ENGG1003 - Monday Week 9

Numerical integration: review and applications

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Lecture overview

1. Review of integration
2. Applications of integration
   - average value of a function
   - area between curves
   - centre of mass
   - probability
3. Interpolation revisited

1) Review of integration

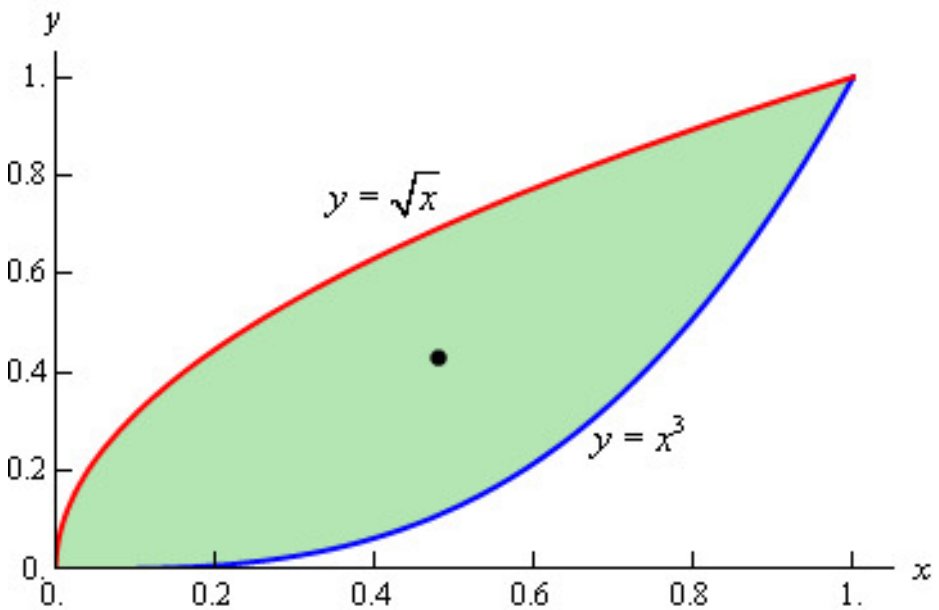
- XXX

2) Applications of integration:
i) average value of a function

- xxx

2) Applications of integration: ii) area between curves

https://tutorial.math.lamar.edu/classes/calcii/centerofmass.aspx



2) Applications of integration: ii) area between curves

- exact area between $f(x) = \sqrt{x}$ and $g(x) = x^3$, domain $[0, 1]$ is $5/12 = 0.4166667$
- using code `areabetweencurves.py` for trapezoidal method, 1000 panels, area is 0.416660

2) Applications of integration: iii) centre of mass

Same example as area between curves, but extend to centre of mass at $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{1}{A}\int_a^b x\left(f(x) - g(x)\right)dx$$

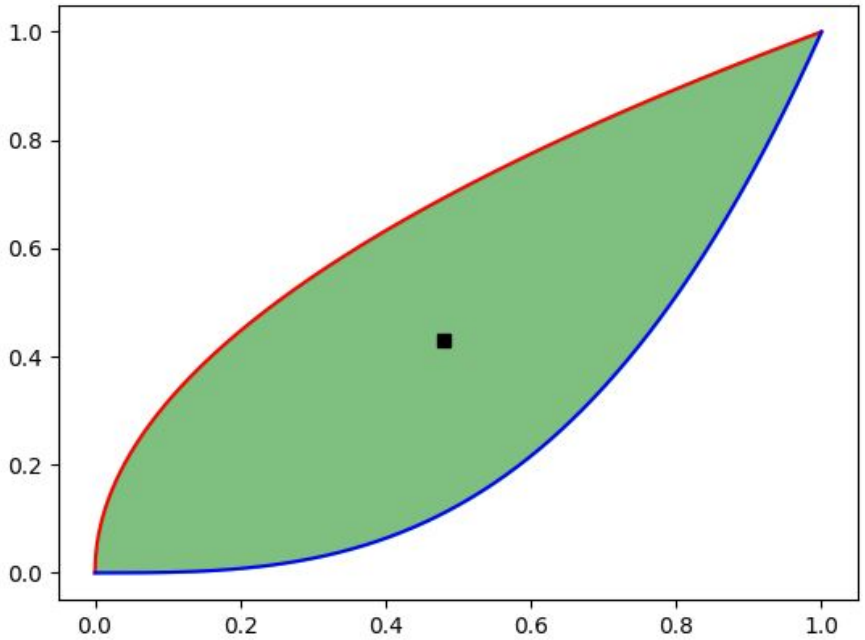
$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}\left([f(x)]^2 - [g(x)]^2\right)dx$$

where

$$A = \int_a^b f(x) - g(x)dx$$

- Python code: `centreofmass.py`

```
Trapezoidal, 100 sub-intervals
Area under f: 0.666463
Area under g: 0.250025
Area between f and g: 0.416438
Centre of mass: (0.4802,0.4287)
Exact centre of mass: (0.4800,0.4286)
```



