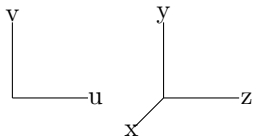
1 First Fundamental Form a surface



Cartesian Coordinate Equation

$$r^2 = x^2 + y^2 + z^2$$

Sphere parametric equation

$$\begin{aligned}
x &= r\sin\alpha\cos\theta \\
y &= r\sin\alpha \\
z &= r\cos\alpha\sin\theta \\
f(\alpha,\theta) &= \begin{cases} x(\alpha,\theta) = r\sin\alpha\cos\theta \\ y(\alpha,\theta) = r\sin\alpha \\ z(\alpha,\theta) = r\cos\alpha\sin\theta \end{cases}
\end{aligned}$$

$$J = \begin{bmatrix} \frac{dx}{d\alpha} & \frac{dx}{d\theta} \\ \frac{dy}{d\alpha} & \frac{dy}{d\theta} \\ \frac{dz}{d\alpha} & \frac{dz}{d\theta} \end{bmatrix} = \begin{bmatrix} r\cos\theta\cos\alpha & -r\cos\alpha\sin\theta \\ r\cos\alpha & 0 \\ -r\sin\alpha\sin\theta & r\cos\alpha\cos\theta \end{bmatrix}$$

$$\begin{aligned}
J^T &= \begin{bmatrix} \frac{dx}{d\alpha} & \frac{dy}{d\alpha} & \frac{dz}{d\alpha} \\ \frac{dx}{d\theta} & \frac{dy}{d\theta} & \frac{dz}{d\theta} \end{bmatrix} = \begin{bmatrix} -r\cos\alpha\sin\theta & 0 & r\cos\alpha\cos\theta \\ r\cos\theta\cos\alpha & r\cos\alpha & -r\sin\alpha\sin\alpha \end{bmatrix} \\
J^T J &= \begin{bmatrix} \frac{dx}{d\alpha} & \frac{dy}{d\alpha} & \frac{dz}{d\alpha} \\ \frac{dx}{d\theta} & \frac{dy}{d\theta} & \frac{dz}{d\theta} \end{bmatrix} \begin{bmatrix} \frac{dx}{d\alpha} & \frac{dx}{d\theta} \\ \frac{dy}{d\alpha} & \frac{dy}{d\theta} \\ \frac{dz}{d\alpha} & \frac{dz}{d\theta} \end{bmatrix} = \begin{bmatrix} x_\alpha & y_\alpha & z_\alpha \\ x_\theta & y_\theta & z_\theta \end{bmatrix} \begin{bmatrix} x_\alpha & x_\theta \\ y_\alpha & y_\theta \\ z_\alpha & z_\theta \end{bmatrix} \\
J^T J &= \begin{bmatrix} x_\alpha x_\alpha + y_\alpha y_\alpha + z_\alpha z_\alpha & x_\alpha x_\theta + y_\alpha y_\theta + z_\alpha z_\theta \\ x_\theta x_\alpha + y_\theta y_\alpha + z_\theta z_\alpha & x_\theta x_\theta + y_\theta y_\theta + z_\theta z_\theta \end{bmatrix} \\
J^T J &= \begin{bmatrix} -r\cos\alpha\sin\theta & 0 & r\cos\alpha\cos\theta \\ r\cos\theta\cos\alpha & r\cos\alpha & -r\sin\alpha\sin\alpha \end{bmatrix} \begin{bmatrix} r\cos\theta\cos\alpha & -r\cos\alpha\sin\theta \\ r\cos\alpha & 0 \\ -r\sin\alpha\sin\alpha & r\cos\alpha\cos\theta \end{bmatrix} \\
J^T J &= \begin{bmatrix} -r\cos\alpha\sin\theta r\cos\theta\cos\alpha + 0 + -r^2\cos\alpha\cos\theta\sin\alpha\sin\theta & a \\ b & c \end{bmatrix}
\end{aligned} \tag{1}$$