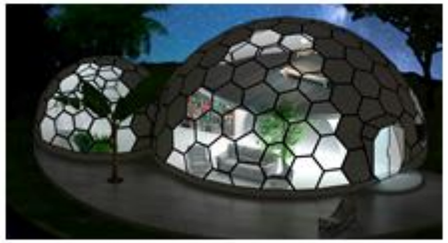
Dome Geometry

Dome-shaped houses have been a popular, if unusual, option for buildings. A dome building has several advantages over a traditional building

- Weight is evenly distributed over the entire structure.
- Larger enclosed volume of space
- Fewer internal support structures.
- Lighter materials can be used.
- More of the surface can be glass.

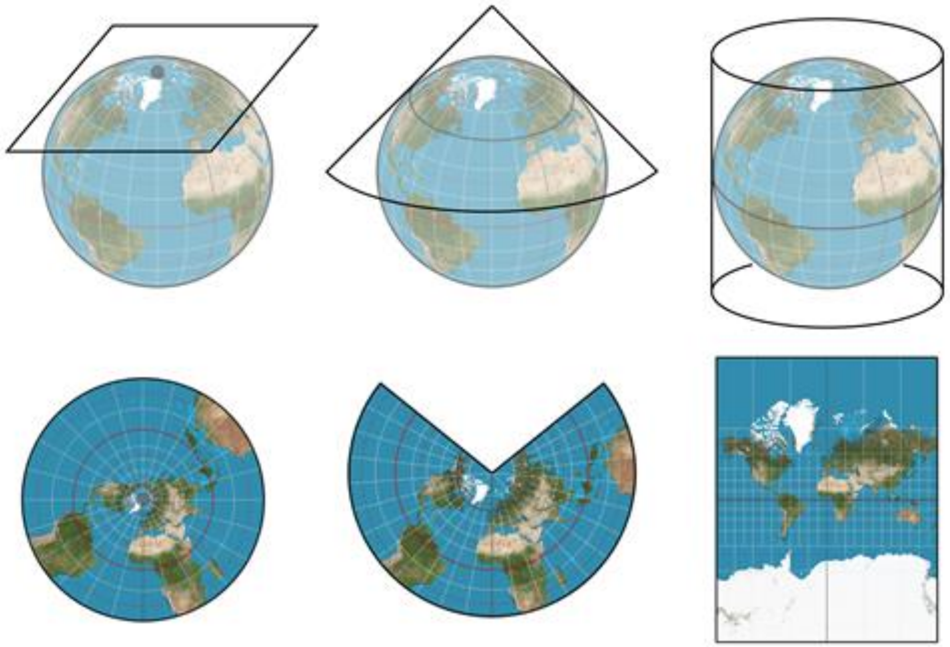
These structures were very popular in the late 70’s and 80’s when hippies (and other earth-loving oddities) were looking to own homes. Being unconventional by choice, they enjoyed the idea of living in a building that would be shunned by their neighbors.

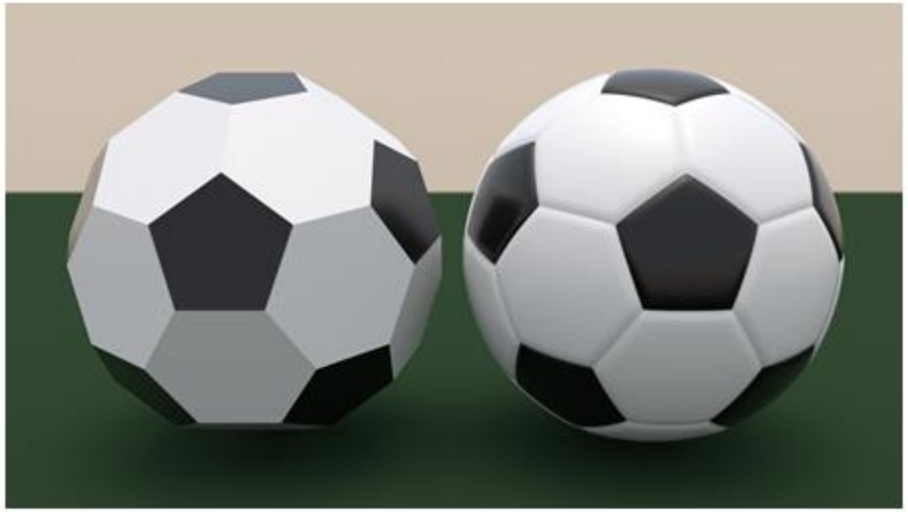
Nevertheless, there is something beautiful and otherworldly about these structures.



