Job Displacement, Remarriage, and Marital Sorting*

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Abstract

We investigate how job displacement affects whom men marry, and study implications for marriage market matching theory. Leveraging quasi-experimental variation from Danish establishment closures, we show that job displacement leads men to break up if matched with low earning women and to re-match with higher earning women. We use a general marriage market search and matching model as conceptual framework, to derive several implications of our empirical findings: (i) husband's and wife's earnings are substitutes, rather than complements on the marriage market (ii) our findings are challenging to reconcile with one-dimensional matching, while consistent with multidimensional matching (iii) a substantial part of the cross-sectional correlation in spouses' incomes arises spuriously from sorting on unobserved characteristics. We highlight the relevance of our results, by contrasting the impact of rising inequality on marital sorting and income inequality in one-dimensional versus multidimensional specifications of our framework.

Keywords: Marriage Market, Sorting, Search and Matching, Job Displacement, Taxation

JEL classification: D10, J12, J63, J65

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1 Introduction

Who marries whom contributes to economic inequality. This idea, which goes back to Becker (1973), has motivated an extensive body of literature that studies empirical patterns of marriage market sorting, and to what extent they contribute to trends in income inequality (see, e.g., Greenwood, Guner, Kocharkov, and Santos, 2015; Eika, Mogstad, and Zafar, 2019). A wide range of studies document strong correlations between spouses' characteristics such as income and education (see, e.g., Becker, 1973), personality traits (Becker, 1973; Dupuy and Galichon, 2014), measures of health (Chiappori, Oreffice, and Quintana-Domeque, 2017a; Guner, Kulikova, and Llull, 2018) or physical attractiveness (Oreffice and Quintana-Domeque, 2010; Chiappori, Oreffice, and Quintana-Domeque, 2012). However, less is known about the origins of these correlations. Consider the case of income. Do individuals directly value the earnings potential of their partners, and make marriage decisions based thereon? Or does the positive income correlation arise through other channels, for example because marriage decisions are based on other (potentially unobserved) characteristics that correlate with income?

In this paper, we offer empirical answers to these questions by estimating the effect of exogenous job displacements on whom displaced men marry (or enter a committed relationship with). To this end we leverage variation from establishment closures in Denmark. Our research design allows us to study how quasi-exogenous changes in a person's employment status and earnings potential affect the characteristics (such as earnings potential, age, and education) of the person they match with on the marriage market. While a wide range of studies have used establishment closures as a source of quasi-exogenous variation, we are the first to exploit this source of variation to analyze marriage market sorting.[1] Based on our findings, we derive implications for marriage market matching theory.

Our empirical design leverages quasi-experimental variation from establishment closures in Denmark between 1980 and 2007. In a difference-in-differences analysis, we compare over 75,000 displaced male workers to a non-displaced control group. We follow treatment and control group over time and compare the evolution in their relationship status, as well as their spouses' (or cohabiting partners') characteristics, such as income, age, and education. We find that displaced men are more likely to separate from their partners, relative to a non-displaced control group. Two thirds of this effect is due to men who stay single, while one third is driven by men who remarry or cohabit with a new partner. Moreover, we find that displaced men who separate from their female partners on average re-match with higher earning new partners, relative to the control group. This effect is not driven by partners' labor supply choices, but is due to men matching with

[1] The wide range of studies, spanning several fields within economics, that uses establishment closures as a source of quasi-exogenous variation goes back to Jacobson, LaLonde, and Sullivan (1993). See, e.g., Gathmann, Helm, and Schönberg (2018), Heining, Schmieder, and von Wachter (2019), and Braxton, Herkenhoff, and Phillips (2020) for recent contributions.

new partners who earn higher hourly wages.[2] We do not find notable effects on other partner characteristics, such as, age, education, or number of children. Robustness checks reveal that our results are not driven by displaced men moving to new firms or municipalities, in which women have a higher average income, or where the sex ratio is skewed towards women. Furthermore we argue that equilibrium effects of plant closures on the marriage market can plausibly be expected to be negligible, based on back-of-the-envelope calculations.

We use our empirical results to reexamine marriage market sorting and the underlying mechanisms that give rise to it. Since Becker (1973) it has been known that marriage market sorting can be explained by complementarities in the utility derived from marriage. Intuitively, likes mate if spouses' characteristics are complements, whereas unlikes mate if spouses' characteristics are substitutes.[3] Following this reasoning, different mechanisms have been proposed to explain why couples tend to be sorted positively on income and education empirically. Examples include complementarities in education (highly educated people value education in their spouse more), complementarities in home production (people who intent to spent similar hours on home production match), or complementarities in raising children.[4] Other mechanisms push towards substitutability of spouses' characteristics and thus negative sorting, such as, substitutability in home production hours (leading to household specialization) or risk sharing (Chiappori et al., 2018; Pilossoph and Wee, 2021). In simple (one-dimensional) models of marriage market matching, there is thus a tight link between complementarities in spouses' characteristics and marriage market sorting.[5]

While much of the literature focuses on one-dimensional matching, recently multimensional environments, in which people face trade-offs between different characteristics they value in their spouse, have gained attention (Dupuy and Galichon, 2014; Chiappori, McCann, and Pass, 2016; Chiappori et al., 2017a). Under multidimensional matching, the link between complementarities in spouses' characteristics and marriage market sorting becomes more complex. For example, a positive correlation between husband's and wife's incomes reflects not only sorting on income, but also sorting on other characteristics that correlate with income (potentially including unobservables). As a consequence, the positive correlation in spouses' incomes may arise due to sorting on

[2] In our data we do not find significant evidence of an added worker effect following job loss. This finding is in accord with several papers that document a small added worker effect following job displacement, e.g., Stephens (2002), Eliason (2011), Birinci (2019), Halla, Schmieder, and Weber (2020).

[3] In more technical terms, marriage market sorting depends on the supermodularity or submodularity of the household production function in the spouses' characteristics.

[4] See Chiappori, Costa-Dias, and Meghir (2018), Chiappori, Iyigun, and Weiss (2009) for models featuring complementarity in education. In Goussé, Jacquemet, and Robin (2017) and Calvo, Lindenlaub, and Reynoso (2021) complementarity in home production hours gives rise to positive sorting. Chiappori, Salanié, and Weiss (2017b) model investments in children's human capital as complements giving rise to positive sorting in the marriage market. These mechanisms are not mutually exclusive and some of the cited studies feature more than one of the described mechanisms.

[5] For example, Calvo et al. (2021) note that a strong role for household specialization is hard to reconcile with positive assortative matching.

correlates of income, even if sorting on income itself is negative (i.e., even if husband's and wife's incomes are substitutes, as predicted by Becker, 1981).[6] Evidence based on exogenous variation is thus needed to disentangle sorting on income from sorting on other characteristics, and to uncover whether husband's and wife's incomes are complements or substitutes on the marriage market.

As conceptual framework we consider a general one-dimensional marriage market search and matching model, which builds on Shimer and Smith (2000) (for applications to marriage markets see, e.g., Jacquemet and Robin, 2013; Goussé et al., 2017; Holzner and Schulz, 2023; Ciscato, 2021). We show that within this framework, it is challenging to reconcile two empirical facts: 1. our empirical finding that displaced men on average match with higher earning women, relative to a non-displaced control group, and 2. the widely documented positive correlation between spouses' incomes. Intuitively, if spouses' incomes are complements the model generates a positive correlation in spouses' incomes but predicts that upon job loss men on average match with lower earning women (at odds with our empirical evidence). By contrast, if spouses' incomes are substitutes the model predicts that upon job loss men on average to match with higher earning women (consistent with our empirical evidence) but generates a negative correlation between spouses' incomes, at odds with the data. We formally show that, under one-dimensional matching, the two empirical facts can neither be reconciled under positive assortative matching (PAM) nor negative assortative matching (NAM).

To realign theory and evidence, we propose a multidimensional extension of the Shimer and Smith (2000) framework. The distinguishing model feature vis-à-vis the one-dimensional framework is that agents consider several characteristics (such as income, age, or physical attractiveness) in their matching decisions, and face trade-offs between them. We define a notion of PAM and NAM in this framework, under which sorting is defined dimension by dimension. PAM can thus arise in one dimension, while NAM arises in another.[7] We show that the proposed framework is consistent with both empirical regularities: the positive correlation between spouses' incomes and our finding that displaced men on average match with higher earning women. Our proposed specification features negative sorting on income and positive sorting on other characteristics, generating the two regularities by the following simple logic: under negative sorting on income (holding other characteristics constant), agents who experience job loss (and thus lose income) tend to match with higher-income partners. At the same time, positive sorting on other characteristics that are positively correlated with income gives rise to a positive correlation between spouses' incomes.

[6] Becker (1981) regards this as a possible reason for the lack of empirical evidence for negative sorting on earnings: "The positive correlation between wage rates of husbands and wives [...] may really be measuring the predicted positive correlation between a husband's wage rate (or his non-market productivity) and his wife's non-market productivity. Many unobserved variables, like intelligence, raise both wage rates and non-market productivity." Lam (1988) offers an alternative explanation for the observed positive correlations despite theoretically-predicted negative sorting on income: the presence of public goods in married households.

[7] See Lindenlaub and Postel-Vinay (2016), who define sorting dimension by dimension in a multidimensional search model of the labor market. The main differences between frictional labor market and marriage market models is that in the latter matching is one-to-one and there is no entry.

The correlation between spouses' incomes is thus spuriously driven by characteristics that are correlated with income. Based on our empirical results we demonstrate that the positive correlation between spouses' incomes cannot be explained purely by sorting on variables observable in our data (specifically, income, age, and education). We show, based on a simple regression analysis, that a substantial share of the measured correlation in spouses' incomes is due to sorting on unobserved characteristics.

Finally, to highlight the policy relevance of our results, we calibrate a one-dimensional as well as a two-dimensional specification of our framework to Denmark. We show that the two-dimensional model aligns with the data while the one-dimensional model is at odds with our empirical findings. We then simulate tax reforms in each calibrated model version. The two model versions make markedly different predictions: Under one-dimensional matching, the marriage market amplifies the effect of tax progressivity on inequality. By contrast, under two-dimensional matching the impact of tax progressivity on inequality is dampened by the marriage market. Our simulations thereby reveal, that whether matching is one-dimensional (at odds with our empirical results) or two-dimensional (consistently with our evidence) make a quantitatively important difference for how the marriage market shapes the long run effects of policy.

The contribution of this paper is threefold. First, we contribute to the literature that documents empirical patterns of marriage market sorting. A wide range of previous work analyzes empirical sorting on wages or labor income (Becker, 1973; Lam, 1988; Wong, 2003) as well as a range of other characteristics, including education, age, health, BMI, and personality traits (Becker, 1973; Oreffice and Quintana-Domeque, 2010; Chiappori et al., 2012; Dupuy and Galichon, 2014; Chiappori et al., 2017a; Guner et al., 2018). Several papers in this literature have studied how the evolution of marital sorting shapes income inequality over time (e.g., Greenwood et al., 2015; Eika et al., 2019).[8] Generally, this literature documents positive correlations between spouses' wages (as well as labor incomes).[9] In this paper, we take a different route by leveraging quasi-experimental variation from job displacements. Our estimates reveal that own income losses lead men to match with higher earning women. These results point towards substitutability between husband's and wife's incomes, rather than complementarity. The measured positive correlation between spouses' incomes, according to our evidence, is driven spuriously by unobserved variables. These findings complement the existing correlational evidence from previous work.

Second, we contribute on the modeling front, by proposing an extension of the Shimer and Smith (2000) marriage market search-and-matching model to multidimensional settings. We thereby contribute to a set of studies that explore multidimensional marriage market matching. E.g.,

[8] See also the methodological contribution by Chiappori et al. (2012), who develop criteria for the suitability of different measures of marriage market sorting, for measuring changes in sorting over time.

[9] Several studies document that not only the raw correlations between spouses' wages (and labor incomes) are positive, but also the respective partial correlations, when various other observed characteristics are held constant (see, e.g., Becker, 1973).

Chiappori et al. (2012) and Dupuy and Galichon (2014) model matching on multiple observed characteristics.[10] Our extended framework adds to this literature as it accounts for multidimensional matching on observed as well as unobserved characteristics, without restricting the distributions of observed and unobserved characteristics to be independent. We show that our empirical results are hard to reconcile with one-dimensional matching, while consistent with multidimensional matching. Moreover, we argue that our empirical findings suggest that sorting on unobserved characteristics (from the researchers perspective) plays an important role. Our proposed extension of the Shimer and Smith (2000) framework captures both these aspects. We show that the extended model is capable of reconciling our empirical findings with theory.

Third, we contribute to a growing literature that takes into account the role of the family in shaping the outcomes of economic policy. A range of papers investigates how household formation and dissolution interact with economic policy, such as taxation (see, e.g., Guner, Kaygusuz, and Ventura, 2012; Bronson and Mazzocco, 2018; Holter, Krueger, and Stepanchuk, 2019; Gayle and Shephard, 2019; Siassi, 2019; Obermeier, 2019), social insurance (Low, Meghir, Pistaferri, and Voena, 2018; Persson, 2020; Schulz and Siuda, 2023), and education policy (Anderberg, Bagger, Bhaskar, and Willson, 2020). Our policy simulations complement these papers, by underscoring the relevance of accounting for marriage market sorting when evaluating policy reforms. Furthermore, our results demonstrate that policy simulations based on models that only match the raw correlation between spouses' labor incomes can be misleading. As shown by our analysis, requiring a model to be consistent with quasi-experimental evidence can alter and, in our case, overturn policy simulation results.

The remainder of our paper is structured as follows: in section 2, we introduce the marriage market matching model with one-dimensional heterogeneity and derive predictions regarding the implications of job displacement for marital sorting. Section 3 describes our data sources and empirical strategy. In section 4, we present our empirical results and show that they are in disagreement with the predictions of the one-dimensional sorting model. We explore multidimensional sorting as a possible avenue to reconcile theory and data in section 5. Section 6 presents the policy experiments and Section 7 concludes.

2 Conceptual Framework

This section introduces a search-and-matching model of the marriage market. We build on the frictional version of the classical Beckerian assignment model developed by Shimer and Smith

[10] Our framework differs from Chiappori et al. (2012) and Dupuy and Galichon (2014) in several ways. By contrast to ours, preferences in their models are specified over a one-dimensional index which combines the various characteristics. Moreover, the matching processes in their models are frictionless while ours is frictional. Importantly, both these papers assume that observed characteristics are independent from unobservables.

(2000), which features two-sided (one-dimensional) heterogeneity and transferable utility.[11] The model serves as a conceptual framework and guides the development of our empirical strategy in Section 3. Several empirical predictions emerge in this framework, and we subsequently test them in Section 4. Finally, we use the model to quantify the implications of our empirical results in Section 6.3.

2.1 Setup

We consider a two-sided matching environment populated by women, denoted by f, and men, denoted by m. Time is continuous, and discounted at rate r. Women and men are fully characterized by their types, $q_f \in Q_f$ and $q_m \in Q_m$. In general, we allow for multidimensional type spaces, assuming that $Q_f = Q_m = \prod_{k=1}^{K} \left[\underline{q}_k, \overline{q}_k\right]$, where each dimension, k, of the Cartesian product represents a distinct type attribute. We will clearly indicate whenever we examine the special case of one-dimensional matching ($K = 1$) or the more general multidimensional case ($K > 1$). Under one-dimensional matching, agent types are summarized by a single attribute (e.g., income or education), whereas multidimensional matching allows for various attributes that may or may not be correlated.

We assume random search. Denote by G_f and G_m the cumulative distribution functions (CDFs) of single women's and men's types.[12] At rate λ_f, a single woman meets a single man drawn from G_m. Vice versa, at rate λ_m, a single man meets a single woman, drawn from G_f. We follow Shimer and Smith (2000) and assume that meeting rates for men and women are proportional to the mass of singles of the other gender, $\lambda_f = \lambda \int dG_m(q_m)$ and $\lambda_m = \lambda \int dG_f(q_f)$, where λ is a common Poisson rate. Upon meeting, female and male agents observe each others' types and jointly decide whether to accept and form a match or to reject and continue the search for a partner.

2.2 Flow Utilities

Single agents' flow value depends on their type q_g ($g \in \{f, m\}$), and is given by the flow utility function $u_g^0(q_g)$.[13] Matched couples of type (q_f, q_m) enjoy the joint flow match value, $f(q_f, q_m)$, which equals the sum of the matched partners' individual flow utilities,

$$f(q_f, q_m) = u_f^1(q_f, q_m) + u_m^1(q_f, q_m), \tag{1}$$

where $u_f^1(q_f, q_m)$ is the flow utility of a type q_f woman matched with a type q_m man (and vice versa for men).

[11] Versions of the Shimer and Smith (2000) model have been applied to study marriage markets, e.g., in Jacquemet and Robin (2013), Goussé et al. (2017), Ciscato (2021), and Holzner and Schulz (2023).

