Firm Productivity, Wages, and Sorting*

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Abstract

We study the link between firm productivity and the wages that firms pay. Guided by a model of labor market sorting with large firms, we infer firm productivity by estimating firm-level production functions, recognizing that worker ability and firm productivity may interact at the match level. Using German data, we find that the most productive firms do not pay the highest wages. Worker transitions are directed toward higher wages, even when this implies moving to a less-productive firm. Productivity sorting, that is, the sorting of high-ability workers into high-productivity firms, is less pronounced than sorting into high-wage firms.

Keywords: Assortative Matching, Labor Market Sorting, Wage Inequality, Job Mobility, Unobserved Heterogeneity, Firm Productivity, Production Function Estimation

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1 Introduction

Using the seminal Abowd et al. (1999) (AKM) approach to decompose wages into worker- and firm-specific components, Card et al. (2013) (CHK) and Song et al. (2019) show that increases in wage inequality are to a large extent driven by increases in the sorting of high-wage workers into high-wage firms. Presumably, the wages that these firms pay are related to their productivity. However, the mapping from productivity to wages is far from clear. On the one hand, highly productive firms may share profits with their workers and pay high wages. On the other hand, wages could be lower due to labor market imperfections or compensating differentials. Less productive (perhaps younger) firms, in turn, may have to pay relatively high wages to retain workers or to expand their workforce if workers have valuable outside options.

The primary contribution of this paper is to shed light on the link between firm productivity and the wages that firms pay. To this end, we estimate firm-level production functions to infer unobserved firm productivity from the data. Importantly, we rely on the wage determination mechanism of a discrete-time model of dynamically optimizing firms, which features multiworker firms, decreasing returns, intrafirm bargaining, matching frictions, and worker–firm complementarities.[1] In the presence of complementarities, a given worker type may contribute substantially to output at some firm types but minimally at others. Our model exhibits a separability property—the wage equation is log-linear in the worker's type—that allows us to use the AKM model to estimate the worker- and firm-specific wage components. These estimates, in turn, allow us to calculate model-consistent labor inputs for production function estimation.

We proceed in three steps. First, we estimate an AKM model on a long panel of German matched employer–employee data. Second, we merge the estimated AKM wage components with detailed German establishment survey data.[2] This allows us to calculate model-consistent labor inputs, which cover all workers at the surveyed establishments, and to more precisely account for the heterogeneity within the workforce. Third, we estimate firm-level production functions and calculate productivity using the refined control-function approach proposed by Ackerberg et al. (2015) (ACF). These authors build on a discrete-time model of dynamically optimizing firms, an approach that has important similarities to our framework. The labor input has dynamic implications, and the estimation procedure allows for interfirm differences in labor adjustment costs and wage setting. One contribution of our paper is that we make transparent the assumptions under which the AKM and ACF approaches can be jointly used to study sorting.

[1]The theory of labor market sorting highlights complementarities between worker ability and firm productivity as an important determinant of wages and the source of positive sorting in the data (Becker, 1973; Shimer and Smith, 2000; Eeckhout and Kircher, 2011, 2018).

[2]We do not observe firms in the legal sense but rather observe establishments, i.e., individual production units. We use the terms firm and establishment interchangeably throughout the paper.

We study the sorting between worker ability (inferred from AKM worker effects) and firm productivity (inferred using ACF), the mapping from firm productivity to wages, and the implications for rising wage inequality. Our main finding is that the most productive firms do not pay the highest wages. Moreover, many low-productivity firms pay relatively high wages. *Productivity sorting*, that is, the sorting of high-ability workers into high-productivity firms, is less pronounced than their sorting into high-wage firms: the average rank correlation between wage-based worker types and productivity-based firm types is 0.07, while the correlation between AKM worker and firm fixed effects is 0.27 in our data.

In fact, the sorting of high-ability workers into high-productivity firms has been decreasing over time. We argue that this development can be explained by the tendency that workers earn lower wages in matches with the most productive firms than in matches with somewhat less productive firms, which has become more pronounced over time. By studying worker transitions, we confirm that workers move toward higher wages, even when this implies moving to a less-productive firm. Consistently, we find that the most productive firms are not the largest in terms of headcount, capital stock, or their total wage bill, despite very high labor productivity (value added per worker). Taken together, our findings suggest that increasing wage sorting could be accompanied by a decrease in the allocative efficiency of the labor market and lower aggregate output.

We rationalize our findings using a combination of theoretical and empirical arguments. Models with large firms, decreasing returns, intrafirm bargaining, and complementarities exhibit a weaker link between wages, firm size, and productivity than models that assume one-worker-one-firm matches. As shown by Stole and Zwiebel (1996), firms may strategically overemploy workers to reduce wages. Cahuc et al. (2008) explore complementarities among heterogeneous worker types and find diverse wage effects in this setting. We abstract from complementarities among workers—this allows us to nest the AKM wage equation—and focus on interactions between worker types and heterogeneous firm productivity at the match level. We find that wages increase in firm productivity at a decreasing rate. To explain decreasing wages at the top, we consider an additional channel: amenities. Positively valued nonwage characteristics of jobs have the potential to explain relatively low wages at the most productive firms. Indeed, we confirm empirically that the most productive firms offer the highest level of amenities.

Last, we show that distinguishing between productivity-based and wage-based firm types is instructive for understanding the sources of increasing wage inequality. A decomposition of wage variance into the variance within and between establishments reveals that the contribution of the between-firm component rose by almost 10% in Germany between 1998 and 2008. This is in line with the findings of Song et al. (2019) for the U.S. Quantitatively, the between-firm inequality is comparable in magnitude to the relatively stable within-firm component of the wage dispersion. However, this picture changes when we decompose the wage variance using the estimated firm productivity and worker ability types. We find that the share of the overall wage variance explained by the between-firm productivity type variance is low in levels and increases by only approximately 4% over time. Its overall contribution to inequality is dwarfed by the variance shares of the within-firm productivity type and between-worker ability type variances. We conclude that productivity sorting contributes less to rising wage inequality than wage sorting.

Contribution to the Literature

We show how the wage determination mechanism of a search-and-matching model with multiworker firms, decreasing returns, intrafirm wage bargaining, and worker–firm complementarities can be used to facilitate the estimation of unobserved firm productivity. We build on Cahuc et al. (2008), who embed the Stole and Zwiebel (1996) intrafirm bargaining framework into the canonical search-and-matching model. Our approach differs from existing structural work in the sorting literature that assumes one-worker-one-firm matches. Under this assumption, the focus lies on worker *quality*, while the *quantity* dimension (i.e., the number of workers) is not considered.[3]

Eeckhout and Kircher (2018) also relax the one-worker-one-firm assumption and study both the quantity and the quality dimensions of production. The firm must decide which worker type(s) to hire and, additionally, how many workers of each type. The main finding is that firms optimally hire multiple workers of exactly one type. This result holds for both frictionless matching and competitive search. In this paper, we are interested in the empirically relevant case of firms that are simultaneously matched with multiple worker types. Thus, we focus on a model with random search, nondegenerate matching sets, and a production structure that is geared toward our empirical approach, whereas Eeckhout and Kircher (2018) study more general production functions.

Our findings contribute to the empirical literature on wage inequality in a number of ways. The aforementioned studies by CHK for Germany and Song et al. (2019) for the U.S. follow the AKM approach. They decompose the wage dispersion into the contributions of unobserved worker ability, firm wage premia, and wage sorting in the labor market. Wage sorting measures the extent to which workers who receive high wages are matched with firms that pay high wages. We show that how one measures firm heterogeneity, that is, by the wages that firms pay or by firm productivity, affects this kind of decomposition. In our data, firms with the highest estimated productivity do not pay the highest wages. For this reason, sorting into high-productivity firms is quantitatively less important for rising inequality than wage sorting.

The main difference between AKM-inspired analyses of labor market sorting and structural analyses is typically the implied (non)monotonicity of the wage equation. Due

[3] Examples are Shimer and Smith (2000), Atakan (2006), Lise et al. (2016), and Lise and Robin (2017). A notable exception is Bagger and Lentz (2019). Here, multiworker firms are present, but production is linear and firm size is limited by search frictions, not decreasing returns.

to complementarities, wages are not a monotonic function of firm type in many theoretical models (e.g., Gautier and Teulings, 2006; Eeckhout and Kircher, 2011; Lise et al., 2016; Hagedorn et al., 2017; Lopes de Melo, 2018). However, the log-linear AKM wage equation implies that wages are strictly increasing in the firm fixed effect for a given worker. Abowd et al. (2018) show that it is possible to find common ground. These authors formally relate the equilibrium quantities derived from the directed-search sorting model of Shimer (2005) to the worker and firm heterogeneity components that can be identified by using the AKM approach. We follow this example and discuss how the wage equation implied by our model maps onto the AKM equation. Moreover, we contribute to the discussion about wage (non)monotonicity by studying how wages evolve across firm productivity types. We find quantitatively important deviations from monotonicity in high- and low-productivity firms. Across firms in the middle of the productivity distribution, however, wages evolve largely monotonically.

Only a small number of related papers in the empirical literature on wage dispersion use non-wage-based measures of firm heterogeneity. Bagger et al. (2014a) estimate firm-level production functions with heterogeneous labor inputs to study wage dispersion using Danish data. Bartolucci et al. (2018) argue that expected firm payoffs provide a summary statistic for firm heterogeneity. They use balance sheet data for a set of Italian firms and rank them by observed profit. Taber and Vejlin (2020) and Bagger and Lentz (2019) rank firms by the share of workers they poach from other firms. Sorkin (2018) applies Google's page ranking algorithm to worker flows. Haltiwanger et al. (2018) and Bertheau et al. (2020) study the cyclical properties of worker flows by using gross output and value added per worker, respectively, as firm quality measures.

Another related body of literature focuses on rent sharing and imperfect competition in the labor market. In line with a different modeling tradition, search frictions and wage bargaining are not explicitly considered. Card et al. (2018) survey this literature and link it to the AKM-inspired literature on wage dispersion. Lamadon et al. (2019) estimate a model of rent sharing, compensating differentials, and monopsony power with U.S. data. Consistent with our findings, these authors find that high-productivity firms pay relatively low wages. They also highlight the importance of amenities for firms' ability to mark down wages and the role of production complementarities in worker sorting. Given the differences in the underlying models and estimation strategies, we view our contribution as complementary to that of Lamadon et al. (2019).

2 Model

This section develops a model of multiworker firms with decreasing returns, intrafirm wage bargaining, worker–firm complementarities, and matching frictions. Here, we discuss the main assumptions and results. Specifically, we show how the model's wage equation maps onto the log-linear AKM wage equation. This step is key for constructing model-consistent firm-level labor inputs using estimated AKM wage components. Moreover, we highlight the assumptions under which our theory is compatible with the ACF approach to production function estimation. All details are relegated to Appendix A.

Consider an economy in which atomistic firms produce a numeraire good using multiple heterogeneous labor inputs. Worker heterogeneity is summarized by $n > 1$ ability types indexed by x. Worker types are time-invariant. Firm productivity, denoted Ω, may change due to idiosyncratic firm-level shocks that evolve according to a stochastic process characterized by the conditional CDF $G(\Omega'|\Omega)$. Workers and firms meet randomly. Conditional on meeting, a match is not guaranteed because the match surplus may be too low. Appendix A.1 provides the details of the assumed matching mechanism.

To set up the firm's problem, we build on Cahuc et al. (2008), who generalize the canonical search-and-matching model to allow for multiworker firms with heterogeneous labor inputs and decreasing returns, strategic interactions, and intrafirm bargaining in the spirit of Stole and Zwiebel (1996). We add heterogeneous firm productivity and a production structure that is consistent with positive worker-firm sorting. To facilitate a simple and transparent link to the empirical models we use (AKM and ACF), we abstract from complementarities between worker types within the same firm.

Furthermore, we make six simplifying assumptions. First, workers and firms are risk-neutral. Second, both worker and firm heterogeneity are one-dimensional.[4] Third, worker ability and firm productivity are known to all market participants and are cardinally measurable. Fourth, infinitely lived workers supply one infinitesimally small unit of labor (no extensive/intensive margin choice) so that labor input is a continuous variable.[5] Fifth, we abstract from capital inputs when discussing the model.[6] Sixth, we present the model in discrete time because the ACF approach to production function estimation also relies on a discrete-time model of dynamically optimizing firms. Time indices are omitted.

A general concave, firm-level production function is

$$Y = F(L, \Omega), \tag{1}$$

where Y is value added and $L = \sum_x xL_x$ is a scalar composite labor input in units of worker ability. It combines all heterogeneous labor inputs L_x, the number of type-x workers employed by the firm. Ω is the current productivity (or TFP) of the firm. Our focus lies on worker–firm sorting, so we assume that output is (log-)supermodular at the match level: the complementarity between firm productivity Ω and worker ability determines

[4]For recent explorations of sorting with multidimensional characteristics, see Lindenlaub (2017), Lindenlaub and Postel-Vinay (2020), and Lise and Postel-Vinay (2020).

[5]We take part-time labor into account when we estimate the production functions in Section 4.2.

[6]Cahuc et al. (2008) explicitly consider a predetermined capital stock, so our model could be extended in this direction. ACF assume a similar capital accumulation mechanism; see Section 4.2.

the contribution of every single match to firm-level output. The marginal product of an additional unit of worker ability is firm-specific. We interpret firm productivity as a "nonrival" resource: we do not consider the span-of-control problem in the optimal allocation of firm resources to heterogeneous workers (Eeckhout and Kircher, 2018).

