

# Lecture 10:

# 3D Parametric Models

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Feb 1, 2018

# Why 3D Deep Learning

- To make money
  - Extra data
- How to relate 2D images with the 3D world that we live in?
- Can we exploit structures and patterns that exist in 3D data but not in images?
  - Geometric Properties
  - 3D data contains structures
  - 3D structures physically should make sense
  - It's easier to describe these physical structures

# Today's Lecture

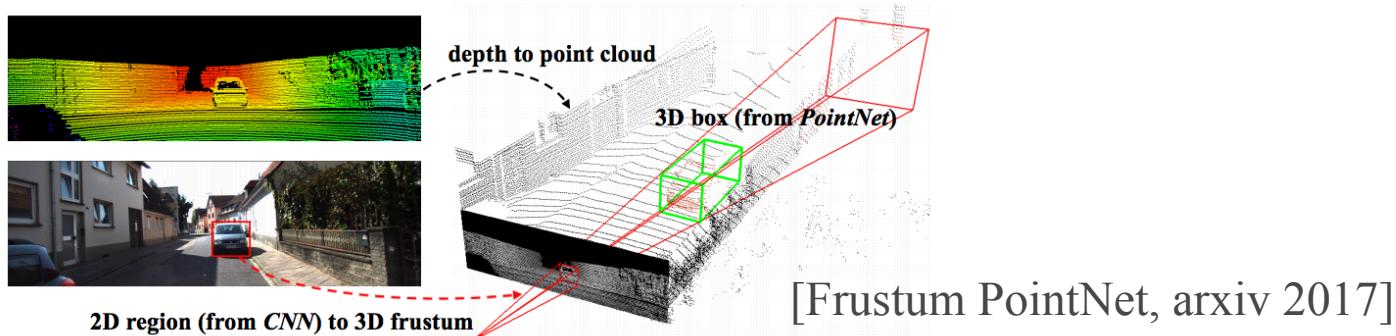
- 3D Mesh Parameterization and Manipulation
  - What makes mesh manipulation possible?
- 3D Parametric Models and Applications
  - Use several tricks
  - Consider what motivates these tricks
  - Can we automate these tricks?
- Brief overview on some of the cuter recent deep learning research using parametric models (2015-Dec. 2017)
  - Research uses these tricks
  - Can we have research that learns tricks?

# Last time

- Learning on unstructured 3D data (point clouds)
- Pros:
  - Easy to capture (depth scanner, multi-view)
  - Generalizable (easy to generate point clouds regardless of data)
  - Simple data structure
  - Great for deep learning!
  - Interesting AI Problems:
    - How can AI understand the 3D world?
    - How can AI relate 2D images to the 3D world?

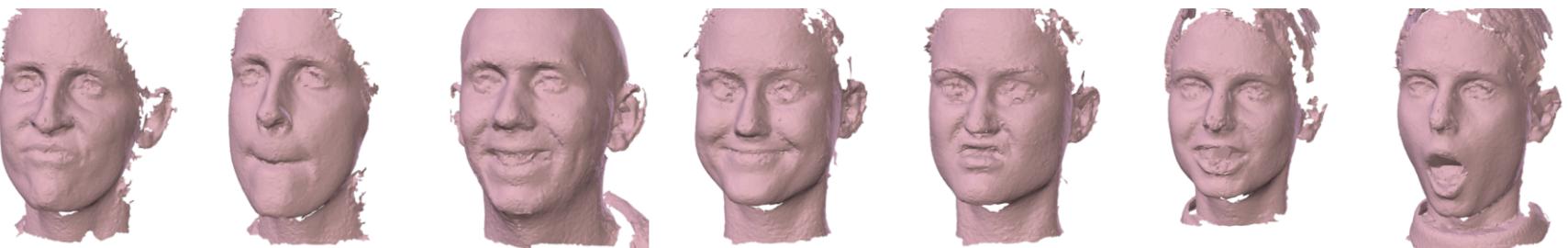
# Last time

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- Pros:
  - Easy to capture (depth scanner, multi-view)
  - Generalizable (easy to generate point clouds regardless of data)
  - Simple data structure
  - Great for deep learning!
  - Useful application\$ (e.g. \$elf driving car\$, robotic\$)



# Drawbacks of Point Clouds

- Noisy and unstructured



[FLAME, SIGGRAPH Asia 2017]

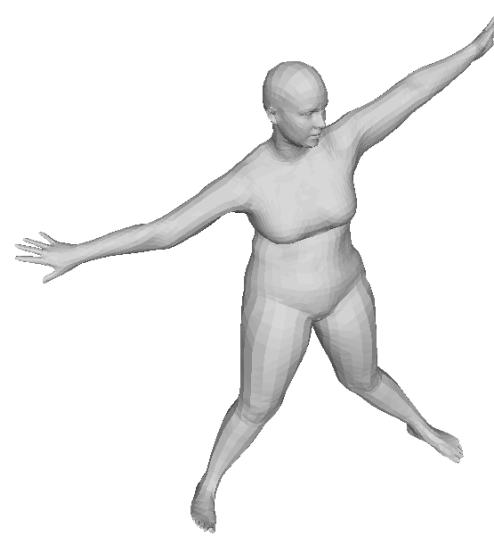
# Drawbacks of Point Clouds

- Noisy and unstructured
- Lack of topology loses information



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# Drawbacks of Point Clouds

- Noisy and unstructured
- Lack of topology loses information
- Difficult to manipulate

## Why deformations?

- Sculpting, customization
- Character posing, animation



[USC CSCI 621]

# Drawbacks of Point Clouds

- Noisy and unstructured
- Lack of topology loses information
- Difficult to manipulate
- Hence, limited application\$\$\$

## Why deformations?

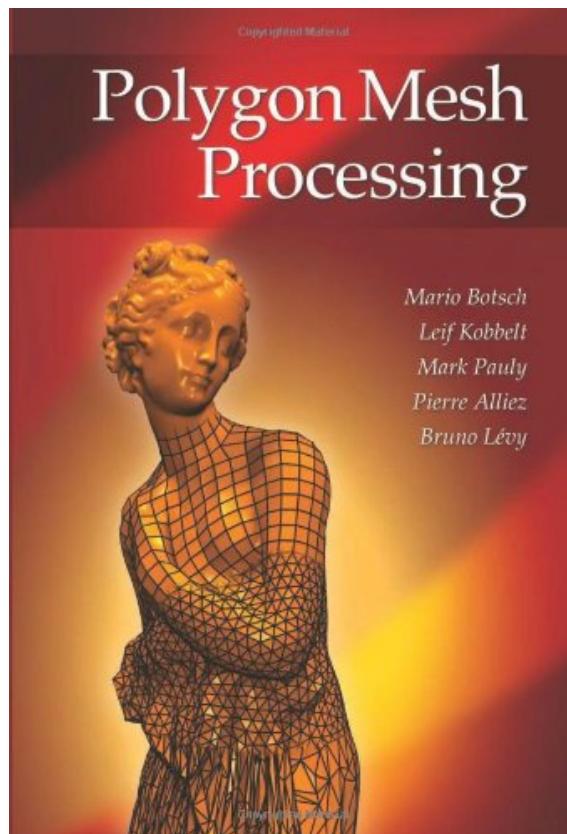
- Sculpting, customization
- Character posing, animation



[USC CSCI 621]

# Polygon Meshes

- All mathematical equations and figures in these slides come from this book or its associated slides

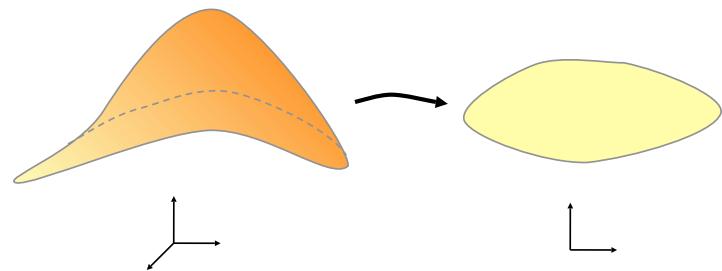


# Parameterized Meshes

- “a parameterization of a 3D surface is a function putting this surface in one-to-one correspondence with a 2D domain” - the Good Book
- Manipulable
- Consider Parameterization of A Family of Shapes:
  - Fixed Correspondences and Topology

## Surface Parameterization

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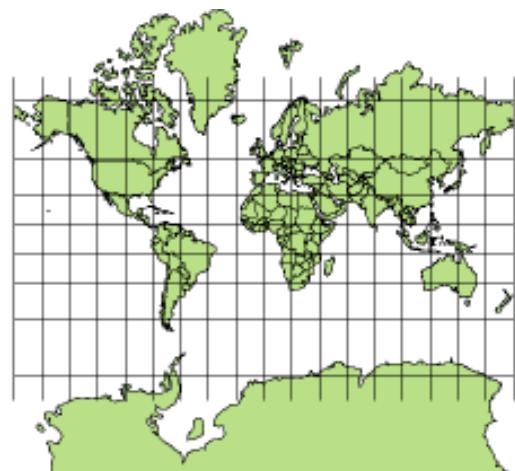
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Christian Rössl, INRIA

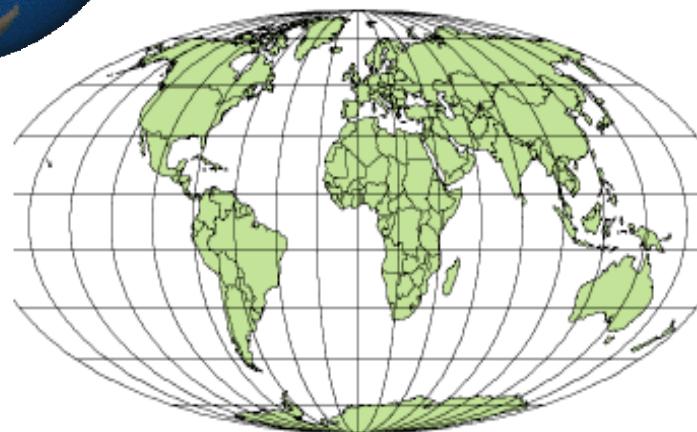
240

# Surface Parameterization

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Mercator-Projektion



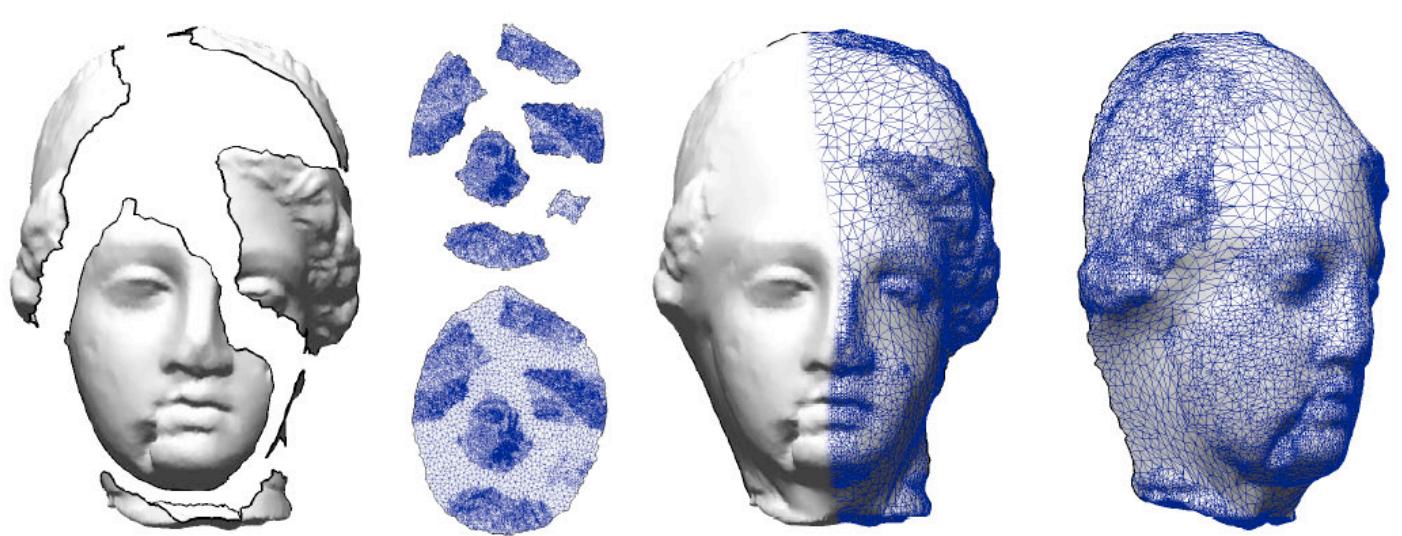
Mollweide-Projektion

[[www.wikipedia.de](http://www.wikipedia.de)]

# Motivation

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- Many operations are simpler on planar domain

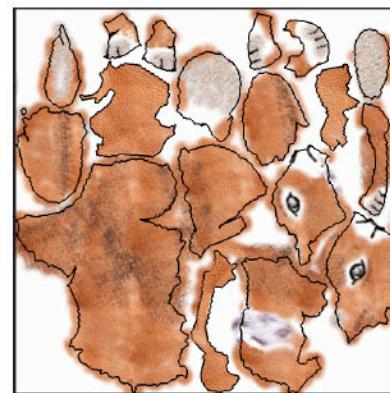
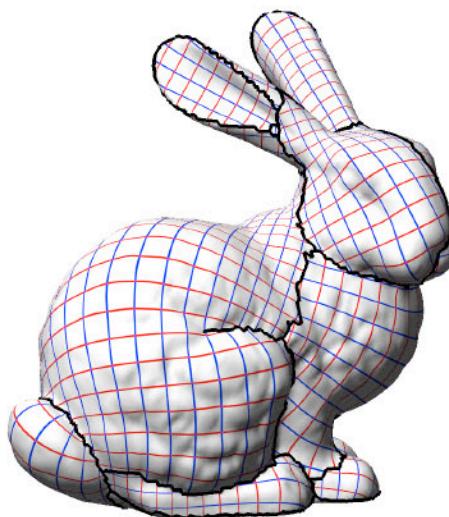