[12] Note that G_f and G_m are equilibrium outcomes, i.e., endogenous objects.

[13] Model objects with superscript 0 refer to singles, whereas objects with superscript 1 refer to matched agents.

2.3 Bellman Equations and Matching

A model agent's decision problem is summarized by two Bellman equations. Denote by $\mathcal{M}(q_g)$ the matching set of a model agent of type q_g. That is, $q_f \in \mathcal{M}(q_m)$ and $q_m \in \mathcal{M}(q_f)$ hold if type q_f women and type q_m men agree to match upon meeting. It follows that the value of being a type q_m single man is given by

$$rV_m^0(q_m) = u_m^0(q_m) + \lambda_m \int_{\mathcal{M}(q_m)} (1-\mu_f)S(q_f, q_m)dG_f(q_f), \tag{2}$$

where $(1-\mu_f)S(q_f, q_m)$ is the the share of marital surplus that type-q_m men receive in a match with a type-q_f women. This Bellman equation states that the value of being single is determined by the flow utility of singlehood and the option value of matching with a partner.

The value for a type-q_m man of being matched with a type-q_f woman is determined by

$$rV_m^1(q_f, q_m) = u_m^1(q_f, q_m) + t_m + \delta(V_m^0(q_m) - V_m^1(q_f, q_m)), \tag{3}$$

where δ is the exogenous separation rate. t_m denotes the intra-household utility transfer, which may be positive or negative.[14]

Given these Bellman equations, the marital surplus is defined as

$$S(q_f, q_m) = V_m^1(q_f, q_m) + V_f^1(q_f, q_m) - V_m^0(q_m) - V_f^0(q_f). \tag{4}$$

The transferable utility assumption entails that the marital surplus can be distributed between spouses without frictions. Couples therefore match upon meeting if (and only if) the marital surplus is weakly positive (i.e. $S(q_f, q_m) \geq 0$). The transfer ensures that both spouses benefit relative to remaining single. The model is closed by assuming that the spouses share the marital surplus by Nash bargaining. Given the female bargaining power μ_f, Nash Bargaining implies that transfers are set such that the wife receives a share $\mu_f S(q_f, q_m)$ of the marital surplus while the husband receives $(1-\mu_f)S(q_f, q_m)$.

2.4 Equilibrium and Sorting

For the one-dimensional case ($K = 1$), Shimer and Smith (2000) prove the existance of an equilibrium which satisfies: 1. *individually optimal behavior:* every agent maximizes her expected payoff, taking all other agents' strategies as given. 2. *steady state:* match creation equals match destruction for each agent type (i.e., for all q_f and all q_m). Shimer and Smith (2000) further characterize

[14] The values of being a single woman or type-q_f women matched with a type-q_m man are defined analogously to (2) and (3). Transfers are constraint to be net-zero, i.e., $t_m = -t_f$.

sorting, by defining the following notions of PAM and NAM, which generalize the corresponding definition for the frictionless case by Becker (1973).[15]

Definition 1. *Consider* $q'_f < q''_f$, $q'_m < q''_m$.

There is PAM if: $q''_f \in \mathcal{M}(q'_m)$ *and* $q'_f \in \mathcal{M}(q''_m) \Rightarrow q'_f \in \mathcal{M}(q'_m)$ *and* $q''_f \in \mathcal{M}(q''_m)$

There is NAM if: $q'_f \in \mathcal{M}(q'_m)$ *and* $q''_f \in \mathcal{M}(q''_m) \Rightarrow q''_f \in \mathcal{M}(q'_m)$ *and* $q'_f \in \mathcal{M}(q''_m)$.

Intuitively, under PAM, whenever two couples, (q'_f, q''_m) and (q''_f, q'_m), can form more positively sorted matches by trading partners, they are willing to do so. By implication, higher-q_m men match on average with higher-q_f women in any PAM equilibrium. That is, $\mathbb{E}[q_f|q_m]$ is weakly increasing in q_m in the population of matched couples. By contrast, higher-q_m men will match on average with lower-q_f women in any NAM equilibrium. That is, $\mathbb{E}[q_f|q_m]$ is weakly decreasing in q_m in the population of matched couples. As a consequence, for the correlation of matched partners' types it holds that

$$\text{PAM} \Rightarrow Corr(q_f, q_m) > 0, \tag{5}$$

$$\text{NAM} \Rightarrow Corr(q_f, q_m) < 0. \tag{6}$$

Using (5) and (6), it is possible to use observed cross-sectional correlations between matched spouses' attributes to draw conclusions about marital sorting patterns. Specifically, under one-dimensional search and matching, $Corr(q_f, q_m) < 0$ is inconsistent with PAM and $Corr(q_f, q_m) > 0$ is inconsistent with NAM. Under the common assumption that agents' types map (one-to-one) into incomes or education levels, the widely documented positive correlations between spouses' income and education levels can be interpreted as evidence that refutes NAM and supports PAM.

2.5 Job Loss and Marriage Market Matching

To link our conceptual framework to the effects of job loss that we estimate in our data, we maintain the assumption of one-dimensional matching (K = 1), and interpret job displacement as a permanent change in an agent's type. Additionally we assume that labor incomes map into agent types by an increasing one-to-one function. Our interpretation is consistent with extensive empirical evidence on the long-term effects of job loss, e.g., wage scarring.[16] Formally, we assume that a man of type q_m who is displaced from his job suffers a permanent type reduction to $q_m - d$, where $d > 0$.

[15]Note that as matching is symmetric, $q_f \in \mathcal{M}(q_m)$ is equivalent to $q_m \in \mathcal{M}(q_f)$. The definition of PAM and NAM thus implies that the respective relationships with q_m and q_f interchanged hold as well.

[16]See our own empirical results in Section 4 as well as previous studies (e.g., Jacobson et al., 1993; Sullivan and von Wachter, 2009).

We use our framework to derive predictions regarding the effects of job displacement that we identify in our empirical analysis: Consider two groups of men (a "treatment group" and a "control group"), observed at two points in time, t_0 and $\tau > t_0$. Suppose men in both groups are matched with a female partner in period t_0. Men in the treatment group are displaced from their jobs in t_0, while men in the control group are not displaced between t_0 and τ. Formally, $q_m(\tau) = q_m(t_0) - d$ for the treated and $q_m(\tau) = q_m(t_0)$ for the control group. Throughout, we impose that the treatment group is small (i.e., of measure zero) so that job displacements impact the displaced agents, but do not induce a transition to a new steady state equilibrium.

We denote by D a treatment indicator, which equals 1 for the (displaced) treatment group, and 0 for the (non-displaced) control group. The CDFs of men's types in either group are denoted by $F(q_m|D = 1)$ and $F(q_m|D = 0)$, respectively. We further denote by D_B an indicator for whether a man experiences a breakup from his t_0-partner between t_0 and τ. D_R denotes an indicator for whether he (re-)matched with a new partner between t_0 and τ.

In our empirical analysis we estimate the following effects of job displacement.

1. The impact of job displacement on breakup-risk:

$$\gamma_B = P(D_B = 1|D = 1) - P(D_B = 1|D = 0).$$

2. The impact of job displacement on which male and female types experience a breakup:

$$\begin{aligned}\gamma_{q_m|B} &= \mathbb{E}[q_m(t_0)|D_B = 1, D = 1] - \mathbb{E}[q_m(t_0)|D_B = 1, D = 0]\\ \gamma_{q_f|B} &= \mathbb{E}[q_f(t_0)|D_B = 1, D = 1] - \mathbb{E}[q_f(t_0)|D_B = 1, D = 0]\end{aligned}$$

3. The impact of job displacement on the risk of staying single after a breakup:

$$\gamma_{R=0|B} = P(D_R = 0|D_B = 1, D = 1) - P(D_R = 0|D_B = 1, D = 0)$$

4. The impact of job displacement on the expected female type a man re-matches with after a breakup:

$$\gamma_{\Delta q_f|R} = \mathbb{E}[q_f(\tau) - q_f(t_0)|D_R = 1, D_B = 1, D = 1] - \mathbb{E}[q_f(\tau) - q_f(t_0)|D_R = 1, D_B = 1, D = 0].$$

Note that both a treatment and a selection margin contribute to $\gamma_{\Delta q_f|R}, \gamma_{q_m|B}, \gamma_{q_f|B}$ and $\gamma_{R=0|B}$: First, job displacement may affect *which types of men* experience a breakup. Second, for a given man, job displacement may have an effect on his propensity to find a partner, or to re-match with a specific q_f-type. We leverage our conceptual framework to derive and test predictions regarding both the treatment and the selection margin.

Based on our conceptual framework we show that the following relationships between marriage market sorting and the described effects of job displacement hold:

Proposition 1. *Consider the described matching environment in steady state equilibrium.*

Under PAM or NAM:

1. *Job displacement increases the separation risk:* $\gamma_B \geq 0$.

2. *Job displacement may increase or decrease the probability of staying single:* $\gamma_{R=0|B}$ *may be positive or negative.*

Under PAM:

3.-a *Job displacement leads men to re-match with women of lower type:* $\gamma_{\Delta q_f|R} \leq 0$.

4.-a *The association between job displacement and partner type is bounded above:* $\gamma_{\Delta q_f|R} \leq \overline{\gamma}_{\Delta q_f|R}$. *The upper bound is given by*

$$\overline{\gamma}_{\Delta q_f|R} = -\int\int_0^d \frac{\partial \mathbb{E}[q_f|q_m]}{\partial q_m}\Big|_{q_m=q-x} dx dF(q|D_R=1, D_B=1, D=1) \leq 0.$$

5.-a *If* $F(q_m|D_B=1, D=1) \leq F(q_m|D_B=1, D=0)$ *holds additionally. Then, on average, women whom displaced men separate from are of higher type than women whom non-displaced men separate from:* $\gamma_{q_f|B} \geq 0$.

Under NAM:

3.-b *Job displacement leads men to re-match with women of higher type:* $\gamma_{\Delta q_f|R} \geq 0$.

4.-b *The association between job displacement and partner type is bounded below:* $\gamma_{\Delta q_f|R} \geq \underline{\gamma}_{\Delta q_f|R}$. *The lower bound is given by*

$$\underline{\gamma}_{\Delta q_f|R} = -\int\int_0^d \frac{\partial \mathbb{E}[q_f|q_m]}{\partial q_m}\Big|_{q_m=q-x} dx dF(q|D_R=1, D_B=1, D=1) \geq 0.$$

5.-b *If* $F(q_m|D_B=1, D=1) \geq F(q_m|D_B=1, D=0)$ *holds additionally. Then, on average, women whom displaced men separate from are of lower type than women whom non-displaced men separate from:* $\gamma_{q_f|B} \leq 0$.

Relationships (*3.-a*) and (*3.-b*) show that in case of PAM or NAM marriage market sorting pins down the sign of the association between job displacement and partner type, $\gamma_{\Delta q_f|R}$. Moreover, relationships (*4.-a*) and (*4.-b*) show that $\gamma_{\Delta q_f|R}$ is bounded away from zero, by bounds that are determined by the slope of $\mathbb{E}[q_f|q_m]$ in q_m.

In Sections 3 and 4 we leverage quasi-experimental variation from plant closures to obtain empirical estimates of γ_B, $\gamma_{q_m|B}$, $\gamma_{q_f|B}$, $\gamma_{\Delta q_f|R}$, and $\gamma_{R=0|B}$. We then leverage the relationships implied by Proposition 1 to confront the described marriage market matching framework with our empirical evidence.

3 Empirical Strategy

Our research design compares roughly 77,000 workers who lose their job due to an establishment closure to a control group of workers who have similar observable characteristics but are unaffected by establishment closures during our sample period. The following subsections describe our data sources, the identification of establishment closures in the data, the empirical matching technique used to define treatment and control group, and the empirical design including regression specifications.

3.1 Data

Our empirical analysis relies on Danish register data, which cover the entire population of men and women living in Denmark between 1980 and 2007.[17] The data are drawn from tax and social security records and include information about employment status, labor income, occupation, work hours as well as marital status and children. The data allow us to analyze both married and unmarried cohabiting couples. Cohabiting couples are defined as two opposite-sex individuals who share the same address, exhibit an age difference of less than 15 years, have no family relationship, and do not share their accommodation with other adults.[18] For employed individuals, the data further identify the establishment a person works at and the number of workers employed at the establishment, among other things.

3.2 Establishment Closures

On the firm side, we rely on the Integrated Database for Labor Market Research (IDA), a register that links workers to firms. To identify establishment closures, we consider establishments that stop operating, i.e., completely shed their workforce. To define the closure year we consider the last three years of establishment operation and pick the first among these years with a workforce reduction of 10% or more. The idea behind this strategy is that the first wave of layoffs can be interpreted as an exogenous shock that affects the workers. Layoffs that occur later, i.e., closer to the final closure date, are likely anticipated by the workers. In our main analysis, we exclude establishments with less than 5 employees in the last period prior to the closure year. These criteria broadly follow the definition of establishment closures in Browning and Heinesen (2012).

3.3 Treatment and Control Group

Based on this definition of establishment closure, we define our group of treated, that is, laid-off workers. Our treatment group consists of men who are employed at a closing establishment in

[17]Our sample stops in 2007 due to a change in the definition of family types in the Danish registers.

[18]This is the official definition of cohabiting couples that Statistics Denmark uses to define family types. Many other studies of couples' behavior that use Danish data rely on it (e.g., Svarer, 2004; Datta Gupta and Larsen, 2007; Datta Gupta and Larsen, 2010; Bruze, Svarer, and Weiss, 2015).

the year of their layoff. To focus on men who are strongly attached to the labor market, we only include men with at least three years of tenure at the closing establishment and who are in the age range 25–45 in the closure year. The time window we consider extends from five years prior to ten years after establishment closure, so ten years after closure the considered men are between 35–55 years old.

Next, we draw a control group. We rely on matched sampling to obtain a group of men who resemble our treatment group in terms of observable characteristics, but who were not laid off as part of an establishment closure during our sample period. More specifically, we draw our control group from all men in our data with tenure of at least three years at establishments that did not close during our observation period. We perform coarse and exact matching (Iacus, King, and Porro, 2012, 2019) to ensure the similarity between treatment and control group in terms of observables. The matching approach discretizes continuous matching variables into bins and then matches each displaced individual with a control individual in the same bin. This method has favorable statistical properties in small samples (see Iacus et al., 2012), as well as the appeal of being straightforward to interpret.

The variables that we perform our exact matching algorithm on are marital status (single, cohabiting, married, divorced), age, children (binary indicator), calendar year, occupation (6 categories), industry (9 groups), establishment size quintiles, and tenure quintiles. We match treatment and control group with respect to each of these variables three years before establishment closure. In the end, we have 77,084 individuals in both the treatment and the control group.

To assess to what extent our matching approach has created balanced treatment and control groups, we compare summary statistics between the groups. To this end, Table 1 displays sample means for a range of observables three years before the establishment closure. The sample means are very close in all cases. The largest percentage difference is approximately 0.7% (for tenure years and number of children). Some of the displayed sample means are significantly different between treatment and control group. Statistical significance arises as a consequence of the large sample size and high precision of the administrative data that leads to very small standard errors. Given the small magnitudes of these differences, we take them to be economically insignificant and conclude that treatment and control groups are balanced in terms of observables.

3.4 Regression Specification

To study how job displacement affects marriage market matching, we estimate the following regression specification, using a set of different outcome variables Y_{it} (described below):

$$Y_{it} = \sum_{\tau=-3}^{10} \alpha_\tau \mathbf{1}\{t = \tau\} + \sum_{\tau=-3}^{10} D_i \beta_\tau \mathbf{1}\{t = \tau\} + e_{it}, \tag{7}$$

Table 1: Summary statistics in year $t = -3$, treatment vs. control group

	Treatment	Control	P-value
Labor income (in DKK)	313,040	311,687	0.005
Partner's labor income (in DKK)	168,728	169,659	0.079
Partner's age	34.19	34.23	0.203
Education (years)	12.55	12.60	0.000
Partner's education (years)	12.10	12.15	0.000
Tenure (years)	4.43	4.39	0.101
No. of children	1.51	1.52	0.146
No. observations	77,084	77,084	

Notes: This table reports summary statistics for treatment and control group in year $t = -3$ relative to job displacement.

where i is an individual index and t is time relative to the year of displacement of the treated individuals. D_i is a dummy variable indicating whether an individual is in the treatment group and e_{it} is the residual error term. The coefficients of interest are β_τ, which correspond to the effect of the displacement on the treatment group relative to the control group in the year of displacement and subsequent years. The key assumption to identify β_τ is that the trend in Y_{it} would have been parallel across treatment and control group in the absence of job displacement.

