Labor Market Dynamics with Sorting*

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Abstract

I study a dynamic search-matching model with two-sided heterogeneity and labor market sorting. Sorting provides novel adjustment margins that amplify the model's response to productivity shocks. First, the match surplus and the incentive to sort increase after a positive shock, especially for high types. Second, matching cutoffs get wider and previously unattractive matches become viable. Third, the endogenous distribution of unemployed worker types shifts down. All three adjustment margins affects forward-looking vacancy posting decisions of firms and augment the response to shocks. The model closely matches time-series moments from US labor market data and produces realistic degrees of wage dispersion and sorting.

Keywords: Search, Matching, Sorting, Mismatch, Aggregate Shocks, Worker Heterogeneity, Firm Heterogeneity, Unemployment Dynamics

JEL Classifications: E24, E32, J63, J64

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1 Introduction

To form a match in the labor market, workers and firms have to spend time and resources. Costly job search can explain the coexistence of unemployed workers and vacant jobs, and this idea forms the core of search-matching models of the labor market (Diamond, 1982; Mortensen, 1982; Pissarides, 1985). Intuitively, these search frictions can be understood as a result of heterogeneity. If workers and firms differ in terms of their skills and productivity, finding the *right match* is costly. Yet, the textbook DMP model is built around representative workers and firms (Pissarides, 2000). At the same time, Shimer (2005) finds that the model fails to generate sufficient volatility in response to shocks.

Existing solutions to the "unemployment-volatility puzzle" reduce the responsiveness of wages to shocks to induce more vacancy posting (Hall, 2005; Hall and Milgrom, 2008; Hagedorn and Manovskii, 2008). However, there is little empirical support for rigid wages of new matches, and these are what matters for job creation (Pissarides, 2009). In this paper, I present an alternative approach to generate amplification. I show that incorporating worker and firm heterogeneity along with the idea that finding the *right match* is costly into an otherwise standard search-matching model can amplify firms' vacancy posting.

The *right match* exists in models of labor market sorting (Becker, 1973). Induced by a production complementarity, positive sorting implies that the optimal, i.e., output-maximizing partner is of the same type. However, search frictions exacerbate the formation of these optimal matches (Shimer and Smith, 2000). With random search, workers and firms trade-off the value of matching against the value of continued search, so not every meeting leads to a match and mismatch arises. In response to productivity shocks, both matching opportunities and the incentives to sort change. These effects go beyond the direct effect of the shock on output and generate amplification.

To decide how many vacancies to post, firms solve a dynamic problem. They form expectations about the future match surplus, which worker types they are willing to match with (and vice versa), and the rates at which they meet these different worker types. Thus, firms' expectations depends on the endogenous distribution of unemployed worker types. In response to a positive productivity shock, the expectations change along all three margins: (i) the surplus increases because output goes up for all potential matches. This effect is most pronounced for matches between high-type workers and high-type firms that disproportionately benefit from the production complementarity (direct effect of sorting). (ii) Firms expect to hire worker types that were previously unwilling to match with them (indirect effect of sorting); (iii) high-type workers are picky in the model due to their high option value of search. Thus, firms that "get access to" high-type workers after a positive shock can expect to meet them at relatively high rates (another indirect effect of sorting). These direct and indirect effects of sorting on firms' vacancy posting constitute a novel amplification mechanism. Thus, I argue that labor market sorting provides a both micro-founded and empirically-supported way to better align search-matching models with the data.

Firms' expectations depend on the distribution of unemployed worker types, so the state space of the model includes an endogenous distribution function. I follow the perturbation approach pioneered by Reiter (2009) and developed further by, among others, Boppart et al. (2018), who show how impulse response functions (IRFs) can be used to efficiently calculate numerical derivatives of complicated functions in heterogeneous-agent models with aggregate shocks. I build on this insight to compute the adjustment path of the match surplus, acceptance sets, and the endogenous distribution. Specifically, I linearize the model around its steady state and define auxiliary state variables that represent the model objects that contain integrals that need to be evaluated numerically. The key to solve and simulate the dynamic sorting model are therefore efficient function to evaluate these integrals, but computation is still time-intensive because the dynamic model needs to be solved for all combinations of worker and firm types (curse of dimensionality).

In the literature on the macro-dynamics of labor markets, the most closely related paper is Lise and Robin (2017). These authors build on Postel-Vinay and Robin (2002) and Robin (2011) to combine a model of on-the-job search with heterogeneity, sorting and aggregate shocks. Using sequential auctions considerably simplifies computation, because the distribution functions drop out of the surplus and, thus, the model's state space. However, this implies that the firm entry decision is static, so vacancy postings immediately adjust in response to shocks and do not depend on the distribution of unemployed worker types. The contribution of this paper is complementary to Lise and Robin (2017). I abstract from on-the-job search and focus on the dynamic firm entry problem, study how it depends on sorting and the endogenous distribution, and show that it generates persistent vacancy dynamics and amplification. The next step in this literature is to analyze a model with both search on the job and dynamic firm entry, which is a fascinating avenue for future research.

Amplification through labor market sorting is compatible with recent findings in the empirical literature. First, evidence for positive sorting has been documented using German, Swedish, Danish, and US data with various methodologies. (Card et al., 2013; Lise et al., 2016; Hagedorn et al., 2017; Bonhomme et al., 2019; Bagger and Lentz, 2019; Lochner and Schulz, 2022). Second, Mueller (2017) documents that in recessions the pool of unemployed shifts toward workers with high wages in their previous job. This is consistent with my model: the unemployment rate of picky high-type workers changes most in response to shocks and the tendency to sort is type-dependent and pro-cyclical, i.e., the incentive to sort is larger for high-type workers and high-type firms and increases disproportionately in booms. Third, Baley et al. (2022) present evidence for counter-cyclical sorting, and explain it with mismatch due to information frictions in a directed search model. My model with full information and random search can generate counter-cyclical sorting as well because matching cutoffs widen in response to positive shocks. Finally, this paper is also related to earlier work that studies the macro-dynamics of labor markets using models with explicit heterogeneity but no sorting (Pries, 2008; Mukoyama, 2019).

The theoretical literature considers two types of sorting models. Depending on the functional form of the production function, sorting can arise based on comparative advantage or absolute advantage. In comparative-advantage sorting models, the worker/firm type does not matter in itself; only the interaction of the types determines output (Marimon and Zilibotti, 1999; Gautier et al., 2010; Gautier and Teulings, 2015). In sorting models with absolute advantage, for example Shimer and Smith (2000), high type workers (firms) produce more than low types, no matter what type of firm (worker) they are matched with. In other words, the production function implies an unambiguous hierarchy, or ranking, of workers and firms. I focus on the hierarchical sorting model because it is empirically more appealing: absolute advantage implies a global ranking of workers and firms, and the data typically used to rank workers and firms in the empirical literature, e.g., education, job tenure, firms size, or value added, are inherently hierarchical.

The remainder of this paper is structured as follows: Section 2 discusses the model and its stationary equilibrium, including comparative statics. Section 3 adds aggregate uncertainty to the model and analyzes the dynamic firm entry problem and (briefly) wage formation. Section 4 discusses the computational strategy and presents results from numerical simulations of the sorting model in comparison to a baseline search and matching model and U.S. labor market data. Section 5 concludes.

2 The Model

2.1 Setup

Time is discrete. Infinitely-lived workers and firms maximize their future discounted income and profit streams, respectively. They are characterized by ex-ante heterogeneity in terms of worker skills and firm productivity. The common discount factor is β. A production complementarity induces positive assortative matching (PAM): for every worker (firm) of a given skill (productivity) level, an optimal firm (worker) to match with exists (Becker, 1973). However, search frictions exacerbate the formation of these optimal matches. Under random search, workers and firms trade-off the value of matching against the option value of continued search upon meeting. Acceptance strategies are summarized by well-defined and mutually-consistent matching sets (Shimer and Smith, 2000). There is full information about the types and the availability of potential matching partners. I focus on one-to-one matches between workers and firms. Thus, I abstract from firm size

Table 1: Distribution functions of matched and unmatched worker and firm types

Distribution of	Relation	Aggregate Stock
Active matches	$g_m(x, y)$	$M = \iint g_m(x, y)\ \mathrm{d}x\mathrm{d}y$
Employed workers	$g_e(x) = \int g_m(x, y)\ \mathrm{d}y$	$E = \int g_e(x)\ \mathrm{d}x$
Unemployed workers	$g_u(x) = g_w(x) - g_e(x)$	$U = \int g_u(x)\ \mathrm{d}x$
Producing firms	$g_p(y) = \int g_m(x, y)\ \mathrm{d}x$	$P = \int g_p(y)\ \mathrm{d}y$
Vacant firms	$g_v(y) \rightarrow$ free entry	$V = \int g_v(y)\ \mathrm{d}y$

and coworker complementarities.[1]

Worker and Firm Heterogeneity

A continuum of workers is endowed with heterogeneous skills $x \in [0, 1]$ that are distributed according to a probability density function (pdf) $g_w(x)$. Similarly, a continuum of firms is endowed with heterogeneous productivity $y \in [0, 1]$ with pdf $g_f(y)$. Both $g_w(x)$ and $g_f(y)$ are exogenous and uniform. Table 1 summarizes how the endogenous distributions of (in)active worker and firm types are related to the exogenous densities. $g_m(x, y)$ is the endogenous two-dimensional joint distribution of active (i.e., producing) (x, y) matches. The distributions of employed workers, $g_e(x)$, and producing firms, $g_p(y)$, can be obtained by integrating out the respective dimension of $g_m(x, y)$. The distribution of unemployed worker types, $g_u(x)$, is obtained by subtracting the distribution of employed workers from $g_w(x)$. The distribution of vacancies, $g_v(y)$, is governed by free entry. The distributions of active matches, employed/unemployed workers, and producing/vacant firms are equilibrium objects. They integrate to the stocks of active matches, M, employed workers, E, unemployed workers, U, producing firms, P, and vacancies, V.