A simple production structure in line with our assumptions is

$$F(L,\Omega)=\left(\sum_{x=1}^{n}(x\times\Omega)L_x\right)^{\beta_l}, \tag{2}$$

where $0<\beta_l<1$ is the output elasticity of the composite labor input. This production function is (weakly) log-supermodular at the match level, in line with the sufficient conditions for positive assortative matching (PAM) derived by Shimer and Smith (2000).[7] Under our assumptions, worker ability units are perfect substitutes at the firm level. That is, output depends on Ω and the number of efficiency units of labor employed. The marginal product of an additional unit of type-x labor is

$$F_x(L,\Omega)=\frac{\partial F(L,\Omega)}{\partial L_x}=x\beta_l\Omega^{\beta_l}L^{\beta_l-1}. \tag{3}$$

Note that worker ability x scales the marginal product at an (L,Ω) firm. We make use of this property below. Furthermore, the marginal product is increasing in firm productivity and the output elasticity of labor but decreasing in the total composite labor input L due to decreasing returns.

As in Cahuc et al. (2008) and ACF, employment is a state variable. The firm's problem is to optimally choose how many vacancies to post given its expected profits from hiring. We assume that vacancies cannot be targeted to specific worker types and are subject to a flow cost c, which could depend on the firm type. In Appendix A.2, we solve the firm's problem and derive the relevant optimality conditions in the steady state.

The bargained wage satisfies the standard Nash sharing rule. The firm's surplus consists of its marginal profit from hiring an additional worker of type x (equation A.11). Its threat point is to renegotiate wages with all other employees (Stole and Zwiebel, 1996). The worker's surplus is the difference between the option values of employment and unemployment (equations A.14 and A.15, respectively). Wage (re)negotiations happen instantaneously so that firm-level employment remains fixed. In Appendix A.3, we show that the outcome of intrafirm wage bargaining in the model is described by the following

[7] Shimer and Smith (2000) establish the existence of an equilibrium in this environment. PAM arises with a log-supermodular match-level production function, while log-submodularity leads to negative assortative matching (NAM). In this empirical paper, we do not attempt to generalize the Eeckhout and Kircher (2018) large-firm sorting conditions for random search models.

differential equation:

$$w(x, L, \Omega) = \alpha \left(F_x(L, \Omega) - L\frac{\partial w(x, L, \Omega)}{\partial L} \right) + (1 - \alpha)(1 - \beta)U(x). \tag{4}$$

This is a discrete-time version of the wage bargaining outcome derived by Cahuc et al. (2008) for the "single labor case". We can utilize their result due to our assumption of perfectly substitutable ability units. α is the workers' bargaining power parameter, and β is the common discount factor. The first term in parentheses shows that the wage of a type-x worker at an (L, Ω) firm is a function of the worker–firm-specific marginal product, $F_x(L, \Omega)$. The second term captures the inframarginal effect that hiring the marginal worker has on all other workers' wages. It mirrors the finding that firms can reduce incumbent workers' wages by increasing employment in the presence of decreasing returns.[8] Absent complementarities between worker types, the inframarginal adjustment reflects decreasing returns only and is unambiguously negative. Finally, the third term in equation (4) captures the worker-specific outside option, which we will return to below. The solution[9] to the differential equation (4) is

$$w(x, L, \Omega) = (1 - \alpha)(1 - \beta)U(x) + \int_0^1 z^{\frac{1-\alpha}{\alpha}} F_x(Lz, \Omega)\mathrm{d}z, \tag{5}$$

indicating that the wage depends on the worker's outside option and an integral expression that combines the worker's marginal product and the inframarginal effect. The latter is weighted by the worker's bargaining power and decreases in the distance to the margin.

2.1 From Theory to Estimation

The model's wage equation makes clear that the wages that firms pay are not directly informative about firm productivity. Equation (5) is nonlinear in Ω, and the effect of productivity on wages is intertwined with firm size and worker bargaining power.

To empirically recover Ω, we propose a two-step approach that combines techniques from the empirical wage dispersion and industrial organization (IO) literatures. First, we estimate the log-linear AKM model with German matched employer–employee data; see Section 3.3. The estimation yields, for all individual workers and firms in the largest connected set, worker fixed effects, firm fixed effects, and a residual. There is a direct correspondence between the AKM wage components and the wage equation in our model.

Consider the worker fixed effects. Both the outside option and the integral expression in (5) are functions of worker ability. Their effects on wages are absorbed by the AKM

[8] Cahuc et al. (2008) allow for unrestricted substitutability/complementarity patterns between worker types. In this case, the effects on coworker wages can be either positive or negative. Firms may strategically over/underemploy specific worker types depending on their contribution to the total wage bill. Smith (1999) studies the efficiency of job creation in such a setting.

[9] We follow Stole and Zwiebel (1996) and Cahuc et al. (2008). Details are relegated to Appendix A.3.

worker fixed effect when both wage components are linear in worker ability. Given our production structure, the integral expression is indeed linear in worker ability x because worker ability merely scales the marginal product of labor. Plugging in (3) yields

$$w(x, L, \Omega) = (1-\alpha)(1-\beta)U(x) + x\int_0^1 z^{\frac{1-\alpha}{\alpha}}\beta_l \frac{F(Lz,\Omega)}{Lz}\mathrm{d}z, \tag{6}$$

where x can be written in front of the integral sign. We show in Appendix A.5 that the worker's outside option in the first term is also linear in x under two additional assumptions. First, the worker's flow value of unemployment, denoted $b(x)$ in the model, must be proportional to x. This is a standard assumption. Second, matching sets must cover the whole type space; that is, there are no unacceptable combinations of worker and firm types. An implication of this assumption combined with our matching technology is that all worker types are unemployed for the same (expected) duration. In Section 4.1, we verify that this assumption holds in our data (conditional on observables). Another implication is that there is no endogenous sorting in the model, despite the complementarity in the production function.

Next, we consider the integral term. How this object is captured by the AKM regression depends on the underlying data-generating process (DGP). For the sake of argument, suppose that our wage equation describes reality without error. The economy is in a steady state, and firm productivity does not change over time. In this case, (a log-affine transformation of) the firm-specific integral term, representing the effects of firm productivity and decreasing returns on the wage, is absorbed by the time-invariant firm fixed effect, and the residuals are zero. With a more complex DGP, the value of the integral term changes over time. In the presence of productivity shocks, there are two effects on wages: a direct effect through the marginal product of labor and an indirect effect that captures the changing employment effect on wages in the presence of decreasing returns. We formally discuss how wages relate to productivity in Appendix A.4. The AKM firm fixed effect captures a time average of these wage effects. Thus, the more persistent the productivity process is, the more informative is the time-invariant firm effect. The firm fixed effect also absorbs firm-specific and time-invariant pay components that are not part of our model, e.g., wage effects of amenities. Additional complications arise in real data, for example, due to measurement error in short panels. Finally, the AKM residual absorbs fluctuations around the time average of the effect of productivity on wages.

To estimate Ω in a model-consistent way, we complement rich survey data on input and output for a representative sample of firms with the estimated AKM wage components of their full workforce and predict their firm-level labor input (wage bill). According to our model and its separability property, the worker fixed effect captures the effect of worker ability on output. This is our primary object of interest. We also include the effects of observable worker characteristics (not included in our model). Moreover, we vary the way in which we include the firm fixed effects, the AKM residuals, or both in the labor input. This allows us to investigate the extent to which the firm-specific and time-(in)variant wage components matter for the inferred productivity measure.

Finally, note that the illustrative model we use to derive equation (6) assumes that only unemployed workers search for jobs. With on-the-job search (OJS), the worker's outside option may be to renegotiate with his or her current employer. Under certain DGPs, this implies that worker type can be identified using wage observations only for hires out of unemployment. The reason is that wages observed after job-to-job (J2J) transitions can be a function of the match surplus at the previous employer and, therefore, might contain a history component.

The key feature that allows us to separate worker type from the integral expression in equation (6) is the linearity of the marginal product of labor in worker type x. This is a direct consequence of our assumed production structure. This "trick" to isolating the worker type and obtaining a log-linear wage equation is well known in the OJS literature. For instance, Postel-Vinay and Robin (2002), Cahuc et al. (2006), and Bagger et al. (2014b) all rely on functional form assumptions to separate worker ability from labor market history.[10] This conceptual relatedness leads us to suspect that a model with large firms, intrafirm bargaining, and the sequential auction OJS framework will, for specific production functions, retain a wage equation that is log-linear in the worker type. In this case, the aforementioned concern regarding wage histories would not apply. However, we leave the establishment of the details of such a model to future research.

A number of recent empirical studies suggest that wage (re)negotiation with current employment as the outside option is of limited importance. Evidence based on the German Job Vacancy Survey shows that most wages are set by wage posting in the German labor market, even for workers who move between firms (Brenzel et al., 2014).[11] DiAddario et al. (2021) show that an augmented AKM model with both origin- and destination-firm effects resembles the wage equation of sequential auction models (Postel-Vinay and Robin, 2002; Bagger et al., 2014b). Using Italian data, these authors find that origin-firm effects contribute little to overall wage variation. They also report a small covariance between the worker fixed effect and the origin-firm effect, which supports the separability of worker ability and labor market history. Last, Caldwell and Harmon (2019) use Danish register data to identify workers' outside options using coworker networks. They find that workers rarely renegotiate their wages in response to changing outside options.

[10]See, for example, the discussion of this model property in Bagger et al. (2014b), p. 1561.

[11]A potential explanation is the prevalence of collective bargaining at the sector or the firm level.

3 Data

Our analysis combines two data sets provided by the Institute for Employment Research (Institut für Arbeitsmarkt- und Berufsforschung, IAB) of the German Federal Employment Agency (Bundesagentur für Arbeit, BA). The first is the "IAB Employee History File" (BeH), which comprises the universe of employment spells recorded by the German social security system. The second data set is the "IAB Establishment Panel" (EP), which is a representative establishment-level survey that can be linked to the BeH.[12]

In this section, we describe the two data sets, explain how we prepare and combine them, and discuss the wage regressions we use to decompose observed wages. These wage components allow us to conduct the model-inspired adjustment of labor inputs at the establishment level, which we then use to estimate productivity. We relegate additional details on sample selection and the imputation procedures to Appendix B.

3.1 Data Sources and Preparation

The BeH employment spell data contain information on worker gender, age, and education[13], as well as start and end dates for the spells, total earnings, and occupation/industry codes. The data cover the vast majority of the German workforce. Only civil servants and self-employed workers who do not pay social security contributions are excluded. Each worker and each establishment have a unique identification number, which allows us to follow workers over time and across establishments.

Regarding sample selection, we largely follow CHK and Lochner et al. (2020a). We start with the universe of employment spells observed between 1998 and 2008. There is no exact information on hours worked in the BeH, so we restrict our sample to full-time employees (males and females) aged 20 to 60. This age restriction circumvents interference from apprenticeship training and early retirement. We define each worker's main job in a given year as his or her job with the highest total wage (including bonus payments).

The EP is a comprehensive annual survey of establishments, that is, single production units such as factories or branches.[14] It provides us with the necessary data to estimate production functions at the establishment level: we observe revenue, intermediate goods purchases (reported as revenue shares), value added (calculated as revenue minus intermediate goods purchases), and net investments in four different categories of capital goods (buildings, production machinery, IT, and transport equipment). We restrict our analysis to EP establishments with nonmissing data on revenue and intermediate goods

[12] The EP is a random sample of all establishments in Germany, stratified according to size, industry, and federal state. See Kölling (2000) and Fischer et al. (2009) for a detailed description of the EP data.

[13] We use four education categories: 1 = "no degree", 2 = "vocational training", 3 = "some college degree", 4 = "university degree".

[14] Of the surveyed establishments, 80% report that they are single-establishment firms.

purchases.[15] To ensure that our results are not driven by outliers, we drop establishments below the 1st and above the 99th percentile of the revenue distribution.

We supplement the EP data with covariates from the "Establishment History Panel" (BHP).[16] These include average wages, headcounts, shares of full-time/part-time workers, and worker shares by skill (education) group. Moreover, the BHP provides administrative information on firm age and a consistent industry classification.

3.2 Imputations

We observe nominal gross daily wages, which we deflate using the consumer price index from the German national accounts. One limitation of the wage data is that earnings are tracked only up to a threshold, the social security contribution assessment ceiling ("Beitragsbemessungsgrenze"). We follow Dustmann et al. (2009) and impute the upper tail of the wage distribution by running a series of tobit regressions, allowing for maximum heterogeneity by fitting the model separately by gender, tenure, education level, and five-year age group (see Appendix B.1). We impute missing and inconsistent education data in the BeH using the methodology proposed in Fitzenberger et al. (2006); see Appendix B.2.

The EP data contain information on net investments. To estimate the level of the capital stock, we use a perpetual inventory method following Müller (2008). This method approximates the establishment-level capital stock by combining net investments with the average economic lives (depreciation rates, available from the national accounts) of the different types of capital goods we observe.