4 Empirical Results

This section presents our empirical results. We establish four main empirical findings: (i) job displacement increases the risk of relationship dissolution, (ii) job displacement particularly increases the relationship dissolution risk for men matched with low earning women, (iii) job displacement increases the probability of being single, as well as the probability of being matched with a new partner (in the 10 years following displacement), (iv) displaced men who separated re-match with higher earning women, compared to non-displaced men.

In Section 4.1 we report the long-run effect of job displacement on employment and earnings. We then document our main empirical findings in Sections 4.2-4.4. Section 4.5 reports effects on additional outcomes (such as age, education, and number of children). We support our results with several robustness checks in Section 4.6, ruling out that our results are driven by men, moving to firms or municipalities with favorable marriage market conditions. We further provide back of the envelope calculations that suggest our results are not driven by marriage market equilibrium effects.

4.1 Labor Income, Employment, Hourly Wages, and Work Hours

In this subsection we report the effects of job displacement on various labor market outcomes. Figure 1 plots estimates of β_τ from estimating Equation (7) using as outcome variables, Y_{it}, an employment dummy (Panel A), annual labor income (Panel B), hourly wages (Panel C), and work hours (Panel D). The regressions for labor income and work hours include zeros for non-employed individuals. The regression for wages are conditional on employment.

Panel A shows that job displacement has a persistent negative impact on labor income that builds up to around -13,000 DKK in the first four years post displacement and persists at around -8,000 DKK ten years after displacement (around -3% of treated men's pre-displacement income). The remaining graphs decompose this effect on labor income into the employment, work hours, and hourly wage margin. Panel B shows that displacement initially reduces the probability of employment by 3 percentage points, implying that a large share of displaced men find a new job within the first year after displacement. This effect persists at around -1 percentage point 10 years after displacement. Panel C shows that displacement reduces hourly wages by around 6 DKK (around 3% of treated men's pre-displacement wage), revealing that while most men find a new job after displacement, these jobs pay significantly lower wages. Panel D shows that displacement also reduces work hours, by around 1.5 weekly hours initially. This effect persists at around -0.5 weekly hours. In sum we find that displaced men experience a persistent reduction in labor income by 3% which is predominantly driven by displaced men switching to jobs that pay lower hourly wages.

4.2 Relationship Status

This subsection establishes that job displacement increases the risk of relationship dissolution. Figure 2 Panel A-C plot estimates of β_τ from estimating Equation (7), using different measures of relationship status as outcome variable, Y_{it}. More specifically, Panel A uses an indicator of whether i is separated from his pre-displacement partner as outcome. Our results in Panel A show that 10 years after displacement, displaced men are 1.3 percentage points more likely to have separated from their pre-displacement partner. Panel B and C decompose this effect into men staying single and men matching with a new partner. Panel B considers an indicator of whether i is single (i.e., not cohabiting and unmarried). Panel C considers an indicator of whether i is matched with a new partner (different from his $t = -3$ partner). Roughly two thirds of the displacement effect on separations thus result in an increase in singles, while one third leads to matches with new partners.

Figure 1: Labor Market Effects of Job Displacement

Notes: The figure shows the effect of establishment closure on different labor market outcomes. The displayed estimates show the impact of job displacement on labor income (Panel A), employment (Panel B), hourly wages (Panel C), and weekly work hours including zeros for non-employed (Panel C). The estimates correspond to estimates of β_τ from equation (7). All estimates are based on a sample of men who experienced an establishment closure between 1980-2007, and the same number of control individuals selected by exact matching. The sample selection criteria and matching algorithm are described in subsection 3.3.

4.3 Which Couples Separate?

Next, we investigate if men's job displacement affects which types of couples separate. Specifically, we ask: are the earnings of women and men in separating couples systematically different between treatment and control group? To adress this question we consider couples' pre-displacement labor incomes. Here we focus on pre-displacement outcomes rather than post-displacement outcomes as we aim to understand pre-existing differences between separating couples in treatment versus control group, i.e., we draw comparisons before the treatment group's labor incomes were affected by job displacement.

Figure 2 Panel D plots the mean differences between separating couples in treatment and control group. The plot shows that the pre-displacement labor income of men in separating couples is not statistically significantly different between treatment and control group (p-value $= 0.16$). In contrast, women in separating couples have significantly lower pre-displacement labor income in the treatment group compared to the control group (point estimate= -1380, p-value $= 0.04$). These results suggest that job displacement particularly increases the relationship dissolution risk for men matched with low earning women.

4.4 Re-matching and Partner Earnings

We now turn to analyzing how job displacement impacts whom (i.e., which types of women) men re-match with. In this section we analyze, if job displacement leads men to re-match with women of higher or lower earnings, compared to their pre-displacement partners. In the subsequent section we report analogous results for other partner characteristics (such as age, education, and number of children from previous relationships).

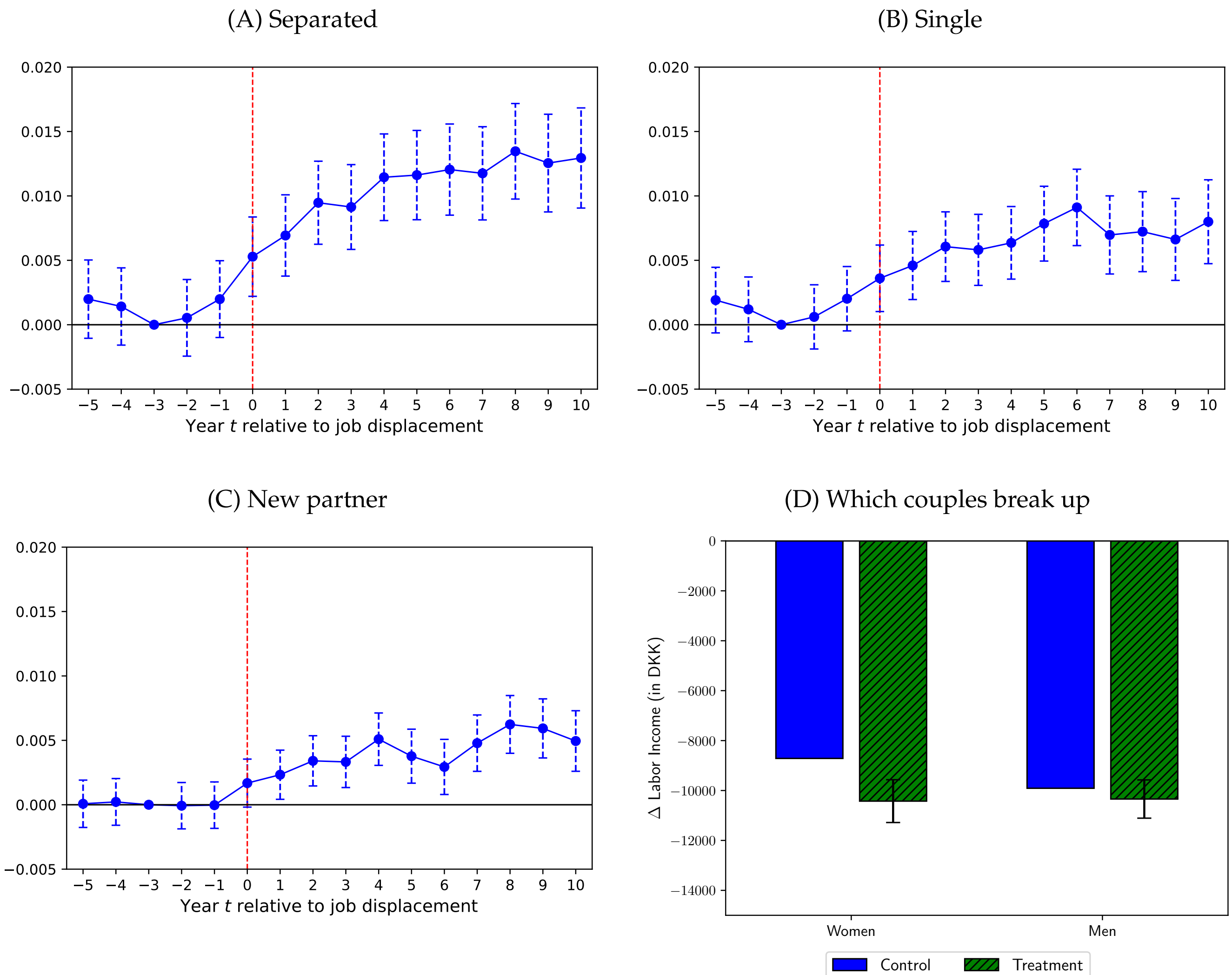
First, we estimate Equation (7) using outcome variables that indicate if men switch to higher, lower or similarly earning women, compared to their last partner. Formally we define

$$\begin{aligned} Y_{it}^{+} &= \mathbf{1}\{Q_{it} > (1+\rho)\widetilde{Q}_{it}\} \\ Y_{it}^{0} &= \mathbf{1}\{(1-\rho)\widetilde{Q}_{it} < Q_{it} < (1+\rho)\widetilde{Q}_{it}\} \\ Y_{it}^{-} &= \mathbf{1}\{Q_{it} < (1-\rho)\widetilde{Q}_{it}\} \end{aligned}$$

where Q_{it} denotes the earnings of individual i's period t partner, $\widetilde{Q}_{it}$ denotes the earnings of i's pre-displacement partner, and ρ is a pre-specified threshold value. Y_{it}^{+} indicates if i switched to a new partner who outearns $i's$ pre-displacement partner by more than $\rho \cdot 100\%$. Y_{it}^{-} indicates if i switched to a new partner who earns less than $(1-\rho)\cdot 100\%$ compared to i's pre-displacement partner. Y_{it}^{0} indicates if i switched to a new partner who earns within a $\pm\rho \cdot 100\%$ range of $i's$ pre-displacement partner. For our main analysis we fix $\rho = 0.05$, and consider as earnings measures both, annual labor incomes as well as hourly wages.

Figure 3 Panel A-C plot estimates of β_τ from estimating Equation (7), using Y_{it}^{+} (Panel A), Y_{it}^{0} (Panel B), and Y_{it}^{-} (Panel C) as outcome variables. Panel A and B show that job displacement increases the probability of matching with (at least 5%) higher earning women, as well as the probability of matching with women within a $\pm 5\%$ earnings range of the pre-displacement partner. Panel C shows that job displacement does not lead to additional matches with (at least 5%) lower earning women, compared to men's pre-displacement partners. Taken together, the results suggest that job displacement leads men to re-match with higher earning women.

Figure 2: Impact of Job Displacement on Relationship Status

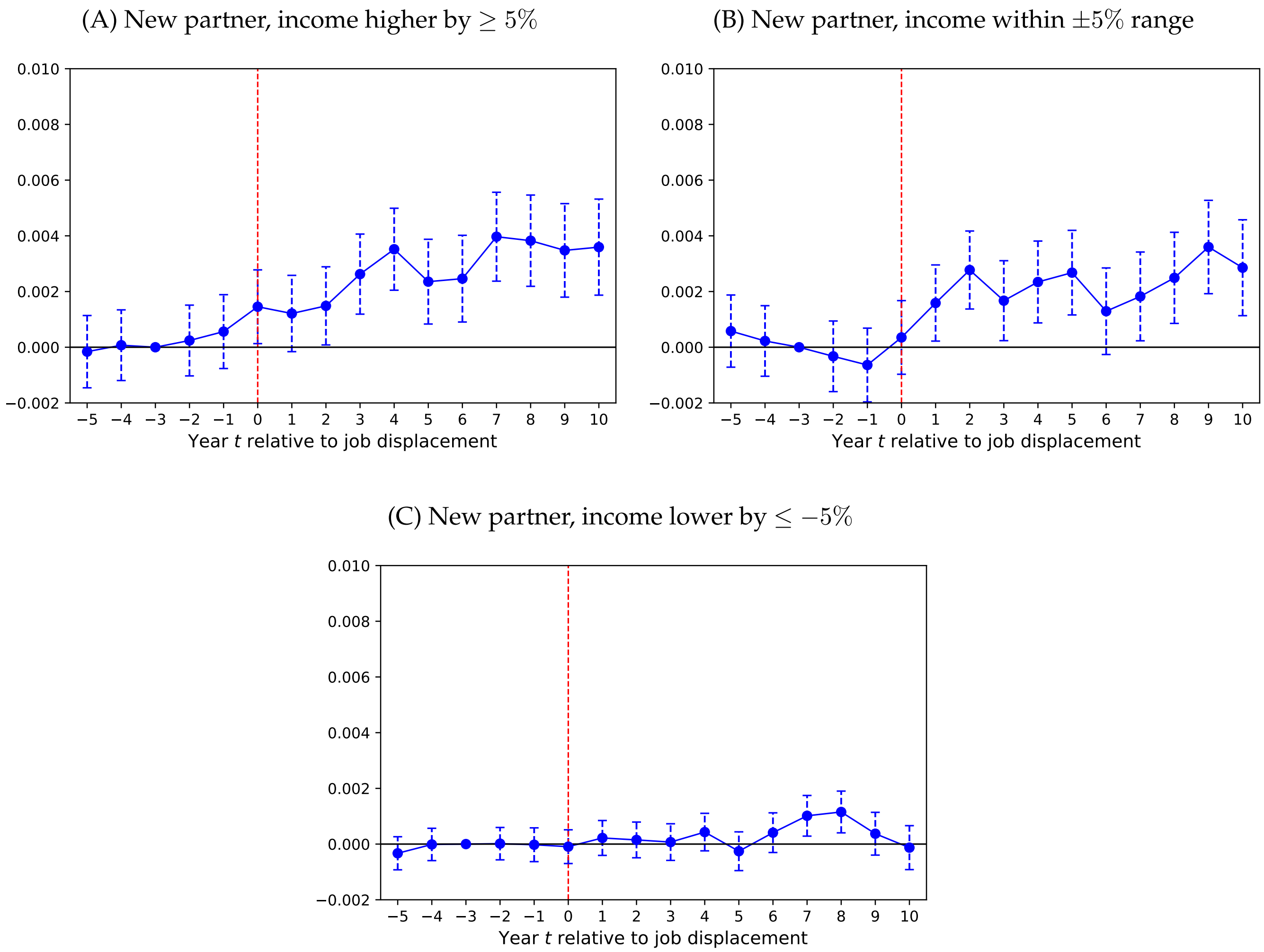


Notes: The figure shows the impact of job displacement on different measures of relationship status. Panel A shows the impact of job displacement on the probability of being separated from the pre-displacement partner. Panel B shows the impact of job displacement on the probability of being single (i.e., unmarried and not cohabiting). Panel C shows the impact of job displacement on the probability of being matched (married or cohabiting) with a new partner who is distinct from the pre-displacement partner. Panel D shows the impact of job displacement on which male and female types (in terms of their labor income) experience a breakup. All estimates correspond to coefficient estimates of β_τ in equation (7) based on our sample of men who experienced an establishment closure between 1980-2007, and the same number of control individuals selected by coarse and exact matching. The sample selection criteria and matching algorithm are described in subsection 3.3.

Next, we assess the magnitude of this effect. To this end, we directly compare re-matching patterns, in terms of partner earnings, between treatment and control group. We estimate the following specification

$$Q_{it} - \widetilde{Q}_{it} = \gamma_0 + \gamma_1 D_{it} + u_{it}, \tag{8}$$

Figure 3: Impact of Job Displacement on Partner Income



Notes: The displayed results show the effect of job displacement on the female type a man re-matches with after a breakup. Panel A shows the impact of job displacement on the probability of matching with a new partner (who is distinct from the pre-displacement partner) who outearns the pre-displacement partner by at least 5%. Panel B shows the impact of job displacement on the probability of matching with a new partner who earns 95% or less of the pre-displacement partner's income. Panel C shows the impact of job displacement on the probability of matching with a new partner who earns within a ±5% range of the pre-displacement partner's income. The estimates correspond to estimates of β_τ in equation (7). All estimates are based on a sample of men who experienced an establishment closure between 1980-2007, and the same number of control individuals selected by exact matching. The sample selection criteria and matching algorithm are described in subsection 3.3.

i.e., we regress the earnings difference between individual i's period t partner and i's pre-displacement partner on a treatment dummy. We consider several specifications, using annual labor income and hourly wages as earnings measure. As a third outcome we consider differences in work hours between i's period t partner and i's pre-displacement partner, to assess the extent to which our results are driven by differences in labor supply. We run these specifications for the post-displacement period (i.e., for $t \geq 0$) and conditional on having re-matched with a new partner

Table 2: Impact of Job Displacement on New Partner's Income, Wage, and Work Hours

	Income	Wage	Work Hours
Treatment	2763.7**	2.263**	0.139
	(1293.57)	(1.08)	(0.18)
Constant	-1111.0	-1.317***	0.405*
	(933.72)	(0.77)	(0.13)
N	59270	35011	45321

Notes: The table reports coefficient estimates of γ_1 in (8). All estimates are based on a sample of men who re-match with a new partner (who is distinct from their pre-displacement partner) after job displacement, and a control group selected by coarse and exact matching. Standard errors are reported in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

(who is distinct from i's pre-displacement partner).[19]

Table 2 summarizes the results. The coefficient estimates confirm that job displacement leads men to re-match with higher earning women. Compared to the control group displaced men experience a statistically significant increase in partner earnings of 2763.7 (p-value $= 0.032$) upon switching to a new partner. The estimation results for wages and work hours show that the effect on labor income is driven by a 2.26 effect on hourly wages (p-value $= 0.035$), while we document a small, statistically insignificant effect on partner work hours. Scaling by the income loss upon job displacement, which we estimate at $\Delta q_m = -8514.244$, yields

$$\frac{\gamma_{\Delta q_f|R}}{\Delta q_m} = -0.33,$$

implying that, among the subgroup of men who experience a breakup and rematch with a new partner, a 1 unit loss in own income is associated with matching with a 0.33 unit higher earning partner.

