1 Newton's Iteration Method

First method to derive Newton's iteratino method

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{def. of limit} \\
f'(x) &\approx \frac{f(x+h)-f(x)}{h} \quad \text{where h is sufficient small} \\
f'(x_0) &\approx \frac{f(x_0+h)-f(x_0)}{h} \\
f'(x_0) &= \frac{y-y_0}{x-x_0} \\
h &= \frac{f(x_0+h)-f(x_0)}{f'(x_0)} \\
h &= \frac{0-f(x_0)}{f'(x_0)} \quad \text{where line } f(x_0+h)=0 \text{ intercepts x-axis} \\
x_1 &= x_0+h \quad \text{second root}
\end{aligned} \tag{1}
$$

Method 2 to derive Newton's iteration method

$$
\begin{aligned}
f'(x_0) &= \frac{y-y_0}{x-x_0} \quad \text{line equation passes pt } (x_0, f(x_0)) \text{ with slop } f'(x_0) \\
y-y_0 &= f'(x_0)(x-x_0) \\
y &= f'(x_0)(x-x_0)+y_0 \\
y-y_0 &= f'(x_0)(x-x_0) \\
x &= \frac{-y_0}{f'(x_0)}+x_0 \quad \text{where } y=0 \\
x &= x_0-\frac{y_0}{f'(x_0)} \\
x_1 &= x_0-\frac{y_0}{f'(x_0)}
\end{aligned} \tag{2}
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