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Homework Assignment 3: Integer Programming Example---Algorithmic Redistricting

1.0 Introduction

Redistricting requires assigning each county to a district in a fair and equitable plan to ensure population balance (one-person-one-vote), district compactness, and equal representation across races within a state. This assignment aims to use integer programming to map voting districts in the state of Washington, where there are 39 counties and 10 legislative districts.

2.0 Data Sources

The data was obtained from US government and nonprofit sources, which minimizes some concerns about its integrity and applicability to the model. The county adjacency data was pulled from the National Bureau of Economic Research website. It has not been updated since 2010. The team pulled the county demographic data from the United States Census Bureau website. This file was last updated in 2022 from the most recent census in 2020. The county boundaries shapefile data was sourced from the Washington State Department of Natural Resources website. The website states the geographic data is provided "as is" to avoid any liability on behalf of the department. The team moved forward with the use of this data to create a visual representation of the model. Data source citations are included in the References section.

3.0 Objective Function

The team attempted a few integer programming approaches to solve this problem. The objective of the first model was to minimize the distances between counties assigned to the same

district by using latitude and longitude data to calculate UTM distances between pairs of county centroids. Using the distance metric did not result in a feasible optimal solution.

The second approach to this model shifted the objective to minimizing the number of counties assigned to each district. The distance between counties was addressed as a constraint utilizing county adjacency data, and the model found a feasible optimal solution. The maps in Appendix A illustrate the suboptimal results of the adjacency constraint.

4.0 Constraints

The model applies several constraints to this problem. The first constraint ensures the entire county population is allocated to a district. The second constraint requires every county to be allocated to a district. The third constraint defines population limits per county. The fourth constraint ensures each district contains only neighboring counties. In the second model, a fifth constraint was added to produce an even distribution of white voters among districts.

To implement the principle of one-person-one-vote, the model attempts to evenly allocate the Washington population between the 10 districts. To find the optimal county population, we calculated the average number of voters across the assigned number of districts. We then applied 90% and 110% factors to derive the higher and lower bound of the population limit. Washington had a population of 7,705,281 in 2022. Based on the goal of creating 10 districts, we determined the average number of voters in each district to be 770,528.

There are three counties in which the total population exceeds 770,528. King County, Pierce County, and Snohomish County have populations of 2,269,675, 921,130, and 827,957, respectively. Before creating the model, we manually assigned these counties to 5 of the 10 districts, represented by the white shape in the model maps in Appendix A. The remaining 36 counties, containing 3,686,519 voters for an average district population of 737,304, needed to be

allocated to the remaining 5 districts. Applying the 90% and 110% factors, we derived lower and upper population bounds of 663,573 and 881,034, respectively.

5.0 Programming

The Python code and output are included in the Github repository. The code utilizes the Python PuLP library to optimize two integer programming problems, one with four constraints and one with the added white population constraint. In both models, the constraints were defined using a series of for loops. The county allocation and population bounds constraints function properly, but the county adjacency and white population distribution constraints require additional tuning.

6.0 Solution

The redistricting plans for both models are included in Appendix B. As mentioned, the three counties that exceeded the average number of voters per district, King County, Pierce County, and Snohomish County, were not included in the programming. The model county assignments for the first model are detailed below:

- **District 1 (6):** Douglas, Grays Harbor, Okanogan, Pacific, Klickitat, Pend Oreille, Spokane, Whitman
- **District 2 (7):** Grant, Island, Whatcom, Yakima
- **District 3 (8):** Adams, Cowlitz, Franklin, Kitsap, Walla Walla, Mason, Skagit
- **District 4 (9):** Chelan, Clallam, Clark, Columbia, Ferry, Garfield, Kittitas, Lincoln, Stevens
- **District 5 (10):** Asotin, Benton, Lewis, San Juan, Skamania, Thurston, Jefferson Wahkiakum

Based on this redistricting plan, the district with the largest population is District 6 with 807,491 people. The district with the smallest population is District 7 with 669,555 people. The solution provides a balanced population among the districts.

The second model attempted to provide equal representation across races. The total white population in Washington state was 89% according to 2022 data. Based on the second model redistricting plan, the district with the highest white population concentration is District 10 at

91% and the district with the lowest white population concentration is District 9 at 88%. This was a minor improvement in white population percentage among districts from the model one results, which output a range of 87-91% white population among districts.