3.3 Wage Regressions

We estimate an AKM-type wage regression on the largest connected set in the BeH data for our period of analysis, 1998–2008. We include both men and women. The connected set contains more than 233 million person-years, corresponding to 35 million individual workers at 3.3 million establishments; 55% of observations are movers between establishments. We use the CHK specification; that is, we estimate a log-linear wage equation for worker i who works at firm $j(i,t)$ in year t:

$$w_{it} = \alpha_i + \psi_{j(i,t)} + x'_{it}\beta + \varepsilon_{it}, \tag{7}$$

where w_{it} are log real daily wages, α_i is the worker fixed effect, $\psi_{j(i,t)}$ is the establishment fixed effect, and x'_{it} contains time-varying controls: an unrestricted set of year dummies and quadratic and cubic terms for age fully interacted with educational attainment.

[15] The main reason for missing information is that some firms choose not to report revenue as their measure of output. This applies mainly to financial institutions and public sector firms.

[16] The BHP covers all establishments with at least one employee subject to social security withholding on a reference date (June 30th). See Spengler (2008) for a detailed description of the BHP data.

ε_{it} is the residual. The regression model is identified for workers who move between establishments.[17] The adjusted R^2 of the regression is 0.92, which is broadly in line with CHK. The correlation between the estimated worker and firm fixed effects, often interpreted as a measure of wage sorting in the labor market, is 0.27 in our time interval. This is slightly higher than what CHK report, likely due to the longer time period and broader sample we consider.[18] Estimated worker and firm fixed effects as well as their covariance could be biased due to limited mobility in the connected set (an incidental parameter problem).[19] We test the parametric correction suggested by Andrews et al. (2008) for two subperiods in our data and find that the limited mobility bias is small.[20]

Table 1 shows the variance decompositions for multiple groups of workers based on the estimated AKM wage components. Column (a) includes all person-years, (b) all women, (c) all men, and (d) all men in West Germany (the CHK sample). We replicate the well-known finding that the majority of the wage variance is explained by unobserved worker heterogeneity: the worker fixed effect explains almost half of the observed variation in wages, slightly more for women and slightly less for men. The second-most important source of variation is the firm fixed effects. They explain roughly one-quarter of the wage variance across the four groups. The third largest determinant is the covariance between worker and firm effects, which explains between 12 and 19% of the wage variance.[21] At only 2%, the share of the wage variance explained by the time-varying observable characteristics is almost negligible. The same is true for the covariances of the observable characteristics with the worker and firm effects. Note that time-invariant covariates such as education are absorbed by the worker effect. The residual explains wage variance shares between 7 and 9% across the four BeH samples.

We use the estimated AKM wage effects for men, women, and both movers and stayers from the full BeH sample (column (a) in Table 1) to construct the labor inputs for the production function estimation at the firm level. Based on the discussion in Section 2.1, we construct wage bill measures that include (exponentiated) worker fixed effects, the

[17] Following CHK, we impute worker fixed effects and residuals for "stayers" (workers who do not change their employer over the entire sample period).

[18] CHK report correlations for shorter time intervals: 0.17 (1996–2002) and 0.25 (2002–2009). Moreover, CHK include only men in the former West Germany, whereas we include both men and women in reunited Germany from 1998–2008.

[19] As shown by Andrews et al. (2008, 2012) and recently revisited by Borovičková and Shimer (2020) and Kline et al. (2020), estimated variances of worker and firm fixed effects are biased upwards, and their covariance is biased downwards.

[20] The fact that we estimate the AKM model on the universe of German register data over a ten-year time window and include both men and women mitigates concerns of severe limited mobility bias due to the large number of observed transitions. Lochner et al. (2020b) and Lachowska et al. (2020) confirm that this bias is limited in data sets with a large number of transitions. The Andrews et al. (2008) correction implies that in our setting, the variance in worker effects decreases by 5%/4% and the variance in firm effects decreases by 4%/3% in the 1998–2002/2003–2008 subperiods, respectively. The covariance between the worker and firm effects increases by 7%/5%.

[21] Interestingly, women are less positively sorted in terms of wages than men. This is in line with what Card et al. (2016) and Bruns (2019) find using Portuguese and German data, respectively.

Table 1: Wage Variance Decompositions

	(a) Regression (7) BeH, full		(b) Regression (7) BeH, women		(c) Regression (7) BeH, men		(d) Regression (7) BeH, men, West	
$\text{Var}(w_{it})$	0.276	(100%)	0.277	(100%)	0.245	(100%)	0.226	(100%)
$\text{Var}(\hat{\alpha}_i)$	0.126	(46%)	0.138	(50%)	0.105	(43%)	0.106	(47%)
$\text{Var}(\hat{\psi}_{j(i,t)})$	0.068	(25%)	0.076	(27%)	0.061	(25%)	0.049	(22%)
$\text{Var}(x'_{it}\hat{\beta})$	0.005	(2%)	0.006	(2%)	0.005	(2%)	0.005	(2%)
$2 \times \text{Cov}(\hat{\alpha}_i, \hat{\psi}_{(i,t)})$	0.049	(18%)	0.032	(12%)	0.047	(19%)	0.037	(16%)
$2 \times \text{Cov}(\hat{\alpha}_i, x'_{it}\hat{\beta})$	0.004	(0%)	0.001	(0%)	0.005	(2%)	0.006	(3%)
$2 \times \text{Cov}(\hat{\psi}_{j(i,t)}, x'_{it}\hat{\beta})$	0.003	(0%)	0.001	(0%)	0.004	(2%)	0.004	(2%)
$\text{Var}(\hat{\varepsilon}_{it})$	0.021	(8%)	0.025	(9%)	0.018	(7%)	0.019	(8%)
Sample mean wage	4.450		4.261		4.553		4.621	
R^2	0.92		0.93		0.91		0.92	
# Observations	233,117,492		82,267,794		150,849,698		123,087,610	

Notes: Variance decompositions for log real daily wages according to regression model (7) for various BeH samples. Mean wages, variances, and covariances are rounded to three decimal places. Source: BeH.

effects of observable characteristics, and, in some specifications, firm fixed effects and residuals to distinguish the different channels through which wages and firm productivity are related according to our theory.

3.4 Final Samples

We use two final samples in the subsequent analysis. We refer to the first sample as *All Matches*. It includes matches that started both before and after 1998. Here, we do not condition on the origin of the match; that is, we do not distinguish between J2J flows and matches formed out of nonemployment (OON). There are 4,695,108 employment spells for 1,344,382 workers employed at 10,004 EP establishments.

We refer to the second sample as *New Matches*. This sample includes matches formed from 1999 onward, and we distinguish between J2J flows and matches formed OON. Thus, we need one initial observation (1998) to determine employment status before the new match. There are 1,656,280 employment spells for 633,831 workers at 9,659 establishments. A total of 64% (1,055,151) of new employment spells are J2J moves from one employer to another, while 36% (601,129) are spells formed OON. Our definition of nonemployment includes marginal employment, unemployment (benefit receipt), and inactivity.[22] Thus, OON spells also include young workers who enter the labor market.

[22] We do not observe UI benefits in the BeH. Thus, unemployment and inactivity are indistinguishable.

In some parts of our empirical analysis, we study changes over time. To this end, we split our sample period (1998–2008) into two subperiods, 1998–2002 and 2003-2008.

4 Ranking Workers and Firms

4.1 The Worker Rankings

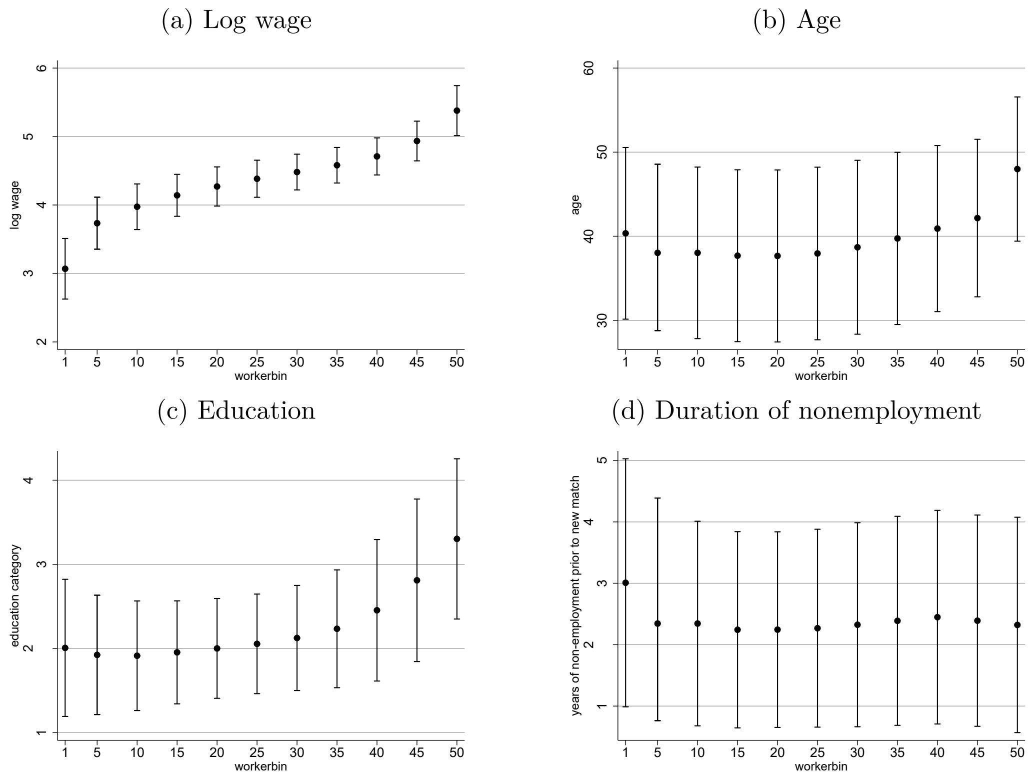
According to our model, the estimated AKM worker fixed effects fully capture the effect of unobserved worker ability on wages. This result can serve as a "microfoundation" for ranking workers based on these wage effects. Using this ability measure, we rank individual workers and create 50 ability bins of equal size.[23] Let $\bar{x}(i)$ denote the ability bin to which worker i belongs. In what follows, individual workers in the same bin are considered to have the same ability type.

To show how the bins summarize worker heterogeneity, we decompose the respective variances of observed wages, age, and education into the shares explained within and between the bins. Little within-bin variance implies homogeneity among the workers along the relevant dimension. The rankings are based on the worker fixed effects, which explain the majority of the wage variation in the data. Thus, the between-bin share of the wage variance is large (70%). For age and education, the picture is quite different. We find that 96% of the age variation and 74% of the education variation are within the bins. Because we control for time-varying age and education effects in the AKM regression, this finding is a direct reflection of the low correlation between the estimated worker effects and the time-varying observables (recall Table 1).

Figure 1 illustrates how log wages, age, and education vary across worker bins. Panel (a) shows that mean wages increase monotonically across worker bins, as expected. In contrast, Panel (b) shows that the mean age across worker bins is relatively flat. Only the highest bin has a somewhat higher mean age. However, due to the large standard deviation, this difference is not significant. Panel (c) shows that the mean education increases above bin 35 but is flat below it. This suggests that highly ranked workers are more likely to have tertiary degrees, while lower ranked workers tend to have vocational training only. Again, however, the differences are hardly significant. Note that the dispersion of education increases among the highest-ranked workers. It is more common to observe high-rank workers with little education than low-rank workers with tertiary degrees. Finally, Panel (d) shows the duration of nonemployment across worker types. The mean observed nonemployment duration is virtually flat. Recall that we control for the effect of observable characteristics in our AKM estimation. The finding that nonemployment durations do not change with worker type supports the assumption that matching sets cover the whole type space, which we made in Section 2.

[23] There are 702,540 individual workers in each of the 50 bins.

Figure 1: Log Wage, Age, Education, and Nonemployment Duration across Worker Bins



Notes: The figures plot the means ± one standard deviation for log wages, age, education, and nonemployment duration for every fifth worker bin. The ages of the individual workers in our sample range from 20 to 60. There were 4 education categories: 1 = "no degree", 2 = "vocational training", 3 = "some college degree", and 4 = "university degree". Data source: BeH.

4.2 The Firm Rankings

We rank firms based on their unobserved productivity, which we infer by estimating production functions at the establishment level. We know from the empirical IO literature that this approach is susceptible to two challenges. The first is "transmission bias", an endogeneity problem (Marschak and Andrews, 1944). Input choices, e.g., the demand for labor or intermediate inputs, are likely correlated with firm productivity.[24] To address this challenge and estimate an unbiased measure of firm productivity, we rely on the ACF version of the control function approach.[25] The key assumption is that intermediate input demand (the control function) is a strictly increasing function of (scalar and unobserved) firm productivity, which may change over time due to shocks. This assumption implies that one can invert the control function and effectively control for unobserved firm productivity by substituting it out of the production function to be estimated. Importantly, the ACF model allows the firm's labor choice to have dynamic implications. This suits our model environment well, as we discuss below.

[24]This is because inputs are demanded optimally by the firm's management based on current firm productivity, which is unobserved by the econometrician.

[25]This approach was originally developed by Olley and Pakes (1996) and refined by Levinsohn and Petrin (2003) and Wooldridge (2009).

The second challenge is that in the presence of heterogeneous worker ability, the quality of the labor inputs varies across firms (Griliches, 1957). Physical units, e.g., headcounts or hours worked, do not reflect differences in worker ability and imply measurement error. This complicates the separate identification of firm productivity and the output elasticity of labor. The empirical IO literature addresses this challenge by controlling for labor quality differences when estimating the production function (e.g., Fox and Smeets, 2011; Irarrazabal et al., 2013). We go one step further and construct model-based labor input measures that isolate the effect of worker ability on output from other wage components, as discussed in Section 2.1.

