

1 Newton's Iteration Method

First method to derive Newton's iterative method

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} && \text{def. of limit} \\f'(x) &\approx \frac{f(x+h) - f(x)}{h} && \text{where } h \text{ is sufficient small} \\f'(x_0) &\approx \frac{f(x_0+h) - f(x_0)}{h} \\f'(x_0) &= \frac{y - y_0}{x - x_0} && (1) \\h &= \frac{f(x_0+h) - f(x_0)}{f'(x_0)} \\h &= \frac{0 - f(x_0)}{f'(x_0)} && \text{where line } f(x_0+h) = 0 \text{ intercepts x-axis} \\x_1 &= x_0 + h && \text{second root}\end{aligned}$$

Method 2 to derive Newton's iteration method

$$\begin{aligned}f'(x_0) &= \frac{y - y_0}{x - x_0} && \text{line equation passes pt } (x_0, f(x_0)) \text{ with slope } f'(x_0) \\y - y_0 &= f'(x_0)(x - x_0) \\y &= f'(x_0)(x - x_0) + y_0 \\y - y_0 &= f'(x_0)(x - x_0) \\x &= \frac{-y_0}{f'(x_0)} + x_0 && \text{where } y = 0 \\x &= x_0 - \frac{y_0}{f'(x_0)} \\x_1 &= x_0 - \frac{y_0}{f'(x_0)} && (2)\end{aligned}$$