

Theorem Definition Collorary Example Remark Note
 Given u, v , u projects onto v , $w = \sigma v$ where σ is the scala we have $v \perp (w - u)$

$$\begin{aligned} v &\perp (\sigma v - u) \\ v^T(\sigma v - u) &= 0 \\ \sigma v^T v - v^T u &= 0 \\ \sigma v^T v &= v^T u \\ \sigma &= \frac{v^T u}{v^T v} \end{aligned}$$

$$\begin{aligned} \sigma v &= \frac{v v^T u}{v^T v} \\ \sigma v &= \frac{v v^T}{v^T v} u \end{aligned}$$

vv^T is the projection matrix which projects u onto v where $|v| = 1$

Concret Example: Let's project $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ onto $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$vv^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Proj_v = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$