ENGG1003 - Monday Week 2

Calculating Pi
Arithmetic
Datatypes

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Before we start...

- Today's lecture is information overload
  - It is a long list of "stuff" to rote learn
  - Half of programming is just "playing with Lego" with this "stuff"
  - There is a lot to learn before we can solve interesting problems
- It will probably push you out of your comfort zone
- Lab experience will rebuild your confidence
  - Much of this content will be used in every lab
- But first, a problem for motivation...

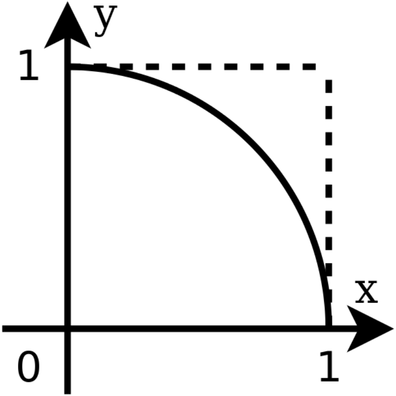
Case Study: Calculating π

- Computers are *really* good at repetitive things
- Lets use this fact to calculate π using a "monte-carlo" method
  - Informally, these are methods which solve problems by repeating the same thing with different inputs until patterns emerge
  - It could repeat millions or billions of times
  - Name comes from the Monaco Principality's high concentration of casinos
- Algorithm pseudocode will be written before an implementation in C

Case Study: Calculating π

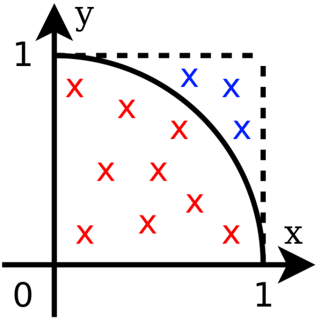
Consider a quadrant of a unit circle ($r = 1$) with a square around it:

- Area of the square $A_1 = 1$
- Area of the circle quadrant $A_2 = \frac{\pi r^2}{4} = \frac{\pi}{4}$
- Ratio of areas $\frac{A_2}{A_1} = \frac{\pi}{4}$
- Therefore $\pi = 4 \times \frac{A2}{A1}$



Case Study: Calculating π

- We can't calculate the area ratio without knowing π
- Estimate it by:
  - Randomly picking many points inside the square
  - Test if the point is inside the circle with $x^2 + y^2 < 1$



- $\pi \approx 4 \times \frac{\text{Number of points which land inside circle}}{\text{Total number of points tested}} = 4 \times \frac{9}{12} = 3$

Algorithm for Calculating π

- How can the above *mathematics* be turned into an *algorithm*?
  - **NB:** You only have to understand the algorithm

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Algorithm for Calculating π

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- The algorithm needs to repeat the *same thing* multiple times
  - This implies use of a loop
  - The loop's *exit condition* needs to be defined
- As the loop repeats, we need to keep track of the following *variables*:
  - The number of points tested
  - The number of points which landed inside the circle
  - The (x, y) coordinates of the point under test

Algorithm for Calculating π

- The number of points tested will be an integer, we will call it `countTotal`
- The number of points found to be inside the circle is also an integer, we will call it `countInside`
- Before these variables are used they should be *initialised*
  - ie: The algorithm will explicitly include `countTotal = 0` and `countInside = 0`
  - So-called *uninitialised* variables have undefined (or random) values

Algorithm for Calculating π

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Algorithm for Calculating π

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  - This implies `IF...ENDIF` *flow control*
- The condition on incrementing `countInside` is $x^2 + y^2 < 1$
- Incrementing a variable in pseudocode takes the form:
  - `variable = variable + 1`
  - This can be read as "variable *becomes* variable plus 1"
  - Maths people would write: $x_{n+1} = x_n + 1$

- The point under test needs two "real" variables: `x` and `y`
  - "Real" comes from mathematics: any number with integer and fractional components. Eg: 1.45
- These values take new random values each loop
- The pseudocode doesn't need to describe how a random number is generated
  - Stating: "`x = a random number between 0 and 1`" is totally acceptable
- At the end of the algorithm the final step will be $\pi = 4 \times \frac{\texttt{countInside}}{\texttt{countTotal}}$

Algorithm for Calculating π

```
BEGIN
  integer countTotal = 0
  integer countInside = 0
  WHILE countTotal < A large number
    x = random number between 0 and 1
    y = random number between 0 and 1
    countTotal = countTotal + 1
    IF x*x + y*y < 1
      countInside = countInside + 1
    ENDIF
  ENDWHILE
  pi = 4*countInside/countTotal
  PRINT pi
END
```

