1 Projection matrix

Given two vectors u, v, find the projection from u onto v.
We can use trigonometry to solve it, but a better solution is using vector.
Given vector $u = \begin{bmatrix} x \\ y \end{bmatrix}$ projects on $v = \begin{bmatrix} x' \\ y' \end{bmatrix}$
The project matrix is

$$p = \frac{uu^T}{u^T u} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} = u^T u \begin{bmatrix} xx & xy \\ yx & yy \end{bmatrix}$$

Or we derived above matrix from:

$$\begin{aligned} P &= \vec{v}\frac{\langle \vec{v}, \vec{u} \rangle}{\|\vec{v}\|} \\ P &= \vec{v}\frac{\vec{v}^T \vec{u}}{\|\vec{v}\|} \\ P &= \vec{v}\vec{v}^T \frac{\vec{u}}{\|\vec{u}\|} \end{aligned} \tag{1}$$

Let's implement it in Haskell

```
u = [[x]
     [y]]
v = [[x', y']]
[ map(\vx -> (head u') ++ vx) v' | u' <- u, v' <- v]

We can use lambda function for string and integer operations:
outerStr::(a->a->a)->[[a]]->[[a]]->[[a]]
outerStr f v r = [ map(\vx -> f (head u') vx) v' | u' <- u, v' <- v]

-- String op
outerStr (++) u v
-- Integer op
outerStr (+) u v
```

1.1 Some properties about Project Matrix

$$\begin{aligned} P = \vec{u}\vec{u}^T \Rightarrow P^T = \left(\vec{u}\vec{u}^T\right)^T = \vec{u}\vec{u}^T = P \\ P^2 = P, P^T = P, \Rightarrow P^2 = P^T \end{aligned} \tag{2}$$