The first principle to understand about dome shapes is that no flat surface can truly represent a curve. Our earth is a great example. There are many maps that have been used to show the surface of the earth on flat paper. They all have problems. As the surface of the earth is stretched over the flat space, there are ***distortions***. Distortions give the map an unfair amount of land in one or more key areas. In the polar projection map, Australia seems to take up as much space as Asia. The world's smallest continent is as large as the world's largest continent. This happens because northern spaces are smaller while southern spaces are larger. All maps suffer from distortions.

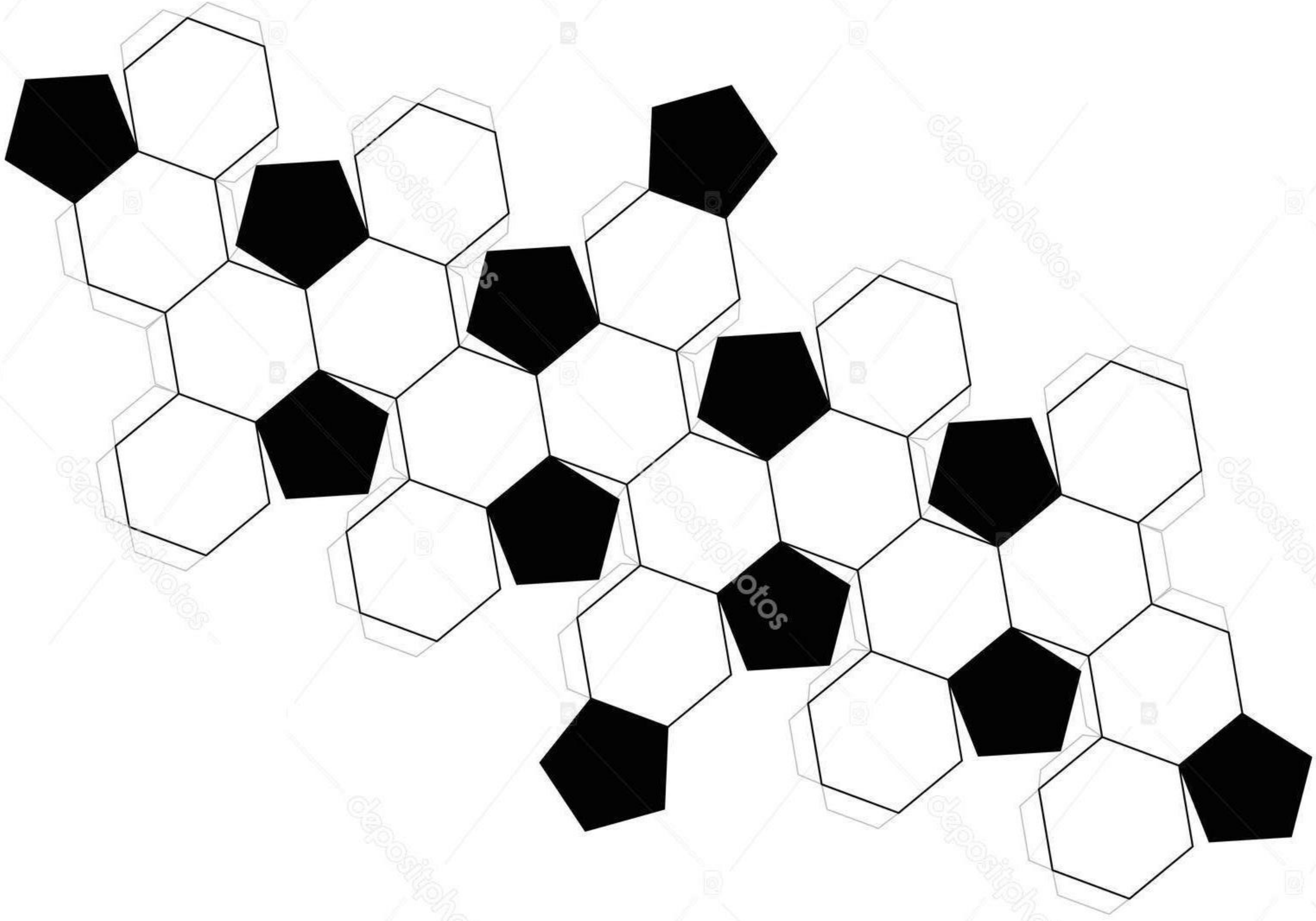
The same thing happens when flat fabric is used to create soccer balls. The hexagon and pentagon shapes do not make a ball shape—they make a truncated icosahedron. Once the pieces are sewn together the ball is inflated and stretched until it is rounded. At that point, the pieces are no longer flat. They have ***deformed*** to make a different shape.





Make your own truncated icosahedron.

