Optimal Weights for Marital Sorting Measures*

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Abstract

Changing distributions of male and female types affect the measurement of education-based marriage market sorting. We develop a weighting strategy that minimizes the distortion of sorting measures due to changing type distributions. The optimal weights reflect that female type distributions have changed relatively more in recent decades. Based on our weighted measure, we document increased sorting in Denmark between 1980 and 2018. Alternative measures suggest flat or decreasing trends.

Keywords: Positive Assortative Mating, Marriage Market Sorting, Homophily, Educational Attainment, Sorting Measures, Aggregation

JEL Classifications: C43, D10, J11, J12

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1 Introduction

An increasing tendency of individuals to marry their like in terms of educational attainment, a phenomenon known as positive assortative mating (PAM, sorting), potentially increases inequality between households (e.g., Kremer, 1997; Fernández and Rogerson, 2001; Breen and Salazar, 2011; Chiappori et al., 2020a). However, the literature disagrees on whether PAM has increased (e.g., Greenwood et al., 2016; Eika et al., 2019; Almar et al., 2023). One reason is that shifting distributions of education-based types distort the measurement of sorting. While recent papers acknowledge this (e.g., Liu and Lu, 2006; Eika et al., 2019; Chiappori et al., 2020b, 2021), how this distortion can be compensated for is an open question.

To answer this question, we provide an in-depth analysis of a widely used sorting measure: the weighted sum of likelihood ratios. This measure captures marital sorting by comparing the observed probability that a man of a given type is married to a woman of the same type to that probability under random matching. The likelihood ratios for these same-type couples are aggregated using weights.

We derive optimal weights that minimize the distortion caused by changing type distributions. The optimal weights reflect that educational outcomes of females increased relatively more in recent decades (consistent with, e.g., Goldin, 2006) and eliminate the dominating effect of female-type-distribution changes on the sorting measure. The optimally weighted measure detects increasing PAM. Conventionally weighted measures suggest flat or decreasing trends because they confound increasing PAM with the increase in the supply of highly educated women. We conclude that it is important to take gender-specific trends in the underlying type distributions into account because they matter for conclusions about sorting trends.

2 Data and Trends

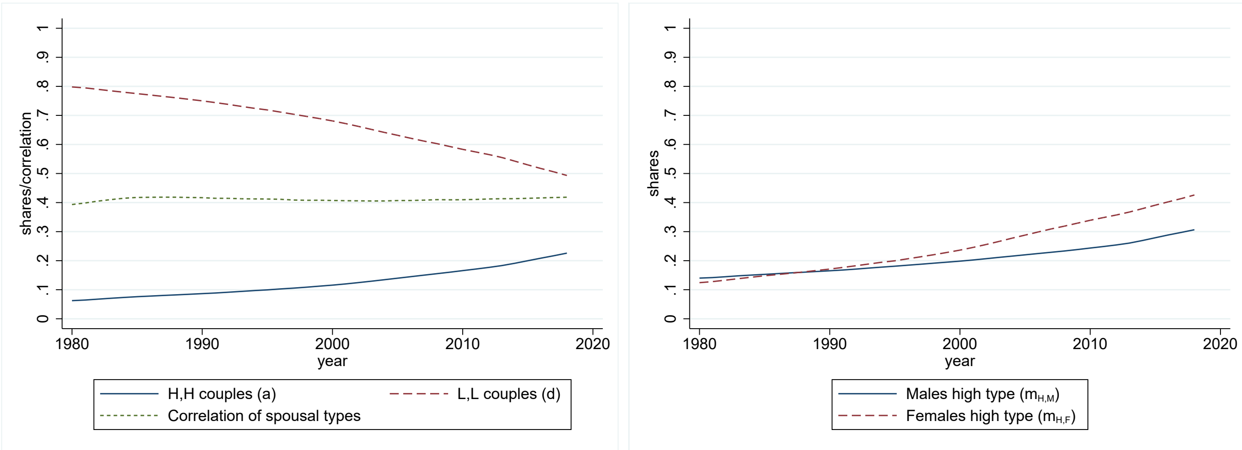
We use Danish data to illustrate how changing type distributions affect the measurement of PAM. The population register contains demographic variables and person IDs for all residents and their (married or cohabiting) partners (Statistics Denmark BEF, 1980–2018).[1] We study the period 1980–2018 and observe on average 1,800,866 individuals in the age range 19–60 per year who are either married to or cohabiting with an individual of the opposite sex. The combined stock of couples is stable over time.[2]

[1] Cohabitation is identified based on a number of criteria: opposite sex, joint children, shared address, less than a 15-year age difference, no family relationship.

[2] Figure A.1 depicts the evolution of the stocks of couple types and their age composition.

Figure 1: Assortative Matches and Marginal Distributions

(a) Couple Shares and Correlation (b) Marginal Distributions



Note: Panel (a) shows how the shares of couples in which both spouses have either high or low educational attainment, (H, H) or (L, L), have evolved over time, along with the cross-sectional correlation of spousal types. Panel (b) shows the evolution of the fraction of highly educated males and females. Section 2 explains how the sample and education-based types are constructed. The symbols a, d, $m_{(H,M)}$, and $m_{(H,F)}$ are introduced in Section 3 and link the data series to the formal analysis of sorting measures.

We use the education register (Statistics Denmark UDDA, 1980–2018) to distinguish between highly educated individuals (bachelor's degrees and above, ISCED 6–8) and individuals with lower educational attainment (compulsory schooling, high school, vocational training, short-cycle tertiary programs, ISCED 1–5). Thus, the education-based type T is either H (high) or L (low). Gender is indexed M (male) and F (female).

