Conformal Mappings in Geometric Algebra

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In 1878 William Kingdon Clifford wrote down the rules for his *geometric algebra*, also known as *Clifford algebra*. We argue in this paper that in doing so he laid down the groundwork that is profoundly altering the language used by the mathematical community to express geometrical ideas. In the real estate business everyone knows that what is most important is **location**. We demonstrate here that in the business of mathematics what is most important to the clear and concise expression of geometrical ideas is **notation**. In the words of Bertrand Russell,

> *…A good notation has a subtlety and suggestiveness which at times make it seem almost like a live teacher.*

Heinrich Hertz expressed much the same thought when he said,

> *One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we originally put into them.*

The development of the real and complex number systems represents a hard-won milestone in the robust history of mathematics over many centuries and many different civilizations [5], [29]. Without it mathematics could progress only haltingly, as is evident from the history of mathematics and even the terminology that we use today. Negative numbers were referred to by Rene Descartes (1596–1650) as "fictitious", and "imaginary" numbers were held up to even greater ridicule, though they were first conceived as early as Heron of Alexandria, the illustrious inventor of the windmill and steam engine during the first century AD [18]. What most mathematicians fail to see even today is that geometric algebra represents the grand culmination of that process with the completion of the real number system to include the concept of direction. Geometric algebra combines the two silver currents of mathematics, geometry and algebra, into a single coherent language. As David Hestenes has eloquently stated,

Herman Gunther Grassmann (1809–1877) was a high school teacher. His far-reaching *Ausdehnungslehre*, "Theory of extension", laid the groundwork for the development of the exterior or outer product of vectors. William Rowan Hamilton (1805–1865) was an Irish physicist, astronomer, and mathematician. His invention of the quaternions as the natural generalization of the complex numbers of the plane to three-dimensional space, together with the ideas of Grassmann, set the stage for William Kingdon Clifford's definition of geometric algebra. William Kingdon Clifford (1845–1879) was a professor of mathematics and mechanics at the University College of London. Tragically, he died at the early age of 34 before he could explore his profound ideas.

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