Missing Knowledge for C Implementation

- More information about arithmetic
  - Relational operators (less/greater-than) look useful
  - Is there a neat way to do `count=count+1`?
  - `countInside` and `countTotal` are both integers. What happens when we divide?
- Datatypes and how they are handled in arithmetic statements
- How do we generate random numbers?
- Syntax for WHILE loops and IF statements

C Arithmetic

- Basic arithmetic was seen in the lab

Operation	C Symbol
Addition	+
Subtraction	−
Multiplication	*
Division	/

Table: Basic arithmetic operators in C

- Complex expressions can be built from these operators and parentheses

C Arithmetic

Examples:

$z = x^2 + 5(y + b)$	`z = x*x + 5*(y + b);`
$u = \frac{x+1}{x-1}$	`u = (x + 1)/(x - 1);`
$v = z^3 + \frac{5(y+b)}{2}$	`v = z*z*z+(5*(y + b))/2;`

- Multiplication is not assumed. If you write `5(y+b)` the compiler will generate a syntax error.
- To be valid C statements the semicolon is required.

C Arithmetic

- C supports two time-saving *unary* operators:
  - Very useful in loops.

Operation	C Syntax	Replaces
Increment	`x++;` or `++x;`	`x = x + 1;`
Decrement	`x--;` or `--x;`	`x = x - 1;`

- It also supports the following shorthand syntax:

`x = x + y;`	`x += y;`
`x = x - y;`	`x -= y;`
`x = x * y;`	`x *= y;`
`x = x / y;`	`x /= y;`

C Arithmetic

What's the difference between `x++` and `++x`?

- `x++` is a *post-increment*
- `++x` is a *pre-increment*
- If they appear in an arithmetic expression, pre-increment is processed *before* the variable is used and post-increment is processed *after* it is used.
- In isolation there is no difference.

Modulus

- Integer division ignores (truncates) any fractional component
- The *modulus* operator provides the remainder after division
  - C uses the `%` character
  - "`a % b`" means "remainder of `a / b`"
- Example:
  - `10 / 3 = 3`
  - `10 % 3 = 1`

Modulus Example - Printing Every *nth* Number

```c
#include <stdio.h>
int main() {
  int x = 0;
  while(x < 1000)
  {
    x++; // Increment x
    if(x%100 == 0) // if(x is divisible by 100)
      printf("%d\n", x);
  }
  return 0;
}
```

Relational Operators

- C supports six *relational* operators:

Operation	C Symbol
Less than	<
Less than or equal to	<=
Greater than	>
Greater than or equal to	>=
Equal to	==
Not equal to	!=

Relational Operators

- The result of a relational operation is `0` or `1`
  - C treats `0` as Boolean FALSE and non-zero as TRUE
- They are typically used as flow control conditions
  - `if(`*condition*`) {`*statements*`}`
  - `while(`*condition*`) {`*statements*`}`
- While we're here: the above is the correct syntax for IF and WHILE flow control in C

`if()` and `while()`

- Syntax for your “cheat sheet”:

```
if(condition) {
  // Do stuff
}
```

```
if(condition) {
  // Do stuff
} else {
  // Do other stuff
}
```

```
while(condition) {
  // Do stuff
}
```

- If there is a single *statement* inside an `if()` or `while()` the { and } are optional

`if()` and `while()`

Example:

```
#include <stdio.h>

main() {
  while(x != 0) {
    scanf("Enter an integer: %d\n", &x);
    if(x % 2 == 0)
      printf("%d is even\n", x);
    else
      printf("%d is odd\n", x);
  }
}
```

Boolean Operators

- We discussed Boolean algebra last week
- It has three operators: AND, OR, NOT
- In C, these are implemented with:

Operator	C Characters
AND	&&
OR	||
NOT	!

- I know the | character as a "pipe" symbol or "vertical slash"
  - Ask your demonstrator to find it on the keyboard

Conditions

- We have seen that `if()` and `while()` statements need *conditions*
- A condition can be any combination of variables, operators, and literals (numbers, defined later)
- The following are all valid:
  - `if(x > 29)`
  - `if(x == 2)`
  - `if(!(x == y))`
  - `if( (x > 5) && (x < 20) )`
  - `if(x = y + z)`
    - Evaluates `(y + z)`, assigns to `x`, uses result as `if()` condition.