Figure 1a shows that the share of (H, H) couples (blue, solid line) increased between 1980 and 2018. Thus, it has become more common to observe couples in which both partners are highly educated. This alone, however, is not evidence of increasing PAM. The share of (L, L) couples (red, dashed line) has decreased at the same time. Moreover, educational attainment has increased, see Panel (b). In 2018, more than 40% (30%) of women (men) are highly educated, compared to approximately 15% in 1980. This shift in the marginal type distributions affects the share of (H, H) couples directly because it became more likely to meet highly educated individuals. Note also that the cross-sectional correlation of couple types in Panel (a) (green, dotted line) is essentially flat. The correlation coefficient conflates changes in couple shares and changes in marginal distributions. We show in Appendix A.2 that the correlation responds to such changes in a highly nonlinear way, which makes the trend of the correlation coefficient uninformative about PAM.[3]

In summary, we need a formal framework to measure PAM and disentangle it from changes in the marginal type distributions.

[3]See also Eika et al. (2019) and Chiappori et al. (2021).

Table 1: Contingency Tables

(a)

M\F	H	L	Marginal
H	a	b	$a+b$
L	c	d	$c+d$
Marginal	$a+c$	$b+d$	1

(b)

M\F	H	L	Marginal
H	a	$m_{(H,M)} - a$	$m_{(H,M)}$
L	$m_{(H,F)} - a$	d	$1 - m_{(H,M)}$
Marginal	$m_{(H,F)}$	$1 - m_{(H,F)}$	1

3 Measurement and Optimal Weights

3.1 The Setup

Table 1a is a contingency table that describes the marriage market allocation. The share of couples in which both spouses have high (low) education is $a > 0$ ($d > 0$). Thus, $a + d$ is the share of sorted couples, while the sum of $b > 0$ and $c > 0$ denotes the share of couples with different levels of education. Intuitively, the higher $a + d$ is relative to $b + c$, the more pronounced PAM. To investigate how changing marginals affect sorting measures, we substitute the share of high-type men $m_{(H,M)}$ for $a+b$ and the share of high-type women $m_{(H,F)}$ for $a+c$ in Table 1b.

3.2 The Weighted Sum of Likelihood Ratios

Based on Table 1b, we define the weighted sum of likelihood ratios as follows:

$$I_{\mathcal{S}} = \frac{a}{m_{(H,M)}m_{(H,F)}} \times w_H + \frac{d}{(1-m_{(H,M)})(1-m_{(H,F)})} \times w_L. \tag{1}$$

PAM is captured by the ratio of the actual shares of sorted couples and the expected shares based on the "supply" of different types. This measure fulfills the formal criteria for sorting measures outlined by Chiappori et al. (2020b, 2021). It aggregates the shares of sorted couples along the diagonal using the weights w_H and w_L. Chiappori et al. (2020b) suggest that these weights can be thought of as a convex combination of the shares of males and females with the same level of education, which depend on the respective marginal distributions. Let $I_{\mathcal{S}}^{convex}$ denote the measure with these weights applied, where $\lambda \in [0, 1]$ is the coefficient on the male marginal distribution:

$$\begin{aligned} I_{\mathcal{S}}^{convex} &= \frac{a}{m_{(H,M)}m_{(H,F)}} \times (\lambda m_{(H,M)} + (1-\lambda)m_{(H,F)}) \\ &+ \frac{d}{(1-m_{(H,M)})(1-m_{(H,F)})} \times (\lambda(1-m_{(H,M)}) + (1-\lambda)(1-m_{(H,F)})). \end{aligned} \tag{2}$$

To investigate the impact of changing shares of sorted couples and marginal distributions, we totally differentiate (2):

$$\Delta I_S^{convex} = \underbrace{\left(\frac{\lambda m_{(H,M)} + (1-\lambda) m_{(H,F)}}{m_{(H,M)} m_{(H,F)}}\right)}_{>0} \Delta a + \underbrace{\left(\frac{\lambda(1-m_{(H,M)}) + (1-\lambda)(1-m_{(H,F)})}{(1-m_{(H,M)})(1-m_{(H,F)})}\right)}_{>0} \Delta d$$
$$+ \underbrace{(1-\lambda)\left(\frac{d}{(1-m_{(H,M)})^2} - \frac{a}{m^2_{(H,M)}}\right)}_{\gtreqless 0} \Delta m_{(H,M)} + \underbrace{\lambda\left(\frac{d}{(1-m_{(H,F)})^2} - \frac{a}{m^2_{(H,F)}}\right)}_{\gtreqless 0} \Delta m_{(H,F)}. \tag{3}$$