4.5 Additional Partner Characteristics

In Table 3 we document treatment effects of job displacement on re-matching patterns in terms of several additional outcome variables. We run specification (8) using age, education (years of schooling), and number of children from previous relationships as outcome variables. The estimation results show that job displacement has a small significant effect on partner age, of -0.14 years, as well as a small significant impact of 0.03 on number of children. The effect on partner

[19] Note that for individual's who are still matched with their pre-displacement partner, by definition $Q_{it} - \widetilde{Q}_{it} = 0$.

Table 3: Impact of Job Displacement on New Partner's Age, Education, and No. of Children

	Age	Education	No. of children
Treatment	-0.140**	-0.0127	0.0306**
	(-2.59)	(-0.91)	(2.95)
Constant	6.454***	0.344***	-0.233***
	(165.90)	(34.32)	(-31.17)
N	71090	67999	71090

Notes: The table reports coefficient estimates of γ_1 in (8). All estimates are based on a sample of men who re-match with a new partner (who is distinct from their pre-displacement partner) after job displacement, and a control group selected by coarse and exact matching. Standard errors are reported in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

education is small and statistically insignificant at -0.01 years of schooling.

4.6 Robustness

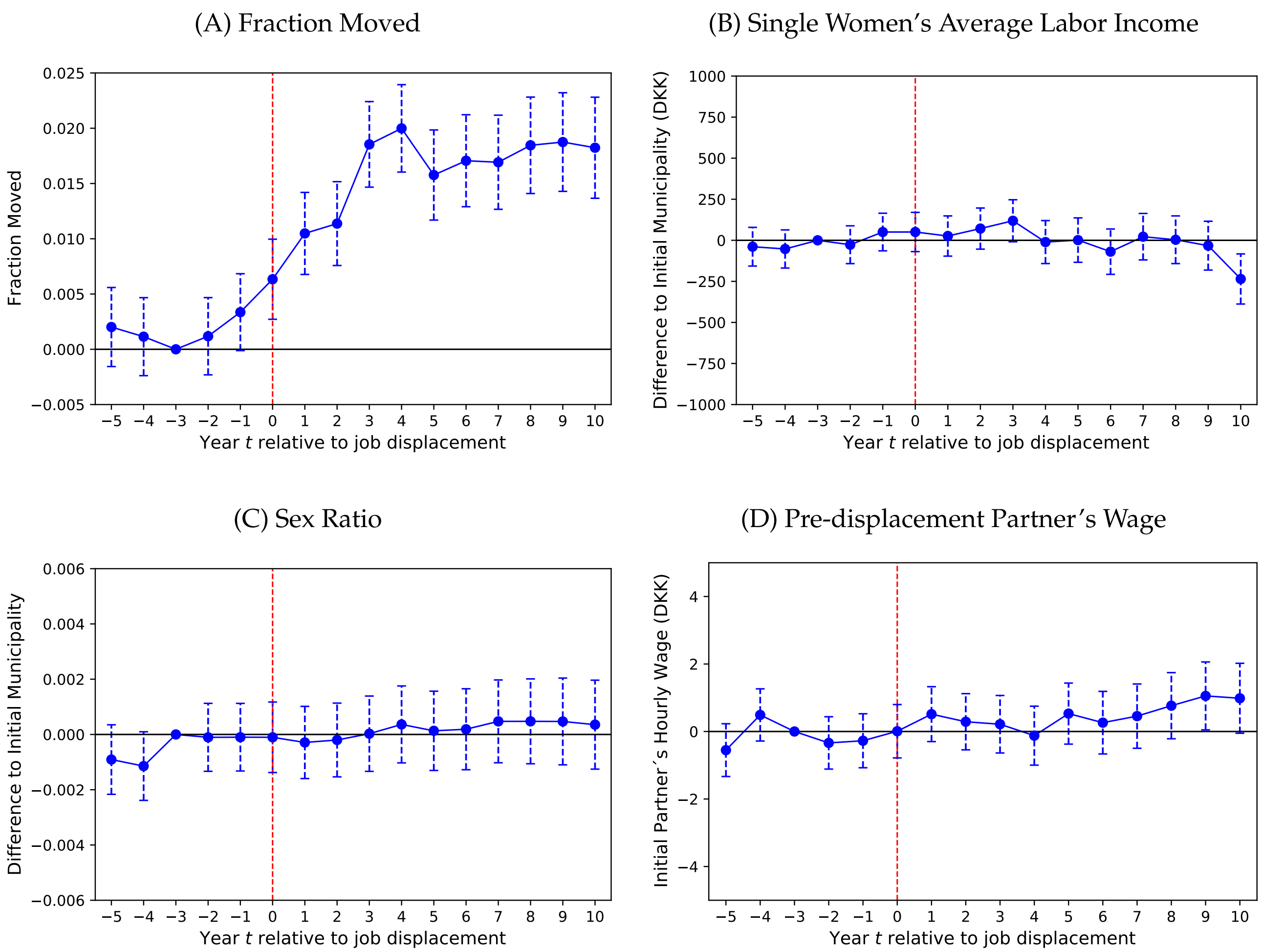
We rule out several alternative explanations for our empirical findings that would be inconsistent with the mechanisms of our conceptual framework.

Moves Across Local Marriage Markets. First, we look at the role of geographical moves. Figure 4 Panel A shows how the displacement affects the propensity to move by estimating our previous specification with an indicator variable for whether the individual has moved to a different region. The figure shows that the impact of displacement on the likelihood of having moved to a different municipality is significantly positive and stabilizes at 1.5 p.p. 3 years after displacement.

Since the figure shows that displaced workers adjust their mobility behavior, it is also important to check whether the regions they move to differ from the choices of the control group. For example, if displaced workers were, for whatever reasons, moving to regions in which women on average earn more than in their previous region, this would partially explain why they find a better partner after their displacement. However, it turns out that there is hardly any difference between the regional earnings of women between the treatment and control group. The difference is very small and not statistically different from zero.

A related issue is that the new regions might differ in the sex ratio. In regions in which there are many women relative to men, it would also be easier to find a better partner than previously. To exclude this explanation, we also ran an event study with the sex ratio of the region as an dependent variable. To capture common age differences between spouses, we construct age-specific sex ratios

Figure 4: Robustness Plots



<u>Notes:</u> Panel A shows the impact of job displacement on the probability of moving to a different municipality. Panel B shows the difference between single women's average labor income in the municipality an individual lives in (in period t) to the average single women's labor income in the municipality where the individual lived in $t = -3$. Panel C shows the difference between the sex ratio ($\frac{\#women}{\#men}$) in the municipality an individual lives in (in period t) to the average sex ratio in the municipality where the individual lived in $t = -3$. The estimates correspond to estimates of β_τ in equation (7). All estimates are based on a sample of men who experienced an establishment closure between 1980-2007, and the same number of control individuals selected by exact matching. The sample selection criteria and matching algorithm are described in subsection 3.3.

which compute the ratio of women relative to men in a 10-year window around the individuals own age.[20] As with the regional earnings, however, there is no difference in the regional sex ratio between treatment and control group after displacement.

Marriage Market Equilibrium Effects of Establishment Closures. A potential challenge to identification is that establishment closures might exert equilibrium effects on the marriage market. Our previous empirical analysis rests on the assumption that establishment closures affect displaced men directly, but do not change the overall composition of the pool of singles on the marriage market. We provide a back of the envelope calculation to demonstrate that we can reasonably expect such equilibrium effects to be small: The workforce of the average closing establishment in our sample is 270 workers. The raw rate at which displaced workers separate from their partners in the 10 years following establishment closure is 0.2. The average inflow of singles into the marriage market over 10 years in the aftermath of an establishment closure hence is approximately $0.2 \times 270 = 54$. This amounts to an influx of 1.4% relative to the average number of singles living in the municipality where the establishment closes (which is 3897).[21] We view this number as a conservative approximation, given the long time horizon (10 years) we consider and given that we look at the number of singles in the municipality of the closing establishment, arguably a lower bound for the size of the local marriage market.

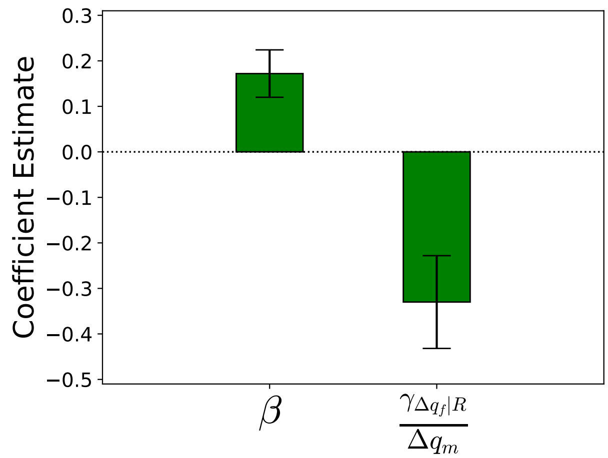
4.7 Confronting Theory and Data

Table 4 summarizes how our empirical results compare to the model predictions derived in Proposition 1. While our empirical evidence from job displacements is consistent with our conceptual framework under NAM, this would imply a negative cross-sectional correlation of matched partners' incomes, which is refuted by our data. PAM, on the other hand, is consistent with a positive correlation of matched partners' incomes, but cannot be reconciled with our evidence from job displacements. When one-dimensional matching is maintained, our conceptual framework cannot simultaneously be qualitatively consistent with our evidence from job displacements as well as the widely documented positive correlation of matched partners' incomes.

[20] For example, for a 40 year old worker, the age-region-specific sex ratio is the ratio between the number of single women between 35 and 45.

[21] We arrive at a similarly small number (0.89%) if we use the median instead of the mean establishment size and number of singles in the municipality.

Figure 5: Correlational Evidence vs. Evidence from Establishment Closures



Notes: The figure displays the regression coefficient of regressing wife's on husband's income, alongside our estimate from establishment closures, $\frac{\gamma_{\Delta q_f|R}}{\Delta q_m}$.

Table 4: Confronting Theory and Data

Impact of job displacement on		Data	NAM	PAM
Breakup-risk	γ_B	$\uparrow$	$\uparrow$	$\uparrow$
Risk of staying single post breakup	$\gamma_{R=0\|B}$	$\uparrow$	$\uparrow/\downarrow$	$\uparrow/\downarrow$
Which female types experience a break up	$\gamma_{q_f\|B}$	$<$	$<$	$>$
Avg. female type a man re-matches with post breakup	$\gamma_{\Delta q_f\|R}$	$>$	$>$	$<$
Cross-sectional income correlation	$Corr(\text{Income}_f, \text{Income}_m)$	>0	<0	>0

5 Reconciling Theory and Data: Multidimensional Matching

As we argued in Section 4.7, our empirical findings cannot be reconciled with one-dimensional matching under PAM or NAM. In this subsection, we argue that under multidimensional matching our conceptual framework is capable of capturing all empirical findings presented in Section 4.

We consider the framework described in Section 2.1 in the multidimensional case, $K > 1$. Our definitions of flow utilities, value functions, and marital surplus from Subsection 2.1 carry over to the case where q_f and q_m are K-dimensional vectors. In the following subsections, we define multidimensional notions of PAM and NAM (dimension by dimension, similar to Lindenlaub and

Postel-Vinay (2016)). We then derive predictions regarding the effects of job displacement under multidimensional matching and derive conditions under which the multidimensional framework is consistent with our empirical evidence.

5.1 Multidimensional Sorting

We extend the definitions of one-dimensional PAM and NAM by Shimer and Smith (2000) (see section 2.4) to the multidimensional case where q_f and q_m are K-dimensional vectors. Denote by $\mathcal{M}(q_m) = \{q_f : S(q_f, q_m) \geq 0\}$ a type-q_m man's multidimensional matching set. It will occasionally be useful to denote q_f by (q_{fi}, q_f^{-i}), where q_{fi} denotes the i-th component and q_f^{-i} denotes all but the i-th components of vector q_f. We define positive and negative assortative mating in dimension i, write PAM(i) and NAM(i), as follows:

Definition 2. *Consider $q'_{fi} < q''_{fi}$, $q'_{mi} < q''_{mi}$.*

There is PAM(i) if for all q_f^{-i}, q_m^{-i}: $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mi}, q_m^{-i})$ and $(q'_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mi}, q_m^{-i})$
$\Rightarrow (q'_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mi}, q_m^{-i})$ and $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mi}, q_m^{-i})$.

There is NAM(i) if for all q_f^{-i}, q_m^{-i}: $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mi}, q_m^{-i})$ and $(q'_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mi}, q_m^{-i})$
$\Rightarrow (q'_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mi}, q_m^{-i})$ and $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mi}, q_m^{-i})$.

To determine how multidimensional matching sets, $\mathcal{M}(q_m)$, are affected when q_{mi} (the i-th component of type q_m), is varied, we require the following additional assumption.

A-1. *For any given q_m and q_f^{-i} there exists a q_{fi}, such that $(q_{fi}, q_f^{-i}) \in \mathcal{M}(q_m)$. For any given q_f and q_m^{-i} there exists a q_{mi}, such that $(q_{mi}, q_m^{-i}) \in \mathcal{M}(q_f)$.*

Assumption A-1 requires that sets of the form $\{q_{fi} : (q_{fi}, q_f^{-i}) \in \mathcal{M}(q_m)\}$ are non-empty. Intuitively, A-1 is satisfied if for a given man with characteristics q_m and a woman with given characteristics q_f^{-i} there exists a realization of characteristic q_{fi} sufficiently favorable such that q_m and (q_{fi}, q_f^{-i}) would agree to match. Leveraging A-1, we establish the following relationship between sorting and multidimensional matching sets that generalizes the corresponding one-dimensional property derived by Shimer and Smith (2000).

Lemma 1. *Under assumption A-1, and given PAM(i) or NAM(i), multidimensional matching sets, $\mathcal{M}(q_m)$, are characterized by one-dimensional sets*

$$\{q_{fi} : (q_{fi}, q_f^{-i}) \in \mathcal{M}(q_m)\} = [a_i(q_{mi}, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})],$$

where

$$q_f \in \mathcal{M}(q_m) \Leftrightarrow q_{fi} \in [a_i(q_{mi}, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})]$$

and a_i, b_i are

(i) weakly increasing in q_{mi} under PAM(i),

(ii) weakly decreasing in q_{mi} under NAM(i).

The proof of Lemma 1 is provided in Appendix A. Intuitively, Lemma 1 states that for given male and female characteristics q_m^{-i} and q_f^{-i}, the remaining i-th dimension of matching set $\mathcal{M}(q_m)$ is an interval with bounds that are weakly increasing in q_{mi} under PAM(i) and weakly decreasing in q_{mi} under NAM(i).

5.2 Job Loss and Multidimensional Matching

To study the effects of job displacement in the described multidimensional matching environment, we interpret job displacement as a permanent change in the i-th dimension of a displaced agent's type. More specifically, a man of type q_m who is displaced from his job suffers a permanent reduction in q_{mi} to $q_{mi} - d$, where $d > 0$. Similarly to the one-dimensional case, we assume that q_{mi} maps into labor income by an increasing one-to-one mapping.[22]

To derive predictions regarding the effects of job displacement in the multidimensional case, we consider the same setup described in Section 2.5, a treatment group displaced in t_0, and a control group that is not displaced in $[t_0, \tau]$. The definition of the effects of job displacement γ_B and $\gamma_{R=0|B}$ carry over from Section 2.5. We further define

$$\begin{aligned}
\gamma_{q_{mi}|B} &= \mathbb{E}\left[q_{mi}(t_0)|D_B = 1, D = 1\right] - \mathbb{E}\left[q_{mi}(t_0)|D_B = 1, D = 0\right] \\
\gamma_{q_{fi}|B} &= \mathbb{E}\left[q_{fi}(t_0)|D_B = 1, D = 1\right] - \mathbb{E}\left[q_{fi}(t_0)|D_B = 1, D = 0\right] \\
\gamma_{\Delta q_{fi}|R} &= \mathbb{E}\left[q_{fi}(\tau) - q_{fi}(t_0)|D_R = 1, D_B = 1, D = 1\right] - \mathbb{E}\left[q_{fi}(\tau) - q_{fi}(t_0)|D_R = 1, D_B = 1, D = 0\right].
\end{aligned}$$

analogously to the corresponding objects from Section 2.5.

