This is simple proof for $\log x + \log y = \log xy$

Proof. $\log a^x + \log a^y = \log a^{x+y}$ or $\log x + \log y = \log xy$

$$
\begin{aligned}
\text{Given:} & \\
a^x a^y &= a^{x+y} \\
\log \text{ both sides} & \\
\log(a^x a^y) &= \log a^{x+y} \\
\log(a^x a^y) &= (x+y)\log a \\
\log(a^x a^y) &= x\log a + y\log a \\
\log(a^x a^y) &= \log a^x + \log a^y \\
\log a^{x+y} &= \log a^x + \log a^y
\end{aligned}
$$

□