Lévy: *Dual Domain Extrapolation*, SIGGRAPH 2003

# Motivation

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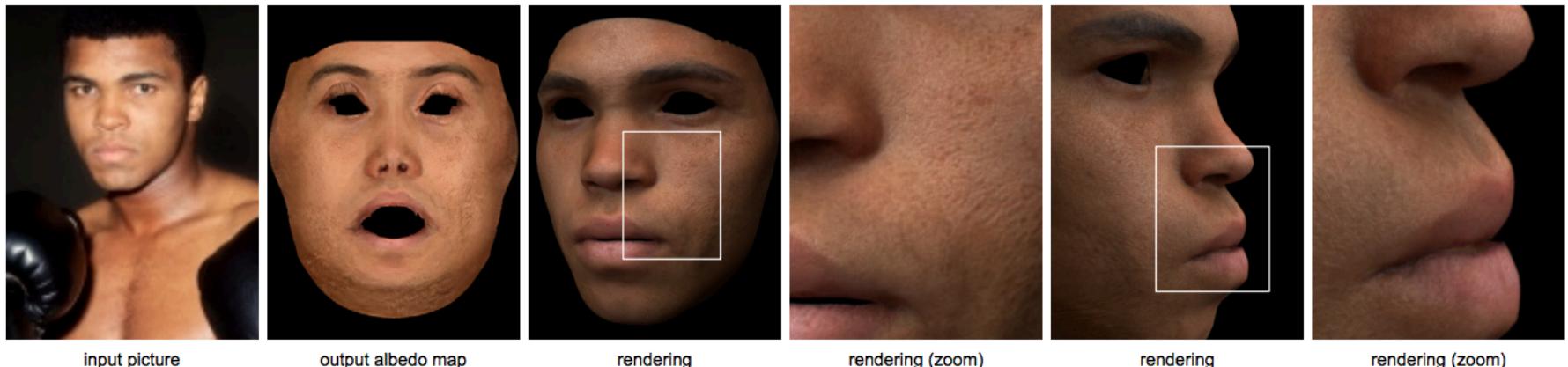
- Texture mapping



Lévy, Petitjean, Ray, and Maillot: *Least squares conformal maps for automatic texture atlas generation*, SIGGRAPH 2002

# OMG Deep Learning!

- Sample application of texture space with deep learning
- Use a deep neural network to render high quality textures using low resolution images
- Style transfer techniques



[Saito et al., CVPR 2017]

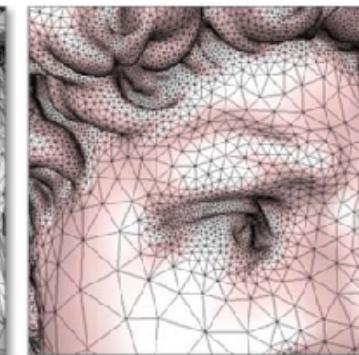
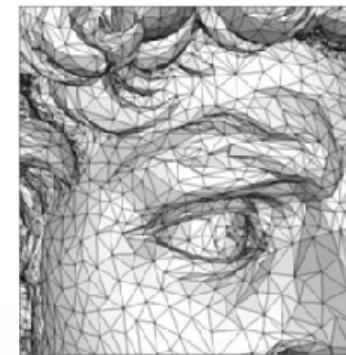
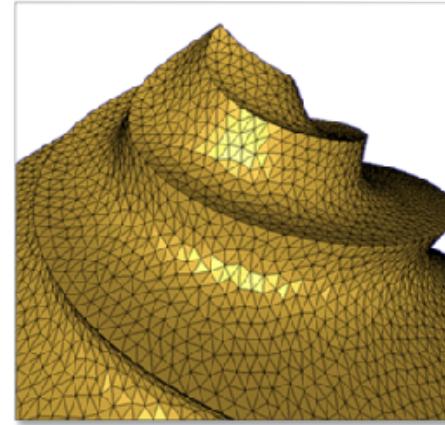
# Discrete Surfaces: Point Sets, Meshes

[USC CSCI 621]

- Flexible
- Suitable for highly detailed scanned data
- No analytic surface
- No inherent “editability”

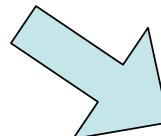
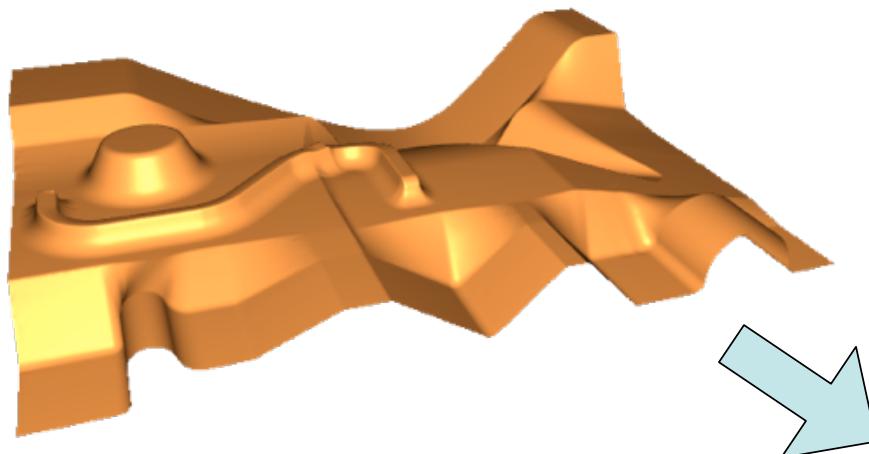


## Mesh Editing

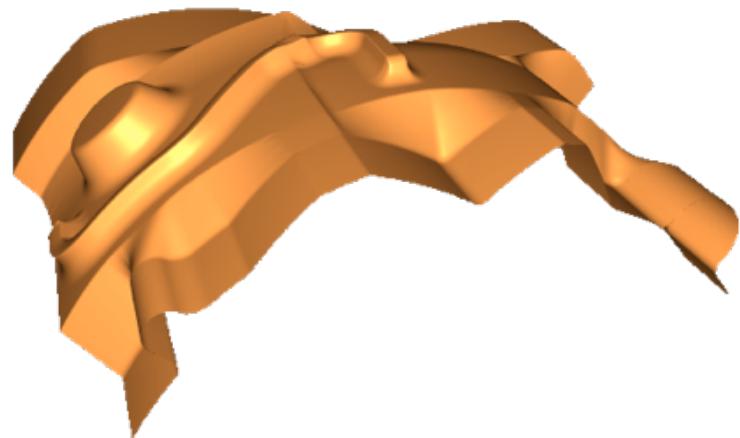


# Mesh Deformation

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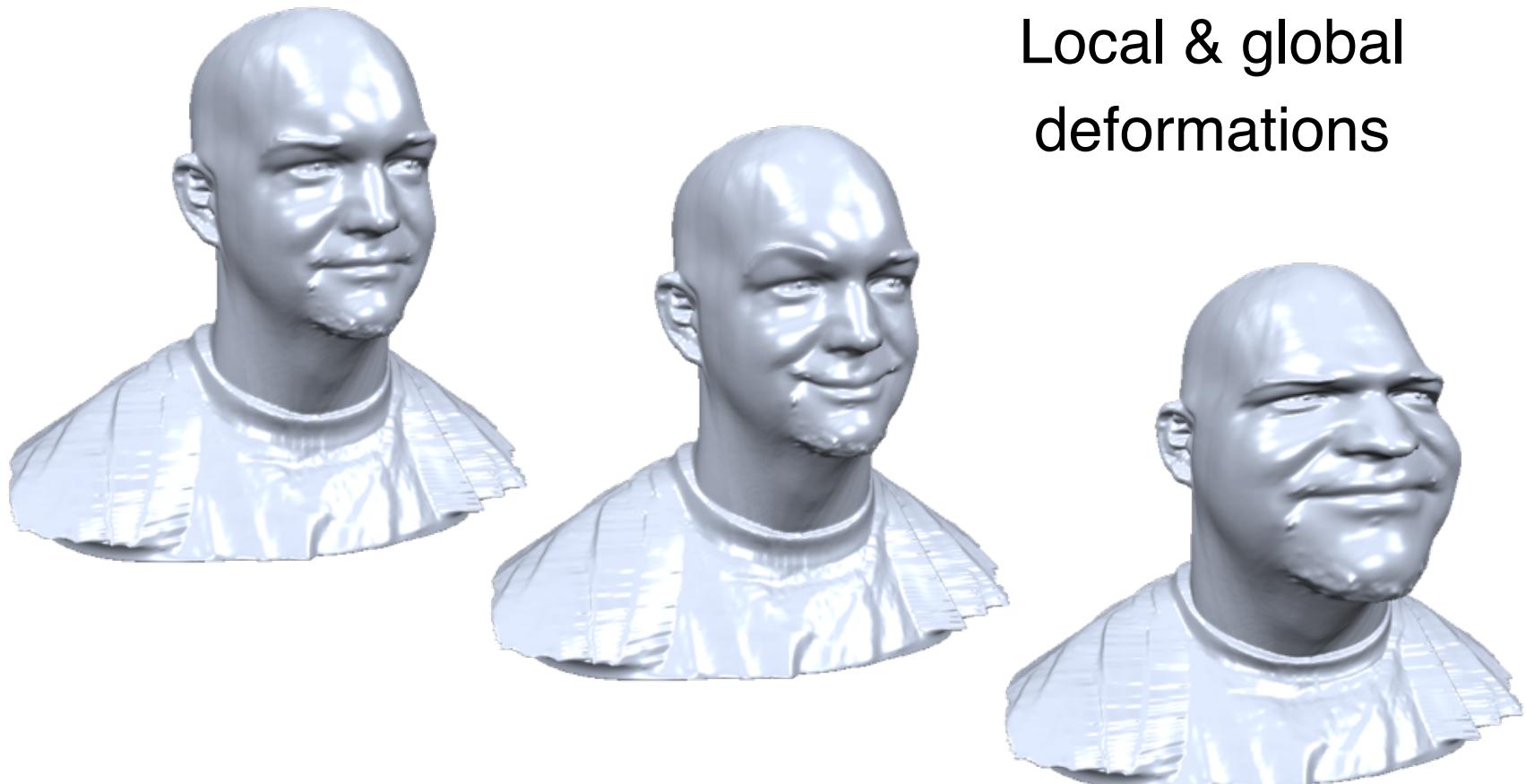


Global deformation  
with intuitive  
detail preservation



# Mesh Deformation

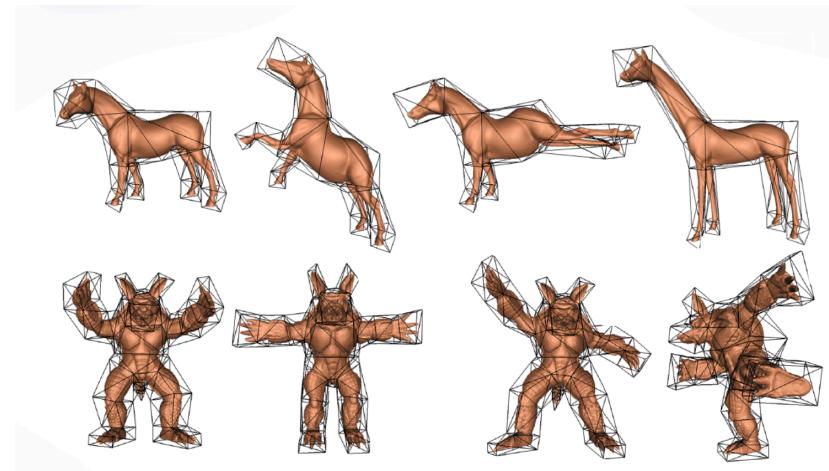
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Local & global  
deformations

# Manipulating Parametric Models

- Surface Deformations
  - Linear Method (Minimize stretching and bending)
  - Non-Linear Method (As rigid as possible)
  - Other methods exist but won't discuss
- Space Deformations
  - Won't Discuss
- Linear Parametric Models

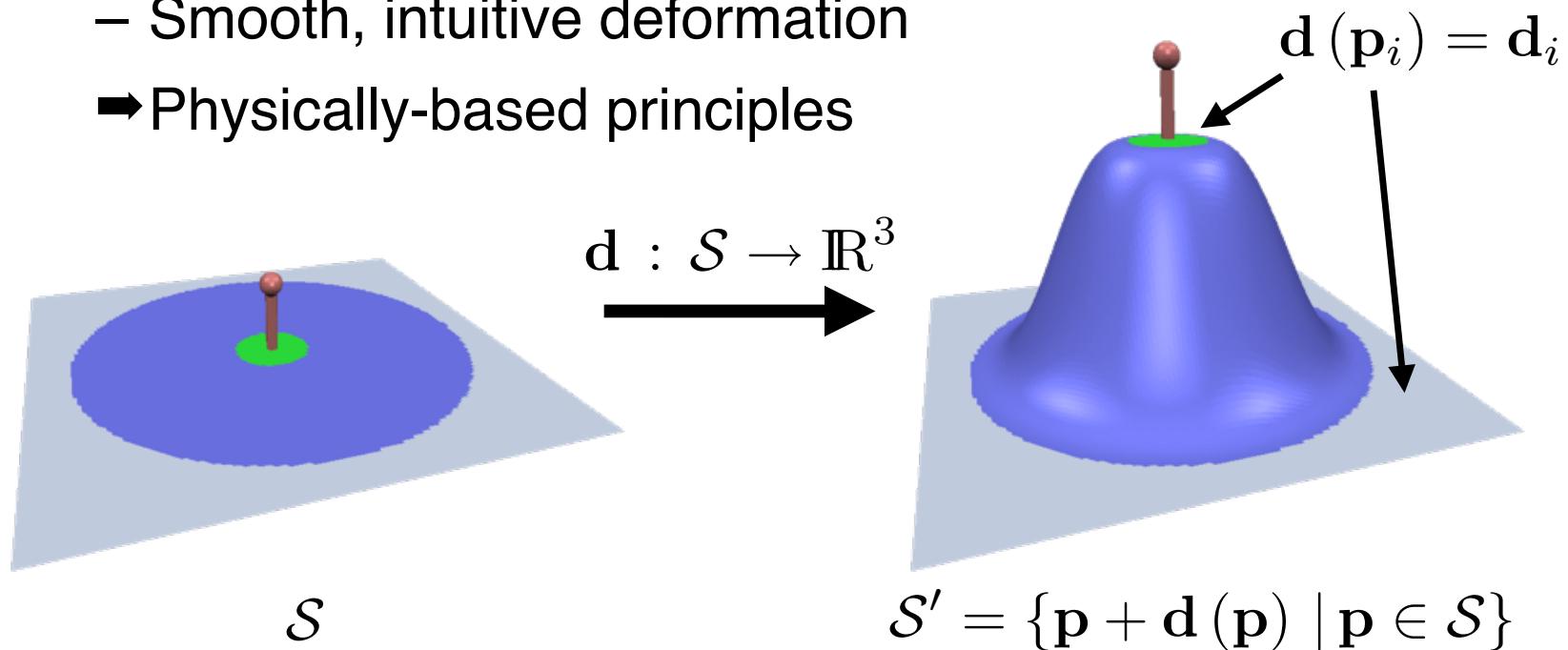


[USC CSCI 621]

