

ENGG1003 - Monday Week 2

Calculating Pi
Arithmetic
Datatypes

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Before we start...

- Today's lecture is information overload
 - ▶ It is a long list of “stuff” to rote learn
 - ▶ Half of programming is just “playing with Lego” with this “stuff”
 - ▶ There is a lot to learn before we can solve interesting problems
- It will probably push you out of your comfort zone
- Lab experience will rebuild your confidence
 - ▶ Much of this content will be used in every lab
- But first, a problem for motivation...

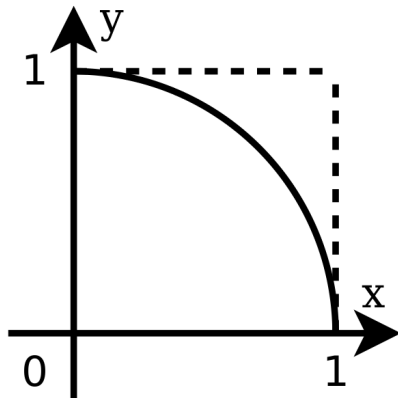
Case Study: Calculating π

- Computers are *really* good at repetitive things
- Lets use this fact to calculate π using a “monte-carlo” method
 - ▶ Informally, these are methods which solve problems by repeating the same thing with different inputs until patterns emerge
 - ▶ It could repeat millions or billions of times
 - ▶ Name comes from the Monaco Principality’s high concentration of casinos
- Algorithm pseudocode will be written before an implementation in C

Case Study: Calculating π

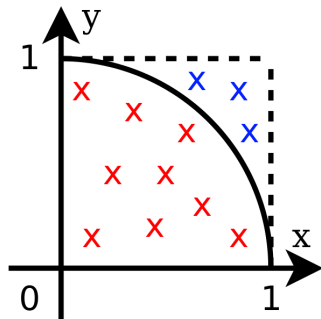
Consider a quadrant of a unit circle ($r = 1$) with a square around it:

- Area of the square $A_1 = 1$
- Area of the circle quadrant $A_2 = \frac{\pi r^2}{4} = \frac{\pi}{4}$
- Ratio of areas $\frac{A_2}{A_1} = \frac{\pi}{4}$
- Therefore $\pi = 4 \times \frac{A_2}{A_1}$



Case Study: Calculating π

- We can't calculate the area ratio without knowing π
- Estimate it by:
 - ▶ Randomly picking many points inside the square
 - ▶ Test if the point is inside the circle with $x^2 + y^2 < 1$



- $$\pi \approx 4 \times \frac{\text{Number of points which land inside circle}}{\text{Total number of points tested}} = 4 \times \frac{9}{12} = 3$$

Algorithm for Calculating π

- How can the above *mathematics* be turned into an *algorithm*?
 - ▶ **NB:** You only have to understand the algorithm

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 - ▶ This implies use of a loop
 - ▶ The loop's *exit condition* needs to be defined

Algorithm for Calculating π

- How can the above *mathematics* be turned into an *algorithm*?
 - ▶ **NB:** You only have to understand the algorithm
- The algorithm needs to repeat the *same thing* multiple times
 - ▶ This implies use of a loop
 - ▶ The loop's *exit condition* needs to be defined
- As the loop repeats, we need to keep track of the following *variables*:
 - ▶ The number of points tested
 - ▶ The number of points which landed inside the circle
 - ▶ The (x, y) coordinates of the point under test

Algorithm for Calculating π

- The number of points tested will be an integer, we will call it `countTotal`
- The number of points found to be inside the circle is also an integer, we will call it `countInside`
- Before these variables are used they should be *initialised*
 - ▶ ie: The algorithm will explicitly include `countTotal = 0` and `countInside = 0`
 - ▶ So-called *uninitialised* variables have undefined (or random) values

Algorithm for Calculating π

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Algorithm for Calculating π

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 - ▶ This implies `IF...ENDIF` *flow control*
- The condition on incrementing `countInside` is $x^2 + y^2 < 1$
- Incrementing a variable in pseudocode takes the form:
 - ▶ `variable = variable + 1`
 - ▶ This can be read as “variable *becomes* variable plus 1”
 - ▶ Maths people would write: $x_{n+1} = x_n + 1$