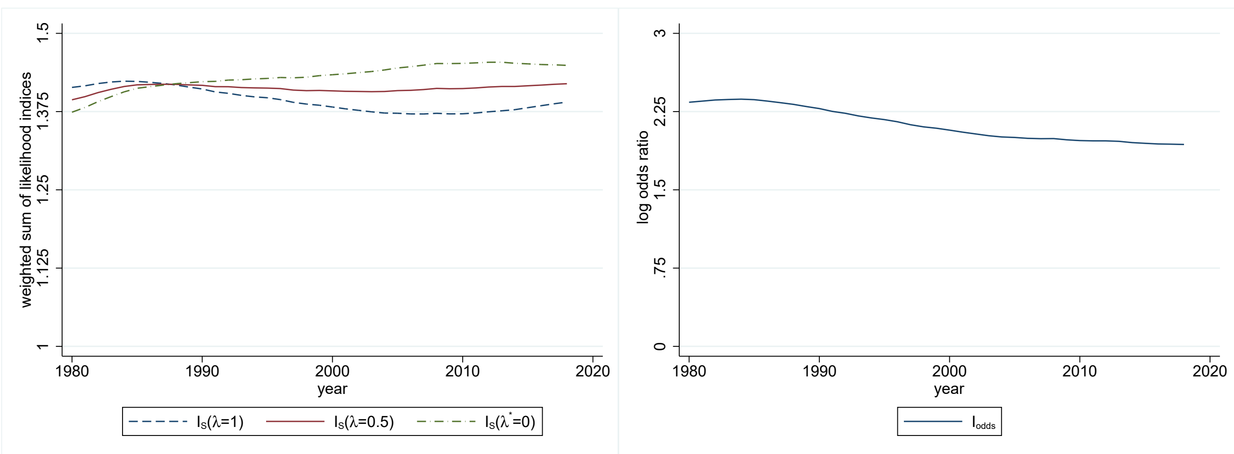
The sorting measure I_S^{convex} is increasing in the shares of sorted couples (a, d) because the coefficients in the first line are unambiguously positive. However, the impact of changing marginal distributions is ambiguous and depends on both the configuration of the contingency table and λ. Thus, the choice of λ allows us to account for gender differences in the effect of changing marginals on measured sorting.

We plot the weighted sum of likelihood ratios I_S^{convex} for different values of λ in Figure 2a. It indicates PAM in all cases because $I_S^{convex} > 1$. However, different values of λ lead to different trends. With weight on changes in the male type distribution ($\lambda = 1$), sorting is decreasing. With weight on changes in the female type distribution ($\lambda = 0$), sorting is increasing. For $\lambda = 0.5$, the trend is flat. Thus, the choice of λ is crucial for conclusions about the trend of PAM.

Figure 2: Sorting Trends Depend on Measures and Weights

(a) Weighted Sum of Likelihood Ratios

(b) Log Odds Ratio



Note: Panel (a) depicts the evolution of the weighted sum of likelihood ratios as defined in Equation (1) for three different values of λ. Panel (b) depicts the evolution of the odds ratio as defined in Equation (5). Section 2 explains how the sample and the education-based types are constructed.

We propose to choose λ to minimize the impact of marginal-distribution changes on the sorting measure. This can be achieved by setting $\lambda \in [0, 1]$ such that the absolute value of the sum of the $\Delta m_{(H,M)}$ and $\Delta m_{(H,F)}$ terms in Equation (3) is minimized:

$$\min_{\lambda} \left| (1-\lambda) \underbrace{\left(\frac{d}{(1-m_{(H,M)})^2} - \frac{a}{m_{(H,M)}^2} \right) \Delta m_{(H,M)}}_{=\gamma_1} + \lambda \underbrace{\left(\frac{d}{(1-m_{(H,F)})^2} - \frac{a}{m_{(H,F)}^2} \right) \Delta m_{(H,F)}}_{=\gamma_2} \right| .$$

This objective function is a convex combination of the two endpoints γ_1 and γ_2. If $sign(\gamma_1) \neq sign(\gamma_2)$, then zero lies between the two endpoints and the optimal λ^* solves $(1-\lambda)\gamma_1 + \lambda\gamma_2 = 0$. If, on the other hand, $sign(\gamma_1) = sign(\gamma_2)$, then the optimal λ^* is the endpoint with the smallest absolute value, either $|\gamma_1|$ or $|\gamma_2|$. In summary:

$$\lambda^* = \begin{cases} 0 & \text{if} \quad sign(\gamma_1) = sign(\gamma_2), \quad |\gamma_2| > |\gamma_1| \\ \frac{\gamma_1}{\gamma_1 - \gamma_2} & \text{if} \quad sign(\gamma_1) \neq sign(\gamma_2) \\ 1 & \text{if} \quad sign(\gamma_1) = sign(\gamma_2), \quad |\gamma_1| > |\gamma_2| . \end{cases} \tag{4}$$

From Figure 1, we know that $\Delta m_{(H,F)} > \Delta m_{(H,M)} > 0$ and that in the base year 1980, $m_{(H,M)} \approx m_{(H,F)}$. Thus, $sign(\gamma_1) = sign(\gamma_2)$ for all years, and λ^* must be either zero or one. In the data, $|\gamma_2| > |\gamma_1|$ because of the larger change in the female marginal type distribution. Thus, $\lambda^* = 0$ is optimal for all years.

We know from Figure 2a that $\lambda^* = 0$ implies increasing sorting. This is because the positive contribution of more sorted high-type couples (term one in Equation (3) is positive) outweighs the negative contributions from fewer sorted low-type couples (term two in Equation (3) is negative) and changing marginal type distributions (term three is negative, and term four drops out with $\lambda^* = 0$ in Equation (3)).[4]

An advantage of the weighted sum of likelihood ratios is that it can be defined for any number of types. In Appendix A.3, we generalize the decision rule (4) for more than two types.

[4]Totally eliminating the effect of changing marginal distributions on the sorting measure—the sum of terms three and four in Equation (3)—would require λ^* to be outside the unit interval. The measure would no longer fulfill the monotonicity property stated in Chiappori et al. (2021) because the measure would decrease in the share of sorted couples; see terms one and two in Equation (3).