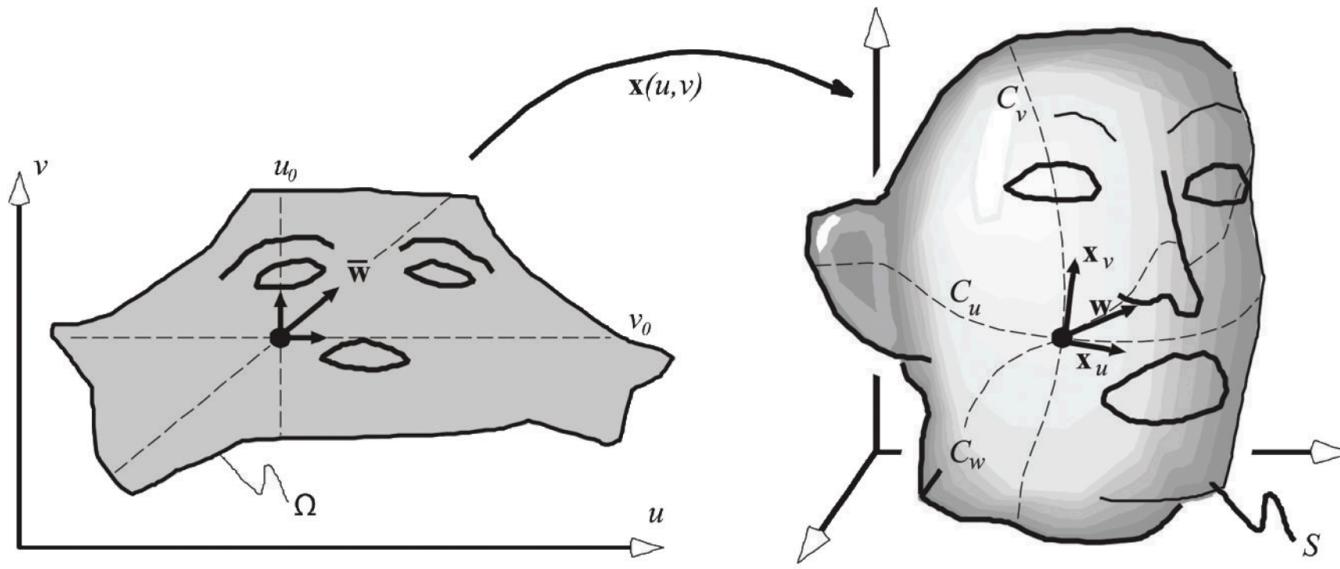
# Modeling Notation

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- Mesh deformation by displacement function  $\mathbf{d}$ 
  - Interpolate prescribed constraints
  - Smooth, intuitive deformation
- Physically-based principles

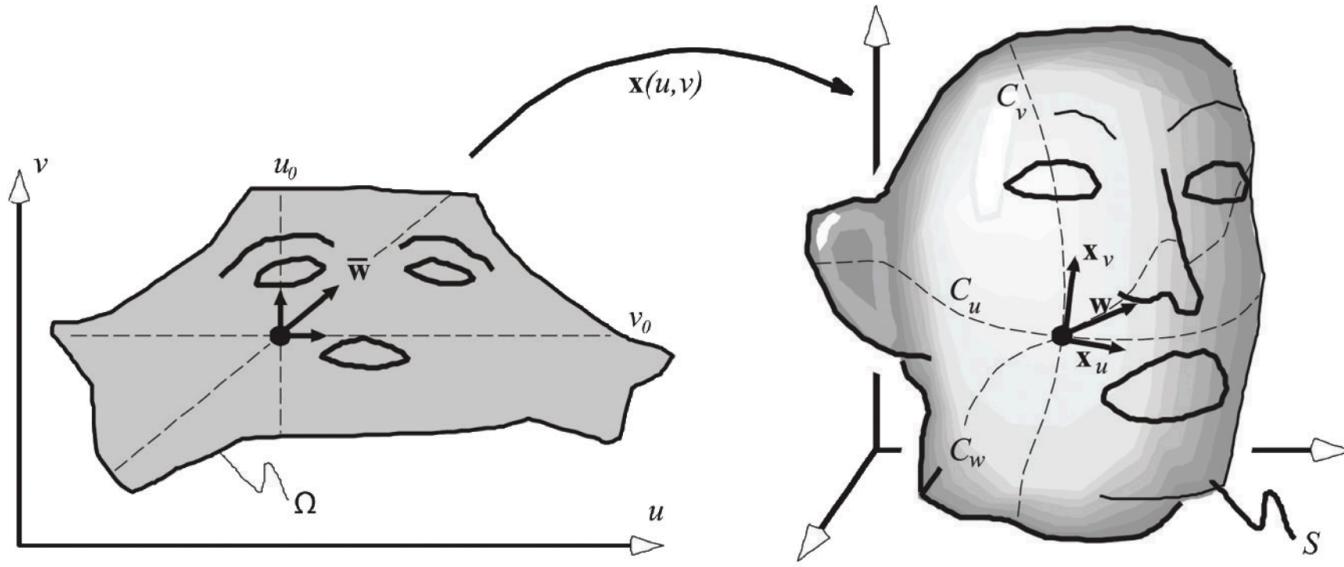


# Differential Geometry



$$\mathbf{x}(u, v) = \begin{pmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{pmatrix}, \quad (u, v) \in \Omega \subset \mathbb{R}^2,$$

# Differential Geometry

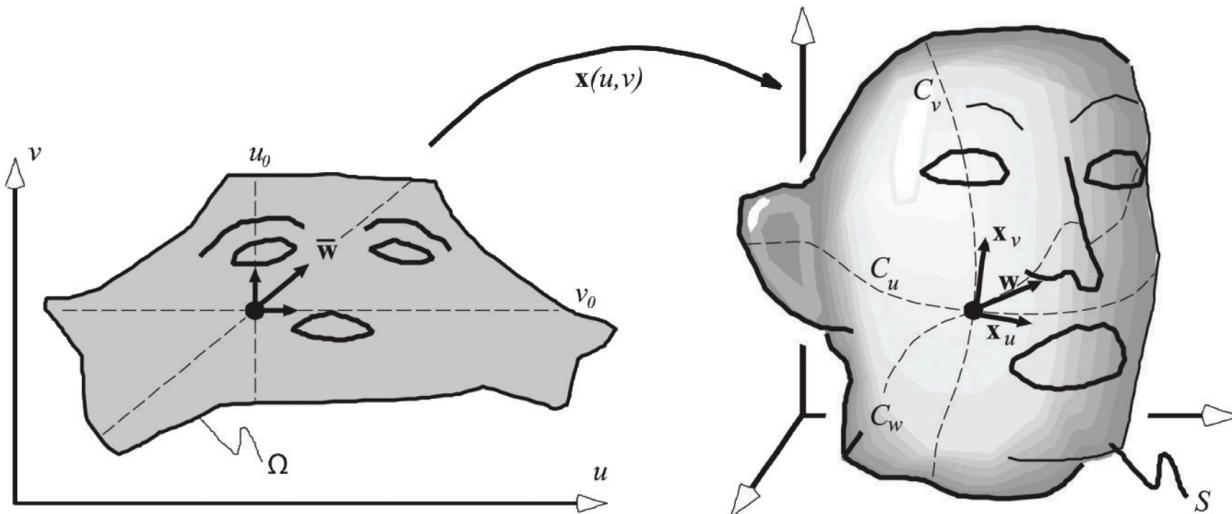


$$\mathbf{x}_u(u_0, v_0) := \frac{\partial \mathbf{x}}{\partial u}(u_0, v_0) \quad \text{and} \quad \mathbf{x}_v(u_0, v_0) := \frac{\partial \mathbf{x}}{\partial v}(u_0, v_0)$$

are, respectively, the tangent vectors of the two *iso-parameter curves*

$$\mathbf{C}_{\mathbf{u}}(t) = \mathbf{x}(u_0 + t, v_0) \quad \text{and} \quad \mathbf{C}_{\mathbf{v}}(t) = \mathbf{x}(u_0, v_0 + t)$$

# Differential Geometry



$$\bar{\mathbf{w}} = (u_w, v_w)^T$$

$$\mathbf{C}_w(t) = \mathbf{x}(u_0 + tu_w, v_0 + tv_w)$$

$$\mathbf{w} = \partial \mathbf{C}_w(t) / \partial t$$

$$\mathbf{w} = \mathbf{J} \bar{\mathbf{w}} \quad \mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{bmatrix} = [\mathbf{x}_u, \mathbf{x}_v]$$

# First Fundamental Form

- The first fundamental form acts as a dot product into parameter space
- Gives us a notion of length and angle

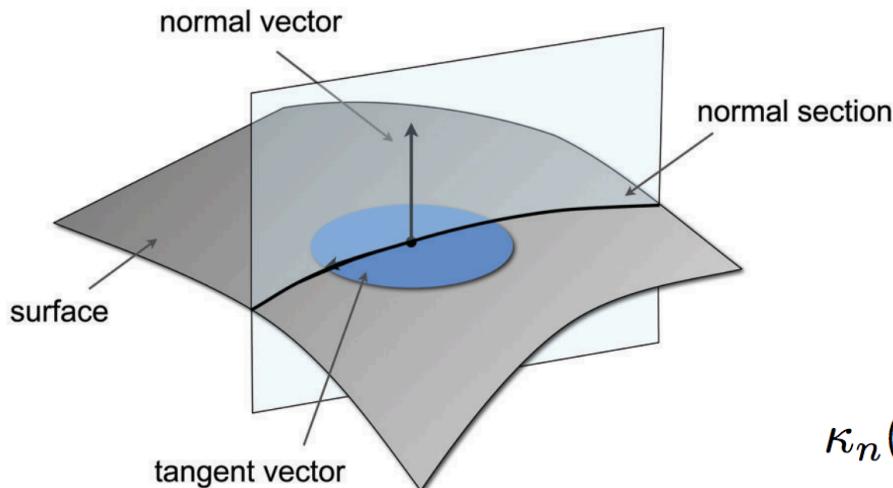
$$\mathbf{w}_1^T \mathbf{w}_2 = (\mathbf{J} \bar{\mathbf{w}}_1)^T (\mathbf{J} \bar{\mathbf{w}}_2) = \bar{\mathbf{w}}_1^T (\mathbf{J}^T \mathbf{J}) \bar{\mathbf{w}}_2$$

$$\mathbf{I} = \mathbf{J}^T \mathbf{J} = \begin{bmatrix} E & F \\ F & G \end{bmatrix} := \begin{bmatrix} \mathbf{x}_u^T \mathbf{x}_u & \mathbf{x}_u^T \mathbf{x}_v \\ \mathbf{x}_u^T \mathbf{x}_v & \mathbf{x}_v^T \mathbf{x}_v \end{bmatrix}$$

$$\begin{aligned} l(a, b) &= \int_a^b \sqrt{(u_t, v_t) \mathbf{I}(u_t, v_t)^T dt} \\ &= \int_a^b \sqrt{E u_t^2 + 2F u_t v_t + G v_t^2} dt. \end{aligned}$$

# Second Fundamental Form

- Second Fundamental Form gives a notion of curvature



$$\mathbf{n} = \frac{\mathbf{x}_u \times \mathbf{x}_v}{\|\mathbf{x}_u \times \mathbf{x}_v\|}$$

$$\kappa_n(\bar{\mathbf{t}}) = \kappa_1 \cos^2 \psi + \kappa_2 \sin^2 \psi$$

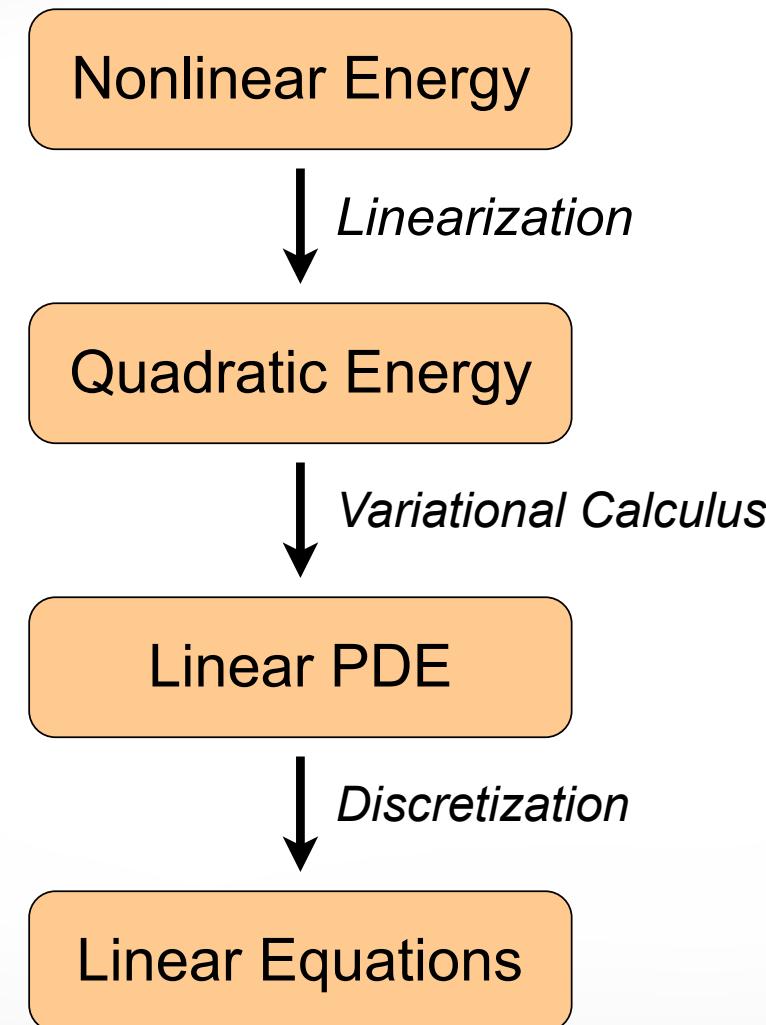
$$\kappa_n(\bar{\mathbf{t}}) = \frac{\bar{\mathbf{t}}^T \mathbf{II} \bar{\mathbf{t}}}{\bar{\mathbf{t}}^T \mathbf{I} \bar{\mathbf{t}}} = \frac{eu_t^2 + 2fu_tv_t + gv_t^2}{Eu_t^2 + 2Fu_tv_t + Gv_t^2}$$

where  $\mathbf{II}$  denotes the *second fundamental form* defined as

$$\mathbf{II} = \begin{bmatrix} e & f \\ f & g \end{bmatrix} := \begin{bmatrix} \mathbf{x}_{uu}^T \mathbf{n} & \mathbf{x}_{uv}^T \mathbf{n} \\ \mathbf{x}_{uv}^T \mathbf{n} & \mathbf{x}_{vv}^T \mathbf{n} \end{bmatrix}.$$