C Arithmetic Operator Precedence

- C has an "order of operations"
- eg: `1+5*2` evaluates to `11`
  - You remember BODMAS / PEDMAS, right?
- Multiplication and division first
- Addition and subtraction second
- Relational operators somewhere below that
- If in doubt: force order with parentheses
  - This makes the code more readable
  - It doesn't cost you anything
  - C compilers understand algebra and will optimise inefficient expressions automatically

Variable Declaration

- In C, all variables are *declared* before use
- Declaration specifies the variable's:
  - Datatype
  - Name
  - An *initialisation value* (optional)
    - Always assume uninitialised variables have random values! Behaviour varies between compilers and target platforms.
  - eg: `int counter = 0;`
    - Type is `int`
    - Name is `counter`
    - Initial value is `0` (optional)
- C is a "strongly-typed" language
  - A variable's type doesn't change

Variable Declaration

- You can declare multiple variables in one line:

```
int a, b, c, d;
```

- You can mix initialised and uninitialised variables:

```
float q, r = 1.0, s = 12e3, t;
```

- All variables declared in this way must be of the same type

Variable Naming Rules

- Variable names:
  - Are case sensitive
  - Must **not** be a C keyword (eg: `int`, `char`, `while`, etc)
  - Must **not** contain special characters (`*-+/`, etc)
  - Must begin with a letter
  - Can contain numbers
  - Can contain underscores
- What you do with those rules is up to you
- Modern practice encourages:
  - camelCaseVariableNames
  - names_with_underscores

Integer Data Types

- There are several integer data types
- They vary by their:
  - Size
  - Support for negative numbers
- C integer types can be 1, 2, 4, or 8 bytes long
  - `int` and `long` sizes vary by platform
  - Larger sizes store larger numbers, use more RAM
- Each type can be *signed* or *unsigned*
  - Unsigned numbers are always positive but you get double the *value range*

Integer Data Types

- The integer data type ranges can be calculated from the data type's size in bits
- For unsigned numbers of bit length n:

$$\max = 2^n - 1 \tag{1}$$

- For signed numbers of bit length n:

$$\max = 2^{(n-1)} - 1 \tag{2}$$
$$\min = -2^{(n-1)} \tag{3}$$

- Signed numbers are stored in *two's complement* format, covered in ELEC1710

Integer Data Types

- C includes the `sizeof()` expression so that a program can discover the size of a data type on a given platform
- On a modern 64-bit Linux desktop machine:

Type	Bytes	Bits	Value Range
`char`	1	8	-128, +127
`short`	2	16	-32768, 32767
`int`	4	32	-2147483648, 2147483647
`long`	8	64	-9223372036854775808, ...

Unsigned Integers

- Unsigned integers are *always positive*
- They are the same size as their signed counterparts
- The `unsigned` keyword placed before the data type makes that variable unsigned
- eg: `unsigned char` is 1 byte and has a value range of `0` to `255`

Overflow

- *Overflow* occurs when the result of a calculation is too big to fit into the target type
- Example: `127 + 1 = -128`

```c
#include <stdio.h>
int main() {
  char x = 127;
  printf("%d\n", x);
  x++;
  printf("%d\n", x);
  return 0;
}
```

Overflow

- Message: make variables as small you can, but no smaller
  - Or just use `long` everywhere and cop the performance hit
- Quite often you need to take a guess at how big a variable needs to be
- Typically a good idea to document software limits due to variable choice

Why Care About Data Types?

- You may be thinking "why not make everything a `long`?"
- Answer: speed and memory
- Smaller types use less RAM
- Arithmetic on a type larger than the target platform's *native size* is slow
- Matters if you store millions of the same type
- Makes a *huge* difference on embedded targets
  - Don't declare 32-bit variables on an 8-bit AVR microcontroller *unless you have to*

Floating Point Data Types

- To store real numbers C has several *floating point* data types
- As with integers, try to use the smallest you can get away with
- `long double` is supported in gcc but...complicated

Type	Size	Range	Precision
float	4	1.2×10^{-38} to 3.4×10^{38}	6 dp
double	8	2.3×10^{-308} to 1.7×10^{308}	15 dp

How good is 15 decimal places?

Paraphrasing NASA:

Calculating the circumference of Earth using 15 decimal places for π results in a truncation error about the size of a molecule.

39 digits of π calculates the circumference of the *visible universe* to within the width of a hydrogen atom ($\approx 10^{-10}$ m).

Floating Point Problems

- Q: Why not *always* use `float` or `double` instead of `int`?
- A: Floating point arithmetic is **NOT** exact
- Example 1: Does this code exit?

```
#include <stdio.h>
int main() {
  float x = 16000000.0;
  while(x < 17000000.0) {
    x = x + 1.0;
    printf("%f\n", x);
  }
}
```

Floating Point Problems

- Example 2: $\tan\left(\frac{\pi}{2}\right)$ is undefined
- But using floating point π can't be represented exactly, so C produces a result anyway

```c
#include <stdio.h>
#include <math.h>
int main() {
  double pi = 3.14159265358979323 8;
  printf("%f\n", tan(pi/2.0));
  printf("%f\n", ftanf((float)pi/2.0));
}
```

Literals

- A *literal* is any number written in the code
- Why not "constant"?
  - That word means something different
- Examples:
  - `x = 5; // 5 is a literal int`
  - `y = 2.0 / z; // 2.0 is a literal double`
- By default:
  - An integer literal is stored as an `int` data type
    - ie: has the value range and arithmetic limits of `int`
  - A floating point literal is stored as `double`