3.3 Alternative Measures

An alternative measure that also fulfills the formal criteria for sorting measures outlined by Chiappori et al. (2020b, 2021) is the log odds ratio. Based on Table 1, it is defined as follows:

$$I_{odds} = \ln\left(\frac{ad}{bc}\right) = \ln\left(\frac{ad}{(m_{(H,M)} - a)(m_{(H,F)} - a)}\right), \tag{5}$$

where the denominator can be written in terms of the marginal distributions using $m_{(H,M)}$ and $m_{(H,F)}$. As before, we totally differentiate I_{odds}.

$$\begin{aligned} \Delta I_{odds} &= \underbrace{\left(\frac{m_{(H,M)}m_{(H,F)} - a^2}{a(m_{(H,M)} - a)(m_{(H,F)} - a)}\right)}_{>0}\Delta a + \underbrace{\left(\frac{1}{d}\right)}_{>0}\Delta d \\ &- \underbrace{\left(\frac{1}{m_{(H,M)} - a}\right)}_{>0}\Delta m_{(H,M)} - \underbrace{\left(\frac{1}{m_{(H,F)} - a}\right)}_{>0}\Delta m_{(H,F)}. \end{aligned} \tag{6}$$

Increasing shares of sorted couples a and d imply higher sorting, while increasing shares of high-type individuals $m_{(H,M)}$ and $m_{(H,F)}$ imply lower sorting. Thus, I_{odds} can decrease over time even with increasing shares a and d if the increase in $m_{(H,M)}$ or $m_{(H,F)}$ is sufficiently large.

We plot I_{odds} in Figure 2b. A log odds ratio > 0 indicates PAM. However, this sorting measure is decreasing over time. The increasing share of (H, H) couples ($\Delta a > 0$) is dominated by a decreasing share of (L, L) couples ($\Delta d < 0$) and increasing shares of highly educated males and females ($\Delta m_{(H,M)} > 0$, $\Delta m_{(H,F)} > 0$); recall Figure 1. Note that the coefficients of the male and female high-type shares are symmetric in (6). Therefore, the measure does not allow for gender-specific effects of changing marginals on measured PAM.

An advantage of the (log) odds ratio is that standardization algorithms can be used to generate uniform marginal distributions while preserving the association patterns in the contingency table (Mosteller, 1968). Tan et al. (2004) show that this procedure leaves the odds ratio unchanged, so sorting patterns can be compared over time despite changing type distributions (see, e.g., Greenwood et al., 2014). However, a limitation of the odds ratio is that it is defined for two types only. The weighted sum of likelihood ratios, which can be defined for any number of types, is, however, not invariant to standardization.[5] Our optimal weighting strategy is a viable alternative to standardization because it facilitates the analysis of sorting trends with more than two types and changing marginal distributions.

[5]As we illustrate in Appendix A.4, standardization changes the sorting measure $I_{\mathcal{S}}$, defined in Equation (1). The reason is that the diagonal elements of the contingency table, which the measure is based on, change while the offsetting changes to the off-diagonal elements are not taken into account.

Finally, Greenwood et al. (2016), Eika et al. (2019), and Almar et al. (2023) use versions of the measure $I_{\mathcal{S}}$ with alternative weights. In Appendix A.5, we show that those weights are not necessarily a convex combination of the male and female marginals. The effect of more sorted couples is thus not guaranteed to be positive. In our data, conclusions based on the optimal λ and the alternative weights used in the literature are similar, i.e., sorting has increased. However, this is coincidental and not guaranteed to hold in other settings.

4 Conclusion

We show how to use gender-specific weights, which compensate for changes in the underlying type distributions, to improve the measurement of education-based marriage market sorting. Because the female type distribution has changed more than its male counterpart in recent decades, attaching the weight to the female side minimizes the distortion of the sorting measure.

We find increasing PAM, while conventionally-weighted measures, which confound increasing PAM with the pronounced increase in the supply of highly educated women, suggest flat or decreasing trends. Thus, the weighting scheme is important, and researchers should use context-dependent weights that are disciplined by the data to study sorting trends.

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Online Appendix

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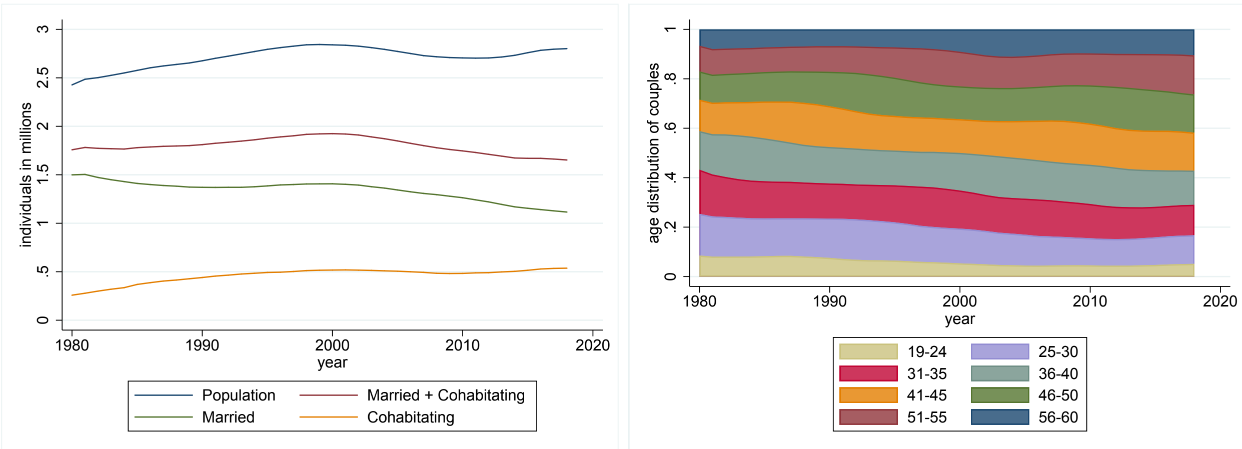
A Appendix

A.1 Additional Results

Figure A.1: Marriage, Cohabitation, Age Composition

(a) Stocks of Couples

(b) Age Composition of Couples



Note: Panel (a) reports the development in numbers of individuals by marital status. Panel (b) plots the age distribution of individuals who are either legally married or cohabiting. Section 2 explains how the sample and the education-based types are constructed.

