Math 501 - Differential Geometry
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7. THE GAUSS-BONNET THEOREM

The Gauss-Bonnet Theorem is one of the most beautiful and one of the deepest results in the differential geometry of surfaces. It concerns a surface S with boundary ∂S in Euclidean 3-space, and expresses a relation between:

- the integral $\int_S K \, d(\text{area})$ of the Gaussian curvature over the surface,
- the integral $\int_{\partial S} \kappa_g \, ds$ of the geodesic curvature of the boundary of the surface, and
- the topology of the surface, as expressed by its Euler characteristic:

$$\chi(S) = \#\,\text{Vertices} - \#\,\text{Edges} + \#\,\text{Faces}\,.$$