A YOUNG PERSON'S GUIDE TO THE HOPF FIBRATION

ZACHARY TREISMAN

The purpose of these notes is to introduce a mathematical structure which goes by the name the Hopf fibration, and demonstrates a number of surprising and beautiful things. The Hopf fibration is a map showing a connection between two spheres, a two dimensional sphere, and a three dimensional sphere. In order to understand this map, we will need to develop a few tools. There will be a few detours along the way, in order to develop enough familiarity with the tools being used so that the student is sufficiently impressed by the structures that are eventually uncovered.

The Hopf fibration and the mathematics that are developed along the way makes for some very interesting visual images. The paintings that have been included here are all the work of Lun-Yi Tsai, an artist, a mathematician, and a good friend. The computer generated images I



FIGURE 1. Hopf Fibration Lun-Yi Tsai