

ENGG1003 - Tuesday Week 1

Algorithms and Pseudocode

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Algorithms

- Informally, an *algorithm* is a series of steps which accomplishes a task
- More accurately, the steps (instructions) must:
 - ▶ Have a strict order
 - ▶ Be unambiguous
 - ▶ Be executable
- “Executable” means that the *target platform* is capable of performing that task.
 - ▶ eg: An industrial welding robot can execute “move welding tip 1 cm left”. A mobile phone can’t.

Algorithms

- An algorithm exists purely as an abstract concept until it is communicated
- We will use:
 - ▶ *Pseudocode* to communicate algorithms to ourselves and other people
 - ▶ The languages C and MATLAB to communicate algorithms to computers
- Pseudocode *can* be very formal, but as engineers we will only use formal rules if required
 - ▶ eg: When documenting algorithms for other people
 - ▶ Your own “working out” can be anything that helps *you*

Algorithm Example 1

Name: Algorithm given to mum to start my car (2015 Tarago)

Result: The vehicle's engine is idling

Initialisation: stand next to the vehicle, key fob in hand

- ➊ Depress the unlock button on the key fob, car will beep twice
- ➋ Place key fob in your pocket
- ➌ Enter the vehicle, sit in the driver's seat
- ➍ Ensure that the gear selector has P engaged
- ➎ Depress the brake pedal
- ➏ Press the engine start button
- ➐ Wait 5 seconds
- ➑ If engine is not idling
 - ▶ Call me

Example Discussion

- Algorithms typically need to feel over-explained
 - ▶ Computers are *really stupid*; get in the habit of over-thinking everything
- The algorithm contained *flow control* in the form of an “if” statement
 - ▶ The final step (“call me”) was *conditional* on the car not starting
- We will discuss conditional logical statements later, but first...

Algorithm Example 2

A wife asks her husband, a programmer, “Could you please go shopping for me and buy one carton of milk, and if they have eggs, get 6?”

A short time later the husband comes back with 6 cartons of milk and his wife asks, “Why did you buy 6 cartons of milk?”

He replies, They had eggs.

Algorithm Example 2a

Lets make this more realistic.

A wife asks her robot helper, “Could you please go shopping for me and buy one carton of milk, and if they have eggs, get 6?”

The robot replies: “Unknown instruction: ‘get 6’.”

Flow Control

- Instructions in an algorithm execute in an ordered list
 - ▶ ie: top to bottom
- Flow Control is any algorithmic mechanism which changes the default “top to bottom” execution behaviour
- We will discuss IF statements and *loops*
- Flow control (almost) always requires a *condition*

Conditions

- Computers don't understand “maybe”
- A *condition* must be absolutely **true** or **false**
- Human examples:
 - ▶ I am within the boundary of the Callaghan campus
 - ▶ I am alive
 - ▶ My net worth is below AU\$100M
- Computer examples:
 - ▶ i is less than 184
 - ▶ x plus y is not equal to zero
 - ▶ Input data has been given to the program
 - ▶ A division by zero has occurred

Code Blocks

- A *block* is a set of instructions which are grouped together
- If a single condition controls multiple instructions they can go together in a block
- A block is typically indicated via indentation
- Eg:

```
IF it is raining
    Pack an umbrella
    Drive to campus instead of walking
    Leave home 40mins early to find parking
ENDIF
```

IF Variants

- There are several versions of IF flow control:
 - ▶ IF ... ENDIF
 - ▶ IF ... ELSE ... ENDIF
 - ▶ IF ... ELSEIF ... ENDIF
- The IF and ELSEIF keywords indicate conditions
- The ELSE keyword is *unconditional*
- Which one you choose depends on need
 - ▶ Is there one thing which is conditional?
 - ▶ Do I need to make a choice between two or more options?
 - ▶ Could nothing be executed?

IF Statement Syntax

- The IF ... ENDIF syntax is:

```
IF condition  
    do some things  
ENDIF
```

- Likewise: IF ... ELSEIF ...
ENDIF syntax is:

```
IF condition1  
    do some things  
ELSEIF condition2  
    do other things  
ENDIF
```

- And finally:

```
IF condition  
    do some things  
ELSE  
    do some things  
ENDIF
```

IF ... ELSEIF

- The IF ... ELSEIF construct can have multiple ELSEIF sections
- A *crucial* point:
 - ▶ Conditions are only tested *if the previous ones fail*
 - ▶ Once a condition is TRUE the others are ignored
 - ▶ ie: IF - ELSE implements a choice priority

Algorithm Example 3 - Quadratic Root Finding

From high school you should know that the equation

$$ax^2 + bx + c = 0 \quad (1)$$

has solutions given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (2)$$

lets write an algorithm which provides real valued solutions to a quadratic equation.