- The point under test needs two “real” variables: x and y
 - ▶ “Real” comes from mathematics: any number with integer and fractional components. Eg: 1.45
- These values take new random values each loop
- The pseudocode doesn’t need to describe how a random number is generated
 - ▶ Stating: “ $x =$ a random number between 0 and 1” is totally acceptable
- At the end of the algorithm the final step will be
$$\pi = 4 \times \frac{\text{countInside}}{\text{countTotal}}$$

Algorithm for Calculating π

BEGIN

integer countTotal = 0

integer countInside = 0

WHILE countTotal < A large number

 x = random number between 0 and 1

 y = random number between 0 and 1

 countTotal = countTotal + 1

 IF $x^2 + y^2 < 1$

 countInside = countInside + 1

 ENDIF

ENDWHILE

pi = $4 \times \text{countInside} / \text{countTotal}$

PRINT pi

END

Missing Knowledge for C Implementation

- More information about arithmetic
 - ▶ Relational operators (less/greater-than) look useful
 - ▶ Is there a neat way to do `count=count+1`?
 - ▶ `countInside` and `countTotal` are both integers. What happens when we divide?
- Datatypes and how they are handled in arithmetic statements
- How do we generate random numbers?
- Syntax for WHILE loops and IF statements

C Arithmetic

- Basic arithmetic was seen in the lab

Operation	C Symbol
Addition	+
Subtraction	-
Multiplication	*
Division	/

Table: Basic arithmetic operators in C

- Complex expressions can be built from these operators and parentheses

C Arithmetic

Examples:

$$\begin{array}{ll} z = x^2 + 5(y + b) & z = x * x + 5 * (y + b) ; \\ u = \frac{x+1}{x-1} & u = (x + 1) / (x - 1) ; \\ v = z^3 + \frac{5(y+b)}{2} & v = z * z * z + (5 * (y + b)) / 2 ; \end{array}$$

- Multiplication is not assumed. If you write $5(y+b)$ the compiler will generate a syntax error.
- To be valid C statements the semicolon is required.

C Arithmetic

- C supports two time-saving *unary* operators:
 - ▶ Very useful in loops.

Operation	C Syntax	Replaces
Increment	<code>x++;</code> or <code>++x;</code>	<code>x = x + 1;</code>
Decrement	<code>x--;</code> or <code>--x;</code>	<code>x = x - 1;</code>

- It also supports the following shorthand syntax:

<code>x = x + y;</code>	<code>x += y;</code>
<code>x = x - y;</code>	<code>x -= y;</code>
<code>x = x * y;</code>	<code>x *= y;</code>
<code>x = x / y;</code>	<code>x /= y;</code>

C Arithmetic

What's the difference between `x++` and `++x`?

- `x++` is a *post-increment*
- `++x` is a *pre-increment*
- If they appear in an arithmetic expression, pre-increment is processed *before* the variable is used and post-increment is processed *after* it is used.
- In isolation there is no difference.

Modulus

- Integer division ignores (truncates) any fractional component
- The *modulus* operator provides the remainder after division
 - ▶ C uses the `%` character
 - ▶ “`a % b`” means “remainder of `a / b`”
- Example:
 - ▶ $10 / 3 = 3$
 - ▶ $10 \% 3 = 1$

Modulus Example - Printing Every *nth* Number

```
1 #include <stdio.h>
2 int main() {
3     int x = 0;
4     while(x < 1000)
5     {
6         x++; // Increment x
7         if(x%100 == 0) // if(x is divisible by 100)
8             printf("%d\n", x);
9     }
10    return 0;
11 }
```

Relational Operators

- C supports six *relational* operators:

Operation	C Symbol
Less than	<
Less than or equal to	<=
Greater than	>
Greater than or equal to	>=
Equal to	==
Not equal to	!=

Relational Operators

- The result of a relational operation is 0 or 1
 - ▶ C treats 0 as Boolean FALSE and non-zero as TRUE
- They are typically used as flow control conditions
 - ▶ `if(condition) {statements}`
 - ▶ `while(condition) {statements}`
- While we're here: the above is the correct syntax for IF and WHILE flow control in C

if () and while ()