# Derivation Steps

[USC CSCI 621]



# Physically-Based Deformation

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- Non-linear stretching & bending energies

$$\int_{\Omega} k_s \boxed{\|\mathbf{I} - \mathbf{I}'\|^2} + k_b \boxed{\|\mathbf{II} - \mathbf{II}'\|^2} \, dudv$$

stretching                          bending

- Linearize energies

$$\int_{\Omega} k_s \left( \|\mathbf{d}_u\|^2 + \|\mathbf{d}_v\|^2 \right) + k_b \left( \|\mathbf{d}_{uu}\|^2 + 2 \|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \right) \, dudv$$

stretching                          bending

# Physically-Based Deformation

---

- Minimize linearized bending energy

$$E(\mathbf{d}) = \int_{\mathcal{S}} \|\mathbf{d}_{uu}\|^2 + 2\|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 d\mathcal{S} \quad f(x) \rightarrow \min$$

- Variational calculus, Euler-Lagrange PDE

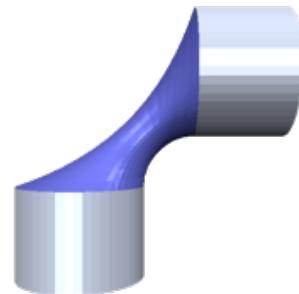
$$\Delta^2 \mathbf{d} := \mathbf{d}_{uuuu} + 2\mathbf{d}_{uuvv} + \mathbf{d}_{vvvv} = 0 \quad f'(x) = 0$$

→ “Best” deformation that satisfies constraints

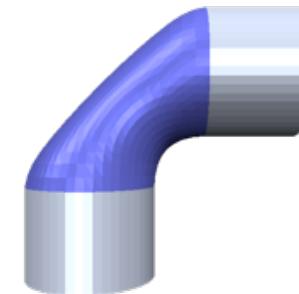
# Deformation Energies

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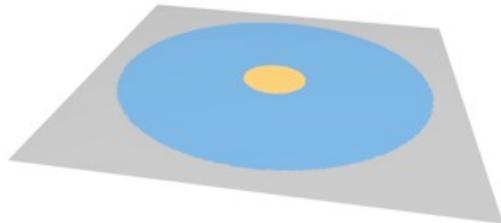
$$-k_s \Delta \mathbf{d} + k_b \Delta^2 \mathbf{d} = 0$$



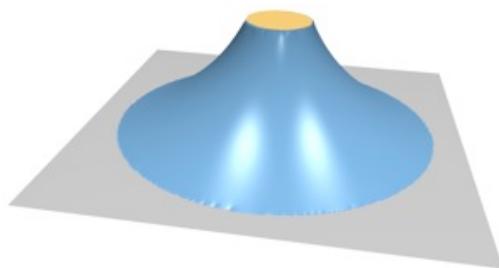
$$\Delta \mathbf{p} = 0$$



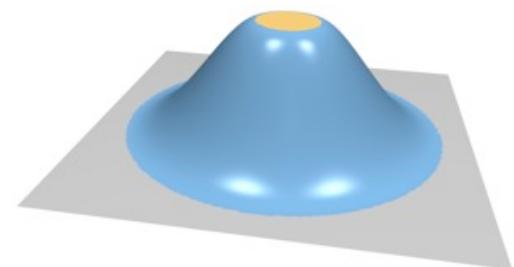
$$\Delta^2 \mathbf{p} = 0$$



Initial state



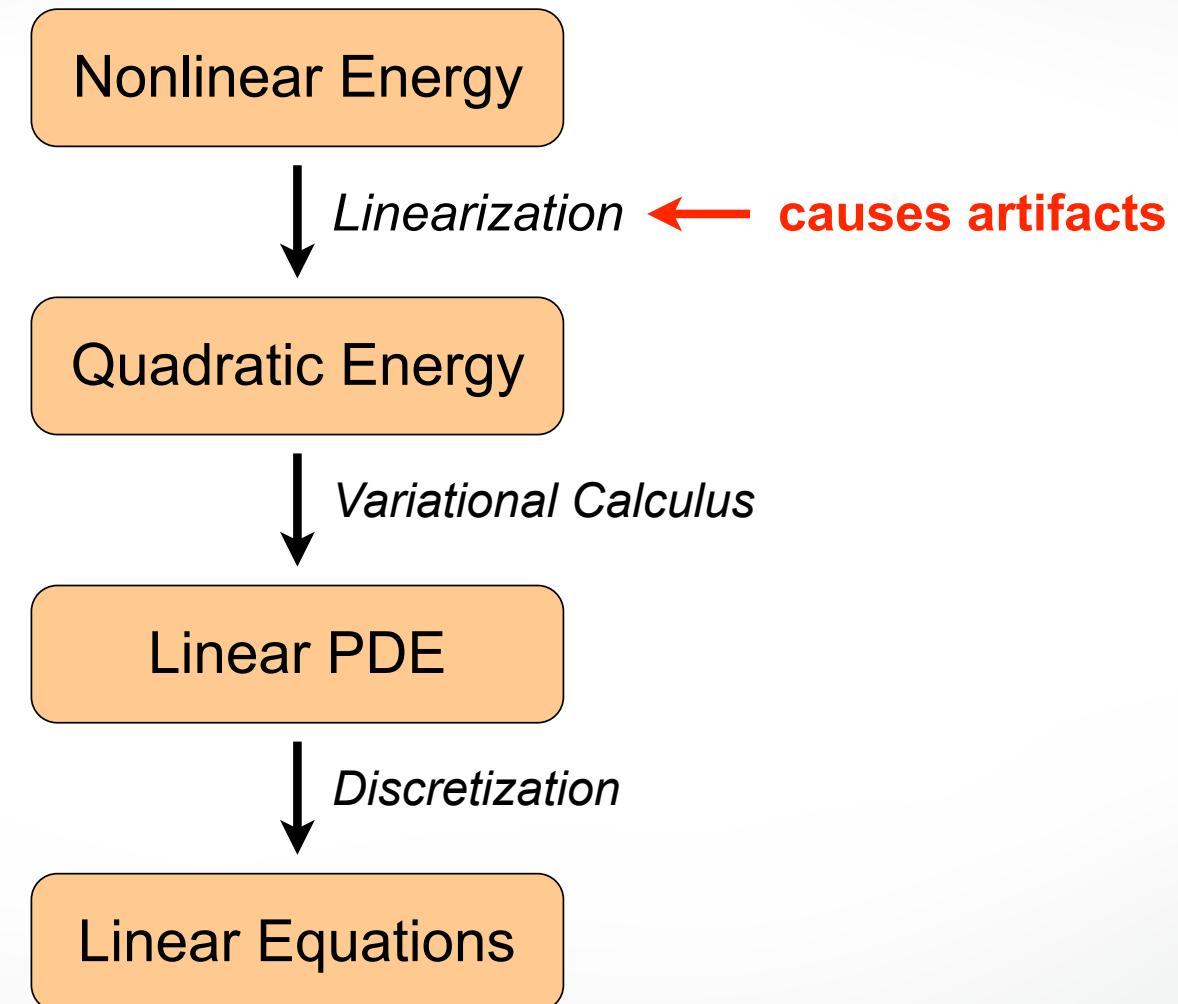
$\Delta \mathbf{d} = 0$   
(Membrane)



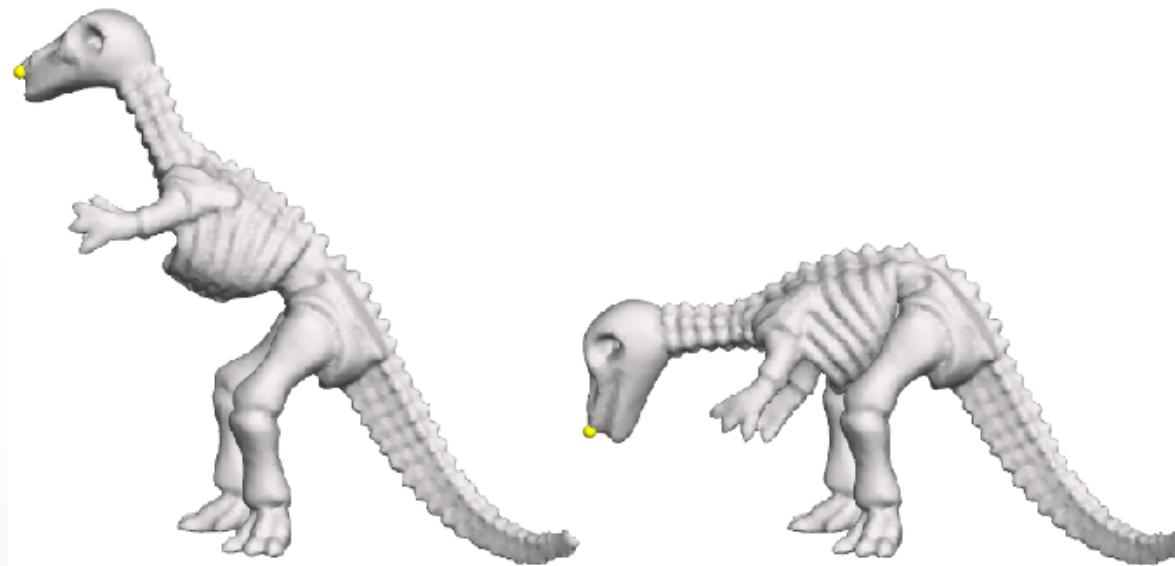
$\Delta^2 \mathbf{d} = 0$   
(Thin plate)

# Linear Approaches

[USC CSCI 621]

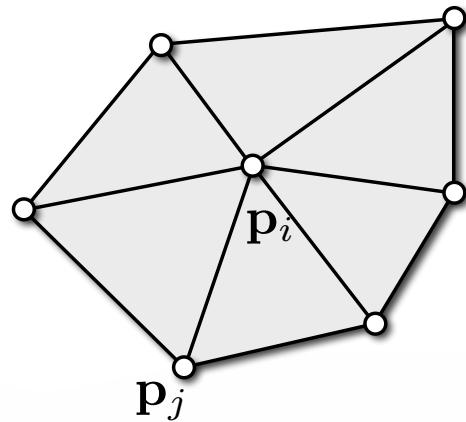


- Smooth large scale deformation
- Local as-rigid-as-possible behavior
  - Preserves small-scale details



- Vertex neighborhoods should deform rigidly

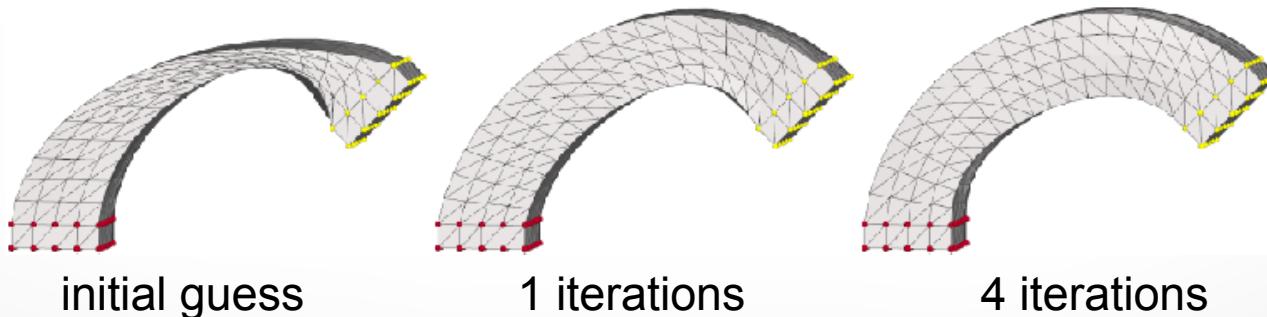
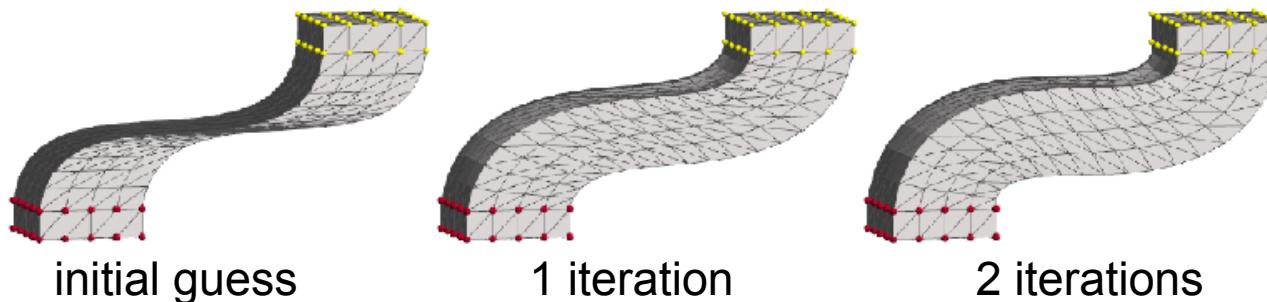
$$\sum_{j \in N(i)} \|(\mathbf{p}'_j - \mathbf{p}'_i) - \mathbf{R}_i (\mathbf{p}_j - \mathbf{p}_i)\|^2 \rightarrow \min$$



