ENGG1003 - Thursday Week 8

Numerical integration

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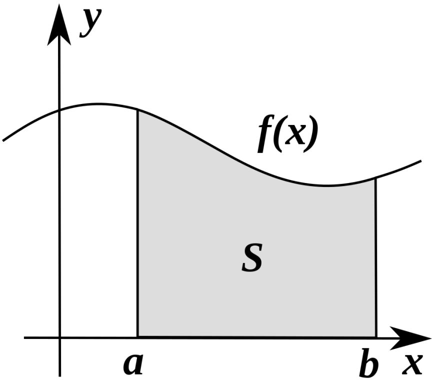
Lecture overview

1. Basic ideas of numerical integration §6.1
    - engineering applications
    - terminology & notation
    - additivity
2. Trapezoidal method §6.2
3. Midpoint method, upper/lower and left/right Riemann sums §6.3
4. Simpson's rule

1) Basic ideas of numerical integration



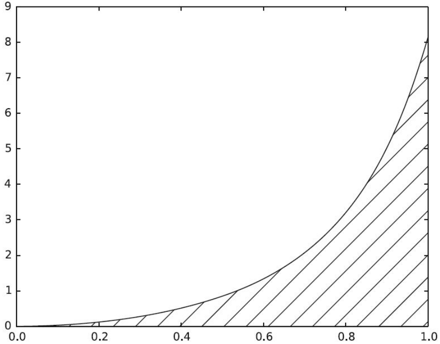
- integral is area under curve

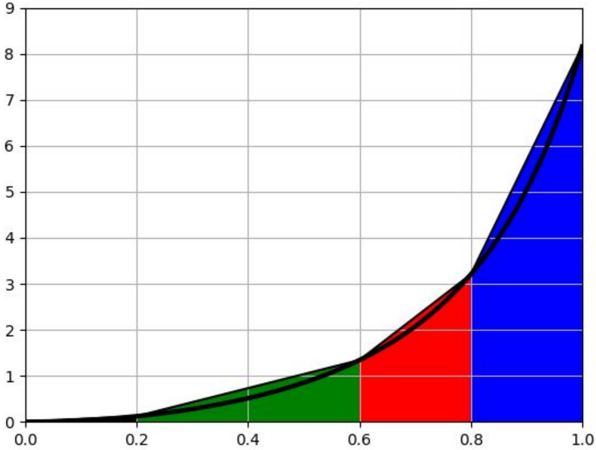
Engineering applications of integration

- 1. Area between curves 2. Distance, Velocity, Acceleration 3. Volume 4. Average value of a function 5. Work 6. Center of Mass 7. Kinetic energy; improper integrals 8. Probability 9. Arc Length 10. Surface Area

2) Trapezoidal method

- Python code for simple example
- use `fill_between` ??

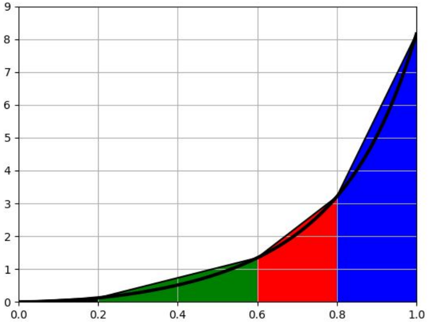




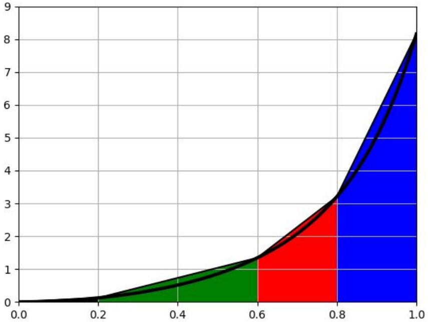
- XXX

Area of trapezoid

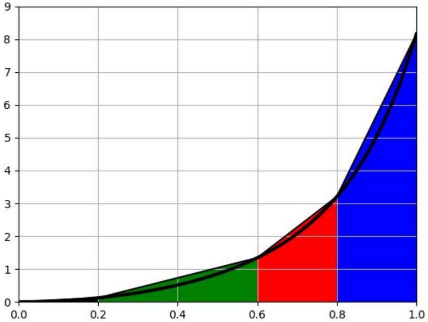
- xxx



$$\int_0^1 v(t)dt \;=\; \int_0^{0.2} v(t)dt + \int_{0.2}^{0.6} v(t)dt + \int_{0.6}^{0.8} v(t)dt + \int_{0.8}^{1} v(t)dt$$



$$
\begin{aligned}
\int_0^1 v(t)dt &= \int_0^{0.2} v(t)dt + \int_{0.2}^{0.6} v(t)dt + \int_{0.6}^{0.8} v(t)dt + \int_{0.8}^{1} v(t)dt \\
&\approx h_1 \frac{v(0)+v(0.2)}{2} + h_2 \frac{v(0.2)+v(0.6)}{2} + \\
&\quad + h_3 \frac{v(0.6)+v(0.8)}{2} + h_4 \frac{v(0.8)+v(1)}{2}
\end{aligned}
$$



$$
\begin{aligned}
\int_0^1 v(t)dt &= \int_0^{0.2} v(t)dt + \int_{0.2}^{0.6} v(t)dt + \int_{0.6}^{0.8} v(t)dt + \int_{0.8}^{1} v(t)dt \\
&\approx h_1 \frac{v(0)+v(0.2)}{2} + h_2 \frac{v(0.2)+v(0.6)}{2} + \\
&\quad + h_3 \frac{v(0.6)+v(0.8)}{2} + h_4 \frac{v(0.8)+v(1)}{2}
\end{aligned}
$$

$$
h_1 = 0.2, \quad h_2 = 0.4, \quad h_3 = 0.2, \quad h_4 = 0.2
$$

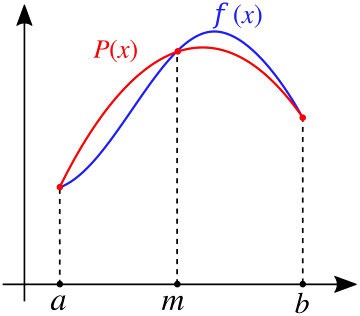
Python code for trapezoidal method

- write as a function
- live demo, experiment with number of panels

3) Midpoint method

- XXX

4) Simpson's rule



Approximate $f(x)$ with parabola $P(x)$, for which integral

$$\int_a^b P(x)dx$$

can be calculated *exactly*

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

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