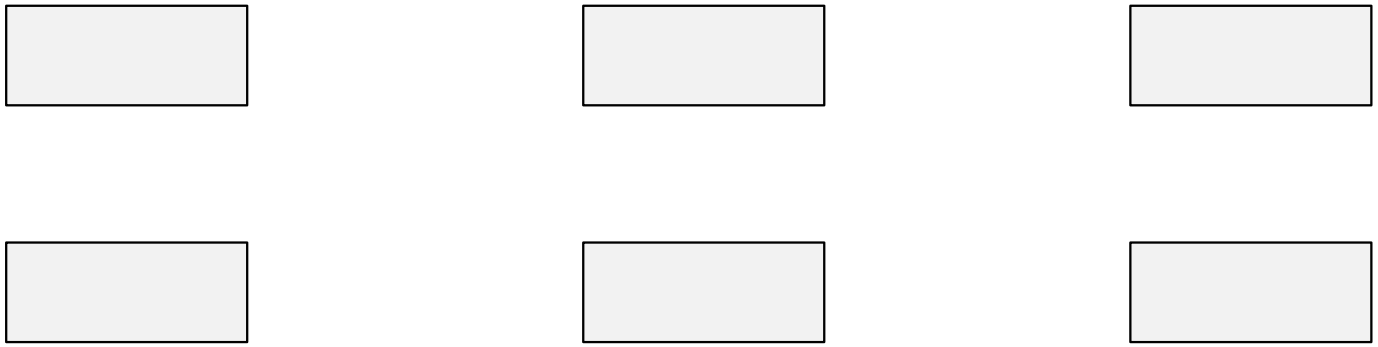


- **How can we select an address to read?**
  - Two addresses so address size is log(2) = 1 bit

6-Bit Memory Array

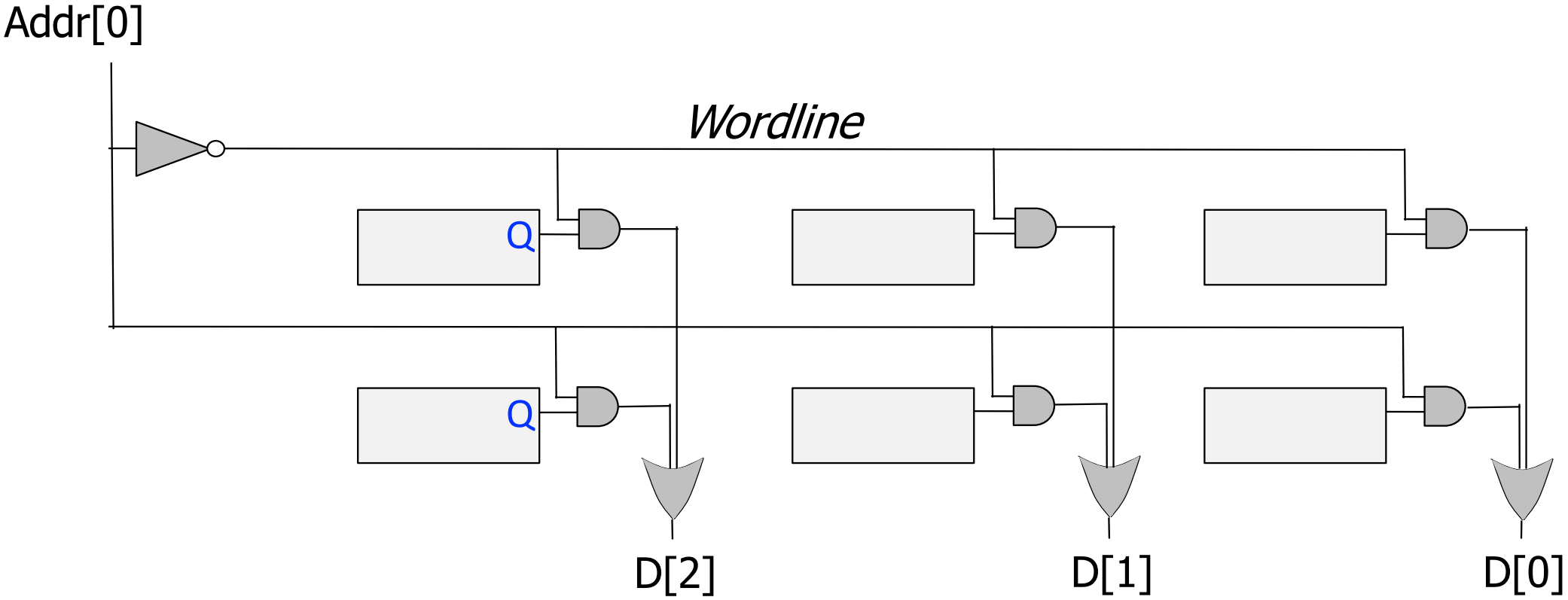
Addr(0)	Bit_2	Bit_1	Bit_0
Addr(1)	Bit_2	Bit_1	Bit_0

Reading from Memory

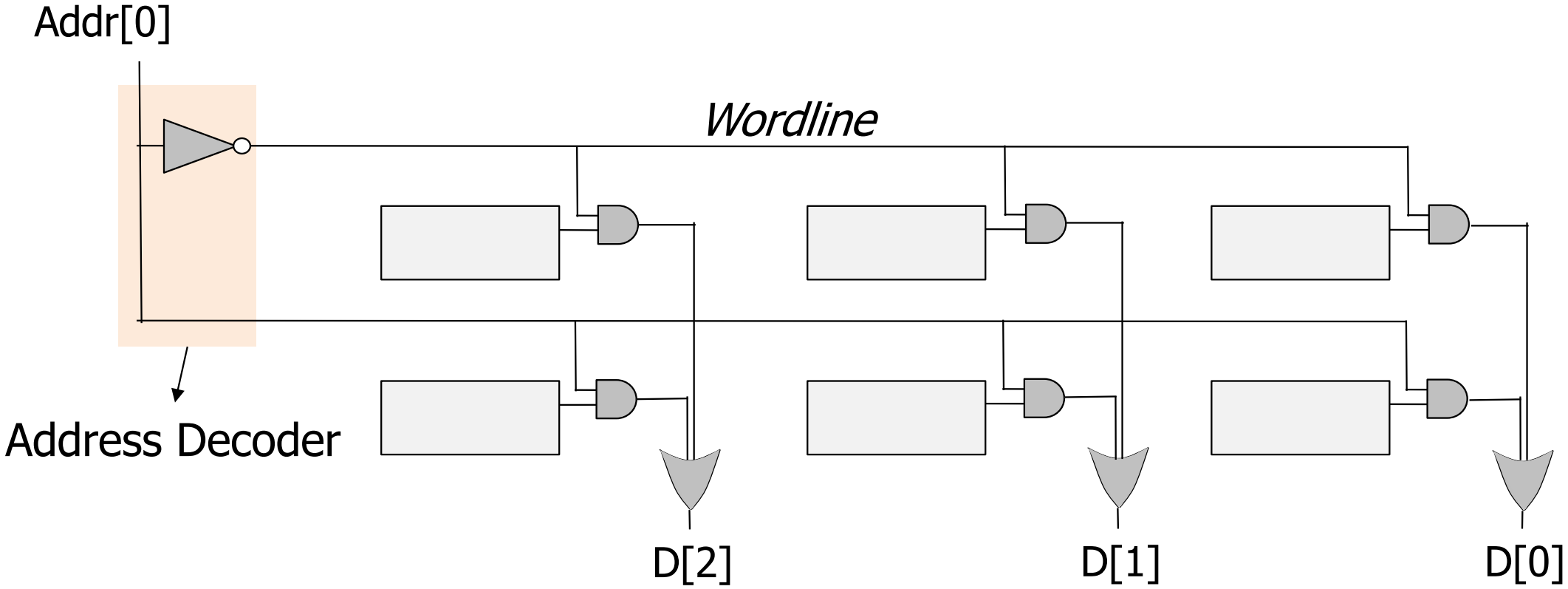


- Each “box” is a **bit cell** storing one bit
  - Can be enabled (like the enabled flip-flop with **EN**)
  - Takes an **i**nput (D_i) and produces an output (**D**)

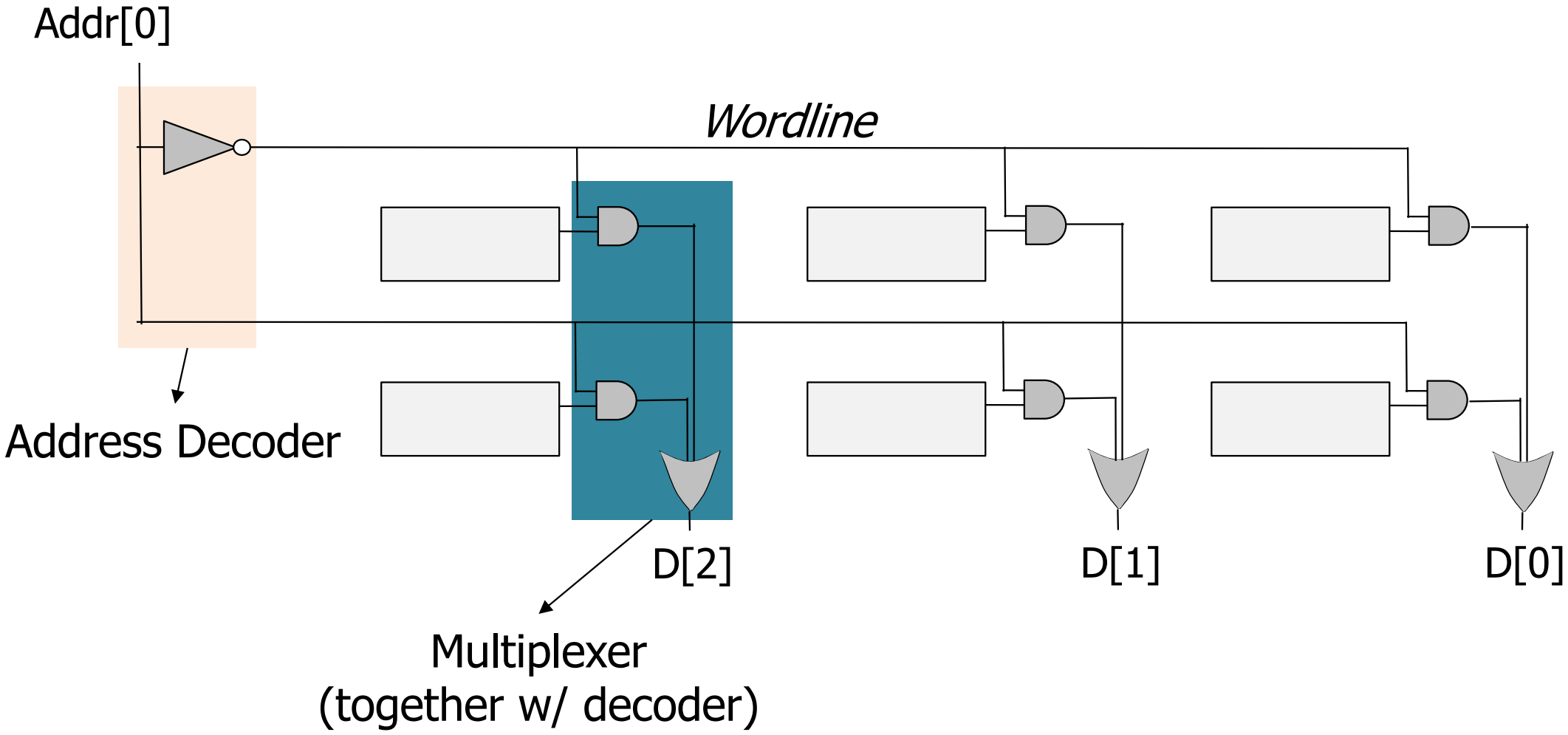
Reading from Memory



Reading from Memory



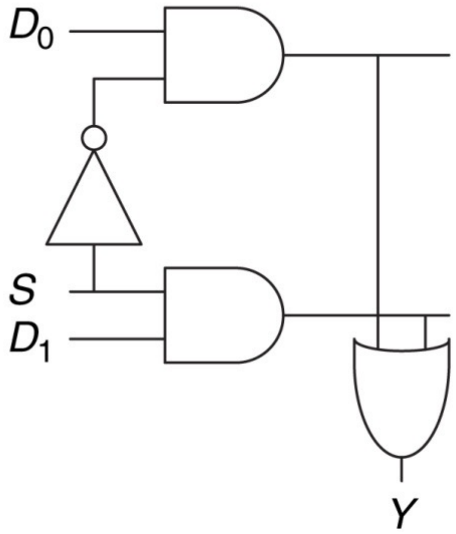
Reading from Memory



Recall: Multiplexer (MUX), or Selector

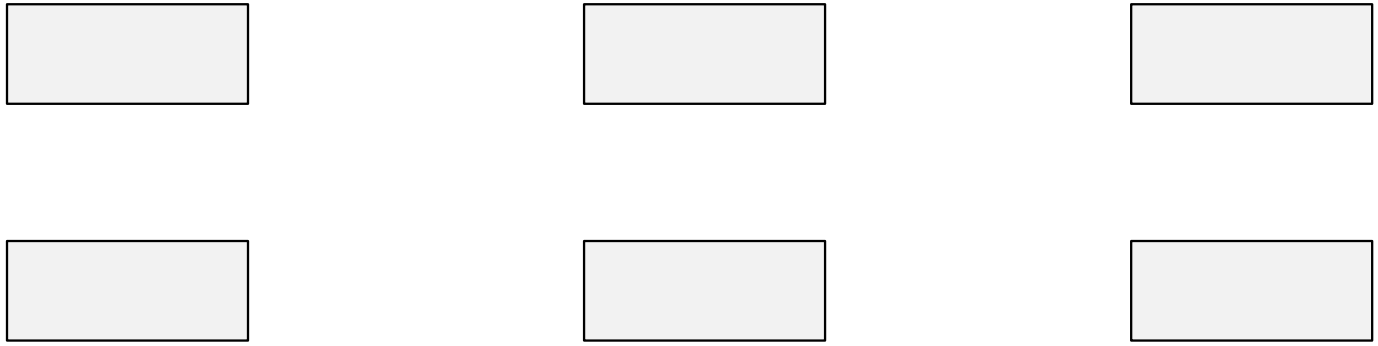
- **Selects** one of the *N* inputs to connect it to the output
  - based on the value of $\log_2 N$-bit control input called select
- Example: 2-to-1 MUX

$$Y = D_0\overline{S} + D_1 S$$



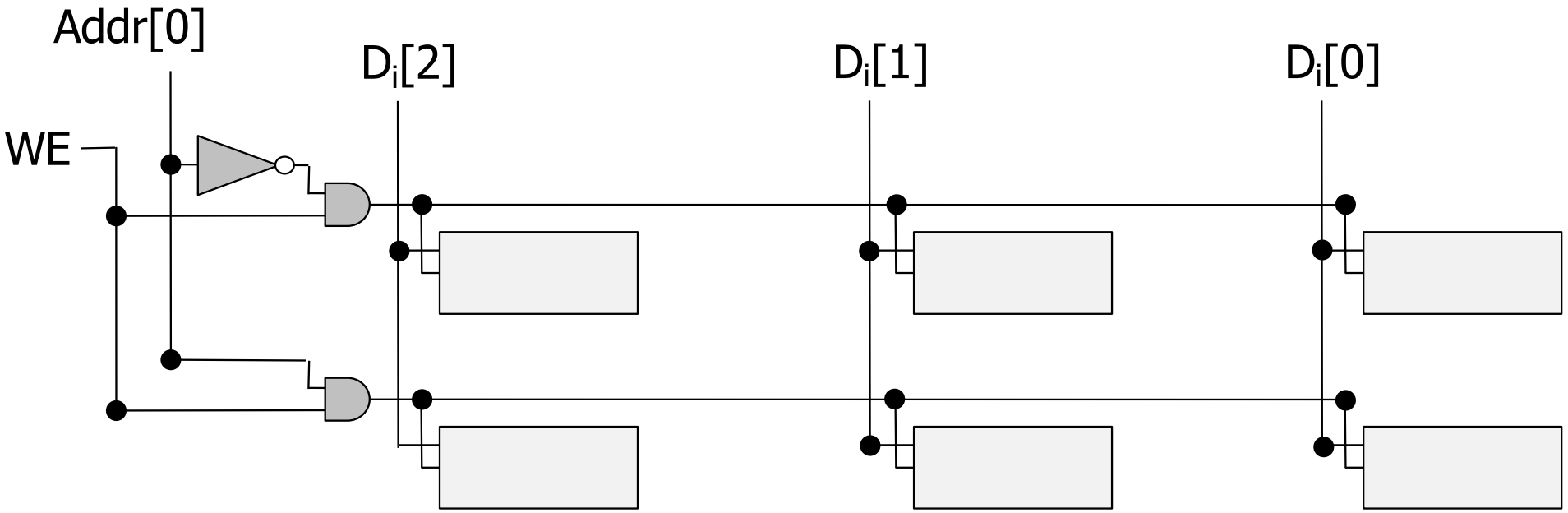
Writing to Memory

- **How can we select an address to write to it?**
  - How should we tell memory our intention to write?
    - We assert the **EN** input of the enabled flip-flop
  - How should we tell memory what data we need to write?
    - We use the **D_i** input of the flip-flop



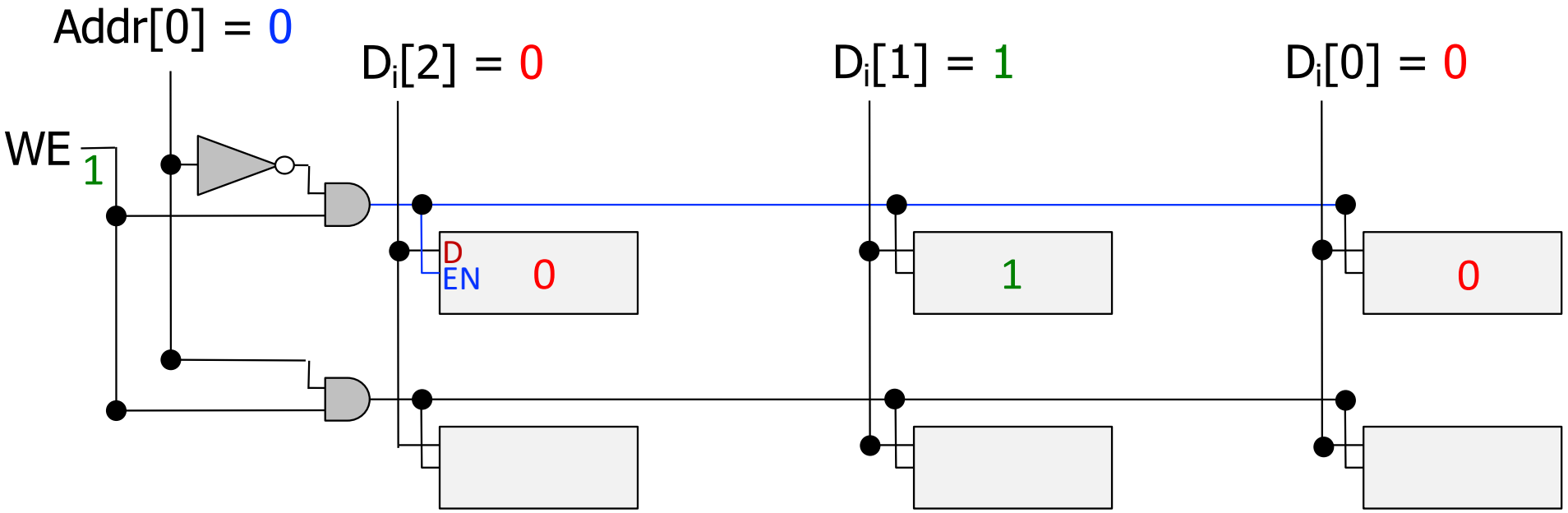
Writing to Memory

- **How can we select an address to write to it?**
  - Input is indicated with D_i



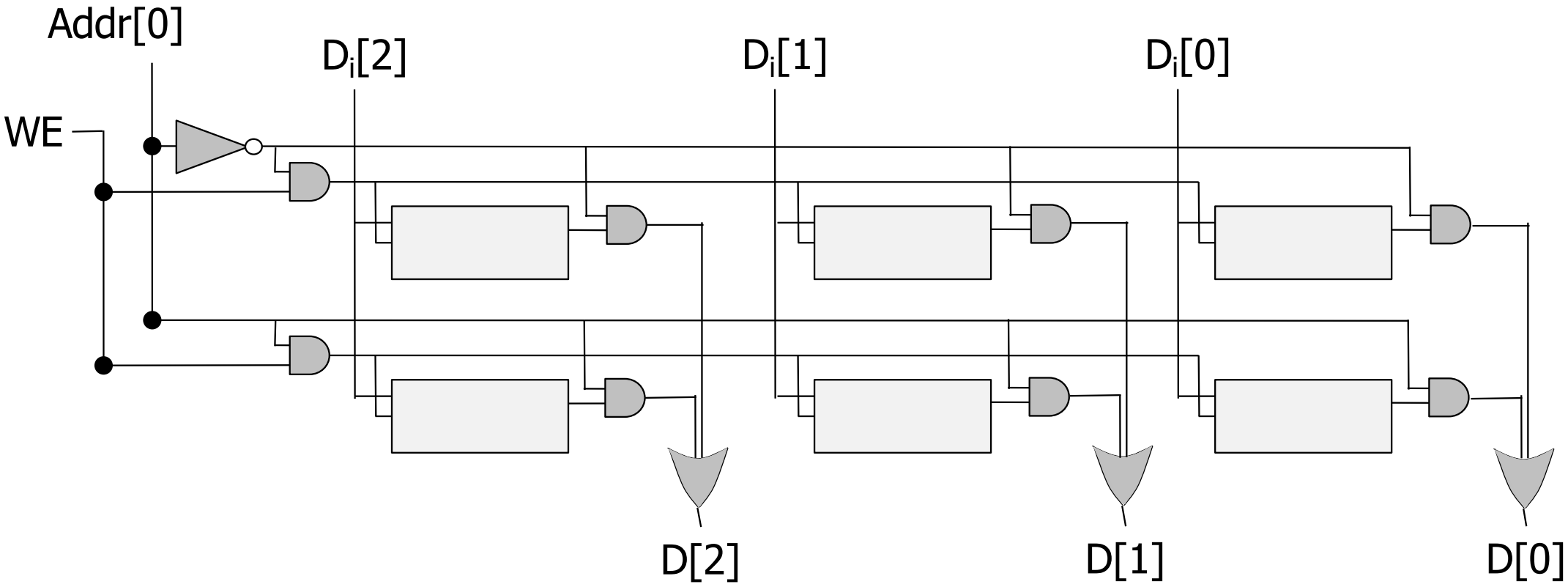
Writing to Memory

- **How can we select an address to write to it?**
  - Input is indicated with D_i



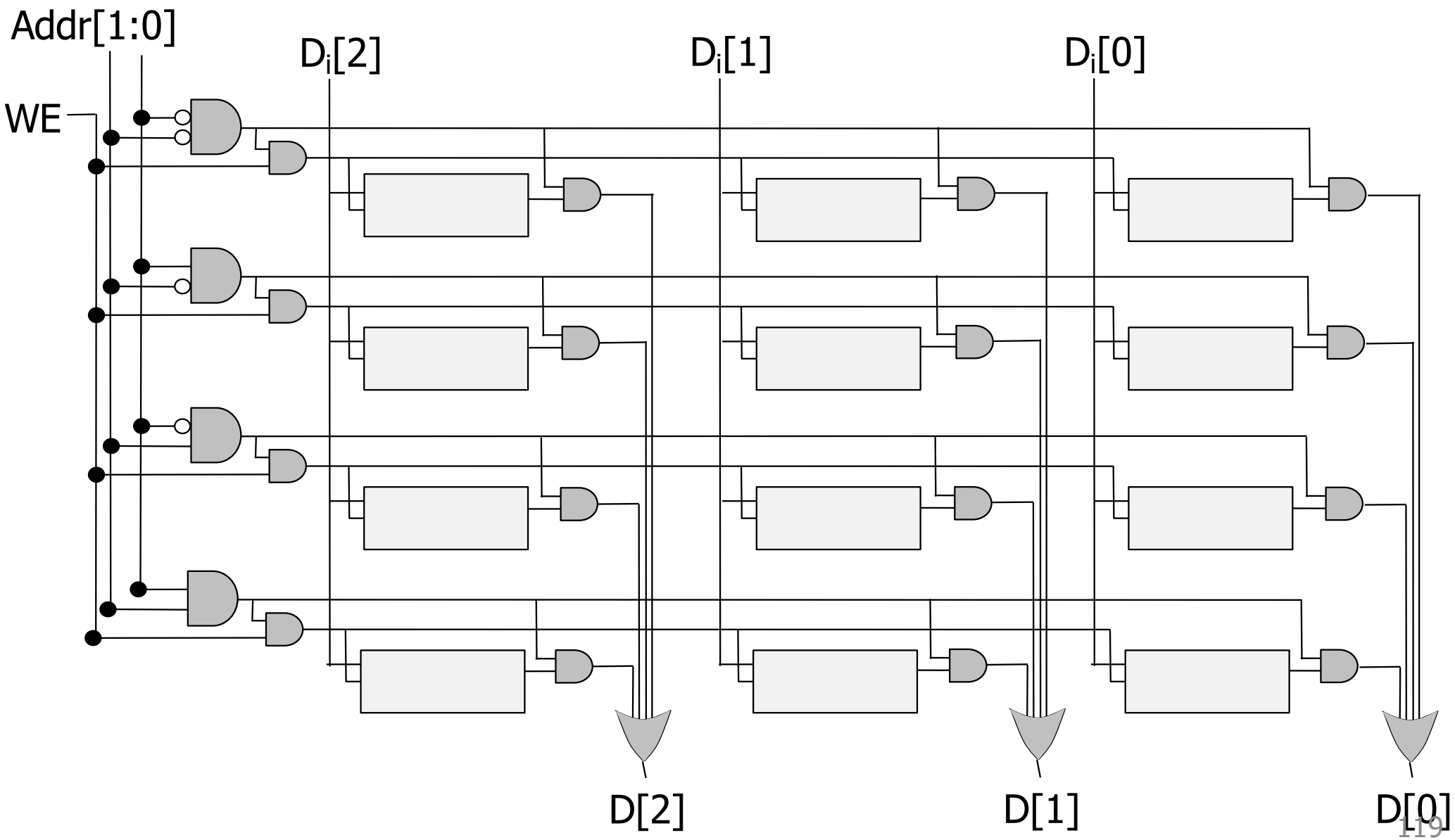
Putting it all Together

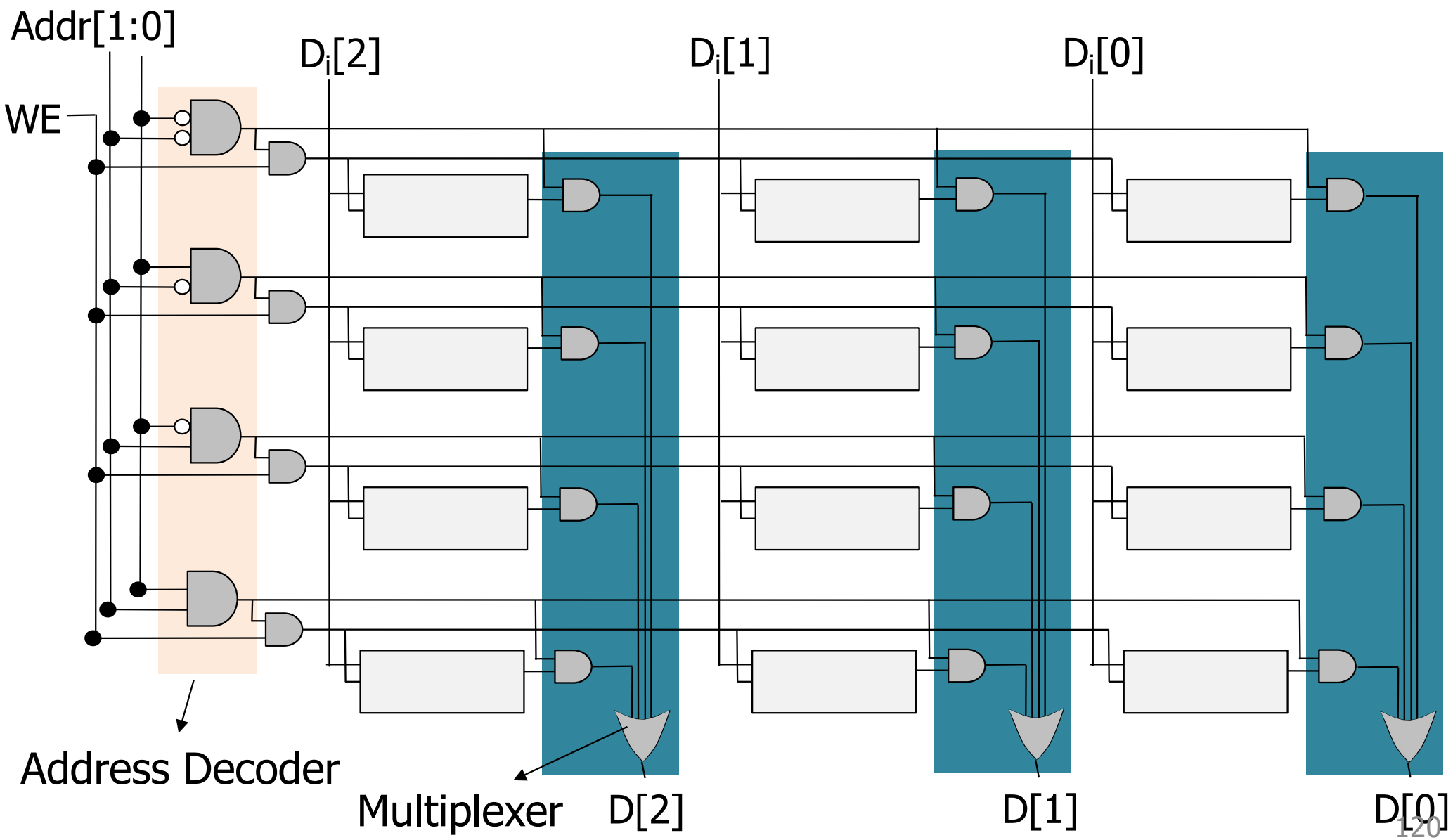
- **Let's enable reading from and writing to a memory array**



