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Clique Community Persistence: A Topological Visual Analysis Approach for Complex Networks

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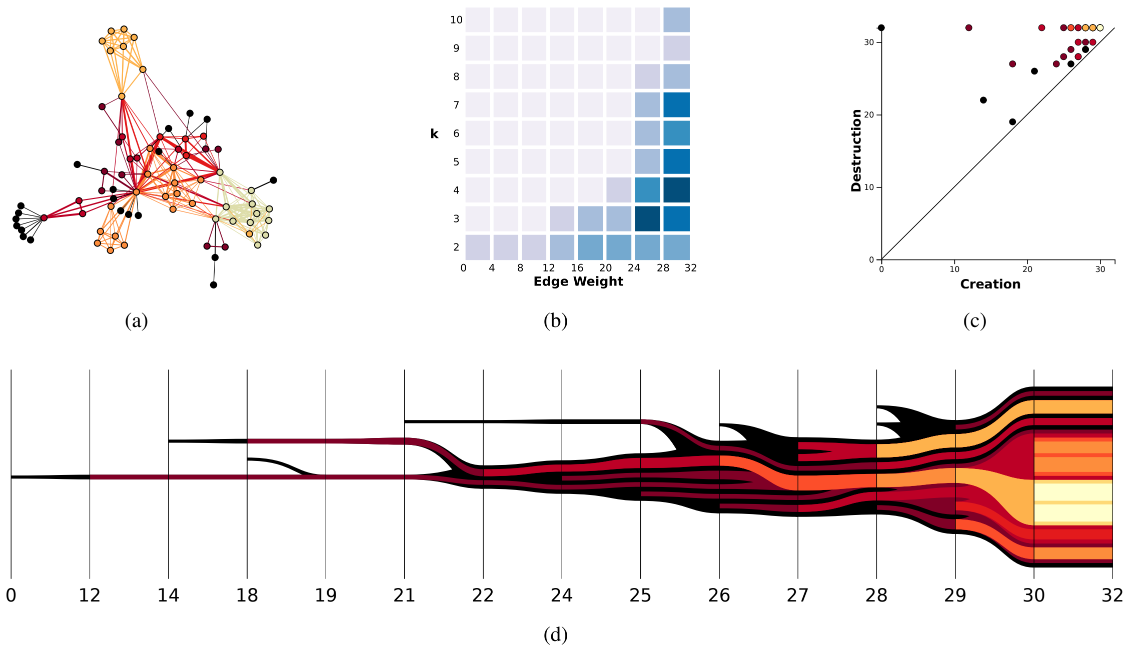

Fig. 1. All components of our proposed approach, shown for the "Les Misérables" co-occurrence network, which we analyze in Section 4.1. If the size of the graph permits it, we show a force-directed graph layout of the network (a), where each vertex is colored according to the maximum degree of their associated clique community. A 2D histogram (b) of the maximum number of individual clique communities for all edge weights and all clique degrees helps in finding relevant edge weight thresholds. The persistence diagram (c) gives an overview of all clique communities and their merging behavior. The nested graph (d) shows how individual clique communities merge when the edge weight of the network increases. Furthermore, it permits tracking the evolution of a single community.

Abstract—Complex networks require effective tools and visualizations for their analysis and comparison. Clique communities have been recognized as a powerful concept for describing cohesive structures in networks. We propose an approach that extends the computation of clique communities by considering persistent homology, a topological paradigm originally introduced to characterize and compare the global structure of shapes. Our persistence-based algorithm is able to detect clique communities and to keep track of their evolution according to different edge weight thresholds. We use this information to define comparison metrics and a new centrality measure, both reflecting the relevance of the clique communities inherent to the network. Moreover, we propose an interactive visualization tool based on nested graphs that is capable of compactly representing the evolving relationships between communities for different thresholds and clique degrees. We demonstrate the effectiveness of our approach on various network types.

Index Terms—Persistent homology, topological persistence, cliques, complex networks, visual analysis.

1 Introduction

Complex network analysis [35, 42, 47] is an active research topic with applications in multiple fields of interest, such as sociology, physics, electrical engineering, biology, and economics. Generally, complex networks are used to represent different kinds of systems that consist of individuals interacting with each other. A *local* analysis often focuses on the connections of a single node and its local relevance. Centrality measures such as betweenness or closeness help identify key nodes. A study of structural properties of the *entire* network, by contrast, concentrates on groups of nodes and their connections. The connectivity of a network can be measured using a large variety of attributes and descriptors such as density, cohesion, diameter, and small-worldness. For this kind of analysis, it is necessary to study *communities* or clusters [16]. Although a concrete definition depends on the application context, a community is usually considered to be a highly-connected group of nodes of the network. All of these concepts augment the description of the local and the global structure of a network. Moreover, they can be used to compare different networks. However, despite the effectiveness of these concepts for evaluating similarities between networks, a tool

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