# 1 Find the projection from $w$ onto $v$.

Dot product of two vectors

$$w \cdot v = \|w\|\|v\| \cos\theta$$

Compute the scalar projection from $w$ onto $v$

$$\frac{w \cdot v}{\|v\|} = \|w\| \cos\theta$$

Multiply the unit vector of $v$

$$proj_v w = \frac{w \cdot v}{\|v\|} \frac{v}{\|v\|}$$

$$proj_v w = \frac{\langle w, v \rangle}{\langle v, v \rangle} v$$

Given vectors: $v_1, v_2, v_3$ find the orthgonal basis for the three vectors.

$$a_1 = v_1 = \frac{\langle a_1, v_1 \rangle}{\langle a_1, v_1 \rangle} a_1 \qquad e_1 = \frac{a_1}{\|a_1\|}$$

$$a_2 = v_2 - \frac{\langle a_1, v_2 \rangle}{\langle a_1, a_1 \rangle} a_1 \qquad e_2 = \frac{a_2}{\|a_2\|}$$

$$a_3 = v_3 - \left( \frac{\langle a_1, v_3 \rangle}{\langle a_1, a_1 \rangle} a_1 + \frac{\langle a_2, v_3 \rangle}{\langle a_2, a_2 \rangle} a_2 \right) \qquad e_3 = \frac{a_3}{\|a_3\|}$$

$$proj_{w_1} v_2 = \frac{\langle u_1, v_2 \rangle}{\langle v_2, v_2 \rangle} v_2 \quad \text{where } w_1 = \text{Span}\{u_1\}$$

$$proj_{w_2} v_3 = \frac{\langle u_1, v_3 \rangle}{\langle v_3, v_3 \rangle} v_3 + \frac{\langle u_2, v_3 \rangle}{\langle v_3, v_3 \rangle} v_3 \quad \text{where } w_2 = \text{Span}\{u_1, u_2\}$$

$$v_1 = \frac{\langle v_1, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1$$

$$v_2 = a_2 + \frac{\langle v_1, v_2 \rangle}{\langle v_1, v_1 \rangle} v_1$$

$$v_3 = a_3 + \left( \frac{\langle v_1, v_3 \rangle}{\langle v_1, v_1 \rangle} v_1 + \frac{\langle v_2, v_3 \rangle}{\langle v_2, v_2 \rangle} v_2 \right)$$

Project $v_1, v_2, v_3$ onto new **orthgonal basis**: $\{e_1, e_2, e_3\}$

$$v_1 = \frac{\langle e_1, v_1 \rangle}{\langle e_1, e_1 \rangle} e_1 \qquad \text{project } v_1 \text{ onto } e_1$$

$$v_2 = \frac{\langle e_1, v_2 \rangle}{\langle e_1, e_1 \rangle} e_1 + \frac{\langle e_2, v_2 \rangle}{\langle e_2, e_2 \rangle} e_2 \qquad \text{project } v_2 \text{ onto } e_1, e_2$$

$$v_3 = \frac{\langle e_1, v_3 \rangle}{\langle e_1, e_1 \rangle} e_1 + \frac{\langle e_2, v_3 \rangle}{\langle e_2, e_2 \rangle} e_2 + \frac{\langle e_3, v_3 \rangle}{\langle e_3, e_3 \rangle} e_3 \qquad \text{project } v_3 \text{ onto } e_1, e_2, e_3$$