Exploring Visualization Methods for Complex Variables

Andrew J. Hanson[1] **and Ji-Ping Sha**[1]

1 **Computer Science Department and Mathematics Department**
Indiana University
Bloomington, IN 47405 USA
{hansona,jsha}@indiana.edu

Abstract

Applications of complex variables and related manifolds appear throughout mathematics and science. Here we review a family of basic methods for applying visualization concepts to the study of complex variables and the properties of specific complex manifolds. We begin with an outline of the methods we can employ to directly visualize poles and branch cuts as complex functions of one complex variable. $\mathbb{C}\mathbf{P}^2$ polynomial methods and their higher analogs can then be exploited to produce visualizations of Calabi-Yau spaces such as those modeling the hypothesized hidden dimensions of string theory. Finally, we show how the study of N-boson scattering in dual model/string theory leads to novel cross-ratio-space methods for the treatment of analysis in two or more complex variables.

1998 ACM Subject Classification I.3.5 Computational Geometry and Object Modeling, J.2 Physical Sciences and Engineering

Keywords and phrases Visualization, Complex Manifolds, High Dimensions

Digital Object Identifier 10.4230/DFU.SciViz.2010.90

1 Introduction

Mathematical visualization of issues involving complex variables is a fundamental problem that, sooner or later, is related to almost any problem in science. Our goal here is to review some general methods that can be used to make the abstract features of complex variables more concrete by exploiting computer graphics technology, and to illustrate these methods with some interesting applications. We begin with a number of general concepts, and conclude with some examples related to problems of mathematical physics motivated by string theory.

The basic methods for the representation of the shapes of homogeneous polynomial equations in $\mathbb{C}\mathbf{P}^2$ were explored in detail in ([3]), and this will be the starting point for many of our basic visualizations. We will also briefly summarize some more recent results of ([4]) treating some geometric objects arising naturally in the complex analysis of integrals appearing in the N-boson scattering amplitudes of the dual models of early string theory.

2 Visualizing Complex Analysis

Complex Numbers

We may think of a complex number in several ways. The most traditional form comes from the observation that, while the trivial equation $x^2 = 1$ can be solved in the domain of real numbers, the closely related equation $x^2 = -1$ cannot: one must introduce an "imaginary

Scientific Visualization: Advanced Concepts.
Editor: Hans Hagen; pp. 90–109
Dagstuhl Publishing
Schloss Dagstuhl – Leibniz Center for Informatics (Germany)