1 Inverse two dimension matrix

Use row reduction to find the inverse of two dimension matrix. If the determinant is non-zero

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Argumented A with identity

$$A' = \begin{bmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{bmatrix}$$

br_1, ar_2

$$A' = \begin{bmatrix} ac & cb & c & 0 \\ ac & ad & 0 & a \end{bmatrix}$$

$r_1 - r_2 \to r_2$

$$A' = \begin{bmatrix} a & b & 1 & 0 \\ 0 & cb - ad & c & -a \end{bmatrix}$$

$(cb - ad)r_2, br_1$

$$A' = \begin{bmatrix} a(cb - ad) & b(cb - ad) & (cb - ad) & 0 \\ 0 & b(cb - ad) & bc & -ba \end{bmatrix}$$

$r_1 - r_2 \to r_1$

$$A' = \begin{bmatrix} a(cb - ad) & 0 & (bc - ad) - bc & ba \\ 0 & b(cb - ad) & bc & -ba \end{bmatrix}$$

br_1, ar_2

$$A' = \begin{bmatrix} ab(cb - ad) & 0 & -abd & ab^2 \\ 0 & ab(cb - ad) & abc & -ba^2 \end{bmatrix}$$

$$\frac{r_1}{ab(cb - ad)} \quad \frac{r_2}{ab(cb - ad)}$$

$$A' = \begin{bmatrix} 1 & 0 & \frac{-abd}{ab(cb-ad)} & \frac{ab^2}{ab(cb-ad)} \\ 0 & 1 & \frac{abc}{ab(cb-ad)} & \frac{-ba^2}{ab(cb-ad)} \end{bmatrix}$$

Simplify a bit

$$A' = \begin{bmatrix} 1 & 0 & \frac{-d}{(cb-ad)} & \frac{b}{(cb-ad)} \\ 0 & 1 & \frac{c}{(cb-ad)} & \frac{-a}{(cb-ad)} \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 0 & \frac{-d}{\det A} & \frac{b}{\det A} \\ 0 & 1 & \frac{c}{\det A} & \frac{-a}{\det A} \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$$