1 Rotate Around Arbitrary Vector in 3D in Right Hand Rule

Given a arbitrary Vector $\vec{v}$ and Vector u, rotate u around $\vec{v}$ in angle θ

1. Compute the projection from u onto $\vec{v}$

$$
\begin{aligned}
\vec{v}^T u &= |\vec{v}||u|\cos\alpha \\
\frac{\vec{v}^T u}{|\vec{v}|} &= |u|\cos\alpha \\
\frac{\vec{v}^T u}{|\vec{v}|}\frac{\vec{v}}{|\vec{v}|} &= |u|\cos\alpha\frac{\vec{v}}{|\vec{v}|} \\
\frac{\vec{v}^T u}{|\vec{v}|}\frac{\vec{v}}{|\vec{v}|} &= |u|\cos\alpha\frac{\vec{v}}{|\vec{v}|} \\
\frac{\vec{v}^T u}{\langle\vec{v},\vec{v}\rangle}\vec{v} &= |u|\cos\alpha\frac{\vec{v}}{|\vec{v}|} \\
\vec{v}_{||} &= \frac{\vec{v}^T u}{\langle\vec{v},\vec{v}\rangle}\vec{v} \\
\vec{v}_{\perp} &= \vec{v} - \vec{v}_{||} \\
\vec{v}_{\perp} &= \vec{v} - \frac{\vec{v}^T u}{\langle\vec{v},\vec{v}\rangle}\vec{v}
\end{aligned}
\tag{1}
$$

2. Compute the cross product of $\vec{v}$ and u e.g. $w = \vec{v} \otimes u$
3. Compute the vector alone the rejection $\vec{v}_{\perp}$ e.g. $\cos\theta\vec{v}_{\perp}$
4. Compute the vector alone the w e.g $\sin\theta|\vec{v}_{\perp}|\frac{w}{|w|}$ 5. Formula can be simplified a bit if $|\vec{v}| = 1$

$$
\begin{aligned}
w &= \vec{v} \otimes u \\
w &= \vec{v} \otimes \vec{v}_{\perp} \\
R(\vec{v}_{\perp}) &= \cos\theta|\vec{v}_{\perp}|\frac{\vec{v}_{\perp}}{|\vec{v}_{\perp}|} + \sin\theta|\vec{v}_{\perp}|\frac{w}{|w|} \\
R(\vec{v}_{\perp}) &= \cos\theta\vec{v}_{\perp} + \sin\theta|\vec{v}_{\perp}|\frac{w}{|w|} \\
R(\theta) &= \vec{v}_{||} + R(\vec{v}_{\perp}) \\
R(\theta) &= \vec{v}_{||} + \cos\theta\vec{v}_{\perp} + \sin\theta|\vec{v}_{\perp}|\frac{w}{|w|}
\end{aligned}
\tag{2}
$$

5. Formula can be simplified if $|\vec{v}| = 1$