1 Intersection of two lines

Given two lines, find the intersection of two lines if they are not parallel
Use a point and a vector to represent a line

$$f(t) = p_0 + s(p_1 - p_0) \quad \text{where} \quad v_1 = p_1 - p_0 \text{ is a vector}$$
$$g(s) = q_0 + t(q_1 - q_0) \quad \text{where} \quad v_2 = q_1 - q_0 \text{ is a vector}$$
$$f(t) = g(s)$$
$$p_0 + s(p_1 - p_0) = q_0 + t(q_1 - q_0)$$
$$p_0 - q_0 = -s(p_1 - p_0) + t(q_1 - q_0)$$
$$p_0 - q_0 = (x_0, y_0)$$
$$p_1 - p_0 = (px, py) \quad \text{s}$$
$$q_1 - q_0 = (qx, qy) \quad \text{t}$$
$$x_0 = -s(px) + t(qx)$$
$$y_0 = -s(py) + t(qy)$$
$$x_0 = s(-px) + t(qx)$$
$$y_0 = s(-py) + t(qy)$$
$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} -px & qx \\ -py & qy \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

Example: let $p_0 = (2,0), p_1 = (2,1), \quad q_0 = (0,0), q_1 = (1,1)$

$$f(s) = (2,0) + s(2,1) \quad \text{where} \quad f(s) = q_0 + t(q_1 - q_0)$$
$$f(t) = (0,0) + t(1,1) \quad \text{where} \quad f(t) = p_0 + s(p_1 - p_0)$$
$$p_0 - q_0 = (2,0) - (0,0) = (2,0)$$
$$p_1 - p_0 = (2,1) - (2,0) = (0,1) \quad \text{where} \quad v_1 = p_1 - p_0 = \overrightarrow{p_0 p_1}$$
$$q_1 - q_0 = (1,1) - (0,0) = (1,1) \quad \text{where} \quad v_2 = q_1 - q_0 = \overrightarrow{q_0 q_1}$$
$$(2,0) = -s(0,1) + t(1,1)$$
$$2 = -s(0) + t1$$
$$0 = -s(1) + t1$$

$$2 = s(0) + t1$$
$$0 = s(-1) + t1$$
$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$
$$s = 2$$
$$t = 2$$