1 Colinear points

Given three points, check whether the three points are colinear or not $p_0(x_0, y_0), p_1(x_1, y_1), p_2(x_2, y_2)$
1. find the equation of two points
2. substitude the third point to the equation and check whether it is zero or not
3. if $f(x, y) = 0$, it is colinear
4. if $f(x, y) > 0$, it is on one side
5. if $f(x, y) < 0$, it is on other side

$$
\begin{aligned}
\frac{y - y_0}{x - x_0} &= \frac{y_1 - y_0}{x_1 - x_0} \\
(y - y_0)(x_1 - x_0) &= (x - x_0)(y_1 - y_0) \\
f(x, y) &= (y - y_0)(x_1 - x_0) - (x - x_0)(y_1 - y_0)
\end{aligned}
\tag{1}
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There is one big problem with the solution. The line can not be vertical line since $x_1 - x_0$ can NOT be zero

Given two pair of points, check whether they are intersected or not, $p_0(x_0, y_0), p_1(x_1, y_2)$ and $p_2(x_2, y_2), p_3(x_3, y_3)$

Line Equation for p_0 and p_1

$$\begin{aligned}
\frac{y - y_0}{x - x_0} &= \frac{y_1 - y_0}{x_1 - x_0} \\
(y - y_0)(x_1 - x_0) &= (x - x_0)(y_1 - y_0) \\
(y - y_0) &= \frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} \\
y &= \frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} + y_0
\end{aligned}$$

Line Equation for p_2 and p_3

$$\begin{aligned}
\frac{y - y_2}{x - x_2} &= \frac{y_3 - y_2}{x_3 - x_2} \\
(y - y_2)(x_3 - x_2) &= (x - x_2)(y_3 - y_2) \\
(y - y_2) &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} \\
y &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + y_2
\end{aligned}$$

Substitude y

$$\begin{aligned}
\frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} + y_0 &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + y_2 \\
\frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + (y_2 - y_0) \\
(x - x_0)(y_1 - y_0) &= (x_1 - x_0)\frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + (x_1 - x_0)(y_2 - y_0) \\
(x - x_0)(y_1 - y_0) &= (x - x_2)\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)} + (x_1 - x_0)(y_2 - y_0) \\
(x - x_0) &= (x - x_2)\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} \\
(x - x_0) &= x\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} \\
x - x\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} + x_0 \\
x\left(1 - \frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)}\right) &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} + x_0 \\
x\frac{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} + x_0 \\
x\frac{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_3 - x_2)(x_1 - x_0)(y_2 - y_0)}{(x_3 - x_2)(y_1 - y_0)} + x_0 \\
x\frac{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_3 - x_2)(x_1 - x_0)(y_2 - y_0)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_3 - x_2)(y_1 - y_0)x_0}{(x_3 - x_2)(y_1 - y_0)} \\
x\left[(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)\right] &= x_2(x_1 - x_0)(y_3 - y_2) + (x_3 - x_2)(x_1 - x_0)(y_2 - y_0) + (x_3 - x_2)(y_1 - y_0)x_0
\end{aligned}$$

Solve for x and let $x = x'$

$$x' = \frac{x_2(x_1 - x_0)(y_3 - y_2) + (x_3 - x_2)(x_1 - x_0)(y_2 - y_0) + (x_3 - x_2)(y_1 - y_0)x_0}{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}$$

Solve for y and let $y = y'$

$$y' = \frac{(x' - x_0)(y_1 - y_0)}{(x_1 - x_0)} + y_0$$