A Bigger Memory Array

4 locations X 3 bits


Addr[1:0]
WE
Di[2]
Di[1]
Di[0]
D[2]
D[1]
D[0]


Addr[1:0]
WE
$D_i[2]$
$D_i[1]$
$D_i[0]$
Address Decoder
Multiplexer
D[2]
D[1]
D[0]

Example: Reading Location 3

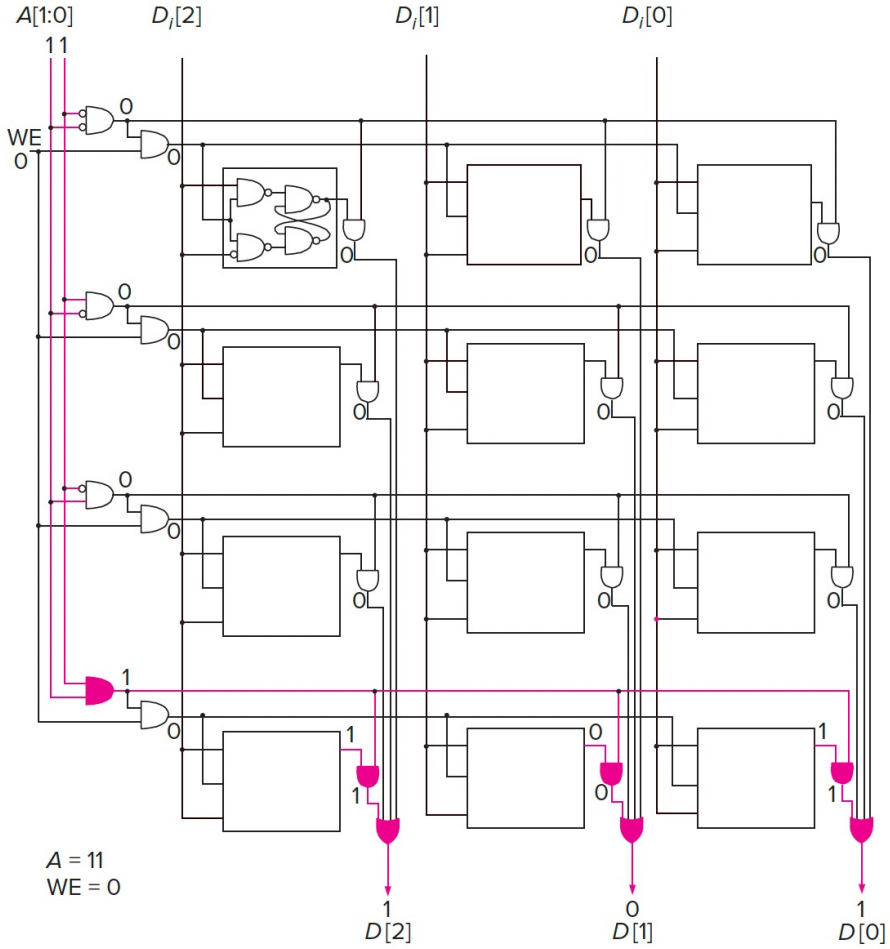
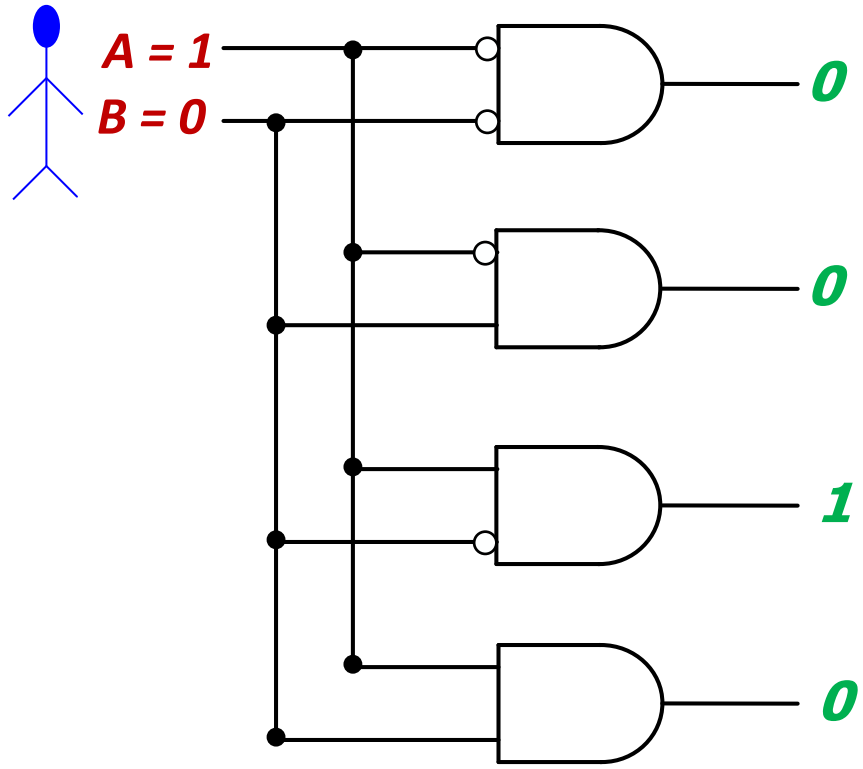


Figure 3.21 Reading location 3 in our 2^2-by-3-bit memory.

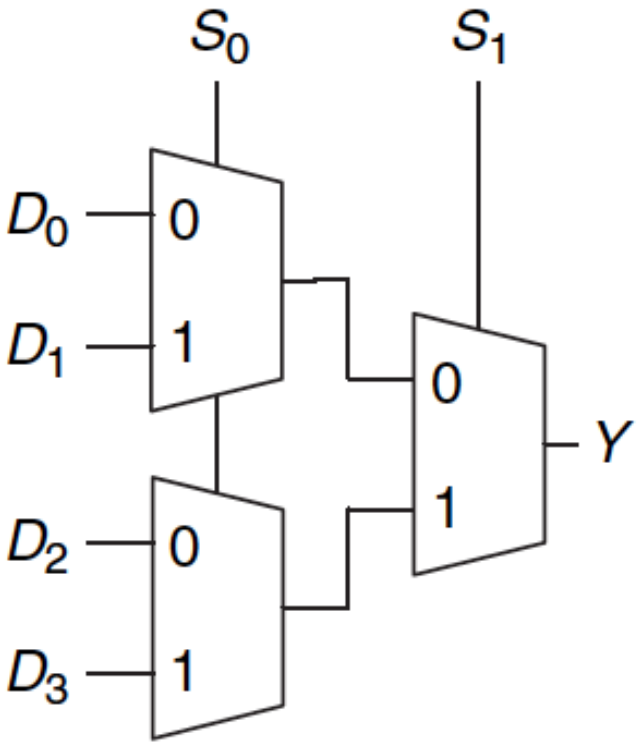
Image source: Patt and Patel, "Introduction to Computing Systems", 3rd ed., page 78.

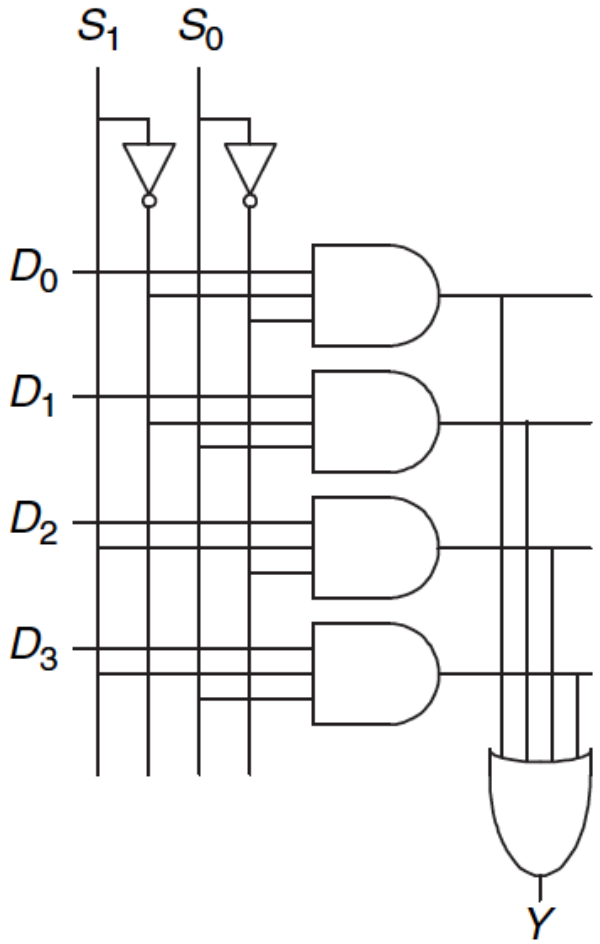
Recall: Decoder

- The decoder is useful in determining how to interpret a bit patter
  - **It could be the address of a location in memory, that the processor intends to read from**
  - **It could be an instruction in the program and the processor needs to decide what action to take (based on *instruction opcode*)**



Recall: A 4-to-1 Multiplexer





Different Types of Storage Elements

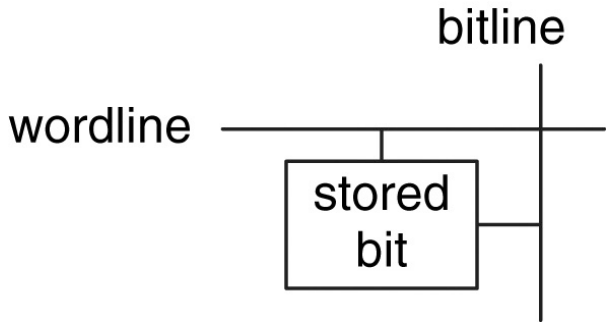
Storage Elements

- Latches and flip-flops
  - Very fast
  - Very expensive (one bit costs tens of transistors)
- Static RAM (SRAM)
  - Relatively fast
  - Expensive (one bit costs 6+ transistors)
- Dynamic RAM (DRAM)
  - Slower, reading destroys content (refresh), needs special process for manufacturing
  - Cheap (one bit costs only one transistor plus one transistor)

Section 5.5 of H&H

Generalized Storage Element

- Stores **one** bit (bit cell)
  - **Wordline** enables (selects) the storage element or bit cell
  - Bit line is used to **read** the stored bit or **write** a new bit

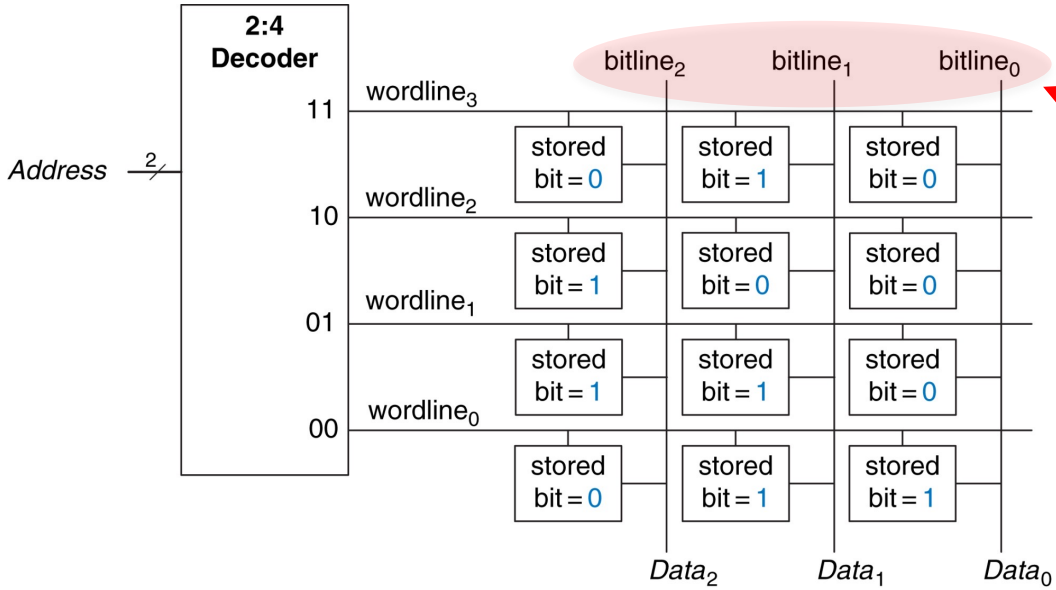


- Why is it called **word line**?
  - Word line **addresses** an entire (unique) word in a row of words
  - We read from and write to an entire word line

Section 5.5 of H&H

Generalized Memory Organization

- A decoder to decode the output
- A multiplexer to select an output (**not shown below**)
  - Can also use **tri-state buffers** instead of AND/OR multiplexer



called bitlines because they produce the stored bits

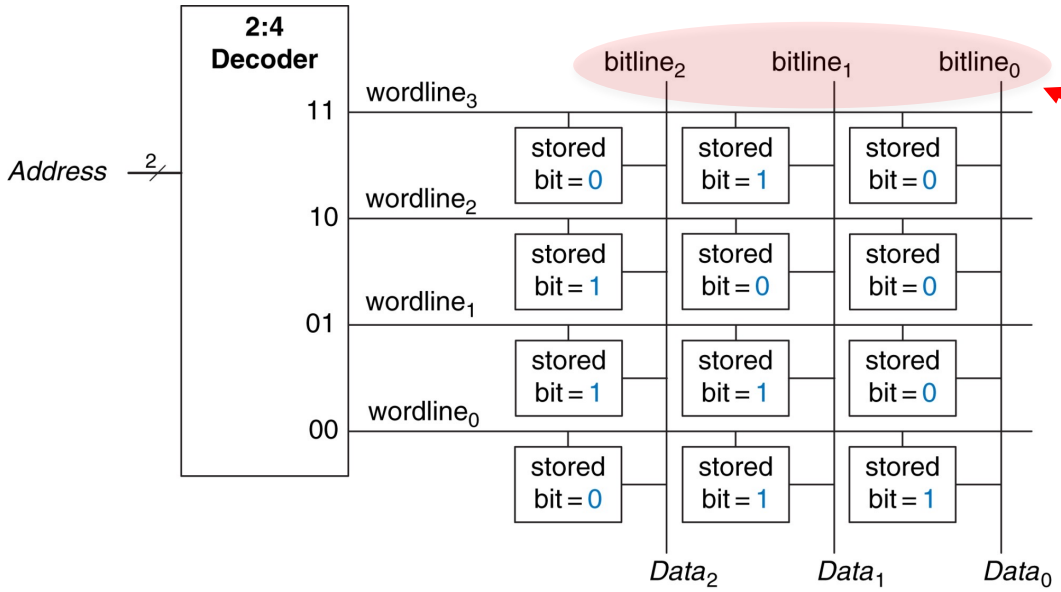
- Storage element can be of any "type"
- **Address/Data** makes up a single **port**

rightmost line is numbered 0

Section 5.5 of H&H

Generalized Memory Organization

- **Bitlines are used for both reading the stored bit and writing a new bit** (specific electrical/circuit details of how are not relevant)



called bitlines because they produce the stored bits

rightmost line is numbered 0

Section 5.5 of H&H

Read/Write Procedure in Detail

- **READ**
  - A wordline is asserted, and the corresponding row of bits cells drives the bitlines HIGH or LOW

- **WRITE**
  - The bitlines are driven HIGH or LOW first and then a wordline is **asserted**, allowing the bitlines values to be stored in that row of bit cells

Section 5.5 of H&H

Recall a Tri-State Buffer

- A tri-state buffer enables gating of different signals onto a wire

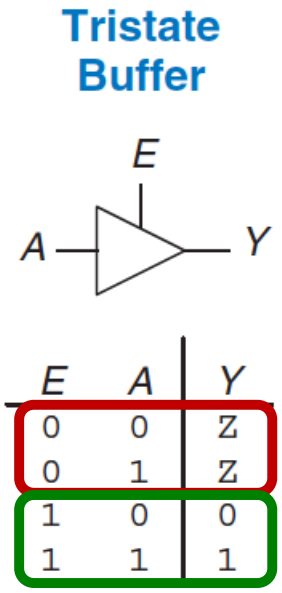


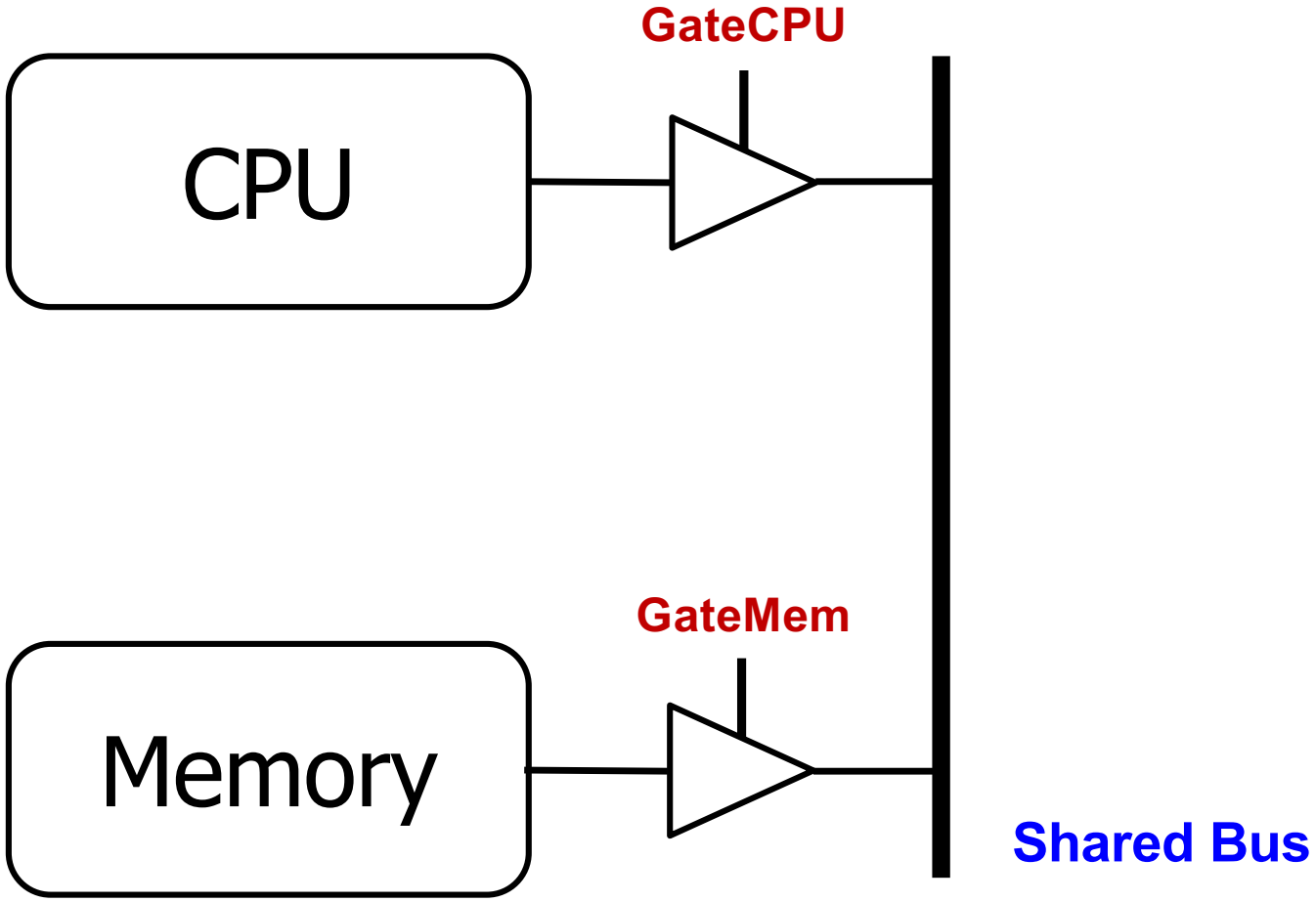
Figure 2.40 Tristate buffer

A tri-state buffer acts like a switch

- When E is LOW, output is a floating signal (Z)
- **Floating:** Signal not driven by any circuit (open circuit, floating wire)

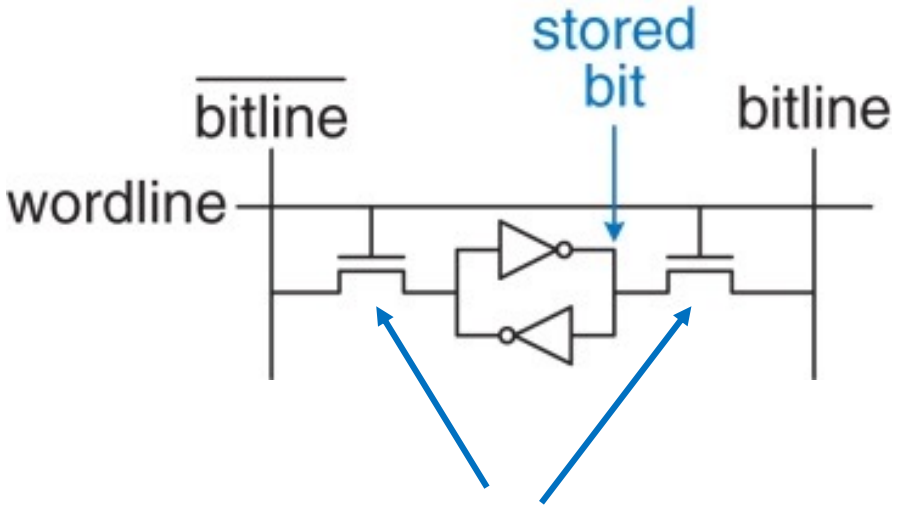
Memory Port with Tri-State Buffer

Homework Exercise!