Production

Let $F(x, y)$ be a non-negative and twice continuously differentiable production function. PAM arises if worker types x and firm types y are complements. Thus, $F(x, y)$ is assumed to be supermodular, i.e., its cross-partial derivatives are positive. Supermodularity of $F(x, y)$ is not sufficient for equilibrium existence in this setting (Shimer and Smith, 2000). Both $F(x, y)$ and the derivatives $F_x(x, y)$ and $F_{xy}(x, y)$ need to be log-supermodular.[2] Following Shimer and Smith (2000) and Teulings and Gautier (2004), I use the following

[1] This is equivalent to assuming that the firms' production function has constant returns to scale at the match level. Thus, firms' aggregate output is equal to the sum of what is produced by every individual worker. Herkenhoff et al. (2018) and Freund (2022) are two recent studies that focus on coworker complementarities.

[2] Taken together, these assumptions imply for any $x' > x$ and $y' > y$, $F(x', y')F(x, y) \geq F(x', y)F(x, y')$, $F_x(x', y')F_x(x, y) \geq F_x(x', y)F_x(x, y')$, and $F_{xy}(x', y')F_{xy}(x, y) \geq F_{xy}(x', y)F_{xy}(x, y')$.

production function that fulfills these conditions:

$$F(x, y) = \mathrm{e}^{xy}. \tag{1}$$

To study the amplification of shocks in this setting, I introduce labor productivity z, which augments output multiplicatively.

$$F(x, y, z) = z\ \mathrm{e}^{xy}. \tag{2}$$

Labor productivity can be understood as a time-varying technology parameter that is common to all matches. In Section 4, z will be stochastic and subject to shocks. In the stationary equilibrium described in the following, z is set to 1 and therefore omitted.

Matches and Separations

Unemployed workers engage in random search. Meetings are governed by a standard Cobb-Douglas matching function with constant returns to scale, $m(U, V) = \vartheta U^{\xi} V^{1-\xi}$, where ξ $(1-\xi)$ is the elasticity of new matches with respect to unemployment (vacancies).[3] ϑ represents matching efficiency. The Poisson meeting rates are functions of aggregate labor market tightness $\theta = {}^{V}\!/_{U}$. $q_v(\theta) = {}^{M(U,V)}\!/_{V}$ is the rate at which vacant firms meet unemployed workers and, correspondingly, $q_u(\theta) = {}^{M(U,V)}\!/_{U}$ is the rate at which unemployed workers meet vacancies. $q_v(\theta)$ is decreasing and $q_u(\theta)$ increasing in θ. An important feature of the sorting model is that not all meetings lead to matches. Due to sorting, the option value of continued search can be higher than the discounted income stream from matching. In this case, the surplus (defined below) is negative. In this model, the matching sets, which include all mutually-acceptable type combinations, capture this logic.

Matches end for two reasons in the sorting model. First, idiosyncratic separation shocks arrive at rate δ. Second, with shocks to labor productivity z, matches may also separate endogenously in response to negative shocks. The reason is that the surplus of marginally profitable matches may become negative in response to a shock.[4]

Surplus

To form an employment relationship, both the worker and the firm have to be willing to match. Formally, this willingness to match is reflected in the match surplus for any (x, y)

[3] Note that using a linear search technology with heterogeneous workers and firms implies congestion effects between different worker and job types. I stick to the Cobb-Douglas matching function for simplicity and comparability to other studies. A quadratic search technology, as used in Shimer and Smith (2000), eliminates this congestion externality. Nöldeke and Tröger (2009) extend Shimer and Smith (2000) to models with linear search technologies.

[4] This mechanism is similar to Mortensen and Pissarides (1994), although aggregate rather than idiosyncratic shocks trigger endogenous separations in the dynamic model of Section 3.

combination. This surplus is defined as the sum of the differences between the values of being matched and unmatched for both workers and firms:

$$\mathcal{S}(x,y) = \mathcal{E}(x,y) - \mathcal{U}(x) + \mathcal{P}(x,y) - \mathcal{V}(y), \tag{3}$$

where $\mathcal{E}(x,y)$ is the value of an employed worker, $\mathcal{U}(x)$ is the value of unemployment, $\mathcal{P}(x,y)$ is the value of a producing firm, and $\mathcal{V}(y)$ is the value of a vacancy. All value functions are spelled out below. When the surplus is positive, both parties agree to form a match. When the surplus is negative, both parties prefer to continue their search. The decision to match (or not) is always mutually consistent.

I use an indicator function to conveniently include the matching sets in the model's value functions. The indicator $\mu(x,y)$ denotes whether an (x,y)-match is acceptable for both workers and firms, which is the case whenever the surplus is positive:

$$\mu(x,y) = \begin{cases} 1 & \text{if } \ \mathcal{S}(x,y) > 0 \\ 0 & \text{if } \ \mathcal{S}(x,y) < 0. \end{cases} \tag{4}$$

Finally, the Nash bargaining solution determines how the match surplus is shared between workers and firms. Their respective surplus shares are equal to the difference of the matched values and the outside options, which are the threat point in the bargaining game. The workers' bargaining power parameter is $\alpha \in (0,1)$.

$$\alpha\mathcal{S}(x,y) = \mathcal{E}(x,y) - \mathcal{U}(x) \tag{5}$$

$$(1-\alpha)\mathcal{S}(x,y) = \mathcal{P}(x,y) - \mathcal{V}(y) \tag{6}$$

The sharing rule also determines the wage. However, wages play no allocative role in the model and specifying them is not necessary to characterize the equilibrium. I briefly discuss the dynamic properties of wages in the sorting model in Section 3 and relegate the derivation of the wage equation to Appendix B.

Value Functions

The value of employment of a type-x worker at a type-y firm is defined by

$$\mathcal{E}(x,y) = W(x,y) + \underbrace{\beta\delta\mathcal{U}(x)}_{\text{separation}} + \underbrace{\beta(1-\delta)\max\{\mathcal{E}(x,y), \mathcal{U}(x)\}}_{\text{continued employment}}, \tag{7}$$

which consists of the flow payment of the match-specific wage, $W(x,y)$, and the discounted value of two contingencies in the next period. With probability δ, the match is hit by a separation shock and the worker receives the value of unemployment, $\mathcal{U}(x)$. With probability $(1-\delta)$, the match survives and the worker receives the maximum of the values of unemployment and employment at firm y. We include the max operator because endogenous separations are possible in the environment with stochastic labor productivity discussed below. There are no endogenous separations in stationary equilibrium.

The value of unemployment for a type-x worker is defined by

$$\begin{aligned}\mathcal{U}(x) = b(x) + \underbrace{\beta(1-q_u(\theta))\mathcal{U}(x)}_{\text{no meeting}} + \underbrace{\beta q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\mu^+(x,y)\mathcal{E}(x,y)\mathrm{d}y}_{\text{successful match}} \\ + \underbrace{\beta q_u(\theta)\mathcal{U}(x)\int_0^1 \frac{g_v(y)}{V}\mu^-(x,y)\mathrm{d}y}_{\text{meet unacceptable firm}}\,.\end{aligned} \tag{8}$$

All unemployed workers receive the type-specific value of home production $b(x)$ with $\frac{\partial b(x)}{\partial x} > 0$. In the following period, the unemployed worker does not meet any firm and remains unemployed with probability $1-q_u(\theta)$. With probability $q_u(\theta)$, a meeting occurs. The firm type y can either be within the matching set of worker type x ($\mu(x,y)=1$) or not ($\mu(x,y)=0$). For clarity of exposition, equation (8) separates the option value of meeting a firm into two parts: (i) the third term represents meetings that lead to the formation of an employment relationship (successful match). Here, $\mu^+(x,y)$ is short for $\mu(x,y)=1$ and $g_v(y)/V$ represents the probability of meeting a specific firm type. (ii) The fourth term represents unsuccessful meetings with unacceptable firm types (no match). Here, $\mu^-(x,y)$ is short for $\mu(x,y)=0$. Note that the limits of integration are 0 and 1 in both terms, so we integrate over all firm types.

