LU Factorization Refference Note
Factor the matrix A to lower L and upper U triangle matrices

$$\begin{aligned} Ax &= b \\ LUx &= b \\ \text{let} \quad Ux &= y \\ x &= U^{-1}y \\ Ly &= b \\ y &= L^{-1}b \\ x &= U^{-1}y = U^{-1}L^{-1}b \end{aligned} \tag{1}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{aligned}
a_{33}x_3 &= b_3 \\
x_3 &= \frac{b_3}{a_{33}} \\
a_{22}x_2 + a_{13}x_3 &= b_2 \\
a_{22}x_2 &= b_2 - a_{13}x_3 \\
a_{22}x_2 &= b_2 - a_{13}\left(\frac{b_3}{a_{33}}\right) \\
x_2 &= \frac{b_2 - a_{13}\left(\frac{b_3}{a_{33}}\right)}{a_{22}} \\
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{11}x_1 &= b_1 - (a_{11}x_2 + a_{12}x_3) \\
x_1 &= \frac{b_1 - (a_{11}x_2 + a_{12}x_3)}{a_{11}} \\
x_1 &= \frac{b_1 - \left(a_{11}\frac{b_2 - a_{13}\frac{b_3}{a_{33}}}{a_{22}} + a_{12}\frac{b_3}{a_{33}}\right)}{a_{11}}
\end{aligned}$$

1 Backward substitute

```
void backsub(int[][] a, int[] x, int[] b){
    for(int i=n-1; i >= 0; i--){
        Double s = 0.0
        for(int j=n-1; j >= i; j--){
            if(j == i){
                x[i] = (b[i] - s)/a[i][j]
            } if(j > i){
                s += a[i][j]*x[j];
            }
        }
    }
}
```

(2)