

1 Intersection of two lines

Given two lines, find the intersection of two lines if they are not parallel

Use a **point** and a **vector** to represent a line

$$f(t) = p_0 + s(p_1 - p_0) \quad \text{where} \quad v_1 = p_1 - p_0 \text{ is a vector}$$

$$f(s) = q_0 + t(q_1 - q_0) \quad \text{where} \quad v_2 = q_1 - q_0 \text{ is a vector}$$

$$p_0 + s(p_1 - p_0) = q_0 + t(q_1 - q_0)$$

$$p_0 - q_0 = -s(p_1 - p_0) + t(q_1 - q_0)$$

$$p_0 - q_0 = (x_0, y_0)$$

$$p_1 - p_0 = (px, py) \quad s$$

$$q_1 - q_0 = (qx, qy) \quad t$$

$$x_0 = -s(px) + t(qx)$$

$$y_0 = -s(py) + t(qy)$$

$$x_0 = s(-px) + t(qx)$$

$$y_0 = s(-py) + t(qy)$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} -px & qx \\ -py & qy \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

Example: let $p_0 = (2, 0)$, $p_1 = (2, 1)$, $q_0 = (0, 0)$, $q_1 = (1, 1)$

$$f(s) = (2, 0) + s(2, 1) \quad \text{where} \quad f(s) = q_0 + t(q_1 - q_0)$$

$$f(t) = (0, 0) + t(1, 1) \quad \text{where} \quad f(t) = p_0 + s(p_1 - p_0)$$

$$p_0 - q_0 = (2, 0) - (0, 0) = (2, 0)$$

$$p_1 - p_0 = (2, 1) - (2, 0) = (0, 1)$$

$$q_1 - q_0 = (1, 1) - (0, 0) = (1, 1)$$

$$(2, 0) = -s(0, 1) + t(1, 1)$$

$$-2 = -s(-0) + t1$$

$$0 = -s(1) + t1$$

$$-2 = s(0) + t1$$

$$0 = s(-1) + t1$$

$$\begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$s = 2$$

$$t = -2$$