Production Function Estimation

We start from the production structure assumed in Section 2, equation 2. We rewrite the equation in terms of the composite labor input in ability units L_{jt} and add capital inputs K_{jt} as well as indices for individual firms j and time t:

$$Y_{jt} = \left(\Omega_{jt} L_{jt}\right)^{\beta_l} \left(\Omega_{jt} K_{jt}\right)^{\beta_k}. \tag{8}$$

β_l and β_k are the output elasticities of labor and capital, respectively. Both inputs are scaled by the firm's current productivity Ω_{jt}. Note that Y_{jt} is value added (revenue minus expenditures on intermediate goods). Intermediate inputs are therefore not part of production function (8). Without assuming constant returns to scale, by the homogeneity of the general Cobb-Douglas function (8), we obtain

$$Y_{jt} = \Omega_{jt}^{\beta_l+\beta_k} L_{jt}^{\beta_l} K_{jt}^{\beta_k}. \tag{9}$$

The sum of the output elasticities in the exponent on Ω_{jt} is irrelevant for the purpose of ranking the firms. Thus, we define $\omega_{jt} = (\beta_l+\beta_k) \ln \Omega_{jt}$ when taking logs. We estimate the following value-added production function, where lowercase letters indicate logarithms:

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + z'_{jt}\gamma + \epsilon_{jt}. \tag{10}$$

We add a constant β_0; the residual ϵ_{jt}, which absorbs transitory shocks; and a vector of additional controls z'_{jt}, including indicators for West German establishments, four firm age categories, the share of part-time workers, and employee representation in management. We also include time and sector fixed effects (32 categories) to account for cyclical fluctuations and differences in demand structures across sectors.

We describe the technical details of the ACF estimation procedure in Appendix C. Here, we discuss the key features that make this approach compatible with the model in Section 2. ACF build on a discrete-time model of dynamically optimizing firms. As in our model, labor input has dynamic implications and therefore enters the firm's dynamic problem as a state variable (cf. equation A.1). We abstract from capital inputs in Section 2, while ACF assume that the capital stock is predetermined by the investment $i_{j,t-1}$ made in the previous period:

$$k_{jt} = \kappa(k_{j,t-1}, i_{j,t-1}). \tag{11}$$

Our model builds on Cahuc et al. (2008), who explicitly consider a capital accumulation process similar to that described by equation (11). Thus, our model and the ACF model are also compatible in the capital dimension.

In the ACF model, the demand for labor and intermediate goods may change in response to contemporaneous firm productivity shocks.[26] The conditional expectation of firm productivity is modeled using a first-order Markov process:

$$\omega_{jt} = E(\omega_{jt}|\omega_{j,t-1}) + \xi_{jt} = \rho\omega_{j,t-1} + \xi_{jt}, \tag{12}$$

where ξ_{jt} is an innovation that is assumed to be uncorrelated with ω_{jt} and k_{jt}. For the estimation, we follow the ACF model and assume that ω_{jt} follows an AR(1) process with parameter ρ. In our model, both the firm's vacancy-posting decision and its matching set depend on current productivity; see Appendix A.2. Moreover, the firm's dynamic problem involves the expected future evolution of productivity. Recall that we abstract from productivity shocks when deriving the wage equation (4) and establishing its linearity in worker type x. This assumption simplifies the derivation but is not critical for our results. As long as the evolution of firm productivity is independent of the employed worker types, the wage equation will still be (log-)linear in x under our assumed production structure.[27]

Finally, the ACF approach allows for (exogenous) across-firm differences in labor adjustment costs and wage setting. This flexibility is key to our approach, as it makes the ACF approach a good fit for models with search, matching, and bargaining. Earlier approaches in the empirical IO literature have needed to make more restrictive assumptions about input prices due to the use of unconditional intermediate input demand as the control function.[28] Intuitively, this is because a firm's unconditional intermediate input demand likely depends on its labor input and, thus, on its wage conditions or adjustment costs. Conditioning on the labor input eliminates this link. In the context of our model,

[26]The firm's information set when making input choices includes all past and present productivity shocks $\{\omega_{j\tau}\}_{\tau=0}^{t}$, but it does not include future productivity shocks.

[27]Consider an environment with productivity shocks and, accordingly, changing labor demand and endogenous separations. In this case, the wage equation contains an additional forward-looking component that captures the expected evolution in firm productivity. Worker type x can still be written in front of the integral sign under the mild assumption that x and the firm productivity process are independent.

[28]As explained in ACF (p. 2431), their approach can handle serially correlated, exogenous, and unobserved shocks to the prices of capital and labor. Levinsohn and Petrin (2003) can only allow for such shocks to the price of capital inputs. Neither approach allows for shocks to the price of intermediates.

Table 2: Production Function Estimation Results

Dependent Variable	*Value Added*				
	(a)	(b)	(c)	(d)	(e)
Labor input	0.7129	0.7131	0.7125	0.7099	0.8573
	(0.0141)	(0.0141)	(0.0141)	(0.0135)	(0.0133)
Capital input	0.1706	0.1703	0.1707	0.1512	0.1358
	(0.0117)	(0.0117)	(0.0117)	(0.0116)	(0.0110)
Labor input variable	AKM Predicted Wage Bill				Headcount
Worker fixed effects	Yes	Yes	Yes	Yes	–
Worker observables	Yes	Yes	Yes	Yes	–
Firm fixed effects	No	Yes	No	Yes	–
Residual	Yes	No	No	Yes	–
Firm-level controls	Yes	Yes	Yes	Yes	Yes
Year FEs	Yes	Yes	Yes	Yes	Yes
Sector FEs	Yes	Yes	Yes	Yes	Yes
Variance of $\hat{\omega}_{jt}$	0.074	0.073	0.072	0.064	0.050
Variance of ϵ_{jt}	0.628	0.627	0.626	0.597	0.583
N	38,598	38,598	38,598	38,598	39,808

Notes: Bootstrapped standard errors (500 iterations) are in parentheses. All estimated coefficients and standard errors are rounded to four decimal places. Data sources: BHP, EP, BeH.

differences in adjustment costs and wage setting across firms arise from two reasons. First, vacancy posting costs are heterogeneous, and firms have limited control over the hiring process.[29] Second, due to decreasing returns to labor and intrafirm bargaining, wages have a firm-size component that varies across firms.[30]

Estimation Results

Table 2 presents the results of the production function estimation. We show five different specifications in which we vary how we control for worker ability. All specifications include the aforementioned firm-level controls as well as year and sector fixed effects.

Specifications (a)–(d) use the predicted wage bills based on different combinations of the estimated AKM wage components. We always include worker fixed effects, which, according to our model, capture the effect of worker ability on output, along with the

[29] With random search, the time that passes until a suitable worker arrives is stochastic.

[30] Our model implies a nondegenerate firm-size distribution due to the heterogeneous productivity.

effects of worker observables. In addition, we include the AKM residual in (a), the firm fixed effect in (b), neither the residual nor the firm fixed effect in (c), and both in (d). The labor input in (d) is thus equal to the total observed wage bill for the firm. For the sake of comparison, we also show results for specification (e), which uses the worker headcount as the labor input measure.

The first interesting finding is the large difference between the estimation results for (a)–(d) and those for (e). The headcount mismeasures the labor input in the presence of worker heterogeneity, so the relatively high estimate of the output elasticity of labor is likely biased upwards in specification (e), while the capital elasticity and the estimated productivity $\hat{\omega}_{jt}$ are biased downwards. Across specifications (a)–(d), the results are very consistent, with an estimated output elasticity of labor of approximately 0.71. Together with the capital elasticity, these estimates imply decreasing returns to scale, in line with our model's assumption.

The second interesting finding is that the AKM wage components beyond the worker fixed and observable effects have little effect on the estimation results. In the light of our model, this is unsurprising. The worker fixed effects capture the effect of worker ability on output, so this wage component is key to understanding how workers contribute to their firm's production process. The firm effect, however, does not directly reflect how workers contribute to production. It captures the time average of the effect that firm productivity has on wages. The more persistent the productivity process is, the more informative the time-invariant firm effect with respect to wage variation. Additionally, firm effects absorb the effect of amenities, which are not clearly related to firm output.

Third, consider the effect of adding the AKM residual. According to our model, the AKM residual absorbs the effect of firm productivity shocks on wages, which have been shown to affect wages ("pass-through").[31] Guiso et al. (2005) show that firms fully insure their workers against transitory productivity shocks but only partially against the more permanent productivity changes captured by $\hat{\omega}_{jt}$. A correlation between the contemporaneous productivity realization and the labor input does not threaten our estimation strategy due to the flexibility of the ACF approach. Notably, adding the residual has little impact on the estimated output elasticities. However, capturing the reaction of wages to shocks in our input measure could still be important. If worker ability and firm productivity are complements in production, the contribution of worker ability to firm output could shrink after a negative productivity shock. For this reason, we select specification (a), which includes the AKM residuals, as our benchmark specification.

Finally, we examine the dispersion and persistence of estimated firm productivity $\hat{\omega}_{jt}$. The estimated variance of 0.074 in our benchmark specification is almost 50% higher than that in the headcount specification (e). It is, however, an order of magnitude smaller than the residual variance. Recall that the residual absorbs transitory shocks, whereas

[31] See also Chan et al. (2021), who study this channel with Danish data and summarize the literature.

Table 3: Firm Ranking Correlations

	$\bar{v}_j$	${}^{\bar{v}_j}/_{\bar{n}_j}$	${}^{\bar{\pi}_j}/_{\bar{n}_j}$	$\bar{n}_j$	$\bar{k}_j$	${}^{\bar{k}_j}/_{\bar{n}_j}$	Workforce education
Correlation with $\hat{\omega}(j)$	0.43	0.25	0.16	0.40	0.24	-0.03	0.08

Notes: The table shows the correlation of the time-invariant estimated firm ranks, $\hat{y}(j)$, with the means of the following firm statistics over time: log value added ($\bar{v}_j$), log value added per worker (${}^{\bar{v}_j}/_{\bar{n}_j}$), profit per worker (${}^{\bar{\pi}_j}/_{\bar{n}_j}$), the log size of the workforce ($\bar{n}_j$), the log capital stock ($\bar{k}_j$), the log capital stock per worker (${}^{\bar{k}_j}/_{\bar{n}_j}$), and workforce education as measured by the mean of the workers' education variable within the firm. Data sources: BHP, EP, BeH.

productivity ω_{jt} is the realization of a persistent stochastic process, and the estimated autocorrelation in $\hat{\omega}_{jt}$ in the benchmark specification is indeed high (0.75). Given the link between firm productivity and AKM firm effects that our model suggests, this high degree of persistence implies that firm fixed effects should be rather stable over time. Recent evidence presented by Lachowska et al. (2020) supports this conjecture.

Firm Rankings

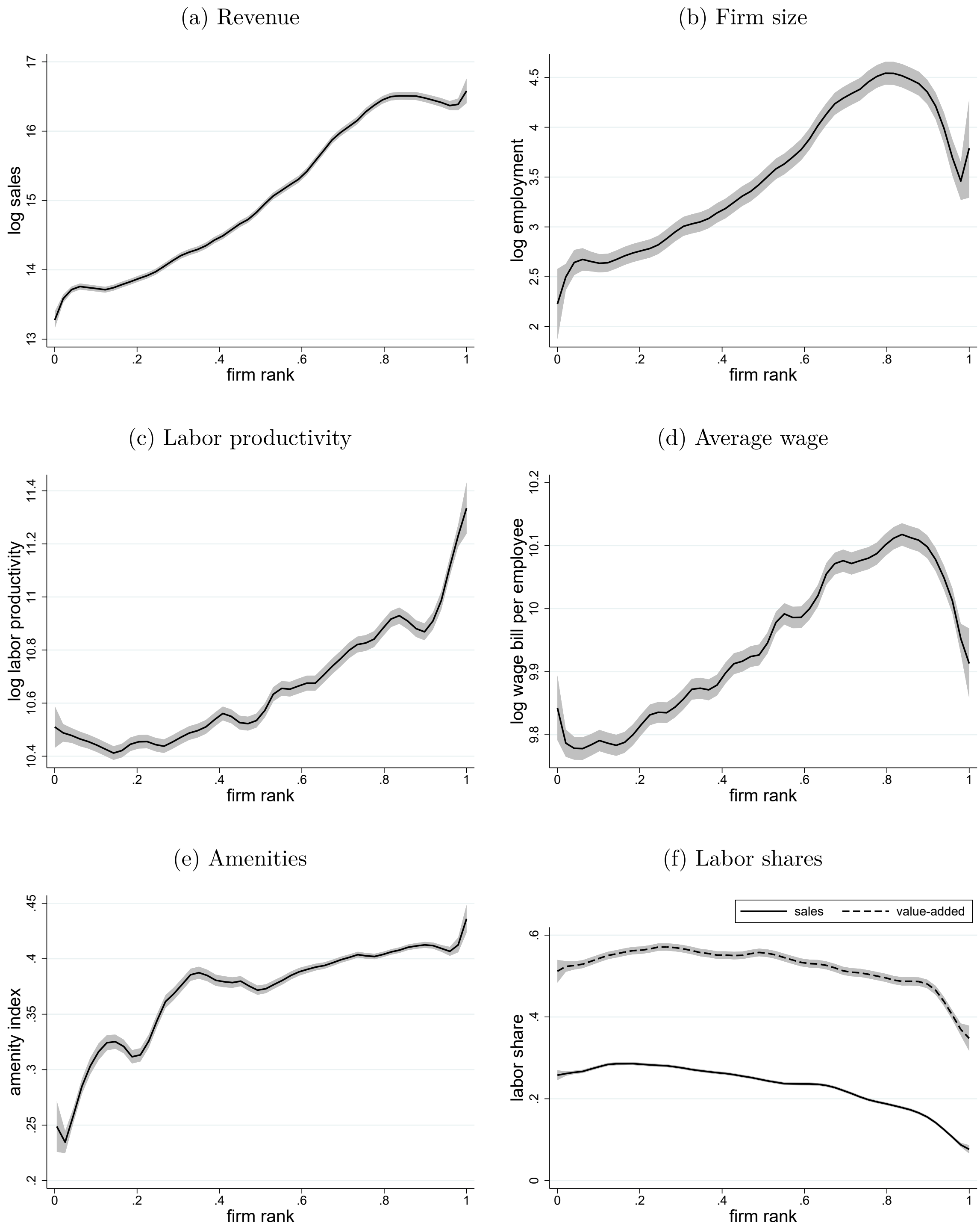
Given the high degree of persistence in the estimated productivity, we rank firms by the firm-level mean. The estimated rank of firm j is denoted $\hat{\omega}(j)$. The use of the mean is in line with the empirical literature on wage inequality and labor market sorting, which typically assumes the permanency of worker and firm types. The fixed effects-based worker types are also time-invariant.

