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Algebraic and Numerical Techniques for the Computation of Matrix Determinants

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Abstract—We review, modify, and combine together several numerical and algebraic techniques in order to compute the determinant of a matrix or the sign of such a determinant. The resulting algorithms enable us to obtain the solution by using a lower precision of computations and relatively few arithmetic operations. The problem has important applications to computational geometry.

Keywords—Evaluation of the determinant, Sign of the determinant, Matrix singularity test, Modular (residue) arithmetic, Rounding error analysis.

1. INTRODUCTION

1.1. The Subject and Some Background

We study the classical problems of the computation of the determinant of a matrix or testing if the determinant vanishes, that is, if the matrix is singular. These problems have a long history (see, for instance, [1–11]) and have recently received a new major motivation, due to their important applications to geometric computations, such as computation of convex hulls and Voronoi diagrams, and testing if the line intervals of a given family have a nonempty common intersection. In such applications, one needs *sign* or *singularity* tests, that is, one needs either to test if $\det A > 0$, $\det A = 0$, or $\det A < 0$, for an $n \times n$ matrix A, or just to test whether $\det A = 0$ or not. In one group of these applications, n is *relatively small* [12,13], ranging from 2 to 10, but

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