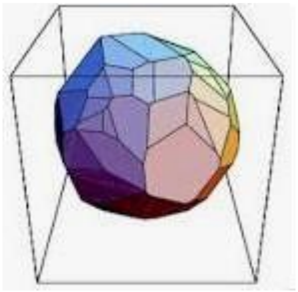
1. Print this image.
2. Cut it out . Be sure to carefully include the lightly draw glue tabs.
3. Fold each edge to make assembly easier.
4. Use glue to fit the polyhedron together.

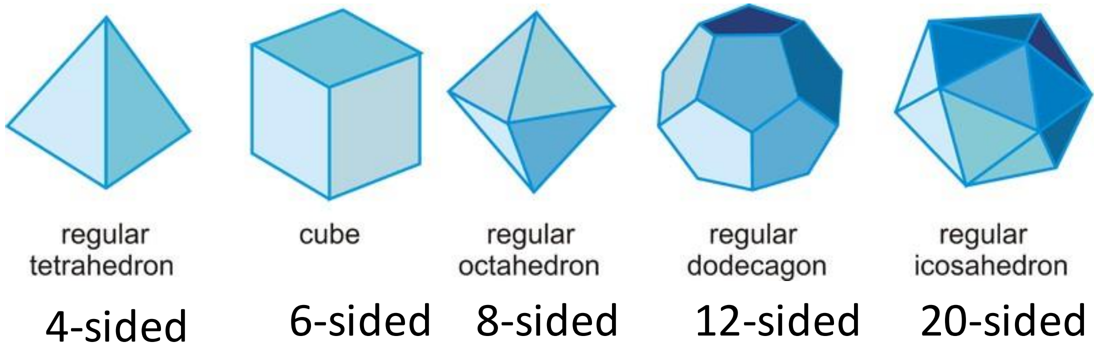


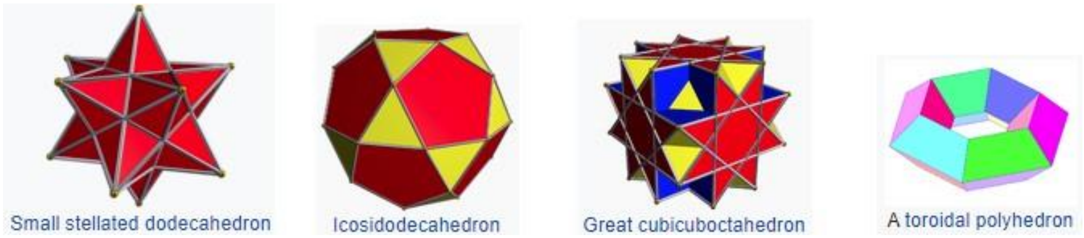
Polyhedrons like these are 3-dimensional shapes made from 2-dimensional shapes. Poly means *many*. Hedron means *face*. There are lots of different polyhedrons with funny names that indicate how many and what kind of sides they have. If you were to take any block of wood and cut flat faces off at any angle you would create and irregular polyhedron.

Regular polyhedrons have the same shape on each side. It is not possible to make a regular polyhedron with 5 sides or 7 sides. Only some polyhedrons can be made.

Polyhedrons don't have to have to be regular. Like the soccer ball, fitting different shapes together can create new shapes. Some polyhedrons can even have faces with **concave** angles. This means they have dips and crevasses.







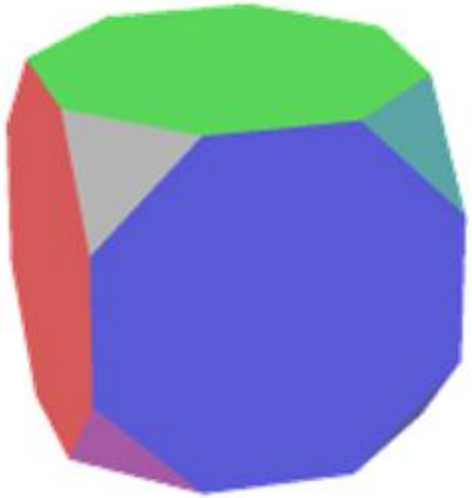
Irregular Polyhedron

This is a craft that needs a sharp knife and an adult. You get to eat it when you are done. ☺

1. Take a medium to large watermelon and set it on a clean surface.
2. Use a very sharp knife to cut the end off the watermelon.
3. Make additional straight cuts to take all the rind off. Be sure these are straight cuts as any curve will ruin the shape.
4. Once you get down to the white you can make smaller straight pieces that will remove those portions.

Talk about the kind of polyhedron you have created before you slice it into edible pieces.

- How many sides does it have?
- What shape are the sides?
- Are the sides regular or irregular?

