COMPSCI 330: Design and Analysis of Algorithms 2/2/2016 and 2/4/2016

Dynamic Programming

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1 Overview

In this lecture we introduce dynamic programming. Dynamic programming is method to quickly solve large problems by first solving intermediate problems, then using these intermediate problems to solve the large problem. We will illustrate the idea of dynamic programming via examples.[1]

2 Longest Increasing Subsequence

We starts with an application of dynamic programming to finding a longest increasing subsequence.

Definition 1. *A subsequence of sequence $x_1, \ldots, x_n$ is some sequence $x_{\phi(1)}, \ldots, x_{\phi(h)}$ such that for all k, $1 \leq k \leq h$, we have $1 \leq \phi(k) \leq n$; and for any x_j in the subsequence, all x_i preceding x_j in the subsequence satisfy $i < j$. An increasing subsequence is a subsequence such that for any x_j in the subsequence, all x_i preceding x_j in the subsequence satisfy $x_i < x_j$. A largest increasing subsequence is a subsequence of maximum length.*

Note that the longest increasing subsequence need not be unique. For example, consider the following subsequence.

$$11 \quad 14 \quad 13 \quad 7 \quad 8 \quad 15 \tag{1}$$

The following is a subsequence.

$$14 \quad 8 \quad 15$$

A longest increasing subsequence of the sequence given in 1 is

$$11 \quad 13 \quad 15$$

In this case, there are also two other longest increasing subsequences:

$$7 \quad 8 \quad 15$$
$$11 \quad 14 \quad 15$$

The problem we will solve is to find a longest increasing subsequence. What kind of subproblem will help with this? Let the input sequence be denoted $v_1, \ldots, v_n$. We have the following two options:

Option 1 v_n is in the subsequence.

Option 2 v_n is not in the subsequence.

[1] Some of the material in this note is from a previous note by Samuel Haney for this class in Fall 2014.