Using LaTeX in category theory

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This document is a quick how-to for the students of the category theory class at the CMU. It contains specific and practical knowledge of LaTeX which will be useful to write homework solutions.

1 Writing your document

1.1 Setting up your document

To get started with LaTeX, section 1 of the Wikibook on http://en.wikibooks.org/wiki/LaTeX provides descriptions of what LaTeX is, how to obtain it and the basics of LaTeX-syntax.

The basic layout of the source code of the document you are currently reading is

```
\documentclass{article}

\title{Using \LaTeX~in category theory}
\author{Egbert Rijke}
\date\today

% Before you begin writing your document you have
% space to set up user-defined commands, or to
% load existing packages containing frequently
% used commands. This part of the file is called
% the preamble.

\begin{document}
\maketitle

% Write the text you want the document to contain
% between the \begin{document} and the
% \end{document} declarations.

\end{document}
```

As soon as the LaTeX-compiler encounters the percent sign (%), the rest of that line including the line-break, is ignored by the compiler. You can use this feature to provide comments within your source code, or to keep your code neat and organized.

The document structure is explained in more detail on http://en.wikibooks.org/wiki/LaTeX/Document_Structure

1.2 User defined macros

User defined commands, also called *macros*, are useful to

- simplify the process of typesetting,
- make the source code of your document (i.e. the `.tex` files) more readable,
- ensure uniformity of notation in your document, or even throughout several documents,
- facilitate change of notation when you're not yet quite sure.

As a rule of thumb, every mathematical concept which needs notation has either a predefined macro (in the case it is very basic), or it has a user defined macro.

The macros defined to write the math in this document are

```
\newcommand{\cat}{%
  \mathbf%
}
\newcommand{\domain}[1]{%
  \mathrm{dom}(#1)%
}
\newcommand{\codomain}[1]{%
  \mathrm{cod}(#1)%
}
\newcommand{\idarrow}[1][]{%
  \mathbf{1}_{#1}%
}
```

Detailed information on defining your own macros can be found on http://en.wikibooks.org/wiki/LaTeX/Macros

1.3 Using the same preamble for several documents

It is a good idea to have a separate file `preamble.tex` for the preamble and include it just above `\begin{document}` with the line

```
\input{preamble}
```

The `\input` command looks for the `.tex` file with the name provided in the braces in the current folder, unless you give it specific instructions to look elsewhere. Thus, the file `preamble.tex` must in this case be in the same folder as the main file.

Using a seperate preamble file allows you to use the same preamble for several documents. This is actually a first step in creating your own packages, see http://en.wikibooks.org/wiki/LaTeX/Creating_Packages.

2 Category theory in LaTeX

2.1 Theorems, definitions and exercises

Theorem environments, which are provided by the `amsthm` package, can be used to declare environments for theorems, lemmas, definitions, exercises, and the like. To declare the environments for definitions and exercises, we have included the lines

```
\theoremstyle{definition}
\newtheorem{defn}{Definition}[section]
\newtheorem{ex}{Exercise}
```

in the preamble.

Now we can start writing definitions. For instance, the definition of a category is written in the `document`-environment as:

```
\begin{defn}
A \emph{category} $\cat{C}$ consists of
\begin{itemize}
\item a collection of objects: $A$, $B$, $C$,
  \ldots
\item a collection of arrows: $f$, $g$, $h$,
  \ldots
\item for each arrow $f$ objects $\domain{f}$ and
  $\codomain{f}$ called the \emph{domain} and
\emph{codomain} of $f$. If $\domain{f}=A$ and
  $\codomain{f}=B$, we also write $f:A\to B$,
\item given $f:A\to B$ and $g:B\to C$, so that
  $\domain{g}=\codomain{f}$, there is an arrow
  $g\circ f:A\to C$,
\item an arrow $\idarrow[A]:A\to A$ for every
  object $A$ of $\cat{C}$,
\end{itemize}
such that
\begin{description}
\item[(Associative law)] for every $f:A\to B$,
  $g:B\to C$ and $h:C\to C$ we have
```

```
\begin{equation*}
h\circ(g\circ f)=(h\circ g)\circ f,
\end{equation*}
\item[(Unit laws)] for every $f:A\to B$ we have
\begin{equation*}
f\circ\idarrow[A]=f=\idarrow[B]\circ f.
\end{equation*}
\end{description}
\end{defn}
```

This results in:

