$$\textbf{Show}\ \ \mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$$

$$\mathrm{tr}(\mathbf{AB}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{i,j}b_{j,i}$$

$$\mathrm{tr}(\mathbf{BA}) = \sum_{j=1}^{n}\sum_{i=1}^{n} b_{j,i}a_{i,j}$$

$$\Rightarrow \mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$$

$$\textbf{Show}\ \ \mathrm{tr}(\mathbf{A+B}) = \mathrm{tr}(\mathbf{A}) + \mathrm{tr}(\mathbf{B})$$

$$\mathrm{tr}(\mathbf{A+B}) = \sum_{i=1}^{n}(a_{i,i} + b_{i,i}) = \sum_{i=1}^{n} a_{i,i} + \sum_{i=1}^{n} b_{i,i} = \mathrm{tr}(\mathbf{A}) + \mathrm{tr}(\mathbf{B})$$

$$\textbf{Show associative of matrix multiplication}\ \ \mathbf{ABC} = \mathbf{A(BC)}$$

$$[\mathbf{(AB)C}]_{i,j} = \sum_{m=1}^{n}\left(\sum_{k=1}^{n} a_{i,k}b_{k,m}\right)c_{m,j} = \sum_{m=1}^{n}\left(\sum_{k=1}^{n} a_{i,k}b_{k,m}c_{m,j}\right)$$

$$[\mathbf{A(BC)}]_{i,j} = \sum_{k=1}^{n} a_{i,k}\left(\sum_{m=1}^{n} b_{k,m}c_{m,j}\right) = \sum_{k=1}^{n}\left(\sum_{m=1}^{n} a_{i,k}b_{k,m}c_{m,j}\right)$$

$$\Rightarrow \mathbf{ABC} = \mathbf{A(BC)}$$