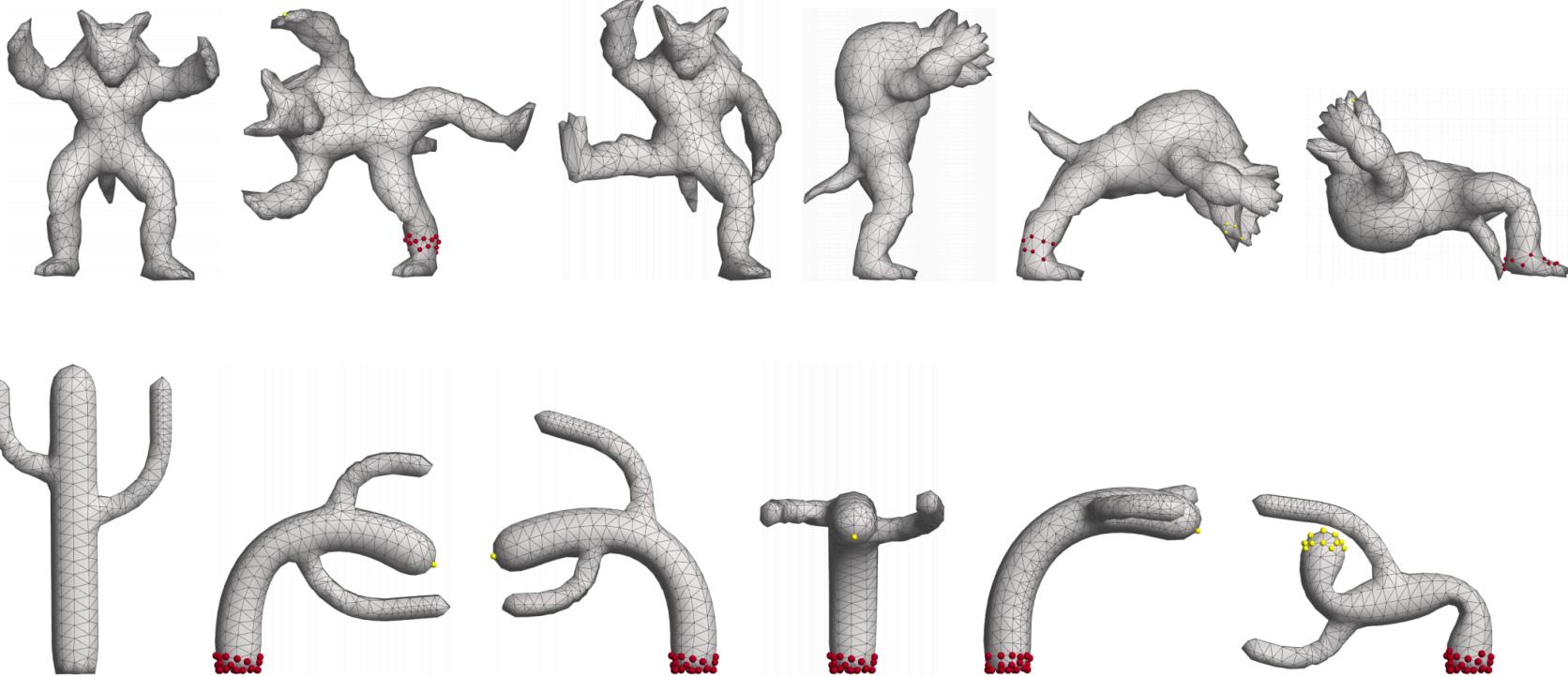
# As-Rigid-As-Possible Modeling

[USC CSCI 621]

- Start from naïve Laplacian editing as initial guess



# As Rigid As Possible Results



[As Rigid As Possible Surface Modeling, SGP 2007]

# In Summary

- Exploit Geometry
- Make use of physical structure
- Minimize Stretching and Bending
- Maximize Rigidity

# Low-Dimensional Parametric Models

- What if we already know what object we are dealing with?
- For 3D Reconstruction, remove the difficulty in guessing the correct shape from a 2D image
- We can introduce a template shape and a parameterization that deforms the template appropriately
  - Create a shape space
- Instead estimating the whole model we estimate the low-dimensional parameterization
- We can parameterize deformations into fast linear operations on a low-dimensional space

# Low-Dimensional Parametric Models

- Pros:
  - Leverage our prior knowledge of the task/our world
  - Low-Dimensional
  - Accurate
  - Won't fall off the shape manifold. Always generate plausible shapes
  - Easy to deform

# Some Parametric Models In Industry

- Some common parametric models include:
  - **Faces**
  - Bodies
  - Hands



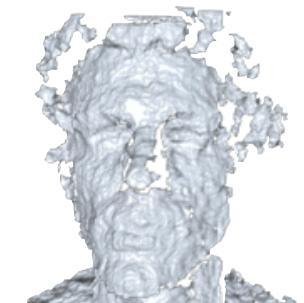
# Performance to Facial Animation [USC CSCI 621]



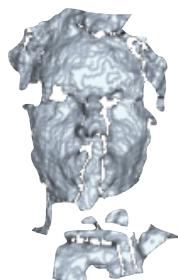


# Requirements for a Practical System

[USC CSCI 621]



1. Real-time performance

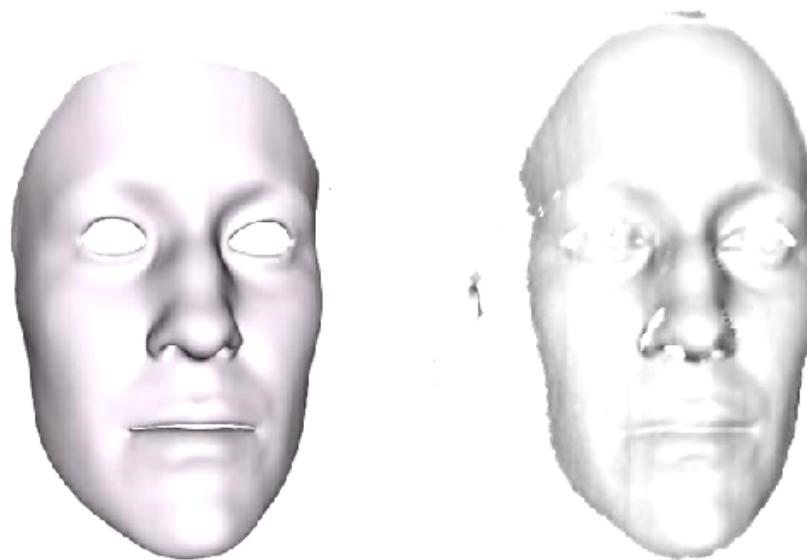


2. Robustness to noise

3. High-level semantics

# Building Expression Space

[USC CSCI 621]

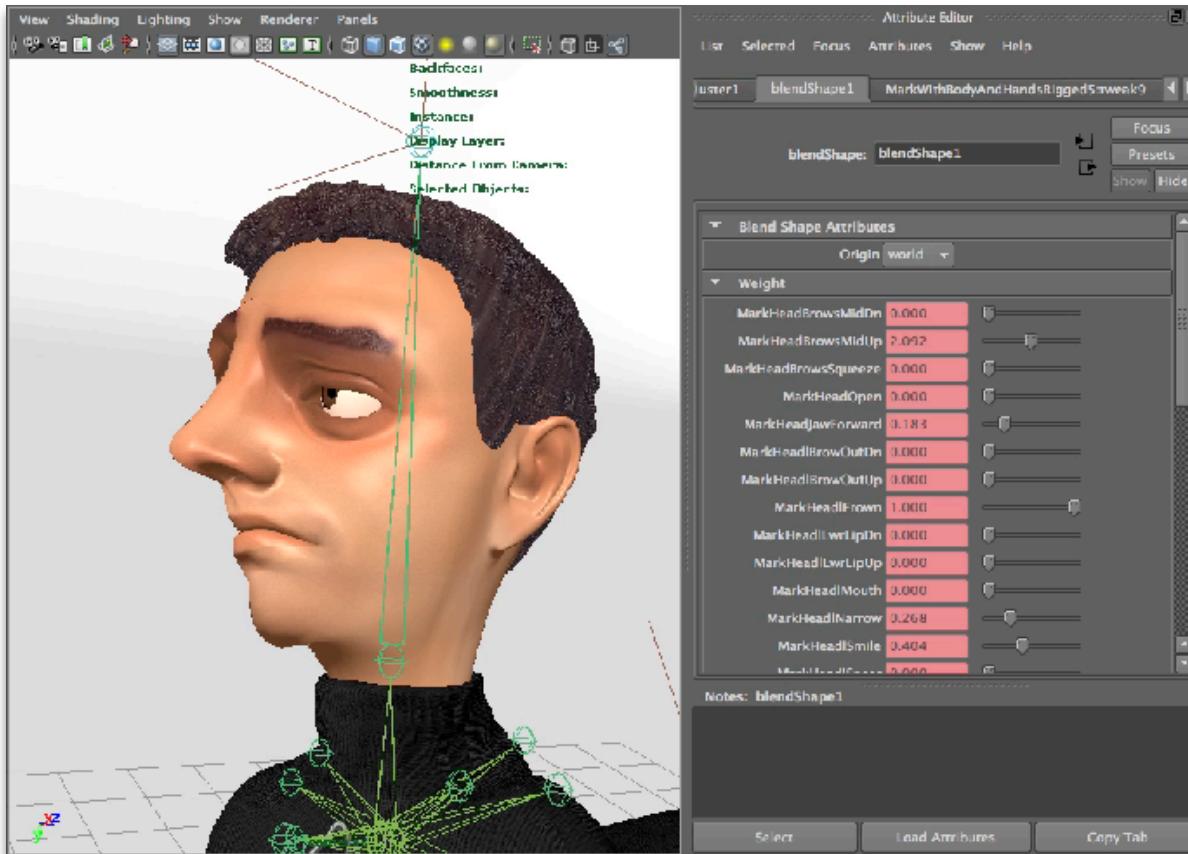


tracked template

input scan

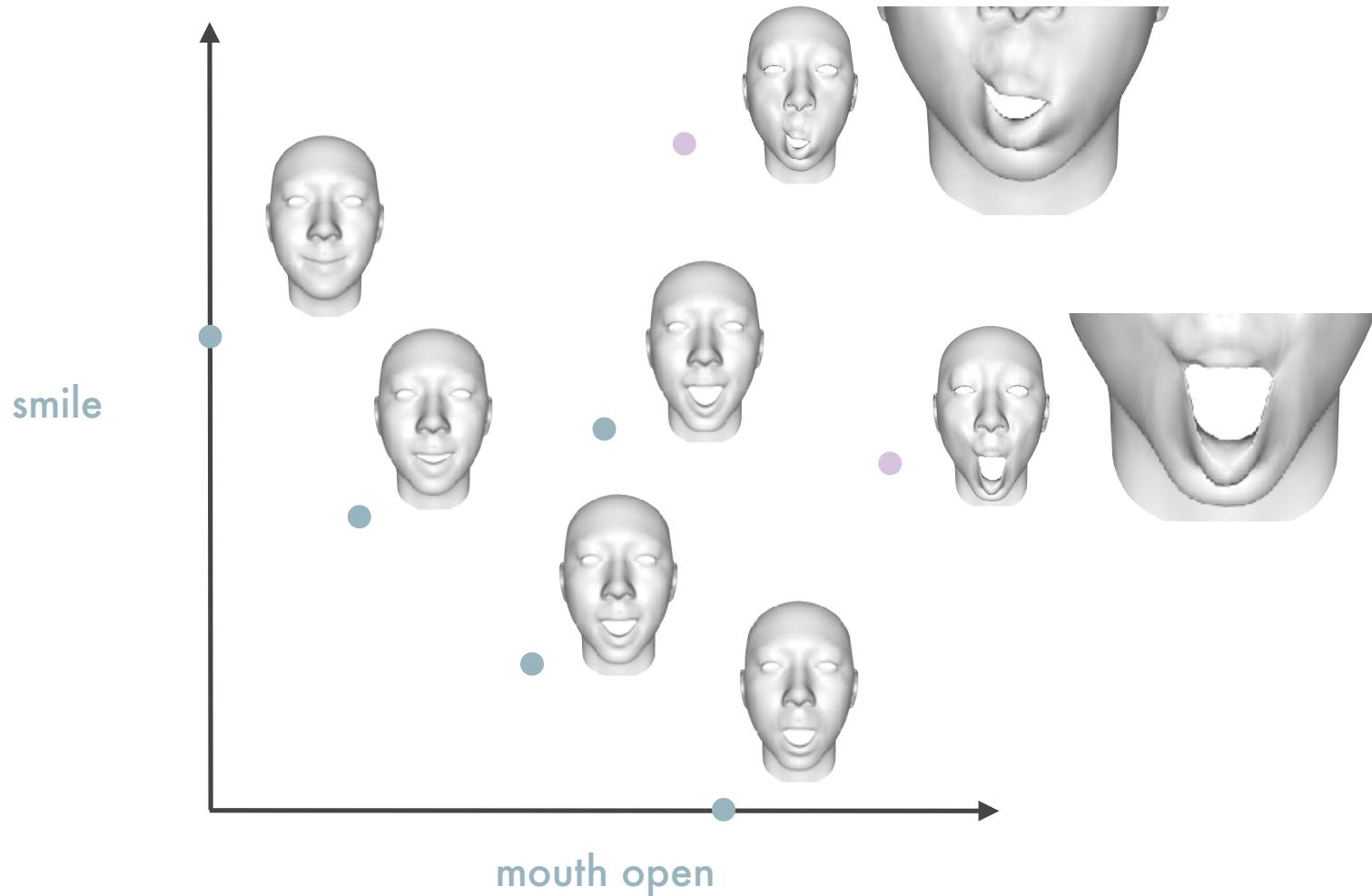
# Rigging & Animation

[USC CSCI 621]



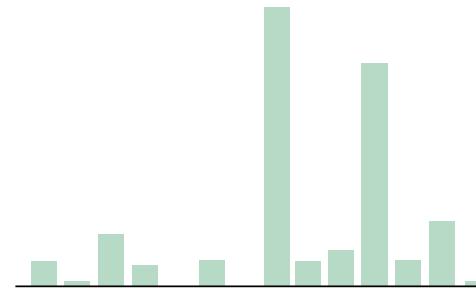
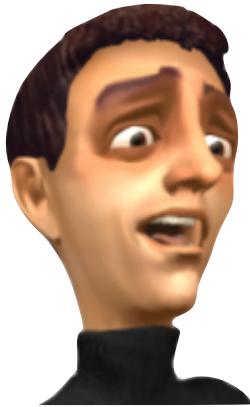
# N-Dim Expression Space

[USC CSCI 621]



# Blendshape Retargeting

[USC CSCI 621]



laughing



...