Definition 2.1. A *category* **C** consists of

- a collection of objects: A, B, C, ...
- a collection of arrows: f, g, h, ...
- for each arrow f objects $\mathrm{dom}(f)$ and $\mathrm{cod}(f)$ called the *domain* and *codomain* of f. If $\mathrm{dom}(f) = A$ and $\mathrm{cod}(f) = B$, we also write $f : A \to B$,
- given $f : A \to B$ and $g : B \to C$, so that $\mathrm{dom}(g) = \mathrm{cod}(f)$, there is an arrow $g \circ f : A \to C$,
- an arrow $\mathbf{1}_A : A \to A$ for every object A of **C**,

such that

(Associative law) for every $f : A \to B$, $g : B \to C$ and $h : C \to C$ we have

$$h \circ (g \circ f) = (h \circ g) \circ f,$$

(Unit laws) for every $f : A \to B$ we have

$$f \circ \mathbf{1}_A = f = \mathbf{1}_B \circ f.$$

2.2 Drawing diagrams

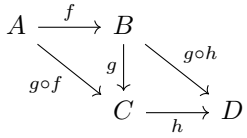
For drawing diagrams, we recommend the `tikz-cd` package. The documentation for the `tikz-cd` package is available on http://www.ctan.org/pkg/tikz-cd. Be sure you use the latest version of `tikz-cd`, because its features and syntax has been changed recently.

As an example of a diagram drawn with `tikz-cd`, the following code displays the diagram that has been used in class to demonstrate the associative law:

```
\begin{equation*}
\begin{tikzcd}
A \arrow[r,"f"]
  \arrow[dr,swap,"g\circ f"]
```

```
    &
B \arrow[dr,"g\circ h"]
    \arrow[d,swap,"g"]
    \\
    {}&
C \arrow[r,swap,"h"]
    &
D
\end{tikzcd}
\end{equation*}
```

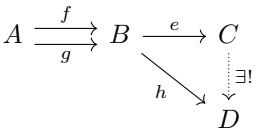
The output of the above code is the diagram



It is also possible to draw parallel arrows, for instance, to display coequalizer diagrams, and dotted arrows to display universal properties. The code

```
\begin{equation*}
\begin{tikzcd}
A \arrow[yshift=.7ex,r,"f"]
    \arrow[yshift=-.7ex,r,swap,"g"]
    &
B \arrow[r,"e"]
    \arrow[dr,swap,"h"]
    &
C \arrow[densely dotted,d,"\exists!"]
    \\
& &
D
\end{tikzcd}
\end{equation*}
```

results in the diagram



When the diagram seems too packed with information, it sometimes helps to separate the rows and columns more by starting the `tikzcd`-environment with the option `column sep=huge` or `row sep=large`, for example. More precisely, the environment would look like

```
\begin{tikzcd}[column sep=huge]
...
\end{tikzcd}
```

The package documentation contains more examples which will prove useful in displaying diagrams.

3 Getting your homework done

The source file of your homework would probably look something like

```
\documentclass{article}
\title{Homework set 1}
\author{Student's name}
\date\today

\usepackage{amssymb,amsthm,amsmath}
\usepackage{tikzcd}

%%%% The exercise environments
\theoremstyle{definition}
\newtheorem{ex}{Exercise}

%%%% My macros
\newcommand{\cat}{%
  \mathbf%
}
\newcommand{\domain}[1]{%
  \mathrm{dom}(#1)%
}
\newcommand{\codomain}[1]{%
  \mathrm{cod}(#1)%
}
\newcommand{\idarrow}[1][]{%
  \mathbf{1}_{#1}%
}

%%%% Macros for specific categories
\newcommand{\Cat}{%
  \cat{Cat}%
}
\newcommand{\Mon}{%
  \cat{Mon}%
}
\newcommand{\Poset}{%
```

```
  \cat{Poset}%
}
\newcommand{\Rel}{%
  \cat{Rel}%
}
\newcommand{\Sets}{%
  \cat{Sets}%
}
\newcommand{\Groups}{%
  \cat{Groups}%
}
\newcommand{\Graphs}{%
  \cat{Graphs}%
}

\begin{document}
\maketitle

\begin{ex}
% Solution to exercise 1
\end{ex}

\begin{ex}
% Solution to exercise 2
\end{ex}

\begin{ex}
\begin{enumerate}
\item % Solution to exercise 3a
\item % Solution to exercise 3b
\item % Solution to exercise 3c
\end{enumerate}
\end{ex}

\end{document}
```

You are free to copy and use the above code.

4 More LaTeX-resources

- Books:
  - The LaTeX companion (2nd edition) by Mittelbach, Goossens and Braams,
  - The TeX-book by Knuth,

  - More math into LaTeX by Grätzer.

- Resources on the web:
  - The main question & answer site with lots of useful tips and tricks is http://tex.stackexchange.com.
  - Whenever you want to use a symbol in your document, but don't know the corresponding LaTeX-command, or even how to call that symbol, DETEXIFY might be an outcome. On the website http://detexify.kirelabs.org/classify.html you can draw your symbol and it will tell you which of the actual LaTeX-symbols it thinks it matches most closely. If this does not help, consult the CTAN comprehensive symbol list, which is available at http://www.ctan.org/tex-archive/info/symbols/comprehensive/

If you have any questions or suggestions you can also email me. To see how things are done, you may also want to have a look at the source of this document. It is available on https://github.com/EgbertRijke/CategoryTheory_Course