Algorithm Example 3 - Quadratic Root Finding

Input: Real numbers a , b , and c

Output: Three numbers:

- 1 The number of solutions, N
- 2 One of the roots, x_1
- 3 The other root, x_2

Behaviour:

- If N is 2 then x_1 and x_2 are different real numbers
- If N is 1 then x_1 is the unique solution and x_2 is undefined
- If N is 0 then x_1 and x_2 are undefined

Algorithm Example 3 - Quadratic Root Finding

```
BEGIN
  INPUT: a, b, c
  D = b^2 - 4ac
  IF D < 0
    N = 0
  ELSEIF D == 0
    N = 1
    x1 = -b / (2a)
  ELSEIF D > 0
    N = 2
    x1 = (-b + sqrt(D)) / (2a)
    x2 = (-b - sqrt(D)) / (2a)
  ENDIF
END
```

- Reasonably formal pseudocode
- The IF ... ELSE IF flow control construct forces exclusive execution of only *one* block
- The first condition that is true causes execution of that block
- Subsequent blocks ignored
- Contains 3 *conditions*

Boolean Algebra Basics

- What if we want more complicated conditions?
Boolean algebra is needed!
- Boolean algebra (or Boolean logic) is a field of mathematics which evaluates combinations of *logical variables* as either true or false
- Boolean *variables* can only take the values **true** (or 1) or **false** (or 0)
- Boolean algebra defines three *operators*:
 - ▶ OR
 - ▶ AND
 - ▶ NOT

Boolean Algebra Basics

- Boolean variables can be allocated any symbols (just like in “normal” algebra)
 - ▶ Typically get upper-case letters
 - ▶ eg: $X = A \text{ OR } B$
- Various symbols can be used for OR/AND/NOT, we will only use the words here
 - ▶ Write them in capitals to remove ambiguity
 - ▶ C and MATLAB have their own symbols for Boolean algebra
 - ▶ Other courses (eg: ELE17100) will use different symbols again

Boolean Operators

- An *operand* is a value on which a mathematical operation takes place
 - ▶ eg: In “1 + 2” the 1 and 2 are operands and + is the operator
- OR - Evaluates true if either operand is true
 - ▶ $X = A \text{ OR } B$
 - ▶ X is true if either one of A or B is true
- AND- Evaluates true only when *both* operands are true
 - ▶ $X = A \text{ AND } B$
 - ▶ X is true only if both A and B are true

Boolean Operators

- OR and AND are *binary* operators
 - ▶ They operate on two operands
 - ▶ From Latin “bini” meaning “two together”
- The NOT operator is *unary*
 - ▶ It only operates on *one* operand
 - ▶ NB: The operand could be a single variable or complex expression
- NOT performs a logical inversion
 - ▶ NOT true = false
 - ▶ NOT false = true

Boolean Condition Examples

- My car needs a service if, since the last service, (more than 6 months has past) OR (more than 15000km have been travelled)
- You will pass this course if (you score 40% or more in the final exam) AND (the weighted sum of all assessments is more than 50%)
- A computer program repeats an algorithm if (there is still data to process) AND (errors have not occurred) AND (NOT (the user has terminated the program))

Algorithm Example 4 - Boolean Conditions

Problem: How can square roots be calculated by a computer?

One Solution: The *Babylonian Method*.

The square root of a , \sqrt{a} , can be found by *iterating*:

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right) \quad (3)$$

until x_n is “close enough” to the true value of \sqrt{a} for our liking.
Execution of this algorithm can use two things:

- 1 The *loop* flow control concept
- 2 Some kind of *stop condition*

Iteration

- In this context iteration is the process of repeatedly applying a formula to the same variables
- Iteration typically creates a sequence of numbers:
 x_0, x_1, \dots, x_n
 - ▶ Eg: The equation $x_n = x_{n-1} + 1$ with a choice of $x_0 = 0$ just counts 1, 2, 3, 4...
- We will study a lot of equations like this in the coming weeks

Square Root By Hand

- Lets find $\sqrt{2}$ “manually”
- In our notation, $a = 2$
- The choice of x_n doesn't *really* matter, lets go with $x_0 = 2$
- Applying the formula $x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$:

$$x_1 = \frac{1}{2} \left(2 + \frac{2}{2} \right) = 1.5$$

$$x_2 = \frac{1}{2} \left(1.5 + \frac{2}{1.5} \right) = 1.4167$$

$$x_3 = \frac{1}{2} \left(1.4167 + \frac{2}{1.4167} \right) = 1.4142$$

Square Root By Spreadsheet

Well that's tedious. Lets try it on a spreadsheet

Questions: When do we stop calculating? How would we write a *stop condition* in computer language terms?

Note that the “difference” is always negative.

Algorithm Example 4 - Boolean Conditions

- For this example we will choose two exit conditions:
 - 1 An acceptable precision is reached
 - 2 An iteration limit is reached
- The resulting Boolean expression is something like:

“If the change between x_n and x_{n+1} is greater than some precision value AND the number of iterations is less than maximum then continue iterating”

Loops

- A *loop* causes an algorithm to execute a given block of instructions multiple times
- Loops typically require an *exit condition*
 - ▶ Without an exit condition they are called *infinite loops*
 - Yes, these have a purpose
- Multiple types of loops
 - ▶ WHILE *condition*...ENDWHILE
 - ▶ DO...WHILE *condition*
 - ▶ FOR *counter* FROM 1 TO *something*

Algorithm Example 4 - Boolean Conditions

- Implementing the square root algorithm:
 - Choose max iterations as 10 and precision as 0.001

```
BEGIN
  INPUT a
  x = a
  xOld = 0 // Why do we do this?
  n = 0
  WHILE (n<10) AND ( (x-xOld) > -0.0001 )
    xOld = x
    x = 1/2*(x + a/x)
    n = n + 1
  ENDWHILE
END
```

- It loops until 10 *iterations* have occurred OR a precision limit is reached

Loop Details

- WHILE conditions are tested before "entering"
- The condition is tested before every repeat
- Variables in the condition should change inside the loop
 - ▶ Try to avoid infinite loops *unless you want one*
- What if we want to force the loop to execute *at least once*?
 - ▶ There exists a DO . . . WHILE loop for this purpose