This is Reddit question in askmath question Check whether the following series is converge or disverge

$$s = \sum_{n=1}^{\infty} \frac{1}{\log n + n^{\frac{1}{2}}}$$

We knew harmonic series is disverge  $\delta$ 

$$\sum_{n=1}^{\infty} \frac{1}{n}$$
 is disverge

If we could show  $\log n + n^{\frac{1}{2}} < n$  where n >= k, k is a constant, then we are done

We know that

$$\log n < \frac{n}{2}$$
 when  $n >= 4$ 

$$n^{\frac{1}{2}} < \frac{n}{2}$$
 where  $n > 4$ 

$$\Rightarrow \ln n + n^{\frac{1}{2}} < \frac{n}{2} + \frac{n}{2} = n \text{ when } n > 4$$

$$\Rightarrow \log n + n^{\frac{1}{2}} < n$$

 $\Rightarrow$ each term in s is greater than the harmonic series's term when n > 4