The flow payoff of a matched firm is match-specific output $F(x,y)$ minus the wage, $W(x,y)$. In the next period, the match breaks up with probability δ or continues with probability $(1-\delta)$. Similar to the value of employment (7), the max operator is included to allow for endogenous separations in response to labor productivity shocks.

$$\mathcal{P}(x,y) = F(x,y) - W(x,y) + \underbrace{\beta\delta\mathcal{V}(y)}_{\text{separation}} + \underbrace{\beta(1-\delta)\max\{\mathcal{P}(x,y),\mathcal{V}(y)\}}_{\text{continued production}} \tag{9}$$

Finally, the value of a vacancy is defined by

$$\begin{aligned}\mathcal{V}(y) = -c(g_v(y)) + \underbrace{\beta(1-q_v(\theta))\mathcal{V}(y)}_{\text{no meeting}} + \underbrace{\beta q_v(\theta)\int_0^1 \frac{g_u(x)}{U}\mu^+(x,y)\mathcal{P}(x,y)\mathrm{d}x}_{\text{successful match}} \\ + \underbrace{\beta q_v(\theta)\mathcal{V}(y)\int_0^1 \frac{g_u(x)}{U}\mu^-(x,y)\mathrm{d}x}_{\text{meet unacceptable worker}}\,.\end{aligned} \tag{10}$$

The flow cost of maintaining an open vacancy is determined by the monotonic function, $c(g_v(y))$, so the cost depends on the measure of vacancies posted and varies across firm types. It represents per-period cost for posting vacancies, screening applications, etc. In the following period, the firm either meets no worker, a worker suitable to fill the job, or an unsuitable worker. $\mu^+(x,y)$ ($\mu^-(x,y)$) indicates that a match will (not) be formed upon meeting and ${g_u(x)}/{U}$ represents the probability of meeting a specific worker type.

Finally, the four value functions (7)–(10) can be simplified by adding and subtracting the value of unemployment and a vacancy, respectively. The surplus sharing rules (5) and (6) can then be plugged in to have the surplus function along with a max operator under the integral signs. Note that $\int_0^1 \frac{g_v(y)}{V} \max\{\mathcal{S}(x,y),0\}\mathrm{d}y$ is equivalent to $\int_0^1 \frac{g_v(y)}{V}\mu(x,y)\mathcal{S}(x,y)\mathrm{d}y$.

$$\mathcal{E}(x,y) = W(x,y) + \beta\left(\mathcal{U}(x) + \alpha(1-\delta)\max\{\mathcal{S}(x,y),0\}\right), \tag{11}$$

$$\mathcal{U}(x) = b(x) + \beta\left(\mathcal{U}(x) + \alpha q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\max\{\mathcal{S}(x,y),0\}\mathrm{d}y\right), \tag{12}$$

$$\mathcal{P}(x,y) = F(x,y) - W(x,y) + \beta\left(\mathcal{V}(y) + (1-\alpha)(1-\delta)\max\{\mathcal{S}(x,y),0\}\right), \tag{13}$$

$$\mathcal{V}(y) = -c(g_v(y)) + \beta\left(\mathcal{V}(y) + (1-\alpha)q_v(\theta)\int_0^1 \frac{g_u(x)}{U}\max\{\mathcal{S}(x,y),0\}\mathrm{d}x\right). \tag{14}$$

2.2 Stationary Equilibrium

Characterization

The stationary equilibrium of the sorting model is characterized by the surplus $\mathcal{S}(x,y)$, which pins down the match indicator function $\mu(x,y)$, and the distributions of unemployment workers and vacancies across types. These equilibrium objects are jointly determined by the surplus value function under free entry, the steady state flow equation, and the job-creation condition, which I now discuss in sequence.

With free entry imposed, the option value of a vacancy, $\mathcal{V}(y)$, is zero. Thus, the firm's option to wait for a better worker does not affect the surplus. Consequently, the surplus value function under free entry is obtained by plugging equations (11)–(13) into (3).

$$\begin{aligned}\mathcal{S}(x,y) = {} & F(x,y) + \beta(1-\delta)\max\{\mathcal{S}(x,y),0\} \\ & - \left(b(x) + \beta\alpha q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\max\{\mathcal{S}(x,y),0\}\ \mathrm{d}y\right).\end{aligned} \tag{15}$$

The surplus depends positively on output and the future discounted surplus, see the first line of equation (15). It depends negatively on the type-dependent outside option of the worker, see the second line of equation (15). This is an important property because it implies that the distributions enter the model's state space.[5] The worker's outside option has two parts. First, the type-specific value of home production $b(x)$. Second, the option

[5]This is the main difference compared to Lise and Robin (2017), who develop a sorting model with on-the-job search and sequential auctions (Postel-Vinay and Robin, 2002). In their model, the distributional terms drop out of the surplus value function.

value of meeting other firms, which depends on the worker's bargaining power α, the meeting rate $q_u(\theta)$, and the surplus integrated over all possible firm types, weighted with the respective probabilities of meeting a specific vacancy, ${}^{g_v(y)}/_V$. Note that the worker's outside option only depends on the worker type x (y is integrated out) and increases in x. Firm productivity enters the right hand side only via output, which increases in firm productivity y.

The steady-state flow condition of the model pins down the distribution of unemployment workers across types, $g_u(x)$:

$$\delta g_m(x,y) = g_u(x)\; q_u(\theta)\; \frac{g_v(y)}{V}\; \mu(x,y). \tag{16}$$

The LHS of equation (16) captures exogenous separations, i.e., the flow from active matches into unemployment for all (x,y) combinations. The RHS captures the workers' flow into employment, which depends on the meeting rate, $q_u(\theta)$, the probability of meeting a specific firm type y, ${}^{g_v(y)}/_V$, and the match indicator $\mu(x,y)$. After integrating out the firm dimension and rearranging, we get the following expression for $g_u(x)$:

$$g_u(x) = \frac{\delta g_w(x)}{\delta + q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\mu(x,y)\mathrm{d}y}. \tag{17}$$

The integral term in the denominator reflects that not all meetings lead to matches. Consequently, the density of unemployed workers differs across types.[6] If there are many firm types that type-x workers are willing to match with (and sufficiently many vacancies), the density of unemployed workers will be relatively low for this type. In turn, if type-x workers are relatively picky, i.e., only willing to match with few firm types, the measure of unemployed workers for this type will be relatively high.

To determine how many vacancies the different firm types post, I derive the job creation condition, which follows from (14) under free entry.

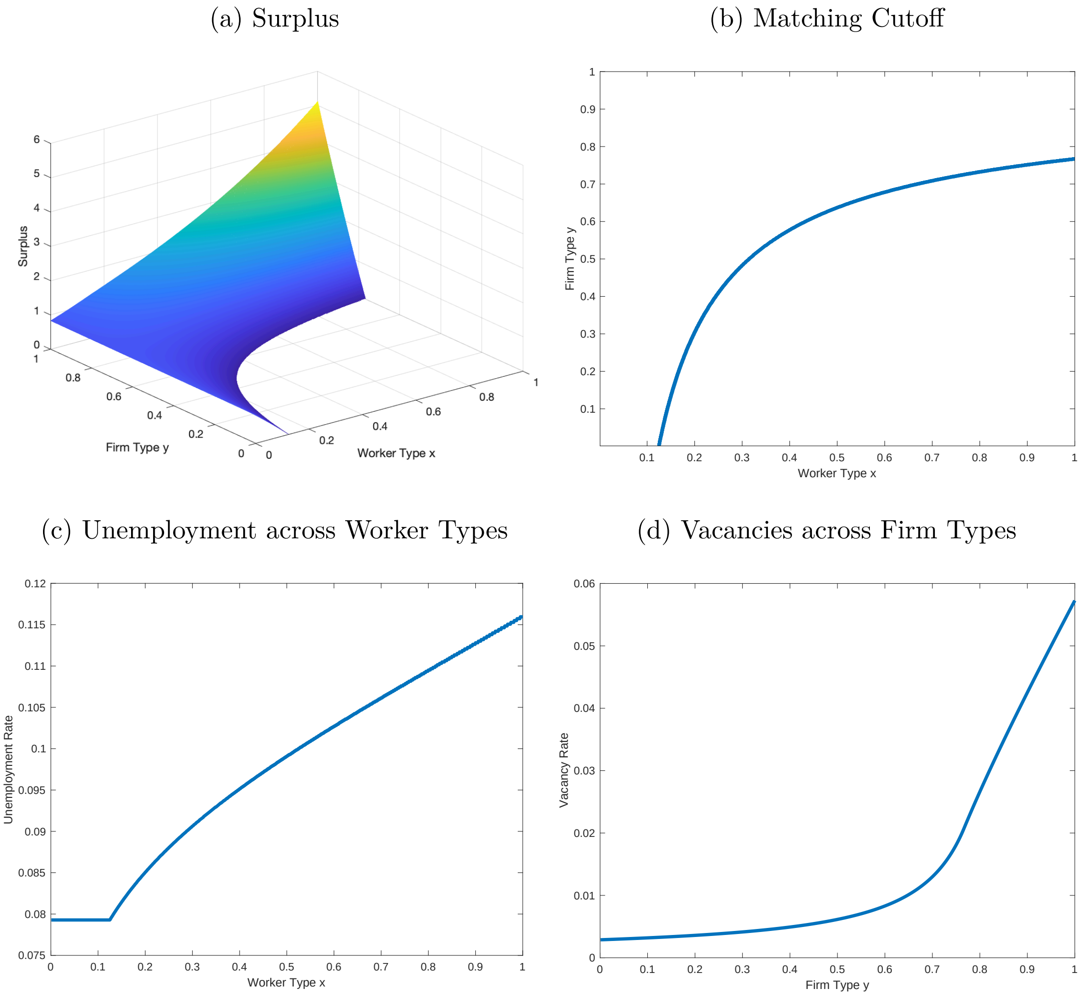
$$c(g_v(y)) = \beta(1-\alpha)q_v(\theta)\int_0^1 \frac{g_u(x)}{U}\mu(x,y)\mathcal{S}(x,y)\mathrm{d}x. \tag{18}$$

