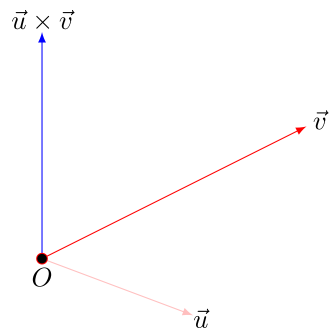
Proof. Zero Proof, Nice Proof, Cool Proof, OOP, FLM Proof □

Given a *parallelogram* $PQRS$ in space, how could you find a vector normal to its plane and with length equal to its area?
Parallelogram PQST and cross product
The area of parallelogram is

$$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}| \sin \alpha$$



The direction of cross product is the right-hand rule
The magnitude of cross product of two vectors is the parallelogram
The dot product of two vectors

$$\vec{u} \circ \vec{v} = |\vec{u}||\vec{v}| \cos \alpha$$

Theorem 1. *Let $\mathbb{V}$ be vector space, and let $S_1 \subseteq S_2 \subseteq \mathbb{V}$. If S_1 is linearly dependent then S_2 is linearly dependent.*

Proof. Let the basis of S_1 is $\mathcal{U} = \{u_1, u_2, \dots u_m\}$ $\because S_1$ is linearly dependent$\therefore \exists w \in S_1 \mid w = a_1\vec{u_1} + a_2\vec{u_2} + \cdots + a_m\vec{u_m}$ $a_1, a_2, \dots, a_m$ are not all zero
Assume the basis of S_2 is $\mathcal{V} = \{\vec{v_1}, \vec{v_2}, \dots, \vec{v_n}\}$ $\because S_1 \subseteq S_2 \therefore \exists$ a subset $\mathcal{U}'$ of $\mathcal{V} \mid \mathcal{U}'$ spans S_1. In other words, w can be written linearly combination of the subset of $\mathcal{V}$. Not all coefficients are zero.
$\implies S_2$ is linearly dependent.
$S_1 \in$S$_2$
$w = a_1\vec{u_1} + a_2\vec{u_2} + \cdots + a_m\vec{u_m}$ not all $a_1, \dots, a_m$ are zero
The basis of $\mathcal{V} = \{v_1, \dots \vec{v_n}\} \implies \vec{u_1}, \vec{u_2}, \dots, \vec{u_m}$ can be written linearly combination of $\{\vec{v_1}, \vec{v_2}, \dots \vec{v_n}\}$
$\implies w = c_1\vec{v_1} + c_2\vec{v_2} + \dots$

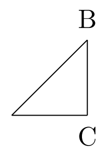
Collorary 1. *Let $\mathbb{V}$ be a vector space, and let $S_1 \subseteq S_2 \subseteq \mathbb{V}$, if S_2 is linearly independent then S_1 is linearly independent.*

Proof. Assume S_1 is linearly dependent, then S_2 is linearly dependent. $\because$ theorem 1 That contracts our assumption S_2 is linearly independent. □

Theorem 2. *Let S be a linearly independent subset of a vector space $\mathbb{V}$, and let $\vec{u}$ be a vector in $\mathbb{V}$ that is not in S. Then $S \cup \{\vec{u}\}$ is linearly dependent if and only if $\vec{u} \in$ span$\{S\}$*

Proof. Prove Theorem 2
Proof: $S \cup \{\vec{u}\}$ is linearly dependent $\Rightarrow \vec{u} \in \text{span}\{S\}$
$\vec{u} \notin S$ and $S \cup \vec{u}$ is linearly dependent $\implies$ $\vec{u}$ can be written linearly combination of the basis of $S \implies \vec{u} \in span\{S\}$

Proof: $\vec{u} \in \text{span}\{S\} \implies S \cup \{\vec{u}\}$ is linearly dependent
$\vec{u} \notin S$ and $\vec{u} \in \text{span}\{S\} \implies \vec{u}$ can be written linearly combination of the basis of $S \implies S \cup \{\vec{u}\}$ is linearly independent. □