Table 3 shows the correlation of estimated firm ranks with several firm-level statistics. The ranks are moderately positively correlated with firm size in terms of the employee headcount (0.40), value added (0.43), value added per worker (0.25), the capital stock (0.24), and profit (0.16). The ranks are virtually uncorrelated with capital per worker (-0.03). These correlations suggest that the largest firms in terms of output, workforce, and assets in our sample are not the most productive firms. The correlation of firm rank with the share of highly educated workers is only 0.08. Thus, highly productive firms are not necessarily characterized by a highly educated workforce.

Figure 2 shows the estimated kernel densities for six firm performance measures across the firm ranks. Revenue (measured by log sales, Panel (a)) mostly increases in rank but becomes relatively flat at the top of the rankings. The most productive firms do not have greater revenue than firms around the 80th percentile of the productivity distribution. In Panel (b), the relation between firm rank and log employment (measured by the headcount) is shown. This size measure increases monotonically until, roughly, the 80th percentile but falls steeply thereafter. In line with the correlations reported above, the most productive firms in our sample are clearly not the largest in terms of employment.

Panel (c) shows that log labor productivity (value added per worker) is relatively flat in the bottom half of the distribution but increases substantially for the most productive

Figure 2: Firm Performance Measures by Estimated Firm Rank



Notes: Estimated univariate kernel densities for selected firm performance measures across estimated firm ranks, normalized to be between zero and one. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is 0.05 for (a)–(e) and 0.04 for (f). The qualitative findings are robust to the bandwidth choice. 95% confidence bands are in gray. Data sources: BHP, EP, BeH.

firms. The labor productivity of the firms at the top of the distribution is twice as high as that of the firms at the bottom. The log wage bill per employee, that is, the average wage, is shown in Panel (d). Among the firms above the 80th percentile, average wages fall. Notably, the largest firms in terms of employee headcount are also situated near the 80th percentile, so average wages are closely related to size but less related to productivity. Interestingly, the least productive firms pay higher average wages than the other firms below the 25th percentile.

Panel (e) links a measure of amenities to firm productivity. We construct a firm-level amenity index based on (up to) 15 EP survey questions. They provide us with indicator variables that relate to the flexibility of working hours, working time accounts, parental benefits, support for young females, equal opportunity agreements, equity participation, and the prevalence of on-the-job training.[32] We control for industry at the 2-digit level and firm size (headcount) in the regression of the amenity index on productivity ranks. Clearly, the most productive firms offer the highest level of amenities.

Finally, Panel (f) shows labor shares, computed using both revenue and value added. Both labor share measures exhibit a hump shape but clearly fall in estimated firm rank. The labor share in terms of revenue (solid) falls from just below 30% for the least productive firms to less than 10% for the most productive firms. For the value added measure (dashed), the labor share falls from 50–60% to approximately 35%. Thus, the average worker obtains a significantly lower share of the output at high-productivity firms.[33]

Binning the Firms

In the next step, we group all individual firms into 15 productivity bins of equal size.[34] Let $\bar{\omega}(j)$ denote the bin to which firm j belongs. In what follows, individual firms in the same bin are considered to be of the same type. As for the worker bins described above, we decompose the variance of some key variables.

The firm bins exhibit a high within-bin wage variance (94% of the total variance). This is a reflection of the fact that we control for heterogeneous worker ability when estimating firm productivity. It also implies that our firms cannot be categorized into high-wage and low-wage firms—all firm types pay dispersed wages to their workers, depending on those workers' ability and contributions to output.

There is also within-bin heterogeneity in terms of output (value added and value added

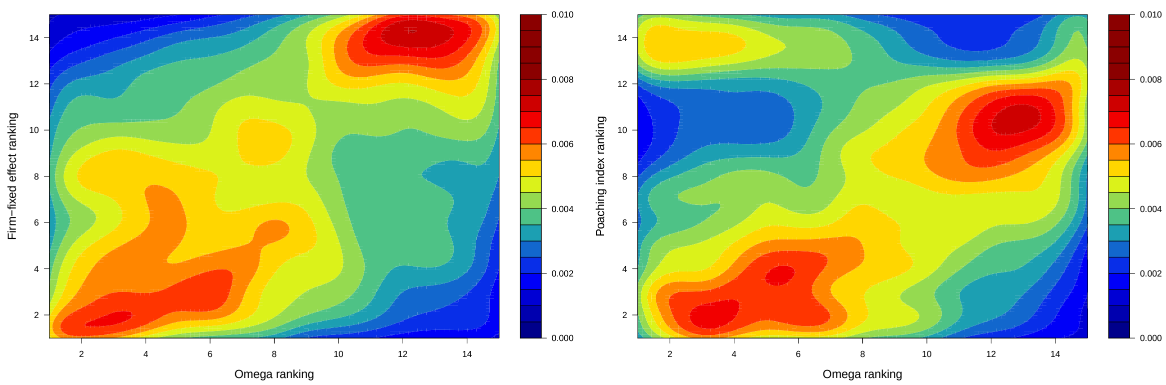
[32]The indicator variables are coded as 1 if the respective amenity exists in the firm and 0 otherwise. For each establishment-year, we sum up the indicators and divide them by the number of available indicators for that year. This results in an index between 0 and 1. The mean is 0.27, and the variance is 0.08.

[33]These low labor shares are interesting given the falling aggregate labor share in many developed economies. In fact, our firm ranking appears to mirror an explanation emphasized in Autor et al. (2017, 2020): the emergence of so-called "superstar" firms.

[34]The EP sample used to estimate firm productivity contains 10,026 individual establishments, so there are 668 establishments in each bin.

Figure 3: Comparison with Alternative Firm Rankings

(a) Firm fixed effects (b) Poaching index



Notes: The two plots depict the contours of the joint empirical distribution of firm-years across combinations of the ω ranking (15 bins) with the AKM firm-effects ranking (15 bins, Panel (a)) and the poaching index ranking (15 bins, Panel (b)). Data sources: BHP, EP, BeH.

per worker) and size (employee headcount and assets). The majority of the variance is always within bins, suggesting that binning does not merely group firms of similar size or production capacity. We also checked for relations between the productivity bins and industry or the prevalence of collective bargaining agreements and employee representation but found none.

Comparison with Alternative Rankings

Finally, we compare our productivity ranking with the results of other ranking techniques used in the literature on wage dispersion and labor market sorting. We create two alternative firm rankings. The first is based on the AKM firm fixed effects and the second on the poaching index used in Bagger and Lentz (2019) and Taber and Vejlin (2020).[35] Correlations with the productivity ranking are positive: 0.280 and 0.119, respectively. To graphically analyze how the rankings are related, we create two sets of 15 firm bins based on both the firm-effects ranking and the poaching ranking. This allows us to compute the empirical distribution of firm-years across the different firm bin combinations. We observe how firms with a given productivity rank are distributed across the firm fixed-effect and poaching rank bins. Figure 3 plots the contours of these empirical distributions.

In Panel (a), the AKM firm-effects bins are depicted on the vertical axis, and the ω ranking bins are depicted on the horizontal axis. The mass of observations is concentrated along the diagonal, which is in line with the positive correlation reported above. In the

[35]The poaching index is based on the idea that high-paying firms poach workers from other firms rather than hiring unemployed workers. We compute this index by comparing the annual number of workers hired directly from other firms to the number of all hires at the firm level and then rank firms based on the firm-level mean of the time-varying poaching index. We use the "Administrative Wage and Labor Market Flow Panel" (AWFP); see Stüber and Seth (2019). In this data set, the aggregated establishment-level worker flows needed to compute the poaching index are readily available.

upper-right quadrant of the plot, the observations are highly concentrated somewhat above the diagonal, reaffirming our observation that the highest-paying firms (here in terms of the AKM wage premia) are located below the top of the productivity distribution. In the lower-left quadrant, the observations are more dispersed. It is not uncommon to observe firm-years in which the estimated AKM wage premium is around the median but estimated productivity is very low and vice versa. Here, the disagreement between the two rankings is large. Figure D.1 in the Appendix shows that the observations in the lower-left quadrant are mainly young and small firms. The high-wage firms in the upper-right quadrant are older and larger firms. Panels (a) and (b) in Figure D.1 show that low-wage (high-wage) firms have on average a low (high) AKM rank, but there is a sizable overlap between the two groups in terms of productivity.

In Panel (b), the mass of observations lies below the diagonal; that is, a firm's poaching rank tends to be lower than its productivity rank. Many high-productivity firms have high poaching ranks, but they are almost never at the top. Additionally, it is not uncommon even for medium-productivity firms to hire mainly OON, as evidenced by the high density of low-poaching rank firms that extends far to the right. Interestingly, the firm years with the highest poaching index are clustered in the upper-left quadrant. Apparently, some low-productivity firms very actively poach workers from other firms. Figure D.2 in the Appendix shows that many of the firms at the top of the poaching index distribution are small and young. One possible explanation is that these firms attempt to grow quickly by poaching workers from other firms. The larger and older firms, which also pay the highest wages, are concentrated in the upper-middle of the poaching rank distribution. They hire a nonnegligible number of their employees OON.

In summary, the comparison of the different rankings shows that firm ranks based on firm wage premia and observed worker mobility are systematically different from our productivity-based firm ranking.

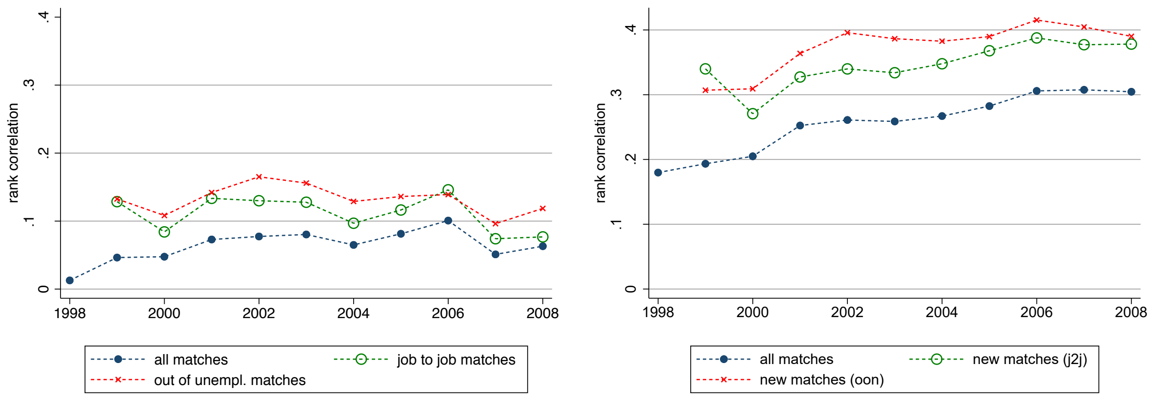
5 Productivity Sorting

To analyze the allocation of workers to firms and measure the degree of productivity sorting, we merge the two data sets containing our estimated worker and firm rankings. In this step, we lose all employment spells at firms that are not part of the EP sample. For this reason, we rebin workers and firms in the merged sample. The number of workers per bin decreases to 26,888. The number of firms per bin is almost unchanged at 667.[36] For all results below, we rely on the two samples defined in Section 3.4: (i) all existing matches and (ii) new matches, which are formed either through J2J flows or OON.

[36] We are unable to merge the employment spell information for 22 out of 10,026 unique establishments.

Figure 4: Rank Correlation Coefficients over Time (1998–2008)

(a) Firm ranking: productivity (b) Firm ranking: AKM firm effects



Notes: Panel (a) shows the Spearman rank correlation coefficients based on our productivity-based (ω) firm ranking. Panel (b) ranks firms using the AKM firm fixed effects. Both plots use the worker ability ranking introduced in Section 4.1. Data sources: BHP, EP, BeH.