Given $g_u(x)$, $\mu(x,y)$, $\mathcal{S}(x,y)$, and the assumed monotonicity of the cost function c, equation (18) can be solved for the density of vacancies for every firm type y. Firms post vacancies until the discounted value on the RHS is lower than the cost implied by $c(g_v(y))$. I assume that c is a convex function to ensure that the distribution of vacancies is non-degenerate.[7] The RHS depends on the worker types that a specific firm type is willing

[6] This implies that steady-state unemployment is higher in the sorting model than in the textbook model for identical meeting and separation rates if there exists at least one (x,y) combination with negative surplus. To see this, integrate out the worker dimension x in (17). The expression $U = {}^{\delta}/\big(\delta + q_u(\theta)\iint \frac{g_v(y)}{V}\mu(x,y)\mathrm{d}y\mathrm{d}x\big)$ is equal to the textbook "Beveridge Curve" equation if the integral is 1, cf. equation (1.5) in Pissarides (2000).

[7] With the assumed production function, high-type firms produce more than low type firms irrespective

Figure 1: Stationary Equilibrium



to match with ($\mu(x, y)$), the surplus and the firm's share ($\mathcal{S}(x, y)$ and $(1 - \alpha)$), and how many unemployed workers of acceptable types the firm can expect to meet ($q_v(\theta)$ and ${}^{g_u(x)}/_{U}$). Once $g_u(x)$ and $g_v(y)$ are known, integrating yields the aggregate stocks of unemployed workers and vacancies, which in turn determine labor market tightness θ and the meeting rates $q_u(\theta)$ and $q_v(\theta)$.

Properties of the Stationary Equilibrium

To study the properties of the sorting model's stationary equilibrium, I solve the model for 1000 discrete worker and firm types using the calibration described in Section 4.2. The computational approach is discussed in Section 4.1.

Panel (a) of Figure 1 shows the surplus across (x, y) combinations. Negative values are

of the worker type they are matched with, so convexity is necessary to discipline the vacancy posting of high-type firms.

set to zero. The surplus reflects the properties of the production function $F(x, y)$, which increases in both the worker and firm type. However, the surplus is negative for a large part of the type space. Specifically, no matches are formed between high-type workers and low-type firms, and even most workers in the lower half of the skill distribution have a relatively threshold firm productivity below which they are unwilling to match. This threshold is depicted by the matching cutoff in Panel (b). The blue line splits the type space into two regions in which $\mu(x, y) = 1$ (lower left, upper left, upper right) and $\mu(x, y) = 0$ (center to lower right) hold, respectively. Workers get increasingly picky as their type increases due to the increasing outside option. The higher the worker type, the higher is value of home production $b(x)$ and the incentive to wait for a high-type firm. The most-skilled workers match only with a relatively small fraction of firm types at the very top (roughly 23% of the types).

Panel (c) demonstrates that the unemployment rate across worker types corresponds to their willingness to match. Based on this calibration, the unemployment rate per worker type is below 8% for low type workers who are willing to match with all firm types. For the most high-skilled workers, the unemployment rate is almost 12%. An important question is whether the pickiness of high-type workers is driven by the utility flow from home production during unemployment, which we allow to be increasing in x, or the option value of continued search, which is related to sorting. To investigate this, I recompute the stationary equilibrium with a constant b and find that the properties of the equilibrium do not change much.[8] So it is indeed the tendency to sort positively that makes workers picky in the model.

Finally, Panel (d) shows the measure of vacancies that the different firm types post. It increases in the type for two reasons related to the production function: (i) more productive firms produce more irrespective of the worker type they hire; (ii) the incentive to sort increases with the type. This directly affects the surplus and, thus, the measure of vacancies posted. However, vacancy posting does not only depend on the surplus but also on the match indicator and the probability to meet specific worker types, recall equation (18). Thus, as the firm type increases along the x-axis in Panel (d), firms "get access to" more workers who are willing to match with them, which further increases the incentive to post vacancies. recall Panel (b). While low-type firms can only match with the 10–20% of worker types at the bottom, high-type firms in the top 20% can expect to match with every worker they meet. Moreover, as high-type workers are more picky, recall Panel (c), the probability of matching with a high-type worker is relatively high conditional on $\mu(x, y) = 1$.

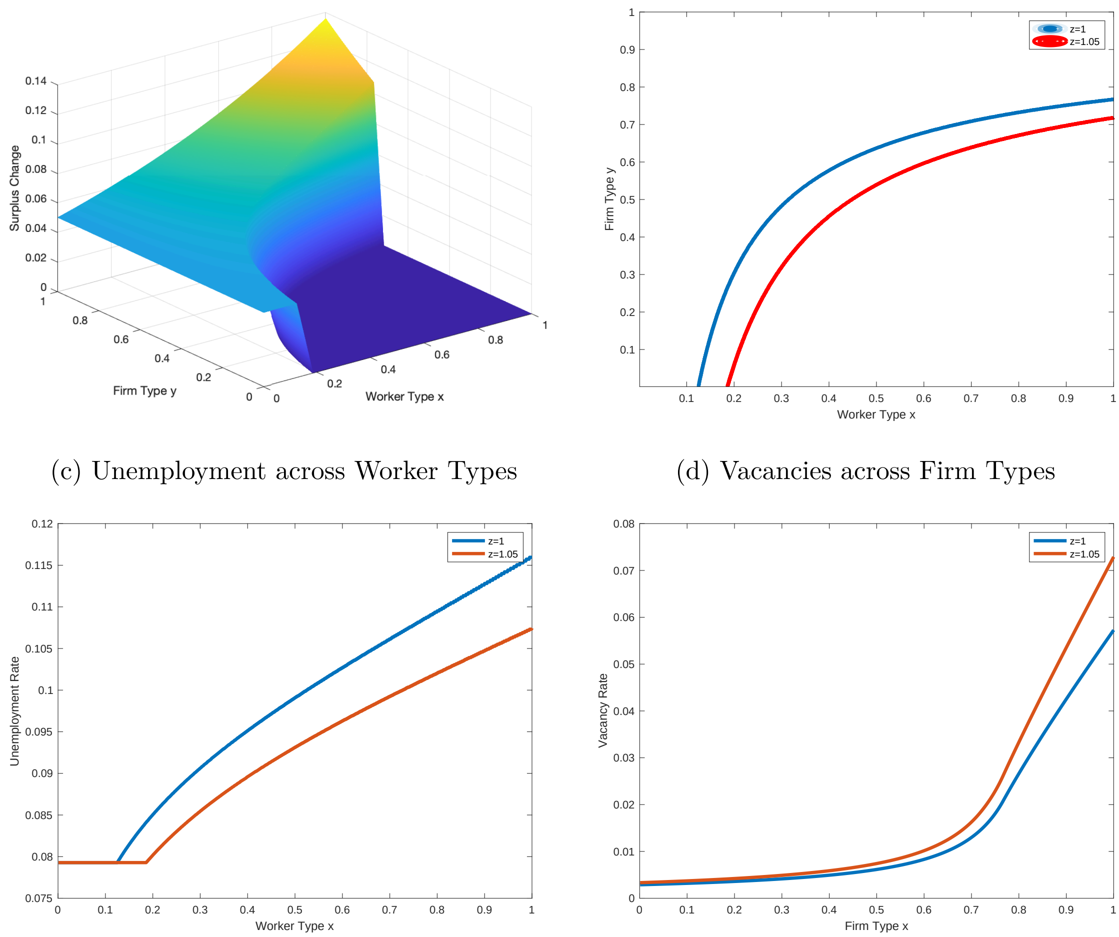
Taken together, vacancies increase non-linearly across firm types for three reasons:

[8] See Appendix Figure A.3. The matching cutoff hardly differs between the constant-b and increasing-b case, so the surplus is not affected much by the specification of b, although unemployment is higher for all worker types when b increases with the type x.

