How to reduce Row Echelon Form, the determinant is changed here Three Axioms for determinant function:
1. Exchange two different rows $\det A = (-1)\det A$
2. Multiply one row with non-zero scalar $\det A = n \det A$
3. Multiply one row with non-zero scalar and add it to other row $\det A = \det A$
Determinant function is linear function in any one column given that the rest of columns are fixed

$$\det \begin{vmatrix} a+a' & b \\ c+c' & d \end{vmatrix} = \det \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \det \begin{vmatrix} a' & b \\ c' & d \end{vmatrix}$$

duplicated $n-1$ copy of first row

```
map(\x -> head A) A
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

```
init A
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

remove first row

```
tail A
```

$$\begin{bmatrix} 4(1 & 2 & 3) \\ 7(1 & 2 & 3) \end{bmatrix} - \begin{bmatrix} 1(4 & 5 & 6) \\ 1(7 & 8 & 10) \end{bmatrix} \Rightarrow D = \begin{bmatrix} 0 & 3 & 6 \\ 0 & 9 & 11 \end{bmatrix}$$

Zero the first column

```
zipWith(\rx ry ->
                let
                    xx = head rx
                    yy = head ry
                in
                    if xx == 0 || yy == 0
                    then ry
                    else zipWith(\x y -> yy*x - xx*y) rx ry
            ) m1 m2
```

```
map(\x -> tail x) D
```

$$\begin{bmatrix} 3 & 6 \\ 9 & 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 6 \end{bmatrix}$$
$$\begin{bmatrix} 3 & 6 \\ 9 & 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 11 \end{bmatrix}$$
$$\begin{bmatrix} 9(3 & 6) \end{bmatrix} - \begin{bmatrix} 3(9 & 11) \end{bmatrix} = \begin{bmatrix} 0 & 21 \end{bmatrix}$$

Final matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 21 \end{bmatrix}$$