5.1 Rank Correlations

We use rank correlation coefficients (Spearman's ρ) to study the association between the estimated worker and firm ranks in the data. The rank correlation is a simple yet informative summary statistic of how workers are allocated to firms. For all matches in all years, we find a significantly positive but relatively low rank correlation of 0.07. For new matches, this correlation is 0.12. The correlation for J2J moves (0.11) is somewhat lower than the correlation for new matches OON (0.13).[37]

The positive rank correlation we find is an indication of PAM based on worker ability and firm productivity. However, the implied degree of productivity sorting is low, both in an absolute sense and compared to earlier studies of wage sorting in German data. CHK and Hagedorn et al. (2017) find correlations of 0.17–0.25 and 0.71, respectively.[38] The key difference between our approach and these earlier studies is that we rank firms based on their estimated productivity, while both CHK and Hagedorn et al. (2017) rely on wages and observed worker mobility to rank firms.

Next, we consider how the estimated rank correlations change over time to clarify the time dynamics in productivity sorting. To this end, we present the rank correlation coefficients for all samples and all years in Figure 4. First, in Panel (a), the estimated rank correlations are calculated using the productivity-based firm ranking. The blue line depicts the correlations for all matches, while the red and green lines depict the

[37] For clarity, the rank correlations presented here are calculated based on the worker and firm bins. Correlations based on worker and firm ranks (not reported) differ only marginally. Table D.1 provides an overview of all estimated rank correlations for different samples and time periods.

[38] CHK use the log-linear AKM model and interpret the correlation between estimated worker and firm effects in the data as a measure of sorting. Hagedorn et al. (2017) study sorting through the lens of a structural model with worker and firm heterogeneity, search frictions, and OJS. Their model allows identification of the sign and strength of sorting without assuming a log-linear wage equation.

correlations for new matches OON and for J2J switches, respectively. For all matches, the degree of sorting first increases over time and then is relatively stable just below 0.1, with a drop in 2007. For both types of new matches, the correlation levels are higher but also fall near the end. The degree of sorting for new matches OON decreased steadily after 2002. For J2J switches, the time dynamics closely mirror those for all matches. This reflects the fact that approximately 65% of all new matches are J2J switches (see also Table D.1). Thus, new matches drive up the rank correlation for all matches in the beginning of the sample period, but once their correlation decreases, the correlation for all matches also levels off.

Panel (b) of Figure 4 facilitates the comparison with wage-based approaches. Here, we compute the same rank correlation coefficients using the same worker ability rankings but using firm rankings that are based on the AKM firm fixed effects. Notably, this wage-based firm ranking implies higher rank correlations (more positive sorting) and a significant increase over time. The correlation for all matches reaches 0.3 near the end of our sample period. These numbers are close in magnitude to the findings of CHK. For new matches, the correlations are even higher, just below 0.4. New matches OON exhibit the highest rank correlations with both the AKM and the productivity-based firm rankings. We delve more deeply into the sources of these changing dynamics for new and all matches in the next subsection.

5.2 Distributional Dynamics

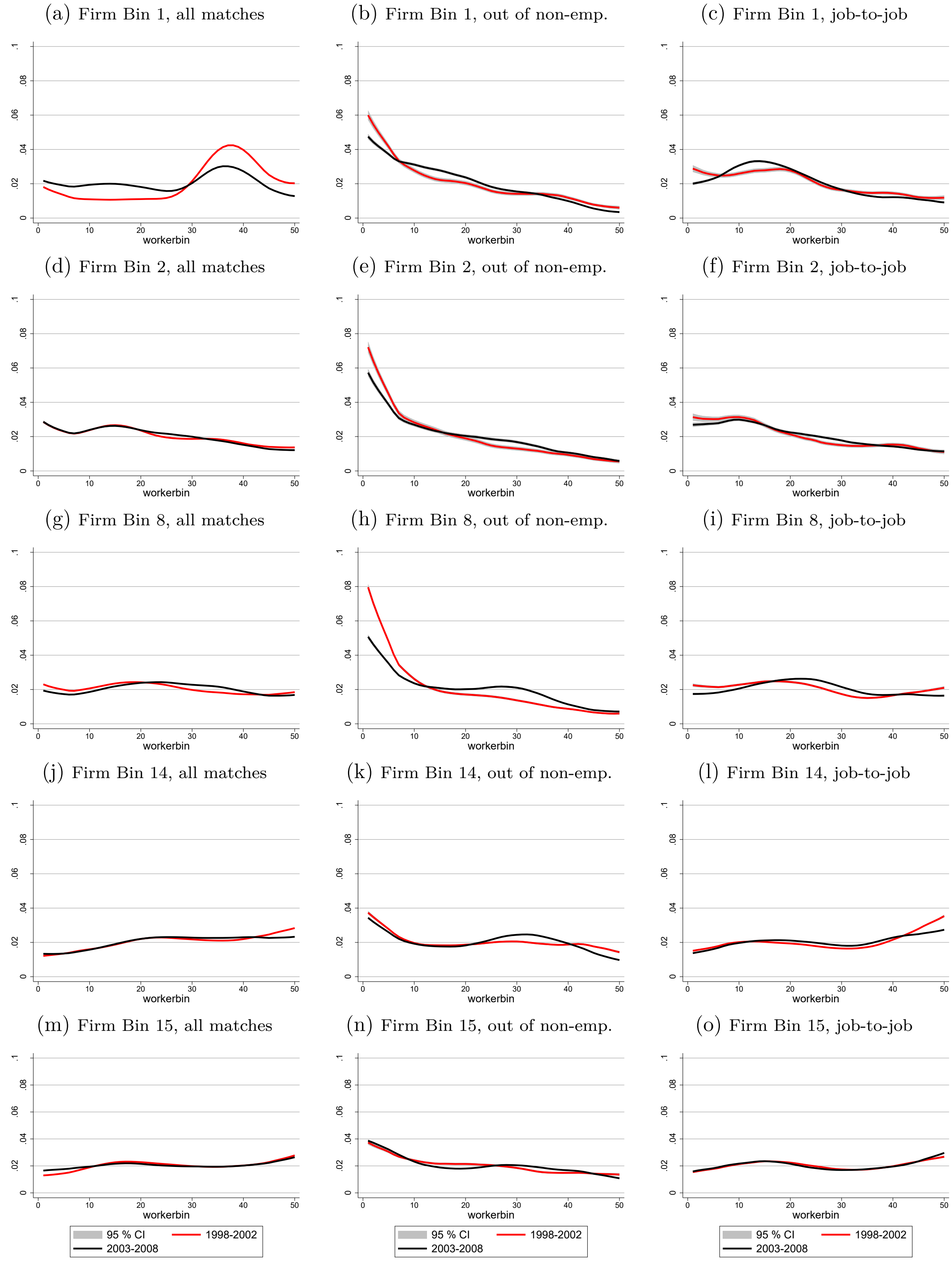
Rank correlations are only a summary statistic for the allocation of workers to firms. To more carefully investigate which worker–firm-type combinations contribute to changes in productivity sorting, we study which worker types the different firm types hire and how this has changed over time. This allows us to precisely track which worker types became “more sorted” and which became “less sorted”. To this end, we estimate univariate density functions for the employed worker types. Figure 5 presents these densities for low-productivity firms (bins 1–2), medium productivity firms (bin 8), and high-productivity firms (bins 14–15).[39] We compare the first half of our sample, 1998–2002 (red line), to the second half, 2003–2008 (black line), and show estimated densities for all matches, new matches OON, and J2J flows, along with the 95% confidence intervals.[40]

Low-type firms (bins 1–2) hire mainly low-ability workers OON. This is where the density of new matches is highest, and it is clearly falling in worker type. This finding is consistent with positive productivity sorting. However, we also find that the average

[39] The estimated univariate densities for the remaining firm bins are available upon request. Appendix Figure D.3 shows the full joint distribution of matches.

[40] In the plots, statistical significance can be determined by an overlap in confidence intervals. This is a conservative approach: it is always true that two statistics with nonoverlapping confidence intervals are significantly different from each other. However, overlapping confidence intervals do not necessarily imply insignificant differences.

Figure 5: Changes in the Worker Type Distributions within Different Firm Bins



(a) Firm Bin 1, all matches
(b) Firm Bin 1, out of non-emp.
(c) Firm Bin 1, job-to-job
(d) Firm Bin 2, all matches
(e) Firm Bin 2, out of non-emp.
(f) Firm Bin 2, job-to-job
(g) Firm Bin 8, all matches
(h) Firm Bin 8, out of non-emp.
(i) Firm Bin 8, job-to-job
(j) Firm Bin 14, all matches
(k) Firm Bin 14, out of non-emp.
(l) Firm Bin 14, job-to-job
(m) Firm Bin 15, all matches
(n) Firm Bin 15, out of non-emp.
(o) Firm Bin 15, job-to-job

Notes: Estimated univariate kernel densities for all new matches conditional on worker bin, time, and match type. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is calculated with Silverman's rule of thumb. Pointwise confidence intervals are calculated using a quantile of the standard normal distribution. Data sources: BHP, EP, BeH.

quality of new hires in low-productivity firms has increased over time. In Panels (b), (c), (e), and (f), the densities for the second half of our sample (black densities) are significantly higher for medium-ability workers hired both OON and due to J2J switches than the red densities. Moreover, the densities for low-ability workers decreased. Regarding the sample of all matches, the densities of worker types hired by bin 1 firms (Panel (a)) have become more uniform over time. That is, in addition to making higher quality new matches, these firms also have sizable outflows of high-ability workers. Taken together, the changes at low-productivity firms confirm the aggregate trend toward less productivity sorting.

For medium-type firms (bin 8), we observe a strong increase in the hiring of medium-ability workers both through poaching and OON; see Panels (h)/(i). For very productive firms (bin 14), the increase in the hiring of medium-ability workers is also present but less pronounced. It is paired with a notable decrease in the hiring of high-ability workers for new matches made both through poaching and OON; see Panels (k)/(l). In sum, the plot for all matches suggests a significant reduction in hiring for high-ability workers and an increase for medium-ability workers, although that increase is smaller than that at medium-productivity firms. For the most productive firms in bin 15, the aforementioned developments are, for the most part, muted; see Panels (n)/(o). We even observe a small but significant increase in new matches with high-ability workers through poaching. The density for all matches does not change for worker types beyond bin 10 (Panel (m)), so the allocation is quite stable at the very top.

This distributional analysis reveals why productivity sorting is relatively stable over time. High-ability workers have, to some extent, been replaced by medium-ability workers. Moreover, low-productivity firms have reduced their hiring of low-ability worker types and increased the quality of their average worker. The stable rank correlation is thus the result of two opposing trends: reduced productivity sorting at the bottom and the top (fewer low-low and high-high matches) and increased sorting in the middle (more medium-medium matches). In the final section, we analyze how these changes in allocation are related to the wages that workers earn at different firms.

6 Wages and Inequality

First, we investigate how wages vary by worker and firm type. Second, we check whether observed worker transitions are directed toward higher wages and/or higher firm productivity. Third, we decompose the wage inequality trend using our productivity-based firm types and ability-based worker types to assess the role of productivity sorting in wage inequality in Germany.

6.1 Wage Variation across Worker and Firm Bins

According to our theory, worker ability and firm productivity are complements in production. Many previous authors have argued that the presence of such complementarities implies that the wage of a given worker type is not necessarily monotonically increasing in firm type.[41] Our approach of controlling for worker ability when estimating firm productivity allows us to test for wage monotonicity by examining how wages vary across different firm types. In Figure 6, we plot the resulting wage–productivity profiles for five groups that include ten worker bins each.[42] All matches are shown in Panel (a), and new matches are shown in Panel (b).[43] We observe quantitatively important deviations from monotonicity in both samples.[44]

The wage–productivity profiles exhibit a characteristic S-shape: low-productivity firms pay relatively high wages. The lowest wages are paid by firms in bins 3–4. Wages then increase monotonically until firm bins 11–12 and decrease thereafter.[45] Moreover, the wage drop at the most productive firms decreases in worker ability. It is most pronounced for low- and medium-ability workers but almost nonexistent for high-ability workers. For new matches in worker bins 41–50, the wage merely levels off. To scrutinize both findings—the observed wage drop at the most productive firms and the fact that the drop decreases in worker ability—we again rely on the estimated AKM wage components. Appendix Figure D.5 reproduces Figure 6 using the firm fixed effects instead of the log wage on the vertical axis. In this case, we observe consistent wage drops at the top across all worker types, including the high-ability workers.

On the one hand, the difference between Figures 6 and D.5 implies that the non-monotonicity at the top of the wage–productivity profiles is driven by firm-specific factors. According to our model, the firm fixed effect captures the time average of the effect of firm productivity on wages, which consists of a direct effect through the marginal product of labor and an indirect effect that captures the changing employment effect on wages in the presence of decreasing returns; see Section 2.1/Appendix A.4. The theory can explain why the firm fixed effect increases in productivity for the most part but not why it decreases at the top. We suspect that positively valued non-wage job character-

[41]See, among others, Gautier and Teulings (2006), Eeckhout and Kircher (2011), Lise et al. (2016), Hagedorn et al. (2017), Lopes de Melo (2018), and Bagger and Lentz (2019).

[42]The plots are based on a series of kernel-weighted local polynomial regressions.

