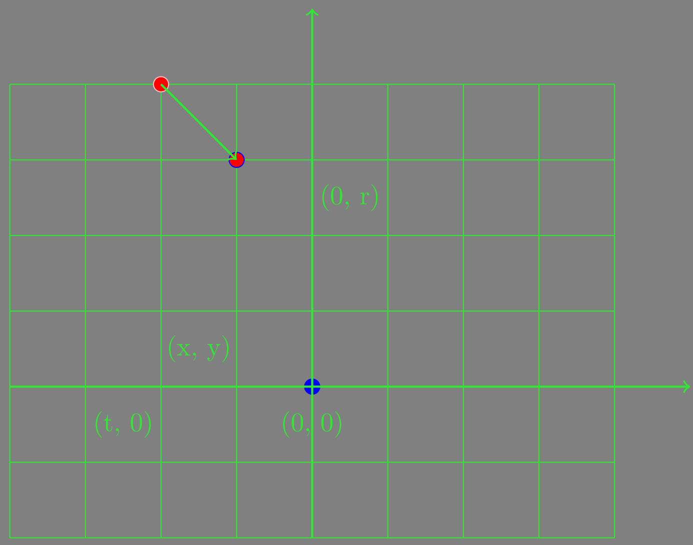
Given a point $p(1,1,1)$ and a vector $\vec{v} = (1,2,3)$.
A line can be defined as $u = p + t\vec{v}$ passing through $p(1,1,1)$ and perpendicular to $\vec{v}$ where $t \in \mathbb{R}$



$$\vec{w} = p'(x,y,z) - p(1,1,1) = \begin{bmatrix} x-1 \\ y-1 \\ z-1 \end{bmatrix}$$
$$[1,2,3] \cdot \vec{w} = 0$$
$$f(x,y,z) = x^2 + y^2 - z = 0$$
$$\nabla f = (\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}) = (2x, 2y, -1)$$
$$\text{at point } p(1,1,1)$$
$$\nabla f = (\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}) = (2, 2, -1)$$
$$\text{Normal at point } p(1,1,1)$$
$$\mathbf{n} = (2-1, 2-1, -1-1) = (1, 1, -2)$$
$$\vec{u} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ let } \vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$f(x) = 0 = x^2$$
$$\lim_{h \to 0} \frac{0-0}{h} = 0$$
$$f(x) = y - x^2 = 0$$
$$\text{The partial derivative of } f(x) = y - x^2 \text{ is } \frac{\partial f}{\partial x} = -2x$$
$$\frac{\partial f}{\partial y} = 1$$
$$\text{However, if } f(x) = 0, \Rightarrow \frac{\partial f}{\partial x} = 0 \quad \text{and} \quad \Rightarrow \frac{\partial f}{\partial x} = 0$$

1 Plane equation in three dimensions

Given a function $x + y + z = 0$ which is just a flat plane and perpendicular to vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$