While the second model achieved a more fair and equitable distribution among population and ethnic backgrounds and evenly distributed the state population among districts, it does not consider voter affiliation and does not effectively group adjacent counties into districts. As a result, this plan is not ready to submit for consideration.

7.0 Maps and Discussion

District maps for both models are included in Appendix A. Comparing these maps to the actual Washington district map visually illustrates the failure of both models' adjacency constraints. Although the second model successfully grouped seven adjacent counties in District 3 (District 8 in the Appendix B tables), the model mostly created groupings of 2-3 counties. In future iterations of the model, the group may explore tightening the adjacency constraint.

The team manually created a district map on Districtr that was able to meet the model's population and county adjacency requirements. However, the Districtr map does not fairly distribute the white population as well as the second model as the percentage of white voters in each district ranges from 49-74.5%. The Districtr map also results in unequal representation of Hispanic and Asian voters and does not factor in voter political party affiliation.

As a result of the shortcomings of the two integer programming models and the Districtr map, we recommend the state of Washington continue using its current district map. If our second model was able to optimize the adjacency constraint and consider the distribution of political party affiliation, Hispanic voters, and Asian voters, we would consider it a fair and equitable representation of the Washington voter population.

References

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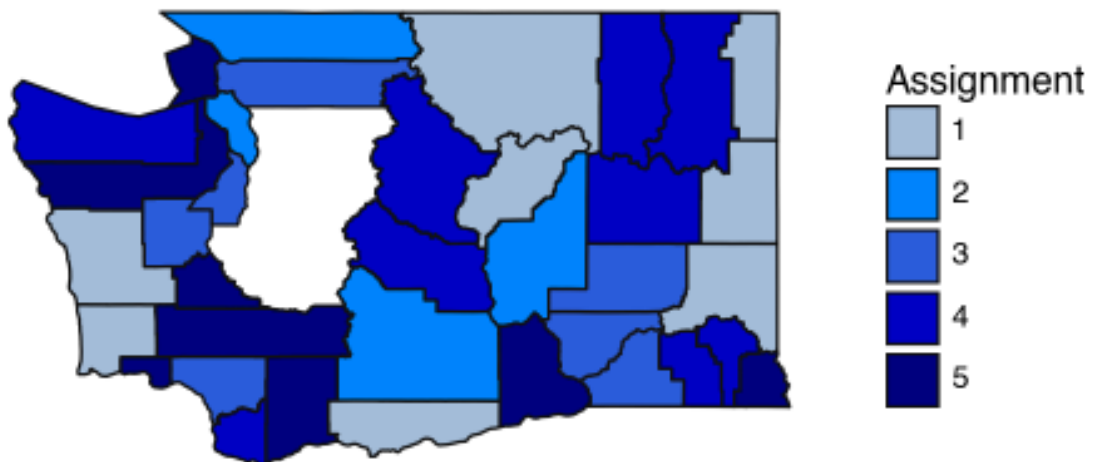   Redistricting Commission. Accessed February 10, 2024.
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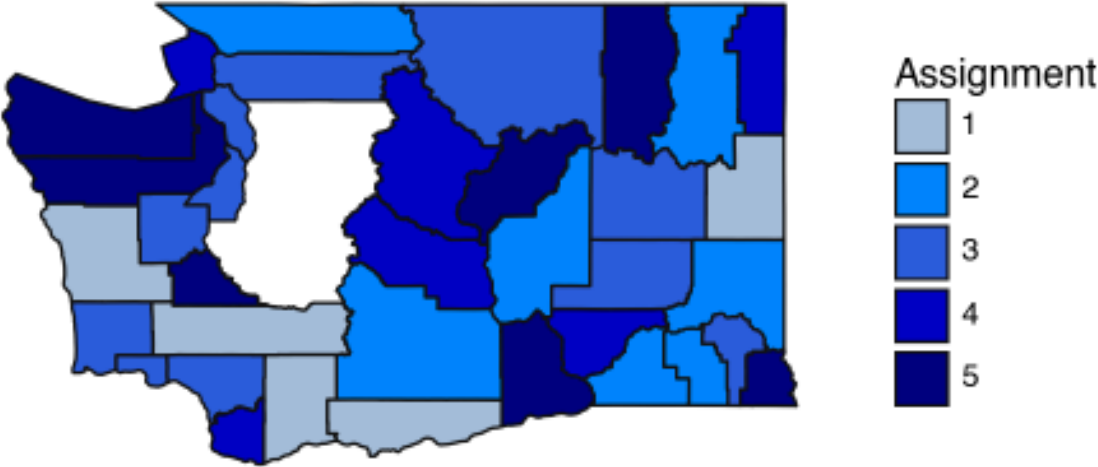
Appendix A
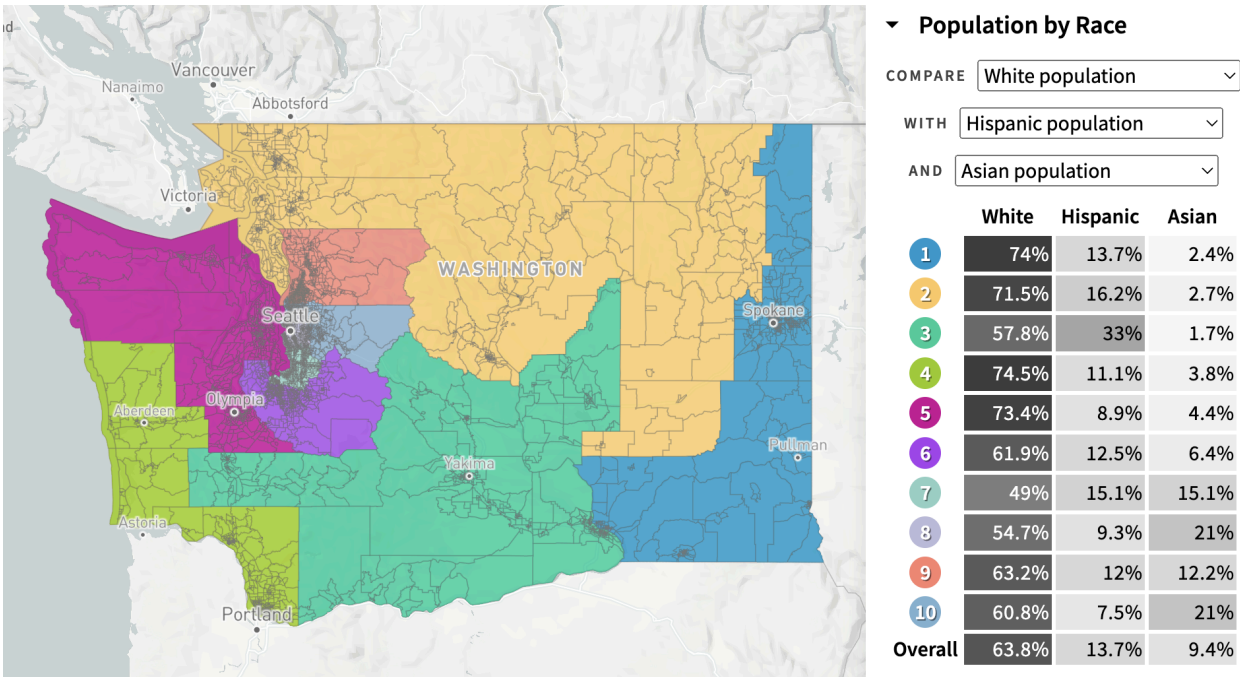
District Maps

