1　Introduction

Affine space is always mysterious for me and it is very confusing and not well defined in High school and University mathematical lecture. It makes your more miserable after you have learned vector Space. However, the confusing makes you wonder what is the difference between Affine Space and vector Space and what is the relationship between them. French mathematician said "An affine space is nothing more than a vector whose origin we try to forget about, by adding translation to linear map"

In vector space, we have zero vector as origin, e.g. in $\mathbb{R}^2$

$$v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

One of the most important property of vector Space is Linear Combination, any vector can be represent by linear combination of their basis, e.g.

$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix} v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} v = 2v_1 + 3v_2 \text{ where } \beta = \{v_1, v_2\} \text{ is called basis}$$

In vector Space, any vector can be represent by linear combination of basis vectors uniquely.

How can we use this concept and come up with similar concept, e.g. [linear combination] for points

Let's review the concept of coordinates system.

A point p is something like $p = (2,3)$ and $(2,3)$ is called the coordinates of point p, how can we define the coordinates of point? There is something called basis vector for the coordinates system. For example, $\mathcal{B} = \{e_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, e_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\}$ is stardard basis in $\mathbb{R}^2$. The coordinates of point is the distance from *Origin* to the intersection which point p is perpendicular to one of the basis vector: e.g. e_1 so coordinates system includes two things: *Origin* and $\mathcal{B} = \{e_1, e_2\}$