ENGG1003 - Thursday Week 9

Normally distributed random numbers

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Lecture overview

1. standard normal distribution (bell curve)
   - pdf, mean $\mu = 0$ and $\sigma = 1$
   - generate using Python
   - histogram
2. using integration to compute probabilities using standard normal distribution
   - area (needs integration) and probability
3. engineering application

1) Standard normal distribution

- Straight into it, generate $100,000$ random numbers generated using `normal` function in numpy's random library
- mean $=$ 0, std $=$ 1

`filename.py`

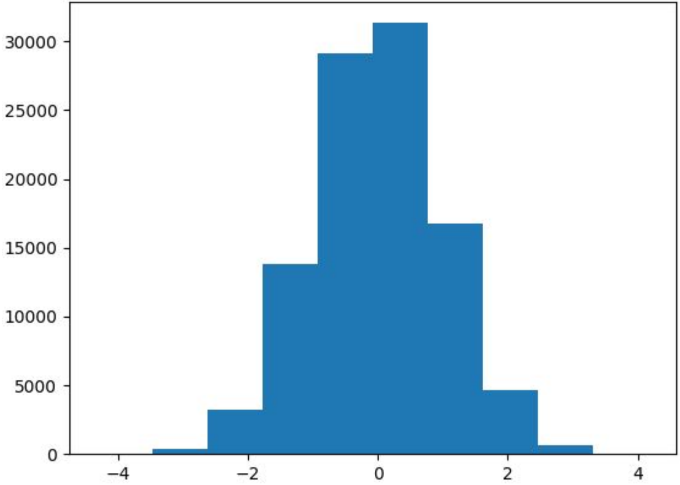
```
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(1)
x = np.random.normal(0.0, 1.0, size=100000)

plt.hist(x, 10)
plt.show()
```

Histogram

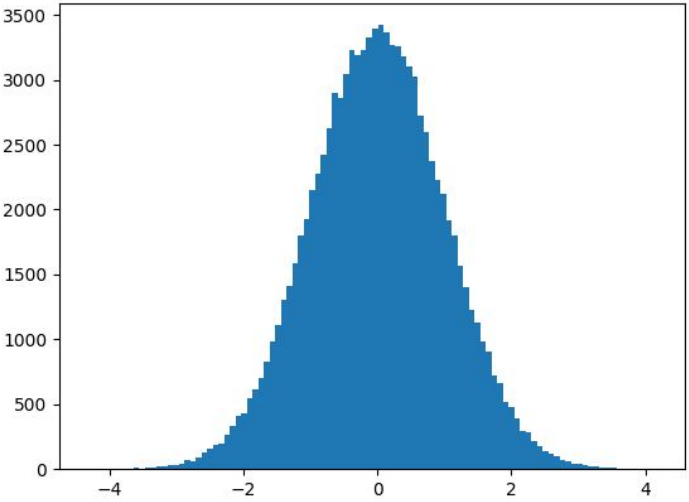
- interpret histogram
- bins, counts, examples
- call hist to return bins—too hard?

- A histogram is a graph showing frequency distributions
- It is a graph showing the number of observations within each given interval.
- To visualize the data set we can draw a histogram with the data we collected
- We will use the Python module Matplotlib to draw a histogram

10 bins

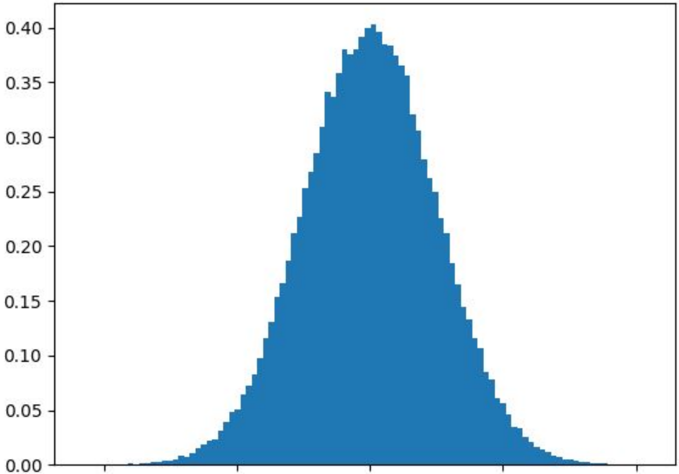


Identical data, but now 100 bins



Identical data, same 100 bins, now area under histogram is normalized to 1

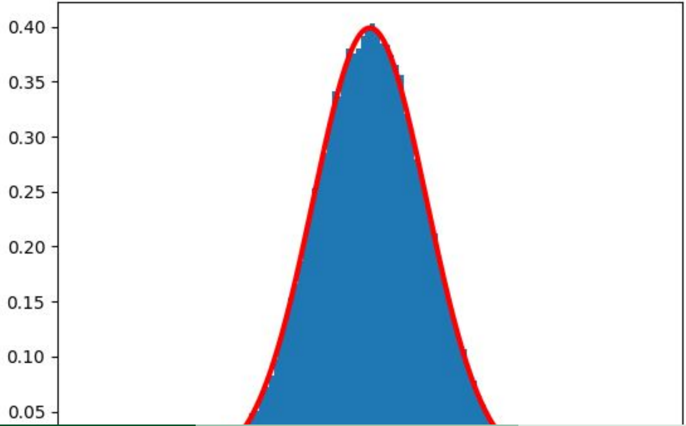
```python
plt.hist(x, 100, density=True)
```



Now with standard normal pdf as red line

```
x = np.linspace(-5,5,num=1000)
f = 1/(np.sqrt(2 * np.pi)) * np.exp(-x**2 / 2)

plt.hist(d, 100, density=True)
plt.plot(x, f, color='r', linewidth=3)
```



Standard normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

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```

- area under $f(x)$ is 1
  - reason for the $1/\sqrt{2\pi}$ factor
- $f(x) \geq 0$ for all x

Probability density functions

- Probability that X takes a value in interval $[a, b]$ is

$$\Pr(a \leq X \leq b) = \int_a^b f(x)dx$$

$$\int_{-\infty}^{\infty} f(x)dx = \Pr(-\infty \leq X \leq \infty) = 1$$

- needs an image

3) Engineering application

- XXX

- xxx

Lecture summary

1. xxx
2. xxx
3. xxx
4. what's next