M392C NOTES: REPRESENTATION THEORY

ARUN DEBRAY MAY 14, 2017

These notes were taken in UT Austin's M392C (Representation Theory) class in Spring 2017, taught by Sam Gunningham. I live-TEXed them using vim, so there may be typos; please send questions, comments, complaints, and corrections to a.debray@math.utexas.edu. Thanks to Kartik Chitturi, Adrian Clough, Tom Gannon, Nathan Guermond, Sam Gunningham, Jay Hathaway, and Surya Raghavendran for correcting a few errors.

Contents

1.	Lie groups and smooth actions: 1/18/17	2
2.	Representation theory of compact groups: 1/20/17	4
3.	Operations on representations: $1/23/17$	6
4.	Complete reducibility: 1/25/17	8
5.	Some examples: 1/27/17	10
6.	Matrix coefficients and characters: 1/30/17	12
7.	The Peter-Weyl theorem: $2/1/17$	13
8.	Character tables: 2/3/17	15
9.	The character theory of $SU(2)$: $2/6/17$	17
10.	Representation theory of Lie groups: 2/8/17	19
11.	Lie algebras: $2/10/17$	20
12.	The adjoint representations: $2/13/17$	22
13.	Representations of Lie algebras: 2/15/17	24
14.	The representation theory of $\mathfrak{sl}_2(\mathbb{C})$: $2/17/17$	25
15.	Solvable and nilpotent Lie algebras: 2/20/17	27
16.	Semisimple Lie algebras: $2/22/17$	29
17.	Invariant bilinear forms on Lie algebras: $2/24/17$	31
18.	Classical Lie groups and Lie algebras: $2/27/17$	32
19.	Roots and root spaces: $3/1/17$	34
20.	Properties of roots: $3/3/17$	36
21.	Root systems: $3/6/17$	37
22.	Dynkin diagrams: 3/8/17	39
23.	Representations of semisimple Lie algebras: $3/10/17$	41
24.	Root data: 3/20/17	43
25.	Representations of $U(n)$ and $SU(n)$: $3/22/17$	44
26.	Example with \mathfrak{sl}_3 : $3/24/17$	45
27.	The Weyl character formula: 3/27/17	47
28.	The Weyl integration formula: 3/29/17	49
29.	Weyl modules for $U(n)$: $3/31/17$	49
30.	: 4/3/17	50
31.	: 4/5/17	50
32.	: 4/7/17	50
33.	Representation theory of $SL_2(\mathbb{R})$: $4/10/17$	50
34.	Principal series: 4/12/17	53
35	Harish-Chandra modules: 4/17/17	55

1