Figure 2: Comparative Statics of a Change in z

(a) Surplus Change (b) Matching Cutoff

(c) Unemployment across Worker Types (d) Vacancies across Firm Types



(i) higher output and surplus; (ii) more worker types are willing to match; (iii) picky high-type workers are more likely to be unemployed, so high-type firms that generate a positive surplus with these workers are relatively likely to match with them.

2.3 Comparative Statics

To explore the sources of amplification in the sorting model, I consider how the model responds to a positive technology shock in a comparative-statics exercise. To this end, I increase the labor productivity parameter z, introduced in equation (2). Figure 2 shows how the change from $z = 1$ (blue curves) to $z = 1.05$ (red curves) affects the model's equilibrium objects.

First, the surplus increases, see Panel (a). Note that the change in surplus increases towards the top. The reason is that the strength of the production complementarity increases in both z and the type.[9] This generates procyclical sorting, that is, the incentive

[9] A property of the production function $F(x, y, z) = z \times e^{xy}$ is that the first derivative also increases

to be optimally matched increases with labor productivity. Additionally, a big change of surplus occurs along the matching cutoff where additional (x, y) combinations become acceptable due to the increased output. The change to the match indicator function $\mu(x, y, z)$ is depicted in Panel (b). It shifts toward the lower-right corner, which means that more matches between high-type workers and low type firms become acceptable. In other words, (high-type) workers become less picky in response to the technology shock. This generates counter-cyclical sorting, that is, mismatch can increase with labor productivity. Which of the two forces dominates in the data is an empirical question, and recent papers document evidence for both counter-cyclical (Baley et al., 2022) and pro-cyclical sorting Mueller (2017).

Panel (c) shows how the density of unemployed workers across types changes. In line with the wider matching cutoff and less picky high-type workers, the unemployment rate decreases most at the top (about one percentage point). Conversely, this implies that in response to a negative shock, unemployment increases most at the top.[10] For low-type workers, the range in which unemployment is minimized (flat density) is extended as more worker types accept all firm types with higher output.

Panel (d) shows how the density of vacancies posted across firm types changes due to higher labor productivity. The first observation is that there is virtually no effect on low-type firms. For firms types between 0.4 and 0.7, the effect on vacancy posting begins to visibly increase in size, and this is due to the fact that these firm types start to benefit from the widening matching cutoff. To see this, compare the horizontal difference between the blue and the red line in Panel (b) for a type 0.1 and a type 0.6 firm. The difference is roughly three times larger for 0.6 firms. This effect can be understood as the "intensive margin" effect (more acceptable matches) of increased productivity on vacancy posting with sorting. For high type firms, the effect on vacancy posting increases further in size. However, above type 0.8, firms do not get access to further worker types because all workers are willing to match with them for both productivity levels. Thus, the further increase in vacancy posting in response to the change in productivity can be seen as the "extensive margin" effect (higher incentive to sort) of increase productivity on vacancy posting with sorting.

Taken together, these comparative statics results show that a significant amplification effect can arise in a dynamic sorting model through the firms' vacancy posting. The effect arises through three margins of adjustment in the firm's entry problem, recall the RHS of (18). In response to a positive shock, the surplus increases and the matching cutoff gets wider. The higher surplus alone leads to more vacancy posting, especially at the top. There is an additional affect of the wider matching set because the firm can now employ

in z and the own type, e.g. $\frac{\partial F(x,y,z)}{\partial x} = F_x(x, y, z) = x\ z\ e^{xy}$.

[10]This is consistent with Mueller (2017), studies the Current Population Survey and documents that in recessions the pool of unemployed workers shifts toward workers with high wages in their previous job.

additional worker types. On top of that, a third effect works through the distribution of unemployed worker types. If the additional worker types have a relatively high incidence of unemployment, vacancy posting will increase even more because the probability of meeting a suitable worker type increases disproportionately. The comparative statics exercise also reveals that there are forces toward both pro- and counter-cyclical sorting in the model (Mueller, 2017; Baley et al., 2022).

3 The Dynamic Sorting Model

In the dynamic sorting model, z is stochastic. Let z_t denote the realization of aggregate labor productivity z in period t. I assume that z follows a Markov process, which is specified and calibrated in Section 4.2. To simplify notation, let z' be next period's realization $(t+1)$. Time subscripts are omitted in the following to avoid notational clutter. I define the state of the system to be $\Omega(g_m(x,y,z),z)$. This state contains the exogenous state variable z and the endogenous state, which can be summarized in the distribution of active matches $g_m(x,y,z)$. All endogenous objects depending on the state have Ω as an argument in the following.

The timing of the model is as follows: a period begins when the state of aggregate labor productivity z is revealed. Workers and firms form their optimal acceptance strategies based on the exogenous state z and the primitives of the model. Both endogenous and exogenous separations take place, and new matches are formed. Workers and firms separated in the same period do not start their search until the next period. Finally, production commences and wages are paid.

Based on these assumptions, the dynamic surplus value function becomes

$$\begin{aligned} \mathcal{S}(x,y,\Omega) &= F(x,y,z) + \beta \mathbb{E}\left[(1-\delta)\max\{\mathcal{S}(x,y,\Omega'),0\}\right] \\ &\quad - \beta \mathbb{E}\left[b(x) - \alpha q_u(\theta(\Omega'))\int_0^1 \frac{g_v(y,\Omega')}{V(\Omega')}\mu(x,y,\Omega')\mathcal{S}(x,y,\Omega')\,\mathrm{d}y\right]. \end{aligned} \tag{19}$$

$F(x,y,z)$ depends on the exogenous state only. Home production $b(x)$ does not depend on z. $\mathbb{E}$ is the expectation operator regarding the future aggregate state. It includes all information available in the period the expectation is formed.

3.1 Job Creation

The firms' dynamic job creation problem is key to understand the amplification result. A firm of type y will post vacancies as long as the expected discounted value of production is at least as high as the convex cost implied by $c(.)$:

$$c(g_v(y,\Omega)) = \beta(1-\alpha)\mathbb{E}\left[q_v(\theta(\Omega'))\int_0^1 \frac{g_u(x,\Omega')}{U(\Omega')}\mu(x,y,\Omega')\mathcal{S}(x,y,\Omega')\ \mathrm{d}x\right]. \tag{20}$$

Thus, vacancy posting depends on an expectation over the firm-specific integral in equation 20. It includes the match indicator function $\mu(x,y,\Omega')$, the surplus function $\mathcal{S}(x,y,\Omega')$, and the probabilities of meeting specific worker types, $\frac{g_u(x,\Omega')}{U(\Omega')}$. As argued above, all three endogenous objects change in response to shocks. Beyond the direct effect of productivity on match the surplus, the match indicator function and the endogenous distribution of unemployed worker types change as well.[11] In case of a positive shock, the number of worker types willing to match with a given firm type increases, so the surplus jumps along the matching cutoff. Moreover, as shown in Section 2.2, high-type workers tend to have higher unemployment rates in the sorting model due to their pickiness. Thus, for the firm types that "get access to" these high-type workers when the matching cutoff widens in response to a positive shock, vacancy posting reacts disproportionately due to the expectation of high meeting rates and future high-surplus matches. This is in line with the comparative statics exercise, which revealed that strong response in vacancy posting occur both for high-type firms and for firm types that are most affected by the widening matching cutoff, recall Figure 2. In Section 4, I study the quantitative importance of the dynamic firm entry problem for the model's ability to generate amplification in response so shocks and match time-series moments of labor market data.

3.2 Wages

I derive the wage equation of the dynamic sorting model in Appendix B.

$$W(x,y,\Omega) = \alpha\left(F(x,y,z) + c(y)\mathbb{E}\left[\frac{\int_0^1 g_v(y,\Omega')\mu(x,y,\Omega')\mathcal{S}(x,y,\Omega')\ \mathrm{d}y}{\int_0^1 g_u(x,\Omega')\mu(x,y,\Omega')\mathcal{S}(x,y,\Omega')\ \mathrm{d}x}\right]\right) + (1-\alpha)b(x). \tag{21}$$

An interesting property is that the match-specific wage $W(x,y,\Omega)$ does not depend on aggregate labor market tightness in the sorting model. Instead, the ratio of the firm's integral term (the denominator of the expectation) and the worker's integral term (the numerator of the expectation) determines the wage. This feature can disconnect match-specific wages from fluctuations in aggregate tightness in the sorting model. Intuitively, what matters for the wage bargain are not the stocks of vacancies and unemployed workers but the alternative matching opportunities of both parties that are summarized by the integral terms.

[11] Compare this to the standard model: $c = \beta(1-\alpha)\mathbb{E}[q_v(\theta(z'))\mathcal{S}(z')]$. Here, vacancy posting costs are constant and absent heterogeneity the firm does not integrate over different worker types and the type-specific meeting probabilities.