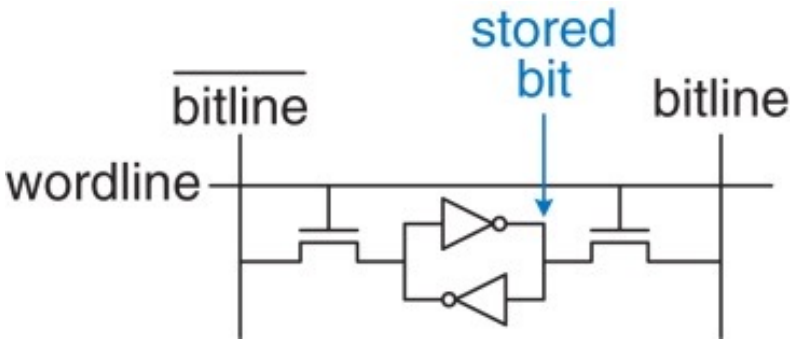
SRAM Bit Cell

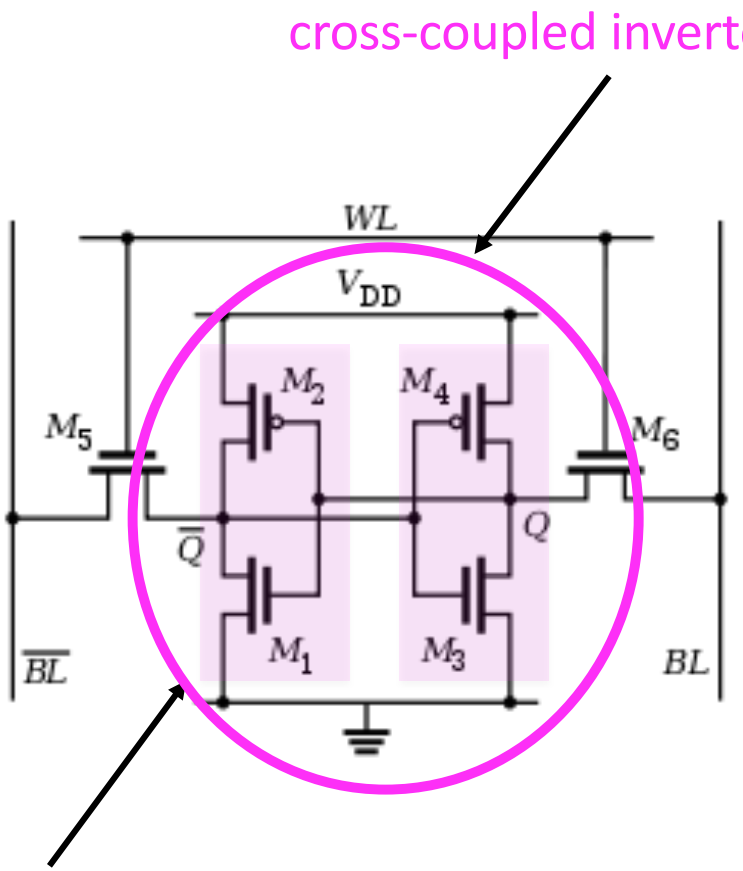
- **SRAM**: **S**tatic **R**andom **A**ccess **M**emory
  - Data bit is stored in a cross-coupled inverter
  - Each cell has two outputs: bitline and **bitline** and $\overline{\textbf{bitline}}$



- When the wordline is asserted, both nMOS transistors are turned **ON**, and data values are **transferred** *to or from* the **bitlines**

SRAM Bit Cell



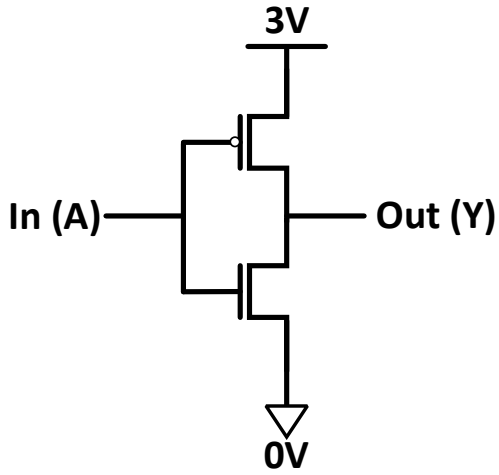


source: https://electronics.stackexchange.com/questions/162466/how-do-the-access-transistors-in-an-sram-cell-work

Recall: MOS NOT Gate (Inverter)

- We have seen a NOT gate at the transistor level
  - If A = **0V** then Y = **3V**
  - If A = **3V** then Y = **0V**

- Interpretation of voltage levels
  - Interpret **0V** as logical (binary) 0 value
  - Interpret **3V** as logical (binary) 1 value



A	P	N	Y
0	**ON**	OFF	1
1	OFF	**ON**	0

$$Y = \bar{A}$$

Random Access Memory (RAM)

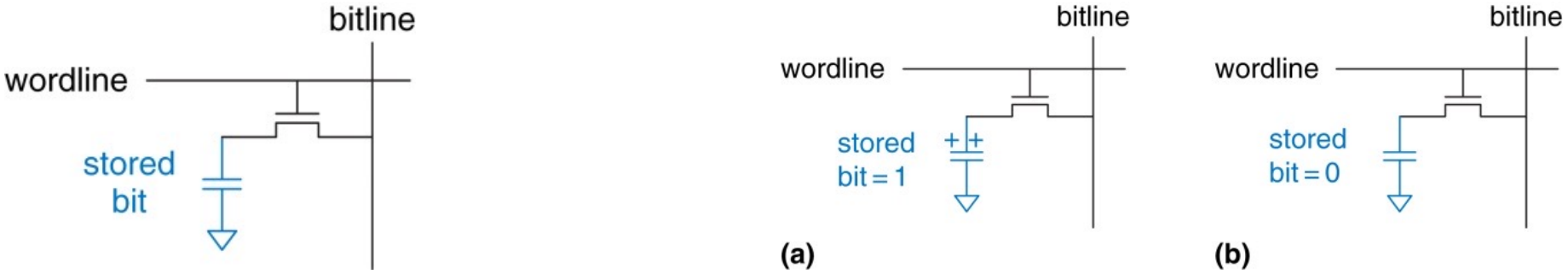
- **RAM**: **R**andom **A**ccess **M**emory
  - Any data word is accessed with the same delay as any other

- Contrast with **S**equential **A**ccess Memory

- **Tape recorder:** Accesses nearby data more quickly than faraway data
- We will cover other storage technologies (**Disk**, **Flash**, **Persistent Memory**) later in the course

DRAM Bit Cell

- **DRAM**: **D**ynamic **R**andom **A**ccess **M**emory
  - Stores a **bit** as the presence or absence of a **charge** on a capacitor
  - The **nMOS** transistor behaves as a **switch** that either connects or disconnects the capacitor from the **bitline**

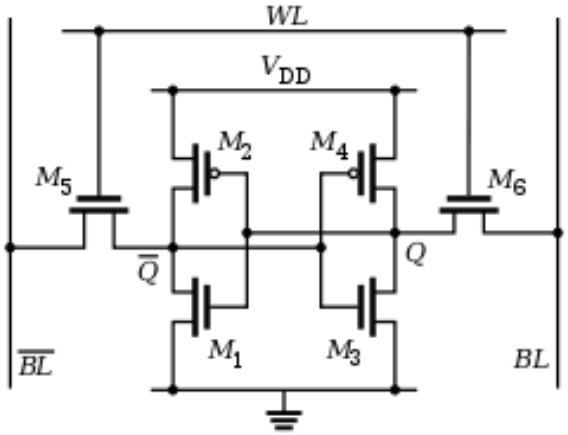


- When the wordline is asserted, the **nMOS transistor** turns **ON**, and the stored bit value **transfers** *to or from* the **bitline**

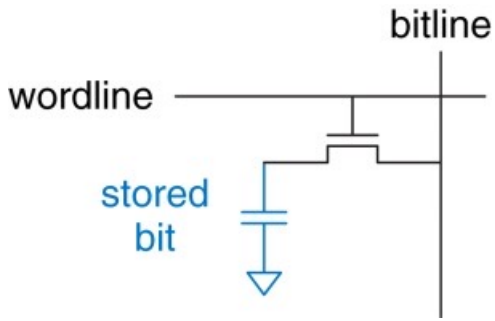
Section 5.5.2 of H&H

Static vs. Dynamic

- Recall the SRAM cell: The **nMOS transistors** (M5 and M6) are driven by cross-coupled inverter connected to **V_{DD}** or **GND**
- DRAM: The capacitor node is not actively driven **HIGH** or **LOW** by a transistor tied to **V_{DD}** or **GND**



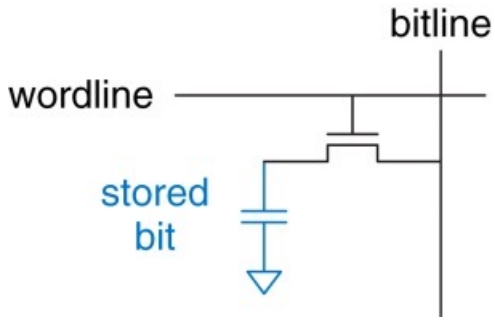
SRAM Cell



DRAM Cell

DRAM Reads and Writes

- **Read:** Upon a read, data values are **transferred** from the capacitor to the bitline.
  - Reading **destroys** the bit value stored on the capacitor, so the data word must be restored (rewritten) after each read
  - **Refresh:** Even when DRAM is not read, the contents must be refreshed (read and rewritten) every **few milliseconds**, because the charge on the capacitor gradually **leaks** away



- **Write:** Upon a **write**, data values are **transferred** from the bitline to the capacitor

DRAM Refresh

- **Refresh:** Even when DRAM is not read, the contents must be refreshed (read and rewritten) every **few milliseconds**, because the charge on the capacitor gradually leaks away
- **Refreshing** consumes extra energy
- **Refreshing** requires costly circuitry. It is a critical drawback of the DRAM technology, especially at large sizes

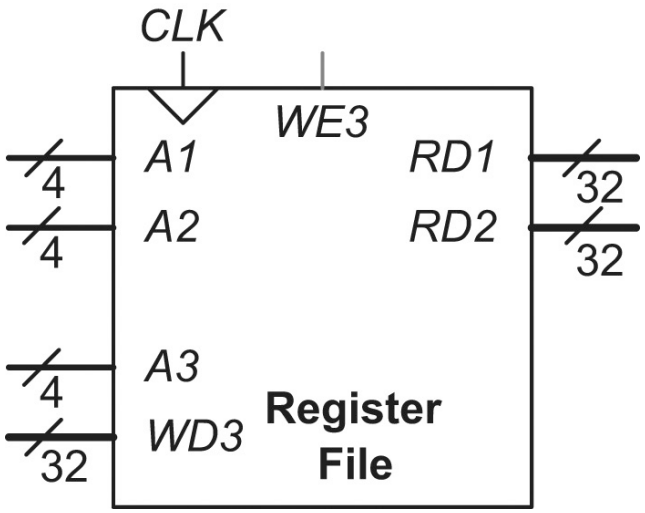
Memory Comparison

- Flip-flops (**20 transistors**)
  - Very fast
  - Very expensive (one bit costs tens of transistors)
- Static RAM (SRAM) (**6 transistors**)
  - Relatively fast
  - Expensive (one bit costs 6+ transistors)
- Dynamic RAM (DRAM) (**1 transistor**)
  - Slower, reading destroys content (refresh), needs special process for manufacturing
  - Cheap (one bit costs only one transistor plus one transistor)

Multi-Ported Memory

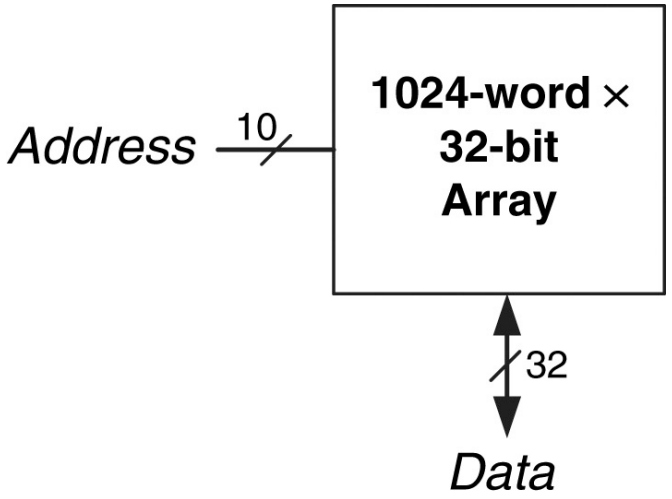
- Each port gives read/write access to one memory address

- Typically, we need to access many addresses simultaneously
  - **Example** with two **read** ports and one **write** port



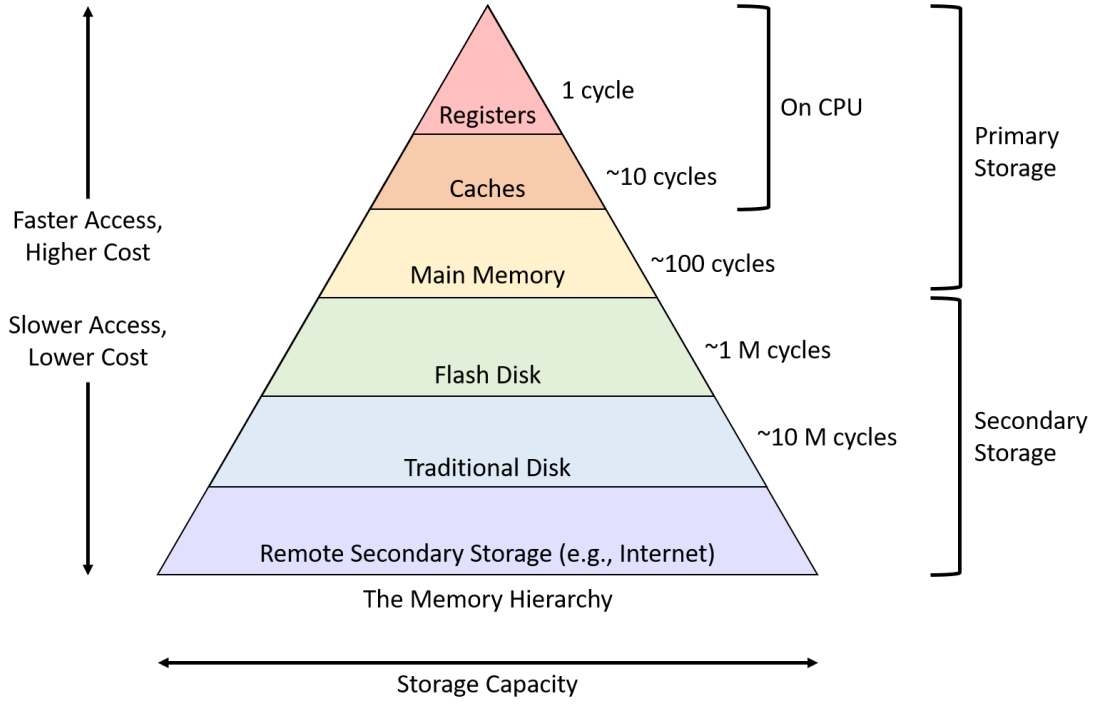
Another Representation of Memory

- What is the memory's **addressability**?
- How big is the **address space**?



Memory Hierarchy

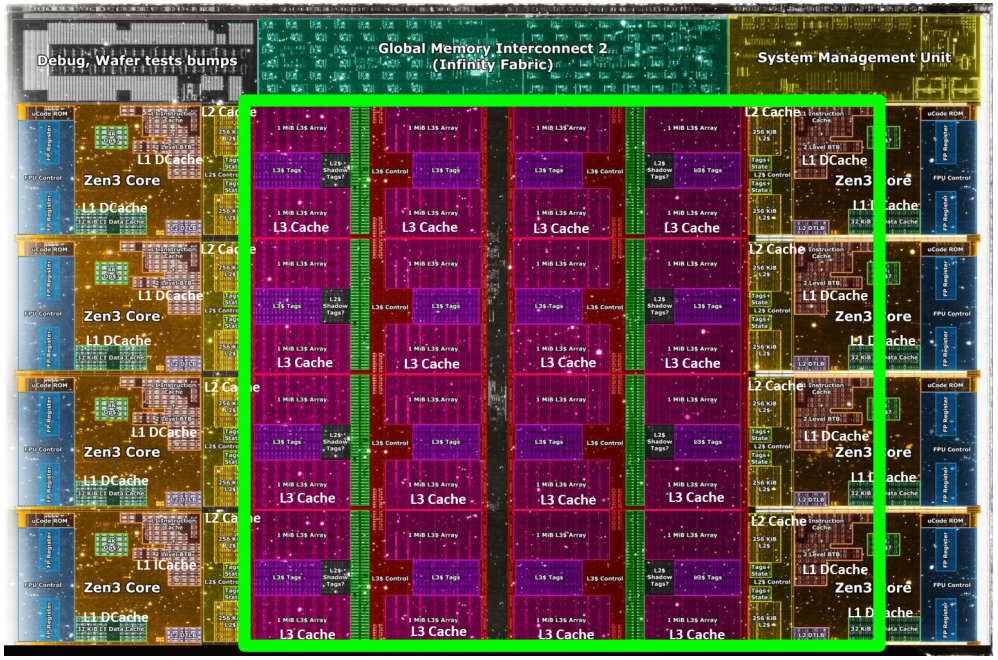
- Large memories use SRAM and DRAM
- Registers inside the **CPU** use flip-flops



- Flip-flops are used in **synchronous sequential circuits** (**next**)

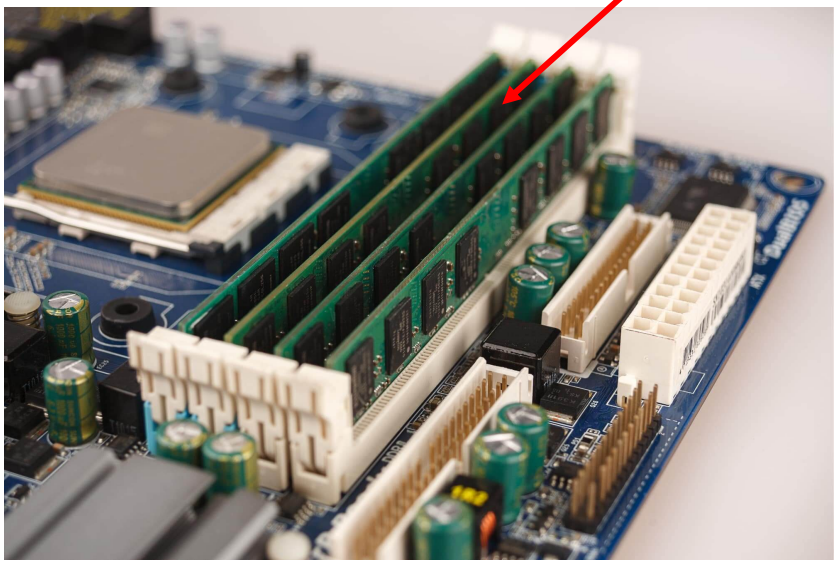
Memory Hierarchy

Inside the CPU chip: SRAM and flip-flops



Outside the CPU chip: DRAM

DRAM Stick or DIMM



Sequential Logic Circuits

- **Week 1 & 2:** Circuits that process information
- **Week 3:** Circuits that can store information, basic storage elements and memory
- **Now:** Digital logic structures that can **both** process information (i.e., make decisions) **and** store information
  - **Decision is based on both input combinations and past inputs**

Sequential Logic Circuits

- We have discovered circuit elements that can **store information**
- Now, we will use these elements to build circuits that **remember past inputs**

Combinational

Only depends on current inputs

Sequential

Opens depending on past inputs

https://www.easykeys.com/228_ESP_Combination_Lock.aspx
https://www.fosmon.com/product/tsa-approved-lock-4-dial-combo

State

- In order for this lock to work, it has to keep track (**remember**) of the past events!
- If passcode is **R13-L22-R3**, sequence of **states** to unlock:
  A. The lock is not open (locked), and no relevant operations have been performed
  B. Locked but user has completed R13
  C. Locked but user has completed R13-L22
  D. Unlocked: user has completed R13-L22-R3

- The **state** of a system is a snapshot of all relevant elements of the system at the moment of the snapshot
  - To open the lock, **states A–D must be completed in order**
  - If anything else happens (e.g., L5), lock **returns** to state A

State Diagram of Sequential Lock

- Completely describes the operation of the sequential lock

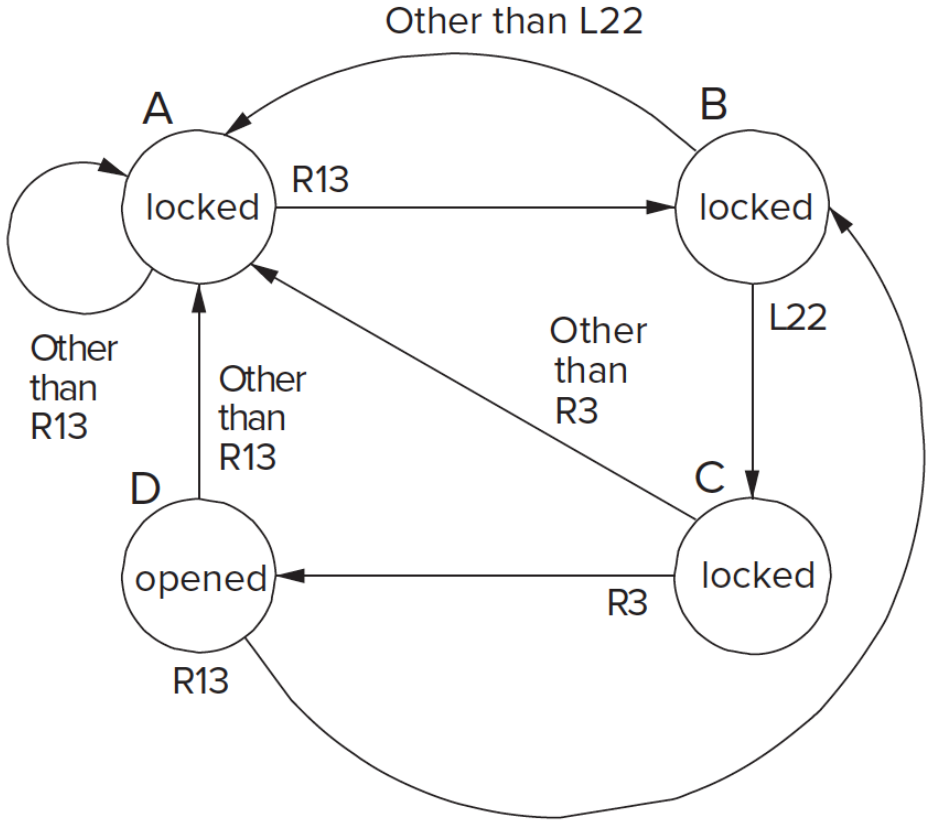


Image source: Patt and Patel, "Introduction to Computing Systems", 2nd ed., page 76.

Another Example: Soft Drink Machine

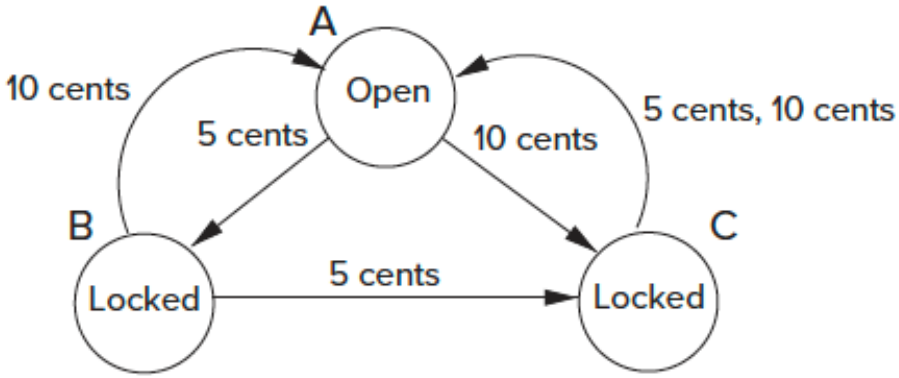


Figure 3.27 State diagram of the soft drink machine.

- There are only **three** possible states:
  A. The lock is open, so a bottle can be (or has been!) removed
  B. The lock is not open, but 5 cents have been inserted.
  C. The lock is not open, but 10 cents have been inserted.

Image source: Patt and Patel, "Introduction to Computing Systems", 2nd ed., page 84.

Another Example: Soft Drink Machine

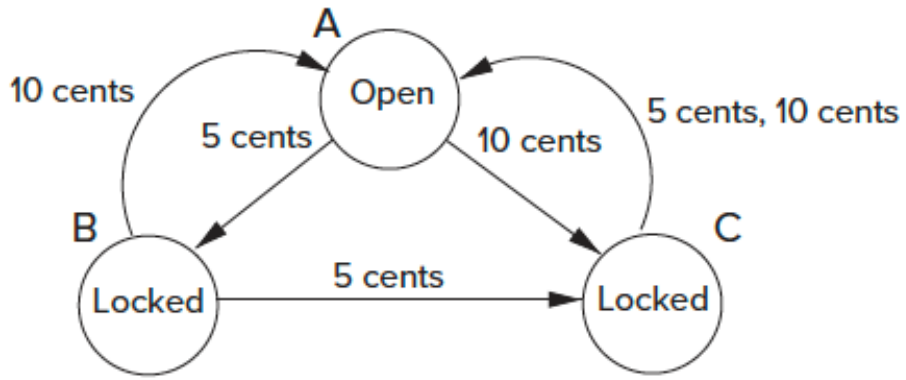


Figure 3.27 **State diagram of the soft drink machine.**

- One possible sequence of states is as follows

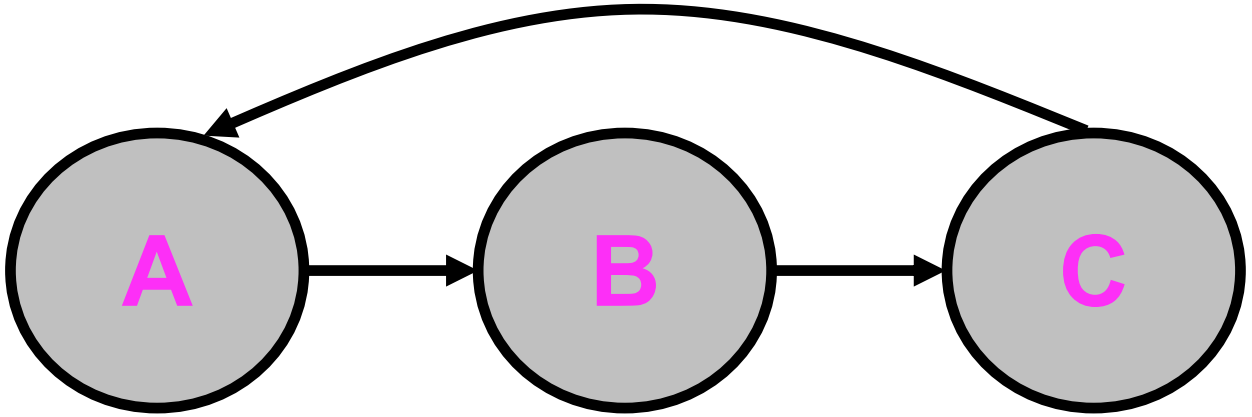
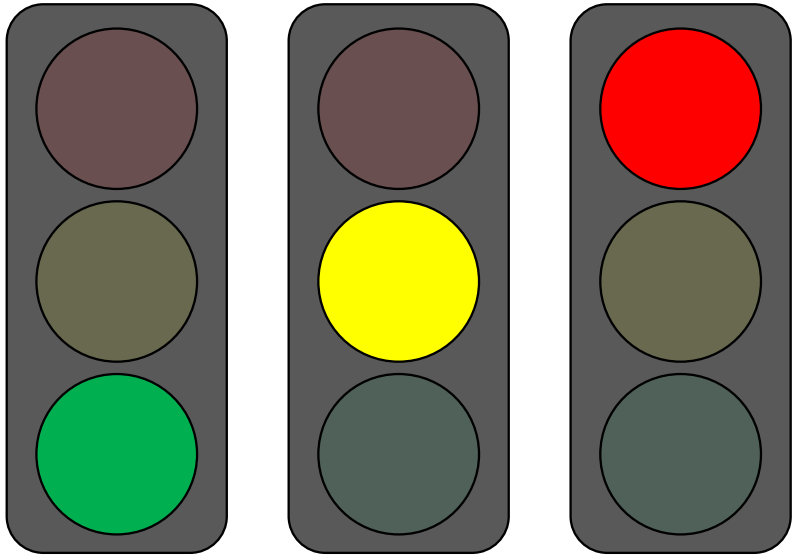


Image source: Patt and Patel, "Introduction to Computing Systems", 2nd ed., page 84.

Another Example: Traffic Light

- There are only three possible states:
  A. Green
  B. Yellow
  C. Red



- The sequence of states is as follows

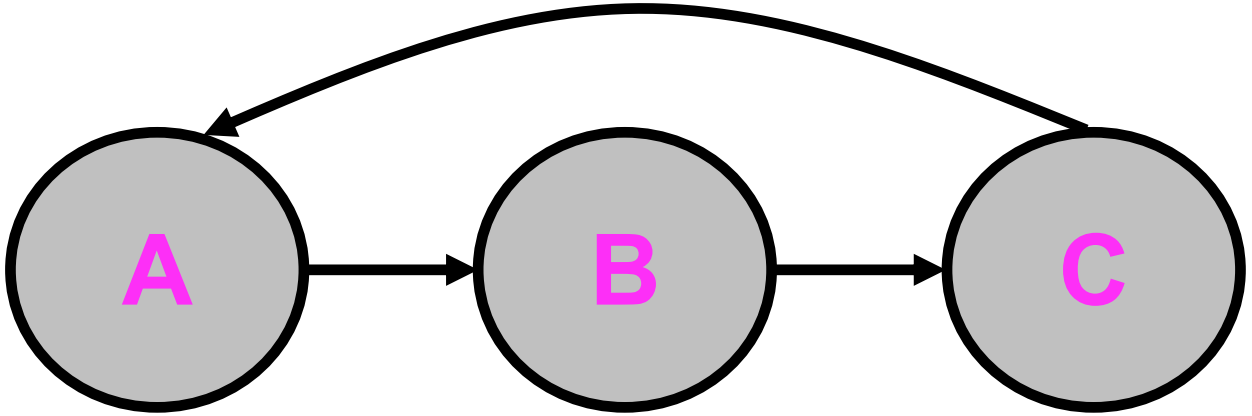
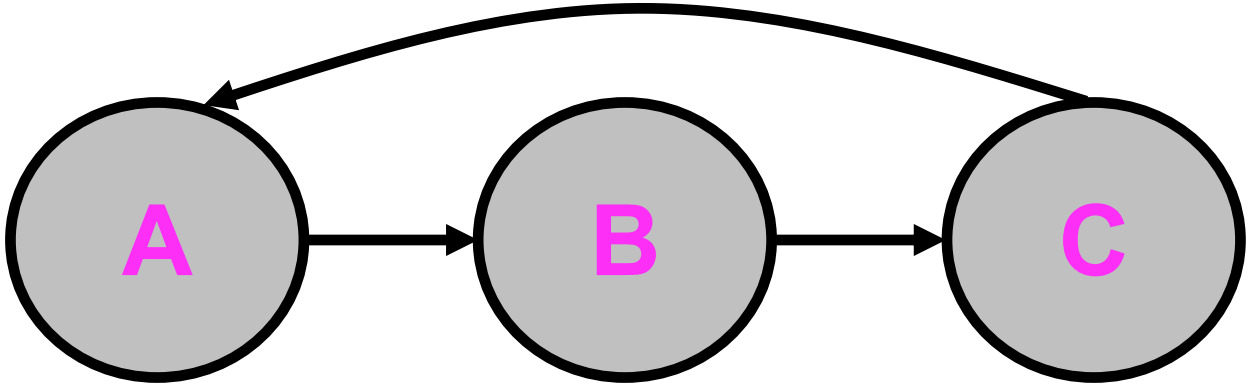


Image source: Patt and Patel, "Introduction to Computing Systems", 2nd ed., page 84.