2) Applications of integration: iv) probability

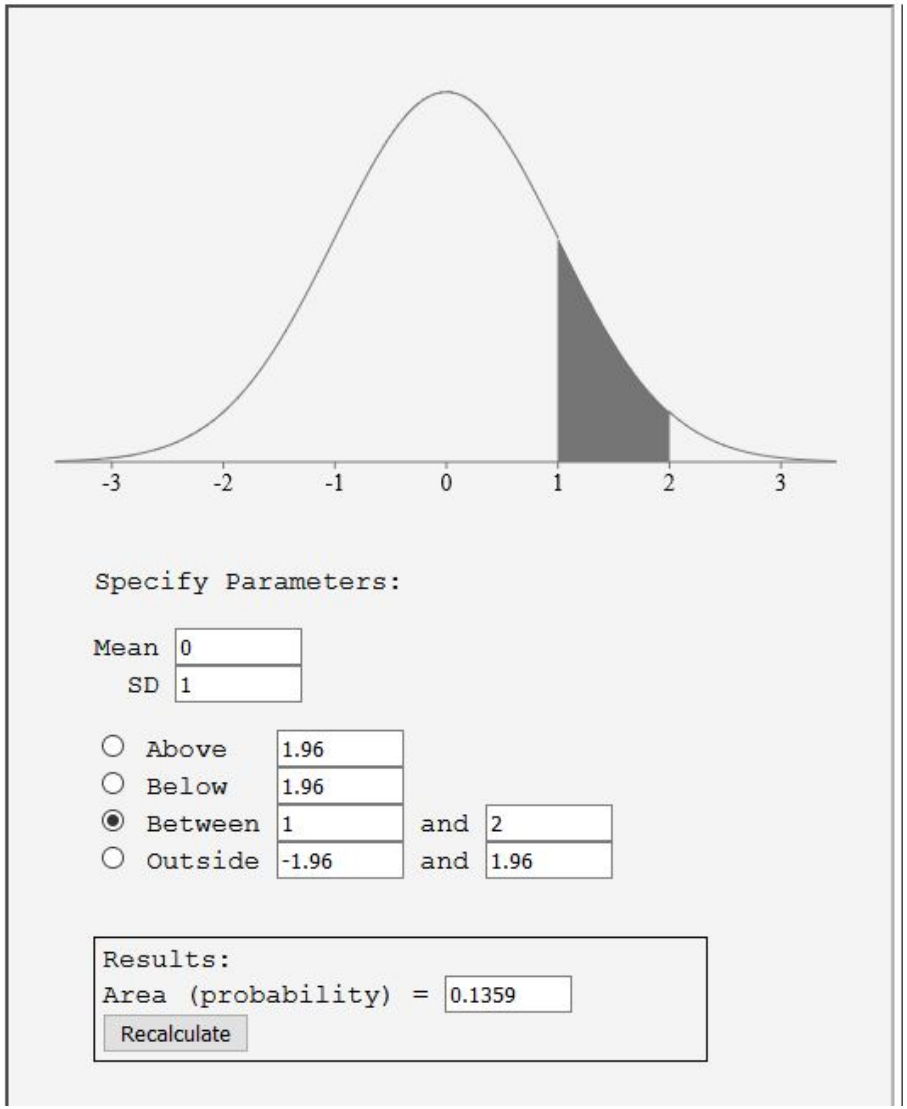
probability density function of normal (or Gaussian) distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

https://onlinestatbook.com/2/calculators/normal_dist.html

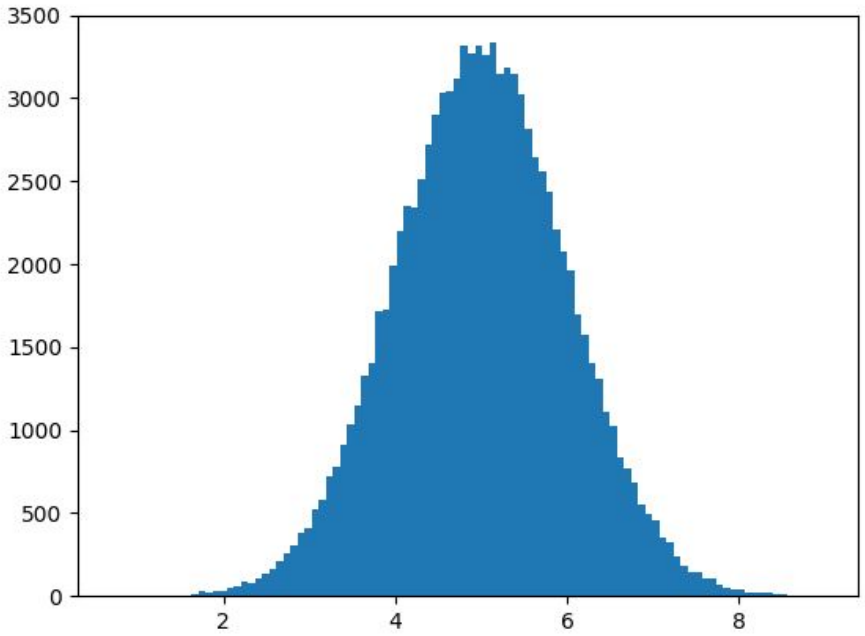
- Python code `normalprob.py`

- normal pdf distribution calculator



Generate normally distributed random numbers in Python

- Python code `generatenormal.py`



3) Interpolation revisited

- XXX

Lecture summary

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- blah
- blah