Geometric algebra: a computational framework for geometrical applications (part I: algebra)

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Abstract

Geometric algebra is a consistent computational framework in which to define geometric primitives and their relationships. This algebraic approach contains all geometric operators and permits specification of constructions in a coordinate-free manner. Thus, the ideas of geometric algebra are important for developers of CAD systems. This paper gives an introduction to the elements of geometric algebra, which contains primitives of any dimensionality (rather than just vectors), and an introduction to three of the products of geometric algebra, the geometric product, the inner product, and the outer product. These products are illustrated by using them to solve simple geometric problems.

Keywords: geometric algebra, Clifford algebra, subspaces, blades, geometric product, inner product, outer product

1 Introduction

In the usual way of defining geometrical objects in fields like computer graphics, robotics and computer vision, one uses vectors to characterize the constructions. To do this effectively, the basic concept of a vector as an element of a linear space is extended by an inner product and a cross product, and some additional constructions such as homogeneous coordinates to encode compactly the intersection of, for instance, offset planes in space. Many of these techniques work rather well in 3-dimensional space, although some problems have been pointed out: the difference between vectors and points [3], and the characterization of planes by normal vectors (which may require extra computation after linear transformations, since the normal vector of a transformed plane is not the transform of the normal vector). These problems are then traditionally fixed by the introduction of data structures and combination rules; object-oriented programming can be used to implement this patch tidily [6].

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