Chapter 2

Linear Algebra

2.1 Introduction

We discuss vectors, matrices, transposes, covariance, correlation, diagonal and inverse matrices, orthogonality, subspaces and eigenanalysis. An alterntive source for much of this material is the excellent book by Strang [58].

2.2 Transposes and Inner Products

A collection of variables may be treated as a single entity by writing them as a *vector*. For example, the three variables x_1, x_2 and x_3 may be written as the vector

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{2.1}$$

Bold face type is often used to denote vectors (scalars - single variables - are written with normal type). Vectors can be written as *column vectors* where the variables go down the page or as *row vectors* where the variables go across the page (it needs to be made clear when using vectors whether $\boldsymbol{x}$ means a row vector or a column vector - most often it will mean a column vector and in our text it will *always* mean a column vector, unless we say otherwise). To turn a column vector into a row vector we use the *transpose* operator

$$\boldsymbol{x}^T = [x_1, x_2, x_3] \tag{2.2}$$

The transpose operator also turns row vectors into column vectors. We now define the *inner product* of two vectors

$$\begin{aligned} \boldsymbol{x}^T\boldsymbol{y} &= [x_1, x_2, x_3] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad (2.3) \\ &= x_1y_1 + x_2y_2 + x_3y_3 \end{aligned}$$