Prove Hermitian matrix has only real eigenvalues

$$\mathbf{A}^* = \mathbf{A}$$

Proof

let eigenvalue $\lambda \neq \vec{0}$ such as

$$\mathbf{A}\vec{v} = \lambda\vec{v}$$

$$\Rightarrow (\mathbf{A}\vec{v})^* = (\lambda\vec{v})^*$$

$$\Rightarrow (\vec{v}^* \mathbf{A}^*) = (\lambda^* \vec{v}^*)$$

Multiply both side by $\vec{v}$

$$\Rightarrow (\vec{v}^* \mathbf{A}^* \vec{v}) = (\lambda^* \vec{v}^* \vec{v})$$

$$\mathbf{A}^* = \mathbf{A}$$

$$\Rightarrow (\vec{v}^* \mathbf{A}\vec{v}) = (\lambda^* \vec{v}^* \vec{v})$$

$$\Rightarrow (\vec{v}^* \lambda\vec{v}) = (\lambda^* \vec{v}^* \vec{v})$$

$$\Rightarrow (\lambda\vec{v}^* \vec{v}) = (\lambda^* \vec{v}^* \vec{v})$$

$$\Rightarrow \lambda = \lambda^*$$

$$\Rightarrow \lambda \in \mathbf{R}$$