Literals

- Integer literals can be in:
  - Decimal: `123`
  - Hexadecimal: `0xA34 // Zero-x`
  - Octal: `O125 // Capital letter O`
  - (Hex and octal are covered in ELEC1710)
- Integer literals can be specified as *unsigned* with the `u` suffix:
  - `938u`
- They can also be declared `long` with the `l` suffix:
  - `3726484l`
  - The compiler will issue a warning if a literal is too big for `int`

Literals

- Floating point literals can be written in many ways:
  - `1.0f // f suffix forces float`
  - `(float)2.3 // Forces float`
  - `1.0 // Default to double`
  - `1e2 // Double, 1 times 10^2`
- `1e2` is known as "e-notation"
  - $\mathtt{XeY} = \mathtt{X} \times 10^{\mathtt{Y}}$
  - I will use it *all the time*
- Forcing literals to `float` is frequently necessary in embedded systems which lack double precision hardware

Mixing Data Types

- C supports arithmetic between different types
- Type conversion is called *casting*
- When types are mixed two things can happen:
  - Types get upgraded automatically (*implicit* type casting)
    - Upgrade path is roughly: `short/char - int - long - long long - float - double`
  - Types get specified manually by the programmer (*explicit* type casting)

Explicit Type Casting

- The data type of a variable (or literal) can be forced to change using *type casting*
- Write the desired type in parentheses before the variable or constant
- Examples:
  - `x = (float)y / k;`
  - `y = (unsigned int)y + 32;`

Format Specifiers

- A *format specifier* controls how `printf();` converts numerical (or textual) data to a series of ASCII characters
- Full details are complex, for now just use:
  - `%d` for integer types
  - `%f` for "fixed decimal place" floating point
  - `%e` for e-notation floating point
  - Cast inside `printf()` to suppress compiler warnings

Format Specifiers

- Casting example:

```c
long i;
// ...
printf("%d\n", (int)i); //Breaks when i>2^31
```

- `%.df` produces `d` decimal places of precision
  - eg:

```c
float x = 1.23456;
printf("%.2f", x); // Prints 1.23
```

- Format specifiers can appear anywhere in the output text, eg:

```c
printf("%f was the result\n", x);
printf("The result %f was calculated\n", x);
```

Format Specifiers

- You can print several numbers at once
- Separate variables with commas
- They are printed in the same order as variables are written
- You can put maths inside `printf();`

```c
printf("%f plus %f is %f\n", x, y, x+y);
```

Integer Division Example

With *all that* out of the way, what is the output of each `printf()` statement?

```
#include <stdio.h>
int main() {
  printf("%d\n", 9/10);
  printf("%f\n", 9/10);
  printf("%f\n", 9.0/10);
  printf("%f\n", 9/10.0);
  printf("%f\n", (float)9/10);
  return 0;
}
```

Listing 1: `intdiv.c`

Random Numbers

- In C there is a *standard library function* which generates random numbers
- We will study functions in more detail later
- To use a library function:
  - Read the function's documentation
  - `#include` the correct *header file*
  - Take note of the *return value* data type
  - Add any *compiler flags* (beyond ENGG1003)
  - Use the function in your code
- Demonstration: read the `rand()` man page
  - Click here

Random Numbers

- Observe that `rand();` requires

```
#include <stdlib.h>
```

- Observe that it returns an `int` between 0 and `RAND_MAX` which, in gcc, is $2^{31}-1$
- Note that is has limits (advanced discussion, beyond ENGG1003)
- For us, using the % operator:

```
x = rand() % (RANGE+1);
```

produces a *good enough* random number between zero and `RANGE`

Random Numbers

- As above, zero to RANGE is produced with:

```
x = rand() % (RANGE+1);
```

- To generate numbers from 1 to RANGE:

```
x = (rand() % RANGE) + 1;
```

- These work because:

```
x = rand() % MAX;
```

  is a random number between zero and (MAX−1)

Random Numbers

- Using the above information, we can roll a standard die with:

```
int dieRoll;
dieRoll = (rand() % 6) + 1;
```

- Or, if you're a D&D fan, roll a D20 with:

```
int D20Roll;
D20Roll = (rand() % 20) + 1;
```

Random Numbers

- `RAND_MAX` is a *constant* defined in `stdlib.h`
- We can try to generate a floating point number from 0 to 1 with a division:

```c
float num;
num = rand()/RAND_MAX; // This is broken
```

- Problem: `rand()` is an integer, `RAND_MAX` is a (large) integer
  - The integer division result will always be zero!
- Solution: use an explicit cast so that a floating point division occurs:

```c
double num;
num = (double)rand()/RAND_MAX;
```

Probably out of time by now, lets implement a π calculator in the lab... *In theory* you have enough information by now.