Geometric algebra: a computational framework for geometrical applications (part II: applications)

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Abstract

Geometric algebra is a consistent computational framework in which to define geometric primitives and their relationships. This algebraic approach contains all geometric operators and permits coordinate-free specification of computational constructions. It contains primitives of any dimensionality (rather than just vectors). This second paper on the subject uses the basic products to represent rotations (naturally incorporating quaternions), intersections, and differentiation. It shows how using well-chosen geometric algebra models, we can eliminate special cases in incidence relationships, yet still have the efficiency of the Plücker coordinate intersection computations.

Keywords: geometric algebra, Clifford algebra, rotation reprensentation, quaternions, dualization, meet, join, Plücker coordinates, homogeneous coordinates, geometric differentiation, computational geometry

1 Introduction

This is the second of two papers introducing geometric algebra. In the previous paper, we introduced *blades*, a computational algebraic representation of *oriented subspaces*, which are the basic element of computation in geometric algebra. We also looked at the geometric product, and two products derived from the geometric product, the inner and outer products. A crucial feature of the geometric product is that it is invertible.

From that first paper, you should have gathered that every vector space with an inner product has a geometric algebra, whether you choose to use it or not. This paper shows how to call upon this structure for the definition of common geometrical constructs, ensuring a totally consistent computational framework. The goal is to show you that this can be done, and that it is compact, directly computational, and transcends the dimensionality of subspaces. We will not use geometric algebra to develop

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