We show that the following relationships between marriage market sorting and the effects of job displacement hold under multidimensional matching:

Proposition 2. *Consider the described matching environment in steady state equilibrium in the multidimensional case, $K > 1$ and suppose A-1 holds.*

Under PAM(i) or NAM(i):

1. *Job displacement increases the separation risk: $\gamma_B \geq 0$.*

[22] Note that other observable attributes than labor income may map one-to-one into q_{mi} and be affected by job displacement as well. The distinguishing feature of the multidimensional case is that other dimensions $j \neq i$ of q_m exist that are not shocked upon job displacement.

2. *Job displacement may increase or decrease the probability of staying single:* $\gamma_{R=0|B}$ *may be positive or negative.*

Under PAM:

3.-a *Job displacement leads men to re-match with women of lower type:* $\gamma_{\Delta q_{fi}|R} \leq 0$.

4.-a *The association between job displacement and partner type is bounded above:* $\gamma_{\Delta q_{fi}|R} \leq \overline{\gamma}_{\Delta q_{fi}|R}$. *The upper bound is given by*

$$\overline{\gamma}_{\Delta q_{fi}|R} = -\int\int\int \frac{\partial \mathbb{E}\left[q_{fi}|q_{mi}, q_m^{-i}, q_f^{-i}\right]}{\partial q_{mi}}\bigg|_{q_{mi}=q-x} dx dG_f^{-i}(q_f^{-i}) dF(q|D_R=1, D_B=1, D=1) \leq 0.$$

5.-a *If* $F(q_{mi}|D_B=1, D=1) \leq F(q_{mi}|D_B=1, D=0)$ *holds additionally. Then, on average, women whom displaced men separate from are of higher type than women whom non-displaced men separate from:* $\gamma_{q_{fi}|B} \geq 0$.

Under NAM:

3.-b *Job displacement leads men to re-match with women of higher type:* $\gamma_{\Delta q_{fi}|R} \geq 0$.

4.-b *The association between job displacement and partner type is bounded below:* $\gamma_{\Delta q_{fi}|R} \geq \underline{\gamma}_{\Delta q_{fi}|R}$. *The lower bound is given by*

$$\underline{\gamma}_{\Delta q_{fi}|R} = -\int\int\int \frac{\partial \mathbb{E}\left[q_{fi}|q_{mi}, q_m^{-i}, q_f^{-i}\right]}{\partial q_{mi}}\bigg|_{q_{mi}=q-x} dx dG_f^{-i}(q_f^{-i}) dF(q|D_R=1, D_B=1, D=1) \geq 0.$$

5.-b *If* $F(q_{mi}|D_B=1, D=1) \geq F(q_{mi}|D_B=1, D=0)$ *holds additionally. Then, on average, women whom displaced men separate from are of lower type than women whom non-displaced men separate from:* $\gamma_{q_{fi}|B} \leq 0$.

Proposition 2 establishes that the claims established in Proposition 1 carry over to the multidimensional case up to minor modifications.

5.3 Cross-Sectional Correlations under Multidimensional Matching

Next, we derive conditions under which the cross-sectional correlation of partners' incomes can be positive even if sorting on income itself (i.e., sorting in the i-th dimension) is negative. Under the proposed conditions the positive correlation between matched spouses' labor incomes may arise spuriously from sorting on the other dimensions of model agent's types, q_f^{-i} and q_m^{-i}.

Formally, we provide conditions under which, the conditional expectation of female labor income, conditional on male labor income for matched couples, $\mathbb{E}[q_{fi}|q_{mi}]$, is increasing (decreasing) q_{mi}, which implies a positive (negative) correlation between q_{fi} and q_{mi}.

To this end we decompose the effect of increased male labor income on $\mathbb{E}[q_{fi}|q_{mi}]$ into a direct effect (DE), capturing the impact of ceteris paribus increasing q_{mi}, holding all other dimensions of the male type, q_m^{-i}, constant, and an indirect effect (IE), that captures the association between q_{mi} and q_m^{-i} in the population of single men. We then derive sufficient conditions on sorting, and on the association between q_{mi} and q_m^{-i} that determine the sign of DE and IE.

We require the following additional assumption on the orientation of matching sets:

A-2. *For any given dimensions i and j, and any $q'_{fi} < q''_{fi}$, $q'_{fj} < q''_{fj}$, $q_f^{-(i,j)}$, and q_m it holds that:*

$$\begin{aligned} & (q'_{fi}, q'_{fj}, q_f^{-(i,j)}) \in \mathcal{M}(q_m) \text{ and } (q''_{fi}, q''_{fj}, q_f^{-(i,j)}) \in \mathcal{M}(q_m) \\ \Rightarrow \quad & (q'_{fi}, q''_{fj}, q_f^{-(i,j)}) \in \mathcal{M}(q_m) \text{ and } (q''_{fi}, q'_{fj}, q_f^{-(i,j)}) \in \mathcal{M}(q_m). \end{aligned}$$

Proposition 3. *Consider the described matching environment in the bi-dimensional case, $K = 2$ and suppose that A-1 and A-2 hold.*

Consider the following decomposition for $q''_{mi} > q'_{mi}$

$$\begin{aligned} \mathbb{E}[q_{fi}|q''_{mi}] - \mathbb{E}[q_{fi}|q'_{mi}] &= \underbrace{\int \mathbb{E}[q_{fi}|q''_{mi}, q_{mj}] - \mathbb{E}[q_{fi}|q'_{mi}, q_{mj}] dG(q_{mj}|q''_{mi})}_{:=\textbf{DE (Direct effect)}} \\ &+ \underbrace{\int \mathbb{E}[q_{fi}|q'_{mi}, q_{mj}] dG(q_{mj}|q''_{mi}) - \int \mathbb{E}[q_{fi}|q'_{mi}, q_{mj}] dG(q_{mj}|q'_{mi})}_{:=\textbf{IE (Indirect effect)}}. \end{aligned}$$

In a bi-dimensional steady state matching equilibrium the following implications hold:

$$\begin{aligned} PAM(i) &\Rightarrow DE \geq 0, \\ NAM(i) &\Rightarrow DE \leq 0. \end{aligned}$$

Given PAM (i) or NAM (i) the following additional implications hold:

$$\begin{aligned} PAM(j) \text{ and } G(q_{mj}|q_{mi}) \text{ is decreasing in } q_{mi} &\Rightarrow IE \geq 0, \\ NAM(j) \text{ and } G(q_{mj}|q_{mi}) \text{ is decreasing in } q_{mi} &\Rightarrow IE \leq 0, \\ PAM(j) \text{ and } G(q_{mj}|q_{mi}) \text{ is increasing in } q_{mi} &\Rightarrow IE \leq 0, \\ NAM(j) \text{ and } G(q_{mj}|q_{mi}) \text{ is increasing in } q_{mi} &\Rightarrow IE \geq 0. \end{aligned}$$

By proposition 3 multidimensional matching is consistent with the observed positive correlation between matched spouses' incomes. Specifically the model can generate a positive correlation between matched spouses' incomes, even when sorting on income is negative, if soting if there is positive sorting on other attributes and income and and other attributes are positively associated. Note that assuming $Corr(q_{mi}, q_{mj}) > 0$ is not sufficient. Instead we assume $G(q_{mi}|q_{mj})$ is increasing, a stronger condition on the association of male income and other attributes, which implies $Corr(q_{mi}, q_{mj}) > 0$. Note that in these cases matched partners' income correlation does not reflect a causal relationship, but arises spuriously. The impact of ceteris paribus changing q_{mi} on marriage market matching is captured by DE, while IE captures the spurious association of matched partners' labor income that is driven by q_{mj}.

Altogether the multidimensional model is thus consistent with our empirical results if 1. sorting on income is negative, $NAM(i)$, which generates that displaced men switch to higher earning female partners relative, to an untreated control group, and 2. sorting on other attributes is positive, $PAM(j)$, and $G(q_{mi}|q_{mj})$ is increasing in q_{mj}, which is consistent with a positive correlation between matched spouses' incomes.

6 Implications

6.1 Implications for the Interpretation of Empirical Matching Patterns

In this subsection, we revisit the widely documented positive correlation in spouses' incomes and wages, in light of our multidimensional matching framework and our evidence from establishment closures. Going back to Becker (1973, 1981), economists have interpreted this positive empirical correlation as indicative of earnings based positive sorting of women and men into marriages.[23] The seminal theory of marriage market matching developed by Becker (1973, 1981), however, predicts positive sorting on "non-market traits" (e.g., IQ, height, attractiveness, ethnic origin), but negative sorting on wages as this maximizes the gains from optimal division of labor in the household.[24]

Various arguments have been made to resolve the apparent discrepancy between the empirical positive correlation in spouses' wages and the theoretical prediction of negative sorting on wages. Becker (1973, 1981) argues that missing wage data for non-working women might bias the observed correlation between spouses' wages toward positive values. Lam (1988) shows in a simple extension of Becker's (1973,1981) framework that joint consumption of a household public good purchased in the market may give rise to positive assortative mating.[25] In recent studies,

[23]As more powerful evidence of earnings based sorting, the partial correlation in spouses' wages, controlling for years of schooling and age, has been documented to be positive, e.g., in Becker (1973, 1981).

[24]Becker (1981) notes: "The strong positive partial correlation between years of schooling is predicted by the theory, but the positive correlation between wage rates is troublesome since the theory predicts a negative correlation when nonmarket productivity is held constant."

[25]This driver of positive assortative mating features, e.g., in Low (2024).

complementarities in spouses' housework hours (e.g., Gayle and Shephard 2019; Calvo et al. 2021) and homophily (e.g., Goussé et al. 2017; Gayle and Shephard 2019; Adda, Pinotti, and Tura 2020) have often been invoked as model mechanisms that generate earnings based positive assortative matching.

Our multidimensional matching framework (presented in Section 5) together with our evidence from establishment closures (see Section 4) offer a perspective that allows to reconcile Becker's (1973,1981) prediction of negative sorting on wages with the positive empirical correlation in spouses' wages, and with most of the above mentioned model mechanisms. Our preferred specification posits that sorting on incomes (and wages), holding all other dimensions of agent type constant, is negative. This is consistent with Becker's (1973,1981) prediction of negative sorting on wages as an artifact of optimal division of labor. At the same time, in our framework the positive cross-sectional correlation in spouses' wages may arise from sorting on other dimensions of agent type, e.g., from complementarities in home productivity (as in Gayle and Shephard 2019 and Calvo et al. 2021) or education homophily (as in Goussé et al. 2017; Gayle and Shephard 2019; and Adda et al. 2020). Our multidimensional matching model offers a unified framework in which these model mechanisms and Becker's (1973,1981) prediction do not negate each other, but can coexist and can serve to simultaneously generate effects consistent with our evidence from job displacements, as well as the widely documented positive correlation of spouses' incomes.

6.2 The Role of Unobserved Characteristics in Explaining Observed Matching Patterns

In our multidimensional matching framework the positive correlation in matched spouses' incomes does not necessarily reflect sorting on income, but may be driven by sorting on other characteristics correlated with but not causally linked to income. In this subsection we decompose the correlation between spouses' incomes into parts driven by different observed characteristics, and a residual term driven by unobserved characteristics. The arguments we use to do this rely on our multidimensional matching framework from Section 5, and on conclusions drawn based on our evidence from job displacements (see Sections 4 and 5).

As a starting point for the decomposition, consider the regression

$$q_{fi} = \beta_0 + \beta_1 q_{mi} + \beta_2' X_m + \beta_3' X_f + \epsilon \tag{9}$$

run on a a sample of couples, where q_{fi}, q_{mi} denote husband's and wife's labor incomes, and X_m, X_f are vectors of observable characteristics other than income.

Suppose that the multidimensional types that women and men match on are $q_f = (q_{fi}, X_f, U_f)$ for women and $q_m = (q_{mi}, X_m, U_m)$ for men, where X_m, X_f are the observed characteristics included in regression (9) and U_f, U_m are characteristics not included in the regression, which may include variables that are unobserved by us, the researchers. Regression (9) estimates the conditional mean $\mathbb{E}[q_{fi}|q_{mi}, X_m, X_f]$, whose dependence on q_{mi} can be decomposed as follows:

$$\mathbb{E}[q_{fi}|q''_{mi}, X_m, X_f] \quad - \quad \mathbb{E}[q_{fi}|q'_{mi}, X_m, X_f] = \Delta_{q_{fi}} + \Delta_{U|q_{fi}},$$

where

$$\Delta_{q_{fi}} \quad = \quad \int \mathbb{E}[q_{fi}|q''_{mi}, X_m, U_m, X_f, U_f] - \mathbb{E}[q_{fi}|q'_{mi}, X_m, U_m, X_f, U_f]dG(U_f, U_m|q''_{mi}, X_m, X_f)$$

and

$$\begin{aligned} \Delta_{U|q_{fi}} \quad = \quad & \int \mathbb{E}[q_{fi}|q'_{mi}, X_m, U_m, X_f, U_f]dG(U_f, U_m|q''_{mi}, X_m, X_f) \\ & - \int \mathbb{E}[q_{fi}|q'_{mi}, X_m, U_m, X_f, U_f]dG(U_f, U_m|q'_{mi}, X_m, X_f). \end{aligned}$$

The first term ($\Delta_{q_{fi}}$) reflects sorting on income, keeping all other characteristics equal. The second term ($\Delta_{U|q_{fi}}$) captures the indirect effect of q_{mi} on q_{fi}, via U_f and U_m.

In section 5 we argued that in our multidimensional matching framework our quasi-experimental evidence from job displacements is consistent with NAM(i) (while being inconsistent with PAM(i)). Note that under NAM(i) it can be shown that $\Delta_{q_{fi}} \leq 0$.[26] This allows us to use regression (9) to estimate a lower bound on $\Delta_{U|q_{fi}}$, the dependence of q_{fi} on q_{mi} that arises due to sorting on U_f, U_m. More specifically, using (9) to estimate the conditional mean $\mathbb{E}[q_{fi}|q_{mi}, X_m, X_f]$, and normalizing $q''_{mi} - q'_{mi} = 1$, it follows that $\beta_1 \leq \Delta_{U|q_{fi}}$. Intuitively, under NAM(i) sorting on income, keeping other characteristics fixed, is negative. Sorting on characteristics not controlled for in the regression must therefore exceed the magnitude of β_1 to rationalize the matching pattern in our data.

In Table 5 we present estimation results from various specifications of regression (9), varying which observed variables are included as dependent variables in the regression (i.e., are part of X_m, X_f) and which ones are not controlled for (i.e., are part of U_m, U_f). The "raw" regression coefficient obtained by regressing wife's on husband's income without including controls is 0.17. Controlling for age or education fixed effects reduces the coefficient estimate by 0.02 (11.6%) and 0.08 (46.5%), respectively. Jointly controlling for age and education fixed effects reduces the estimate by 0.1 (58%). Leveraging that β_1 is a lower bound on $\Delta_{U|q_{fi}}$, these estimates imply $\Delta_{U|q_{fi}} \geq 0.073$. I.e., at least 42.4% $(= 0.073/0.172 \cdot 100\%)$ of the "raw regression coefficient" is due to sorting on characteristics not controlled for in the regression (i.e., characteristics other than income, age, and education) potentially including characteristics that are typically unobserved by researchers.[27]

[26] This follows directly from the first step of the proof of Proposition 4.

[27] This may include characteristics that are unavailable in the Danish register data, but that other researchers have measured and studied, such as Oreffice and Quintana-Domeque (2010), Dupuy and Galichon (2014), Chiappori et al. (2017a), Fisman, Iyengar, Kamenica, and Simonson (2006).

Table 5: Regressing wife's on husband's income, controlling for age and education

	(1)	(2)	(3)	(4)
Husband's labor income ($\hat{\beta}_1$)	0.172***	0.154***	0.089***	0.073***
	(0.000521)	(0.000523)	(0.000539)	(0.000541)
Covariates				
Male education FE	No	No	Yes	Yes
Female education FE	No	No	Yes	Yes
Male age FE	No	Yes	No	Yes
Female age FE	No	Yes	No	Yes
Observations	3,180,802	3,118,538	3,086,225	3,086,225

Notes: This table reports coefficient estimates of β_1 from equation (9) for varying sets of control variables X_f and X_m. All specifications are estimated on our full sample of married or cohabiting couples, observed between 1980 and 2007. Standard errors are reported in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

6.3 Counterfactual Simulations

In this section, we show that our results have important implications for understanding the relationship between inequality and assortative matching. We study the impact of a major tax reform which replaces the current Danish tax schedule by the US schedule in our one-dimensional and two-dimensional models. In a nutshell, we show that the marriage market plays a very different role in the 1D and in the 2D model. In the 1D model, where individuals directly match on income, the marriage market *amplifies* the effects of the reform (relative to keeping the distribution of couples fixed). In the 2D model, where the positive income correlation results from unobserved characteristics, it *dampens* the effects of the reform. These results illustrate that understanding the reasons behind the positive income correlation in the data—whether it directly results from income complementaries or whether it is a byproduct of sorting on other characteristics—has significant policy implications.