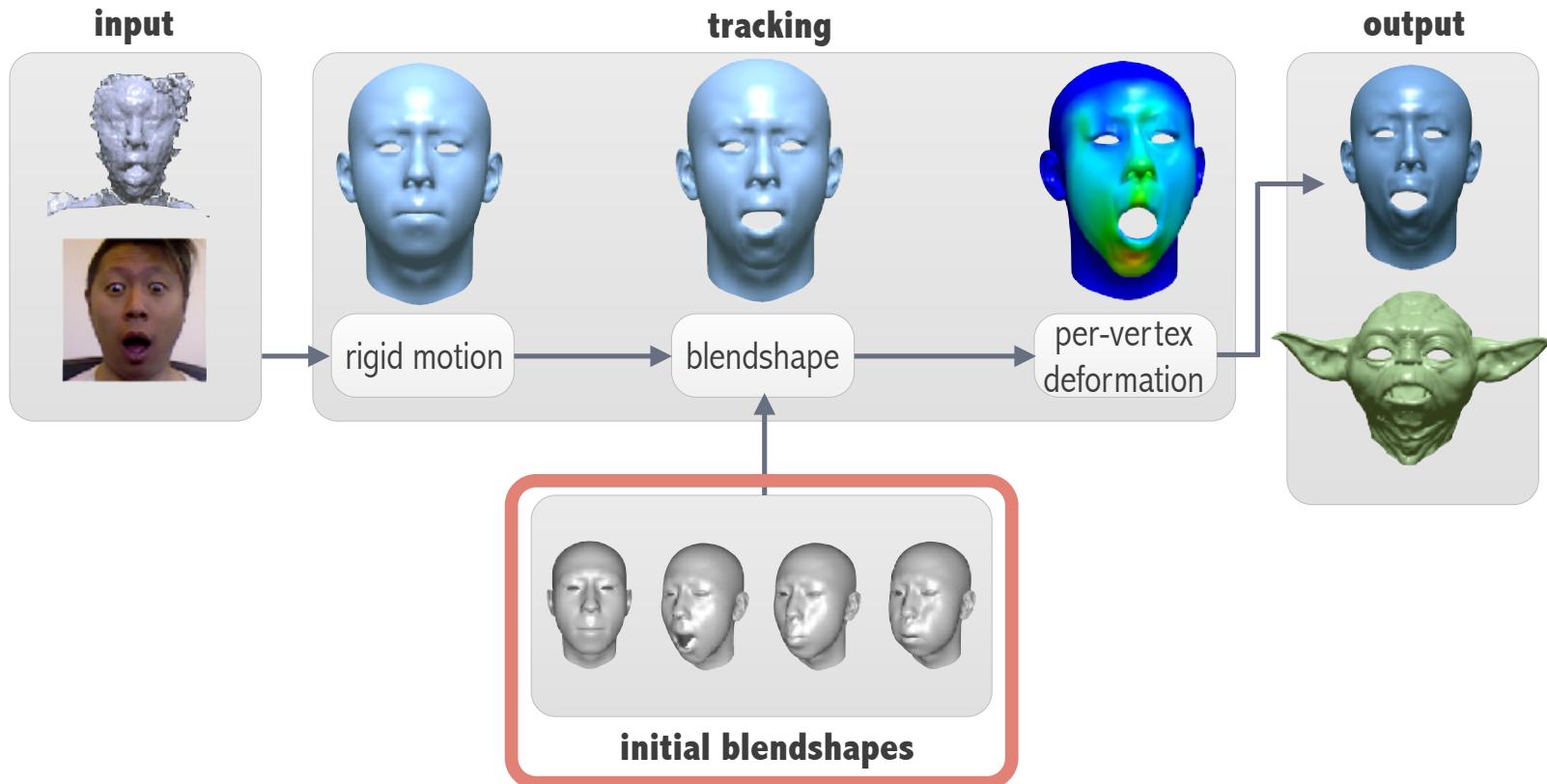
many blendshapes

SIGGRAPH 2010

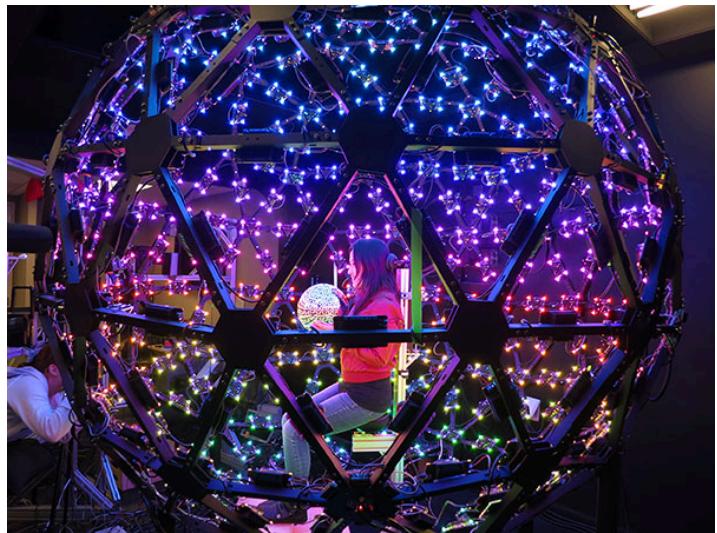
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# Pipeline Overview

[USC CSCI 621]



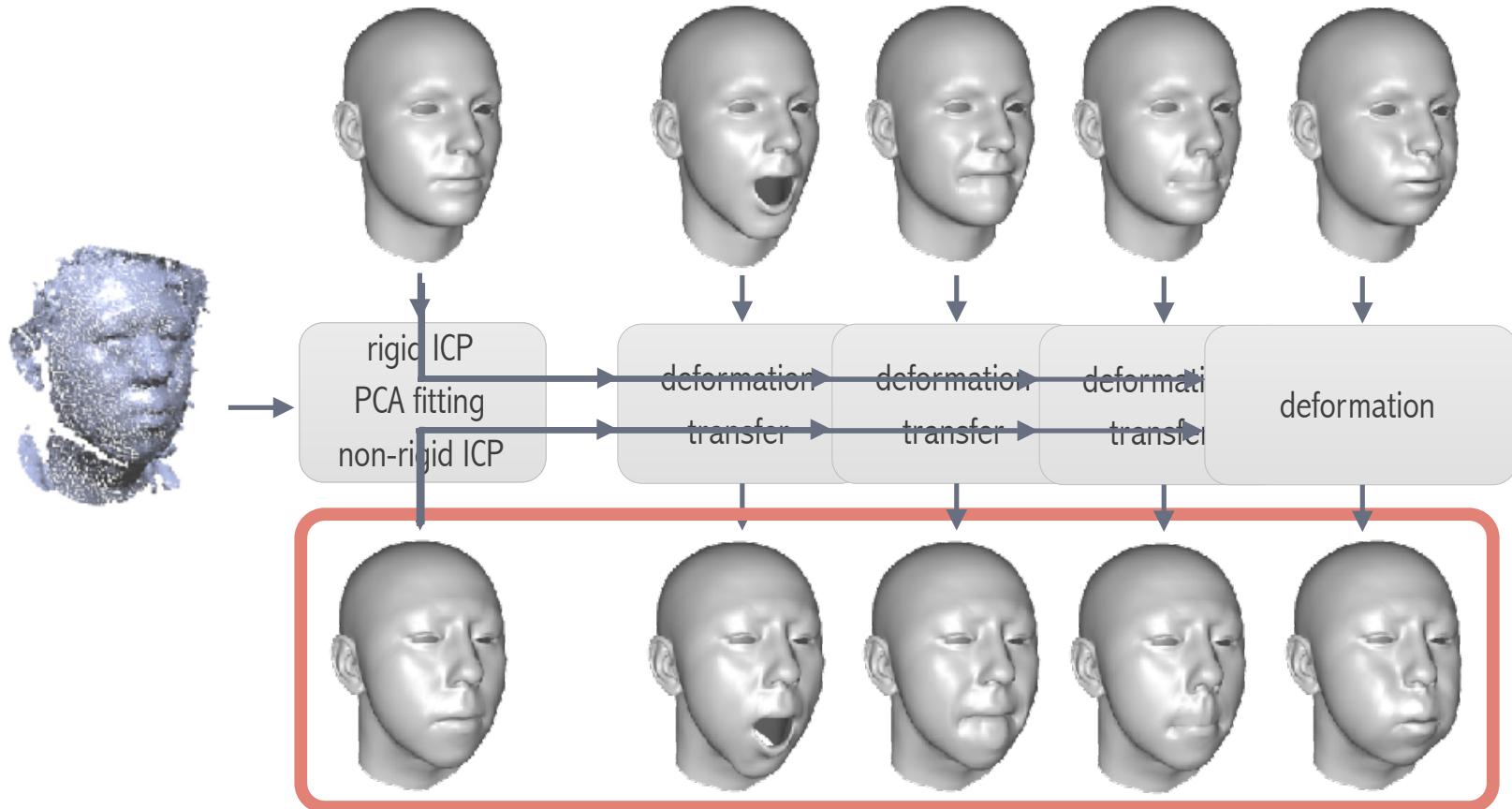
# Hollywood Face Models



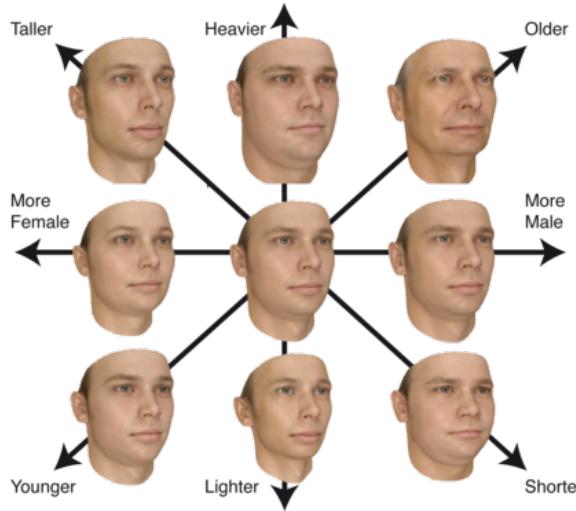
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# Building Initial Blendshape Model

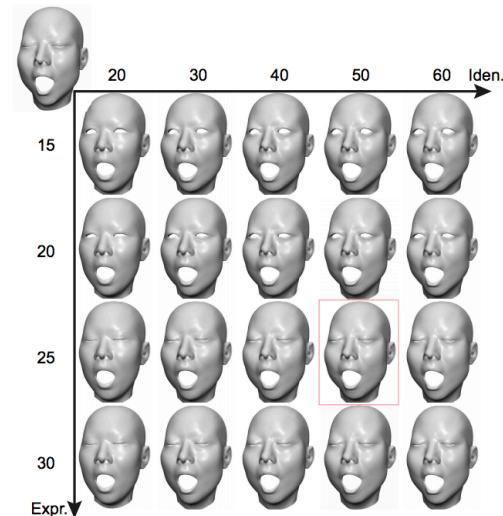
[USC CSCI 621]



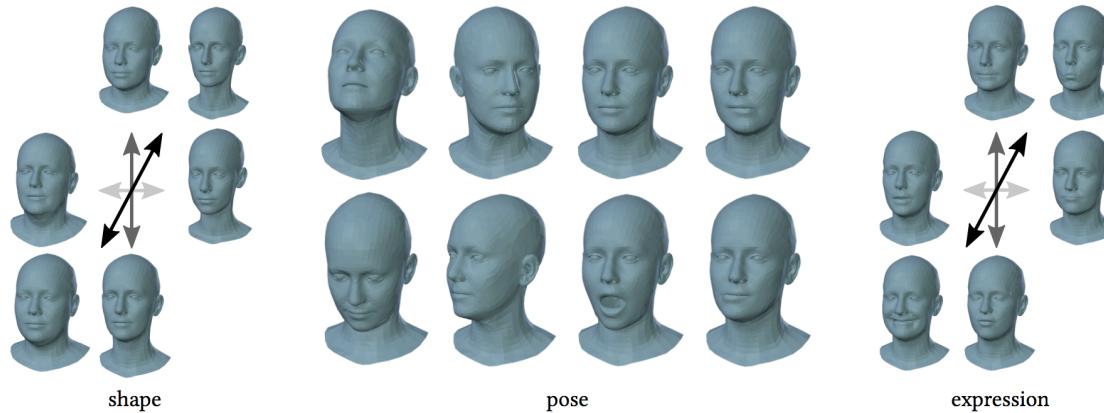
# Public Face Models



[Basel Face Model, 1997, 2009, 2017]



[FaceWarehouse, TOG 2014]



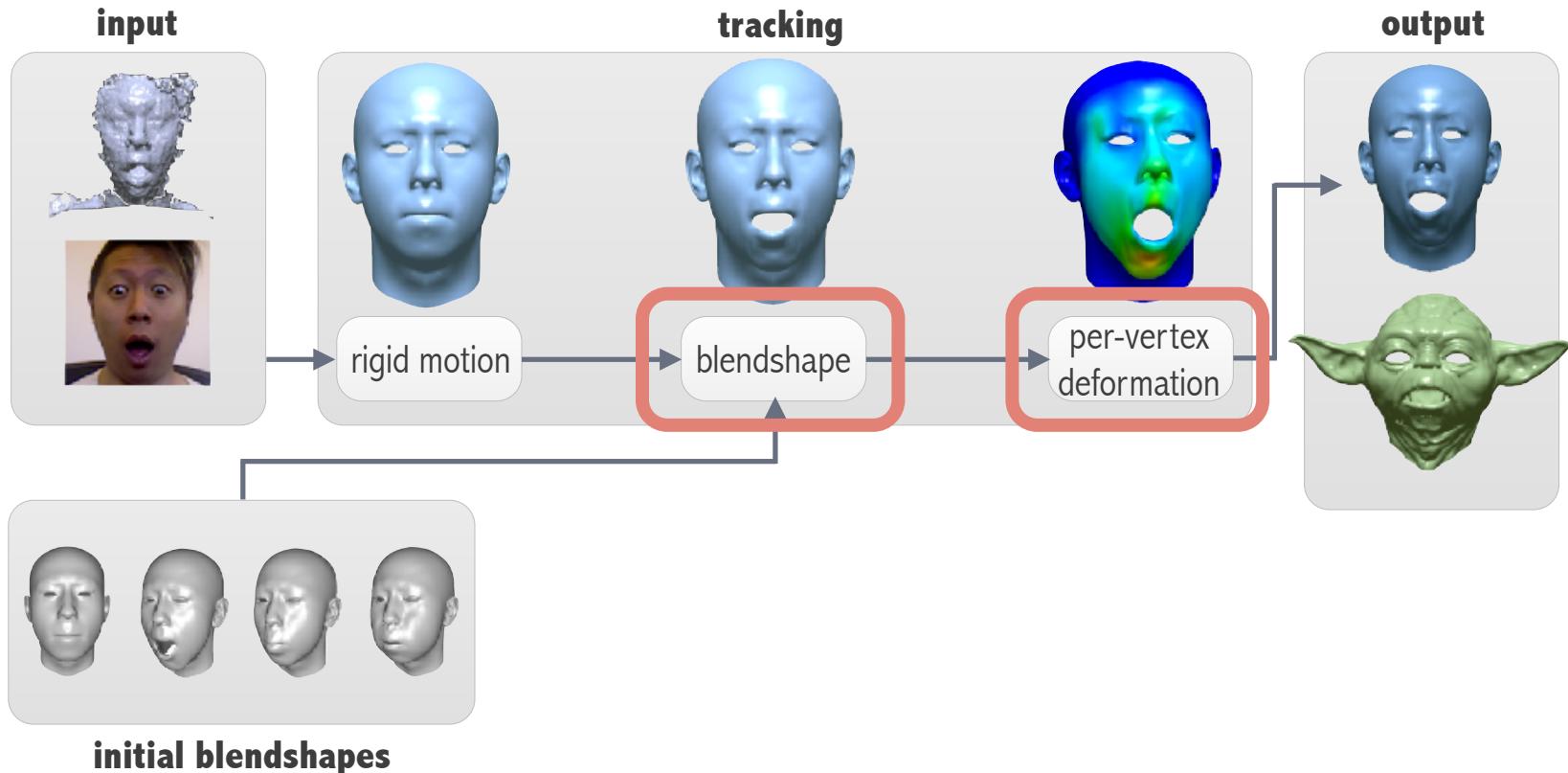
[FLAME, SIGGRAPH Asia 2017]

# Building Initial Blendshape Models

- Exploit commonalities among face data
- Create a low-dimensional shape space that you can interpolate inside of
- We have used data to largely automate the process
- Difficult future work:
  - Are there even more basic commonalities among nature that we can exploit?
  - Can we learn to exploit these basic commonalities automatically?

