ENGG1003 - Monday Week 8

Solving nonlinear algebraic equations

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Lecture overview

1. Solving nonlinear algebraic equations pp. 175-176
   - general setting
   - two problems: flight time, fluid level
2. Bisection method §7.4
3. Secant method §7.3
   - Newton–Raphson method
4. Extensions
   - bisection & secant methods: re-write as functions
   - timing code in Python
   - speed comparisons

1) Solving nonlinear algebraic equations

- *linear* equations: $ax + b = 0$
  - solution $x = -b/a$
- *nonlinear* equations
  - quadratic $ax^2 + bx + c = 0$: solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - cubic and quartic (orders 3 and 4): exact solutions exist but are *very* complicated
  - quintic (order 5) equations: exact solutions *do not exist* in general, proving that needs *serious* mathematics
- most equations in engineering applications have no exact "pen and paper" solutions!

Numerical solutions to equations

"An approximate answer to the right problem is worth a good deal more than an exact answer to an approximate problem"
—John Tukey

General problem: find x satisfying

$$f(x) = 0$$

where $f(x)$ is a formula involving x

Example

$$f(x) = e^{-x}\sin(x) - \cos(x)$$

has solution $x = 7.85359326$ because

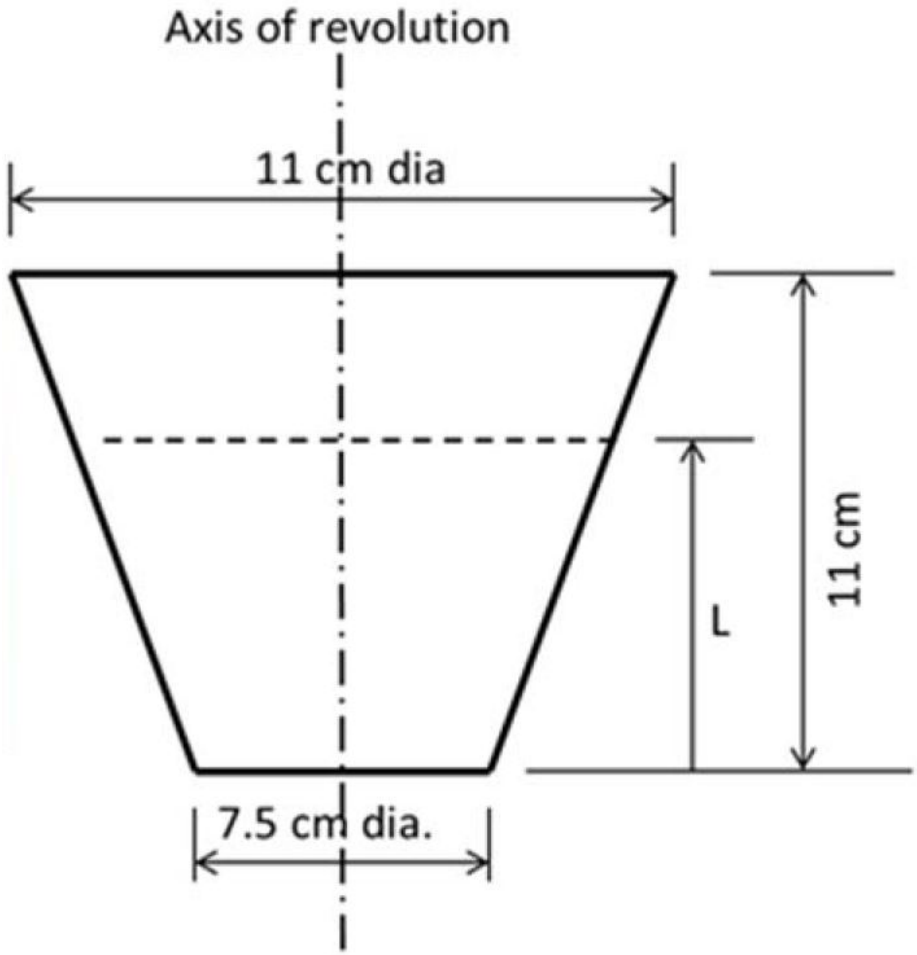
$$e^{-7.85359326}\sin(7.85359326) - \cos(7.85359326) = 0.00$$

Flight time

- one more time!

Fluid level

image of measuring cup



Fluid level

- cup dimension figure
- water in dam, coal in stockpile
- volume V (mL) depends on depth L as follows, *presented without proof:*

$$V = 0.0268L^3 + 1.884L^2 + 44.15L$$

- Question: depth L when cup holds 500 mL of water?
- solve $f(L) = 0$ where

$$F(L) = 0.0268L^3 + 1.884L^2 + 44.15L - 500$$

2) Bisection method

- basic idea: visualisation

- bisection method: key equations

- bisection method: pseudocode

- bisection method: Python code
- live demo

- bisection method: simulation results

3) Secant method

- basic idea: visualisation

- secant method: key equations

- secant method: pseudocode

- secant method: Python code
- live demo

- secant method: simulation results

Newton–Raphson method

- XXX

4) Extensions: re-write methods as functions

- bisection re-write as function

- secant re-write as function

Timing code in Python

- import time
- `tStart = time.perf_counter()`
- (code)
- `tStop = time.perf_counter()`
- elapsedTime = tStop-tStart

Speed comparisons: bisection vs. secant

- xxx
- xxx

Lecture summary

- Solving nonlinear algebraic equations
- Bisection method
- Secant method
  - Newton–Raphson method
- Extensions

More information

- Newton–Raphson method in §7.2 textbook
  - known as Newton's method
  - needs differential calculus MATH1110

- §7.3 and §7.4 text for "optimised" versions of bisection and secant methods, which minimise number of function calls

- extension: proof of measuring cup volume
  `https://www.sjsu.edu/me/docs/hsu-Chapt`