Asynchronous vs. Synchronous State Changes

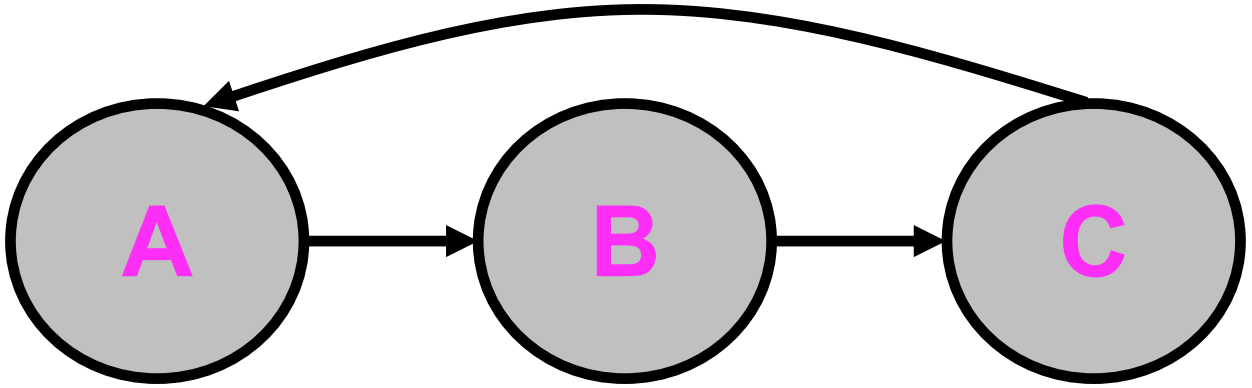
- Sequential lock we saw is an asynchronous machine
  - State transition occur when they occur
  - There is nothing that synchronizes when each state transition must occur
- Most modern computers are synchronous machines
  - State transitions take place after fixed units of time
  - Controlled in part by a clock, as we will see soon
- These are two different design paradigms, with **tradeoffs**

Changing State: The Notion of Clock (I)

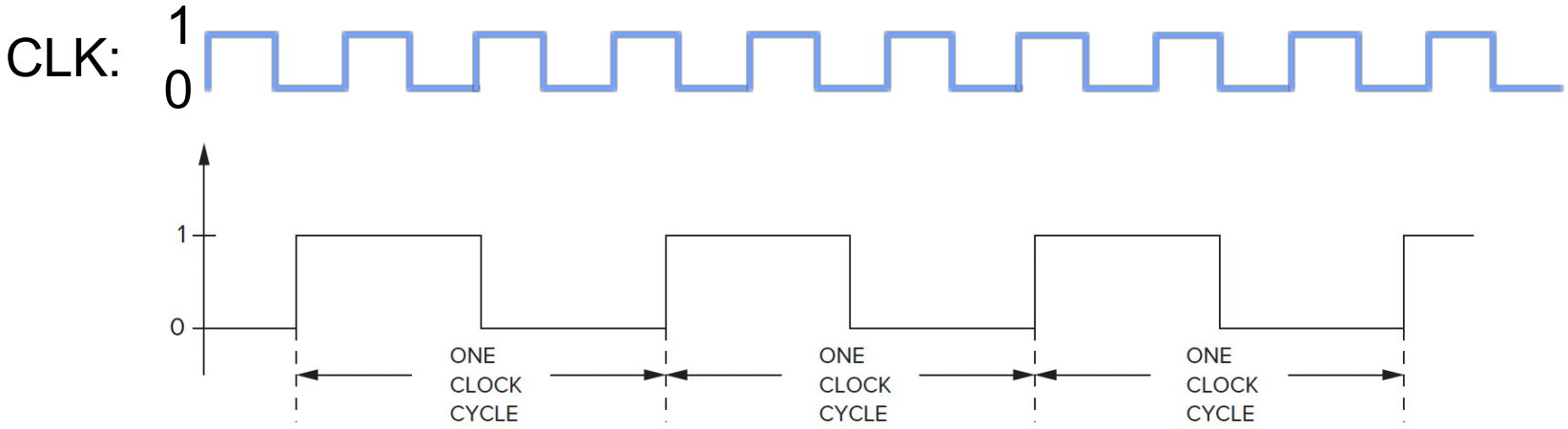


- When should the vending machine change state from **A** to **B**?
- When should the traffic light change from one state to another?

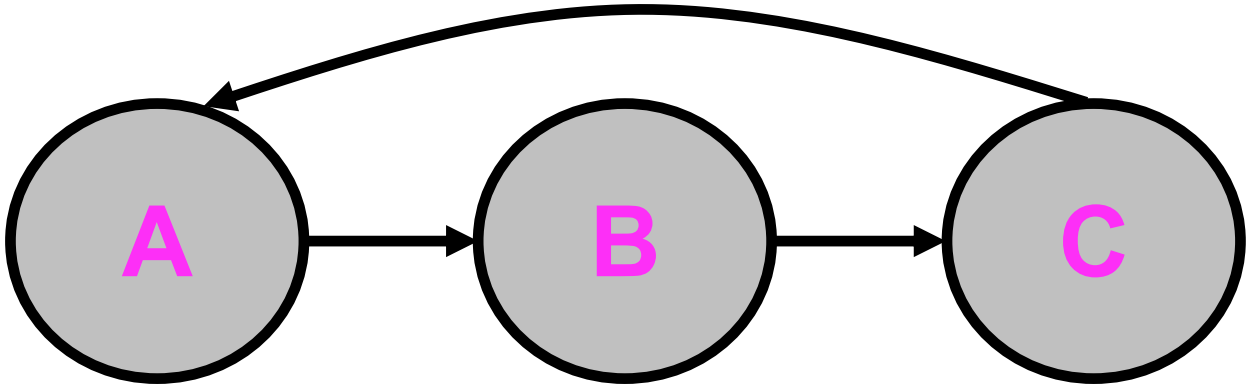
Changing State: The Notion of Clock (I)



- When should the machine change state from **A** to **B**?
- We need a **clock** to dictate when to change state
  - Clock alternates between 0 & 1



Changing State: The Notion of Clock (I)



- When should the machine change state from **A** to **B**?
- We need a **clock** to dictate when to change state
  - Clock alternates between 0 & 1

- At the start of a clock cycle (), system state changes
  - During a clock cycle, the state stays constant
  - The machine stays in a specific state an equal amount of time

Changing State: The Notion of Clock (II)

- **Clock** is a general mechanism that triggers transition from one state to another in a (synchronous) sequential circuit
- Clock synchronizes state changes across many sequential circuit elements
- Combinational logic evaluates for the length of the clock cycle
- Clock cycle should be chosen to accommodate maximum combinational delay

Asynchronous vs. Synchronous State Changes

- Sequential lock we saw is an asynchronous machine
  - State transition occur when they occur
  - There is nothing that synchronizes when each state transition must occur

- Most modern computers are synchronous machines
  - State transitions take place after fixed units of time
  - Controlled in part by a clock, as we will see soon

- These are two different design paradigms, with **tradeoffs**
  - Synchronous control can be easier to get correct when the system consists of **many components and many states**
  - Asynchronous control can be more efficient (no clock **overheads**)

We will assume synchronous systems in this course

"the art of directing the **simultaneous** performance of several players or singers by the use of gesture." Wikipedia, Conducting

We will assume synchronous systems in this course

Finite State Machines

Compulsory Reading: Section **3.4** of H&H

Finite State Machines

- What is a **Finite State Machine** (FSM)?
  - **A discrete-time model** of a stateful system
  - Each state of the system at a given time

- An **FSM** pictorially shows
  1. The set of all possible states that a system can be in
  2. How the system transitions from one state to another

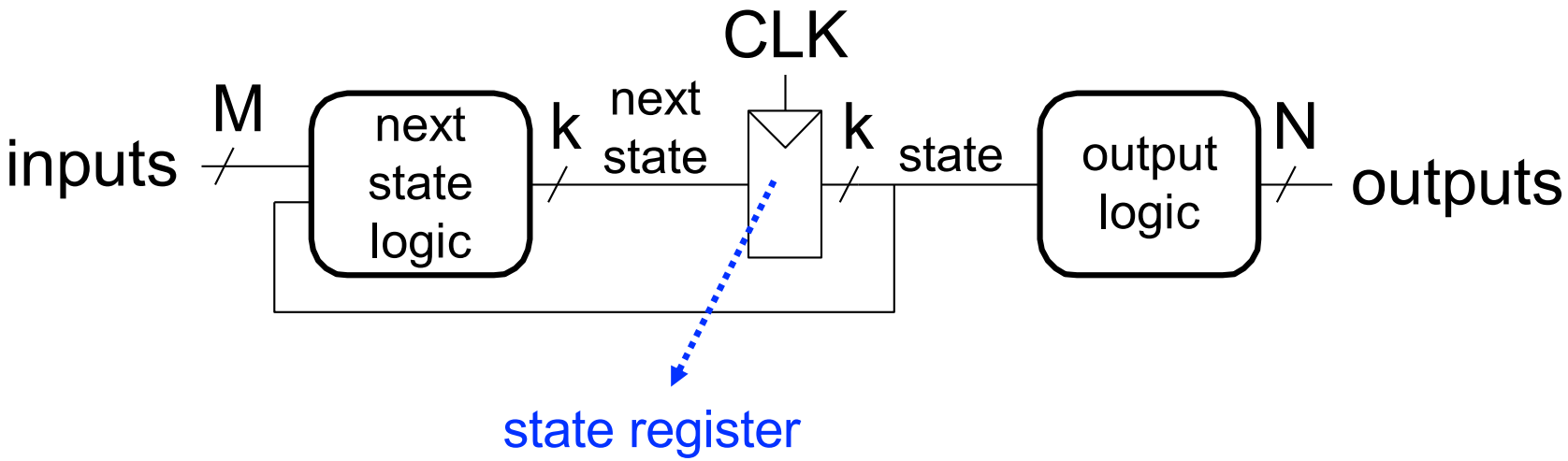
- An **FSM** can model
  - A traffic light, an elevator, microwave, microprocessor, fan speed, car lock

FSMs Consist of:

- **Five elements:**
  - A **finite** number of states
    - ***State***: snapshot of all relevant elements of the system at the time of the snapshot
  - A **finite** number of external inputs
  - A **finite** number of external outputs
  - An explicit specification of all state transitions
    - How to get from one state to another
  - An explicit specification of what determines each external output value
    - E.g., If state is A, then output is RED; If state is B, output is YELLOW

Finite State Machines (FSMs)

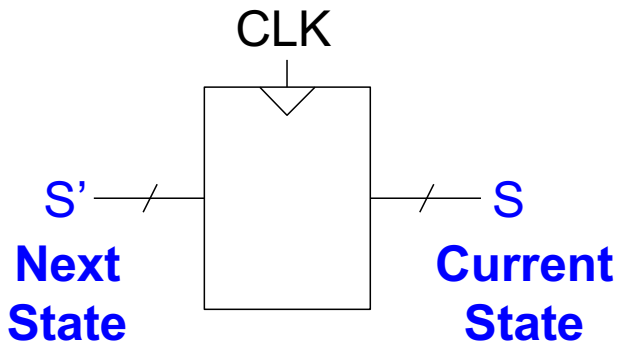
- Each FSM consists of three separate parts:
  - next state logic
  - state register
  - output logic



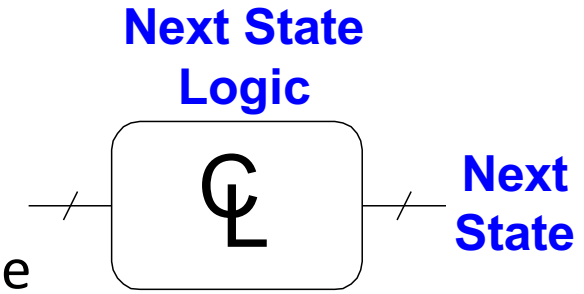
Section 3.4 of H&H

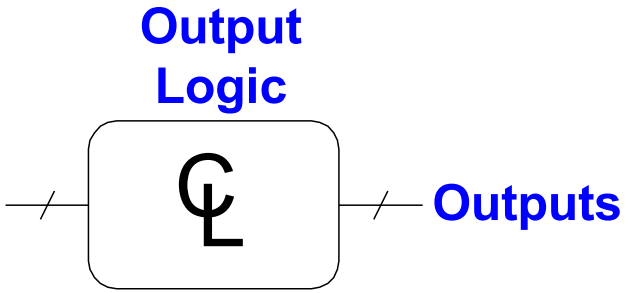
FSMs Consist of:

- **Sequential circuits**
  - State register(s)
    - Store the current state and
    - Provide the next state at the clock edge



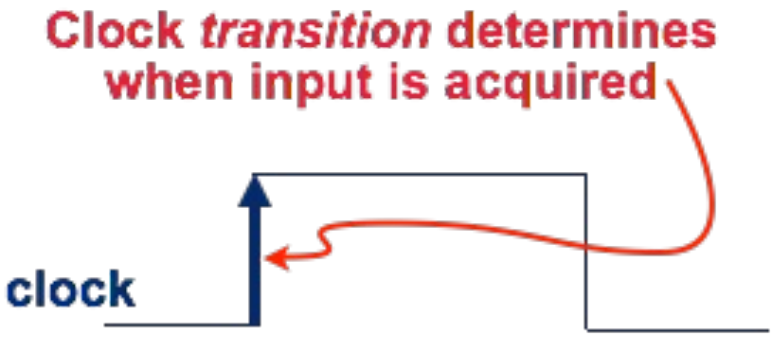
- **Combinational Circuits**
  - Next state logic
    - Determines what the next state will be
  - Output logic
    - Generates the outputs





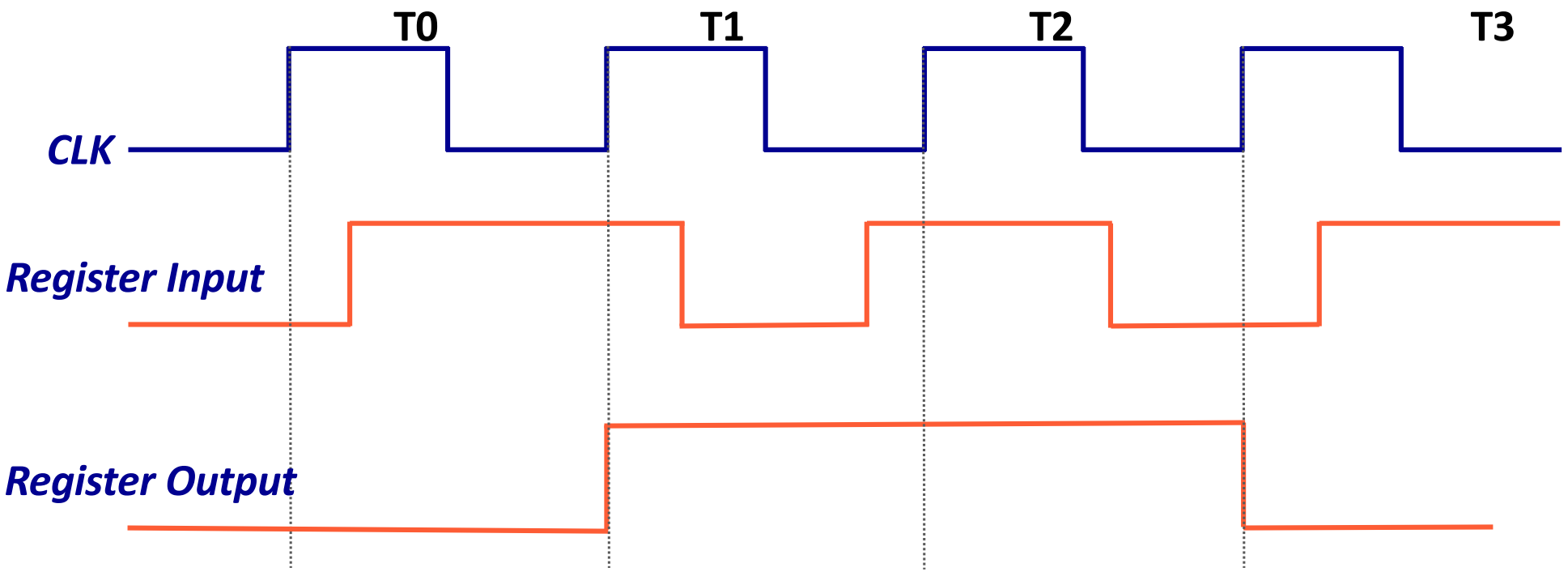
State Register

- We need flip-flops (not latches) to implement state register. **Why?**
- Properties of state register
  - We need to store data at the **beginning** of every clock cycle



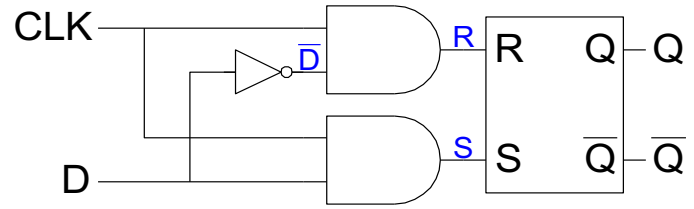
State Register

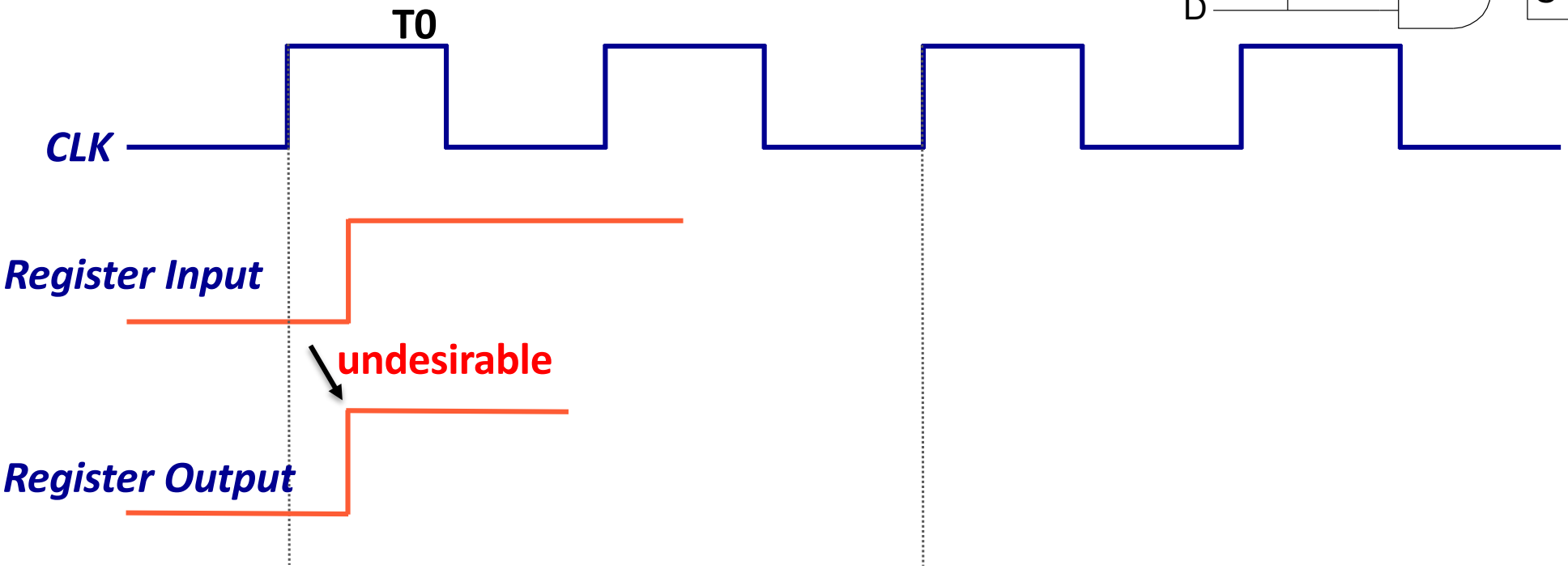
- We need flip-flops (not latches) to implement state register. **Why?**
- Properties of state register
  - The data must be **available** during the **entire clock cycle**



The Problem with Latches

- We cannot simply wire a clock to **CLK** input of a latch
  - Whenever the clock is **HIGH**, the latch propagates **D** to **Q**
  - **The latch is transparent**





State Register uses Flip-Flops

- **D** (input) is **observable** at **Q** (output) **only** at the **beginning of the next** clock cycle

- **Q** is **available for the full clock cycle**

Implementing FSMs

Traffic Light Controller

The Next Example is from H & H: Section 3.4

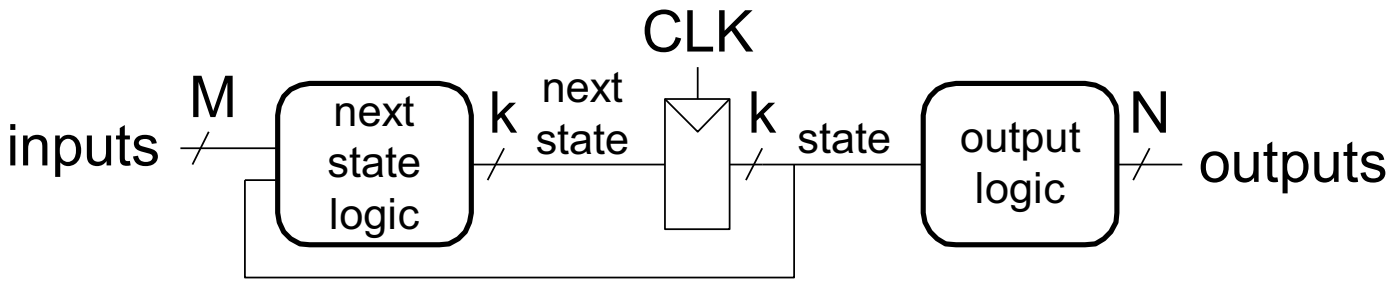
Acknowledgement: *Selection of Slides from Digital Design and Computer Architecture, Onur Mutlu, ETH Zurich, Spring 2022*
https://safari.ethz.ch/digitaltechnik/spring2022/doku.php?id=schedule

Finite State Machines (FSMs)

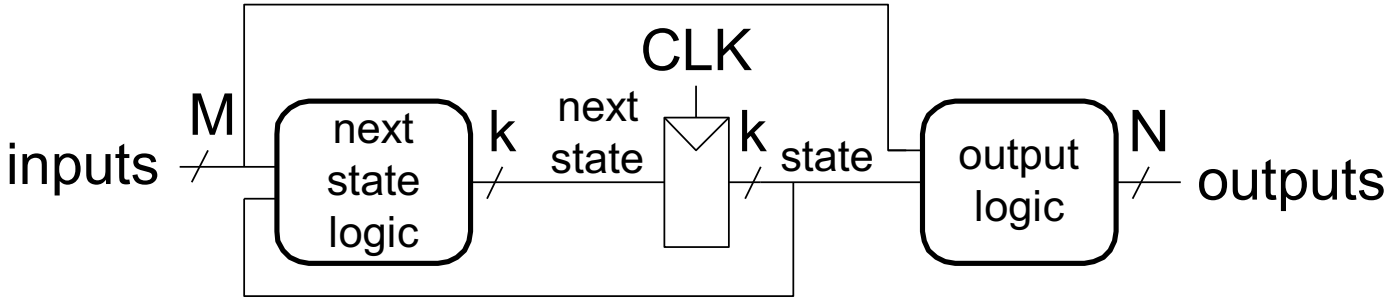
- Next state is determined by the current state and the inputs
- Two types of finite state machines differ in the **output logic**:
  - **Moore FSM**: outputs depend only on the current state
  - **Mealy FSM**: outputs depend on the current state and inputs

Finite State Machines (FSMs)

Moore FSM

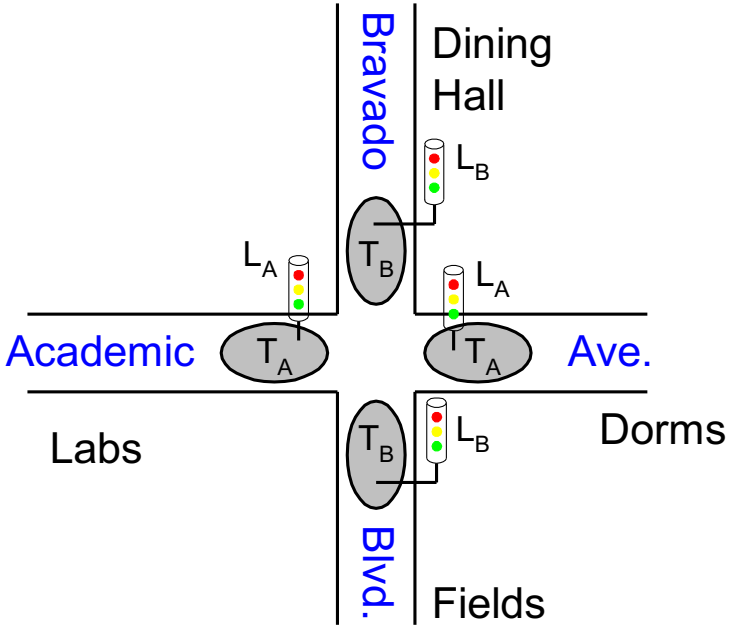


Mealy FSM



Finite State Machine Example

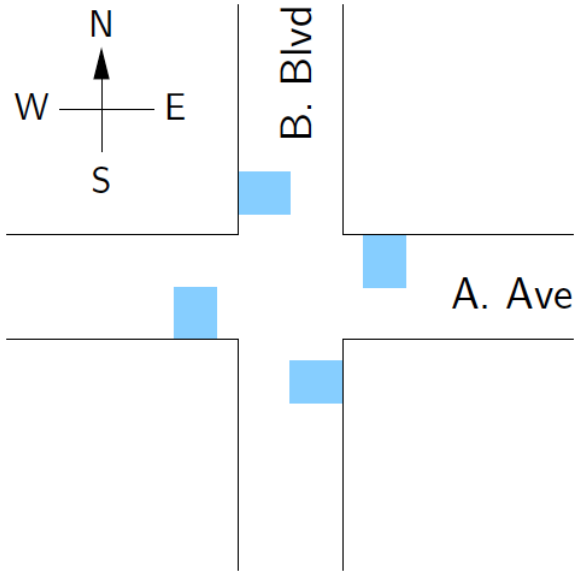
- "Smart" traffic light controller
  - **2 inputs**:
    - Traffic sensors: T_A , T_B (TRUE when there's traffic)
  - **2 outputs**:
    - Lights: L_A , L_B (Red, Yellow, Green)
  - State can change every 5 seconds
    - Except if green and traffic, stay green



From H&H Section 3.4.1

Finite State Machine Example

- Traffic sensors are built into the road
- Each sensor indicates if a street is empty or there are vehicles nearby

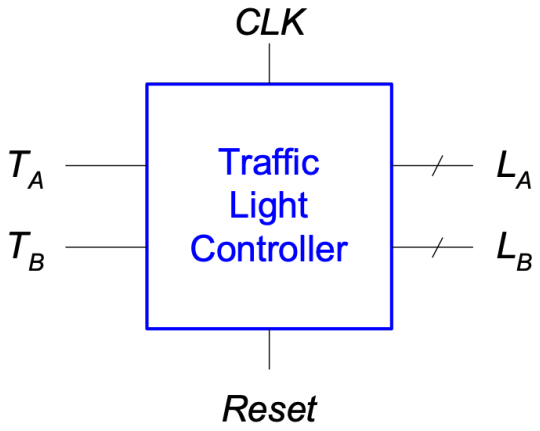


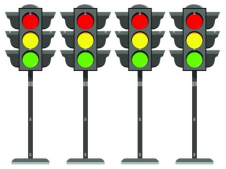
T_A = (eastbound traffic on A) OR (westbound traffic on A)

