ENGG1003 - Tuesday Week 3

More Sequence Examples
Maybe More Flow Control

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Assessment Task Rules

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Easy(ish) Assessment Task Example

Write a C program which generates a sequence of numbers:

$$x_1, x_2, x_3, \ldots$$

with the iterative equation:

$$x_n = 3x_{n-1} + 2x_{n-2}$$

and initial conditions:

$$x_1 = 3,\ x_2 = 1$$

The program should exit after printing (x_8 or an $x_n > 100$).

Easy(ish) Assessment Task Example

The program's output format is:

```
n x<newline>
```

For the values given, the output is:

```
1 3.000000
2 1.000000
3 9.000000
4 29.000000
5 105.000000
```

Easy(ish) Assessment Task Example

- What do we need to do?
  - Set up variables
  - Give some initial values
  - Implement the equation
  - Print the initial values
  - Write a `while()` loop
  - Get the exit condition correct
  - Print results
  - Wrap the whole thing in `main()`

Set up variables

Question didn’t specify, but lets assume `float`

```
float xn, xnm1, xnm2;
int n;
```

Give some initial values

Question gave us:

$$x_1 = 3, \; x_2 = 1$$

Be careful with `xnm1` and `xnm2`, where are we starting?

```
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3
```

Implement the equation

$$x_n = 3x_{n-1} + 2x_{n-2}$$

```
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3

xn = 3.0*xnm1 + 2*xnm2;
```

That calculates x_3, but how does the program "advance in time"?

Implement the equation

Shift all the variables “forward in time” with:

```c
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3

xn = 3.0*xnm1 + 2*xnm2;
xnm2 = xnm1;
xnm1 = xn;
```

Print the initial values

```c
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3

// x1 and x2 given so just hard code n
printf("1 %f\n", xnm2);
printf("2 %f\n", xnm1);

xn = 3.0*xnm1 + 2*xnm2;
xnm2 = xnm1;
xnm1 = xn;
```

Write a `while()` loop

We need to calculate x_n more than once, so:

```c
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3

// x1 and x2 given so just hard code n
printf("1 %f\n", xnm2);
printf("2 %f\n", xnm1);

while( /* something */ ) {
  xn = 3.0*xnm1 + 2*xnm2;
  xnm2 = xnm1;
  xnm1 = xn;
}
```

Get the exit condition correct

The value of n goes from 1 to 8, and xn must remain below 100:

```c
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3
// x1 and x2 given so just hard code n
printf("1 %f\n", xnm2);
printf("2 %f\n", xnm1);
while( (n <= 8) && (xn < 100) ) {
  xn = 3.0*xnm1 + 2*xnm2;
  xnm2 = xnm1;
  xnm1 = xn;
  n++;
}
```

Print results

```c
float xn, xnm1 = 1, xnm2 = 3;
int n = 3; // The first unknown is x for n=3
// x1 and x2 given so just hard code n
printf("1 %f\n", xnm2);
printf("2 %f\n", xnm1);
while( (n <= 8) && (xn < 100) ) {
  xn = 3.0*xnm1 + 2*xnm2;
  xnm2 = xnm1;
  xnm1 = xn;
  n++;
  printf("%d %f\n", n, xn);
}
```

Wrap the whole thing in `main()`

NB: This code still has errors. Search for and debug live.

```c
#include <stdio.h>
main() {
  float xn, xnm1 = 1, xnm2 = 3;
  int n = 3; // The first unknown is x for n=3
  // x1 and x2 given so just hard code n
  printf("1 %f\n", xnm2);
  printf("2 %f\n", xnm1);
  while( (n <= 8) && (xn < 100) ) {
    xn = 3.0*xnm1 + 2*xnm2;
    xnm2 = xnm1;
    xnm1 = xn;
    n++;
    printf("%d %f\n", n, xn);
  }
```

Is the solution optimal?

- Some marks are allocated to reducing variable count
- It tests your understanding of how the = operation works
- Lets look at the maths:

```
xn = 3.0*xnm1 + 2*xnm2;
xnm2 = xnm1;
xnm1 = xn;
n++;
printf("%d %f\n", n, xn);
```

- Do we need *all* those variables?