4 Numerical Simulations and Empirical Analysis

In this paper, I judge the model's empirical performance by its ability to match aggregate time-series moments of US labor market data. To this end, I run numerical simulations of a calibrated model in a stochastic environment where aggregate shocks to labor productivity z drive the business cycle. I test whether a dynamic search-matching model with two-sided heterogeneity and sorting can overcome the unemployment-volatility puzzle (Shimer, 2005) due to the additional sorting-related adjustment mechanisms that I have discussed.

4.1 Computation

To introduce my computational approach, I first discuss how the stationary equilibrium discussed in Section 2.2 is computed. Second, I explain how this heterogeneous-agent model with aggregate shocks can be out of steady state. I use MATLAB and Dynare (Adjemian et al., 2022).

Stationary Equilibrium

I approximate the stationary equilibrium of the sorting model using value function iteration on a discrete grid. After initialization, the first step is to compute the fixed point of (15). In practice, it is necessary to allow for some smoothing of the acceptance strategy $\mu(x, y)$ along the cutoff to ensure convergence. I allow for mixed strategy solutions close to the cutoff, following Hagedorn et al. (2017). In the second step, knowing $\mathcal{S}(x, y)$ and $\mu(x, y)$, I use the steady-state flow condition (16) to solve for the endogenous distribution of unemployed worker types, $g_u(x)$. The third and final step is to calculate the measure of vacancies $g_v(y)$ along with the stocks U, V, and the meeting rates. I solve (18) numerically and integrate over $g_u(x)$ and $g_v(y)$ to compute the stocks of unemployment workers and vacancies.

The solution algorithm alternates between computing the fixed point of the surplus function for all (x, y) combinations and updating the distributions until convergence is achieved.[12] Due to vectorization and fast solvers, the stationary equilibrium can be computed in a matter of seconds for a grid of 100×100 types. This performance is critical for computing the model's response to shocks in Section 4. Note that uniqueness is not guaranteed in the Shimer and Smith (2000) class of models, so I verify that the surplus function is contracting in the parameter space I consider by repeatedly solving the model for different initial conditions.[13]

[12] That is, until the absolute difference of the surplus between two iterations is less than 10^{-6}.

[13] This approach is similar to Shimer and Smith (2000), Lise and Robin (2017), and Hagedorn et al. (2017), among others. I would like thank Robert Shimer for sharing the code used to produce the numerical results in Shimer and Smith (2000).

Out-of-Steady-State Dynamics

The dynamic sorting model has a complex state space Ω, which consists of the exogenous state z (labor productivity) and the endogenous state $g_m(x, y, z)$ (joint distribution of active matches). All other endogenous objects—the surplus, the matching sets, the distributions of unmatched worker and firm types, the stocks U and V, the arrival rates $q_u(\theta)$ and $q_v(\theta)$, and aggregate labor market tightness θ—follow from the state of the system Ω.

Keeping track of Ω's evolution is computationally challenging because it contains endogenous distributions and is thus infinite-dimensional. To tackle this challenge, I follow the perturbation approach pioneered by Reiter (2009) and developed further by, among others, Boppart et al. (2018), who show how impulse response functions (IRFs) can be used to efficiently calculate numerical derivatives of complicated functions in heterogeneous-agent models with aggregate shocks. I build on this insight to compute the adjustment paths of the match surplus, and the endogenous distributions in a computational procedure that is otherwise standard. I linearize the model around its steady state and define auxiliary state variables for the model objects that contain integrals over high-dimensional endogenous objects. This concerns in particular the RHS of equation (20), which determines vacancy posting.[14]

To keep track of the deviations of the auxiliary state variables from there steady state values, I resolve the model conditional on every draw of the exogenous state z and compute the numerical differentials of the auxiliary state variables with respect to z for all (x, y) and z combinations. The key to success for this method is a very fast algorithm that solves the model using the algorithm outlined above, but simulating the dynamic sorting model is still time-intensive because the adjustment paths of the integral terms need to computed for all combinations of worker and firm types (curse of dimensionality). Once the numerical derivatives of the integral terms are known, one can use external functions in Dynare to call the solver from within the "model block". Dynare is also very helpful for the underlying standard computations, e.g., checking stability of the system and computing policy functions.

Since I make heavy use of discretization and approximations to simulate the model, I need to check the reliability of my computational approach. It is well known that log-linearization around the steady state is error prone in nonlinear systems. Fortunately, the calibrated labor productivity process used in the context of U.S. labor market dynamics is not very volatile. The calibrated standard deviation of z is only 2%, so the model always remains in relatively close vicinity to the steady state. I check my computations by plugging simulated data back into the Bellman equations to discover whether the data solve them. The mean computational error I make is quite small at 3.8%. This value is

[14] In an earlier version of this paper that focused on wage dynamics in addition to firm entry, I additionally defined auxiliary state variables for the two integral terms that show up in the wage equation (21).

very close to the error one makes when solving simple dynamic search and matching models using log-linearization and perturbation, so my method of dealing with the additional complexity of the sorting model's state space does not appear to significantly increase computational errors.[15] Figures A.1 and A.2 in the Appendix show the distribution of computational errors and their positive correlation of the errors with z.

4.2 Calibration Based on Business Cycle Properties

As in Shimer (2005), a time period is set to be one quarter. Table 2 shows the calibration of the model based on the U.S. labor market data used for the simulation exercise and for the analysis of the stationary equilibrium in Section 2.2. To ensure comparability with the results in Shimer (2005), identical parameter values are used whenever possible. A value of 0.1 for the separation rate translates into an average employment spell of about 2.5 years during the United States in the relevant period (1951–2003). The quarterly discount rate is set to 0.012, representing an annual interest rate of roughly 5%. The discount factor (as it appears in the model equations) is thus $\beta = {}^{1}/_{1.012} \approx 0.99$. The matching function elasticity is set to 0.72, in line with Shimer (2005), which is within the empirically supported range (Petrongolo and Pissarides, 2001). I set the bargaining parameter equal to the matching function elasticity, that is, I follow the Hosios (1990) condition for socially efficient vacancy posting in the decentralized equilibrium.[16]

Several parameters need to be recalibrated in the sorting model. I allow the value of nonmarket activity $b(x)$ to increase in the worker type. I calibrate it to be $0.223 \times \max_x F(x, y)$. This implies that home production $b(x)$ has a mean of 40% of the output a worker of type x can produce in his optimal match. This assumption is a natural extension of Shimer (2005), who assumes a constant b of 0.4 when output is normalized to 1. The efficiency parameter of the aggregate matching function, ϑ, needs to be increased in the sorting model to take into account that not all meetings result in matches. A value of 2 implies, along with the other parameter values, that the *net* job finding rate, that is, the rate of matches that are formed after a meeting, is close to the value Shimer (2005) constructs from the data, which is 1.355 (quarterly). The calibrated economy has a steady-state unemployment rate of about 7.8%.

The convex vacancy posting cost function takes the following form:

$$c(g_v(y)) = \frac{c_0}{1+c_1} g_v(y)^{1+c_1}$$

c_0 and c_1 are set to the values shown in Table 2 to target a steady-state aggregate labor

[15] Petrosky-Nadeau and Zhang (2016) show that solving the representative agent search and matching model in Hagedorn and Manovskii (2008) via log-linearization and perturbation creates a mean computational error of 3.75%. I find a mean error of 3.84% with slightly more dispersion.

[16] I do not adapt the Hosios condition for the sorting model, i.e., this parametrization does not necessarily imply that vacancy posting is socially efficient.

Table 2: Parameter values for the quarterly calibration of the search and matching model for the U.S. labor market (1951–2003)

Parameter	Symbol	Value	Source
Discount factor	β	0.99	Shimer (2005)
Separation rate	δ	0.1	Shimer (2005)
Workers' bargaining power	α	0.72	Shimer (2005)
Matching function elasticity	ξ	0.72	Shimer (2005)
Matching function constant	ϑ	2	Calibrated to match
Value of nonmarket activity	$b(x)$	$0.223 \times \max_x F(x, y)$	steady-state unemp.
Vacancy posting costs	c_0	0.03	Calibrated to match
	c_1	0.4	steady-state θ.
First order autocorrelation	ρ	0.765	Hagedorn
Standard deviation	σ_ϵ	0.013	& Manovskii (2008)

market tightness of 1.[17]

The stochastic labor productivity process z is normalized to 1 in steady state and calibrated to resemble empirical labor productivity in the United States over the relevant period of time. I follow Hagedorn and Manovskii (2008) and set it up as a first-order autoregressive process:

$$z_t = \rho\, z_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2). \tag{22}$$

$\rho \in (0, 1)$ captures the degree of first-order autocorrelation of the AR(1) process. Innovations are drawn from a Gaussian distribution with mean 0 and standard deviation σ_ϵ. Both parameters are set to match quarterly U.S. labor productivity.[18] All values in Table 2 are based on quarterly data. Shimer's (2005) simulation results as well as my own results are reported as deviations from a HP trend, which is conventional in the literature.[19]

Using the calibration in Table 2, the model produces wage dispersion and labor market sorting of magnitudes that are comparable to benchmarks reported in the literature. The mean-min ratio is 1.35, which is close to the empirically-supported range of 1.5–2

[17] In practice, I set c_0 to the value estimated in Lise and Robin (2017) and adjust c_1 to target $\theta = 1$.