T_B = (northbound traffic on B) OR (southbound traffic on B)

Finite State Machine Example

- Inputs T_A and T_B
  - Returns TRUE if there are cars on the road
  - Returns FALSE if the road is empty
- Outputs $L_{A1:0}$ and $L_{B1:0}$
  - Each set of lights receive 2-bit digital inputs from the traffic light controller specifying whether it should be: RED, YELLOW, GREEN

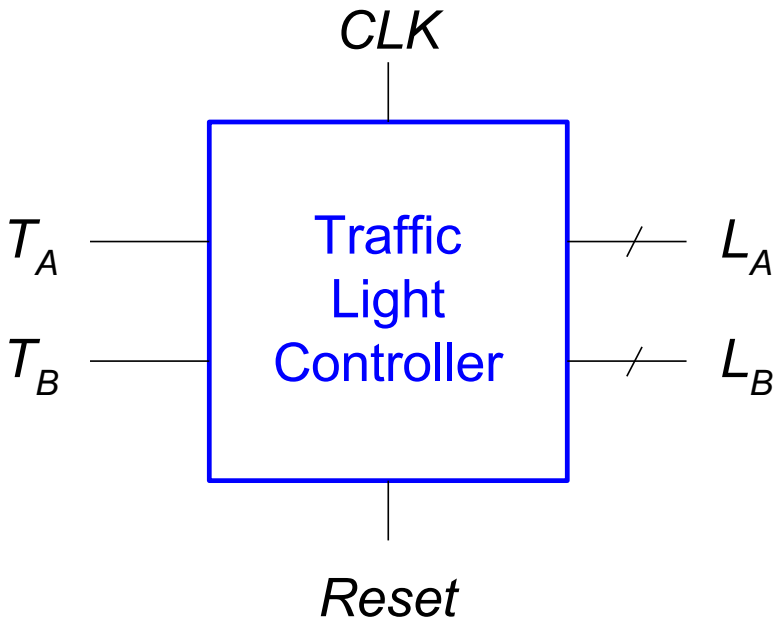




Output	Encoding
GREEN	00
YELLOW	01
RED	10

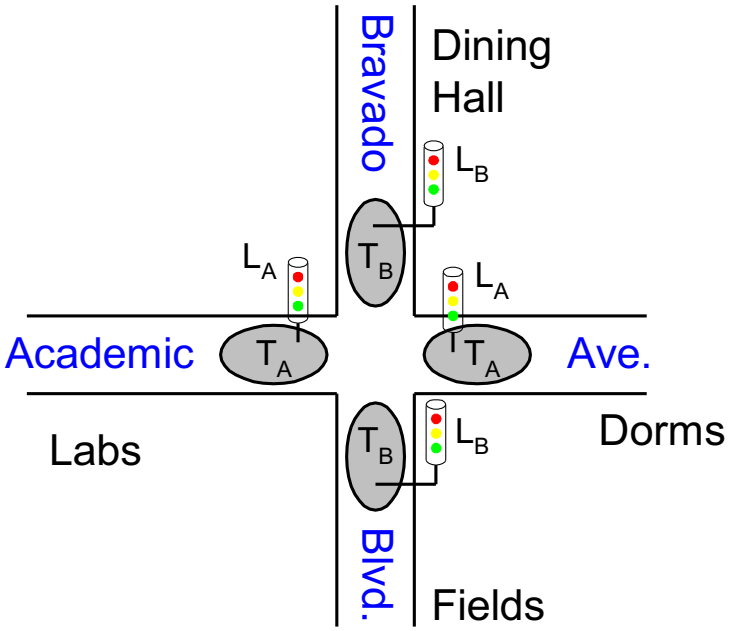
Finite State Machine Blackbox

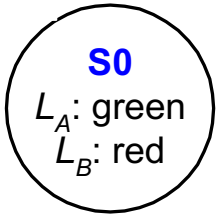
- **Inputs:** CLK, Reset, T_A , T_B
- **Outputs:** L_A , L_B



Finite State Machine Diagram

- **Moore FSM:** outputs labeled in each state
  - **States:** Circles
  - **Transitions:** Arrows (Arcs)

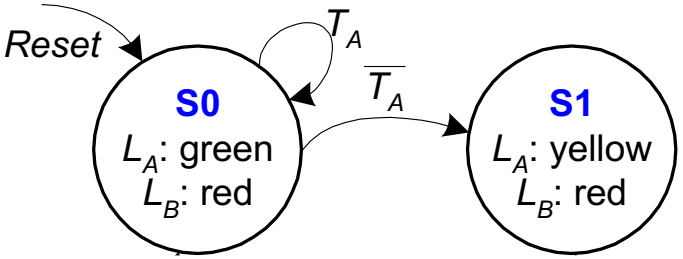


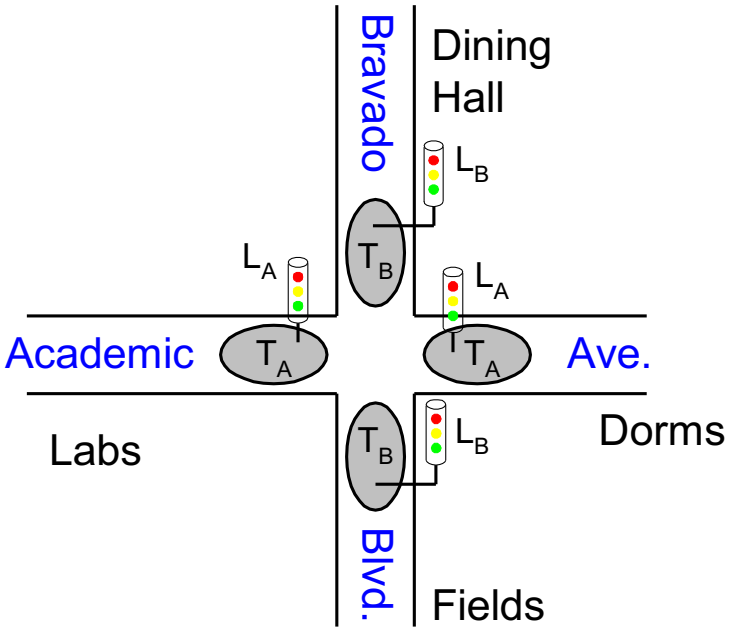


Finite State Machine Diagram

- **Moore FSM:** outputs labeled in each state
  - **States:** Circles
  - **Transitions:** Arrows (Arcs)

→ From "current state" **S0** to "next state" **S1**





Finite State Machine Diagram

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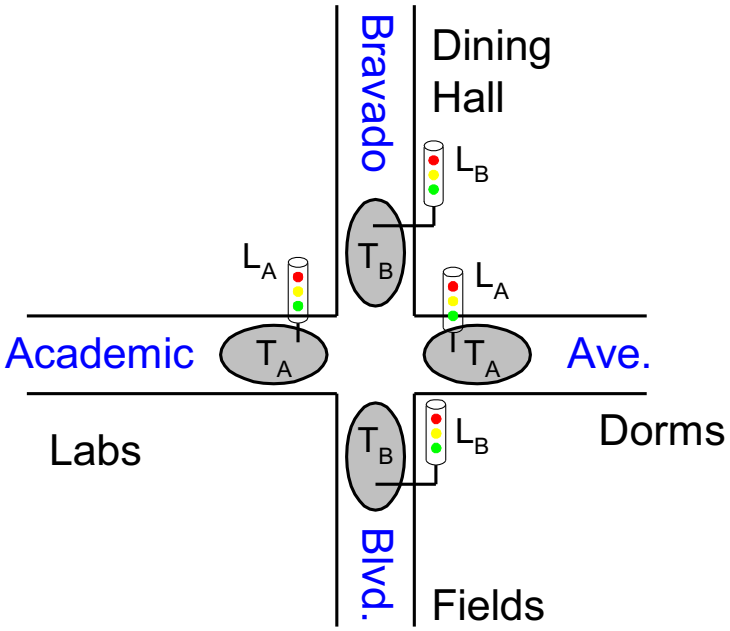
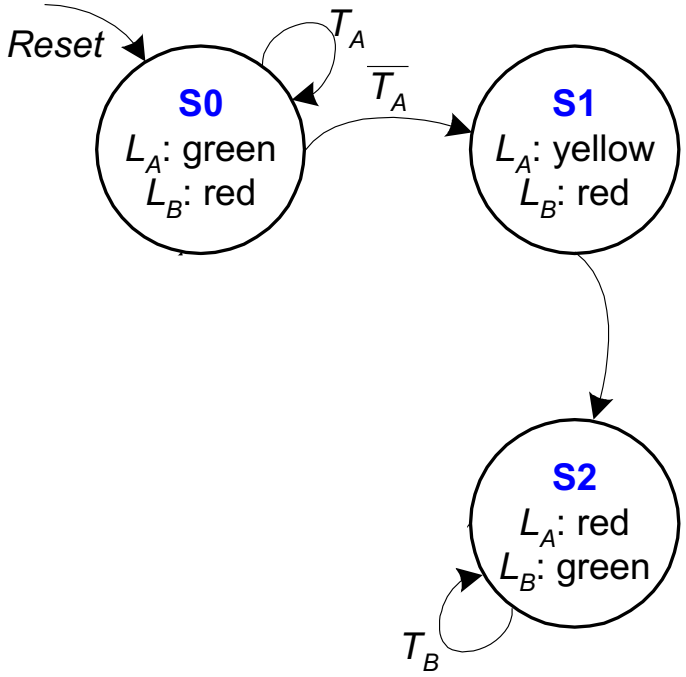
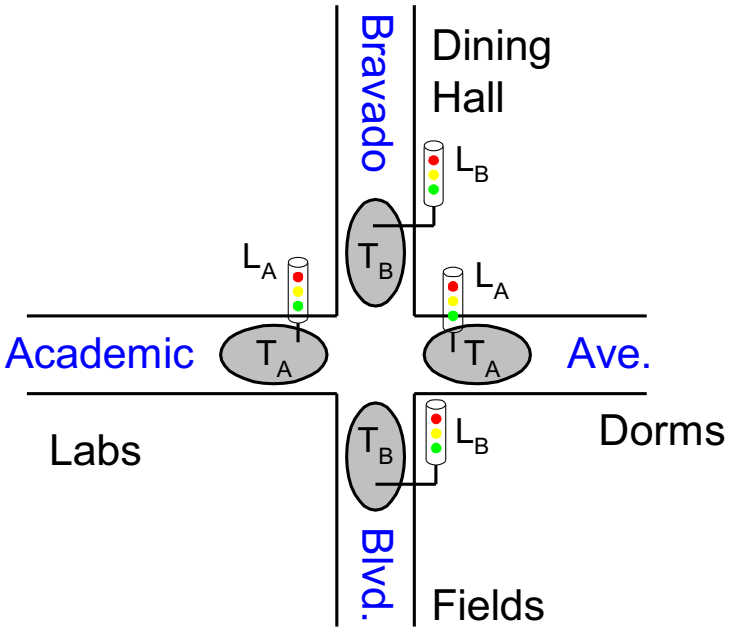
  - **States:** Circles
  - **Transitions:** Arrows (Arcs)

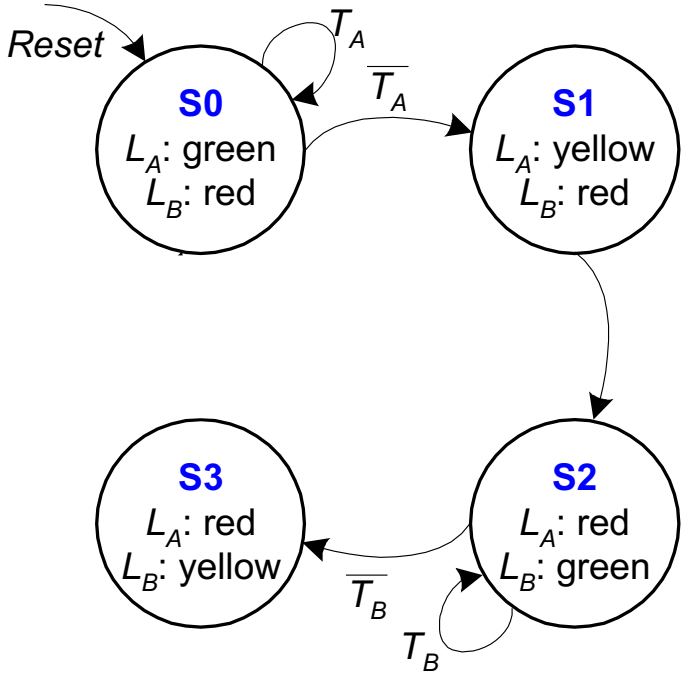




Finite State Machine Diagram

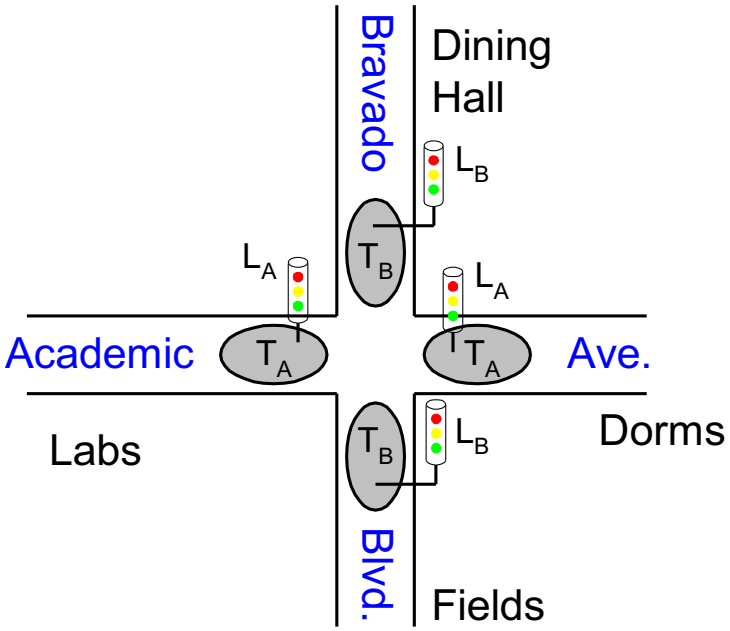
- **Moore FSM:** outputs labeled in each state
  - **States:** Circles
  - **Transitions:** Arrows (Arcs)

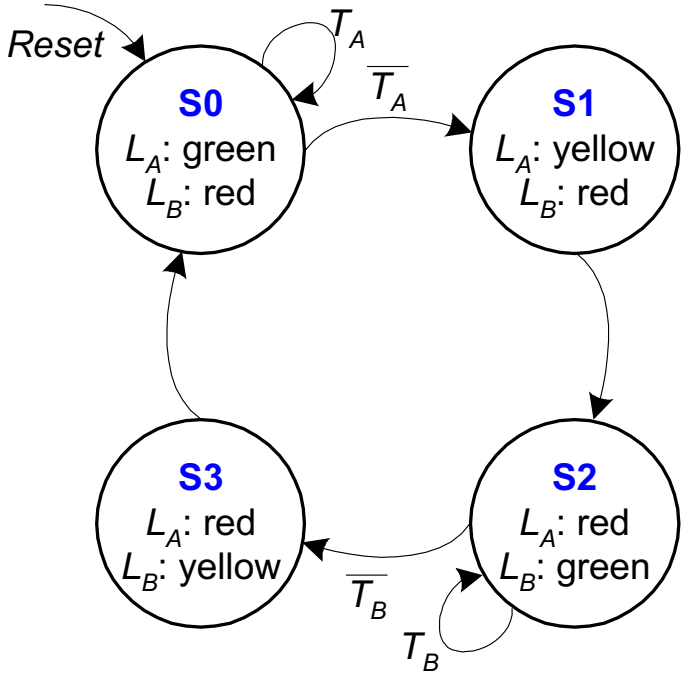




Finite State Machine Diagram

- **Moore FSM:** outputs labeled in each state
  - **States:** Circles
  - **Transitions:** Arrows (Arcs)

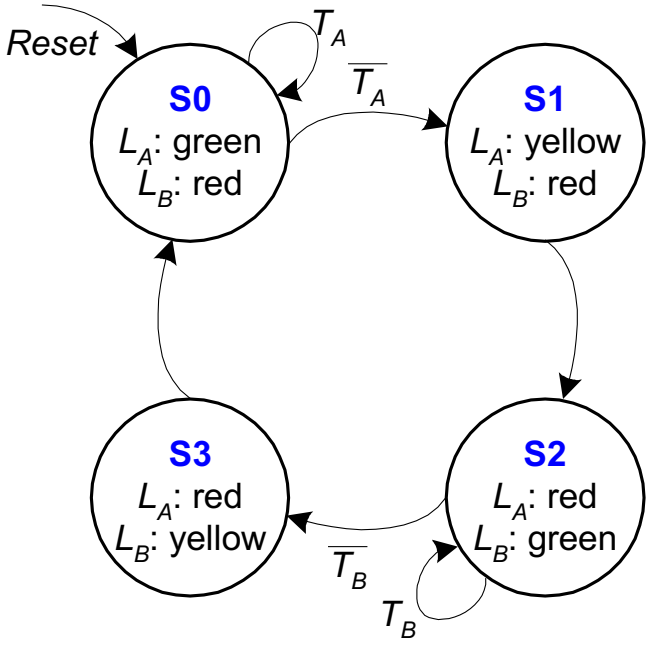




Finite State Machine:

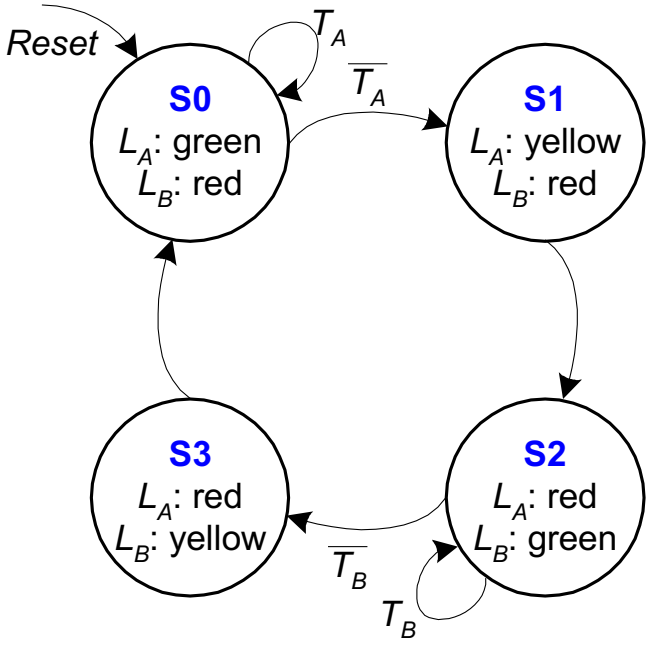
State Transition Table

FSM State Transition Table



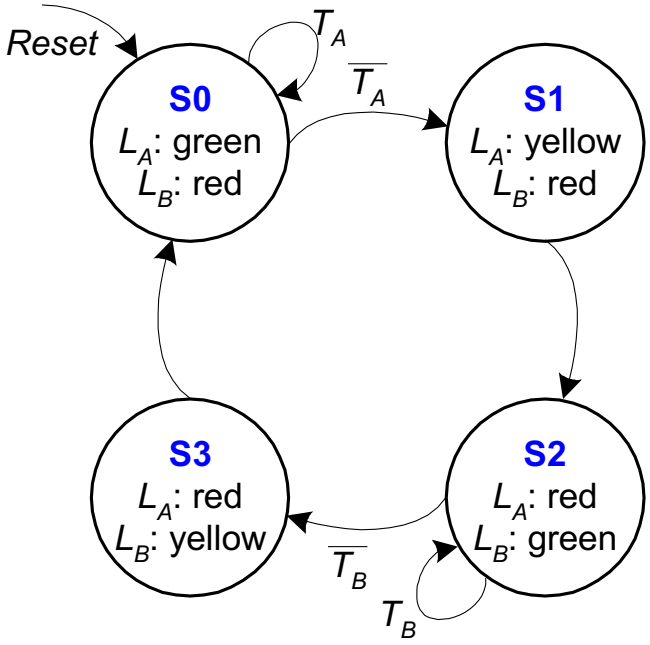
Current State	Inputs		Next State
S	T_A	T_B	S'
S0	0	X	
S0	1	X	
S1	X	X	
S2	X	0	
S2	X	1	
S3	X	X	

FSM State Transition Table



Current State	Inputs		Next State
S	T_A	T_B	S'
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0

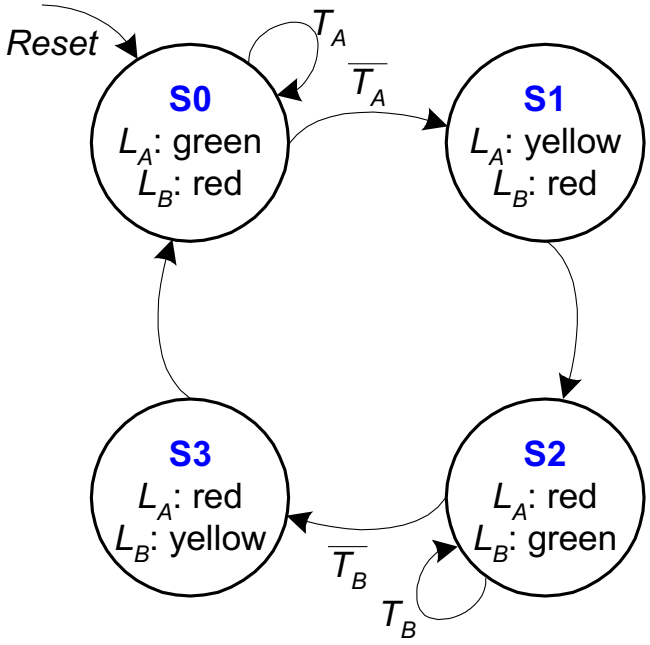
FSM State Transition Table



Current State	Inputs		Next State
S	T_A	T_B	S'
S0	0	X	S1
S0	1	X	S0
S1	X	X	S2
S2	X	0	S3
S2	X	1	S2
S3	X	X	S0

State	Encoding
S0	00
S1	01
S2	10
S3	11

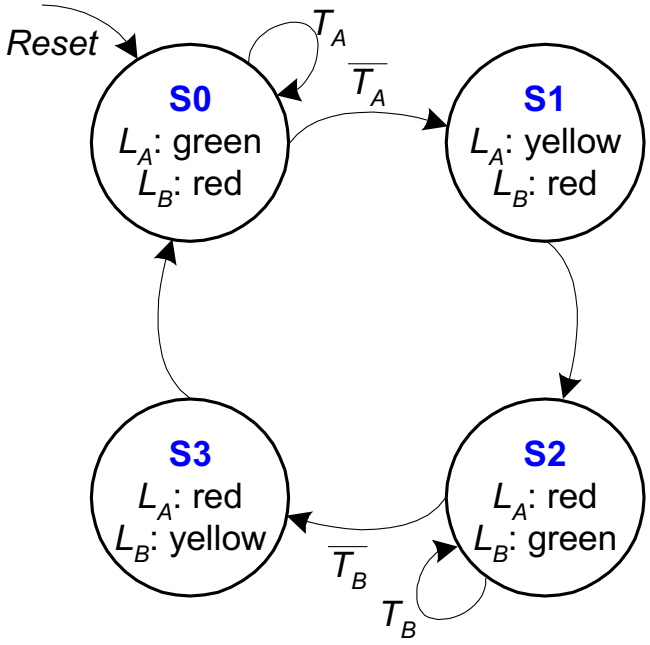
FSM State Transition Table



Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

FSM State Transition Table

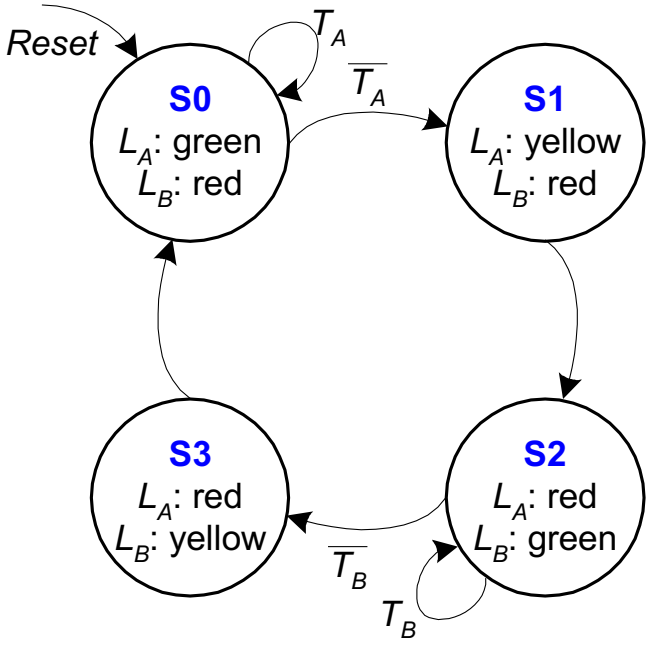


Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

S'_1 = ?

