$$\textit{Show cosets of Ideal } \mathbf{I} \textit{ form a group under addition}$$

$$\mathbf{r} \in \mathbf{R} \textit{ and } s \in \mathbf{I}$$

$$r + s \in \mathbf{I}$$

$$0 \in \mathbf{I}$$

$$\textit{Show } \sqrt{2\sqrt{2\sqrt{2}}}... \textit{ approach } 2$$

$$\sqrt{2} = \mathbf{2}^{\frac{1}{2}}$$

$$\sqrt{2\sqrt{2}} = \mathbf{2}^{\frac{1}{2}+\frac{1}{2^2}}$$

$$\sqrt{2\sqrt{2\sqrt{2}}} = \mathbf{2}^{\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}}$$

$$\textit{Show the limit of } \mathbf{S}_n = \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^n} = 1 \quad \textit{when } n \to \infty$$

$$\mathbf{S}_n = \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^n}$$

$$\Rightarrow \frac{1}{2}\mathbf{S}_n = \frac{1}{2}(\frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^n})$$

$$\Rightarrow \frac{1}{2}\mathbf{S}_n = (\frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{n+1}})$$

$$\Rightarrow (1) - (2) = \mathbf{S}_n - \frac{1}{2}\mathbf{S}_n = \frac{1}{2} - \frac{1}{2^{n+1}}$$

$$\Rightarrow \mathbf{S}_n(1 - \frac{1}{2}) = \frac{1}{2} - \frac{1}{2^{n+1}}$$

$$\Rightarrow \frac{1}{2^{n+1}} \to \mathbf{0} \quad \textit{when } n \to \infty$$

$$\Rightarrow \mathbf{S}_n \to \mathbf{1} \quad \textit{when } n \to \infty$$

$$\Rightarrow \lim_{n\to\infty} \mathbf{2}^{\mathbf{S}_n} = 2 \quad \textit{when } n \to \infty$$