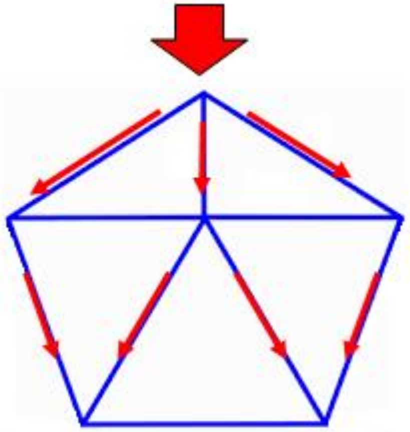


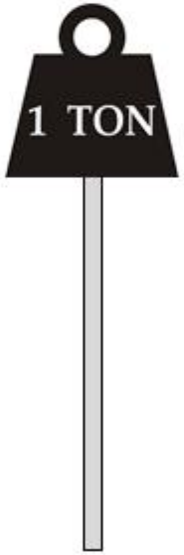
For any dome, we will start with the regular twenty-sided polyhedron. Each side of this shape is an equilateral triangle. This makes it easy to build this shape. Each face and edge of the structure is exactly the same.

Any build made with this shape is going to be strong and able to support the weight of its top by distributing any downward force. The edge materials take that force to the lower vertices and from there down again. This is the same thing that an egg does with its shell. As long as the edge can take the force without breaking the whole structure is safe.

The problem with this shape is that it depends on those long edge materials. Forces that push along the length of the edge can shatter it. Forces that push down from the middle of an edge can bend it. The longer the edges are, the more likely they are to break. To keep the edges short and strong, the polygon must stay small, too.

regular icosahedron

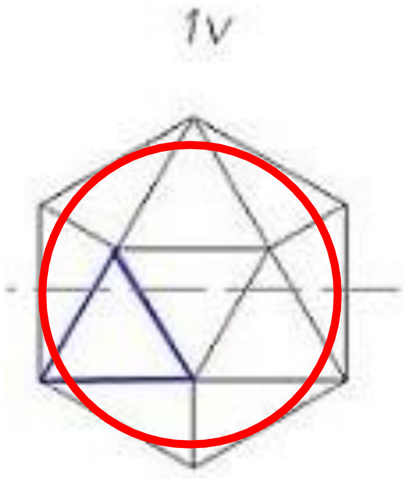


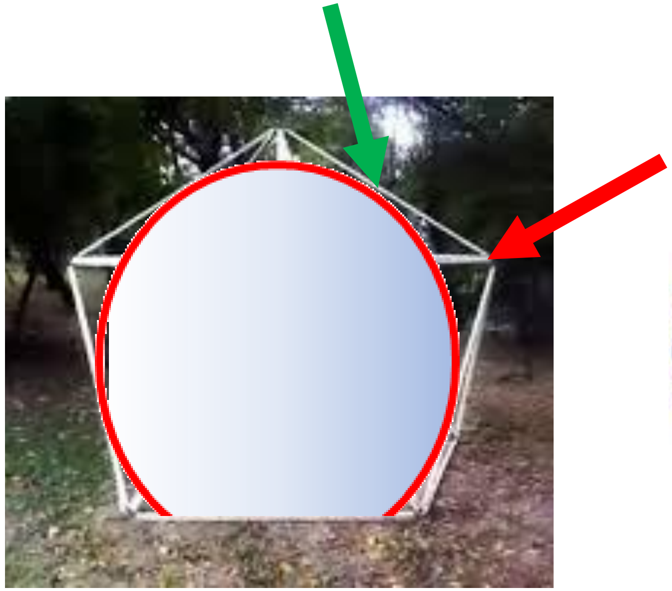


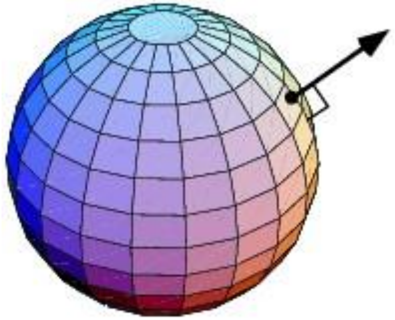
The ideal shape for distributing and strengthening a structure is the sphere or ovoid. Like a circle or oval, the shape is smoothly curved. Eggs are built in this way. Each little speck of the egg is like one face of the shape, but they are extremely small. The distribution of forces is almost perfect.

Any edge can break, but the shorter the edges the stronger it is. The polyhedron can be much better if it had more faces. Twenty faces sounds like a lot, but it really isn’t that many. If the structure were made like an egg with thousands of small triangular faces the distribution of force would be much smoother.