- Syntax for your “cheat sheet”:

```
1 if(condition) {  
2     // Do stuff  
3 }
```

```
1 if(condition) {  
2     // Do stuff  
3 } else {  
4     // Do other stuff  
5 }
```

```
1 while(condition) {  
2     // Do stuff  
3 }
```

- If there is a single *statement* inside an if () or while () the { and } are optional

if() and while()

Example:

```
1 #include <stdio.h>
2
3 main() {
4     while(x != 0) {
5         scanf("Enter an integer: %d\n", &x);
6         if(x % 2 == 0)
7             printf("%d is even\n", x);
8         else
9             printf("%d is odd\n", x);
10    }
11 }
```

Boolean Operators

- We discussed Boolean algebra last week
- It has three operators: AND, OR, NOT
- In C, these are implemented with:

Operator	C Characters
AND	& &
OR	
NOT	!

- I know the | character as a “pipe” symbol or “vertical slash”
 - ▶ Ask your demonstrator to find it on the keyboard

Conditions

- We have seen that `if()` and `while()` statements need *conditions*
- A condition can be any combination of variables, operators, and literals (numbers, defined later)
- The following are all valid:
 - ▶ `if(x > 29)`
 - ▶ `if(x == 2)`
 - ▶ `if(!(x == y))`
 - ▶ `if((x > 5) && (x < 20))`
 - ▶ `if(x = y + z)`
 - Evaluates `(y + z)`, assigns to `x`, uses result as `if()` condition.

C Arithmetic Operator Precedence

- C has an “order of operations”
- eg: $1+5*2$ evaluates to 11
 - ▶ You remember BODMAS / PEDMAS, right?
- Multiplication and division first
- Addition and subtraction second
- Relational operators somewhere below that
- If in doubt: force order with parentheses
 - ▶ This makes the code more readable
 - ▶ It doesn't cost you anything
 - ▶ C compilers understand algebra and will optimise inefficient expressions automatically

Variable Declaration

- In C, all variables are *declared* before use
- Declaration specifies the variable's:
 - ▶ Datatype
 - ▶ Name
 - ▶ An *initialisation value* (optional)
 - Always assume uninitialised variables have random values!
Behaviour varies between compilers and target platforms.
 - ▶ eg: `int counter = 0;`
 - Type is `int`
 - Name is `counter`
 - Initial value is 0 (optional)
- C is a “strongly-typed” language
 - ▶ A variable's type doesn't change

Variable Declaration

- You can declare multiple variables in one line:

```
1 int a, b, c, d;
```

- You can mix initialised and uninitialised variables:

```
1 float q, r = 1.0, s = 12e3, t;
```

- All variables declared in this way must be of the same type

Variable Naming Rules

- Variable names:
 - ▶ Are case sensitive
 - ▶ Must **not** be a C keyword (eg: `int`, `char`, `while`, etc)
 - ▶ Must **not** contain special characters (`*-+/,` etc)
 - ▶ Must begin with a letter
 - ▶ Can contain numbers
 - ▶ Can contain underscores
- What you do with those rules is up to you
- Modern practice encourages:
 - ▶ `camelCaseVariableNames`
 - ▶ `names_with_underscores`

Integer Data Types

- There are several integer data types
- They vary by their:
 - ▶ Size
 - ▶ Support for negative numbers
- C integer types can be 1, 2, 4, or 8 bytes long
 - ▶ `int` and `long` sizes vary by platform
 - ▶ Larger sizes store larger numbers, use more RAM
- Each type can be *signed* or *unsigned*
 - ▶ Unsigned numbers are always positive but you get double the *value range*

Integer Data Types

- The integer data type ranges can be calculated from the data type's size in bits
- For unsigned numbers of bit length n :

$$\text{max} = 2^n - 1 \quad (1)$$

- For signed numbers of bit length n :

$$\text{max} = 2^{(n-1)} - 1 \quad (2)$$

$$\text{min} = -2^{(n-1)} \quad (3)$$

- Signed numbers are stored in *two's complement* format, covered in ELEC1710

Integer Data Types

- C includes the `sizeof()` expression so that a program can discover the size of a data type on a given platform
- On a modern 64-bit Linux desktop machine:

Type	Bytes	Bits	Value Range
<code>char</code>	1	8	-128, +127
<code>short</code>	2	16	-32768, 32767
<code>int</code>	4	32	-2147483648, 2147483647
<code>long</code>	8	64	-9223372036854775808, ...