A.2 The Correlation Coefficient

Following Chiappori et al. (2021), the correlation coefficient in the 2×2 case as described in Table 1b can be written in the following way:

$$I_{corr} = \frac{ad - (m_{H,M} - a)(m_{H,F} - a)}{\sqrt{m_{H,M}(1 - m_{H,M})m_{H,F}(1 - m_{H,F})}}. \tag{A.1}$$

Applying the same approach as in the main text, we totally differentiate (A.1); define $\Theta = \sqrt{m_{H,M}(1 - m_{H,M})m_{H,F}(1 - m_{H,F})}$, and obtain the following:

$$
\begin{aligned}
\Delta I_{corr} \quad = \quad & \underbrace{\left(\frac{d-2a+m_{H,M}+m_{H,F}}{\Theta}\right)}_{>0}\Delta a+\underbrace{\left(\frac{a}{\Theta}\right)}_{>0}\Delta d \qquad\qquad \text{(A.2)}\\
+ & \underbrace{\left(\frac{-(m_{(H,F)}-a)\Theta-\frac{\sqrt{m_{(H,F)}}(1-2m_{(H,M)})\sqrt{1-m_{(H,F)}}}{2\sqrt{m_{(H,M)}}\sqrt{1-m_{(H,M)}}}(ad-(m_{(H,M)}-a)(m_{(H,F)}-a))}{\Theta}\right)}_{\gtreqless 0}\Delta m_{(H,M)}\\
+ & \underbrace{\left(\frac{-(m_{(H,M)}-a)\Theta-\frac{\sqrt{m_{(H,M)}}(1-2m_{(H,F)})\sqrt{1-m_{(H,M)}}}{2\sqrt{m_{(H,F)}}\sqrt{1-m_{(H,F)}}}(ad-(m_{(H,M)}-a)(m_{(H,F)}-a))}{\Theta}\right)}_{\gtreqless 0}\Delta m_{(H,F)}.
\end{aligned}
$$

As can be seen from Equation (A.2), the impact of changing marginal distributions $m_{(H,M)}$ and $m_{(H,F)}$ on the correlation coefficient is ambiguous and highly nonlinear.

A.3 Generalization

An advantage of the I_S^{convex} sorting measure is its generalizability to more than two types. Consider a marriage market with N types of males and N types of females. Table A.1 shows the generalized contingency table for this case.

Table A.1: Generalized Contingency Table

M\F	1	2	$\cdots$	$n-1$	n	Marginal
1	a_{11}	a_{12}	$\cdots$	$a_{1,n-1}$	$m_{1M}-\sum_{i=1}^{n-1} a_{1i}$	m_{1M}
2	a_{21}	a_{22}	$\cdots$	$a_{2,n-1}$	$m_{2M}-\sum_{i=1}^{n-1} a_{2i}$	m_{2M}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
$n-1$	$a_{n-1,1}$	$a_{n-1,2}$	$\cdots$	$\ddots$	$\vdots$	$m_{n-1,M}$
n	$m_{1F}-\sum_{i=1}^{n-1} a_{i1}$	$m_{2F}-\sum_{i=1}^{n-1} a_{i2}$	$\cdots$	$\cdots$	a_{nn}	$1-\sum_{i=1}^{n-1} m_{iM}$
Marginal	m_{1F}	m_{2F}	$\cdots$	$m_{n-1,F}$	$1-\sum_{i=1}^{n-1} m_{iF}$	1

The generalized version of I_S^{convex} can be written as follows:

$$
\begin{aligned}
I_S^{convex} = & \sum_{i=1}^{n-1}\frac{a_{ii}}{m_{iM}m_{iF}}\left(\lambda m_{iM}+(1-\lambda)m_{iF}\right)\\
& +\frac{a_{nn}}{(1-\sum_{i=1}^{n-1} m_{iM})(1-\sum_{i=1}^{n-1} m_{iF})}\left(\lambda\left(1-\sum_{i=1}^{n-1} m_{iM}\right)+(1-\lambda)\left(1-\sum_{i=1}^{n-1} m_{iF}\right)\right)
\end{aligned} \tag{A.3}
$$

As in the 2×2 case in (2), we take the weighted sum over all the diagonal cells of the contingency table divided by the product of the respective male and female marginal distributions.

Next, we totally differentiate Equation (A.3):

$$\begin{aligned}
\Delta I_S^{convex} = & \sum_{k=1}^{n-1} \underbrace{\left(\frac{\lambda m_{kM} + (1-\lambda) m_{kF}}{m_{kM} m_{kF}}\right)}_{>0} \Delta a_{kk} \\
& + \underbrace{\left(\frac{\lambda \left(1 - \sum_{i=1}^{n-1} m_{iM}\right) + (1-\lambda)\left(1 - \sum_{i=1}^{n-1} m_{iF}\right)}{\left(1 - \sum_{i=1}^{n-1} m_{iM}\right)\left(1 - \sum_{i=1}^{n-1} m_{iF}\right)}\right)}_{>0} \Delta a_{nn} \\
& + (1-\lambda) \underbrace{\sum_{k=1}^{n-1} \left(\frac{a_{nn}}{\left(1 - \sum_{i=1}^{n-1} m_{iM}\right)^2} - \frac{a_{kk}}{m_{kM}^2}\right) \Delta m_{kM}}_{=\xi_1 \gtreqless 0} \\
& + \lambda \underbrace{\sum_{k=1}^{n-1} \left(\frac{a_{nn}}{\left(1 - \sum_{i=1}^{n-1} m_{iF}\right)^2} - \frac{a_{kk}}{m_{kF}^2}\right) \Delta m_{kF}}_{=\xi_2 \gtreqless 0}.
\end{aligned} \tag{A.4}$$

