Supplemental Materials for EE203001 Students

II. Determinant Functions

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1 Three Axioms for a Determinant Function

Let d be a scalar-valued function on the space $M_{n\times n}$ of all $n\times n$ matrices. Let $A_1, A_2, \ldots, A_n$ be the n rows of an $n\times n$ matrix A. We will denote the value $d(A)$ of A under the function d as $d(A) = d(A_1, A_2, \ldots, A_n)$, indicating that d is a scalar-valued function of n n-dimensional row vectors $A_1, A_2, \ldots, A_n$. The scalar-valued function d is called a determinant function of order n if it satisfies the following three axioms:

Axiom 1. *Homogeneity in each row.* If matrix B is obtained from matrix A by multiplying one row, says the ith row, of A by a scalar α, then $d(B) = \alpha\ d(A)$, i.e.,

$$d(A_1, \ldots, \alpha A_i, \ldots, A_n) = \alpha\ d(A_1, \ldots, A_i, \ldots, A_n).$$

Axiom 2. *Invariance under row addition.* If matrix B is obtained from matrix A by adding one row, says the kth row, of A to another row, says the ith row, of A, then $d(B) = d(A)$, i.e.,

$$d(A_1, \ldots, A_i + A_k, \ldots, A_k, \ldots, A_n) = d(A_1, \ldots, A_i, \ldots, A_k, \ldots, A_n).$$

Axiom 3. *The determinant of the identity matrix is one.*

$$d(I_{n\times n}) = d(e_1, e_2, \ldots, e_n) = 1.$$

The primary purpose of this supplement is to show that there exists such a determinant function d of order n and it is unique. Before being able to do so, we will derive important properties of such a determinant function d if exists.