

What is that?

$$\begin{aligned}
& \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \\
&= \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \\
&= \begin{bmatrix} \langle r_1, u_1 \rangle & \langle r_1, u_2 \rangle & \langle r_1, u_3 \rangle \\ \langle r_2, u_1 \rangle & \langle r_2, u_2 \rangle & \langle r_2, u_3 \rangle \\ \langle r_3, u_1 \rangle & \langle r_3, u_2 \rangle & \langle r_3, u_3 \rangle \end{bmatrix} \\
&= \begin{bmatrix} b \langle r_1, u_1 \rangle & b \langle r_1, u_2 \rangle & b \langle r_1, u_3 \rangle \\ a \langle r_2, u_1 \rangle & a \langle r_2, u_2 \rangle & a \langle r_2, u_3 \rangle \\ \langle r_3, u_1 \rangle & \langle r_3, u_2 \rangle & \langle r_3, u_3 \rangle \end{bmatrix} \\
&= \begin{bmatrix} b & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \langle r_1, u_1 \rangle & \langle r_1, u_2 \rangle & \langle r_1, u_3 \rangle \\ \langle r_2, u_1 \rangle & \langle r_2, u_2 \rangle & \langle r_2, u_3 \rangle \\ \langle r_3, u_1 \rangle & \langle r_3, u_2 \rangle & \langle r_3, u_3 \rangle \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \langle r_1, u_1 \rangle & \langle r_1, u_2 \rangle & \langle r_1, u_3 \rangle \\ \langle r_2, u_1 \rangle & \langle r_2, u_2 \rangle & \langle r_2, u_3 \rangle \\ \langle r_3, u_1 \rangle & \langle r_3, u_2 \rangle & \langle r_3, u_3 \rangle \end{bmatrix} \\
&= \begin{bmatrix} \langle br_1, u_1 \rangle & \langle br_1, u_2 \rangle & \langle br_1, u_3 \rangle \\ \langle br_1, u_1 \rangle - \langle ar_2, u_1 \rangle & \langle br_1, u_2 \rangle - \langle ar_2, u_2 \rangle & \langle br_1, u_3 \rangle - \langle ar_2, u_3 \rangle \\ \langle r_3, u_1 \rangle & \langle r_3, u_2 \rangle & \langle r_3, u_3 \rangle \end{bmatrix} \tag{1}
\end{aligned}$$

$$\begin{aligned}
A &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\
&= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \textcolor{red}{0} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\
&\Rightarrow \left\langle \begin{bmatrix} x \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \right\rangle = 0 \\
&\Rightarrow a_{11}x + a_{21} = 0 \quad \Rightarrow x = \frac{-a_{21}}{a_{11}}
\end{aligned}$$