We apply the same logic as in the 2×2 case (Equation (4)) to derive a decision rule for λ in the general case:

$$\lambda^* = \begin{cases} 0 & \text{if } sign(\xi_1) = sign(\xi_2), \quad |\xi_1| < |\xi_2| \\ \frac{\xi_1}{\xi_1 - \xi_2} & \text{if } sign(\xi_1) \neq sign(\xi_2) \\ 1 & \text{if } sign(\xi_1) = sign(\xi_2), \quad |\xi_1| > |\xi_2| \end{cases} \tag{A.5}$$

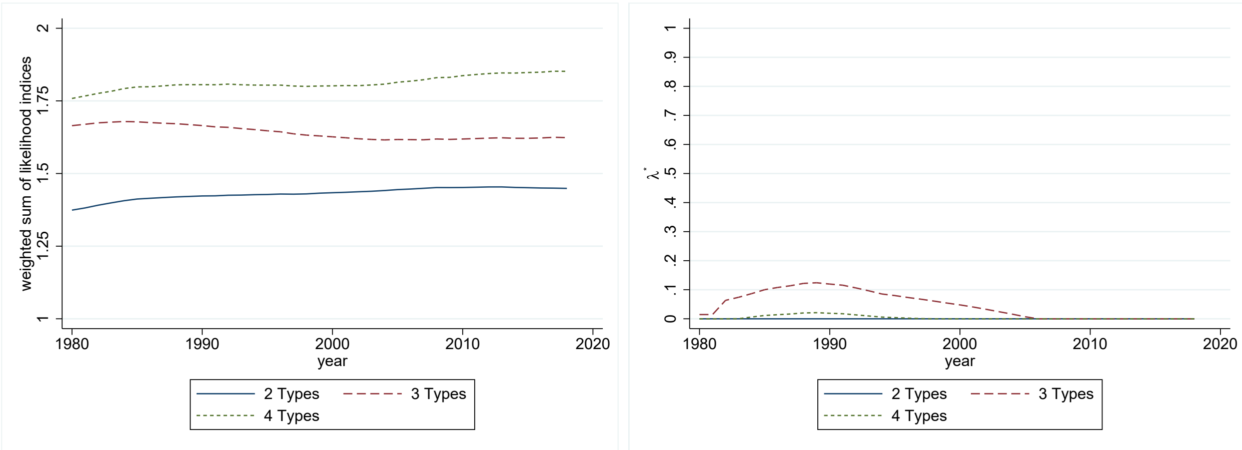
Here, the objective function is a convex combination of the two endpoints ξ_1 and ξ_2 defined in Equation (A.4), which are equivalent to γ_1 and γ_2 in the 2×2 case.

In addition to λ^* and $I_S^{\lambda^*}$ for $N = 2$ (H = Tertiary, L = Non-tertiary), we compute the optimal weights and the sorting measure for $N = 3$ (Tertiary, Secondary, Primary), and $N = 4$ (Master/PhD, Bachelor, Secondary, Primary). We show the development of I_S^{convex} for all cases in Figure A.2, Panel (a). The values for λ^* are shown in Panel (b).

Sorting is positive irrespective of the number of types because the indices in Figure A.2, Panel (a), are always greater than one. For the two-type case (blue solid line), we obtain the exact same trend as in Figure 2, Panel (a) with $\lambda = 0$ (green dash-dotted line) because $\lambda^* = 0$ in the two-type case (blue solid line in Figure A.2, Panel (b)). Interestingly, in the three-type case, we see a slightly decreasing extent of sorting (although almost flat) and some intermediate values for λ^* (red dashed lines in Figure A.2). Hence, the increasing extent of sorting is not driven

Figure A.2: Marriage Market Sorting and Optimal Weights with Multiple Types

(a) Sorting Trends with Multiple Types (b) Optimal λ with Multiple Types



Note: Panel (a) shows the development of the sorting measure $I_S^{\lambda^*}$ for $N = 2$, $N = 3$, and $N = 4$. Panel (b) shows the values of λ^* according to the derived decision rule (A.5). Section 2 explains how the sample and the education-based types are constructed.

by sorting trends within the lower levels of education, i.e., primary and secondary education. Instead, increasing sorting is driven by more sorting among highly educated individuals. This is evident from the increasing trend for the four-type case (green short-dashed line) in Figure A.2, Panel (a). In this case, λ^* is zero or close to zero throughout (see Figure A.2, Panel (b)), which is consistent with the more pronounced changes in the female type distribution.

