

# 1 How to Solve quadratic equation

$$ax^2 + bx + c = 0 \quad \text{such as} \quad a \neq 0$$

Divide the equation by a such as

$$\begin{aligned} \frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\ \Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\ \Rightarrow x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} &= 0 \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} &= 0 \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{c}{a} \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \\ \Rightarrow \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \Rightarrow \left(x + \frac{b}{2a}\right) &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ \Rightarrow x &= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ \Rightarrow x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$