Unsigned Integers

- Unsigned integers are *always positive*
- They are the same size as their signed counterparts
- The `unsigned` keyword placed before the data type makes that variable unsigned
- eg: `unsigned char` is 1 byte and has a value range of 0 to 255

Overflow

- *Overflow* occurs when the result of a calculation is too big to fit into the target type
- Example: $127 + 1 = -128$

```
1 #include <stdio.h>
2 int main() {
3     char x = 127;
4     printf("%d\n", x);
5     x++;
6     printf("%d\n", x);
7     return 0;
8 }
```

Overflow

- Message: make variables as small you can, but no smaller
 - ▶ Or just use `long` everywhere and cop the performance hit
- Quite often you need to take a guess at how big a variable needs to be
- Typically a good idea to document software limits due to variable choice

Why Care About Data Types?

- You may be thinking “why not make everything a long?”
- Answer: speed and memory
- Smaller types use less RAM
- Arithmetic on a type larger than the target platform’s *native size* is slow
- Matters if you store millions of the same type
- Makes a *huge* difference on embedded targets
 - ▶ Don’t declare 32-bit variables on an 8-bit AVR microcontroller *unless you have to*

Floating Point Data Types

- To store real numbers C has several *floating point* data types
- As with integers, try to use the smallest you can get away with
- `long double` is supported in gcc but...complicated

Type	Size	Range	Precision
<code>float</code>	4	1.2×10^{-38} to 3.4×10^{38}	6 dp
<code>double</code>	8	2.3×10^{-308} to 1.7×10^{308}	15 dp

How good is 15 decimal places?

Paraphrasing NASA:

Calculating the circumference of Earth using 15 decimal places for π results in a truncation error about the size of a molecule.

39 digits of π calculates the circumference of the *visible universe* to within the width of a hydrogen atom ($\approx 10^{-10}$ m).

Floating Point Problems

- Q: Why not *always* use float or double instead of int?
- A: Floating point arithmetic is **NOT** exact
- Example 1: Does this code exit?

```
1 #include <stdio.h>
2 int main() {
3     float x = 16000000.0;
4     while(x < 17000000.0) {
5         x = x + 1.0;
6         printf("%f\n", x);
7     }
8 }
```

Floating Point Problems

- Example 2: $\tan\left(\frac{\pi}{2}\right)$ is undefined
- But using floating point π can't be represented exactly, so C produces a result anyway

```
1 #include <stdio.h>
2 #include <math.h>
3 int main() {
4     double pi = 3.141592653589793238;
5     printf("%f\n", tan(pi/2.0));
6     printf("%f\n", ftanf((float)pi/2.0));
7 }
```

Literals

- A *literal* is any number written in the code
- Why not “constant”?
 - ▶ That word means something different
- Examples:
 - ▶ `x = 5; // 5 is a literal int`
 - ▶ `y = 2.0 / z; // 2.0 is a literal double`
- By default:
 - ▶ An integer literal is stored as an `int` data type
 - ie: has the value range and arithmetic limits of `int`
 - ▶ A floating point literal is stored as `double`

Literals

- Integer literals can be in:
 - ▶ Decimal: 123
 - ▶ Hexadecimal: 0xA34 // Zero-x
 - ▶ Octal: 0125 // Capital letter O
 - ▶ (Hex and octal are covered in ELEC1710)
- Integer literals can be specified as *unsigned* with the `u` suffix:
 - ▶ 938u
- They can also be declared `long` with the `l` suffix:
 - ▶ 3726484l
 - ▶ The compiler will issue a warning if a literal is too big for `int`

Literals

- Floating point literals can be written in many ways:
 - ▶ `1.0f` // `f` suffix forces float
 - ▶ `(float)2.3` // Forces float
 - ▶ `1.0` // Default to double
 - ▶ `1e2` // Double, 1 times 10^2
- `1e2` is known as “e-notation”
 - ▶ $XeY = X \times 10^Y$
 - ▶ I will use it *all the time*
- Forcing literals to `float` is frequently necessary in embedded systems which lack double precision hardware