One could speculate that the higher level of sorting with more types is due to a mechanical effect of adding more types. This is not the case. The overall level of sorting increases because a higher extent of sorting among high types is uncovered with more granular types. To understand why there is no mechanical effect, consider the following example. Using the notation from the generalized contingency table (Table A.1), in which the highest type has index 1, we compare the sorting measures for the $N = 2$ and $N = 3$ cases under $\lambda^* = 0$

$$I_S^{convex}(N = 2) = \frac{a_{11}}{m_{1M}} + \frac{a_{22}}{1 - m_{1M}}, \tag{A.6}$$

and

$$I_S^{convex}(N = 3) = \frac{a_{11}}{m_{1M}} + \frac{a_{22}}{m_{2M}} + \frac{a_{33}}{1 - m_{1M} - m_{2M}}. \tag{A.7}$$

Let the top category be divided into two separate categories when we go from $N = 2$ to $N = 3$. The terms related to the bottom category remain unchanged, so the second term in Equation (A.6) and the third term in Equation (A.7) can be ignored when comparing sorting levels. For $N = 2$, assume, as in Table A.2, that two-thirds of men are in the top category and that all top-category men are married to top-category women: Thus, ignoring the bottom

Table A.2: The 2×2 case - Example

Male\Female	1	2	Marginal
1	2/3	0	2/3

category, we obtain $I_S^{convex}(N=2) = \frac{a_{11}}{m_{1M}} = 1$. Now, what happens to I_S^{convex} when we divide the top category into two separate categories depends on the extent of sorting within the previous top category. Consider two cases. First, there could be no sorting within the previous top category, so the two-thirds of men would be uniformly distributed across matches in categories 1 and 2; see Panel (a) of Table A.3. However, there could also be perfect sorting

Table A.3: The 3×3 case - Example

(a) No sorting within

Male\Female	1	2	3	Marginal
1	1/6	1/6	0	1/3
2	1/6	1/6	0	1/3

(b) Perfect sorting within

Male\Female	1	2	3	Marginal
1	1/3	0	0	1/3
2	0	1/3	0	1/3

within the previous top category; see Panel (b) of Table A.3. In this case, the third of men in the new category 1 would be matched with category-1 women, and the third of men in the new category 2 would be matched with category-2 women.

If no sorting is revealed within the top category, the sorting measures $I_S^{convex}(N=2)$ and $I_S^{convex}(N=3)$ are identical: $I_S^{convex}(N=3) = \frac{a_{11}}{m_{1M}} + \frac{a_{22}}{m_{2M}} = \frac{1}{2} + \frac{1}{2} = 1 = I_S^{convex}(N=2)$. This proves that there is no mechanical effect of adding another type that increases the sorting measure. In contrast, if perfect sorting is revealed within the top category, $I_S^{convex}(N=3)$ is twice as high as $I_S^{convex}(N=2)$: $I_S^{convex}(N=3) = \frac{a_{11}}{m_{1M}} + \frac{a_{22}}{m_{2M}} = 1+1 = 2$. The examples considered here are two extreme cases, but they show that introducing another type can either leave the sorting measure unchanged or increase it, depending on the configuration of the contingency table and the sorting patterns within categories. In our setting, going from $N=2$ to $N=4$ indeed uncovers positive sorting within tertiary-education subgroups, i.e., among graduates with master's/PhD degrees. This explains the ranking of sorting measures in Figure A.2, Panel (a).

A.4 The Standardization Approach

An alternative approach to address changing type distributions is to standardize the contingency tables that are to be compared. Introduced by Mosteller (1968), this approach has been applied to study marriage market sorting by Greenwood et al. (2014). The idea is to rescale the base- (e.g., 1980) and end-year (e.g., 2018) contingency tables through a series of iterations known as the Sinkhorn and Knopp (1967) (SK) algorithm to obtain uniform marginal distributions. The standardization approach can be used to study the evolution of marriage market sorting with changing type distributions under the condition that the sorting measure itself is not affected by the standardization.

On the one hand, Tan et al. (2004) find that the log odds ratio (I_{odds}, Equation (5)) is invariant to the iterations of the SK algorithm. However, its applicability is limited by the restriction to two types. On the other hand, the weighted sum of likelihood ratios ($I_{\mathcal{S}}$, Equation (1)) can be computed for any number of types but is not invariant to the iterations of the SK algorithm. Consider the numerical example in Table A.4. Panel (a) shows an illustrative 3×3 contingency table. We use the SK algorithm (see table notes for further details) to rescale the Panel (a) contingency table to the Panel (b) standardized contingency table, which has uniform marginal distributions.

Table A.4: Standardization Example

(a) Example Contingency Table

Male\Female	1	2	3	Marginal
1	0.30	0.05	0.05	0.40
2	0.01	0.24	0.10	0.35
3	0.02	0.03	0.20	0.25
Marginal	0.33	0.32	0.35	1.00

(b) Standardized Contingency Table

Male\Female	1	2	3	Marginal
1	0.28	0.03	0.02	0.33
2	0.01	0.25	0.07	0.33
3	0.04	0.05	0.24	0.33
Marginal	0.33	0.33	0.33	1.00

Note: Panel (a) shows an illustrative example of a 3×3 contingency table. We apply seven rounds of the SK algorithm to Panel (a) to arrive at Panel (b) with uniform marginal distributions. Each round contains three steps. Step 1: multiply the sum of the rows by 3 (chosen to arrive at a uniform distribution with each type representing 1/3), and divide through the rows with these numbers. Step 2: for the updated contingency table, sum the columns, multiply by 3, and divide through the columns with these numbers. Step 3: evaluate the distance between the row marginals and the target marginals. Stop if the target is reached. Reiterate otherwise. Our target was 0.33.

Consider the $\begin{bmatrix} 1,1 & 1,2 \\ 2,1 & 2,2 \end{bmatrix}$ submatrices in Panels (a) and (b) of Table A.4. The log odds ratio has a constant value of 4.97 before and after the standardization. The weighted sum of likelihood ratios, which we compute according to Equation (A.8) for this example, changes from 2.23 (the sum of the diagonal is 0.74) in Panel (a) to 2.28 (the sum of the diagonal is 0.76) in Panel (b). The reason is that diagonal elements of the contingency table change, and the offsetting changes to the off-diagonal elements are not taken into account. Thus, the weighted sum of likelihood ratios is not invariant to the standardization procedure. When the task is to study the evolution of marriage market sorting with more than two types, which precludes the use of the log odds ratio, the weighted sum of likelihood ratios cannot be combined with standardization to compensate for changing type distributions. In that case, the optimal weighting strategy developed in this paper is a viable alternative because it enables us to analyze sorting trends with more than two types and changing marginal distributions.