Parts of the polyhedron match the smooth shell we want. The problem are the pointy parts of the structure. If we could just smooth those out it would help. Dome structures work to reduce those points.







Paper Polyhedron

You will use hot glue, newspaper, tape, and bamboo skewers to make two polygons and compare their strength.

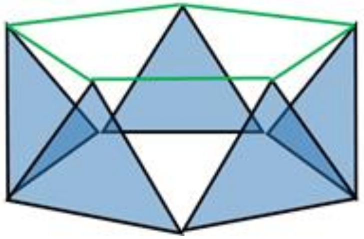
Take large sheets of newspaper and roll them into thin tubes. You need to create

- Six very long tubes (>2 ft) of equal length
- Thirty-six short tubes (<1 ft) of equal length

Some tips:

- Using a skewer may help to get things started.
- Put a couple of pieces of clear tape around each tube to keep the shape. Put the tape on either side of any cuts you make.
- The tubes don’t need to be very thick. About pencil width will work well. Longer tubes may have to be thicker.

Use the hot glue to make a one triangle with three of the long tubes. The remaining tubes will glue to each corner and a fourth point to make a regular tetrahedron. Repeat the procedure with six small tubes.



Start the icosahedron by making 5 triangles from the shorter tubes. Make them into a ring by gluing five more tubes across the tops and gluing the lower corners together. Add five more tubes to the top to form a small pyramid. Do the same on the bottom.

You can play with the shapes and handle them. You can use the same technique to make other polyhedrons. You can also blue construction paper over each triangle to decorate your shapes. Hang them in your room. Ask your child to answer some questions:

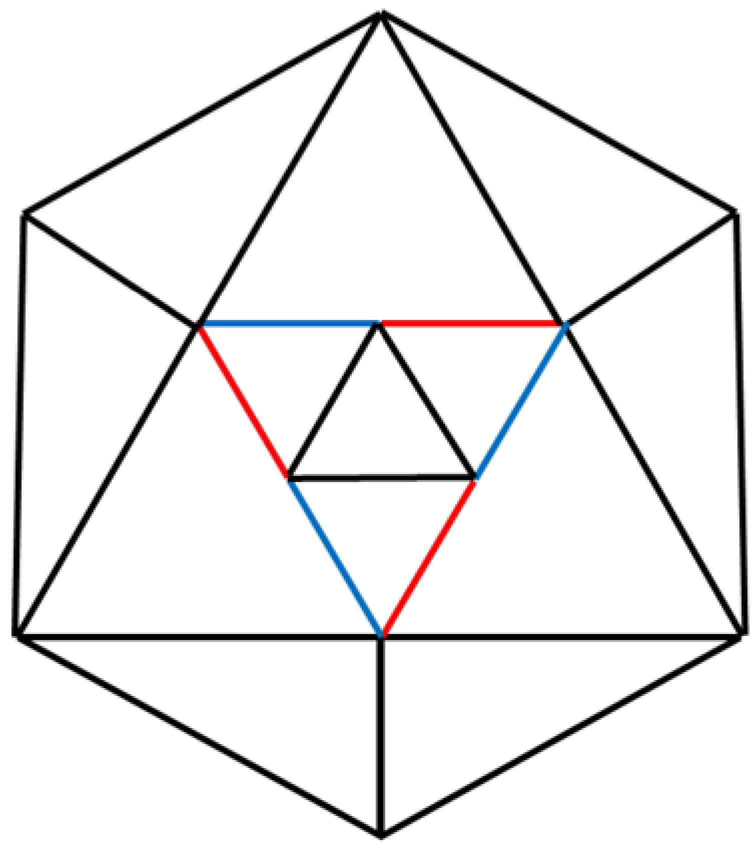
- How strong are the shapes?
- How much pressure would it take to break the shape?
- Which is stronger? Why?

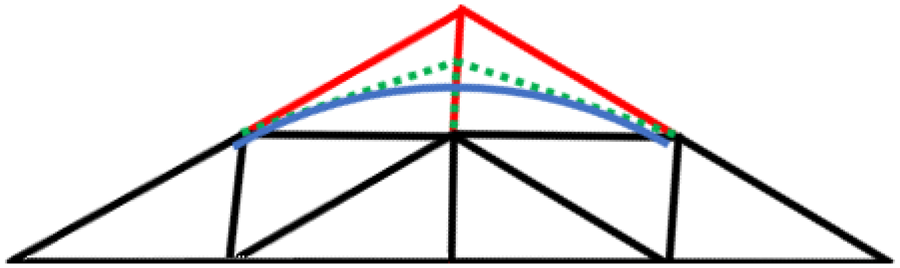
The polyhedron is made from equal edges, but to improve the curve we start by cutting each edge in half. Connecting those edges creates four triangles for each of the existing faces. Instead of 20 faces, we have 80. This will give us more flexibility to create a smoother curve.