[18] Shimer (2005), Hornstein et al. (2005), and Hagedorn and Manovskii (2008) report the parameter values necessary to represent U.S. labor productivity "as seasonally adjusted quarterly real average output per person in the non-farm business sector constructed by the BLS" (Hagedorn and Manovskii (2008), p. 1694).

[19] The Hodrick-Prescott (HP) filter is a technique for decomposing the trend and the cyclical component of a time series (Hodrick and Prescott, 1997). Shimer (2005) sets the smoothing parameter of the filter to $\lambda = 10^5$ instead of to the more common value of $\lambda = 1600$ for quarterly data. This makes the cyclical component less volatile and more persistent. I use the same value as Shimer to generate comparable moments. Hornstein et al. (2005) point out that a more volatile trend, using the common smoothing parameter $\lambda = 1600$ for quarterly data, "has almost no effect on the relative volatilities" (p. 23).

Table 3: Actual and simulated standard deviations of labor market variables

	Standard deviations	U	V	θ	$q_u(\theta)$	z	$F(x,y,z)$
1.	U.S. data	0.190	0.202	0.382	0.118	0.02	-
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010	0.02	-
3.	No sorting, no heterogeneity	0.009	0.026	0.035	0.010	0.02	-
4.	Sorting, hierarchical model	0.102	0.277	0.380	0.168	0.02	0.06

Note: Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28, 39. Calculated based on quarterly U.S. data, 1951–2003. Rows 3 & 4: Standard deviations of simulated data from my model with and without sorting. All moments come from HP-filtered data with $\lambda = 10^5$.

identified in Hornstein et al. (2011) and much higher than the value that can be generated with the canonical search-matching model.[20] Spearman's rank correlation coefficient, a measure of sorting, is 0.095. That is, the extent of positive sorting is not high despite the production complementarity. Mismatch is substantial. This is in line with evidence for the US presented by Lise et al. (2016), who report a small degree of positive sorting. Lochner and Schulz (2022) use German data and measure sorting by correlating firm types based on estimated firm productivity and worker types based on estimated worker ability. They find a correlation of 0.07.

4.3 The Amplification Effect of Sorting

The Shimer Puzzle revolves around the canonical search-matching model's ability (or lack thereof) to explain the volatility of the unemployment rate, the vacancy rate, aggregate labor market tightness, and the job-finding rate over the business cycle. The search-matching model with sorting produces sufficient amplification in response to shocks and can match the data. Second moments of simulated time series data are of the same order of magnitude as the volatility observed in U.S. labor market data for the relevant period of time. In particular, the simulated standard deviations of unemployment, vacancies, labor market tightness, and the job-finding rate are much closer to empirical second moments than simulated data from standard search and matching models. Table 3 compares my results to those of Shimer (2005) and the empirical data moments.

The first two rows of Table 3 show the well-known unemployment-volatility puzzle emphasized by Shimer (2005). The standard deviations of unemployment, U, vacancies, V, labor market tightness, θ, and the job-finding rate, $q_u(\theta)$, in simulated time series data miss the empirical standard deviations by a factor of 10 to 20. I first replicate Shimer's results by simulating a model from without worker/firm heterogeneity and sorting. That is, I set output to 1 and use the same calibration as Shimer (2005). Vacancy posting costs c and the value of home production b are constants in this case. reassuringly, the

[20]The canonical model generates a mean-min ratio of at most 1.05 according to Hornstein et al. (2011).

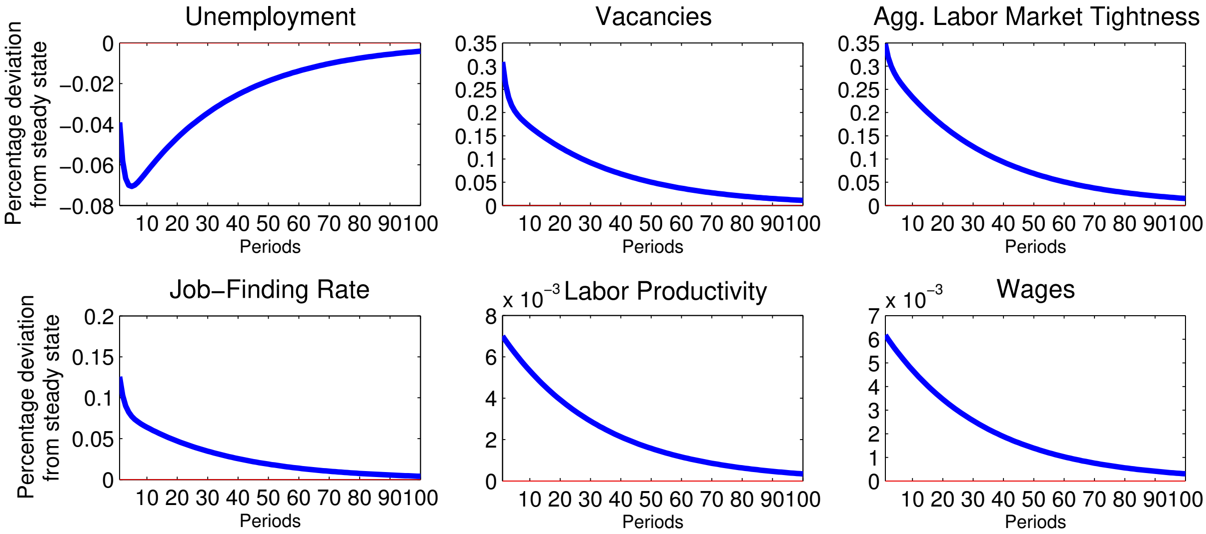
results in the third row of Table 3 are nearly identical to Shimer's results.

My main result is reported in the fourth row of Table 3. The second moments of simulated time series data from the model with sorting are much closer to the data than are those from the standard model without sorting. The standard deviation of the HP-filtered time series of labor market tightness (0.380) is very close to the data (0.382). The standard deviations of vacancies (V) and the job-finding rate (q^u_t)) are also much higher than in the baseline model; they even overshoot their empirical counterparts to some extent. This is evidence for strong amplification through firms' vacancy posting, as conjectured in Section 2.3. The standard deviation of unemployment (U) is amplified as well but remains somewhat lower than the empirical value. This is unsurprising in a sorting model. As shown in Section 2.3, the response of the unemployment rate differs across worker types. For high-type workers, unemployment falls disproportionately in response to a positive shock because their outside option changes the most. In contrast, low-type workers are willing to match with all firm types, so the effect of the outside option is muted form them. To improve upon the sorting model's ability to fully match the dynamics of the unemployment rate, one promising approach would be to drop the assumption that the underlying exogenous distribution of worker types, $g_w(x)$ is uniform.

Finally, note also that overall match-specific output, $F(x, y, z)$), fluctuates more in the sorting model than z. This is due to the fact that, as explained in Section 2.3, the production function implies that a higher z increases the incentive to sort (procyclical sorting). It becomes relatively more valuable to be optimally matched as z increases, so $F(x, y, z)$ is more volatile in response to shocks than z alone.

To illustrate the dynamics of the sorting model, Figure 3 shows impulse response functions of six key variables: unemployment, vacancies, aggregate labor market tightness, and the job-finding rate, as well as the autoregressive labor productivity process and wages. In response to a positive shock to labor productivity, unemployment falls by about 6 percentage points initially and shows a hump-shaped return to steady state. Vacancy posting, job finding, and aggregate tightness of the labor market show strong positive reactions directly after the shock. All impulse responses show a high degree of persistence as well as realistic correlations and cyclical properties. For instance, unemployment and vacancies move in opposite directions in response to the shock and are thus highly negatively correlated (Beveridge Curve). Note also the less-than-proportional adjustment of wages in response to the labor productivity shock: the initial adjustment of wages is roughly 85% of the jump in labor productivity in the depicted example, so in contrast to the textbook model, wages do not increase one-to-one with labor productivity. This is due to the fact that wages are partly shielded from aggregate fluctuations in this model, recall Section 3.2.

Figure 3: Impulse Response Functions of Key Variables in the Search and Matching Model with Sorting



5 Conclusions

I propose a search-matching model with two-sided heterogeneity and sorting and study the amplification of productivity shocks in this framework. The model provides new adjustment mechanisms in response to shocks that are explicitly related to sorting. In particular, firms' vacancy posting decisions are affected along three margins. First, In response to a positive shock, the surplus increases for all matches, and this effect increases in the worker and the firm type. Second, due to higher surplus, the matching cutoff gets wider. That is, previously unattractive matches become viable and firms can expect to employ additional worker types. Third, the incidence of unemployment varies across worker types. High-type workers are picky and thus more likely to be unemployed, so firms that get access to high-type workers as the matching cutoff widens show a particularly strong vacancy response.