- In this case: yes

```c
xn = 3.0*xnm1 + 2*xnm2;
xnm2 = xnm1;
xnm1 = xn;
n++;
printf("%d %f\n", n, xn);
```

- We can't overwrite `xnm1` before shifting it into `xnm2`
- Result must be stored in `xn` first

Another Isolated Example

- What if the equation was:

$$x_n = 0.2x_{n-1}$$

- This will *work*:

```
xn = 0.2*xnm1;
xnm1 = xn;
```

- But because we never need `xnm1` elsewhere this is more optimal:

```
xn = 0.2*xn;
```

- Marks (above a pass) may be allocated to variable optimisation

Hard Assessment Task Example

Write a C program which generates two sequences of numbers:

$$x_0, x_1, x_2, \ldots$$
$$y_0, y_1, y_2, \ldots$$

with the coupled iterative equations:

$$\begin{aligned} x_n &= 0.6x_{n-1} + 0.2y_{n-1} \\ y_n &= 0.1x_{n-1} + 0.9y_{n-1} \end{aligned}$$

and initial conditions:

$$x_0 = 5$$
$$y_0 = 0$$

Hard Assessment Task Example

$$x_n = 0.6x_{n-1} + 0.2y_{n-1}$$
$$y_n = 0.1x_{n-1} + 0.9y_{n-1}$$

- Lets have an attempt at implementing the equations
- We need *at least* two variables:
  - `float xn`
  - `float yn`
- Lets also use two “previous” variables:
  - `float xnm1`
  - `float ynm1`

Hard Assessment Task Example

$$
\begin{aligned}
x_n &= 0.6x_{n-1} + 0.2y_{n-1} \\
y_n &= 0.1x_{n-1} + 0.9y_{n-1}
\end{aligned}
$$

- Our calculation code can then be:

```
xn = 0.6*xnm1 + 0.2*ynm1;
yn = 0.1*xnm1 + 0.9*ynm1;
xnm1 = xn;
ynm1 = yn;
```

- **Question:** Do we need all these variables?

Hard Assessment Task Example

$$x_n = 0.6x_{n-1} + 0.2y_{n-1}$$
$$y_n = 0.1x_{n-1} + 0.9y_{n-1}$$

- **Counter-question:** What is wrong with this?

```
xn = 0.6*xn + 0.2*yn;
yn = 0.1*xn + 0.9*yn;
```

Hard Assessment Task Example

$$x_n = 0.6x_{n-1} + 0.2y_{n-1}$$
$$y_n = 0.1x_{n-1} + 0.9y_{n-1}$$

- **Counter-question:** What is wrong with this?

```
xn = 0.6*xn + 0.2*yn;
yn = 0.1*xn + 0.9*yn;
```

- Why doesn't mathematics convert into code?

Hard Assessment Task Example

- Mathematics is *instant*

Hard Assessment Task Example

- Mathematics is *instant*
- Code is evaluated line by line

Hard Assessment Task Example

- Mathematics is *instant*
- Code is evaluated line by line
- Variables can *change* between lines, resulting in the wrong equation being implemented
- The previous slide was *actually* doing:

$$x_n = 0.6x_{n-1} + 0.2y_{n-1}$$
$$y_n = 0.1x_n + 0.9y_{n-1}$$

```
xn = 0.6*xn + 0.2*yn;
yn = 0.1*xn + 0.9*yn;
```

Hard Assessment Task Example

- Observe the correct subscripts:

$$
\begin{aligned}
x_n &= 0.6x_{n-1} + 0.2y_{n-1} \\
y_n &= 0.1x_{n-1} + 0.9y_{n-1}
\end{aligned}
$$

- In the 2nd equation we need x_{n-1} but the first equation would destroy that value
- We *must* use an extra variable to store x_{n-1} for y_n to be calculated correctly

Hard Assessment Task Example

- Aside: You may see coupled equations vaguely like this in signals and systems theory

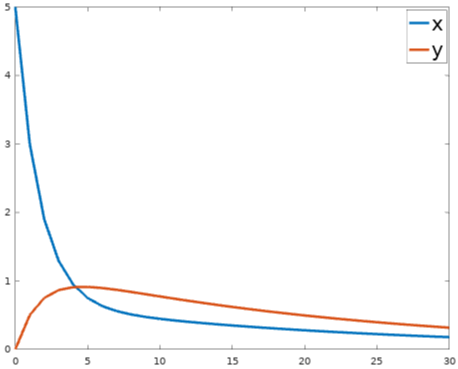
$$x_n = 0.6x_{n-1} + 0.2y_{n-1}$$
$$y_n = 0.1x_{n-1} + 0.9y_{n-1}$$

- Lets write C code with the minimum variables:

```c
xtmp = x; // store xn before we lose it
x = 0.6*x + 0.2*y;  // Original xn value lost
y = 0.1*xtmp + 0.9*y; // stored xn used, yn
```

- ...And implement

Results



This graph was generated by copying the output data into a file then loading it in MATLAB.