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RECENT ADVANCES IN COMPUTATIONAL CONFORMAL GEOMETRY

XIANFENG DAVID GU*, FENG LUO†, AND SHING-TUNG YAU‡

Abstract. Computational conformal geometry focuses on developing the computational methodologies on discrete surfaces to discover conformal geometric invariants. In this work, we briefly summarize the recent developments for methods and related applications in computational conformal geometry. There are two major approaches, holomorphic differentials and curvature flow. Holomorphic differential method is a linear method, which is more efficient and robust to triangulations with lower quality. Curvature flow method is nonlinear and requires higher quality triangulations, but it is more flexible. The conformal geometric methods have been broadly applied in many engineering fields, such as computer graphics, vision, geometric modeling and medical imaging. The algorithms are robust for surfaces scanned from real life, general for surfaces with different topologies. The efficiency and efficacy of the algorithms are demonstrated by the experimental results.

Keywords: Computational Conformal Geometry, Holomorphic Differential, Curvature Flow

1. Introduction. Computational conformal geometry focuses on developing the computational methodologies on discrete surfaces to discover conformal geometric invariants. Computational conformal geometry is an emerging field, which combines differential geometry, algebraic topology, complex analysis, Riemann surface theory, algebraic geometry with computer science. It has broad applications in many fields in both pure theoretic research, such as mathematics, theoretic physics, and engineering applications, such as mechanics, computer graphics, computer vision, geometric modeling, network and medical imaging.

Classical computational complex analysis focuses on the mappings among domains on the complex plane $\mathbb{C}$. Classical methods for constructing conformal mappings include Schwarz-Christoffel maps, osculation method, polynomial expansion method, circle packing method and many other methods. For details, we refer readers to [1] and [2] for a more thorough discussion.

With the development of 3D data acquisition technologies, huge amount of surface and volumetric data have been accumulated. For example, the state of art of 3D scanner based on phase shifting technology can capture dynamic surfaces with a quarter of million samples per frame, at the frame rate as high as 180 per second. Figure 1 illustrates one example of the acquired human face surface. The geometric surfaces are stored and represented in computers as polyhedral surfaces, most commonly simplicial complexes with a piecewise linear embedding in the Euclidean space,

*Computer Science Department, State University of New York at Stony Brook, Stony Brook, NY USA.
†Mathematics Department, Rutgers University, Piscataway, NJ USA.
‡Mathematics Department, Harvard University, Cambridge, MA USA.