This is Reddit question in askmath question Check whether the following series is converge or disverge

$$s = \sum_{n=1}^{\infty} \frac{1}{\log n + n^{\frac{1}{2}}}$$

We knew harmonic series is disverge $\delta \epsilon$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$
 is disverge

If we could show $\log n + n^{\frac{1}{2}} < n$ where n >= k, k is a constant, then we are done.

We know that

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$$\log n < \frac{n}{2} \text{ when } n >= 4$$

$$n^{\frac{1}{2}} < \frac{n}{2} \text{ where } n > 4$$

$$\Rightarrow \ln n + n^{\frac{1}{2}} < \frac{n}{2} + \frac{n}{2} \text{ when } n > 4$$

$$\Rightarrow \log n + n^{\frac{1}{2}} < n$$

$$\Rightarrow \operatorname{each term in s is greater than the harmonic series's term when } n > 4$$