

Cross Product

$$\begin{aligned}
\begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} \times \begin{bmatrix} a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} &= \begin{bmatrix} i & j & k \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = (-1)^{1+1}i \det m_i + (-1)^{1+2}j \times \det m_j + (-1)^{1+3}k \times \det m_k \\
&= i \det m_i - j \det m_j + k \det m_k \\
&= a_{22}a_{33}i - a_{23}a_{32}j + ka_{21}a_{33}
\end{aligned}$$

where

$$\begin{aligned}
m_i &= \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \\
m_j &= \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} \\
m_k &= \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}
\end{aligned}$$