

# 1 Intersection of two lines

Given two lines, find the intersection of two lines if they are not parallel

$$\begin{aligned}f(t) &= p_0 + s(p_1 - p_0) \\f(s) &= q_0 + t(q_1 - q_0) \\p_0 + s(p_1 - p_0) &= q_0 + t(q_1 - q_0) \\p_0 - q_0 &= -s(p_1 - p_0) + t(q_1 - q_0) \\p_0 - q_0 &= (x_0, y_0) \\p_1 - p_0 &= (px, py) \quad s \\q_1 - q_0 &= (qx, qy) \quad t \\x_0 &= s(px) - t(qx) \\y_0 &= s(py) - t(qy) \\\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} &= \begin{bmatrix} px & -qx \\ py & -qy \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \\p_0 &= (2, 0) \\p_1 &= (2, 1) \\q_0 &= (0, 0) \\q_1 &= (1, 1) \\f(s) &= (2, 0) + s(2, 1) \\f(t) &= (0, 0) + t(1, 1) \\p_0 - q_0 &= (2, 0) - (0, 0) = (2, 0) \\p_1 - p_0 &= (2, 1) - (2, 0) = (0, 1) \\q_1 - q_0 &= (1, 1) - (0, 0) = (1, 1) \\(2, 0) &= -s(0, 1) + t(1, 1) \\-2 &= s(-0) + t1 \\0 &= s(-1) + t1 \\\begin{bmatrix} -2 \\ 0 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} \\s &= 2 \\t &= 2\end{aligned}$$