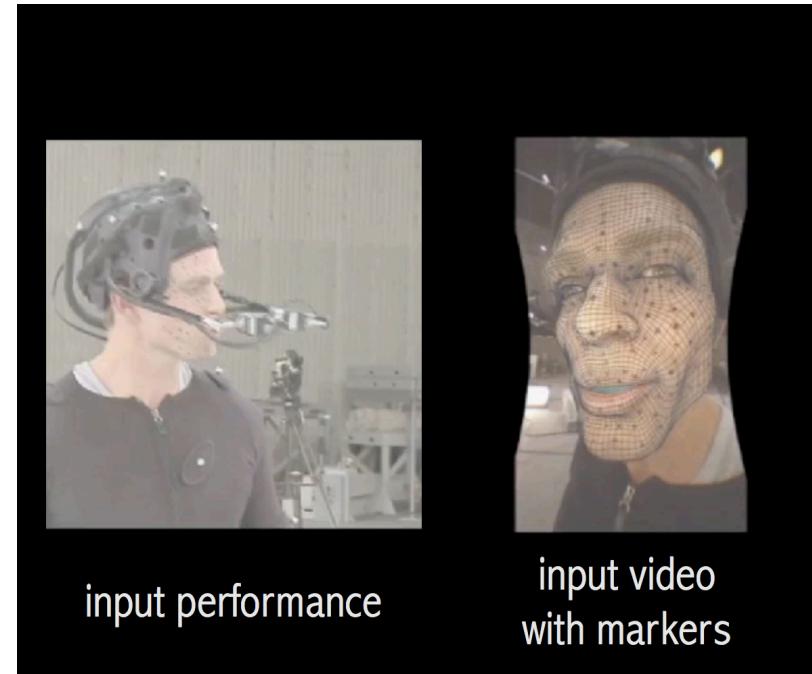
# Pipeline Overview

[USC CSCI 621]



# Solving for Pose and Blend-shapes

- First extract facial landmarks



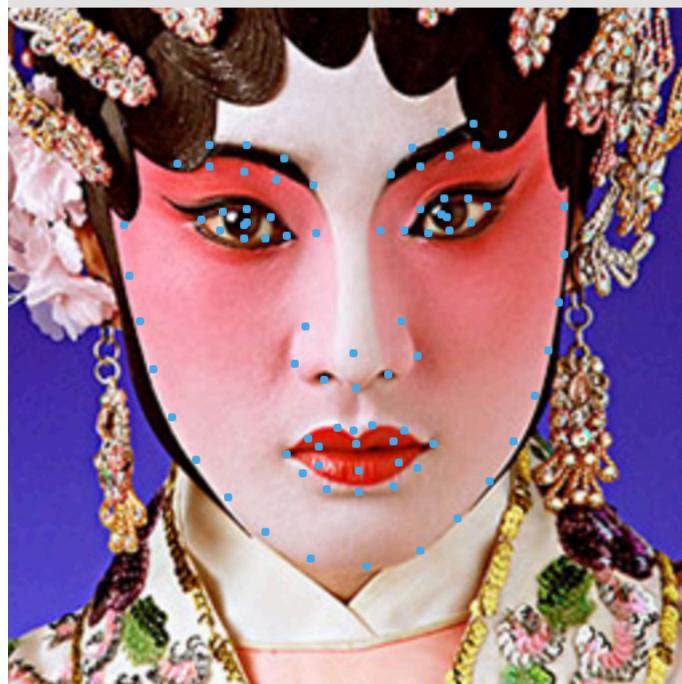
# Solving for Pose and Blend-shapes

- First extract facial landmarks
- Markerless capture
- Democratizing human body digitization!



[Face++]

# Solving for Pose and Blend-shapes



[Face<sup>++</sup>]

Given a set of  $(p^i, q^i)$  correspondences where  $p^i$  denotes a pixel in the input image and  $q^i$  denotes its corresponding vertex in the 3DMM, we minimize the following energy function defined over the parameters  $\mathcal{X} = (f, \mathbf{R}, \mathbf{t}, \alpha_{id}, \alpha_{exp})$ :

$$E(\mathcal{X}) = E_{data}(\mathcal{X}) + E_{reg}(\mathcal{X}), \quad (3)$$

$$E_{data}(\mathcal{X}) = \sum_i w_i \|p^i - \Pi_f(\mathbf{R}S_{q^i} + \mathbf{t})\|^2,$$

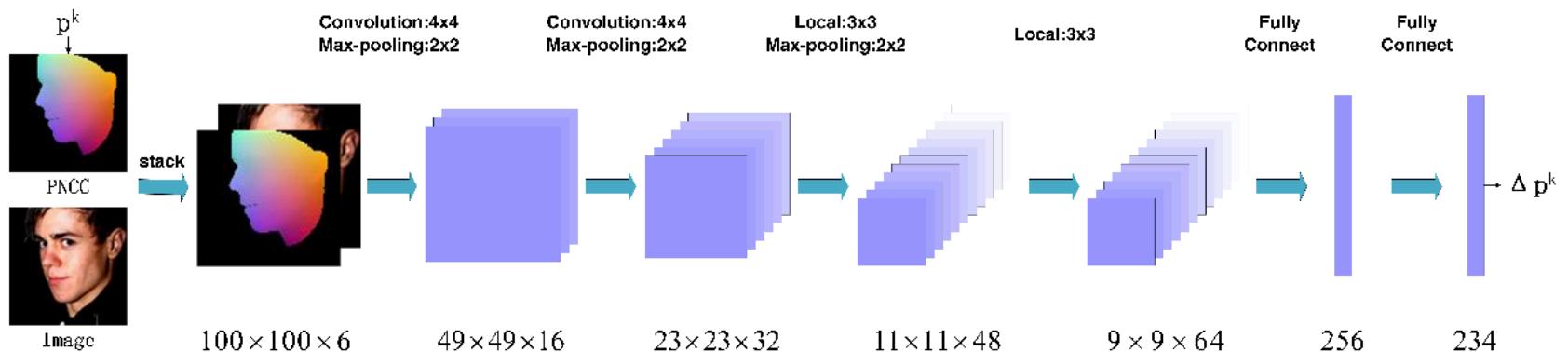
$$E_{reg}(\mathcal{X}) = w_{id} \sum_{id,i} \left( \frac{\alpha_{id,i}}{\sigma_{id,i}} \right)^2 + w_{exp} \sum_{exp,i} \left( \frac{\alpha_{exp,i}}{\sigma_{exp,i}} \right)^2,$$

$$S_{q^i} = (\bar{S} + A_{id}\alpha_{id} + A_{exp}\alpha_{exp})_{q^i}$$

[Yu et al., ICCV 2017]

# Solving for Pose and Blend-shapes...

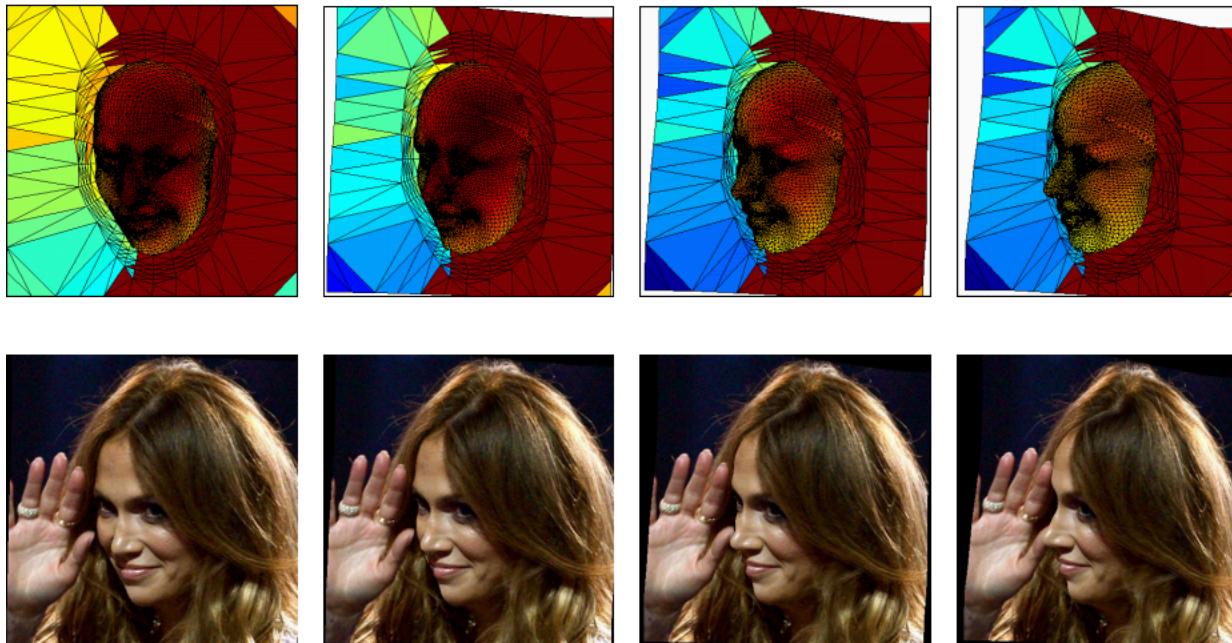
- ...with deep learning!
- 3DFFA directly learns pose and shape parameters



[3DDFA, CVPR 2016]

# Solving for Pose and Blend-shapes...

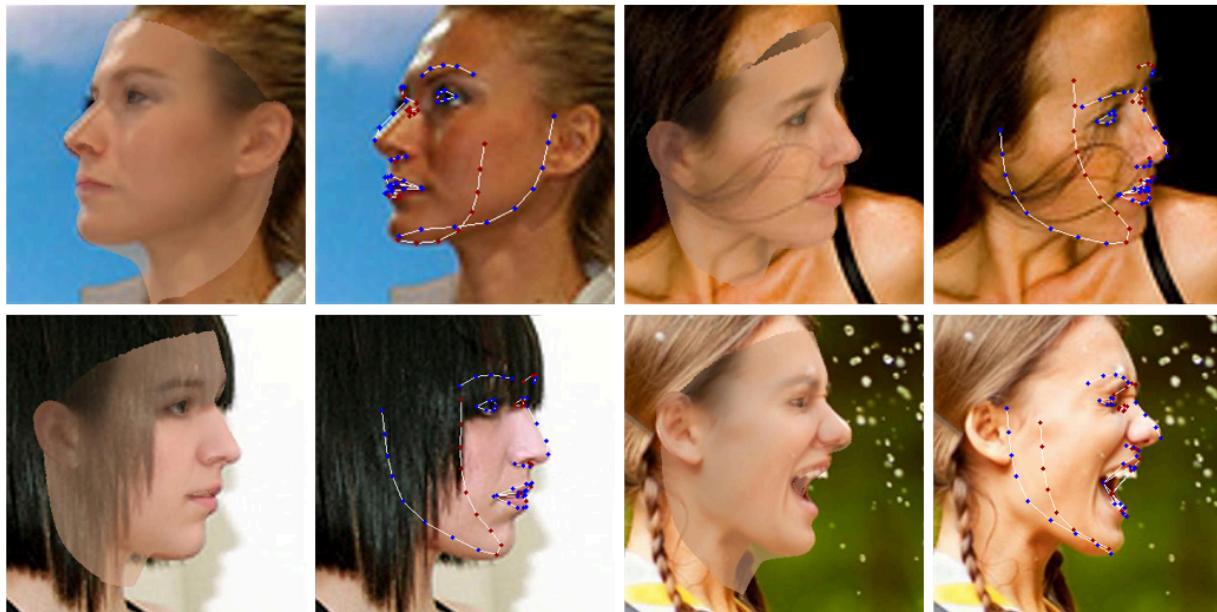
- ...with deep learning!
- 3DFFA directly learns pose and shape parameters
- Allows face-tracking for side-faces and difficult occlusions because deep learning



[3DDFA, CVPR 2016]

# Solving for Pose and Blend-shapes...

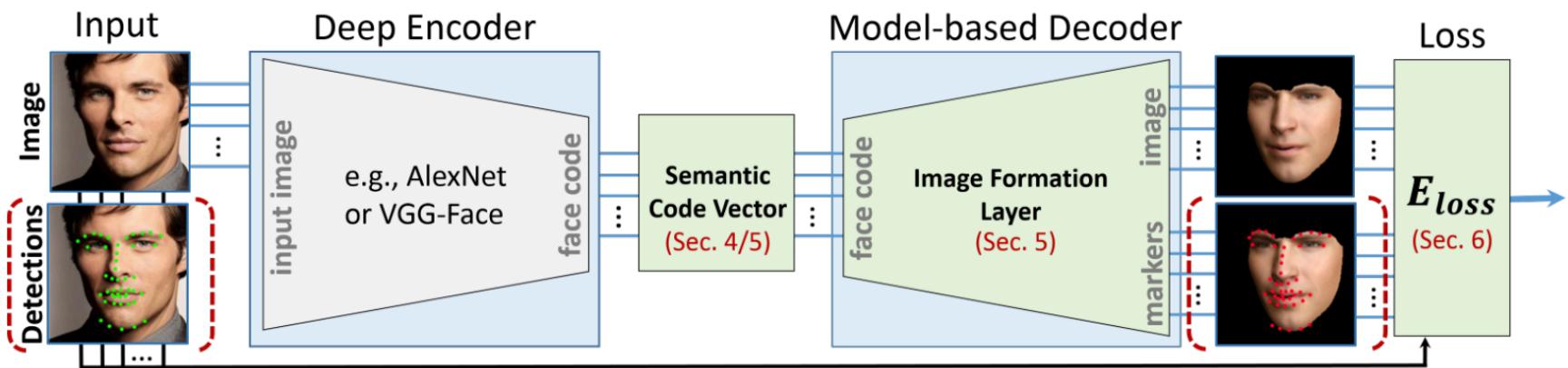
- ...with deep learning!
- 3DFFA directly learns pose and shape parameters
- Allows face-tracking for side-faces and difficult occlusions because deep learning