Just cutting each face into 4 doesn't change the overall structure. The smaller edges help, but the new shape hasn't gotten any closer to the oval we want.

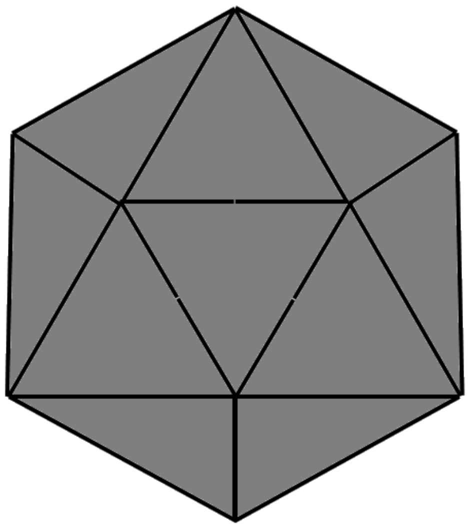
Let's focus on one of those pointy corners. The new triangles are just as pointy, but what if we changed the triangles? If we lengthen the length of those sides, the point will collapse closer to the desired curve. So that is the last change we make.

The new triangles will add more faces to the polyhedron and because they have longer sides, we get new faces that are closer to the perfect curve we want.

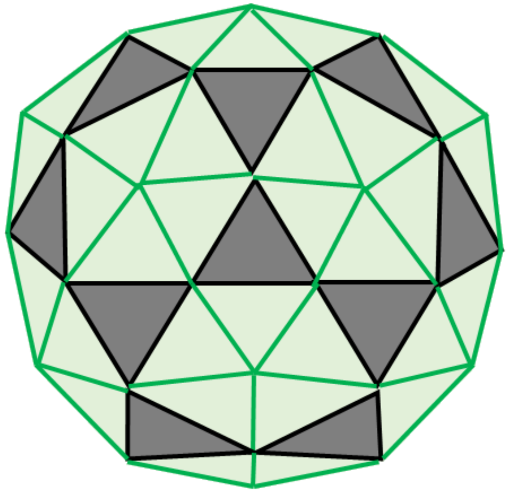




The polyhedron we started with was made of 20 equilateral triangles, but the changes we made have not created equilateral triangles. The new shape still has equilaterals in the center of the old ones. These are half as large as the original.



The new triangles have two sides that are a little bit longer. This makes those pointy ends a little less pointy, but it also means that we have two different kinds of triangles now.



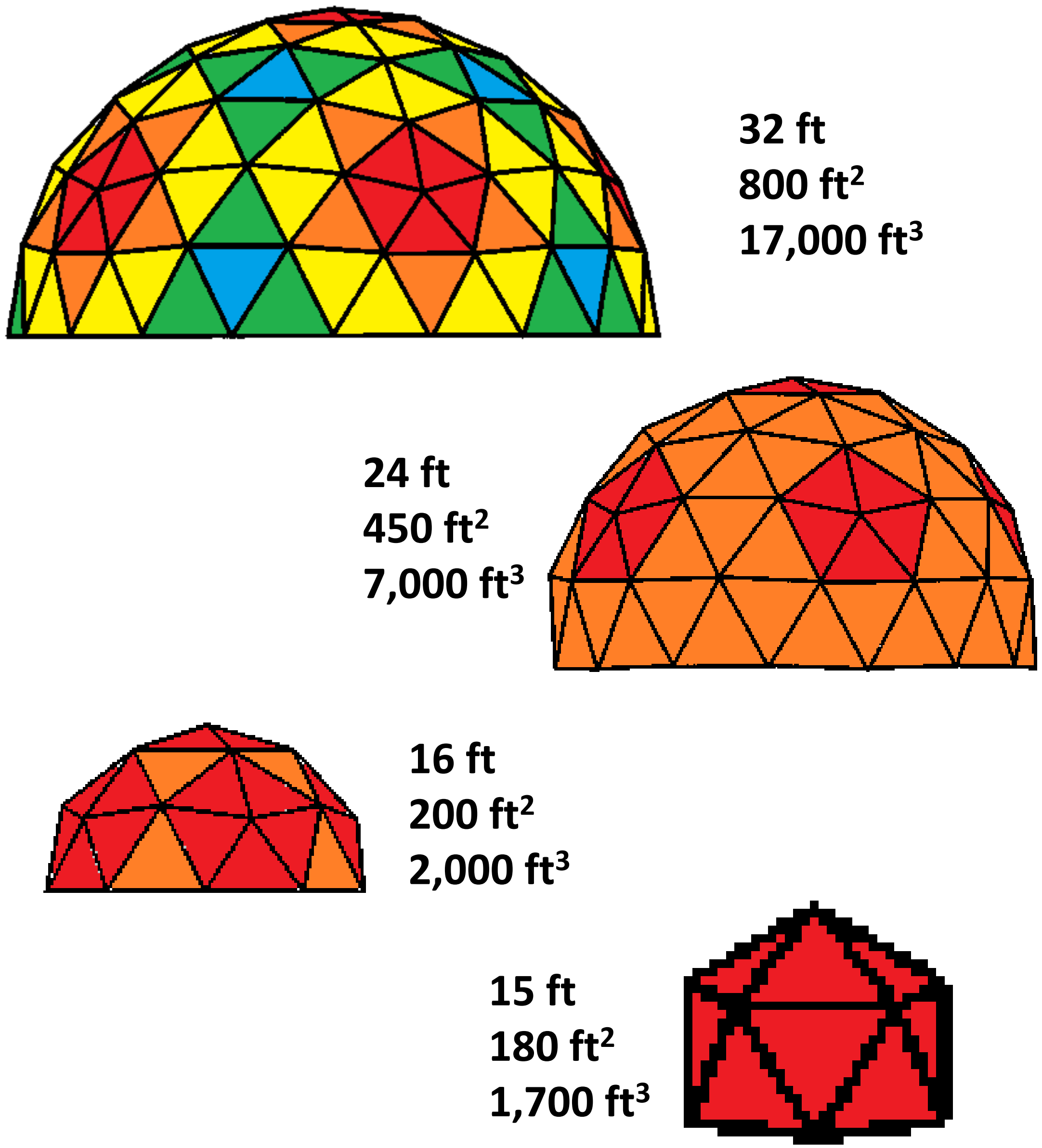
Because this is an icosahedron that has every line cut in two pieces, the new shape is knowns as a 2V. It’s still not perfectly round. We could cut each triangle again and create a better approximation of a sphere.