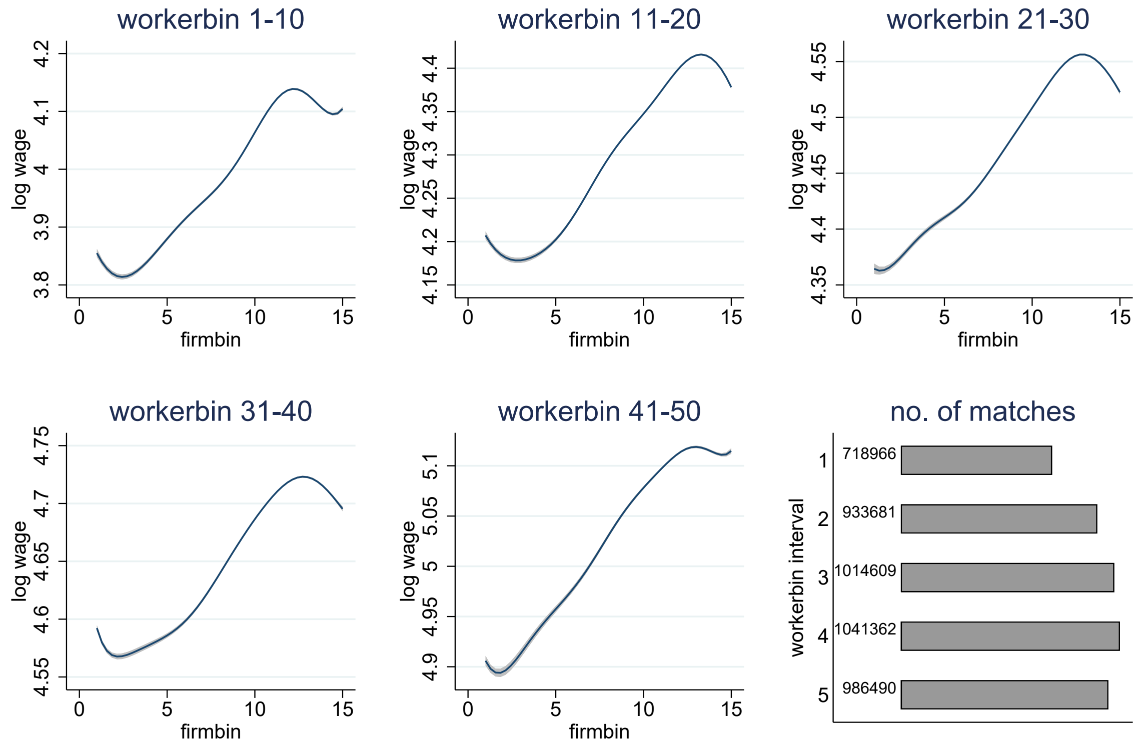
[43]We relegate plots for matches OON and J2J moves to Appendix Figure D.4. The results we discuss here do not depend on this distinction.

[44]The wage drop is most pronounced for medium-ability workers. In the sample of all matches, the average wage loss from being employed in a bin 15 firm instead of a bin 12 firm for a bin 11–20 worker in terms of (deflated) log daily wages is approximately 4% (approximately 1,177 euros.) For new matches of workers in bins 11–20, the wage difference between working in a bin 1 firm and a bin 3 firm is approximately 10% (approximately 2,316 euros).

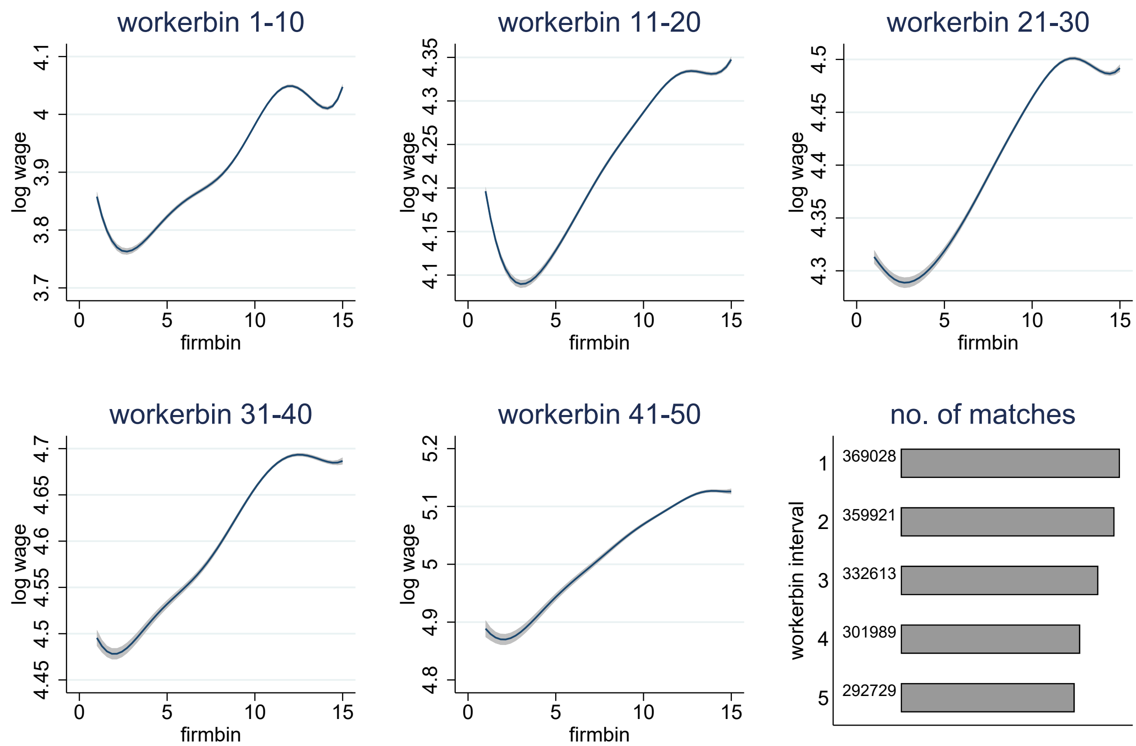
[45]The maximum around firm bin 12 can be related to the firm performance measures shown in Figure 2, Section 4.2. The firms in bin 12 (around the 80th percentile of the productivity distribution) pay the highest average wages and are the largest firms in terms of headcount. The firms above are smaller, pay less on average, and exhibit greater labor productivity as well as lower labor shares.

Figure 6: Wage–Productivity Profiles

(a) All Matches



(b) New Matches



Notes: The plots show the estimated wage–productivity profiles across firm bins for all matches (a) and new matches (b). The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. The 95% confidence bands are in gray. Data sources: BHP, EP, BeH.

istics (amenities) are behind this decrease. Recall that Figure 2e shows that the most productive firms offer the highest level of amenities, which could enable them to pay relatively low wages. This interpretation is consistent with recent findings by Lamadon et al. (2019), who estimate a model of imperfect competition on U.S. data.

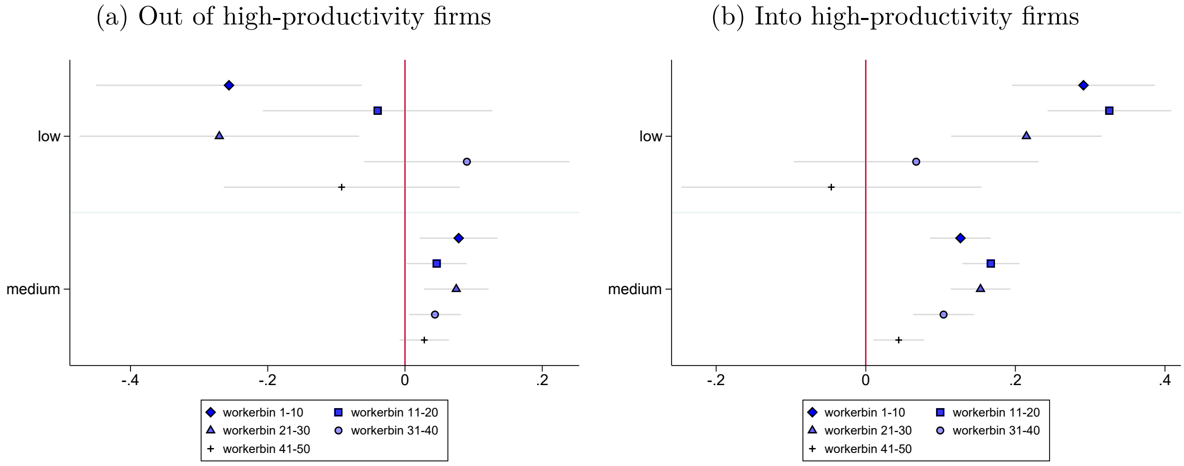
On the other hand, the fact that the wage decrease is mitigated for high-ability workers suggests that the link between wages and productivity is more complex than the relatively simple wage setting mechanism in our model. We examine the AKM residuals in Figure D.6 and indeed observe large positive residuals for matches between the most productive firms and high-ability workers. These positive wage effects overcompensate the (suspected) negative effect of amenities. Complementarities among worker types, which we abstract from to obtain a log-linear wage equation, could explain the positive residuals. Moreover, the residual absorbs the effect of productivity fluctuations on wages according to the model. The quantitative importance of this pass-through channel might be higher at the most productive firms, and it might also covary with worker ability. These mechanisms cannot be explained with the simple production structure of our model and require further investigation in future research.

Finally, we present two robustness checks for the wage–productivity profiles. First, concerns about their validity might be related to potential sampling error in the data used to estimate worker and firm ranks. On the worker side, we rely on the universe of employment spells available through the German social security registers, so there is no worker sampling. On the firm side, we rely on EP (firm survey) data, which might be prone to sampling error. To alleviate this concern, we re-estimate the wage–productivity profiles using 20 random subsamples (drawn with replacement and clustered at the firm level). The S-shape is robust; see Figure D.7. Second, we confirm in Figure D.8 that the nonmonotonicities are not related to tenure effects. We re-estimate the wage–productivity profiles using the first match-year only. If anything, the wage drop at the top is more pronounced in this case. Additionally, Figure D.9 shows that firm types based on AKM firm fixed effects do not reveal any nonmonotonicities, as one would expect.

6.2 Worker Transitions

The nonmonotonicities that we find in Figure 6 reflect the mean wage differences across firm types for a given set of worker types. The next step is to check whether observed worker transitions between firm bins are consistent with these wage-productivity profiles: Do workers transition toward higher wages, even if this implies switching to a less-productive employer? In this case, transitions out of the most productive firms should lead to wage gains, at least for a subset of destination firm bins. Similarly, moving up the productivity ladder should yield wage gains, but they should be decreasing in the origin firm bin and might even be negative for transitions to the most productive firms.

Figure 7: Wage Changes for Observed Transitions



Notes: The plots show estimated coefficients and 95% confidence intervals (robust standard errors) from a linear regression of individual-level wage differences among transitioning workers on dummies for origin and destination firm bins. The sample consists of new matches (J2J switches, no intermittent nonemployment spell) for five groups of worker types. The depicted coefficients are for transitions out of (Panel (a)) and into (Panel (b)) high-productivity firms (bins 13–15). The vertical axis captures the destination/origin firm bin groups: low (bins 1–3) and medium (bins 4–12). Data sources: BHP, EP, BeH.

Figure 7 shows that wage changes for observed J2J transitions support these conjectures. We regress log-wage differences for the same five groups of worker bins on a set of origin and destination firm bin dummies.[46] To simplify the graphical illustration, we also group firm bins into high (13–15), medium (4–12), and low (1–3) bins. Panel (a) shows the estimated wage changes for transitions out of high-productivity firms. Low- and medium-type workers experience significant wage gains of 7.8% (bins 1–10), 4.6% (bins 11–20), 7.5% (bins 21-30), and 4.4% (bins 31–40) when they transition from high-productivity to medium-productivity firms. For the highest-ability worker types, the wage differences are not significantly different from zero, which is consistent with the muted wage drop for these worker types. Wage differences for transitions from high-productivity to low-productivity firms are quite noisy and in most cases not significantly different from zero. Overall, we find that transitions down the firm productivity ladder lead to wage gains for many worker types and that even transitions into low-productivity firms do not necessarily come with a negative wage change. These findings suggest that job mobility and sorting are guided by wages and not by firm productivity. This also explains the trend of decreasing productivity sorting at the top (see Section 5.2).

Panel (b) presents estimated wage changes for transitions into high-productivity firms in a similar manner. As conjectured, we find significant positive wage changes for most worker types, and they decrease in the origin firm type. For workers in bins 31–50, we cannot reject the hypothesis that transitioning from a low- to a high-productivity firm

[46]We measure the difference between the wage during the last spell in the pretransition firm and the wage during the first spell in the posttransition firm.

yields no wage gain, which is in line with the S-shaped wage-productivity profiles. Low- to medium-ability workers (1–30) can expect significant wage gains of more than 20% when moving from a low-productivity to a high-productivity firm. When the origin firm is of medium productivity, wage gains are between 4.4% and 16.7% and decreasing in worker ability. Based on the wage drops at the top in Figure 6, one might expect that some of these transitions lead to wage cuts. This is not the case, reaffirming that workers tend to move toward higher wages.

In summary, wage changes for observed transitions align well with the patterns displayed in Figure 6. Workers select jobs to maximize their wages, even when this implies a downward move along the firm productivity ladder. Upward transitions also tend to yield positive wage effects. We do not observe workers accepting wage cuts to move to more productive firms. We again test the robustness of our findings by re-estimating the wage changes using 20 random subsamples (drawn with replacement and clustered at the firm level). All findings are robust; see Figure D.10.