1 Chinsse math problem

Theorem 3. *Given $x^x \div x = x^2$ find x*
$x \neq 0 \implies x^2 \neq 0$ *then* $x^x = x^3$
case 1: x = 3
case 2: x = 1
case 3: x = -1

Example 1. *If* $a^n = 1$ *then* $a \neq 0, n = 0$ *or* $a = 1, n$ *can be any number*
$$\begin{cases} a \neq 0, n = 0 \\ a = 1, n \text{ can be any number} \\ a = -1, n = 2k \quad k \in \mathbb{N} \end{cases}$$

2 Interview question

Find all Integer a, b satifiy the equation $\sqrt{a} + \sqrt{b} = \sqrt{2023}$
First attempt:

$$\begin{aligned}
\sqrt{a} + \sqrt{b} &= \sqrt{2023} \\
(\sqrt{a} + \sqrt{b})^2 &= \sqrt{2023}^2 \\
a + b + 2\sqrt{ab} &= 2023 \\
\sqrt{ab} &= \frac{2023 - (a+b)}{2} \\
ab &= \left(\frac{2023 - (a+b)}{2}\right)^2 \\
ab &= \frac{2023^2 + (a+b)^2 - 2 \times 2023(a+b)}{4} \\
4ab &= 2023^2 + (a+b)^2 - 2 \times 2023(a+b) \\
4ab &= 2023^2 + a^2 + b^2 + 2ab - 2 \times 2023(a+b) \\
0 &= 2023^2 + (a-b)^2 - 2 \times 2023(a+b)
\end{aligned}$$

Second attempt:

$$\begin{aligned}
\sqrt{a} + \sqrt{b} &= 17\sqrt{7} \\
\sqrt{a} &= 17\sqrt{7} - \sqrt{b} \\
&\text{Square both sizes} \\
a &= 2023 + b - 2 \times 17\sqrt{7b} \\
a &= 2023 + b - 34\sqrt{7b}
\end{aligned}$$

3 Cambridge Math Interview Question

Graphic the equation $y^2 - y = x^2 - x$

$$\begin{aligned}
y^2 - y &= x^2 - x \\
y^2 - x^2 &= y - x \\
(y+x)(y-x) &= y - x \\
&\text{There two cases:} \\
y &= x \\
y + x &= 1 \quad \text{if } y \neq x
\end{aligned} \tag{1}$$

4 World cup quarter-final

Croatia	Argentina	Tue, Dec 13, 11:00am
Morocco	Frence	Wed, Dec 14, 11:00am

Croatia	Brazil	Fri, Dec 9, 7:00am
Netherland	Argentina	Fri, Dec 9, 11:00am
Morocco	Portual	Sat, Dec 10, 7:00am
England	Frence	Sat, Dec 10, 11:00am

