Vectors and Matrices

Appendix A

Vectors and matrices are notational conveniences for dealing with systems of linear equations and inequalities. In particular, they are useful for compactly representing and discussing the linear programming problem:

$$\text{Maximize} \sum_{j=1}^{n} c_j x_j,$$

subject to:

$$\begin{aligned} \sum_{j=1}^{n} a_{ij} x_j &= b_i \qquad (i = 1, 2, \ldots, m), \\ x_j &\geq 0 \qquad (j = 1, 2, \ldots, n). \end{aligned}$$

This appendix reviews several properties of vectors and matrices that are especially relevant to this problem. We should note, however, that the material contained here is more technical than is required for understanding the rest of this book. It is included for completeness rather than for background.

A.1 VECTORS

We begin by defining vectors, relations among vectors, and elementary vector operations.

Definition. A *k-dimensional vector y* is an ordered collection of k real numbers $y_1, y_2, \ldots, y_k$, and is written as $y = (y_1, y_2, \ldots, y_k)$. The numbers y_j $(j = 1, 2, \ldots, k)$ are called the *components* of the vector y.

Each of the following are examples of vectors:

i) $(1, -3, 0, 5)$ is a four-dimensional vector. Its first component is 1, its second component is -3, and its third and fourth components are 0 and 5, respectively.
ii) The coefficients $c_1, c_2, \ldots, c_n$ of the linear-programming objective function determine the n-dimensional vector $c = (c_1, c_2, \ldots, c_n)$.
iii) The activity levels $x_1, x_2, \ldots, x_n$ of a linear program define the n-dimensional vector $x = (x_1, x_2, \ldots, x_n)$.
iv) The coefficients $a_{i1}, a_{i2}, \ldots, a_{in}$ of the decision variables in the ith equation of a linear program determine an n-dimensional vector $A^i = (a_{i1}, a_{i2}, \ldots, a_{in})$.
v) The coefficients $a_{1j}, a_{2j}, \ldots, a_{nj}$ of the decision variable x_j in constraints 1 through m of a linear program define an m-dimensional vector which we denote as $A_j = (a_{1j}, a_{2j}, \ldots, a_{mj})$.