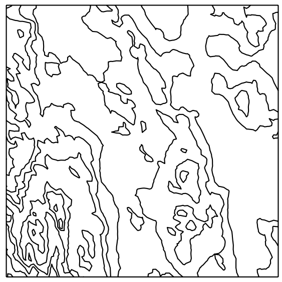
9 Delaunay Triangulations

Height Interpolation

When we talked about maps of a piece of the earth's surface in previous chapters, we implicitly assumed there is no relief. This may be reasonable for a country like the Netherlands, but it is a bad assumption for Switzerland. In this chapter we set out to remedy this situation.

We can model a piece of the earth's surface as a *terrain*. A terrain is a 2-dimensional surface in 3-dimensional space with a special property: every vertical line intersects it in a point, if it intersects it at all. In other words, it is the graph of a function $f : A \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ that assigns a height $f(p)$ to every point p in the *domain*, A, of the terrain. (The earth is round, so on a global scale terrains defined in this manner are not a good model of the earth. But on a more local scale terrains provide a fairly good model.) A terrain can be visualized with a perspective drawing like the one in Figure 9.1, or with contour lines—lines of equal height—like on a topographic map.



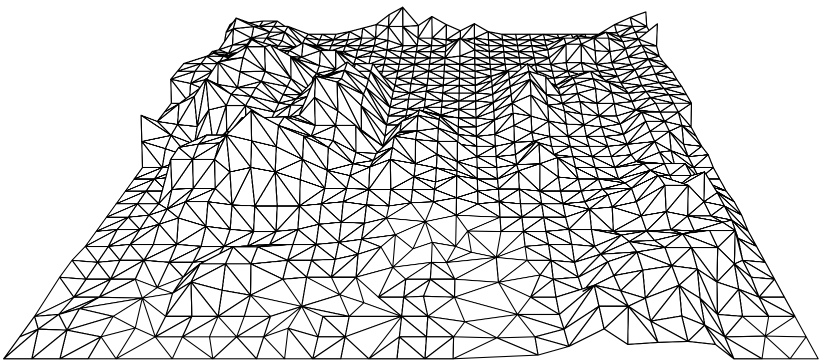


Figure 9.1
A perspective view of a terrain

Of course, we don't know the height of every point on earth; we only know it where we've measured it. This means that when we talk about some terrain, we only know the value of the function f at a finite set $P \subset A$ of sample points. From the height of the sample points we somehow have to approximate the height at the other points in the domain. A naive approach assigns to every $p \in A$ the height of the nearest sample point. However, this gives a discrete terrain, which