

# *Chebyshev Polynomials*



Reading

Problems

## Differential Equation and Its Solution

The Chebyshev differential equation is written as

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + n^2 y = 0 \quad n = 0, 1, 2, 3, \dots$$

If we let  $x = \cos t$  we obtain

$$\frac{d^2 y}{dt^2} + n^2 y = 0$$

whose general solution is

$$y = A \cos nt + B \sin nt$$

or as

$$y = A \cos(n \cos^{-1} x) + B \sin(n \cos^{-1} x) \quad |x| < 1$$

or equivalently

$$y = A T_n(x) + B U_n(x) \quad |x| < 1$$

where  $T_n(x)$  and  $U_n(x)$  are defined as Chebyshev polynomials of the first and second kind of degree  $n$ , respectively.