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Visualization of 4D Vector Field Topology

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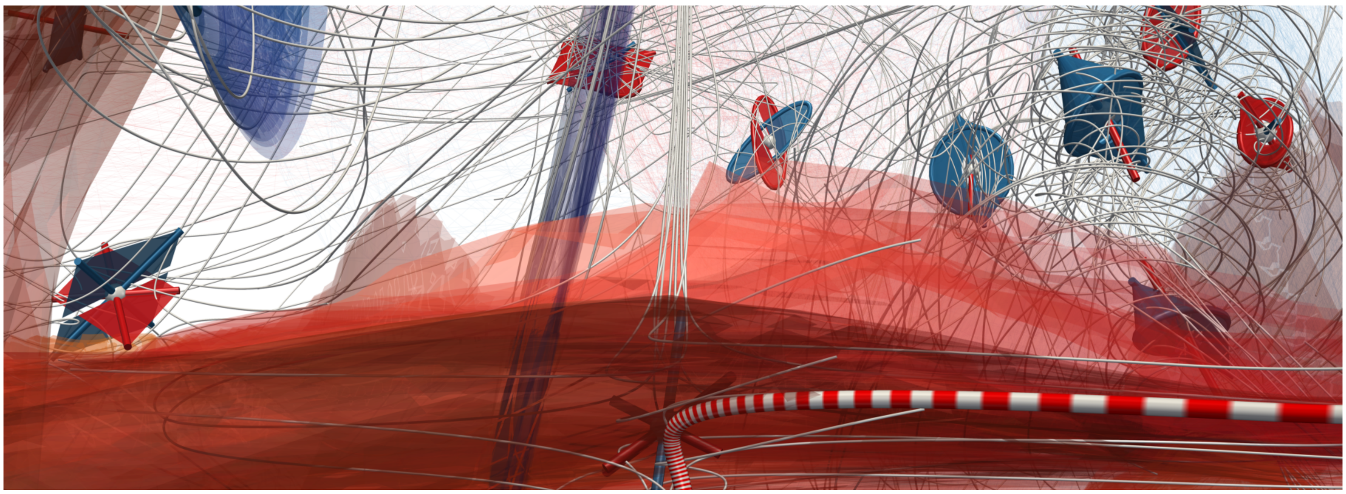

Figure 1: *3D projection of topological structures of a 4D vector field, with critical points depicted by 4D glyphs, their stable and unstable invariant manifolds (blue and red, respectively) by volumes in 4D, together with 4D streamlines for context. The 4D camera moves in 4D space along a selected streamline (striped red–white), while maintaining a view which minimizes the amount of clutter in the 3D projection.*

Abstract

In this paper, we present an approach to the topological analysis of four-dimensional vector fields. In analogy to traditional 2D and 3D vector field topology, we provide a classification and visual representation of critical points, together with a technique for extracting their invariant manifolds. For effective exploration of the resulting four-dimensional structures, we present a 4D camera that provides concise representation by exploiting projection degeneracies, and a 4D clipping approach that avoids self-intersection in the 3D projection. We exemplify the properties and the utility of our approach using specific synthetic cases.

CCS Concepts

•***Human-centered computing*** → ***Visualization techniques;*** •***Applied computing*** → *Mathematics and statistics;*

1. Introduction

Many physical phenomena in our everyday life, that involve force or motion, are accessible by the concept of a two-dimensional or three-dimensional time-independent vector field. There are, however, cases, where a fourth dimension is required for appropriate modeling, leading to four-dimensional vector fields. Since vector fields correspond to differential equations [Asi93] and therefore to dynamical systems, any continuous deterministic dynamical system with four-dimensional phase space, and any four-dimensional differential equation, represents a four-dimensional vector field. Besides mathematics, there is a wide range of problems that lead to such four-dimensional dynamical systems, including the change of concentration of four substances due to chemical reactions, or the evolution of four competing species. It is, however, nowadays often the case that such problems are investigated by only a few simulations instead of dense ensembles, i.e., simulations are obtained only for a few initial conditions or parameter choices. Such sets of simulations result in four-dimensional phase spaces whose domain is, however, less than four-dimensional, and thus do not necessitate four-dimensional analysis. Nevertheless, a prominent source for four-dimensional vector fields is already today the phase space in physics, consisting of position and momentum. It is $2n$-dimensional for a problem in n-space, and is used to represent the motion of inertial objects due to, e.g., forces.