Theorem Collorary Example Remark Note

# 1 Intersection of two lines

Given two pair of points, check whether they are intersected or not, $p_0(x_0, y_0), p_1(x_1, y_2)$ and $p_2(x_2, y_2), p_3(x_3, y_3)$

$$
\begin{aligned}
&\text{Line Equation for } p_0 \text{ and } p_1 \\
\frac{y - y_0}{x - x_0} &= \frac{y_1 - y_0}{x_1 - x_0} \\
(y - y_0)(x_1 - x_0) &= (x - x_0)(y_1 - y_0) \\
(y - y_0) &= \frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} \\
{\color{red} y} &= \frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} + y_0
\end{aligned}
$$

$$
\begin{aligned}
&\text{Line Equation for } p_2 \text{ and } p_3 \\
\frac{y - y_2}{x - x_2} &= \frac{y_3 - y_2}{x_3 - x_2} \\
(y - y_2)(x_3 - x_2) &= (x - x_2)(y_3 - y_2) \\
(y - y_2) &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} \\
y &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + y_2
\end{aligned}
$$

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\begin{aligned}
\text{Substitude } y& \\
\frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} + y_0 &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + y_2 \\
\frac{(x - x_0)(y_1 - y_0)}{(x_1 - x_0)} &= \frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + (y_2 - y_0) \\
(x - x_0)(y_1 - y_0) &= (x_1 - x_0)\frac{(x - x_2)(y_3 - y_2)}{(x_3 - x_2)} + (x_1 - x_0)(y_2 - y_0) \\
(x - x_0)(y_1 - y_0) &= (x - x_2)\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)} + (x_1 - x_0)(y_2 - y_0) \\
(x - x_0) &= (x - x_2)\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} \\
(x - x_0) &= x\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} \\
x - x\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} + x_0 \\
x\left(1 - \frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)}\right) &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} + x_0 \\
x\frac{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_1 - x_0)(y_2 - y_0)}{(y_1 - y_0)} + x_0 \\
x\frac{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_3 - x_2)(x_1 - x_0)(y_2 - y_0)}{(x_3 - x_2)(y_1 - y_0)} + x_0 \\
x\frac{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} &= x_2\frac{(x_1 - x_0)(y_3 - y_2)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_3 - x_2)(x_1 - x_0)(y_2 - y_0)}{(x_3 - x_2)(y_1 - y_0)} + \frac{(x_3 - x_2)(y_1 - y_0)x_0}{(x_3 - x_2)(y_1 - y_0)} \\
x\left[(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)\right] &= x_2(x_1 - x_0)(y_3 - y_2) + (x_3 - x_2)(x_1 - x_0)(y_2 - y_0) + (x_3 - x_2)(y_1 - y_0)x_0 \\
\text{Solve for } x \text{ and let } x = x'& \\
x' &= \frac{x_2(x_1 - x_0)(y_3 - y_2) + (x_3 - x_2)(x_1 - x_0)(y_2 - y_0) + (x_3 - x_2)(y_1 - y_0)x_0}{(x_3 - x_2)(y_1 - y_0) - (x_1 - x_0)(y_3 - y_2)} \\
\text{Solve for } y \text{ and let } y = y'&
\end{aligned}
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