5 Addition table

$789 + 54 = 843$

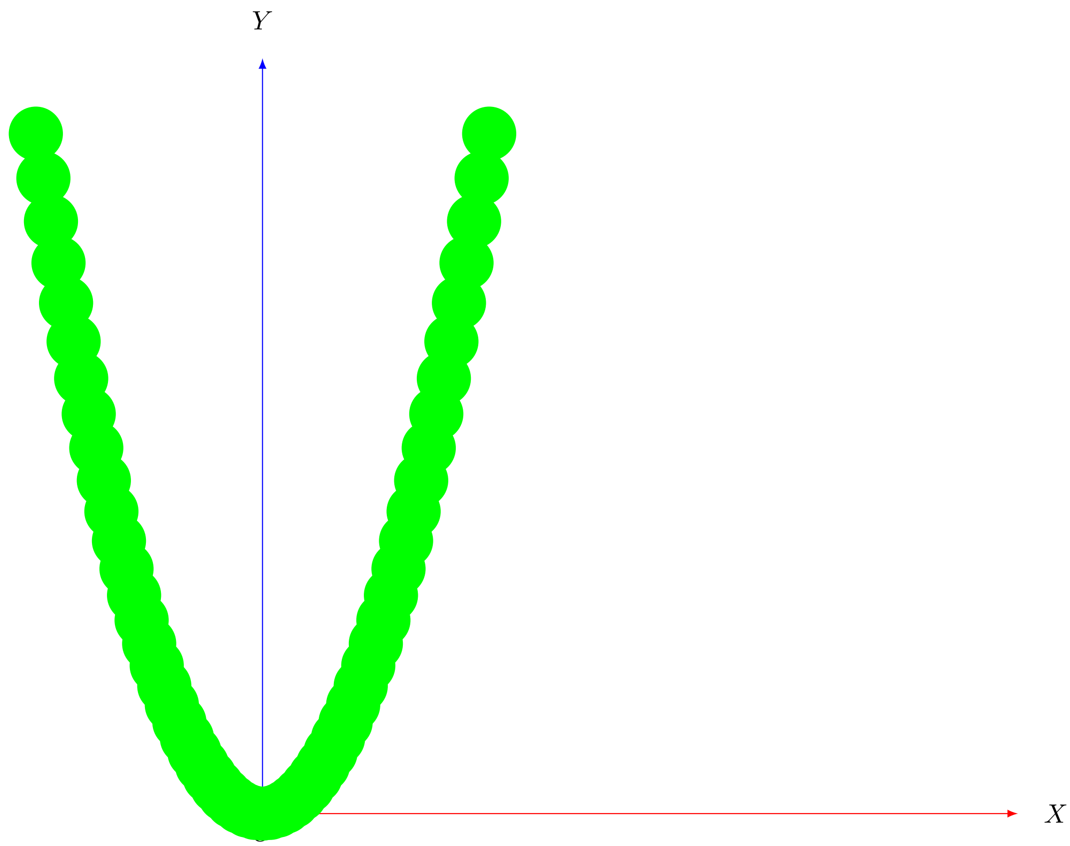
	$1 + 7$	$\leftarrow 1 + 13$	$\leftarrow 0 + 13$
	7	8	9
$+$		5	4
	8	4	3

6 Multiplication table

		$3, 9 \leftarrow 35 + 4 \mod 10$	$4, 5 \leftarrow 40 + 5 \mod 10$	$4, 5 \leftarrow 45 \mod 10$	
			$3, 1 \leftarrow 28 + 3 \mod 10$	$3, 5 \leftarrow 32 + 3 \mod 10$	$1, 6 \leftarrow 36 \mod 10$
			7	8	9
				5	4
		3	1	5	6
	3	9	5	5	
	4	2	7	0	6

7 Partition an array with a pivot

Example 2. *Given an array { 2, 3, 9, 1, 4 } and choose 4 as a pivot*
left: 2, 3, 1
right: 9
{ left pivot right }
new array: {2, 3, 1, 4, 9}



8 Multiply all numbers but the current one

1	2	3	4	5		
	2	3	4	5		
	2	3	4	5		
	2	3	4	5	1	

Problem 1. *Prove there is infinite many prime*

Proof. Assume there are finite many primes such as

$$p_1, p_2, \dots, p_n \quad where p_n \text{ is the maximum prime}$$

Let $m = p_1 \times p_2 \dots p_n$
Then $p_1 \times p_2 \times \cdots \times p_n + 1$ must be composite number.
$\implies \frac{p_1 \times p_2 \times \cdots \times p_n + 1}{p_k} = \frac{p_1 \times p_2 \times \cdots \times p_n}{p_k} + \frac{1}{p_k} \in \mathbb{N}$
But $\frac{1}{p_k}$ can not be $\mathbb{N}$
$\implies$ our assumption must be false. $\square$

Proof. $n \mid ab$ then $n \mid a$ or $n \mid b$ where $\gcd a, b = 1 \quad a, b, n \in \mathbb{N}$ $\square$

9 Haskell format string libraries

9.1 text-format and text-format-simple

There are **text-format** and **text-format-simple**

```
import Text.Format
let str = format "str={0} number={1}" ["dog", show 100]
```

9.2 Text.Printf

Text.Printf is similar to C printf, but I can not figure out how to convert it to String.

```
import Text.Printf
printf "str=%s number=%d" "dog" 100
```