I argue that adding to sorting and heterogeneity to search-matching models is a both micro-founded and empirically supported complement to exiting approaches to analyze labor market data. The sorting model produces sufficient amplification in response to shocks and overcomes the unemployment-volatility puzzle. Second moments of simulated time series data are of the same order of magnitude as the volatility observed in U.S. labor market data. For future research, it should be noted that the model in this paper is a stylized model that has been designed to describe a novel adjustment mechanism. Many simplifying assumptions should be relaxed, and the model's ability to match cross-sectional facts in addition to time series facts need to be scrutinized.

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A Additional Results

A.1 Computational Errors

To check the accuracy of the computational method described in Section 4.1, I plug simulated data from the dynamic sorting model back into the Bellman equations of the model. Ideally, the simulated data would solve these equations exactly. However, I make heavy use of discretization and approximation techniques, so it is reasonable to expect some imprecision. For convenience, I use the wage equation for this test because it contains both the firms' and the workers' integral terms:

$$W(x,y,\Omega) - \alpha \left(F(x,y,z) + c(g_v(y,\Omega))\mathbb{E}\left[\frac{\int_0^1 g_v(y,\Omega') \max\{\mathcal{S}(x,y,\Omega'),0\}\,\mathrm{d}y}{\int_0^1 g_u(x,\Omega') \max\{\mathcal{S}(x,y,\Omega'),0\}\,\mathrm{d}x}\right]\right) \\ - (1-\alpha)b(x) \stackrel{?}{=} 0.$$

Solving the dynamic sorting model by log-linearization and perturbation results in a mean computational error of 3.84%. The 2.5th, 50th, and 97.5th percentiles of the distribution are −15.0%, 4.67%, and 17.6%, respectively. This distribution is slightly left skewed due to the fact that the model's response to shocks is not symmetric around the steady state, for example, because of endogenous separations, which only happen after negative shock. Figures A.1 and A.2 show a histogram of the computational errors and a scatter plot that shows the positive correlation of the errors with z.

A recent paper by Petrosky-Nadeau and Zhang (2016) can serve as a benchmark for the size and distributions of the errors. The authors show that solving a representative agent search and matching model via log-linearization and perturbation—they use Hagedorn and Manovskii (2008) as an example—creates a mean computational error of 3.75% with the 2.5th, 50th, and 97.5th percentiles of the distribution being −11.1%, −3.66%, and 8.76%, respectively. I conclude from this that the errors resulting from the computational approach in this paper lead to errors of an expectable magnitude, even though the errors I find are slightly more dispersed than what Petrosky-Nadeau and Zhang (2016) find for a representative agent search and matching model.

Figure A.1: Histogram of Computational Errors

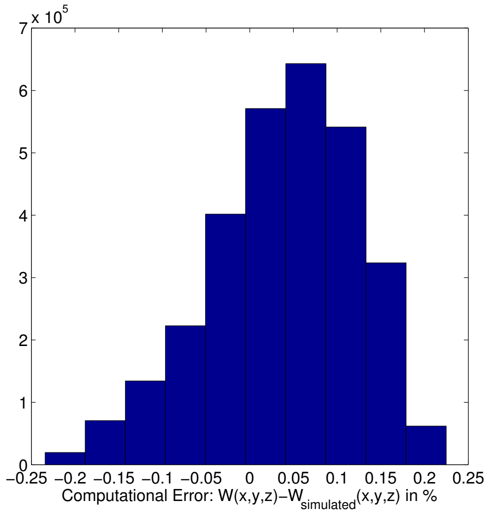
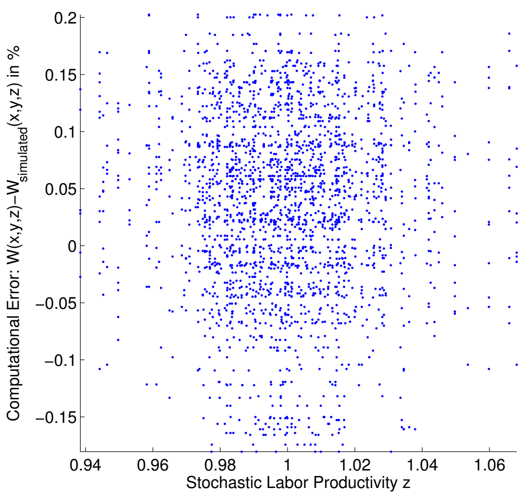


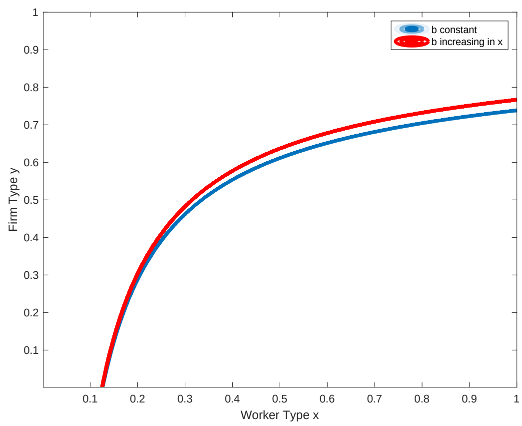
Figure A.2: Scatter Plot of Computational Errors and Stochastic Labor Productivity z



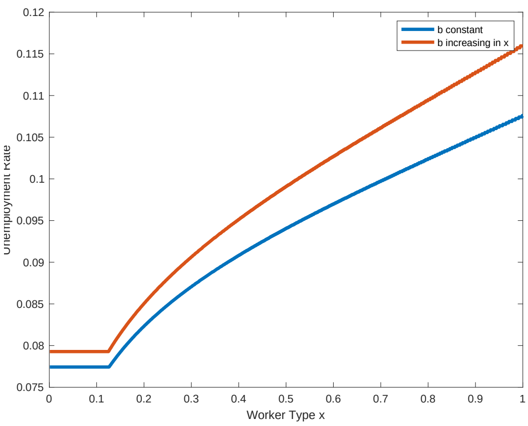
A.2 Stationary Equilibrium with constant b

Figure A.3: Stationary Equilibrium with constant b

(a) Matching Cutoffs



(b) Unemployment across Worker Types



B Wage Formation

To derive match-specific wages in the sorting model, I start with the Nash bargaining solution and impose free entry ($\mathcal{V}(y) = 0$):

$$\mathcal{E}(x, y, \Omega) - \mathcal{U}(x, \Omega) = \frac{\alpha}{1-\alpha}\mathcal{P}(x, y, \Omega). \tag{23}$$

Plugging in the value functions and maximizing the Nash product yields an expression for the match-specific wage in the dynamic model, $W(x, y, \Omega)$:

$$\begin{aligned} W(x, y, \Omega) &= \alpha F(x, y, z) + (1-\alpha)b(x) \\ &\quad + (1-\alpha)\beta\alpha\mathbb{E}\left[q_u(\theta(\Omega'))\int_0^1 \frac{g_v(y, \Omega')}{V(\Omega')}\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y\right]. \end{aligned} \tag{24}$$

As in the standard model, the wage is a convex combination of match-specific output, $F(x, y, z)$, and the worker's outside option. In the sorting model, the integral term deserves attention: the outside option depends on the expected value of the surplus with all other potential employers in the matching set, weighted by the distribution. Thus, the higher the surplus and the higher the probability of meeting other firm types that the worker is willing to match with, the higher the outside option and the bargained wage. After factoring out α, Equation (20) can be used to bring the firm's integral term into the denominator of the expectation:

$$\begin{aligned} W(x, y, \Omega) = \alpha &\left(F(x, y, z) + c(g_v(y, \Omega))\mathbb{E}\left[\theta(\Omega')\frac{\int_0^1 \frac{g_v(y,\Omega')}{V(\Omega')}\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y}{\int_0^1 \frac{g_u(x,\Omega')}{U(\Omega')}\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}x}\right]\right) \\ &+ (1-\alpha)b(x). \end{aligned} \tag{25}$$

Now, the same logic applies from the firm's perspective: the expected value of the surplus, the matching set, and the distribution of other unemployed worker types influence the negotiated wage negatively through the denominator: the more workers are available for the firm to match with and the higher the surplus, the lower is bargained wage with a specufic worker type x. Note that aggregate labor market tightness $\theta(\Omega')$ in front of the quotient cancels out with ${}^1\!/_{V(\Omega')}$ in the numerator and ${}^1\!/_{U(\Omega')}$ in the denominator:

$$W(x, y, \Omega) = \alpha\left(F(x, y, z) + c(y)\mathbb{E}\left[\frac{\int_0^1 g_v(y, \Omega')\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y}{\int_0^1 g_u(x, \Omega')\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}x}\right]\right) + (1-\alpha)b(x), \tag{26}$$

so the match-specific wage $W(x, y, \Omega)$ does not depend on aggregate labor market tightness in the sorting model. The ratio of the two integral term determines how changes in the aggregate state affect the wage.