[3DDFA, CVPR 2016]

# More face models and deep learning

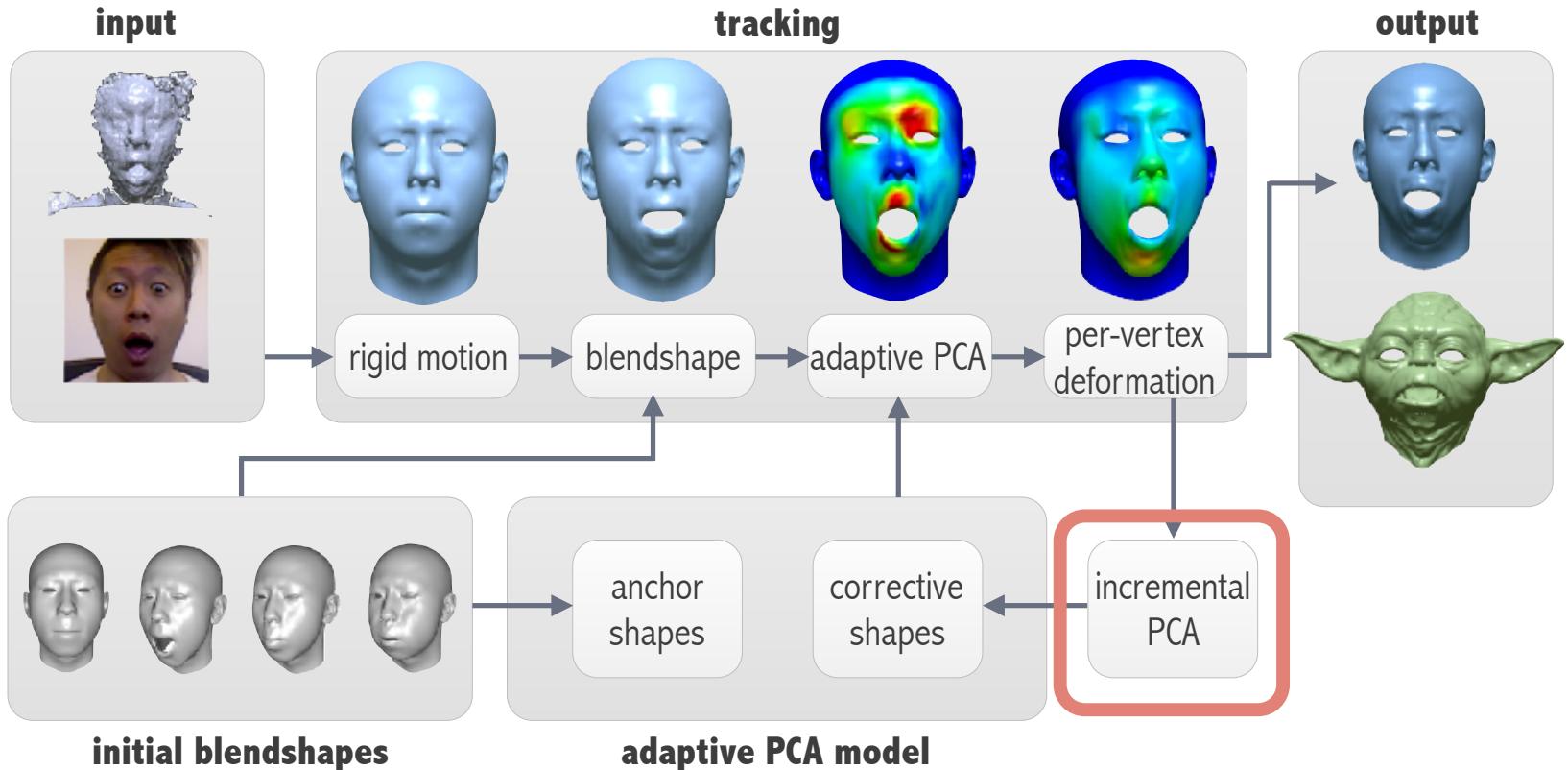
- Unsupervised deep learning also possible using parametric models
- Passes knowledge of the model into the encoder



[MOFA, ICCV 2017]

# Pipeline Overview

[USC CSCI 621]



# Doing it all...

- ...with deep learning!

Name	Description
input	Input $1 \times 240 \times 320$ image
conv1a	Conv $3 \times 3$ , $1 \rightarrow 64$ , stride $2 \times 2$ , ReLU
conv1b	Conv $3 \times 3$ , $64 \rightarrow 64$ , stride $1 \times 1$ , ReLU
conv2a	Conv $3 \times 3$ , $64 \rightarrow 96$ , stride $2 \times 2$ , ReLU
conv2b	Conv $3 \times 3$ , $96 \rightarrow 96$ , stride $1 \times 1$ , ReLU
conv3a	Conv $3 \times 3$ , $96 \rightarrow 144$ , stride $2 \times 2$ , ReLU
conv3b	Conv $3 \times 3$ , $144 \rightarrow 144$ , stride $1 \times 1$ , ReLU
conv4a	Conv $3 \times 3$ , $144 \rightarrow 216$ , stride $2 \times 2$ , ReLU
conv4b	Conv $3 \times 3$ , $216 \rightarrow 216$ , stride $1 \times 1$ , ReLU
conv5a	Conv $3 \times 3$ , $216 \rightarrow 324$ , stride $2 \times 2$ , ReLU
conv5b	Conv $3 \times 3$ , $324 \rightarrow 324$ , stride $1 \times 1$ , ReLU
conv6a	Conv $3 \times 3$ , $324 \rightarrow 486$ , stride $2 \times 2$ , ReLU
conv6b	Conv $3 \times 3$ , $486 \rightarrow 486$ , stride $1 \times 1$ , ReLU
drop	Dropout, $p = 0.2$
fc	Fully connected $9720 \rightarrow 160$ , linear activation
output	Fully connected $160 \rightarrow N_{\text{out}}$ , linear activation



[Laine et al. , SCA 2017]

# Some Parametric Models

- Some common parametric models include:
  - Faces
  - **Bodies**
  - Hands

# SMPL (SIGGRAPH Asia 2015)

## SMPL: Skinned Multi-Person Linear model

Matthew Loper

Naureen Mahmood

Javier Romero

Gerard Pons-Moll

Michael J. Black



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MAX-PLANCK-GESELLSCHAFT  
SIGGRAPH Asia 2015

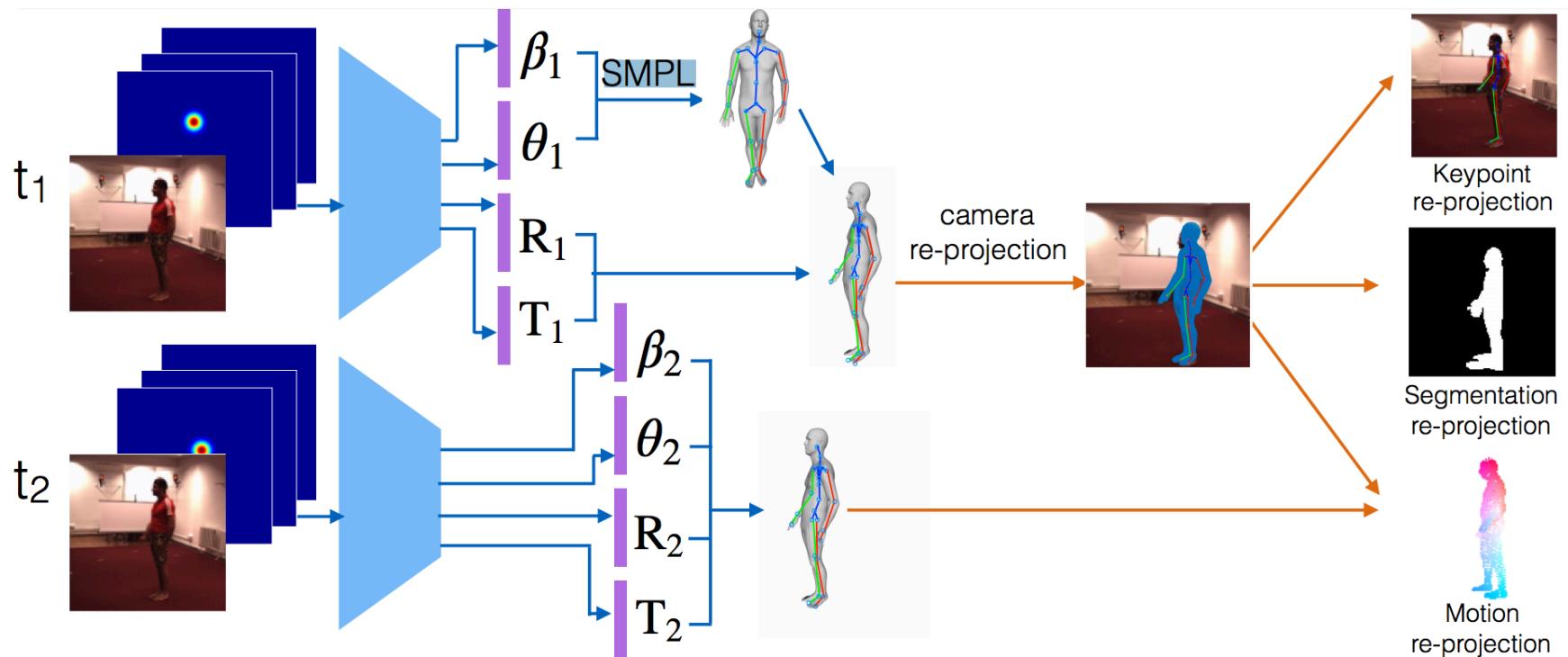
# SIMPLify (ECCV 2016)

- Learn to fit a human body model to a single image
- First estimate pose points using CNN (only CNN methods perform well)
- Fit body parameters to pose points



# Fitting Parameters...

- ...using deep learning

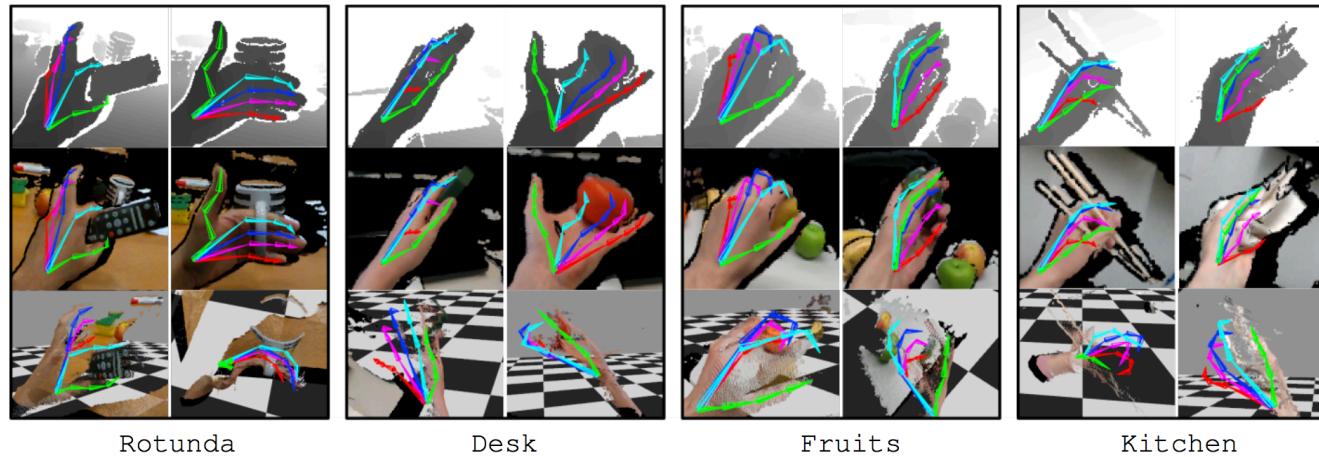
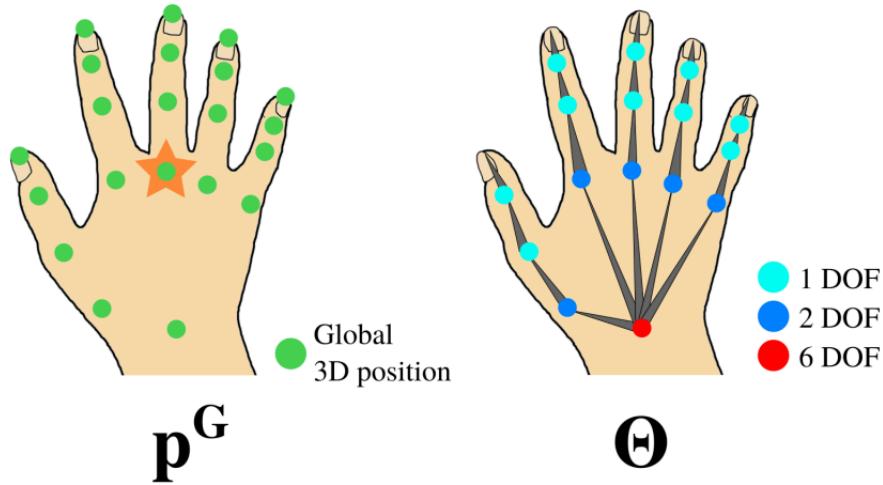


[Tung et al., NIPS 2017]

# Some Parametric Models

- Some common parametric models include:
  - Faces
  - Bodies
  - **Hands**

# Hand Models



[Mueller et al., ICCV 2017]

# Conclusion

- 3D structures carry a geometric and physical meaning that classical geometers and graphics people have exploited
- Many graphics/geometry problems make use of knowledge and assumptions of our world for better performance
  - Exploit commonalities
  - Low-Dimensional Parametric Models
  - Easy Deformation
  - Easy 3D estimation from a single image
- Deep learning is everywhere
- Can we learn these assumptions?