A.5 Alternative Weights

In this appendix, we first demonstrate the equivalence of the weights used in Greenwood et al. (2016), Eika et al. (2019), and Almar et al. (2023). Second, we show how these alternative weights compare to those derived in this paper.

Greenwood et al. (2016) divide the sum of the diagonal elements (trace) of the matrix formed by the contingency table by the trace of the counterfactual matrix under random matching. They do not use an explicit weighting scheme. In our notation, their sorting measure is

$$I_S^{trace} = \frac{a+d}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})}. \tag{A.8}$$

We first show that I_S^{trace} is mathematically equivalent to the weighted sum of likelihood ratios used in Eika et al. (2019) and Almar et al. (2023). In our notation and for the 2×2 case, their weights are

$$w_H = \frac{m_{(H,M)}m_{(H,F)}}{m_{(H,M)}m_{(H,F)} + m_{(L,M)}m_{(L,F)}} \tag{A.9}$$

and

$$w_L = \frac{m_{(L,M)}m_{(L,F)}}{m_{(H,M)}m_{(H,F)} + m_{(L,M)}m_{(L,F)}} = \frac{(1-m_{(H,M)})(1-m_{(H,F)})}{m_{(H,M)}m_{(H,F)} + m_{(L,M)}m_{(L,F)}}. \tag{A.10}$$

We plug these weights into the definition of the weighted sum of likelihood ratios according to Equation (1). The products of the marginal distributions cancel out, and we are left with the sorting measure

$$\frac{a+d}{m_{(H,M)}m_{(H,F)} + m_{(L,M)}m_{(L,F)}} = \frac{a+d}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})} = I_S^{trace}, \tag{A.11}$$

which is exactly the Greenwood et al. (2016) measure. Although their weighting is not explicit, the random matching counterfactual in the denominator takes the marginal distributions and their changes over time into account.

Next, we rewrite this sorting measure as a weighted sum of likelihood ratios with weights of the same form as in Equation (2):

$$
\begin{aligned}
I_{\mathcal{S}}^{trace} &= \frac{a}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})} \times \frac{m_{(H,M)}m_{(H,F)}}{m_{(H,M)}m_{(H,F)}} \\
&+ \frac{d}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})} \times \frac{(1-m_{(H,M)})(1-m_{(H,F)})}{(1-m_{(H,M)})(1-m_{(H,F)})} \\
&= \frac{a}{m_{(H,M)}m_{(H,F)}} \times \frac{m_{(H,M)}m_{(H,F)}}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})} \\
&+ \frac{d}{(1-m_{(H,M)})(1-m_{(H,F)})} \times \frac{(1-m_{(H,M)})(1-m_{(H,F)})}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})}.
\end{aligned}
$$

This result can be generalized to the $n \times n$ case as described in Appendix A.3.

We now turn to comparing the weights implied by $I_{\mathcal{S}}^{trace}$ to those used in $I_{\mathcal{S}}^{convex}$. The implied weights of $I_{\mathcal{S}}^{trace}$ are not necessarily a convex combination of the male and female marginals. To see this, define λ^{trace} as the λ that equalizes $I_{\mathcal{S}}^{trace}$ and $I_{\mathcal{S}}^{convex}$:

$$
\lambda^{trace} = \left(\frac{m_{(H,M)}m_{(H,F)}}{m_{(H,M)}m_{(H,F)} + (1-m_{(H,M)})(1-m_{(H,F)})} - m_{(H,F)} \right) \frac{1}{m_{(H,M)} - m_{(H,F)}}. \tag{A.12}
$$

Evidently, λ^{trace} can lie outside the unit interval. In the data, $\lambda^{trace} < 0$ holds until 1987, which is the last year in which males had a higher share of high types than females ($m_{(H,M)} > m_{(H,F)}$). From 1988, $\lambda^{trace} > 1$ holds. For these values of λ, the effect of more sorted couples on the measure is not necessarily positive; see Equation (3).

In our data, conclusions based on $I_{\mathcal{S}}^{convex}$ with λ^* and $I_{\mathcal{S}}^{trace}$ turn out to be similar, i.e., sorting has increased, but this is coincidental and not guaranteed to hold in other settings. To scrutinize this finding, we compare the weights used in $I_{\mathcal{S}}^{\lambda^*}$ and $I_{\mathcal{S}}^{trace}$. It holds that $w_H^{\lambda^*} > w_H^{trace}$, which implies that $I_{\mathcal{S}}^{\lambda^*} > I_{\mathcal{S}}^{trace}$. The reason is that the likelihood ratio for (H,H) couples is larger than that for (L,L) couples, i.e., $\frac{a}{m_{(H,M)}m_{(H,F)}} > \frac{d}{(1-m_{(H,M)})(1-m_{(H,F)})}$. However, we see a stronger increase in $I_{\mathcal{S}}^{trace}$ than in $I_{\mathcal{S}}^{\lambda^*}$ because the increase in w_H is larger for $I_{\mathcal{S}}^{trace}$, i.e., $0 < \Delta w_H^{\lambda^*} < \Delta w_H^{trace}$. Thus, both $I_{\mathcal{S}}^{trace}$ and $I_{\mathcal{S}}^{\lambda^*}$ increase over time.