FSM State Transition Table

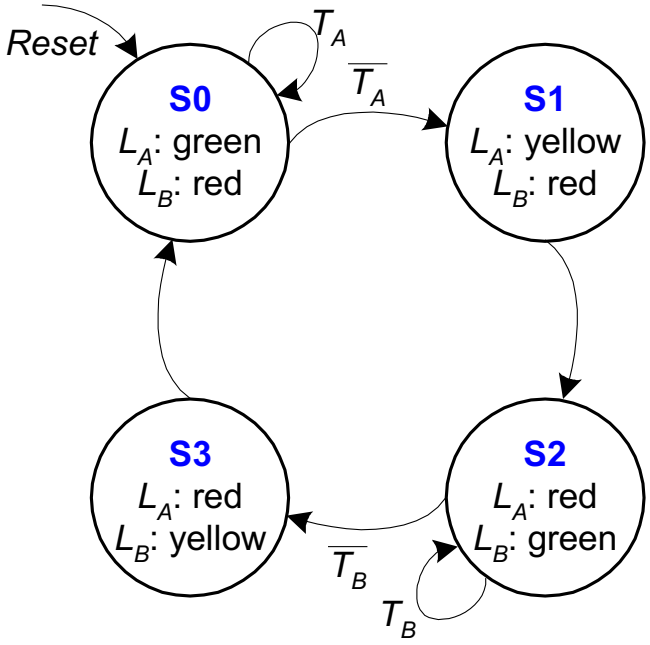


Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = (\overline{S}_1 \cdot S_0) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B) + (S_1 \cdot \overline{S}_0 \cdot T_B)$$

FSM State Transition Table



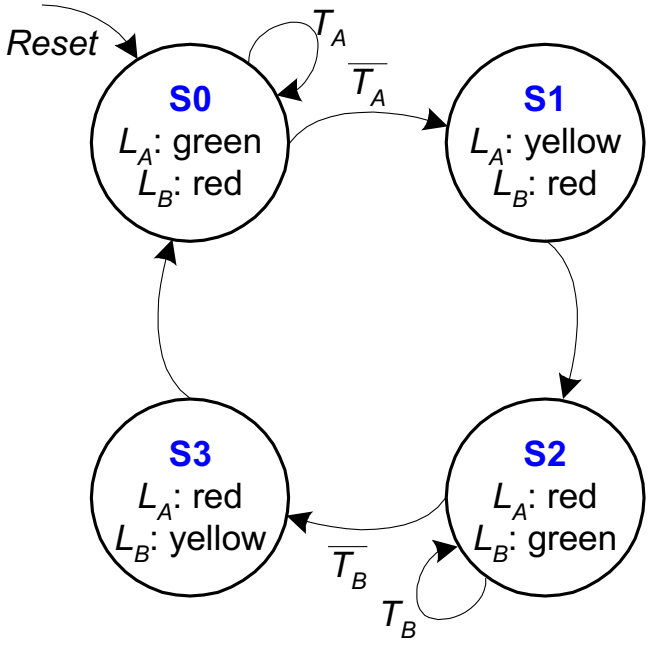
Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = (\overline{S}_1 \cdot S_0) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B) + (S_1 \cdot \overline{S}_0 \cdot T_B)$$

$$S'_0 = ?$$

FSM State Transition Table



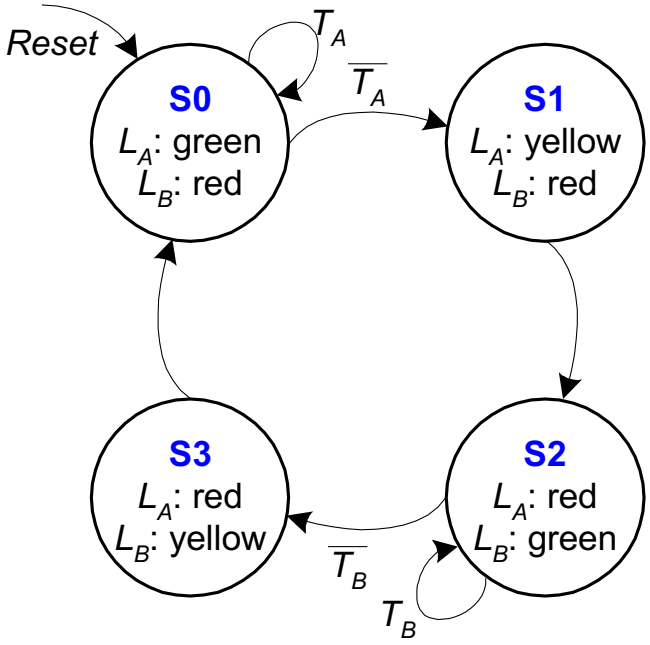
Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

$$S'_1 = (\overline{S}_1 \cdot S_0) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B) + (S_1 \cdot \overline{S}_0 \cdot T_B)$$

$$S'_0 = (\overline{S}_1 \cdot \overline{S}_0 \cdot \overline{T}_A) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B)$$

FSM State Transition Table



Current State		Inputs		Next State	
S_1	S_0	T_A	T_B	S'_1	S'_0
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

State	Encoding
S0	00
S1	01
S2	10
S3	11

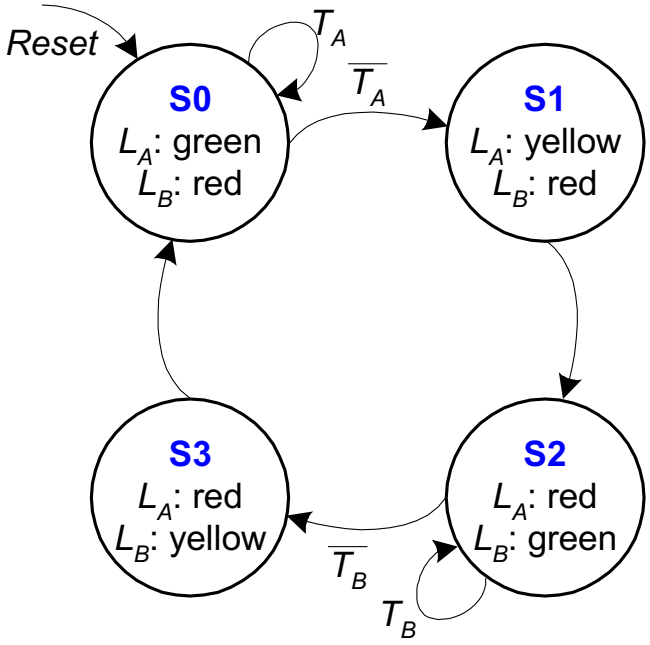
$$S'_1 = S_1 \text{ xor } S_0 \quad \textbf{(Simplified)}$$

$$S'_0 = (\overline{S}_1 \cdot \overline{S}_0 \cdot \overline{T}_A) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B)$$

Finite State Machine:

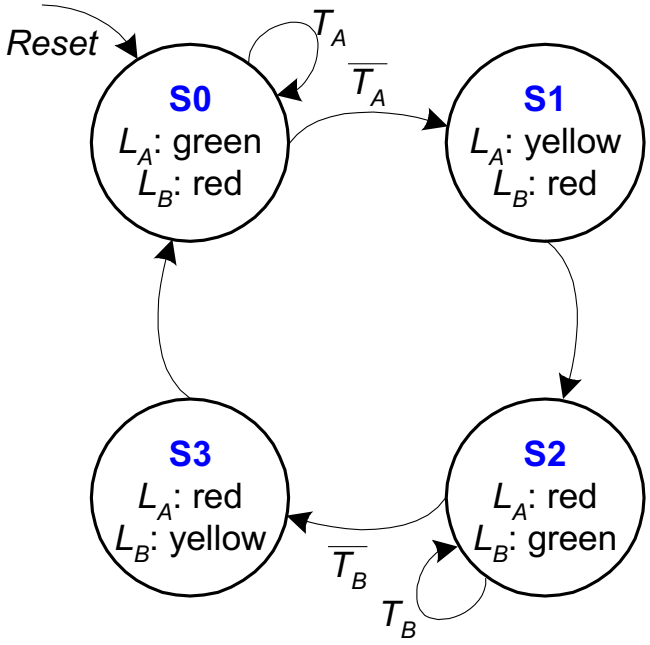
Output Table

FSM Output Table



Current State		Outputs	
S_1	S_0	L_A	L_B
0	0	green	red
0	1	yellow	red
1	0	red	green
1	1	red	yellow

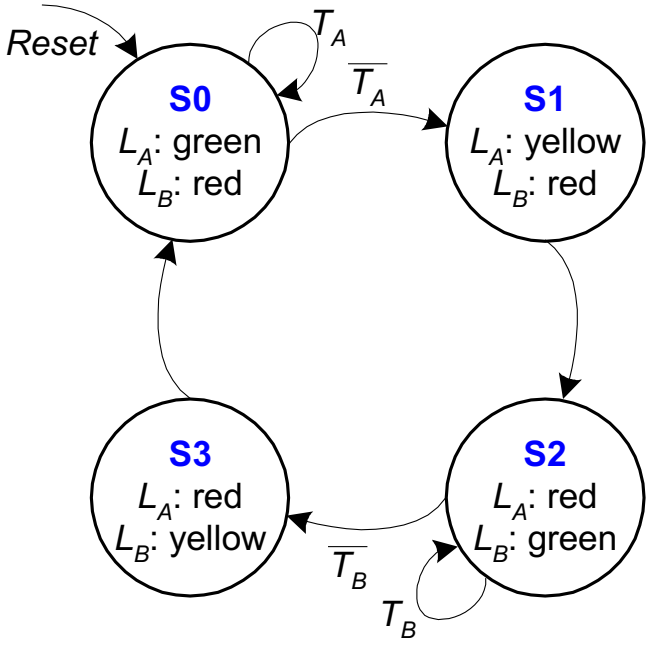
FSM Output Table



Current State		Outputs	
S_1	S_0	L_A	L_B
0	0	green	red
0	1	yellow	red
1	0	red	green
1	1	red	yellow

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

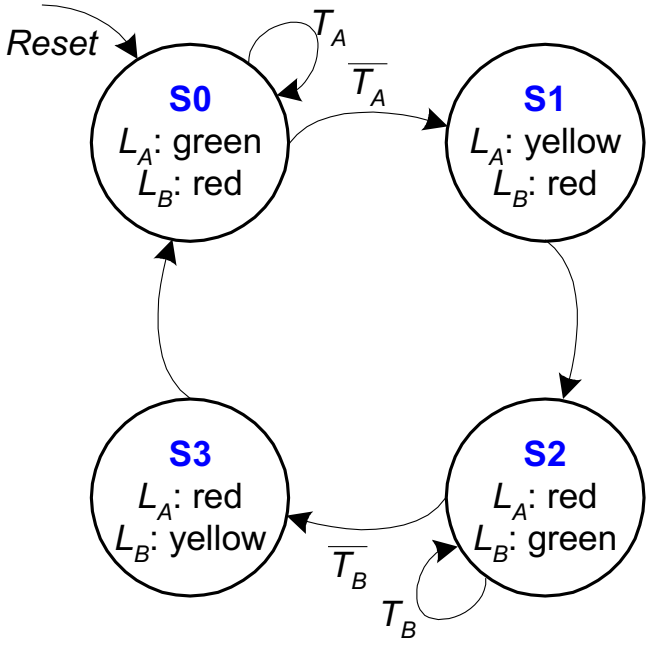


$L_{A1} = S_1$

Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

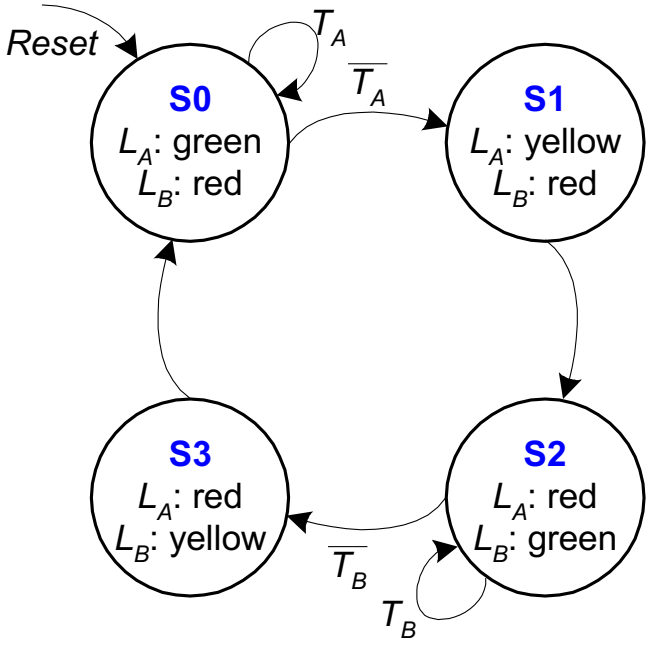


Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

$$L_{A1} = \overline{S_1}$$
$$L_{A0} = \overline{S_1} \cdot S_0$$

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table

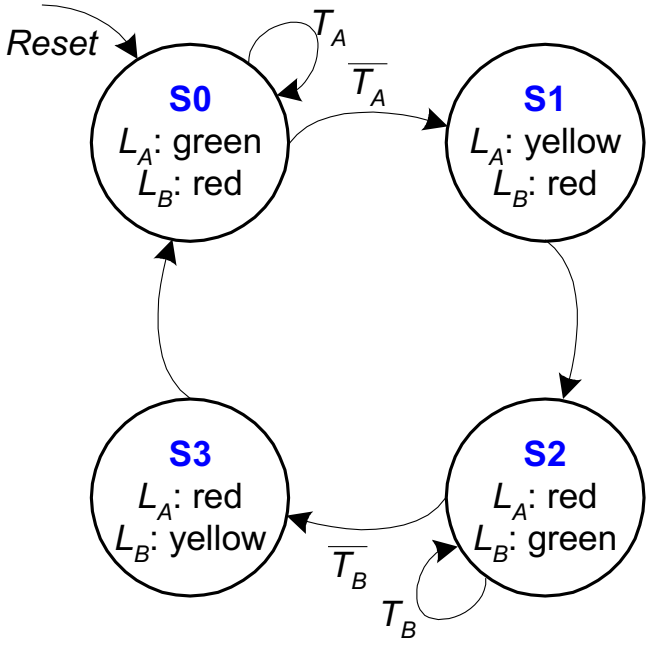


Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

$$L_{A1} = \overline{S_1}$$
$$L_{A0} = \overline{S_1} \cdot S_0$$
$$L_{B1} = \overline{S_1}$$

Output	Encoding
green	00
yellow	01
red	10

FSM Output Table



Current State		Outputs			
S_1	S_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

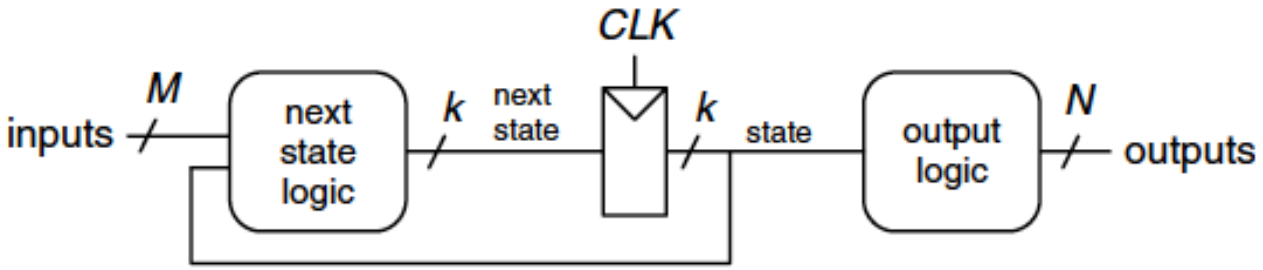
$$L_{A1} = \overline{S_1}$$
$$L_{A0} = \overline{S_1} \cdot S_0$$
$$L_{B1} = \overline{S_1}$$
$$L_{B0} = S_1 \cdot S_0$$

Output	Encoding
green	00
yellow	01
red	10

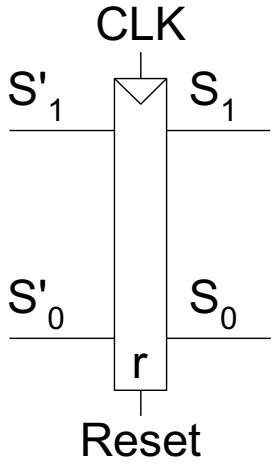
Finite State Machine:

Schematic

FSM Overview

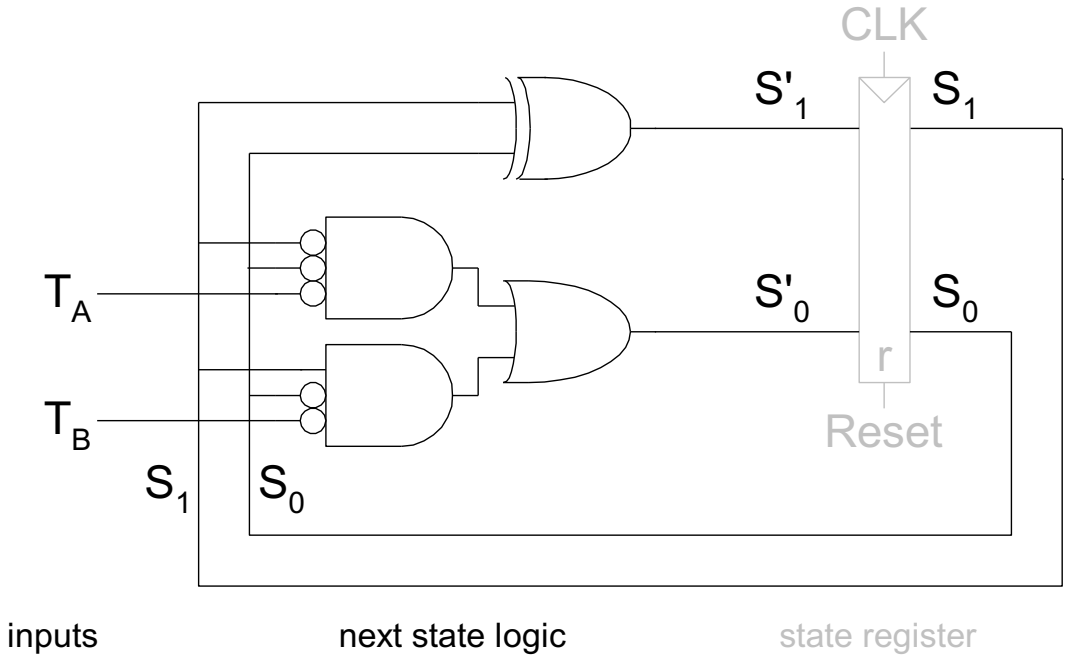


FSM Schematic: State Register



state register

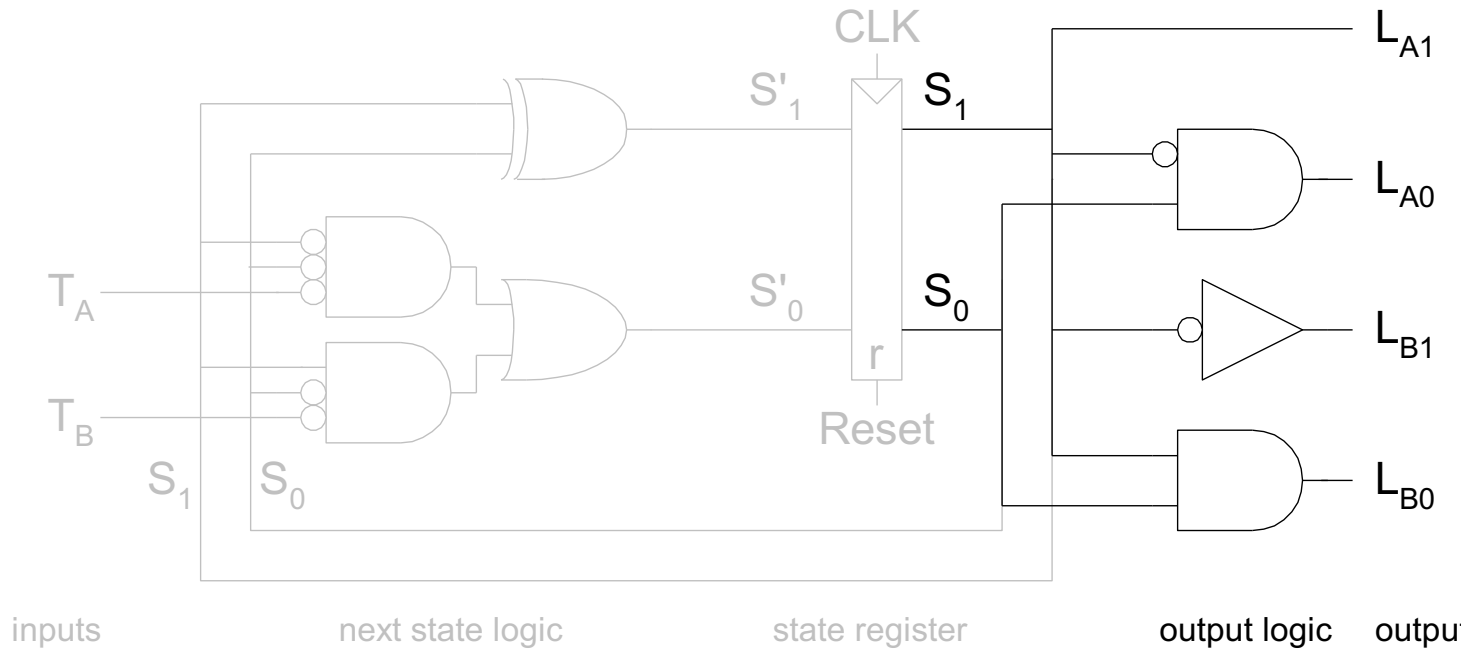
FSM Schematic: Next State Logic



$$S'_1 = S_1 \text{ xor } S_0$$

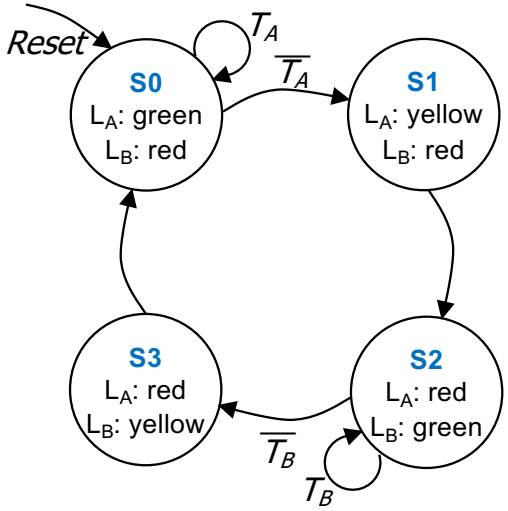
$$S'_0 = (\overline{S}_1 \cdot \overline{S}_0 \cdot \overline{T}_A) + (S_1 \cdot \overline{S}_0 \cdot \overline{T}_B)$$

FSM Schematic: Output Logic



$$L_{A1} = \overline{S_1}$$
$$L_{A0} = \overline{S_1} \cdot S_0$$
$$L_{B1} = \overline{S_1}$$
$$L_{B0} = S_1 \cdot S_0$$