Washington District Map - Actual
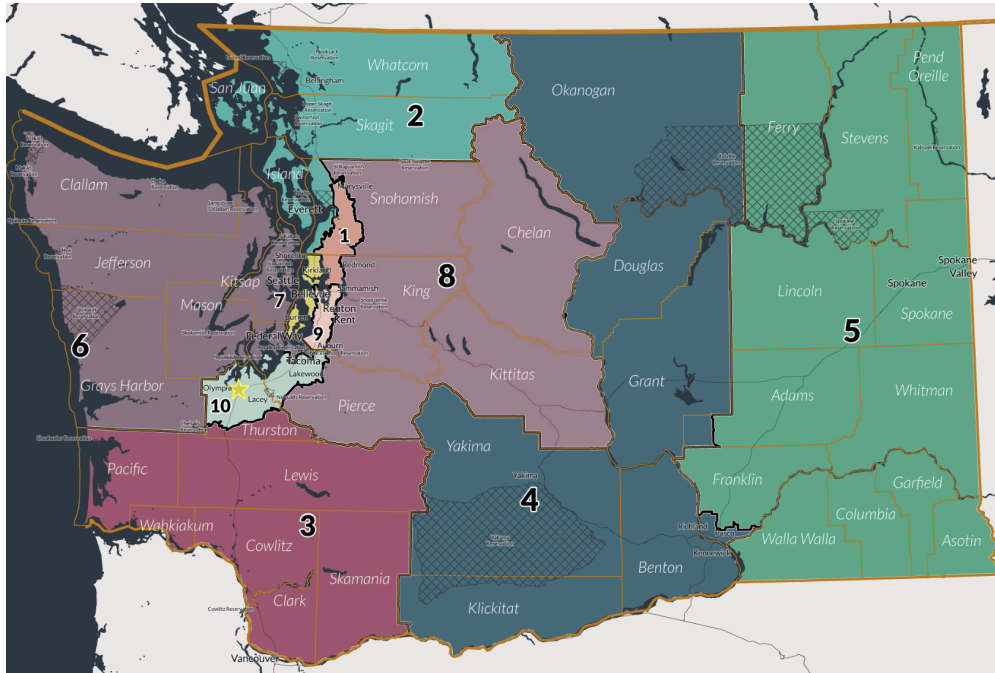
Washington District Map - Model 1

Washington District Map - Model 2 (% White Population Constraint)

Washington District Map - Districtr

	White	Hispanic	Asian
1	74%	13.7%	2.4%
2	71.5%	16.2%	2.7%
3	57.8%	33%	1.7%
4	74.5%	11.1%	3.8%
5	73.4%	8.9%	4.4%
6	61.9%	12.5%	6.4%
7	49%	15.1%	15.1%
8	54.7%	9.3%	21%
9	63.2%	12%	12.2%
10	60.8%	7.5%	21%
Overall	63.8%	13.7%	9.4%

Appendix B

Redistricting Plans

Model 1

District	Assigned Counties	Total Population	White Population	% of White Population
1	King County	756,558	677,974	90%
2	King County	756,558	677,974	90%
3	King County	756,559	677,974	90%
4	Pierce County	921,130	805,004	87%
5	Snohomish County	827,957	738,461	89%
6	Douglas County, Grays Harbor County, Okanogam County, Pacific County, Klickitat County, Pend Oreille County, Spokane County, Whitman County	807,491	732,600	91%
7	Grant County, Island County, Whatcom County, Yakima County	669,555	586,838	88%
8	Chelan County, Clallam County, Clark County, Columbia County, Ferry County, Garfield County, Kittitas County	761,536	672,584	88%
9	Clark County, Franklin County, Pend Oreille County, Chelan County, San Juan County, Kittitas County, Lincoln County, Stevens County	774,614	698,435	90%
10	Asotin County, Benton County, Lewis County, San Juan County, Skamania County, Thurston County, Jefferson County, Wahkiakum County	673,323	600,778	89%

Model 2 (% White Population Constraint)

District	Assigned Counties	Total Population	White Population	% of White Population
1	King County	756,558	677,974	90%
2	King County	756,558	677,974	90%
3	King County	756,559	677,974	90%
4	Pierce County	921,130	805,004	87%
5	Snohomish County	827,957	738,461	89%
6	Ferry County, Douglas County, Thurston County, Jefferson County, Asotin County, Clallam County, Benton County	684,199	608,493	89%
7	Clark County, Franklin County, Pend Oreille County, Chelan County, San Juan County, Kittitas County	754,660	671,735	89%
8	Wahkiakum County, Skagit County, Okanogan County, Mason County, Pacific County, Adams County, Lincoln County, Kitsap County, Island County, Garfield County, Cowlitz County	772,113	690,190	89%
9	Stevens County, Walla Walla County, Whatcom County, Yakima County, Grant County, Columbia County, Whitman County	743,652	654,592	88%
10	Lewis County, Skamania County, Grays Harbor County, Klickitat County, Spokane County	731,895	666,225	91%