Each step adds to the number of cuts. The 3V and 4V shapes are closer and closer to the shape of the sphere.

This pictures show the difference between 1V, 2V, 3V, and 4V domes. First let's look at the colors. Each color represents a different sized triangle. The five-sided red triangles are always the smallest. Across the spectrum from red to blue each triangle has one or more sides that are larger.

Now let's consider the size of the building. The measurements on these pictures are based on a 5 ft edge length. The largest edge in each structure is just 5 feet long. As you can see, a 2V doesn't increase the size of the dome much, but the larger number of vertices make 3V and 4V much larger.

People have built whole houses out of a 4V. It's large enough to support two or even three stories inside.



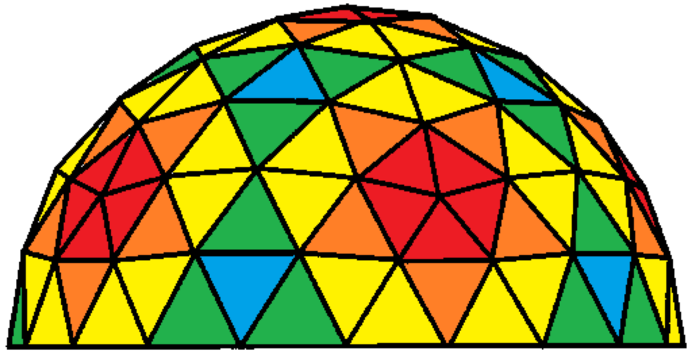
Studying the Dome House

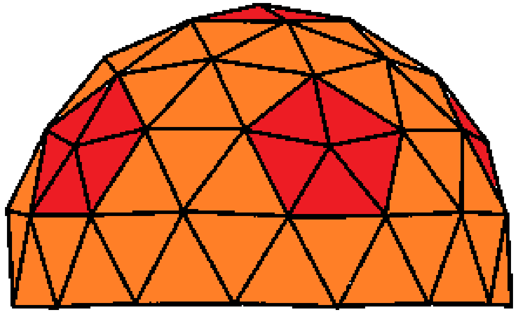
Count out the total number or triangles in each dome.

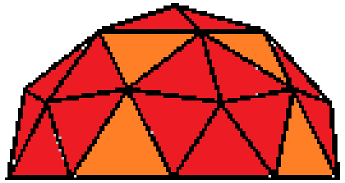
- How many of each kind?
- How many all together?
- How many in a full sphere?

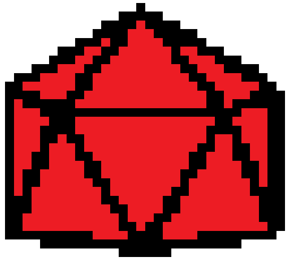
Consider what you might build inside of a dome. Think about these things. Draw a picture and write about what you designed.