FSM Timing Diagram



CLK

Reset

T_A

T_B

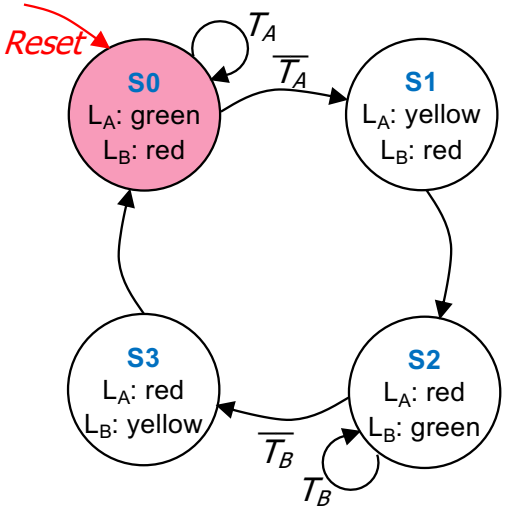
$S'_{1:0}$

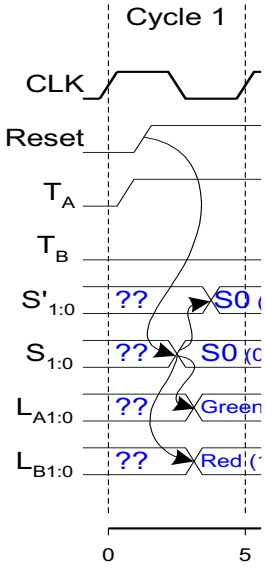
$S_{1:0}$

$L_{A1:0}$

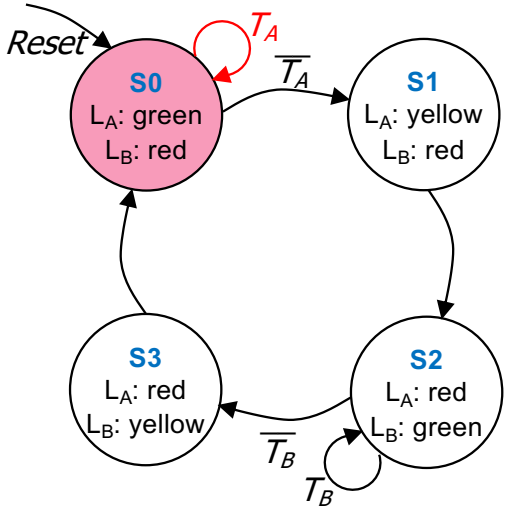
$L_{B1:0}$

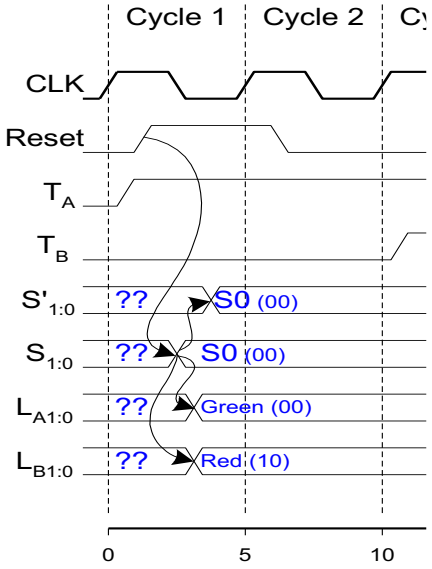
FSM Timing Diagram



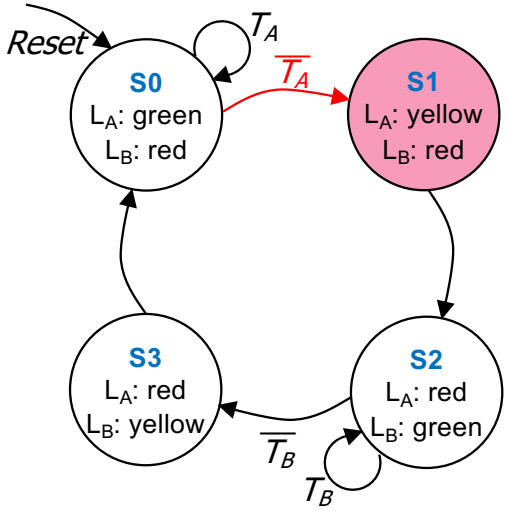


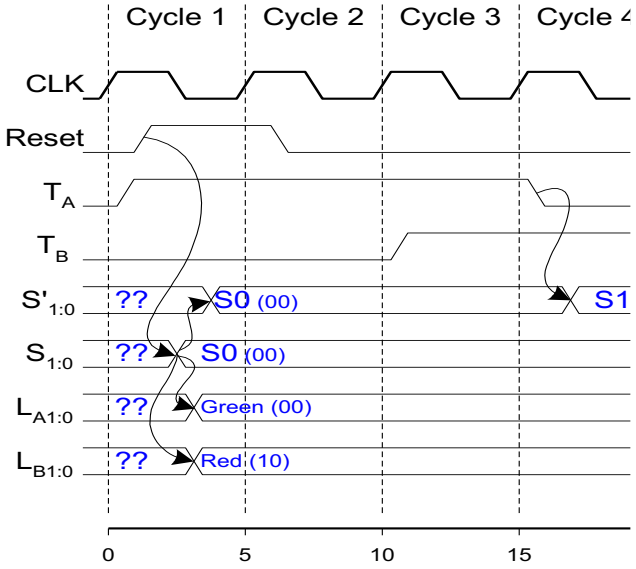
FSM Timing Diagram



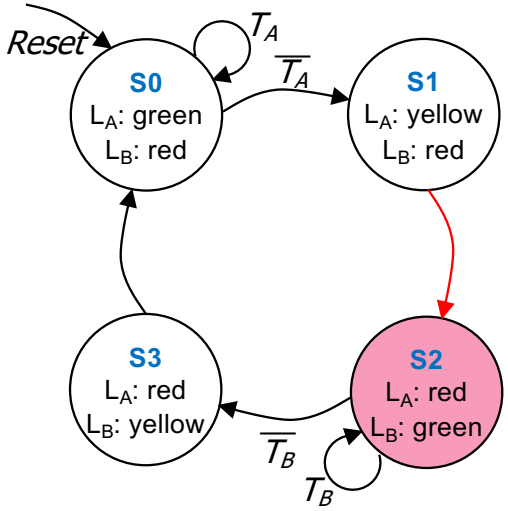


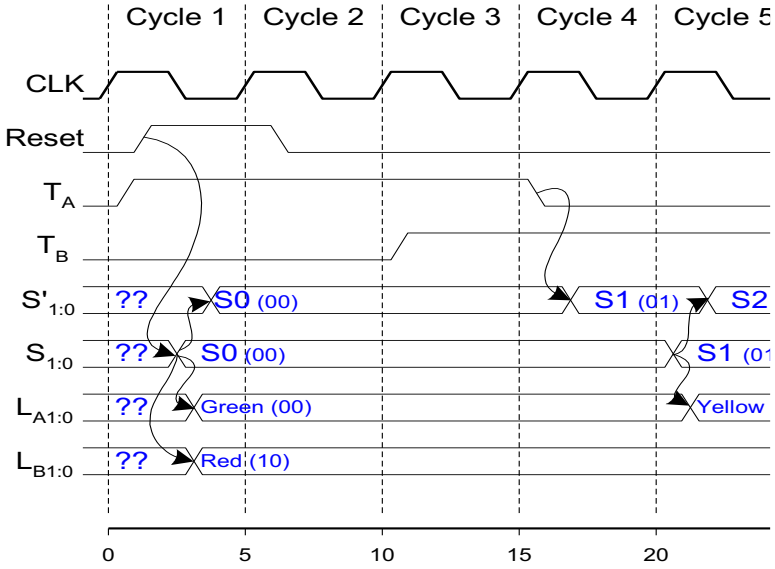
FSM Timing Diagram



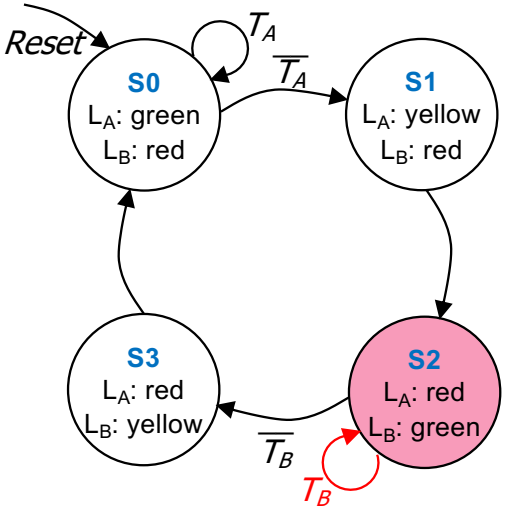


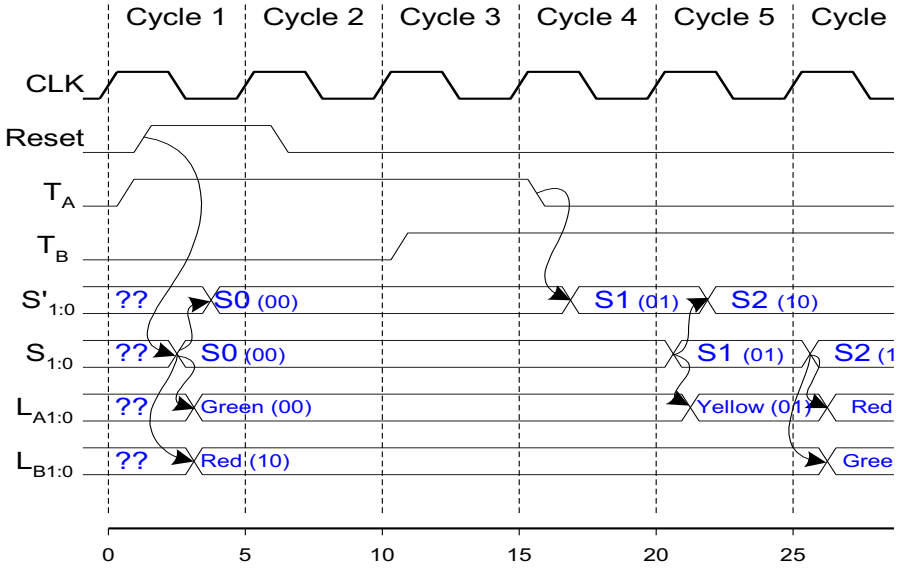
FSM Timing Diagram



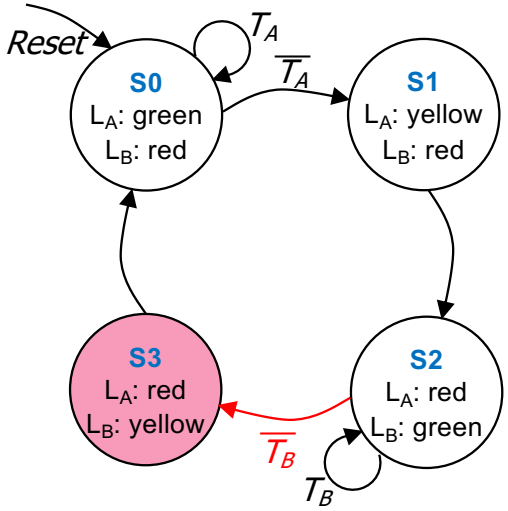


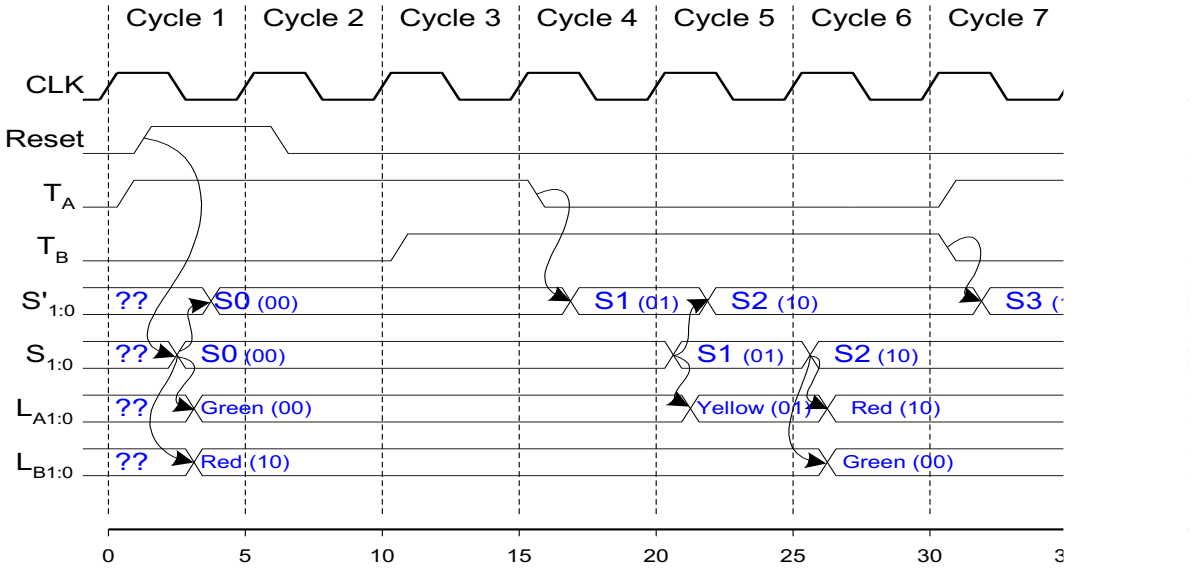
FSM Timing Diagram



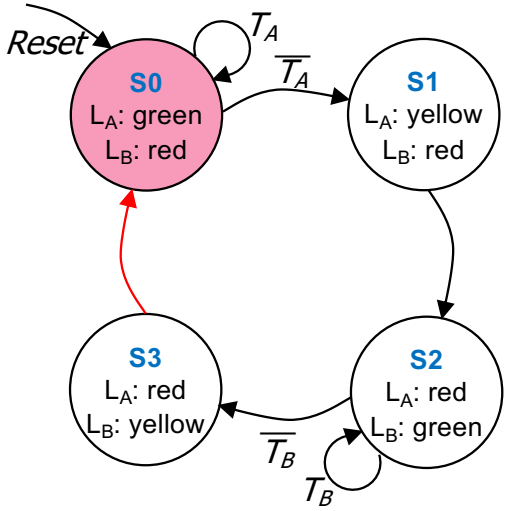


FSM Timing Diagram

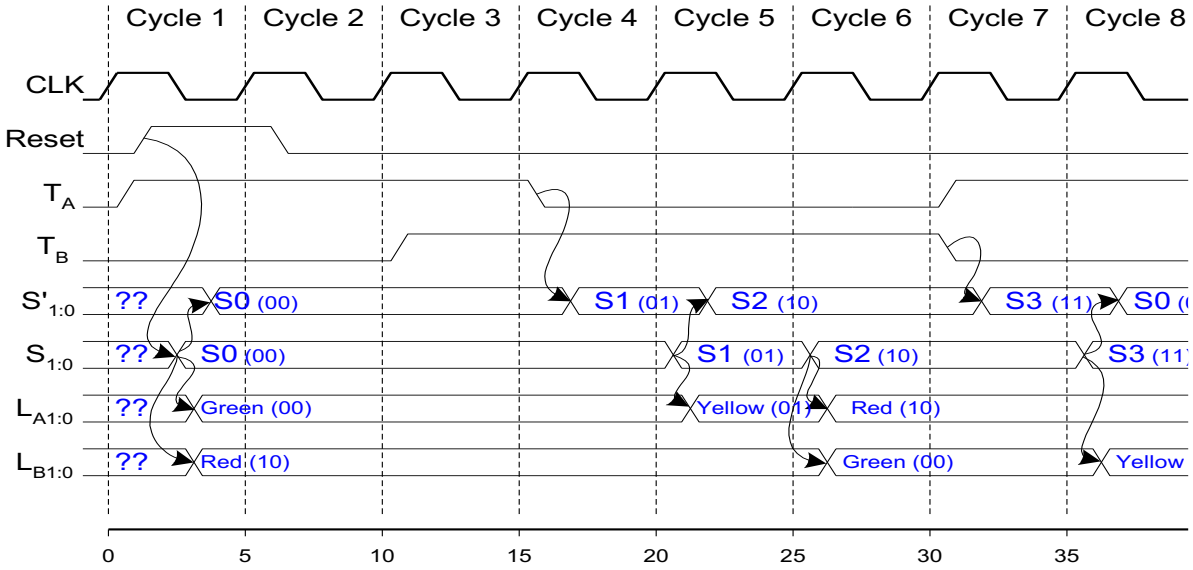




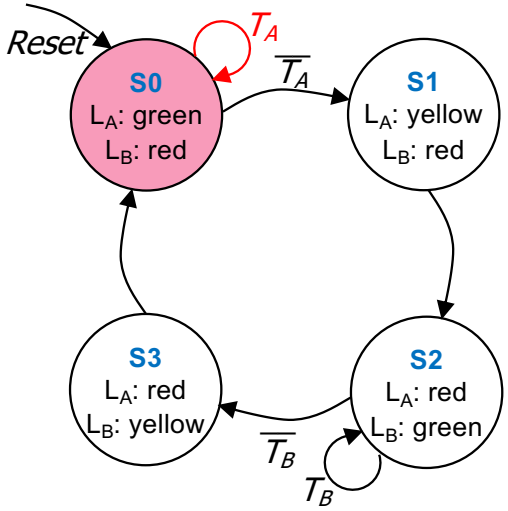
FSM Timing Diagram

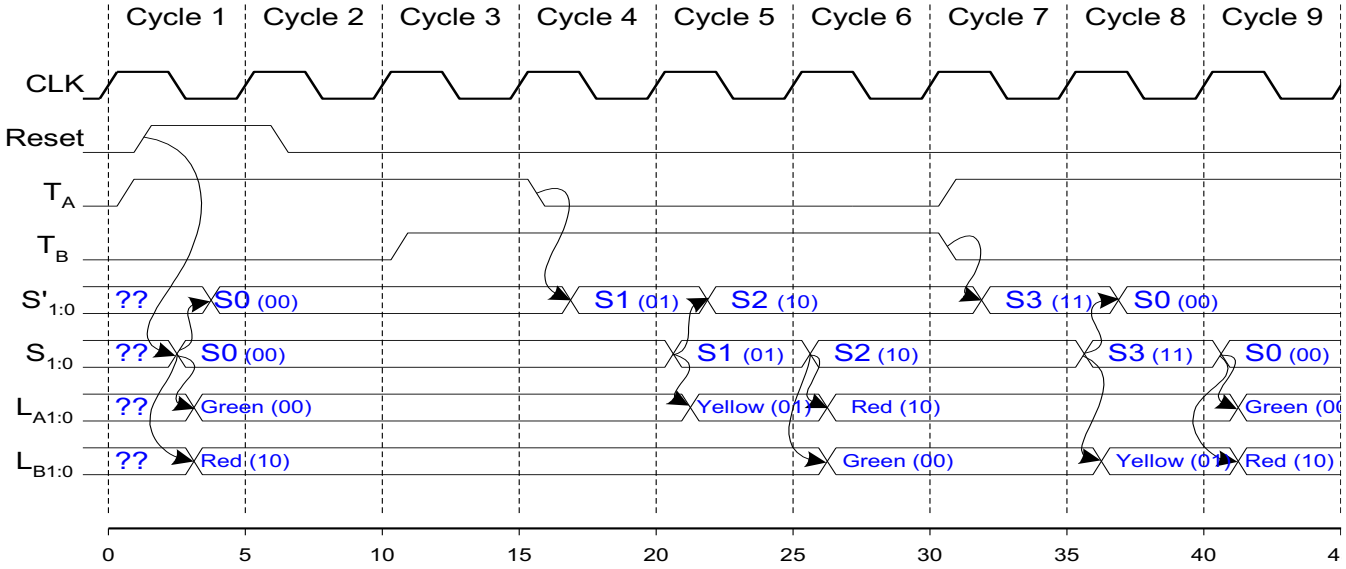


This is from H&H Section 3.4.1

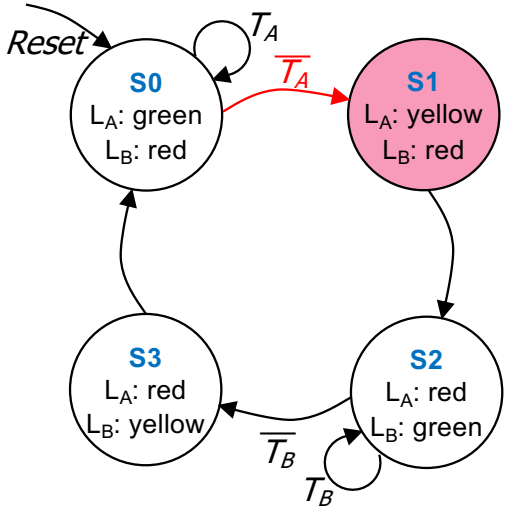


FSM Timing Diagram

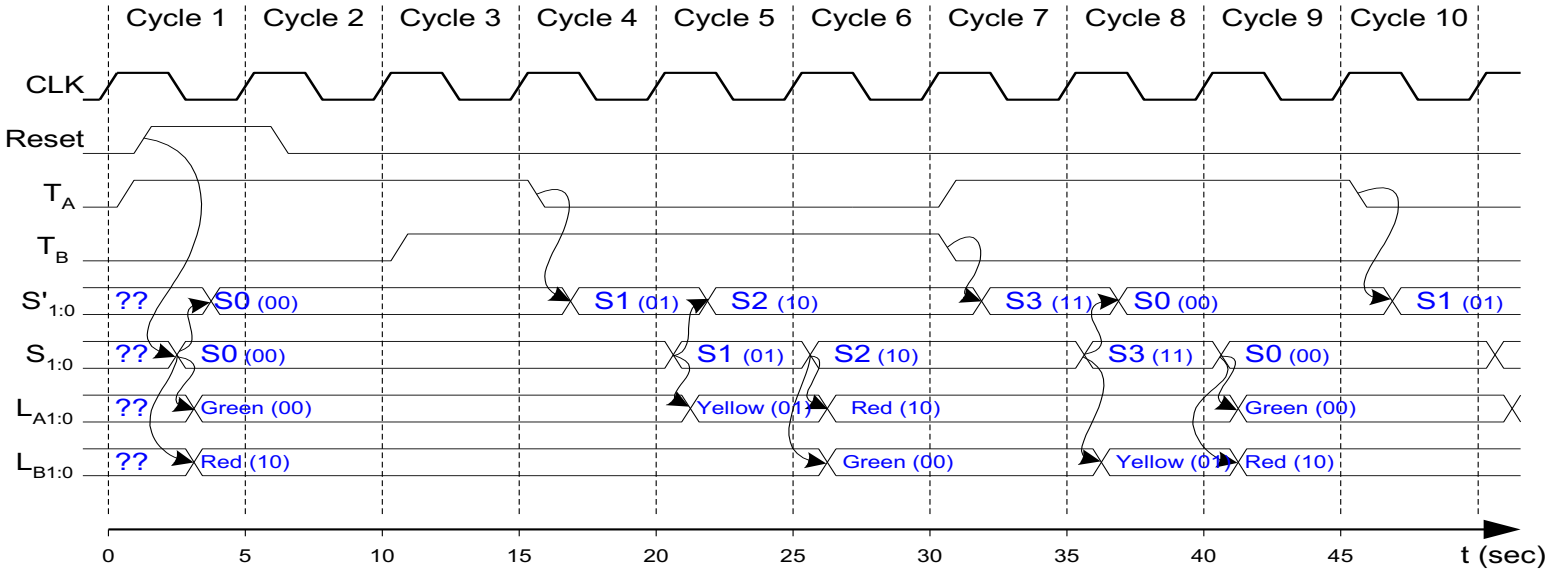




FSM Timing Diagram



See H&H Chapter 3.4



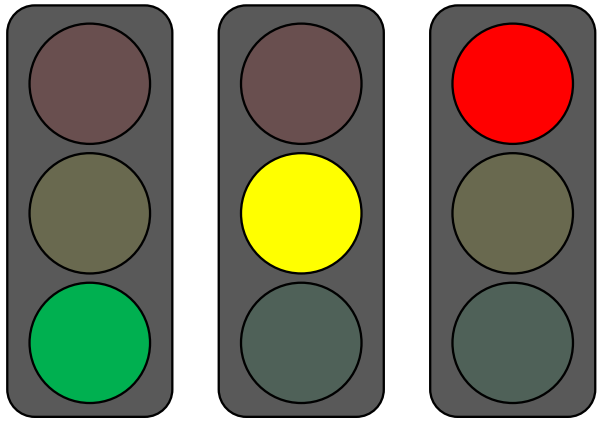
Finite State Machine:

State Encoding

FSM State Encoding

- How do we encode the state bits?
  - Three common state binary encodings with different tradeoffs
    - **Fully Encoded**
    - **1-Hot Encoded**
    - **Output Encoded**

- Let's see an example traffic light with 3 states
  - Green, Yellow, Red



FSM State Encoding

1. **Binary Encoding (Full Encoding):**
   - Use the minimum possible number of bits
     - Use $log_2(num\_states)$ bits to represent the states
   - *Example state encodings:* 00, 01, 10
   - **Minimizes** # flip-flops, but not necessarily output logic or next state logic
2. **One-Hot Encoding:**
   - Each bit encodes a different state
     - Uses *num_states* bits to represent the states
     - Exactly 1 bit is "hot" for a given state
   - *Example state encodings:* 0001, 0010, 0100, 1000
   - **Simplest design process** – very automatable
   - **Maximizes** # flip-flops, **minimizes** next state logic

FSM State Encoding

1. **Binary Encoding (Full Encoding):**
   - Use the minimum possible number of bits
     - Use
   - *Examp*
   - **Minim** logic
2. **One-H**
   - Each b
     - Use
     - Exa
   - *Example state encodings:* 0001, 0010, 0100, 1000
   - **Simplest design process** – very automatable
   - **Maximizes** # flip-flops, **minimizes** next state logic

The designer must carefully choose an encoding scheme to optimize the design under given constraints

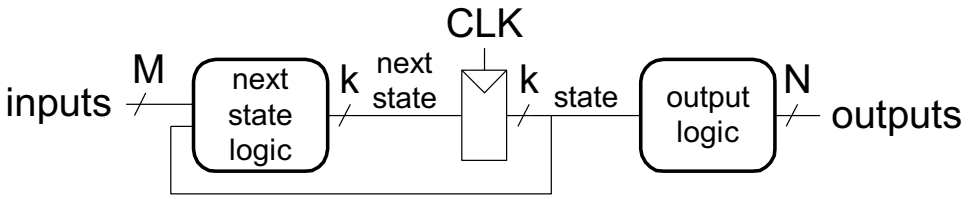
Moore vs. Mealy Machines

Section 3.4.3 of H&H

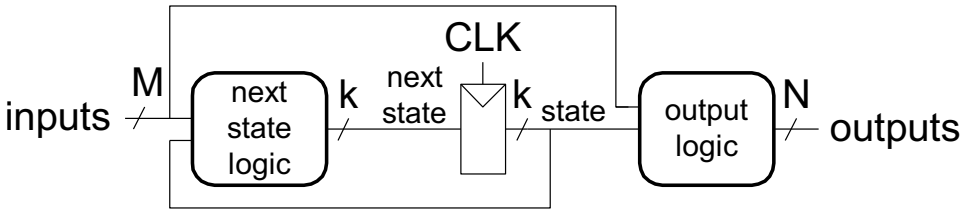
Recall: Moore vs. Mealy Machines

- Next state is determined by the current state and the inputs
- Two types of FSMs differ in the **output logic**:
  - **Moore FSM**: outputs depend only on the current state
  - **Mealy FSM**: outputs depend on the current state and the inputs

Moore FSM



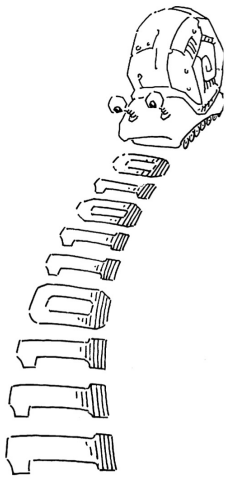
Mealy FSM



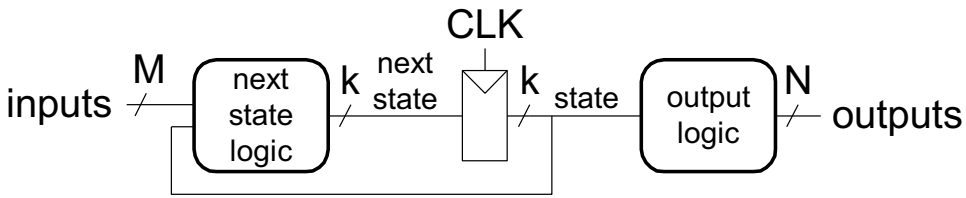
Section 3.4.3 of H&H

Moore vs. Mealy Examples

- Alyssa P. Hacker has a snail that crawls down a paper tape with 1's and 0's on it.
- The snail smiles whenever the last four digits it has crawled over are 1101.
- Design Moore and Mealy FSMs of the snail's brain.



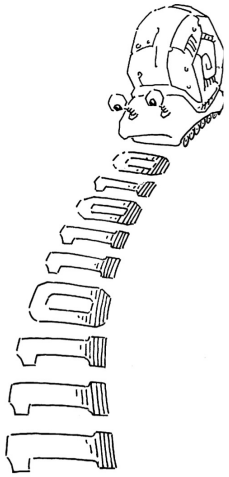
Moore FSM



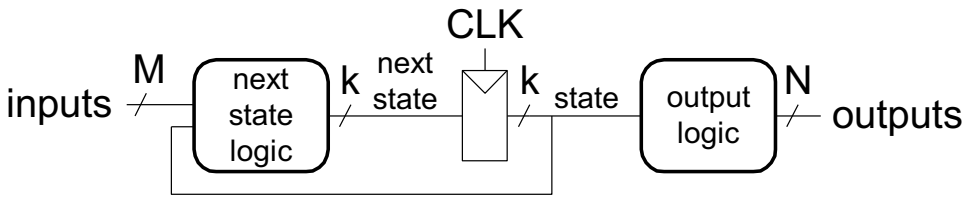
Example 3.7 of H&H

Moore vs. Mealy Examples

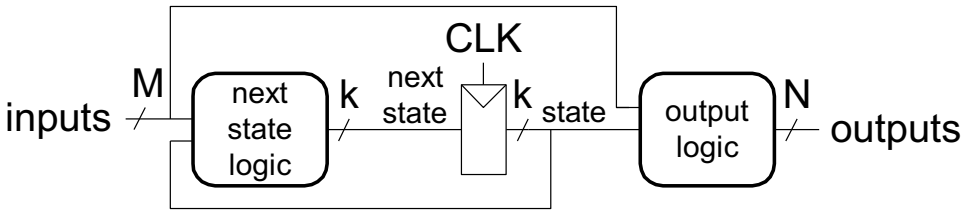
- Alyssa P. Hacker has a snail that crawls down a paper tape with 1's and 0's on it.
- The snail smiles whenever the last four digits it has crawled over are 1101.
- Design Moore and Mealy FSMs of the snail's brain.



Moore FSM

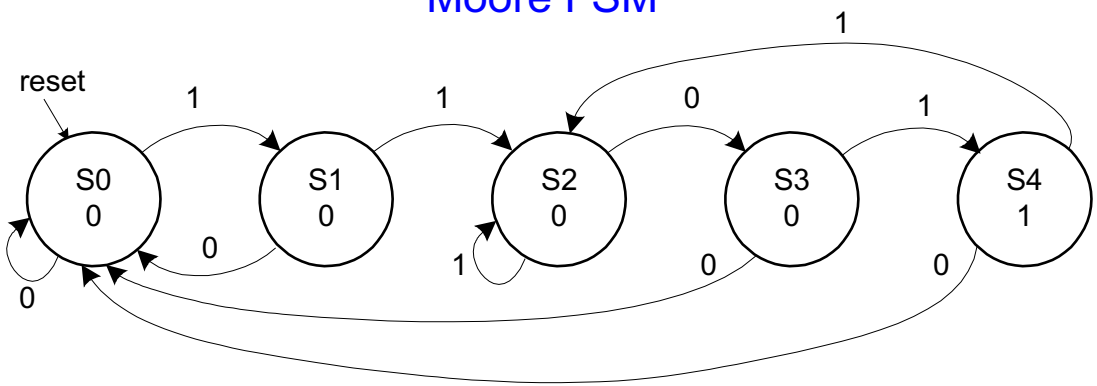


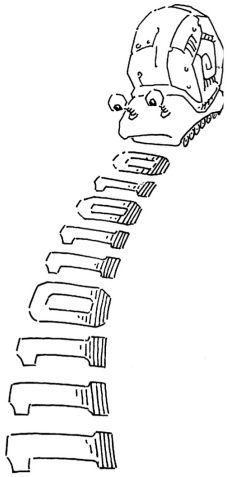
Mealy FSM



State Transition Diagrams

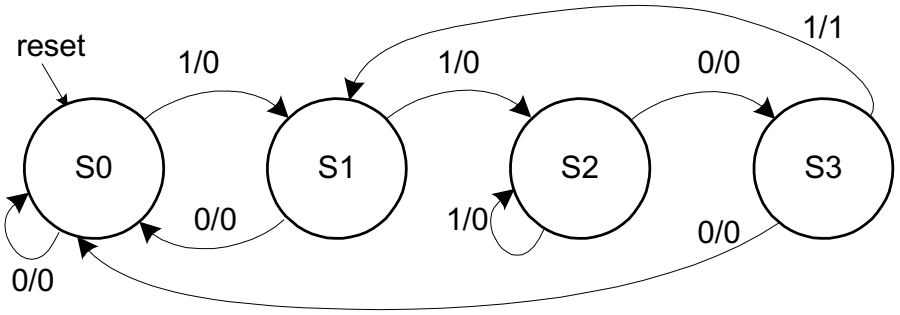
Moore FSM





What are the tradeoffs?

Mealy FSM



FSM Design Procedure (I)

Step # 1: State transition diagram

- Formalize the specification and remove ambiguity

Step # 2: Derive the next state logic

- Binary encoding for states
- State transition (truth) table
- Minimized Boolean equations for next state logic

Step # 3: Derive the output logic

- Binary encoding for outputs
- Output table & Boolean equations

Step # 4: Turn the Boolean equations into logic gate implementation

- Next state logic & output logic

FSM Design Procedure (II)

- **Determine** all possible states of your machine
- **Develop** a **state transition diagram**
  - Generally, this is done from a textual description
  - You need to:
    - 1) determine the **inputs** and **outputs** for each **state** and
    - 2) figure out how to get from one state to another
- **Approach**
  - Start by defining the **reset state** and what happens from it – this is typically an easy point to start from
  - Then continue to add **transitions** and **states**
  - Picking **good state names** is very important
  - Building an FSM is **like** programming (but it *is not* programming!)
    - An FSM has a sequential "control-flow" like a program with conditionals and goto's
    - The if-then-else construct is controlled by one or more inputs
    - The outputs are controlled by the state or the inputs
  - In hardware, we typically have many concurrent FSMs

What We Covered Until Now

- The concept of state
- State diagrams
- Asynchronous vs. synchronous state changes
- Synchronous sequential circuits
- FSMs
  - Components, transition diagram, tables, equations, schematic
  - State transition diagrams
  - State encoding
  - Moore vs. Mealy
  - Design procedure

Why are Arbitrary Sequential Circuits a Bad Idea?