Model Specification In order to solve the model numerically and simulate counterfactual tax policies, we make additional assumptions on the functional forms of the surplus functions. Like before, q_f and q_m are female and male labor income, while x_f and x_m are indices of other individual characteristics. We use the following specifications for the utility functions in the 1D and 2D model:

$$\text{1D-Model} \qquad \begin{array}{lll} \text{Couples:} & f_{1D}(q_f, q_m) & = \gamma_1^{1D} C(\frac{q_f + q_m}{2}) - \gamma_2^{1D}(q_f - q_m)^2 \\ \text{Singles:} & u_s^S(q_s) & = \frac{1}{2} C(q_s), \quad s \in \{f, m\} \end{array} \tag{10}$$

$$\text{2D-Model:} \quad \begin{aligned} &\text{Couples:} \quad f_{2D}(q_f, x_f, q_m, x_m) &&= \gamma_1^{2D} C(\tfrac{q_f+q_m}{2}) - \gamma_2^{2D}(x_f - x_m)^2 \\ &\text{Singles:} \quad u_s^S(q_s) &&= \tfrac{1}{2} C(q_s), \quad s \in \{f, m\}, \end{aligned} \tag{11}$$

Here, $C(.)$ is the CRRA utility function.[28] We motivate these functional forms through a setting in which individuals derive utility from consumption and the characteristics of their partner. In both models, the CRRA term reflects consumption utility, where we assume that consumption in marriage is a public good for both partners. The sorting terms capture all other motives for marriage which are unrelated to consumption.

Our quantitative model also adds an idiosynchratic "love shock" z which is an i.i.d. shock drawn upon meeting a potential partner. The love shock represents all other reasons to form a relationship, which are not directly captured by the model, and leads to empirically more plausible marriage patterns.[29] The shock has mean zero and variance σ_z and follows a distribution with CDF $G(z)$. Given $G(z)$, singles who meet start a relationship with the following probability:

$$\alpha(v_f, v_m) \equiv \Pr\{s(v_f, v_m) + z > 0 \mid v_f, v_m\} = 1 - G[-s(v_f, v_m)] \tag{12}$$

Marital surplus is specified as $s(v_f, v_m) = f(v_f, v_m) - s_m(v_m) - s_f(v_f)$, where the latter two terms represent the values of singlehood.[30] We solve both models numerically by iterating on the distributions of single types and the values of being single, (G_f, G_m, V_f^S, V_m^S).

We assume that model agents match on net income. For the purpose of simulating changes in tax policy it becomes relevant to specify the relationship between gross and net income. We denote gross incomes by $\tilde{q}_f$ and $\tilde{q}_m$. Net income equals $q_s = \tilde{q}_s - T(q_s)$, where

$$T(\tilde{q}_s) = \max\left((1 - \psi_1 \tilde{q}_s^{-\psi_2}) \cdot \tilde{q}_s, 0\right)$$

is the tax schedule. The parametrization of the tax schedule follows Heathcote, Storesletten, and Violante (2017). ψ_2 controls the progressivity of the tax system and ψ_1 shifts the level of taxation. As Denmark is a country with individual taxation of partners, we assume that the tax function is applied to each partner individually.[31]

Calibration We calibrate a 1D as well as a 2D specification our model to Denmark. We fix the discount rate at 0.97. The separation rate, δ, is estimated directly to match mean rate at which

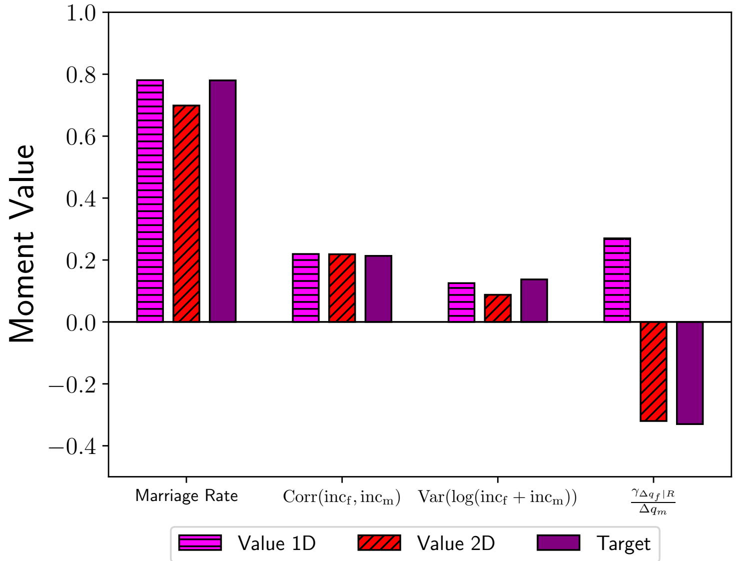
[28]We set the risk aversion parameter to 1.5.

[29]For example, in the 1D model, income would perfectly predict whether a meeting results in a relationship in the absence of the love shock.

[30]Essentially, this is generalization of results presented in Jacquemet and Robin (2013) for the one-dimensional case.

[31]Note that we abstract from the transfer system by ruling out negative values of the tax function, as transfers are typically means-tested and based on *family* rather than *individual* income, which would complicate the model.

Figure 6: Model fit



Notes: The figure displays simulated values from the onedimensional and the bidimensional quantitative model, as well as the targeted data moments.

singles match with new partners in our data. The remaining model parameters are calibrated by targeting empirical moments computed from Danish adminstrative data.[32] We target moments that reflect the mariage market equilibrium, as well as the variance of log-gross household income among married couples, as a measure of economic inequality.[33] Our calibration procedure selects values for the free parameters of our model that minimize the euclidian distance between model moments and empirical moments.

Both the 1D as well as the 2D model specification provide a good fit to the selected empirical moments. We closely match the empirical correlation between matched spouses' incomes. Our measure of inequality, the simulated variance of log gross income, is slightly lower than in the data. Both models match the fraction of people who are matched with a spouse, while the rate at which matches are formed in the 2D specification is slightly lower than in the data.

Simulation Results We now turn to the results from several counterfactual policy simulations. In particular, we consider the impact of changes in tax progressivity on income inequality. Increasing tax progressivity is a common tool for counteracting income inequality (see, e.g., Heathcote et al. (2017)). In this context our simulations are aimed at contrasting policy effects in the 2D model that is consistent with our empirical results, and the 1D model, that is refuted by our data. Our

[32]We compute these moments from pooled longitudinal data, spanning the time window of our main empirical analysis, 1980–2007.

[33]In using this measure of income inequality we follow, e.g., Blundell, Pistaferri, and Preston (2008).

Table 6: Policy Simulations: Impact of Tax Reforms on Inequality

	$\text{Var}[\log(q_f^{\text{net}} + q_m^{\text{net}})]$	
	1D model	2D Model
(1) Danish Tax Schedule	0.033	0.031
(2) U.S. Tax Schedule, marital sorting fixed	0.043	0.039
(2) U.S. Tax Schedule	0.045	0.039
Fraction due to marital sorting	0.11	-0.08

simulation results show that our findings are relevant for understanding the link between tax progressivity, income inequality and marital sorting.

We study several hypothetical tax reforms through the lense of the calibrated versions of the 1D as well as the 2D specification of our marriage market search and matching framework. We start from a status quo scenario, in which the tax schedule is fixed at an approximation of the real world tax system in Denmark (this tax schedule is given by $\psi_1 = 0.65$ and $\psi_2 = 0.15.$, i.e., the same parameters that we use in our calibration). We simulate moving from the Danish tax schedule to an approximation of the U.S. tax system. Table 6 contrasts the impact of each simulated tax reform on economic inequality in the 1D specification versus the 2D specification of our model. Moreover we use our model to isolate the effect of taxes on income inequality that is mediated via the marriage market. Specifically, by simulating tax reforms while keeping marital sorting fixed, we quantify the impact of the considered tax reforms on income inequality, if the marriage market was unresponsive. Table 6 shows that increasing tax progressivity generally reduces income inequality (in the 1D as well as the 2D specification of our model). The magnitudes and contribution of the marriage market to this effect, however, differ substantially between the 1D model which is refuted by our empirical results and the 2D model which is consistent with our data. We can isolate the impact of this reduction on inequality by introducing the progressive tax while holding the distribution of married couples fixed. This also leads to a reduction in inequality, as shown in table 6, but the reduction is not as strong as in the full model, where we allow the distribution to adjust. We can compute the relative importance of the marriage market adjustment by dividing the difference in inequality between the two policy experiments (column (2) - (3)) by the difference in inequality between no taxation and a progressive tax (column (3) - (1)). The interpretation of this number is fraction of reduction in inequality which is due to the marriage market, which turns out to be sizable in the 1D model.

7 Conclusion

In this paper, we have investigated the impact of job displacement on the remarriage outcomes of workers. Leveraging quasi-experimental variation from establishment closures in Denmark, we find that workers who experience a displacement shock are more likely to separate from their current partner if she is low earning and on average find a new partner with *higher* earnings than their previous partner, relative to an untreated control group.

We show that these findings are hard to reconcile within a general one-dimensional search and matching framework which generates positive assortative matching on income in the cross-section. Instead we propose a multidimensional search and matching framework, in which sorting on income is negative ceteris paribus, when keeping all other dimensions fixed, while the positive cross-sectional correlation between spouses incomes arises from sorting in other dimensions that correlate with income. We further compare the multidimensional search and matching framework that is consistent with our empirical evidence, to the commonly used one-dimensional model specification that is at at odds with our empirical findings. We that the multidimensional framework implies an important role fort sorting on unobservables, and predicts a more nuanced relationship between marital sorting and economic inequality compared to the standard one-dimensional search and matching model.

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Appendix

Job Displacement, Remarriage, and Marital Sorting

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A Proofs and Derivations

Proof of proposition 1: We start by proving that under PAM or NAM, $\gamma_B \geq 0$:

As men in the control group by definition are not displaced between period t_0 and τ, their types are unchanged between these points in time, i.e., $q_m(\tau) = q_m(t_0)$. A control group couple that was matched in period τ, therefore continues to have the identical (non-negative) marital surplus it had in t_0.

It follows that no endogenous breakups occur in the control group. Exogenous breakups, by assumption, occur at rate δ. The overall probability that a man in the control group experiences a breakup from his t_0-partner between t_0 and τ is thus given by:

$$P\left(D_B = 1 | D = 0\right) = 1 - e^{-\delta(\tau - t_0)} \tag{A.1}$$

Note that this holds under PAM as well as under NAM.

In the treatment group, by contrast, men's types change between t_0 and τ due to job displacement. Specifically, $q_m(\tau) = q_m(t_0) - d < q_m(t_0)$.

For a given man with pre-displacement type $q_m(t_0)$, job displacement will lead to a breakup if it changes the couples' marital surplus from weakly positive to negative, or equivalently if $q_f(t_0) \in \mathcal{M}\left(q_m(t_0)\right)$ but $q_f(t_0) \notin \mathcal{M}\left(q_m(t_0) - d\right)$.

Shimer and Smith (2000) show that under NAM or PAM matching sets are closed intervals, $\mathcal{M}(q_m) = [a(q_m), b(q_m)]$, with interval bounds, $a(q_m), b(q_m)$, that are weakly increasing in $q_m$und PAM and that are weakly decreasing under NAM. It follows under PAM that job displacement leads to a breakup for a man of pre-displacement type q_m if and only if he is matched with a woman of type $q_f \in (\max\{b(q_m - d), a(q_m)\}, b(q_m)]$.

Similarly, it follows under NAM that job displacement will lead to a breakup for a man with pre-displacement type q_m if and only if he is matched with a woman of type $q_f \in [a(q_m), \min\{a(q_m - d), b(q_m)\})$.

Additionally, breakups occur exogenously at rate δ under PAM as well as under NAM.

It follows that under PAM the overall probability that a man in the treatment group experiences a breakup from his t_0-partner between t_0 and τ is given by:

$$P\left(D_B = 1|D=1\right) = \underbrace{1 - e^{-\delta(\tau - t_0)}}_{\text{prob. of exogenous breakups}} + \underbrace{\int G_f(b(q_m(t_0))) - G_f((\max\{b(q_m(t_0)-d), a(q_m(t_0))\})dF(q_m(t_0)|D=1)}_{\text{prob. of endogenous breakups}}. \tag{A.2}$$

Note that $G_f(b(q_m(t_0))) - G_f((\max\{b(q_m(t_0)-d), a(q_m(t_0))\})$ is the mass of men of type $q_m(t_0)$ matched with a woman of type $q_f \in (\max\{b(q_m - d), a(q_m)\}, b(q_m)]$, i.e., the mass of $q_m(t_0)$-type men who experience an endogenous breakup after displacement.

Similarly, under NAM, the overall probability that a man in the treatment group experiences a breakup from his t_0-partner between t_0 and τ is:

$$P\left(D_B = 1|D=1\right) = \underbrace{1 - e^{-\delta(\tau - t_0)}}_{\text{prob. of exogenous breakup}} + \underbrace{\int G_f(\min\{a(q_m(t_0)-d), b(q_m(t_0))\}) - G_f(a(q_m(t_0)))dF(q_m(t_0)|D=1)}_{\text{prob. of endogenous breakup}}, \tag{A.3}$$

From (A.1), (A.2), and (A.3) it follows that under PAM as well as under NAM
$\gamma_B = P\left(D_B = 1|D=1\right) - P\left(D_B = 1|D=0\right) \geq 0$. This concludes the proof of statement *1*.

Next, we turn to proving that under PAM the impact of job displacement on partner type, $\gamma_{\Delta q_f|R}$, is weakly negative and bounded above by

$$\overline{\gamma}_{\Delta q_f|R} = -\int\int_0^d \frac{\partial \mathbb{E}[q_f|q_m]}{\partial q_m}\Big|_{q_m = q-x} dx dF(q|D_R = 1, D_B = 1, D = 1) \leq 0.$$

Denote by D_δ an indicator that equals 1 for men who experience an exogenous breakup between t_0 and τ, and 0 for all other men. Consider men in the treatment group of pre-displacement type q_m

who separate from their t_0-partner and rematch with a new partner between t_0 and τ. The average female type this group of men is matched with in t_0 can be written as weighted average:

$$\mathbb{E}\left[q_f(t_0)|D_R=1,D_B=1,D=1,q_m(t_0)=q_m\right]=$$

$$\mathbb{E}\left[q_f(t_0)|D_R=1,D_B=1,D=1,q_m,D_\delta=1\right]P(D_\delta=1|D_R=1,D_B=1,D=1,q_m)$$

$$+\mathbb{E}\left[q_f(t_0)|D_R=1,D_B=1,D=1,q_m,D_\delta=0\right]P(D_\delta=0|D_R=1,D_B=1,D=1,q_m)$$

$$\begin{aligned}
&=\frac{1-e^{-\delta(\tau-t_0)}}{1-e^{-\delta(\tau-t_0)}+G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}\cdot\frac{1}{G_f(b(q_m))-G_f(a(q_m))}\int\limits_{a(q_m)}^{b(q_m)}q_f dG_f(q_f)\\
&+\frac{G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}{1-e^{-\delta(\tau-t_0)}+G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}\\
&\cdot\frac{1}{G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}\int\limits_{\max\{b(q_m-d),a(q_m)\}}^{b(q_m)}q_f dG_f(q_f)\\
&=\frac{1-e^{-\delta(\tau-t_0)}}{1-e^{-\delta(\tau-t_0)}+G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}\mathbb{E}\left[q_f|a(q_m)<q_f<b(q_m)\right]\\
&+\frac{G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}{1-e^{-\delta(\tau-t_0)}+G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}\mathbb{E}\left[q_f|\max\{b(q_m-d),a(q_m)\}<q_f<b(q_m)\right]
\end{aligned}\tag{A.4}$$

Next we turn to computing the corresponding average for period τ, taking into account that men in the treatment group are displaced in t_0. Their type when re-matching with a new partner in $(t_0,\tau]$ is therefore q_m-d, and the average female type they are matched with in τ is:

$$\begin{aligned}
\mathbb{E}\left[q_f(\tau)|D_R=1,D_B=1,D=1,q_m(t_0)=q_m\right] &= \frac{1}{G_f(b(q_m-d))-G_f(a(q_m-d))}\int\limits_{a(q_m-d)}^{b(q_m-d)}q_f dG_f(q_f)\\
&= \mathbb{E}\left[q_f|a(q_m-d)<q_f<b(q_m-d)\right].
\end{aligned}\tag{A.5}$$