- What would you put on the roof? Would it be glass or shingles? Maybe a little of both?
- Is there room for a second floor?
- Where would you put the stairs? The door?









Text

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Student activity

Together activity

Mastery check

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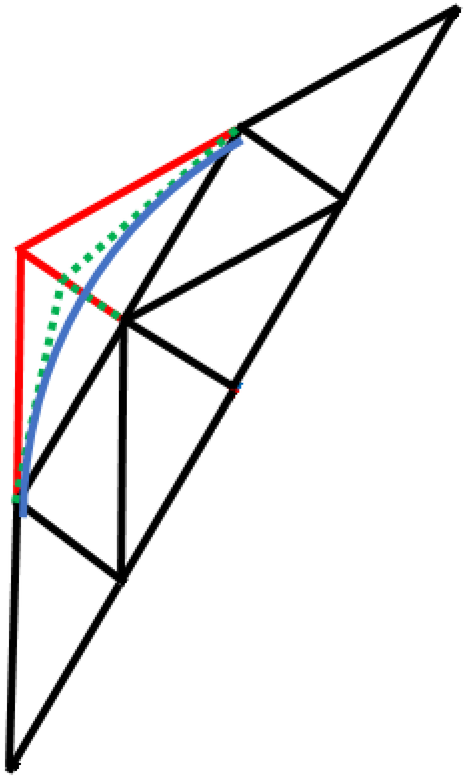

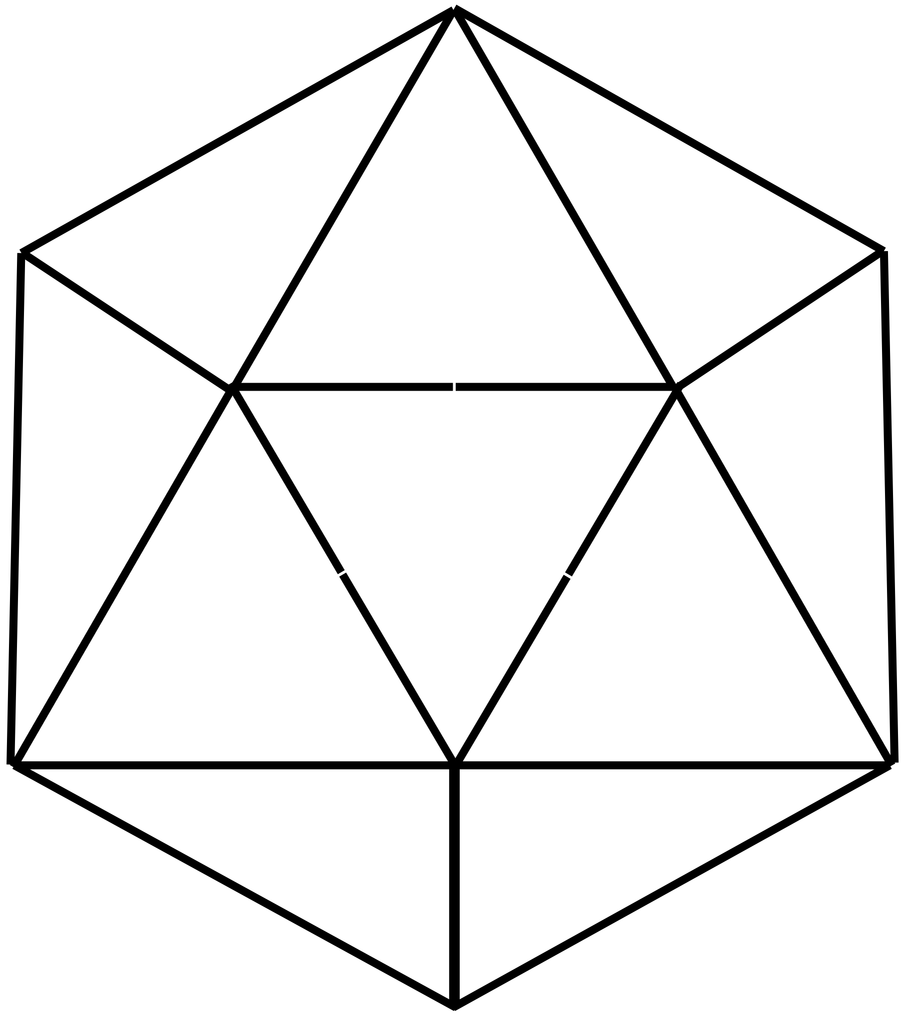

Text