Lectures on Universal Algebra

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1 Algebras

In this first section we will consider some common features of familiar algebraic structures such as groups, rings, lattices, and boolean algebras to arrive at a definition of a general algebraic structure.

Recall that a group **G** consists of a nonempty set G, along with a binary operation $\cdot : G \to G$, a unary operation $^{-1} : G \to G$, and a constant 1_G such that

- $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all x, y, $z \in G$,
- $x \cdot x^{-1} = 1_G$ and $x^{-1} \cdot x = 1_G$ for all $x \in G$,
- $1_G \cdot x = x$ and $x \cdot 1_G = x$ for all $x \in G$.

A group is abelian if it additionally satisfies:

$x \cdot y = y \cdot x$ for all x, $y \in G$.

A ring **R** is a nonempty set R along with binary operations $+$, $\cdot$, a unary operation $-$, and constants 0_R and 1_R which satisfy

- R, along with $+$, $-$, and 0_R is an abelian group.
- $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all x, y, $z \in G$.
- $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$ and $(y + z) \cdot x = (y \cdot x) + (z \cdot x)$ for all x, y, $z \in G$.