

1 Curvature for any curve

Definition 1 (What is curvature). *A curvature is the amount by which a curve deviates from a straight line. Or in simply term, a curve is how far apart from a straight line.*

1.1 What is the curvature of table, soccer and horse saddle

1.2 Curvature in real life

Drive your car in a **straight road** or in a **curve road**.

Definition 2 (Osculating Circle). *What is Osculating Circle? Osculating Circle is the maximum circle which is just touch one point on the curve.*

2 Find intersection of **circle** and **parabola**

Circle Equation

$$x^2 + (y - r)^2 = r^2$$

Parabola Equation

$$y = x^2$$

Expand (1)

$$x^2 + y^2 + r^2 - 2ry = r^2$$

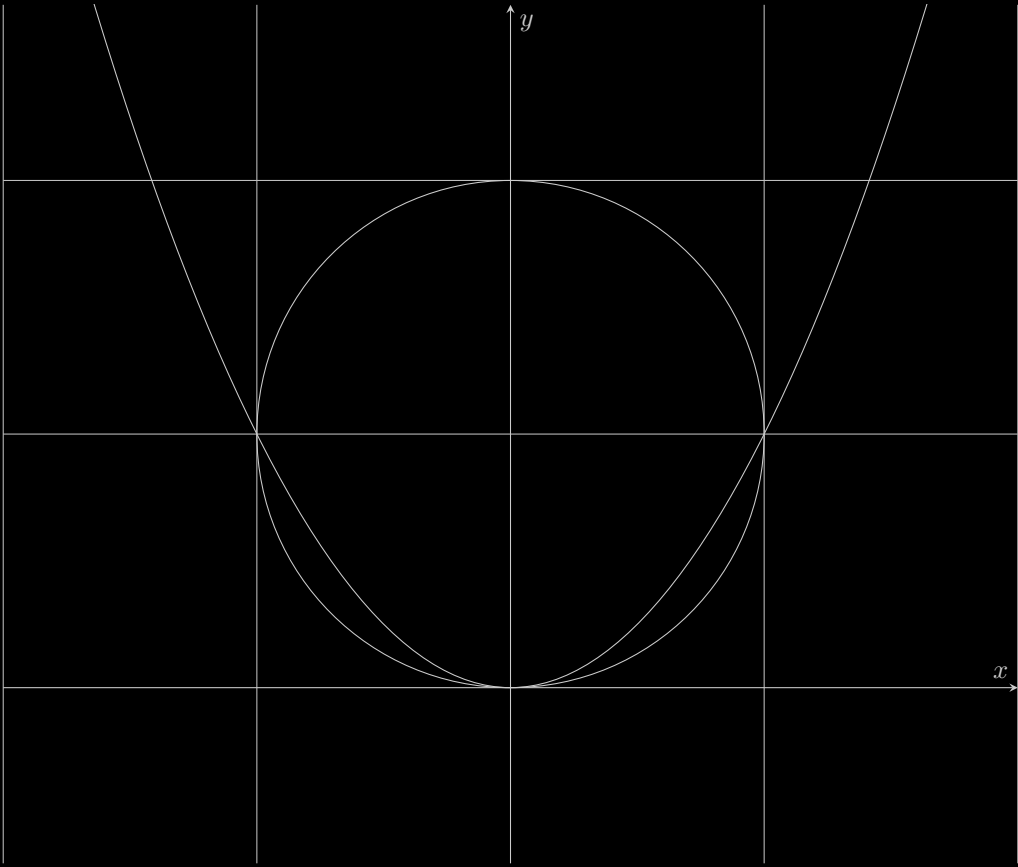
Sub (2) into (1)

$$\begin{aligned} x^2 + x^4 + r^2 - 2rx^2 &= r^2 && \text{where} \quad \text{Sub } y = x^2 \\ x^2 + x^4 - 2rx^2 &= 0 \\ 1 + x^2 - 2r &= 0 && \text{where} \quad x \neq 0 \\ x^2 &= 2r - 1 \end{aligned}$$

