Chapter 3

Direct Sums, Affine Maps, The Dual Space, Duality

3.1 Direct Products, Sums, and Direct Sums

There are some useful ways of forming new vector spaces from older ones.

Definition 3.1. Given $p \geq 2$ vector spaces $E_1, \ldots, E_p$, the product $F = E_1 \times \cdots \times E_p$ can be made into a vector space by defining addition and scalar multiplication as follows:

$$\begin{aligned} (u_1, \ldots, u_p) + (v_1, \ldots, v_p) &= (u_1 + v_1, \ldots, u_p + v_p) \\ \lambda(u_1, \ldots, u_p) &= (\lambda u_1, \ldots, \lambda u_p), \end{aligned}$$

for all $u_i, v_i \in E_i$ and all $\lambda \in \mathbb{R}$.

With the above addition and multiplication, the vector space $F = E_1 \times \cdots \times E_p$ is called the *direct product* of the vector spaces $E_1, \ldots, E_p$.