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A Sweepline Algorithm for Voronoi Diagrams

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Abstract. We introduce a geometric transformation that allows Voronoi diagrams to be computed using a sweepline technique. The transformation is used to obtain simple algorithms for computing the Voronoi diagram of point sites, of line segment sites, and of weighted point sites. All algorithms have $O(n \log n)$ worst-case running time and use $O(n)$ space.

Key Words. Voroni diagram, Delaunay triangulation, Sweepline algorithm.

1. Introduction. The Voronoi diagram of a set of sites in the plane partitions the plane into regions, called Voronoi regions, one to a site. The Voronoi region of a site s is the set of points in the plane for which s is the closest site among all the sites.

The Voronoi diagram has many applications in diverse fields. One application is solving closest-site queries. Suppose we have a fixed set of sites and a query point, and would like to know the closest site to the query point. If the Voronoi diagram of the set of sites is constructed, then this problem has been reduced to determining the region containing the query point. If in fact the number of query points is large relative to the number of sites, then the construction of the Voronoi diagram is worthwhile. The papers by Preparata [16] and by Green and Sibson [8] contain references to other applications.

We present simple sweepline algorithms for the construction of Voronoi diagrams when sites are points and when sites are line segments. The proposed algorithms are based on the sweepline technique [17], [20]. The sweepline technique conceptually sweeps a horizontal line upward across the plane, noting the regions intersected by the line as the line moves. Computing the Voronoi diagram directly with a sweepline technique is difficult, because the Voronoi region of a site may be intersected by the sweepline long before the site itself is intersected by the sweepline. Rather than compute the Voronoi diagram, we compute a geometric transformation of it. The transformed Voronoi diagram has the property that the lowest point of the transformed Voronoi region of a site appears at the site itself. Thus the sweepline algorithm need consider the Voronoi region of a site only when the site has been intersected by the sweepline. It turns out to be easy to reconstruct the real Voronoi diagram from its transformation; in fact in practice the real Voronoi diagram would be constructed, and the transformation computed only as necessary. The sweepline algorithms compute the Voronoi diagram of n sites in time $O(n \log n)$ and space usage $O(n)$.

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