For the control group, by contrast, as men's types are unchanged between t_0 and τ, the corresponding expressions are given by:

$$\begin{aligned}
\mathbb{E}\left[q_f(t_0)|D_R=1,D_B=1,D=0,q_m(t_0)=q_m\right] &= \mathbb{E}\left[q_f(\tau)|D_R=1,D_B=1,D=0,q_m(t_0)=q_m\right] \\
&= \frac{1}{G_f(b(q_m))-G_f(a(q_m))}\int\limits_{a(q_m)}^{b(q_m)} q_f dG_f(q_f) \\
&= \mathbb{E}\left[q_f|a(q_m)<q_f<b(q_m)\right]. \qquad \text{(A.6)}
\end{aligned}$$

Using (A.4), (A.5), and (A.6) it follows for $\gamma_{\Delta q_f|R}$ that

$$\begin{aligned}
\gamma_{\Delta q_f|R} = \quad & \int \mathbb{E}\left[q_f(\tau)-q_f(t_0)|D_R=1,D_B=1,D=1,q_m\right] dF(q_m|D_R=1,D_B=1,D=1) \\
& -\int \mathbb{E}\left[q_f(\tau)-q_f(t_0)|D_R=1,D_B=1,D=0,q_m\right] dF(q_m|D_R=1,D_B=1,D=0) \\
= \quad & \int \frac{1-e^{-\delta(\tau-t_0)}}{1-e^{-\delta(\tau-t_0)}+G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})} \\
& \cdot\Bigg(\mathbb{E}\left[q_f|a(q_m-d)<q_f<b(q_m-d)\right]-\mathbb{E}\left[q_f|a(q_m)<q_f<b(q_m)\right]\Bigg) \\
& dF(q_m|D_R=1,D_B=1,D=1) \\
& +\int \frac{G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})}{1-e^{-\delta(\tau-t_0)}+G_f(b(q_m))-G_f(\max\{b(q_m-d),a(q_m)\})} \\
& \cdot\Bigg(\mathbb{E}\left[q_f|a(q_m-d)<q_f<b(q_m-d)\right]-\mathbb{E}\left[q_f|\max\{b(q_m-d),a(q_m)\}<q_f<b(q_m)\right]\Bigg) \\
& dF(q_m|D_R=1,D_B=1,D=1) \\
\leq \quad & \int \mathbb{E}\left[q_f|a(q_m-d)<q_f<b(q_m-d)\right] \\
& -\mathbb{E}\left[q_f|a(q_m)<q_f<b(q_m)\right] dF(q_m|D_R=1,D_B=1,D=1) \\
= \quad & \int \mathbb{E}\left[q_f|q_m-d\right]-\mathbb{E}\left[q_f|q_m\right] dF(q_m|D_R=1,D_B=1,D=1) \\
= \quad & -\int\int\limits_0^d \frac{\partial \mathbb{E}\left[q_f|q_m\right]}{\partial q_m}\Big|_{q_m=q-x} dx dF(q|D_R=1,D_B=1,D=1) \\
= \quad & \overline{\gamma}_{\Delta q_f|R},
\end{aligned}$$

where the weak inequality follows as[34]

$$\mathbb{E}\left[q_f|a(q_m) < q_f < b(q_m)\right] \leq \mathbb{E}\left[q_f|\max\{b(q_m - d), a(q_m)\} < q_f < b(q_m)\right]. \tag{A.7}$$

As shown by Shimer and Smith (2000), $\mathbb{E}\left[q_f|q_m\right]$ is weakly increasing in q_m under PAM, from which $\overline{\gamma}_{\Delta q_f|R} \leq 0$ follows. This concludes the proof of statements *3.-a* and *3.-b*.

By analogous steps it can be shown that under NAM $\gamma_{\Delta q_f|R}$ is weakly positive and bounded below by $\underline{\gamma}_{\Delta q_f|R} \geq 0$ (statements *4.-a* and *4.-b*).

Finally, we prove that under PAM, if $F(q_m|D_B = 1, D = 1) \leq F(q_m|D_B = 1, D = 0)$, then $\gamma_{q_f|B} \geq 0$. As noted above, under PAM $\mathcal{M}(q_m) = [a(q_m), b(q_m)]$ with interval bounds that are weakly increasing in q_m (see Shimer and Smith (2000)). By implication, under PAM $\mathbb{E}\left[q_f|a(q_m) < q_f < b(q_m)\right]$ is weakly increasing in q_m. From $F(q_m|D_B = 1, D = 1) \leq F(q_m|D_B = 1, D = 0)$ it follows that[35]

$$\begin{aligned} & \int \mathbb{E}\left[q_f|a(q_m) < q_f < b(q_m)\right] dF(q_m|D_B = 1, D = 1) \\ \geq \quad & \int \mathbb{E}\left[q_f|a(q_m) < q_f < b(q_m)\right] dF(q_m|D_B = 1, D = 0). \end{aligned} \tag{A.8}$$

Using (A.4) and (A.6) it follows for $\gamma_{q_f|B}$ that

$$\begin{aligned} \gamma_{q_f|B} \quad = \quad & \int \mathbb{E}\left[q_m(t_0)|D_B = 1, D = 1, q_m\right] dF(q_m|D_B = 1, D = 1) \\ & - \int \mathbb{E}\left[q_m(t_0)|D_B = 1, D = 0\right] dF(q_m|D_B = 1, D = 0) \\ = \quad & \int \frac{1 - e^{-\delta(\tau - t_0)}}{1 - e^{-\delta(\tau - t_0)} + G_f(b(q_m)) - G_f(\max\{b(q_m - d), a(q_m)\})} \mathbb{E}\left[q_f|a(q_m) < q_f < b(q_m)\right] \\ & + \frac{G_f(b(q_m)) - G_f(\max\{b(q_m - d), a(q_m)\})}{1 - e^{-\delta(\tau - t_0)} + G_f(b(q_m)) - G_f(\max\{b(q_m - d), a(q_m)\})} \\ & \mathbb{E}\left[q_f|\max\{b(q_m - d), a(q_m)\} < q_f < b(q_m)\right] dF(q_m|D_B = 1, D = 1) \\ & - \int \mathbb{E}\left[q_f|a(q_m) < q_f < b(q_m)\right] dF(q_m|D_B = 1, D = 0) \end{aligned}$$

[34]Note that in general for any random variable X, and $a \leq a'$ it holds that $\mathbb{E}[X|a \leq X \leq b] \leq \mathbb{E}[X|a' \leq X \leq b]$.

[35]Note that in general, if $F_1(x) \geq F_2(x)$ for all x, then $\int h(x)dF_2(x) \geq \int h(x)dF_1(x)$ for any weakly increasing measurable function $h(x)$.

$$
\begin{aligned}
&\geq \int \mathbb{E}\left[q_f | a(q_m) < q_f < b(q_m)\right] dF(q_m | D_B = 1, D = 1) \\
&\quad - \int \mathbb{E}\left[q_f | a(q_m) < q_f < b(q_m)\right] dF(q_m | D_B = 1, D = 0) \\
&\geq 0\,,
\end{aligned}
$$

where the first weak inequality follows by (A.7) and the second follows by (A.8). This concludes the proof of statement *5-a*. Statement *5-b* can be proved by analogous steps. □

Proof of Lemma 1: Define $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i}) := \{q_{fi} : (q_{fi}, q_f^{-i}) \in \mathcal{M}(q_m)\}$. We proceed by first proving that any set $\mathcal{M}_i$ is a convex set and then show that its bounds are weakly increasing under PAM (i).[36]

1. $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i})$ is convex:

Consider $q'_{fi} < q''_{fi} < q'''_{fi}$, with q'_{fi} and q'''_{fi} in $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i})$, i.e.,

$$(q'_{fi}, q_f^{-i}) \in \mathcal{M}(q_m), \tag{A.9}$$

$$(q'''_{fi}, q_f^{-i}) \in \mathcal{M}(q_m). \tag{A.10}$$

Now consider $\mathcal{M}(q_f)$. By A-1 there exists a $\hat{q}_{mi}$ such that $(\hat{q}_{mi}, q_m^{-i}) \in \mathcal{M}(q''_{fi}, q_f^{-i})$. As matching is symmetric, equivalently:

$$(q''_{fi}, q_f^{-i}) \in \mathcal{M}(\hat{q}_{mi}, q_m^{-i}). \tag{A.11}$$

In case $\hat{q}_{mi} = q_{mi}$, (A.11) yields $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q_{mi}, q_m^{-i})$ and we have shown convexity of $\mathcal{M}_i$. Now suppose $\hat{q}_{mi} < q_{mi}$ then PAM (i) together with (A.9) and (A.11) implies $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q_m)$. If $\hat{q}_{mi} > q_{mi}$ the same follows from PAM (i), together with (A.10) and (A.11). In each case we have shown that $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q_{mi}, q_m^{-i})$, and thus that $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i})$ is convex. $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i})$ is thus an interval described by bounds $a_i(q_{mi}, q_m^{-i}, q_f^{-i})$, $b_i(q_{mi}, q_m^{-i}, q_f^{-i})$.

2. $a_i(q_{mi}, q_m^{-i}, q_f^{-i})$ and $b_i(q_{mi}, q_m^{-i}, q_f^{-i})$ are weakly increasing in q_{mi} under $PAM(i)$:

[36] Note that $\mathcal{M}_i$ is bounded, as it is a subset of $\left[\underline{q}_i, \overline{q}_i\right]$ by assumption.

b_i is weakly increasing in q_{mi}: Suppose not, then $b_i(q'_{mi}, q_m^{-i}, q_f^{-i}) > b_i(q''_{mi}, q_m^{-i}, q_f^{-i})$ for some $q'_{mi} < q''_{mi}$. Note that as $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i}) = [a_i(q_{mi}, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})]$ it follows that $b_i(q'_{mi}, q_m^{-i}, q_f^{-i}) \in \mathcal{M}_i(q'_{mi}, q_m^{-i}, q_f^{-i})$ and $b_i(q''_{mi}, q_m^{-i}, q_f^{-i}) \in \mathcal{M}_i(q''_{mi}, q_m^{-i}, q_f^{-i})$. Equivalently $(b_i(q'_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}) \in \mathcal{M}((q'_{mi}, q_m^{-i}))$ and $(b_i(q''_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}) \in \mathcal{M}(q''_{mi}, q_m^{-i})$. By PAM(i) this constellation implies $(b_i(q'_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}) \in \mathcal{M}(q''_{mi}, q_m^{-i})$. Equivalently, $b_i(q'_{mi}, q_m^{-i}, q_f^{-i}) \in \mathcal{M}_i(q''_{mi}, q_m^{-i}, q_f^{-i})$, in contradiction to $b_i(q''_{mi}, q_m^{-i}, q_f^{-i})$ being the upper bound of $\mathcal{M}_i(q''_{mi}, q_m^{-i}, q_f^{-i})$.

That a_i is weakly increasing in q_{mi} follows by similar steps that yield, $a_i(q''_{mi}, q_m^{-i}, q_f^{-i}) \in \mathcal{M}_i(q'_{mi}, q_m^{-i}, q_f^{-i})$, in contradiction to $a_i(q'_{mi}, q_m^{-i}, q_f^{-i})$ being the lower bound of $\mathcal{M}_i(q'_{mi}, q_m^{-i}, q_f^{-i})$.

The proof that $a_i(q_{mi}, q_m^{-i}, q_f^{-i})$ and $b_i(q_{mi}, q_m^{-i}, q_f^{-i})$ are weakly decreasing in q_{mi} under $NAM(i)$ proceeds analogously. □

Proof of Proposition 2: We first prove that under PAM(i) or NAM(i), $\gamma_B \geq 0$.

As men in the control group are not displaced, their types are unchanged between t_0 and τ, i.e., $q_m(\tau) = q_m(t_0)$. It follows that no endogenous breakups occur in the control group, while exogenous breakups occur at rate δ. Like in the one-dimensional case, the probability that control group couples break up between t_0 and τ is thus given by

$$P\left(D_B = 1 | D = 0\right) = 1 - e^{-\delta(\tau - t_0)} \tag{A.12}$$

under PAM as well as under NAM.

In the treatment group, the *i*-th dimension of men's type changes between t_0 and τ due to job displacement. Specifically, $q_{mi}(\tau) = q_{mi}(t_0) - d < q_{mi}(\tau)$. For a given man, with pre-displacement type $q_m(t_0)$, job displacement leads to a breakup if and only if $q_f(t_0) \in \mathcal{M}\left((q_m^i, q_m^{-i})\right)$ and $q_f(t_0) \notin \mathcal{M}\left((q_m^i - d, q_m^{-i})\right)$. By Lemma 1, equivalently $q_{fi} \in [a_i(q_{mi}, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})]$ and $q_{fi} \notin [a_i(q_{mi} - d, q_m^{-i}, q_f^{-i}), b_i(q_{mi} - d, q_m^{-i}, q_f^{-i})]$. Further, $a_i(q_{mi}, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})$ are weakly increasing in q_{mi} under PAM(i) and weakly decreasing in q_{mi} under NAM(i).

It follows under PAM(i) that job displacement leads to a breakup for a man of pre-displacement type q_m if and only if he is matched with a q_f-type woman, such that

$$q_{fi} \in \left(\max\{b_i(q_{mi} - d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}, b_i(q_{mi}, q_m^{-i}, q_f^{-i})\right].$$

Similarly, under NAM job displacement leads to breakup for a man of pre-displacement type q_m if and only if he is matched with a woman of type q_f, such that

$$q_{fi} \in \left[a_i(q_{mi}, q_m^{-i}, q_f^{-i}), \max\{a_i(q_{mi} - d, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})\}\right).$$

Additionally, breakups occur exogenously at rate δ under PAM(i) as well as under NAM(i).

Denote by $G_{fi}(q_{fi})$ the marginal CDF of q_{fi}, by $G_f^{-i}(q_f^{-i})$ the joint CDF of q_f^{-i}, and by $G_{fi}(q_{fi}|q_f^{-i})$ the marginal CDF of q_{fi} conditional on q_f^{-i}.

Under PAM(i) the overall probability that a man in the treatment group experiences a breakup between t_0 and τ is:

$$\begin{aligned} P(D_B = 1|D = 1) &= 1 - e^{-\delta(\tau - t_0)} \\ &\quad + \int\int G_{fi}\left(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}\right) \\ &\quad - G_{fi}\left(\max\{b_i(q_{mi} - d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i}\right) dG_f^{-i}\left(q_f^{-i}\right) dF(q_m) \end{aligned} \tag{A.13}$$

Similarly, under NAM(i) the overall probability that a man in the treatment group experiences a breakup between t_0 and τ is:

$$\begin{aligned} P(D_B = 1|D = 1) &= 1 - e^{-\delta(\tau - t_0)} \\ &\quad + \int\int G_{fi}\left(\min\{a_i(q_{mi} - d, q_m^{-i}, q_f^{-i}), b_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i}\right) \\ &\quad - G_{fi}\left(a_i(q_{mi} - d, q_m^{-i}, q_f^{-i}))|q_f^{-i}\right) dG_f^{-i}\left(q_f^{-i}\right) dF(q_m). \end{aligned} \tag{A.14}$$

From (A.12), (A.13), and (A.14) it follows that under PAM(i) as well as under NAM(i) $\gamma_B = P\left(D_B = 1|D = 1\right) - P\left(D_B = 1|D = 0\right) \geq 0$, concluding the proof of statement *1*.

Next, we turn to proving that under PAM(i), $\gamma_{\Delta q_{fi}} \leq 0$.

