Scalable Locally Injective Mappings

MICHAEL RABINOVICH and ROI PORANNE
ETH Zurich
and
DANIELE PANOZZO
New York University
and
OLGA SORKINE-HORNUNG
ETH Zurich

We present a scalable approach for the optimization of flip-preventing energies in the general context of simplicial mappings, and specifically for mesh parameterization. Our iterative minimization is based on the observation that many distortion energies can be optimized indirectly by minimizing a family of simpler proxy energies. Minimization of these proxies is a natural extension of the local/global minimization of the ARAP energy. Our algorithm is simple to implement and scales to datasets with millions of faces. We demonstrate our approach for the computation of maps that minimize a conformal or isometric distortion energy, both in two and three dimensions. In addition to mesh parameterization, we show that our algorithm can be applied to mesh deformation and mesh quality improvement.

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1. INTRODUCTION

Mappings are an essential tool in computer graphics and geometry processing. One of the most basic uses, and the main focus of this paper, is *mesh parameterization*. Many practical applications, such as texture mapping, remeshing, shape correspondence and attribute transfer rely on the computation of a low-distortion parameterization. The problem has been extensively studied, and a plethora of algorithms have been devised. Linear methods were proposed first, providing efficient ways to compute parameterizations, but only able to ensure injectivity of the map when the mesh boundary is fixed a priori, which induces a high distortion. As more powerful processors became available, nonlinear optimization became tractable, allowing one to compute free boundary, injective or bijective maps of a very high quality. Still, current nonlinear approaches typically require long computation times and do not scale well to large datasets, such as detailed scanned surfaces like the one in Fig. 1.

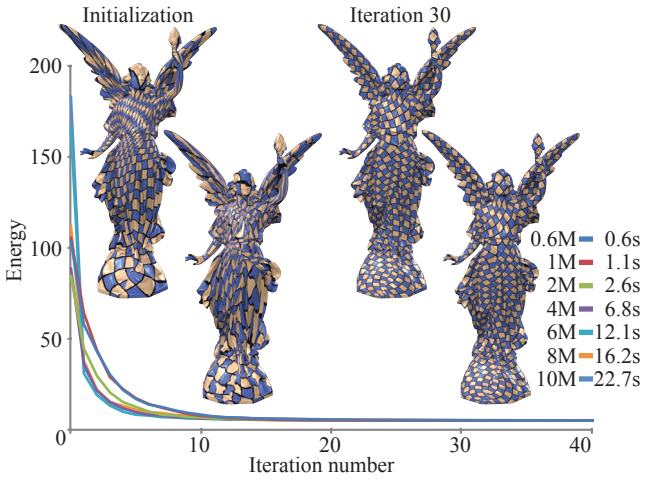

Fig. 1. We compare the behavior of our algorithm on progressively simplified versions of the Lucy model. We observe that the number of iterations required is not dependent on the resolution of the mesh. The time required per iteration in seconds appears on the righthand side of the image.

In this paper, we propose a simple algorithm that combines the benefits of the two approaches: it scales well to large datasets with millions of elements (Fig. 1) and minimizes state-of-the-art nonlinear energies (Fig. 2). In particular, we focus on minimizing *flip-preventing* energies and we propose an algorithm that is guaranteed to produce parameterizations without any flipped elements.

The key idea of our method is to minimize the nonlinear energy using much simpler proxy functions that permit the use of a local/global approach. Our algorithm scales well to large datasets even using a single core, and it can take advantage of the recent developments in parallel solution of linear systems to be deployed on multi-core architectures. While we are unable to provide a strict bound on the convergence rate, we experimentally found that the number of iterations required by our method is related to the geometric surface complexity and is not affected by the tessellation density

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