$x = \pm\sqrt{2r - 1}$ where $r \neq \frac{1}{2}$

3 Intersection of three points between **circle** and **parabola**

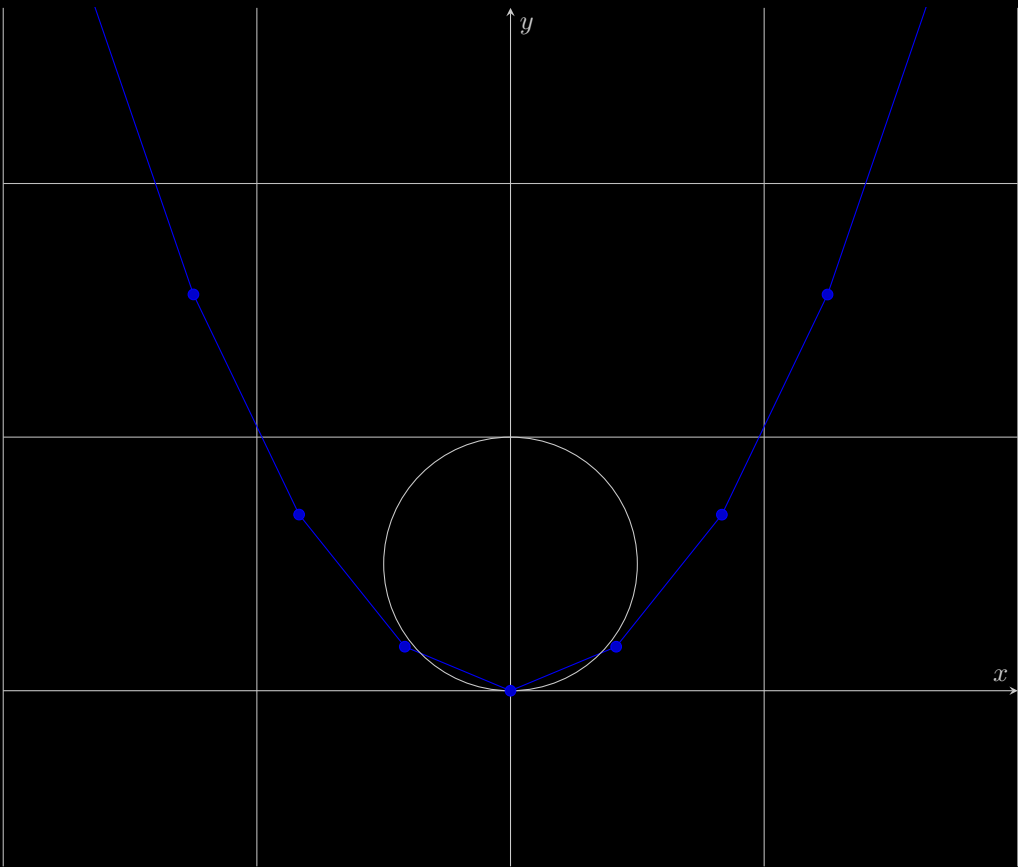
If $r > \frac{1}{2}$



4 Intersection of only ONE points between **circle** and **parabola**

If $r \rightarrow \frac{1}{2}$

$\Rightarrow x^2 + (y - \frac{1}{2})^2 = \frac{1}{2}$ is an **Osculating Circle**



5 Validation Osculating Circle

If $r = \frac{1}{2}$

$x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2$ circle

$y = x^2$ parabola

Sub parabola into circle

$$\begin{aligned} y + (y - \frac{1}{2})^2 &= (\frac{1}{2})^2 \\ y + y^2 - y + (\frac{1}{2})^2 &= (\frac{1}{2})^2 \\ y^2 &= 0 \\ \Rightarrow x = 0 \quad \wedge \quad y &= 0 \end{aligned}$$

(1)

Proof. Circle $x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2$ and Parabola $y = x^2$ intersects only ONE point

If $r = \frac{1}{2}$

$x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2$ circle

$y = x^2$ parabola

Sub parabola into circle

$$\begin{aligned} y + (y - \frac{1}{2})^2 &= (\frac{1}{2})^2 \\ y + y^2 - y + (\frac{1}{2})^2 &= (\frac{1}{2})^2 \\ y^2 &= 0 \\ \Rightarrow x = 0 \quad \wedge \quad y &= 0 \end{aligned}$$

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