

# 1 Rotate Around Arbitrary Vector in 3D in right hand rule

Given a arbitrary Vector  $\vec{v}$  and Vector  $u$ , rotate  $u$  around  $\vec{v}$  in angle  $\theta$

1. Compute the projection from  $u$  onto  $\vec{v}$

$$\begin{aligned}
 \vec{v}^T u &= |\vec{v}| |u| \cos \alpha \\
 \frac{\vec{v}^T u}{|\vec{v}|} &= |u| \cos \alpha \\
 \frac{\vec{v}^T u}{|\vec{v}|} \frac{\vec{v}}{|\vec{v}|} &= |u| \cos \alpha \frac{\vec{v}}{|\vec{v}|} \\
 \frac{\vec{v}^T u}{|\vec{v}|} \frac{\vec{v}}{|\vec{v}|} &= |u| \cos \alpha \frac{\vec{v}}{|\vec{v}|} \\
 \frac{\vec{v}^T u}{\langle \vec{v}, \vec{v} \rangle} \vec{v} &= |u| \cos \alpha \frac{\vec{v}}{|\vec{v}|} \\
 \vec{v}_{||} &= \frac{\vec{v}^T u}{\langle \vec{v}, \vec{v} \rangle} \vec{v} \\
 \vec{v}_{\perp} &= \vec{v} - \vec{v}_{||} \\
 \vec{v}_{\perp} &= \vec{v} - \frac{\vec{v}^T u}{\langle \vec{v}, \vec{v} \rangle} \vec{v}
 \end{aligned} \tag{1}$$

2. Compute the cross product of  $\vec{v}$  and  $u$  e.g.  $w = \vec{v} \otimes u$
3. Compute the vector along the rejection  $\vec{v}_{\perp}$  e.g.  $\cos \theta \vec{v}_{\perp}$
4. Compute the vector along the  $w$  e.g.  $\sin \theta |\vec{v}_{\perp}| \frac{w}{|w|}$

$$\begin{aligned}
 w &= \vec{v} \otimes u \\
 R(\vec{v}_{\perp}) &= \cos \theta |\vec{v}_{\perp}| \frac{\vec{v}_{\perp}}{|\vec{v}_{\perp}|} + \sin \theta |\vec{v}_{\perp}| \frac{w}{|w|} \\
 R(\vec{v}_{\perp}) &= \cos \theta \vec{v}_{\perp} + \sin \theta |\vec{v}_{\perp}| \frac{w}{|w|} \\
 R(\theta) &= \vec{v}_{||} + R(\vec{v}_{\perp}) \\
 R(\theta) &= \vec{v}_{||} + \cos \theta \vec{v}_{\perp} + \sin \theta |\vec{v}_{\perp}| \frac{w}{|w|}
 \end{aligned} \tag{2}$$