1 LU Factorization

Factor the matrix A to lower L and upper U triangle matrices

$$\begin{aligned} Ax &= b \\ LUx &= b \\ \text{let} \quad Ux &= y \\ x &= U^{-1}y \\ Ly &= b \\ y &= L^{-1}b \\ x &= U^{-1}y = U^{-1}L^{-1}b \end{aligned} \tag{1}$$

$$\begin{aligned} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \\ a_{33}x_3 &= b_3 \\ x_3 &= \frac{b_3}{a_{33}} \\ a_{22}x_2 + a_{13}x_3 &= b_2 \\ a_{22}x_2 &= b_2 - a_{13}x_3 \\ a_{22}x_2 &= b_2 - a_{13}\left(\frac{b_3}{a_{33}}\right) \\ x_2 &= \frac{b_2 - a_{13}\left(\frac{b_3}{a_{33}}\right)}{a_{22}} \\ a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{11}x_1 &= b_1 - (a_{11}x_2 + a_{12}x_3) \\ x_1 &= \frac{b_1 - (a_{11}x_2 + a_{12}x_3)}{a_{11}} \\ x_1 &= \frac{b_1 - \left(a_{11}\frac{b_2 - a_{13}\frac{b_3}{a_{33}}}{a_{22}} + a_{12}\frac{b_3}{a_{33}}\right)}{a_{11}} \end{aligned} \tag{2}$$

2 Example

$$\begin{aligned} \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} &= \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned} \tag{3}$$

$$\begin{aligned} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} &= \begin{bmatrix} 6 \\ 9 \\ 7 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{aligned} \tag{4}$$

Proof. coming soon □

3 Backward substitute

```
Haskell code here
```

```
    /**
        backward substitute

        a11 a12 a13 x[0] = b[0]
            a22 a23 x[1] = b[1]
                a33 x[2] = b[2]
    */
    public static void backwardSubstitute(Double[][] a, Double[] x, Double[] b){
        // check null here
        int height = a.length;
        int width = a[0].length;
        for(int h = height - 1; h >= 0; h--){
            int s = 0;
            for(int w = width - 1; w >= h; w--){
                if(w == h){
                    x[h] = (b[h] - s)/a[h][w];
                }else{
                    s += a[h][w]*x[w];
                }
            }
        }
    }
```