

Prove Hermitian matrix has only real eigenvalues

$$\mathbf{A}^* = \mathbf{A}$$

Proof

let eigenvalue $\lambda \neq 0$ such as

$$\mathbf{A}\vec{v} = \lambda\vec{v}$$

$$\Rightarrow (\mathbf{A}\vec{v})^* = (\lambda\vec{v})^*$$

$$\Rightarrow (\vec{v}^* \mathbf{A}^*) = (\lambda^* \vec{v}^*)$$

Multiply both side by \vec{v}

$$\Rightarrow (\vec{v}^* \mathbf{A}^* \vec{v}) = (\lambda^* \vec{v}^* \vec{v})$$

$$\mathbf{A}^* = \mathbf{A}$$

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$$\Rightarrow (\vec{v}^* \lambda \vec{v}) = (\lambda^* \vec{v}^* \vec{v})$$

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$$\Rightarrow \lambda = \lambda^*$$

$$\Rightarrow \lambda \text{ is real}$$