Denote by D_δ an indicator that equals 1 for men who experience an exogenous breakup between t_0 and τ, and 0 for all other men. Consider men in the treatment group of pre-displacement type q_m, who separate from their t_0-partner and rematch with a new partner between t_0 and τ. Moreover, condition on the t_0-partner's type in all but the i-th dimension, $q_f^{-i}(t_0) = q_f^{-i}$. The conditional mean of the t_0-partner's type in the i-th dimension can be written as weighted average:

$$
\begin{aligned}
&\mathbb{E}\left[q_{fi}(t_0)|D_R=1, D_B=1, D=1, q_m(t_0)=q_m, q_f^{-i}(t_0)=q_f^{-i}\right] = \\
&\mathbb{E}\left[q_{fi}(t_0)|D_R=1, D_B=1, D=1, q_m, q_f^{-i}, D_\delta=1\right] P(D_\delta=1|D_R=1, D_B=1, D=1, q_m, q_f^{-i}) \\
&+\mathbb{E}\left[q_{fi}(t_0)|D_R=1, D_B=1, D=1, q_m, q_f^{-i}, D_\delta=0\right] P(D_\delta=0|D_R=1, D_B=1, D=1, q_m, q_f^{-i}) \\
&= \frac{1-e^{-\delta(\tau-t_0)}}{1-e^{-\delta(\tau-t_0)} + G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
&\cdot \frac{1}{G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(a_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i})} \int\limits_{a_i\left(q_{mi}, q_m^{-i}, q_f^{-i}\right)}^{b_i\left(q_{mi}, q_m^{-i}, q_f^{-i}\right)} q_{fi} dG_{fi}(q_{fi}|q_f^{-i}) \\
&+\frac{G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})}{1-e^{-\delta(\tau-t_0)} + G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
&\cdot \frac{1}{G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
&\int\limits_{\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}}^{b_i(q_{mi}, q_m^{-i}, q_f^{-i})} q_{fi} dG_{fi}(q_{fi}|q_f^{-i}) \\
&= \frac{1-e^{-\delta(\tau-t_0)}}{1-e^{-\delta(\tau-t_0)} + G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
&\cdot \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_m^{-i}, q_f^{-i}) < q_{fi} < b_i(q_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}\right] \\
&+\frac{G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})}{1-e^{-\delta(\tau-t_0)} + G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
&\cdot \mathbb{E}\left[q_{fi}|\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\} < q_{fi} < b_i(q_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}\right]
\end{aligned} \tag{A.15}
$$

Taking into account that treatment group men are displaced in period t_0, the corresponding average for period τ is:

$$
\mathbb{E}\left[q_{fi}(\tau)|D_R=1, D_B=1, D=1, q_m, q_f^{-i}\right]
$$

$$= \frac{1}{G_{fi}(b_i(q_{mi}-d, q_m^{-i}, q_f^{-i})) - G_{fi}(a_i(q_{mi}-d, q_m^{-i}, q_f^{-i}))} \int\limits_{a_i\left(q_{mi}-d, q_m^{-i}, q_f^{-i}\right)}^{b_i\left(q_{mi}-d, q_m^{-i}, q_f^{-i}\right)} q_{fi} dG_{fi}(q_{fi}|q_f^{-i})$$

$$= \mathbb{E}\left[q_{fi}|a_i(q_{mi}-d, q_m^{-i}, q_f^{-i}) < q_{fi} < b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), q_f^{-i}\right]. \tag{A.16}$$

For the control group, by contrast, men's types are unchanged between t_0 and τ. The corresponding expressions therefore are:

$$\mathbb{E}\left[q_{fi}(t_0)|D_R = 1, D_B = 1, D = 0, q_m, q_f^{-i}\right]$$

$$= \mathbb{E}\left[q_{fi}(\tau)|D_R = 1, D_B = 1, D = 0, q_m, q_f^{-i}\right]$$

$$= \frac{1}{G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})) - G_{fi}(a_i(q_{mi}, q_m^{-i}, q_f^{-i}))} \int\limits_{a_i\left(q_{mi}, q_m^{-i}, q_f^{-i}\right)}^{b_i\left(q_{mi}, q_m^{-i}, q_f^{-i}\right)} q_{fi} dG_{fi}(q_{fi}|q_f^{-i})$$

$$= \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_m^{-i}, q_f^{-i}) < q_{fi} < b_i(q_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}\right] \tag{A.17}$$

Using (A.15), (A.20), and (A.17) it follows for $\gamma_{\Delta q_{fi}}$ that

$$\begin{aligned}
\gamma_{\Delta q_{fi}|R} \;=\;& \int \mathbb{E}\left[q_{fi}(\tau) - q_{fi}(t_0)|D_R = 1, D_B = 1, D = 1, q_m, q_f^{-i}\right] dG_f^{-i}(q_f^{-i}) dF(q_m|D_R = 1, D_B = 1, D = 1) \\
& - \int \mathbb{E}\left[q_{fi}(\tau) - q_{fi}(t_0)|D_R = 1, D_B = 1, D = 0, q_m, q_f^{-i}\right] dG_f^{-i}(q_f^{-i}) dF(q_m|D_R = 1, D_B = 1, D = 0) \\
=\;& \int\int \frac{1 - e^{-\delta(\tau - t_0)}}{1 - e^{-\delta(\tau - t_0)} + G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
& \left(\mathbb{E}\left[q_{fi}|a_i(q_{mi}-d, q_m^{-i}, q_f^{-i}) < q_{fi} < b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), q_f^{-i}\right] - \right. \\
& \left. \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_m^{-i}, q_f^{-i}) < q_{fi} < b_i(q_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}\right]\right) \\
& + \frac{G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})}{1 - e^{-\delta(\tau - t_0)} + G_{fi}(b_i(q_{mi}, q_m^{-i}, q_f^{-i})|q_f^{-i}) - G_{fi}(\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\}|q_f^{-i})} \\
& \left(\mathbb{E}\left[q_{fi}|a_i(q_{mi}-d, q_m^{-i}, q_f^{-i}) < q_{fi} < b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), q_f^{-i}\right] - \right. \\
& \left. \mathbb{E}\left[q_{fi}|\max\{b_i(q_{mi}-d, q_m^{-i}, q_f^{-i}), a_i(q_{mi}, q_m^{-i}, q_f^{-i})\} < q_{fi} < b_i(q_{mi}, q_m^{-i}, q_f^{-i}), q_f^{-i}\right]\right)
\end{aligned}$$

$$
\begin{aligned}
& dG_f^{-i}(q_f^{-i})dF(q_m|D_R=1,D_B=1,D=1) \\
\leq\; & \int\int\Bigg(\mathbb{E}\left[q_{fi}|a_i(q_{mi}-d,q_m^{-i},q_f^{-i})<q_{fi}<b_i(q_{mi}-d,q_m^{-i},q_f^{-i}),q_f^{-i}\right] \\
& -\mathbb{E}\left[q_{fi}|a_i(q_{mi},q_m^{-i},q_f^{-i})<q_{fi}<b_i(q_{mi},q_m^{-i},q_f^{-i}),q_f^{-i}\right]\Bigg)dG_f^{-i}(q_f^{-i})dF(q_m|D_R=1,D_B=1,D=1) \\
=\; & \int\int\mathbb{E}\left[q_{fi}|q_{mi}-d,q_m^{-i},q_f^{-i}\right]-\mathbb{E}\left[q_{fi}|q_{mi},q_m^{-i},q_f^{-i}\right]dG_f^{-i}(q_f^{-i})dF(q_m|D_R=1,D_B=1,D=1) \\
=\; & \int\int\mathbb{E}\left[q_{fi}|q_{mi}-d,q_m^{-i},q_f^{-i}\right]-\mathbb{E}\left[q_{fi}|q_{mi},q_m^{-i},q_f^{-i}\right]dG_f^{-i}(q_f^{-i})dF(q_m|D_R=1,D_B=1,D=1) \\
=\; & -\int\int\int\frac{\partial\mathbb{E}\left[q_{fi}|q_{mi},q_m^{-i},q_f^{-i}\right]}{\partial q_{mi}}\Bigg|_{q_{mi}=q-x}dxdG_f^{-i}(q_f^{-i})dF(q|D_R=1,D_B=1,D=1) \\
=\; & \overline{\gamma}_{\Delta q_{fi}|R},
\end{aligned}
$$

where the weak inequality follows as

$$
\begin{aligned}
& \mathbb{E}\left[q_{fi}|a_i(q_{mi},q_m^{-i},q_f^{-i})<q_{fi}<b_i(q_{mi},q_m^{-i},q_f^{-i}),q_f^{-i}\right] \\
\leq\; & \mathbb{E}\left[q_{fi}|\max\{b_i(q_{mi}-d,q_m^{-i},q_f^{-i}),a_i(q_{mi},q_m^{-i},q_f^{-i})\}<q_{fi}<b_i(q_{mi}-d,q_m^{-i},q_f^{-i}),q_f^{-i}\right].
\end{aligned}
\tag{A.18}
$$

By Lemma 1 under PAM(i)

$$
\mathbb{E}\left[q_{fi}|q_{mi},q_m^{-i},q_f^{-i}\right]=\mathbb{E}\left[q_{fi}|a_i(q_{mi},q_m^{-i},q_f^{-i})<q_{fi}<b_i(q_{mi},q_m^{-i},q_f^{-i}),q_f^{-i}\right]
$$

is weakly increasing in q_{mi}, from which $\overline{\gamma}_{\Delta q_{fi}|R}\leq 0$ follows. This concludes the proof of statements *3.-a* and *3.-b*.

By analogous steps it can be shown that under NAM(i) $\gamma_{\Delta q_{fi}|R}$ is weakly positive and bounded below by $\overline{\gamma}_{\Delta q_{fi}|R}\geq 0$ (statements *4.-a* and *4.-b*). □

Lemma 2. *Given the assumptions of Lemma 1 and A-2, under PAM(j) or NAM(j)*

$$
\{q_{fi}:(q_{fi},q_f^{-i})\in\mathcal{M}(q_m)\}=[a_i(q_{mi},q_{mj},q_m^{-i,j},q_f^{-i}),b_i(q_{mi},q_{mj},q_m^{-i,j},q_f^{-i})],
$$

where a_i,b_i are

(i) increasing in q_{mj} under PAM(j),

(ii) decreasing in q_{mj} under NAM(j).

Proof of Lemma 2: We start by proving that for any $q'_{fi} < q''_{fi}$, $q'_{mj} < q''_{mj}$, q_f^{-i}, and q_m^{-j}:

$$\begin{aligned} & (q'_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mj}, q_m^{-j}) \text{ and } (q''_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mj}, q_m^{-j}) \\ \Rightarrow \quad & (q'_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mj}, q_m^{-j}) \text{ and } (q''_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mj}, q_m^{-j}). \end{aligned} \tag{A.19}$$

Under PAM(j) it follows by Lemma 1 that:

$$(q'_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mj}, q_m^{-j}) \Leftrightarrow q_{fj} \in [a_j(q''_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j}), b_j(q''_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j})],$$

with a_j, b_j weakly increasing in q_{mj}. It follows that $a_j(q'_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j}) \leq a_j(q''_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j})$. In the special case $q_{fj} = a_j(q'_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j})$, it follows trivially that $(q'_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mj}, q_m^{-j})$. Outside this special case, it holds that $a_j(q'_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j}) < q_{fj}$. It follows that there exists a $\breve{q}_{fj} < q_{fj}$ such that $\breve{q}_{fj} \in [a_j(q'_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j}), b_j(q'_{mj}, q_m^{-j}, q'_{fi}, q_f^{-i,j})]$, or equivalently $(q'_{fi}, \breve{q}_{fj}, q_f^{-i,j}) \in \mathcal{M}(q'_{mj}, q_m^{-j})$. Together with $(q''_{fi}, q_{fj}, q_f^{-i,j}) \in \mathcal{M}(q'_{mj}, q_m^{-j})$ by A-2, $(q'_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mj}, q_m^{-j})$ is implied (the first part of the right hand side of implication A.19).

By analogous steps, using PAM(j) together with Lemma 1 and A-2, it can be shown that $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q'_{mj}, q_m^{-j})$ implies $(q''_{fi}, q_f^{-i}) \in \mathcal{M}(q''_{mj}, q_m^{-j})$, proving the second part of implication A.19.

By Lemma 1 we have

$$\mathcal{M}_i(q_{mi}, q_{mj}, q_m^{-i,j}, q_f^{-i}) = [a_i(q_{mi}, q_{mj}, q_m^{-i,j}, q_f^{-i}), b_i(q_{mi}, q_{mj}, q_m^{-i,j}, q_f^{-i})]$$

for $\mathcal{M}_i(q_{mi}, q_m^{-i}, q_f^{-i}) := \{q_{fi} : (q_{fi}, q_f^{-i}) \in \mathcal{M}(q_m)\}$. Next, we use A.19 to show that under PAM(j) b_i is weakly increasing in q_{mj}: Suppose not, then $b_i(q_{mi}, q'_{mj}, q_m^{-i,j}, q_f^{-i}) > b_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i})$ for some $q'_{mj} < q''_{mj}$.

From $\mathcal{M}_i(q_{mi}, q_{mj}, q_m^{-i,j}, q_f^{-i}) = [a_i(q_{mi}, q_{mj}, q_m^{-i,j}, q_f^{-i}), b_i(q_{mi}, q_{mj}, q_m^{-i,j}, q_f^{-i})]$ it follows that $b_i(q_{mi}, q'_{mj}, q_m^{-i,j}, q_f^{-i}) \in \mathcal{M}_i(q_{mi}, q'_{mj}, q_m^{-i,j}, q_f^{-i})$ and $b_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i}) \in \mathcal{M}_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i})$. Equivalently, $(b_i(q_{mi}, q'_{mj}, q_m^{-i}, q_f^{-i}), q_f^{-i}) \in \mathcal{M}((q_{mi}, q'_{mj}, q_m^{-i,j}))$ and $(b_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i}), q_f^{-i}) \in \mathcal{M}((q_{mi}, q''_{mj}, q_m^{-i,j}))$.

By A.19 this constellation implies $(b_i(q_{mi}, q'_{mj}, q_m^{-i,j}, q_f^{-i}), q_f^{-i}) \in \mathcal{M}((q_{mi}, q''_{mj}, q_m^{-i,j}))$, or equivalently, $b_i(q_{mi}, q'_{mj}, q_m^{-i,j}, q_f^{-i}) \in \mathcal{M}_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i})$, in contradiction to $b_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i})$ being the upper bound of $\mathcal{M}_i(q_{mi}, q''_{mj}, q_m^{-i,j}, q_f^{-i})$.

By similar steps it can be shown that a_i is weakly increasing in q_{mj}.

The proof that a_i, b_i are weakly decreasing in q_{mj} under NAM(j) proceeds analogously. □

Proof of Proposition 3: We first show that PAM(i) implies $DE \geq 0$. By Lemma 1

$$\begin{aligned}\mathbb{E}\left[q_{fi}|q_{mi}, q_{mj}, q_{fj}\right] &= \frac{1}{G_{fi}(b_i(q_{mi}, q_{mj}, q_{fj})) - G_{fi}(a_i(q_{mi}, q_{mj}, q_{fj}))} \int\limits_{a_i\left(q_{mi}, q_{mj}, q_{fj}\right)}^{b_i\left(q_{mi}, q_{mj}, q_{fj}\right)} q_{fi} dG_{fi}(q_{fi}|q_{fj}) \\ &= \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_{mj}, q_{fj}) < q_{fi} < b_i(q_{mi}, q_{mj}, q_{fj}), q_{fj}\right].\end{aligned}$$

where, under PAM(i), $a_i(q_{mi}, q_{mj}, q_{fj})$ and $b_i(q_{mi}, q_{mj}, q_{fj})$ are weakly increasing in q_{mi} implying the same for $\mathbb{E}\left[q_{fi}|q_{mi}, q_{mj}, q_{fj}\right]$. It follows that

$$E[q_{fi}|q_{mi}, q_{mj}] = \int \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_{mj}, q_{fj}) < q_{fi} < b_i(q_{mi}, q_{mj}, q_{fj}), q_{fj}\right] dG_{fj}(q_{fj})$$

is also weakly increasing in q_{mi}, and

$$DE = \int E[q_{fi}|q''_{mi}, q_{mj}] - E[q_{fi}|q'_{mi}, q_{mj}] dG_{mj}(q_{mj}|q''_{mi}) \geq 0.$$

By analogous steps it follows that NAM(i) implies $DE \leq 0$.

Next, we establish that under PAM(j) if $G_{mj}(q_{mj}|q_{mi})$ is weakly decreasing in q_{mi}, $IE \geq 0$ follows. By Lemma 1

$$\mathbb{E}\left[q_{fi}|q_{mi}, q_{mj}, q_{fj}\right] = \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_{mj}, q_{fj}) < q_{fi} < b_i(q_{mi}, q_{mj}, q_{fj}), q_{fj}\right].$$

By Lemma 2 $a_i(q_{mi}, q_{mj}, q_{fj})$ and $b_i(q_{mi}, q_{mj}, q_{fj})$ are weakly increasing in q_{mj} under PAM (j), implying the same for $\mathbb{E}\left[q_{fi}|q_{mi}, q_{mj}, q_{fj}\right]$. It follows that

$$E[q_{fi}|q_{mi}, q_{mj}] = \int \mathbb{E}\left[q_{fi}|a_i(q_{mi}, q_{mj}, q_{fj}) < q_{fi} < b_i(q_{mi}, q_{mj}, q_{fj}), q_{fj}\right] dG_{fj}(q_{fj})$$

weakly increasing in q_{mj}. As $G(q_{mj}|q''_{mi})$ first order stochastically dominates $G(q_{mj}|q'_{mi})$ this implies

$$IE = \int \mathbb{E}[q_{fi}|q'_{mi}, q_{mj}]dG(q_{mj}|q''_{mi}) - \int \mathbb{E}[q_{fi}|q'_{mi}, q_{mj}]dG(q_{mj}|q'_{mi}) \geq 0.$$

The remaining implications for IE follow analogously. $\square$