Projection Matrix

March 12, 2024

Given two vectors u, v, projects u onto v, $w = \sigma v$ where σ is a scala we have $u \perp (w - u)$

$$
\begin{aligned}
v \perp (\sigma v - u) \\
v^T(\sigma v - u) = 0 \\
\sigma v^T v - v^T u = 0 \\
\sigma v^T v = v^T u \\
\sigma = \frac{v^T u}{v^T v}
\end{aligned}
$$

$$
\begin{aligned}
\sigma v = \frac{v v^T u}{v^T v} \\
\sigma v = \frac{v v^T}{v^T v} u
\end{aligned}
$$

vv^T is the projection matrix which projects u onto v where $|v| = 1$

Concret Example: Project $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ onto $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Projection Matrix: $vv^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$Proj_v = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$