Mixing Data Types

- C supports arithmetic between different types
- Type conversion is called *casting*
- When types are mixed two things can happen:
 - ▶ Types get upgraded automatically (*implicit* type casting)
 - Upgrade path is roughly: short/char - int - long - long long - float - double
 - ▶ Types get specified manually by the programmer (*explicit* type casting)

Explicit Type Casting

- The data type of a variable (or literal) can be forced to change using *type casting*
- Write the desired type in parentheses before the variable or constant
- Examples:
 - ▶ `x = (float)y / k;`
 - ▶ `y = (unsigned int)y + 32;`

Format Specifiers

- A *format specifier* controls how `printf()` ; converts numerical (or textual) data to a series of ASCII characters
- Full details are complex, for now just use:
 - ▶ `%d` for integer types
 - ▶ `%f` for “fixed decimal place” floating point
 - ▶ `%e` for e-notation floating point
 - ▶ Cast inside `printf()` to suppress compiler warnings

Format Specifiers

- Casting example:

```
1 long i;  
2 // ...  
3 printf("%d\n", (int)i); //Breaks when i>2^31
```

- `%.df` produces `d` decimal places of precision

▶ eg:

```
1 float x = 1.23456;  
2 printf("%.2f", x); // Prints 1.23
```

- Format specifiers can appear anywhere in the output text, eg:

```
1 printf("%f was the result\n", x);  
2 printf("The result %f was calculated\n", x);
```

Format Specifiers

- You can print several numbers at once
- Separate variables with commas
- They are printed in the same order as variables are written
- You can put maths inside `printf()` ;

```
1 printf("%f plus %f is %f\n", x, y, x+y);
```

Integer Division Example

With *all that* out of the way, what is the output of each `printf()` statement?

```
1 #include <stdio.h>
2 int main() {
3     printf("%d\n", 9/10);
4     printf("%f\n", 9/10);
5     printf("%f\n", 9.0/10);
6     printf("%f\n", 9/10.0);
7     printf("%f\n", (float)9/10);
8     return 0;
9 }
```

Listing 1: `intdiv.c`

Random Numbers

- In C there is a *standard library function* which generates random numbers
- We will study functions in more detail later
- To use a library function:
 - ▶ Read the function's documentation
 - ▶ `#include` the correct *header file*
 - ▶ Take note of the *return value* data type
 - ▶ Add any *compiler flags* (beyond ENGG1003)
 - ▶ Use the function in your code
- Demonstration: read the `rand()` man page
 - ▶ Click [here](#)

Random Numbers

- Observe that `rand()` ; requires

```
1 #include <stdlib.h>
```

- Observe that it returns an `int` between 0 and `RAND_MAX` which, in gcc, is $2^{31} - 1$
- Note that it has limits (advanced discussion, beyond ENGG1003)
- For us, using the `%` operator:

```
1 x = rand() % (RANGE+1);
```

produces a *good enough* random number between zero and `RANGE`

Random Numbers

- As above, zero to RANGE is produced with:

```
1 x = rand() % (RANGE+1);
```

- To generate numbers from 1 to RANGE:

```
1 x = (rand() % RANGE) + 1;
```

- These work because:

```
1 x = x % y;
```

is an integer between zero and y for any x

Random Numbers

- Using the above information, we can roll a standard die with:

```
1 int dieRoll;  
2 dieRoll = (rand() % 6) + 1;
```

- Or, if you're a D&D fan, roll a D20 with:

```
1 int D20Roll;  
2 D20Roll = (rand() % 20) + 1;
```

Random Numbers

- `RAND_MAX` is a *constant* defined in `stdlib.h`
- We can try to generate a floating point number from 0 to 1 with a division:

```
1 float num;  
2 num = rand()/RAND_MAX; // This is broken
```

- Problem: `rand()` is an integer, `RAND_MAX` is a (large) integer
 - ▶ The integer division result will always be zero!
- Solution: use an explicit cast so that a floating point division occurs:

```
1 double num;  
2 num = (double)rand()/RAND_MAX;
```


Probably out of time by now, lets implement a π calculator in the lab... *In theory* you have enough information by now.