Chapter 1

Various Ways of Representing Surfaces and Basic Examples

Lecture 1.

a. First examples. For many people, one of the most basic images of a surface is the surface of the Earth. Although it looks flat to the naked eye (at least in the absence of any striking geographic features), we learn early in our lives that it is in fact round, and that its shape is very well approximated by a sphere. Geometrically, the sphere is defined as the locus of points at a fixed distance, called the *radius*, from a given point, the centre. Using Cartesian coordinates and putting the origin at the centre, we derive the familiar equation

$$x^2 + y^2 + z^2 = R^2, \tag{1.1}$$

where R is the radius; the sphere is the set of all points in $\mathbb{R}^3$ whose coordinates (x, y, z) satisfy this equation.

Many other familiar shapes can also be defined geometrically and represented as the set of solutions of a single equation, as in (1.1). For example, the (round) cylinder is the locus of points at a fixed distance from a given straight line. If the line is taken to be the z-axis and the