Reading: Section **3.4** of H&H

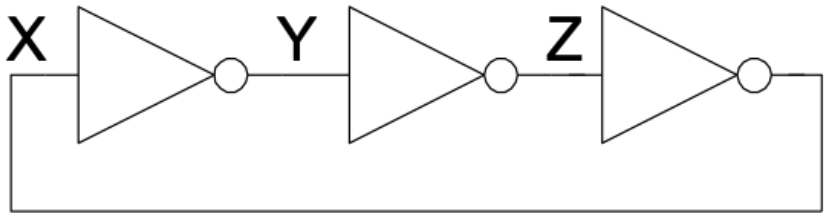
Arbitrary Sequential Circuits

- **Important:** We need to **discipline** ourselves and only build *synchronous sequential* circuits
- State is **synchronized** at the clock edges
- Let's examine an arbitrary sequential circuits

Section 3.3 of H&H

Ring Oscillator

- What does the circuit below do?
  - **One output**, **No inputs**



Assume X = 0
- Y = 1, Z = 0, X = 1
- Inconsistent with original assumption
- **Oscillates** between 0 and 1

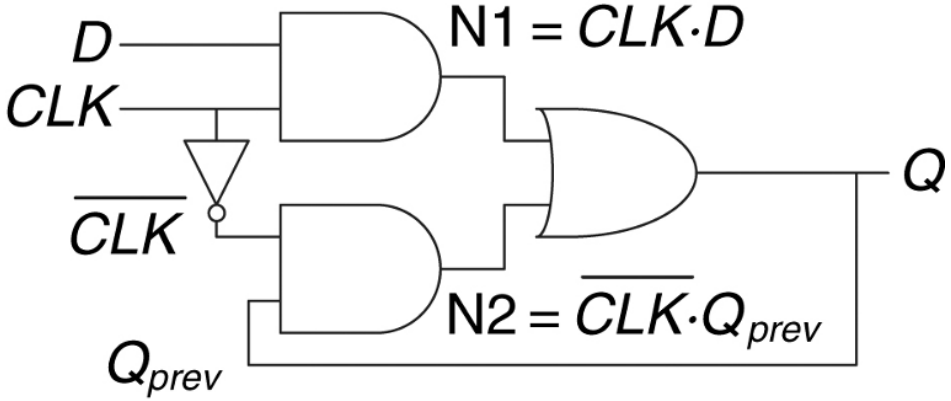
- No stable state (**Unstable** or **Astable**)
- X **oscillates** b/w **0** and **1**
- If the propagation delay of each inverter is **1 ns**, then can you plot X? What is the clock period?
- What is the period (repetition time)?

Example 3.3 of H&H

Cooked-up D Latch Optional Self-Study

- What does the circuit below do?
  - CLK = D = 1 (*transparent*, Q = 1)
  - CLK = 0 (Q = Q_{prev})

$$Q = CLK \cdot D + \overline{CLK} \cdot Q_{prev}$$



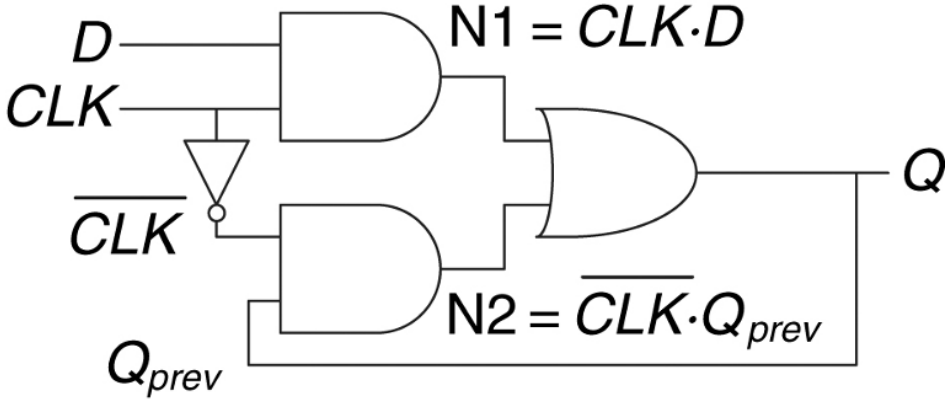
Example 3.4 (page 119) of H&H

Cooked-up D Latch Optional Self-Study

- What does the circuit below do?
  - CLK = D = 1 (*transparent*, Q = 1)
  - CLK = 0 (Q = Q_{prev})

CLK	D	Q_{prev}	Q
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

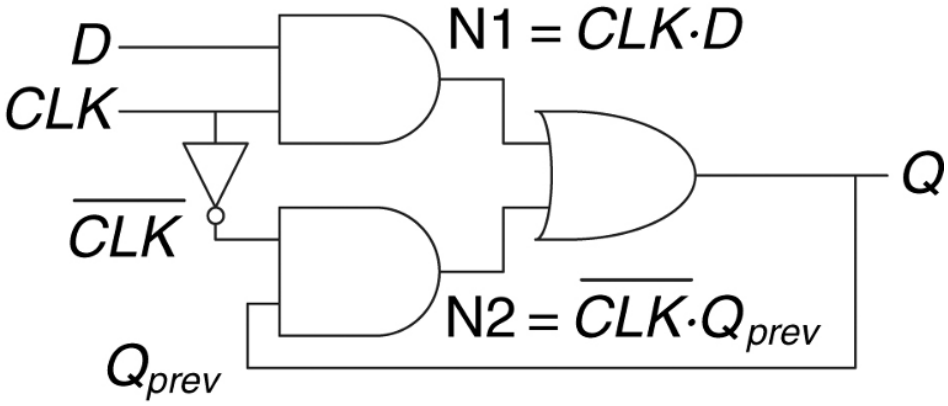
$$Q = CLK \cdot D + \overline{CLK} \cdot Q_{prev}$$



Example 3.4 (page 119) of H&H

Cooked-up D Latch Optional Self-Study

- What if $t_{INV} >> t_{AND}$ and $t_{INV} >> t_{OR}$?
  - Node N1 and Q may both fall before $\overline{CLK}$ changes/rises
  - Q gets stuck at 0
  - This is known as a *race condition: The path through CLK to Q is faster than $\overline{CLK}$ to Q*



Example 3.4 (page 119) of H&H

Takeaways

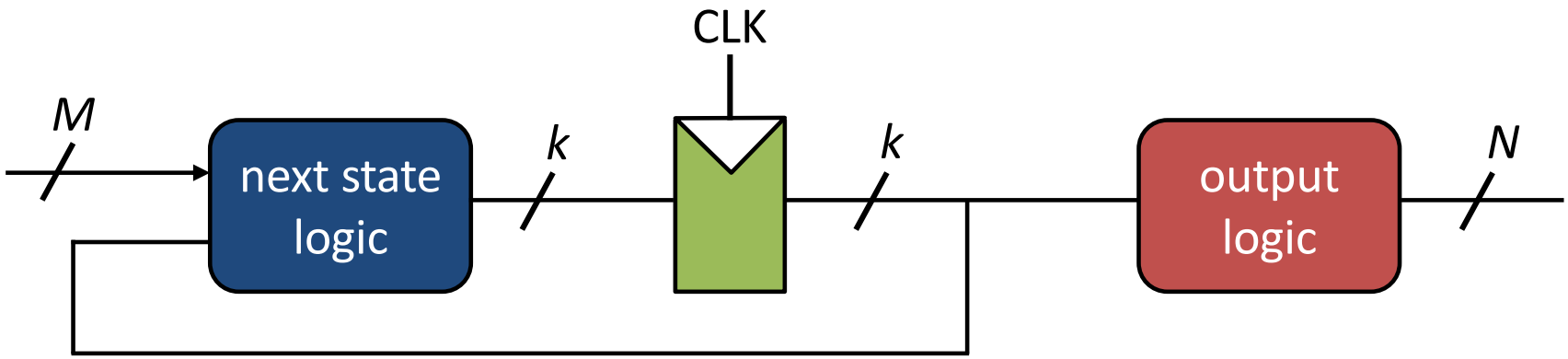
- When outputs are fed back directly back to inputs, these are called **cyclic** paths

- Combinational logic has **NO** cyclic paths
  - Outputs settle after propagation delay

- Circuits with cyclic paths are called **asynchronous** circuits
  - Difficult to analyze
  - Timing issues (race conditions, oscillations)
  - May work in one set of conditions (e.g., temperature) but not in another

Synchronous Sequential Circuits

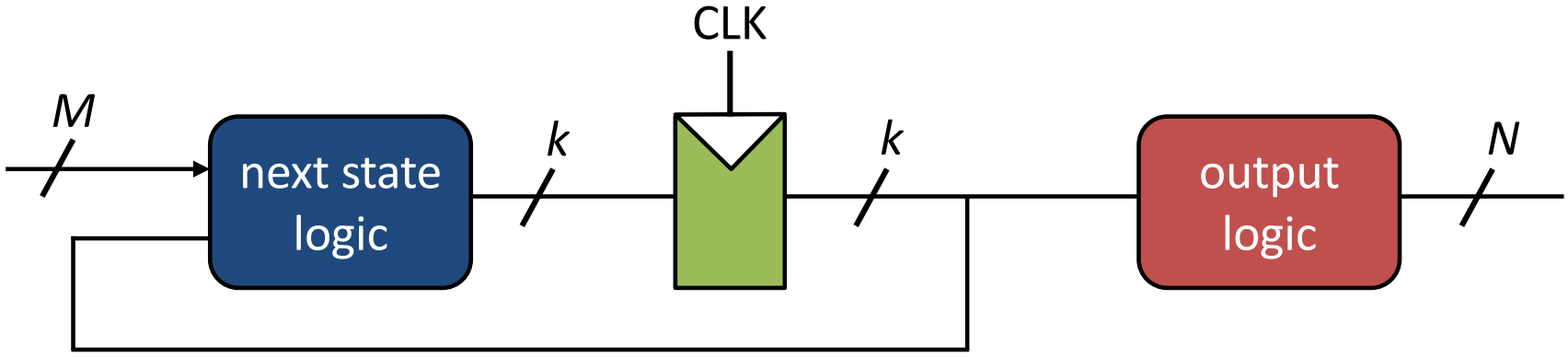
- What is the problem with asynchronous circuits?
  - *Cyclic paths lead to races and unstable behavior*
- **Solution:** Break the cyclic paths by **inserting** registers somewhere in the path
- Registers contain **state**, and **synchronized** to the clock



Section 3.3.2 of H&H

Synchronous Sequential Circuits

- What is the problem with asynchronous circuits?
  - *Cyclic paths lead to races and unstable behavior*
- **Solution:** Break the cyclic paths by **inserting** registers somewhere in the path
- Registers contain **state**, and **synchronized** to the clock

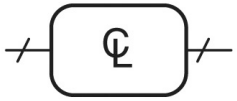


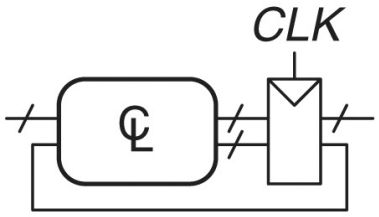
General Rule: *If the clock is sufficiently slow, so that the inputs to all registers settle before the next clock edge, all races are eliminated*

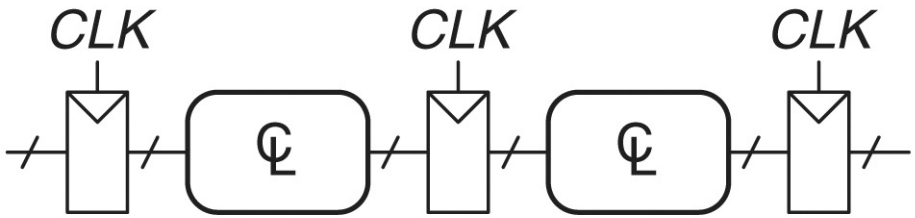
Composition Rules

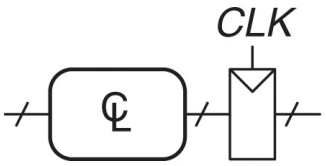
- Every circuit element is either a register or a combinational element
- At least one element is a register
- All registers receive the same clock signal
- Every cyclic path contains at least one register

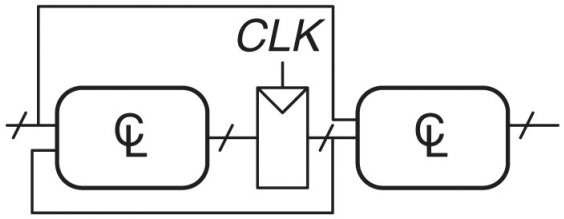
Which circuits are synchronous sequential?

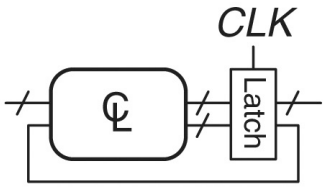


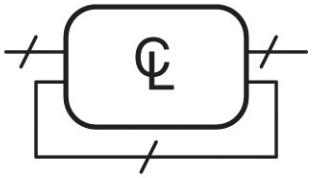


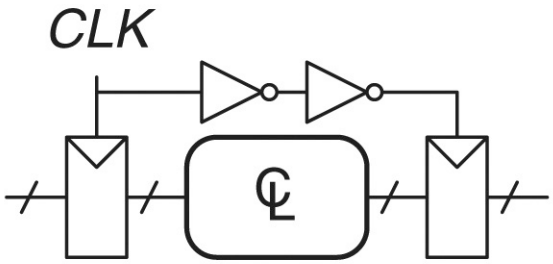












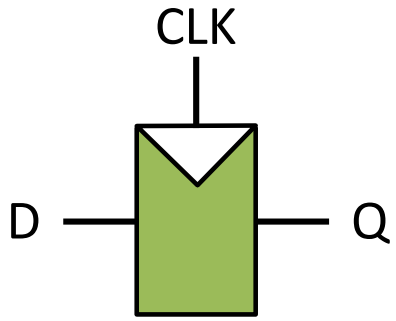
Example 3.5 of H&H

Timing Issues in Sequential Circuits

Reading: Section **3.5** of H&H (More detailed than what we need in this course)

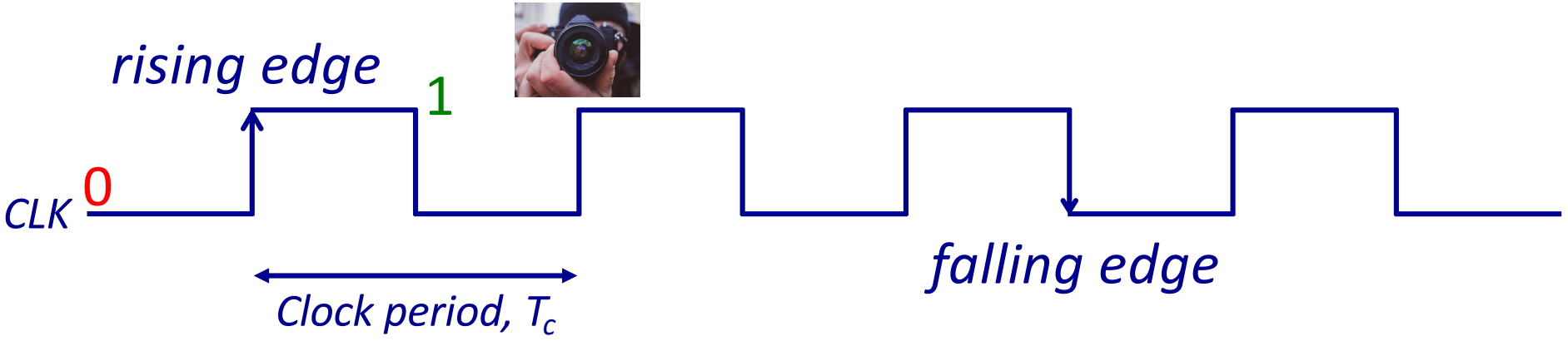
Timing in Sequential Circuits

- We need to understand three aspects of timing specification
  - Clock-to-Q propagation delay
  - Setup time
  - Hold time



Recall the Clock

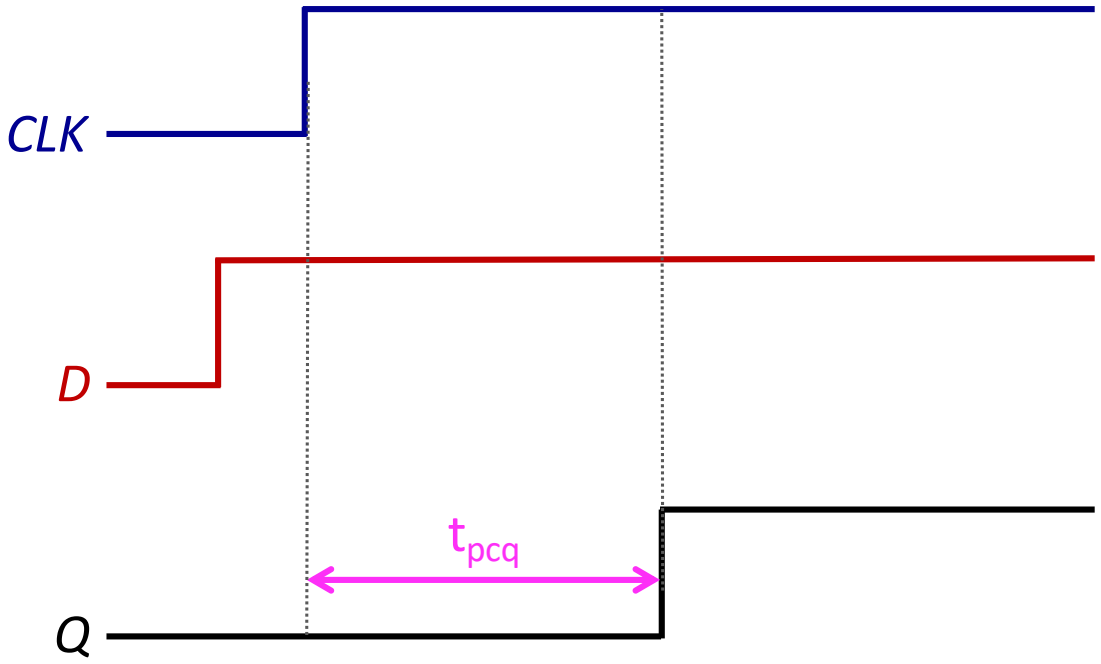
- Output does not change instantly when the **rising edge** arrives
- Input need to stay **stable** for some time period for the flip-flop to take a **reliable** photograph

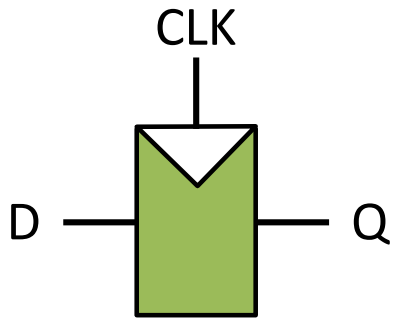


Frequency = $1/T_c$

Clock-to-Q Propagation Delay

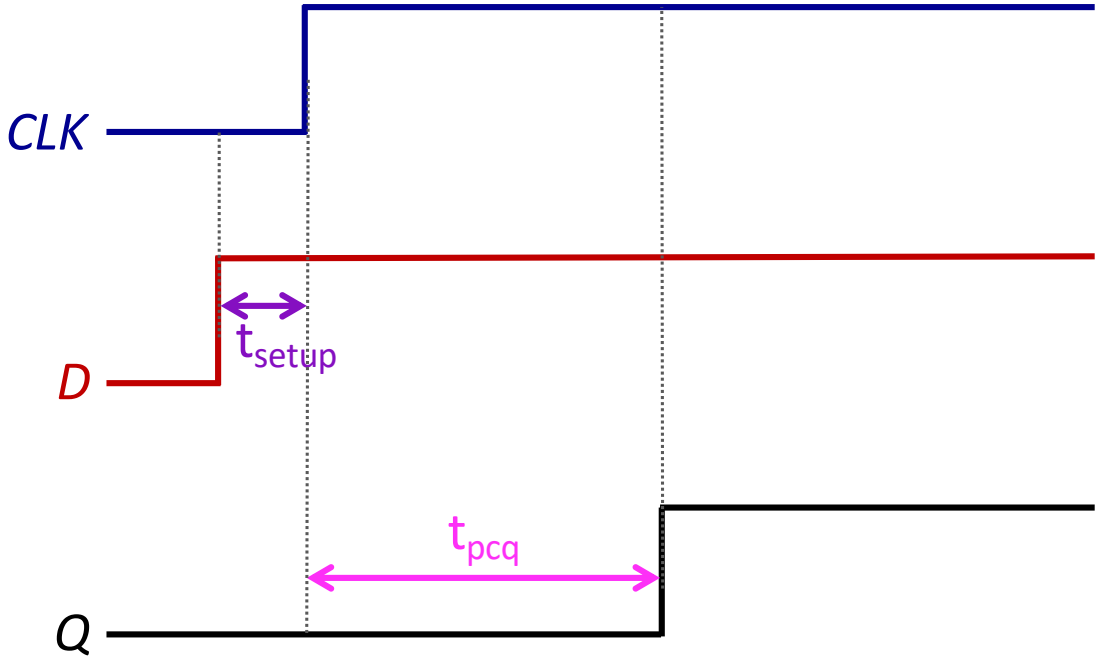
- When the clock rises, the time it takes for the output to **settle** to the final value (t_{pcq})

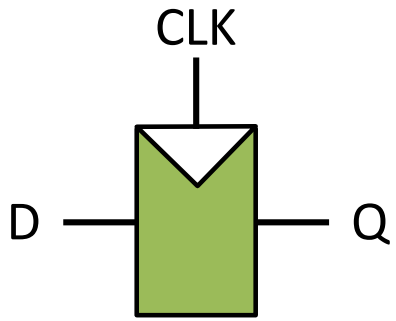




Setup Time

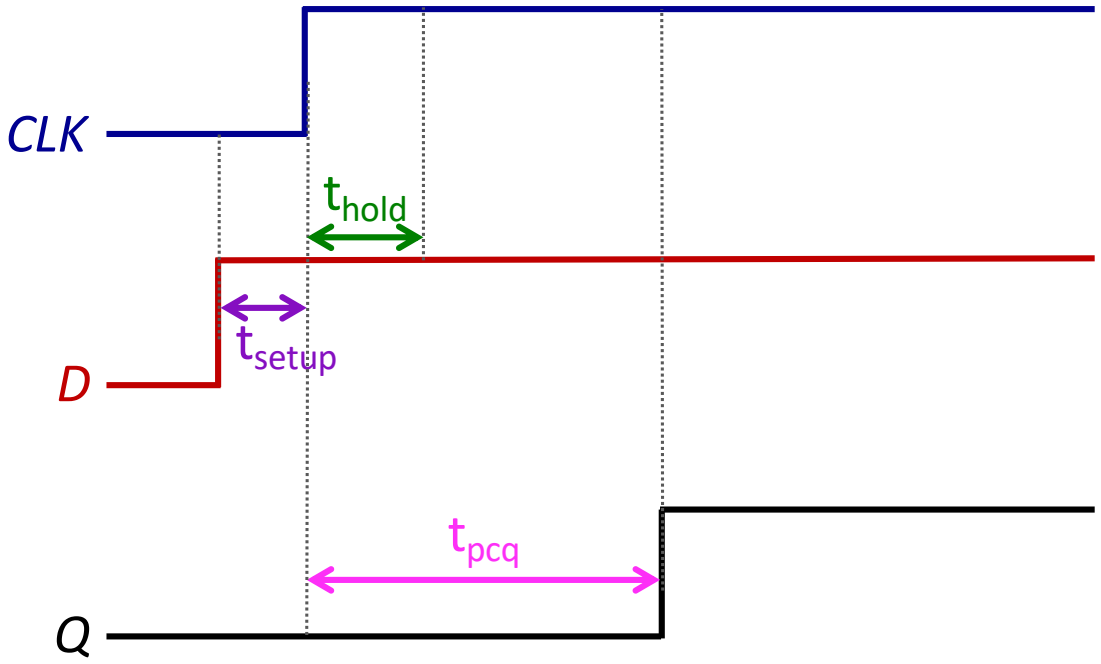
- For the circuit to sample its input correctly, the input must have **stabilized** at least some setup time, t_{setup}, before the rising edge of the clock

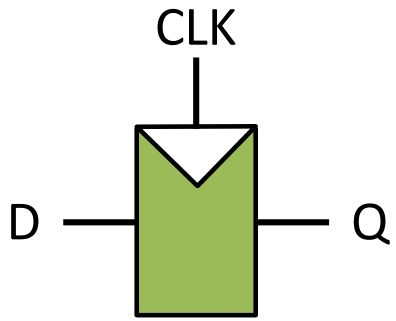




Hold Time

- The input must remain **stable** for at least some hold time (t_{hold}) after the rising edge of the clock





Aperture Time

- The sum of the **setup** and **hold** times is called the aperture time of the circuit

- It is the total time for which the input must remain stable

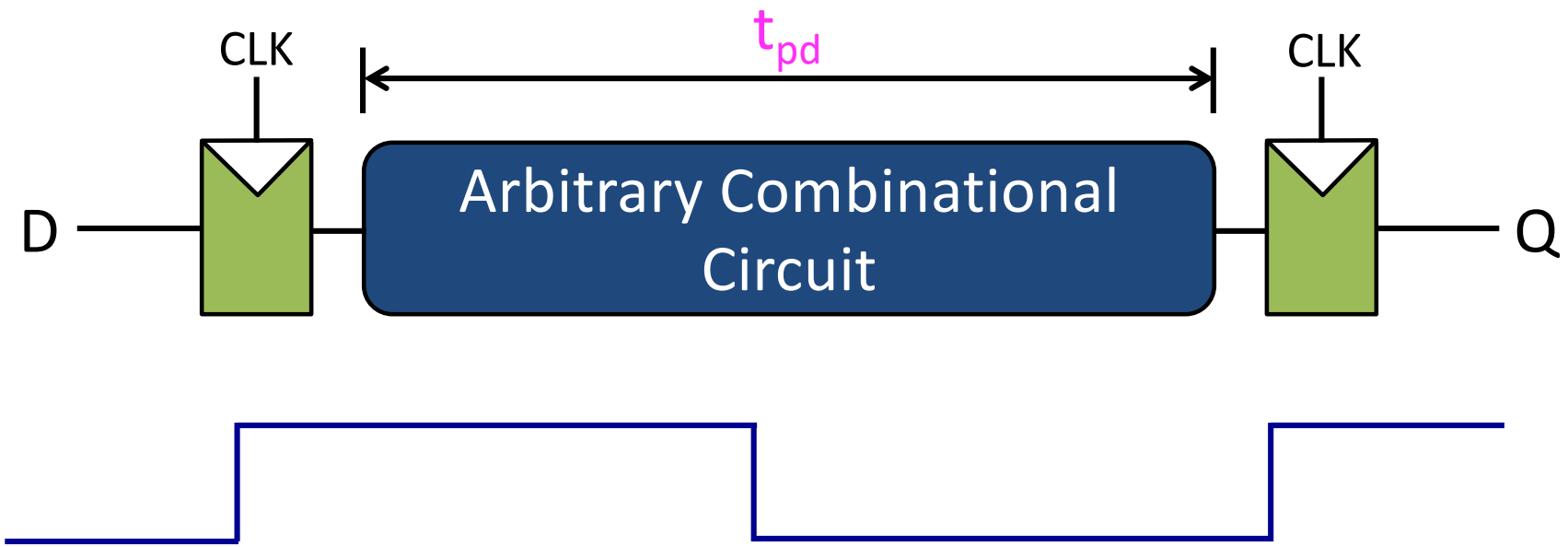
Technology and Hold Time

- It is a reasonable assumption that modern flip-flops have a **hold time** close to zero

- We can ignore hold time in subsequent discussions

Example

- What is the **clock period** for the circuit below for it to work correctly?
  - t_{pd}
  - $t_{pd} + t_{pcq}$
  - $t_{pd} + t_{pcq} + t_{setup}$



Sequencing Overhead

- $t_{pcq} + t_{setup}$ is called the **sequencing** overhead of the flipflop

- $T_c = t_{pd} + t_{pcq} + t_{setup}$
  - Ideally, the entire clock period should be spent doing useful work (i.e., processing done by the combinational circuit)
  - The **sequencing** overhead of the flip-flop **cuts** into this time