



Derived circle parametic equation line equatin

$$\begin{aligned}
 \text{line equation from } (0,r) \text{ to } (t, 0) \quad & \frac{y-0}{x-t} = \frac{y-r}{x-0} \\
 \Rightarrow xy &= (y-r)(x-t) \\
 \Rightarrow xy &= xy - rx - yt + rt \\
 \Rightarrow rx &= rt - yt = t(r-y) \\
 \Rightarrow rx &= t(r-y) \\
 \Rightarrow x &= \frac{t}{r}(r-y) \tag{1}
 \end{aligned}$$

Let the circle equation to be

$$\begin{aligned}
 x^2 + y^2 &= r^2 \tag{2} \\
 \Rightarrow \left( \frac{t}{r}(r-y) \right)^2 + y^2 &= r^2 \\
 \Rightarrow \left( \frac{t}{r} \right)^2 (r-y)^2 + y^2 &= r^2 \\
 \Rightarrow t^2(r^2 + y^2 - 2ry) + r^2y^2 &= r^4 \\
 \Rightarrow t^2r^2 + t^2y^2 - 2rt^2y + r^2y^2 &= r^4 \\
 \Rightarrow (t^2 + r^2)y^2 - 2rt^2y + t^2r^2 - r^4 &= 0
 \end{aligned}$$

$$\Rightarrow a = (t^2 + r^2) \quad b = -2rt^2 \quad c = t^2r^2 - r^4$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow y = \frac{2rt^2 \pm \sqrt{4r^2t^4 - 4(t^2 + r^2)(t^2r^2 - r^4)}}{2(t^2 + r^2)}$$

$$\Rightarrow y = \frac{rt^2 \pm \sqrt{r^2t^4 - (t^4r^2 - t^2r^4 + t^2r^4 - r^6)}}{(t^2 + r^2)}$$

$$\Rightarrow y = \frac{rt^2 \pm \sqrt{r^2t^4 - t^4r^2 + t^2r^4 - t^2r^4 + r^6}}{(t^2 + r^2)}$$

$$\Rightarrow y = \frac{rt^2 \pm \sqrt{t^2r^4 - t^2r^4 + r^6}}{(t^2 + r^2)}$$

$$\Rightarrow y = \frac{r(t^2 \pm r^2)}{(t^2 + r^2)}$$

$$y = r \text{ or } y = \frac{r(t^2 - r^2)}{(t^2 + r^2)}$$

$$\text{If } y = r \Rightarrow \begin{cases} x = \frac{t}{r}(r - y) = 0 \\ y = r \end{cases}$$

$$\text{If } y = \frac{r(t^2 - r^2)}{(t^2 + r^2)} \Rightarrow \begin{cases} x = \frac{t}{r}(r - y) & (2) \\ y = \frac{r(t^2 - r^2)}{(t^2 + r^2)} & (3) \end{cases}$$

Sub(3) into (2)

$$\Rightarrow x = \frac{t}{r} \left( r - \frac{r(t^2 - r^2)}{t^2 + r^2} \right)$$

$$\Rightarrow x = t \left( 1 - \frac{(t^2 - r^2)}{t^2 + r^2} \right)$$

$$\Rightarrow x = \frac{2tr^2}{t^2 + r^2}$$

$$\Rightarrow \begin{cases} x = r \frac{2tr}{t^2 + r^2} \\ y = r \frac{(t^2 - r^2)}{t^2 + r^2} \end{cases} \quad \text{or} \quad \begin{cases} x = 0 \\ y = r \end{cases}$$