6.3 Wage Inequality

Song et al. (2019) argue that two-thirds of the increase of wage inequality in the U.S. from 1978 to 2013 can be attributed to increasing pay differences between firms. Using the AKM approach, they show that increasing wage sorting (high-wage workers into high-wage firms) and increasing segregation of workers contribute roughly equally to firms' rising contribution to wage inequality. Panel (a) of Figure 8 reveals that this trend is also present in the German data. We use establishment identifiers to decompose the variance in wages into the respective shares explained within and between establishments. Between 1998 and 2008, wage inequality grew because the between-firm component (blue line) grew by approximately 10%. The within-establishment contribution (red line) is of similar magnitude but stable over time.

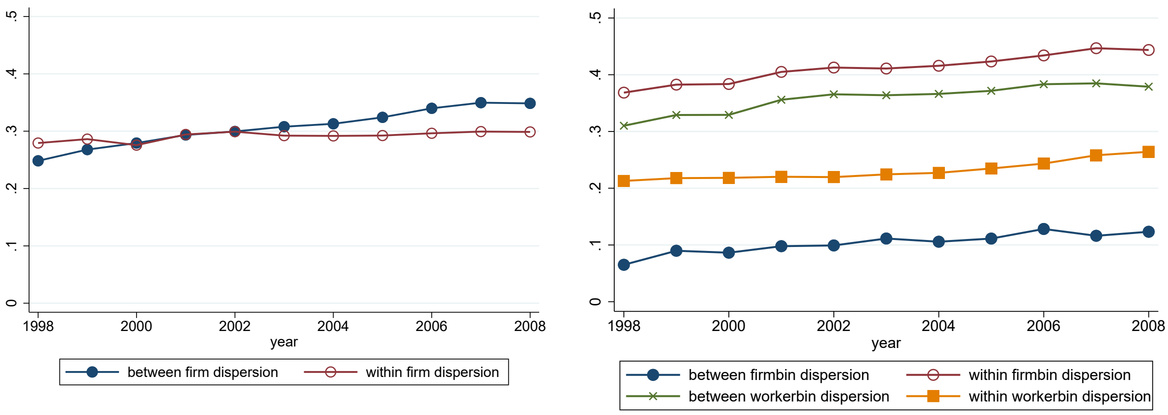
In Panel (b), we decompose wage inequality based on our productivity-based firm and ability-based worker types. The wage variance between the firm bins (blue line) does increase but only by approximately 3%. It contributes little to overall inequality. The wage variance within firm bins (red line) is quantitatively more important. Worker segregation, that is, wage differences between worker bins (green line), is also a major contributing factor. Both components increase over time at a higher rate than the between-firm bin inequality. The contribution of within-worker bin inequality (orange line) is moderately large and increasing but only in the second half of our sample. We conclude that increasing pay differences between firm productivity types are not the main driving force behind increasing wage inequality in Germany.

To scrutinize the increasing between-worker bin and between-firm bin components, we show how the wage profiles depicted in Figure 6 changed between the two subperiods con-

Figure 8: Decomposition of Wage Dispersion over Time

(a) Song et al. (2019) replication

(b) Our approach



Notes: Panel (a) shows yearly decompositions of the wage dispersion using establishment identifiers. Panel (b) shows yearly decompositions of the wage dispersion into the respective contributions of within- and between-estimated worker bin variance and within- and between-estimated firm bin variance. Data sources: BHP, EP, BeH.

sidered earlier, 1998–2002 and 2003–2008; see Figure D.11. We observe large differences in wage growth across worker types. These differences correspond to increasing worker segregation, that is, the increase in the between-worker variance component in Figure 8b. For low-type workers (bins 1–10), wages decreased in matches with all firm types above the very bottom and by more than 10% (all matches, firm bins 2–4). For the next set of worker bins (11–20), wages shrank at low-productivity firms but were relatively stable at more productive firms. For high-type workers (bins 41–50), wages increased in matches with almost all firm types, and most notably with firms at the top, by more than 7% (all matches, firm bin 15). For medium-type workers (bins 21–40), wages increased at more-productive firms and decreased at less-productive firms. Notably, the nonmonotonic wage humps at the most productive firms became more pronounced over time. This can explain the decreasing sorting of high-ability workers into high-productivity firms. Moreover, the wage–productivity profiles became steeper. This implies increasing between-firm wage inequality, consistent with Figure 8b.

7 Conclusions

Based on the wage equation from a sorting model with multiworker firms, two-sided heterogeneity, and random search, we propose a novel strategy to estimate unobserved firm productivity. Our approach is inspired by the empirical IO literature and relies on estimated AKM wage components to measure the contribution of heterogeneous worker ability to firm-level output. We study productivity sorting, wages across firm productivity types, and inequality trends in the German labor market.

Our analysis reveals a number of novel empirical facts. Productivity sorting is positive, low, and relatively stable over time. At the most productive firms, sorting decreases as high-ability workers become more likely to work at slightly less productive firms that pay higher wages. We confirm that most worker transitions are directed toward higher wages, including when this implies moving down the firm productivity ladder. Low-productivity firms pay relatively high wages, perhaps to induce growth or to retain workers. If workers move away from high-productivity firms to increase their wages, two side effects of increasing wage sorting could be decreasing allocative efficiency and lower aggregate output. Thus, we argue that our approach is a useful complement to the well-established wage-based analysis of sorting in the literature.

We have used our model to clarify the assumptions needed to link the AKM and ACF approaches. For future research, it would be promising to estimate the full model and use it for counterfactual analysis. In this context, OJS and more general production structures would be natural and useful extensions. Moreover, an analysis of the link between labor market sorting and increasing wage inequality, on the one hand, and much-debated trends such as falling labor shares, rising market concentration, and increasing monopoly and monopsony power of certain firms, on the other hand, seems promising.

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Online Appendix

(not for publication)

A Model Details

A.1 Matching Technology

Due to random search, firms cannot target their vacancies to specific worker types. They post vacancies v subject to a productivity-dependent cost $c(\Omega)$. Meetings are generated by a Cobb-Douglas matching function with constant returns to scale (Pissarides, 2000; Petrongolo and Pissarides, 2001). Without loss of generality, let worker ability x and firm productivity Ω be distributed uniformly on $[0, 1]$. Meeting rates are functions of aggregate labor market tightness, $\theta = {}^{V}\!/\!{}_{U}$, where $V = \int g_v(\Omega)\mathrm{d}\Omega$ and $U = \int g_u(x)\mathrm{d}x$ are the aggregated numbers of vacancies and unemployed workers, respectively. $q_v(\theta)$ is the rate at which firms meet workers and $q_u(\theta)$ is the rate at which unemployed workers meet vacancies.

$g_v(\Omega)$ is the PDF of vacancies at type Ω firms and $g_u(x)$ is the PDF of unemployed workers of type x. In this environment, a match is not guaranteed conditional on meeting. Suppose a type x worker meets a productivity Ω firm. Both parties may prefer to continue searching in case the match surplus $S(x, L, \Omega)$, defined below, is negative. Note that the surplus depends on the worker type, the firm type, and the total composite labor input. In steady state, existing matches can only end at the exogenous rate δ. Endogenous separations may happen out of steady state in case a shock to firm productivity reduces the surplus with specific worker types below zero.

A.2 The Firm's Problem

In the outlined environment, the profit flow of a firm with productivity Ω solves the following Bellman equation. The firm's problem is to maximize output given the current composite labor input and productivity less the total wage bill and hiring costs. Current employment is a state variable. The firm controls future discounted profits by posting costly vacancies, given its expected evolution of productivity:

$$\Pi(L, \Omega) = \max_{v} \left\{ F(L, \Omega) - \sum_{x=1}^{n} w(x, L, \Omega)L_x - vc(\Omega) + \beta \int \Pi(L', \Omega')\mathrm{d}G(\Omega'|\Omega) \right\}. \quad \text{(A.1)}$$

This profit flow is maximized subject to n constraints that capture the evolution of employment for every worker type x at the firm:

$$L'_x = (1 - \delta)L_x + vq_v(\theta)\frac{g_u(x)}{U}\mu(x, L, \Omega) \ \forall x. \quad \text{(A.2)}$$

${}^{g_u(x)}/_{U}$ is the probability that conditional on meeting the worker is of type x. The indicator function $\mu(x, L, \Omega)$ returns the value one if a match of a type x worker and productivity Ω firm with composite labor input L has a strictly positive surplus and zero otherwise. In case $\mu(x, L, \Omega) = 0$ no additional type x workers are hired.

The match surplus is defined as

$$S(x, L, \Omega) = J_x(L, \Omega) + E(x, \Omega) - U(x), \tag{A.3}$$

and depends on three option value equations defined below. Thus, the indicator $\mu(x, L, \Omega)$ is defined as

$$\mu(x, \Omega) = \begin{cases} 1 & \text{if} \quad S(x, L, \Omega) > 0 \\ 0 & \text{if} \quad S(x, L, \Omega) \leq 0. \end{cases} \tag{A.4}$$

Below, we indicate $\mu(x, L, \Omega) = 1$ ($\mu(x, L, \Omega_j) = 0$) by writing $\mu^+(x, L, \Omega)$ ($\mu^-(x, L, \Omega)$).

Optimality Conditions

We closely follow Cahuc et al. (2008) and define the marginal value of an additional worker of type x at a firm with productivity Ω and workforce L as

$$J_x(L, \Omega) = \frac{\partial \Pi(L, \Omega)}{\partial L_x}. \tag{A.5}$$

The marginal product of type x labor (MPL) at a productivity-Ω firm is

$$F_x(L, \Omega) = \frac{\partial F(L, \Omega)}{\partial L_x}. \tag{A.6}$$

The FOC of the maximization problem (A.1) with respect to v is

$$0 = -c(\Omega) + q_v(\theta)\frac{g_u(x)}{U}\mu(x, L, \Omega)J_x(L', \Omega'). \tag{A.7}$$

The envelope theorem implies

$$J_x(L, \Omega) = \frac{\partial F(L, \Omega)}{\partial L_x} - \sum_{k=1}^{n} L_k \frac{\partial w_k(L, \Omega)}{\partial L_x} - w(x, L, \Omega) + \beta(1 - \delta)J_x(L', \Omega'). \tag{A.8}$$

Assuming a steady state where $L' = L$ and $\Omega' = \Omega$, (A.7) can be rewritten as

$$J_x(L, \Omega) = \frac{c(\Omega)}{q_v(\theta)\frac{g_u(x)}{U}\mu^+(x, L, \Omega)}, \tag{A.9}$$

so for every worker type within the firm's matching set ($\mu(x, L, \Omega) = 1$), the marginal profit is equal to the expected recruitment cost at the optimal level of employment. In case a type x worker is not part of the firm's matching set, $\mu(x, L, \Omega) = 0$, marginal profits are undefined. Integrating the worker type out of (A.9) yields the firm's expected marginal profit of posting a vacancy:

$$J(L, \Omega) = \frac{c(\Omega)}{q_v(\theta) \int \frac{g_u(x)}{U} \mu^+(x, L, \Omega) \mathrm{d}x}. \tag{A.10}$$

Applying the steady state assumption to (A.8) yields

$$J_x(L, \Omega) = \frac{F_x(L, \Omega) - w(x, L, \Omega) - \sum_{k=1}^n L_k \frac{\partial w(k,L,\Omega)}{\partial L_x}}{1 - \beta(1 - \delta)}, \tag{A.11}$$

so the marginal profit can also be expressed as the discounted marginal product, net of the individual wage and net of the effect of the marginal hire on the total wage bill.

Equating (A.9) and (A.11), one gets

$$F_x(L, \Omega) = w(x, L, \Omega) + \frac{c(\Omega)(1 - \beta(1 - \delta))}{q_v(\theta) \frac{g_u(x)}{U} \mu(x, L, \Omega)} + \sum_{k=1}^n L_k \frac{\partial w(k, L, \Omega)}{\partial L_x}, \tag{A.12}$$

so the MPL of worker type x at a (L, Ω) firm equals the wage plus expected turnover costs and the marginal effect of the additional worker on the total wage bill.

A.3 Wage Determination

To derive the wage equation, we rely on the Nash sharing rule

$$\frac{\alpha}{1 - \alpha} J_x(L, \Omega) = E(x, L, \Omega) - U(x), \tag{A.13}$$

where $\alpha \in (0, 1)$ is the workers' common bargaining parameter. The RHS captures the worker's surplus of working at a firm with productivity Ω and workforce L relative to the worker's outside option, the value of unemployment, $U(x)$. The firm's surplus consists of the marginal profits of hiring an additional worker of type x, $J_x(L, \Omega)$, as defined above. Its threat point is to fire the worker and renegotiate wages with all other employees (Stole and Zwiebel, 1996). Following Cahuc et al. (2008), we assume that wages are continuously and instantaneously (re)negotiated, so L is fixed during (re)negotiations.

In steady state, the worker's value of employment is

$$E(x, L, \Omega) = w(x, L, \Omega) + \underbrace{\beta \delta U(x)}_{\text{separation}} + \underbrace{\beta(1 - \delta) E(x, L, \Omega)}_{\text{continued employment}}. \tag{A.14}$$

The value of unemployment is

$$
\begin{aligned}
U(x) = b(x) + \underbrace{\beta(1-q_u(\theta))U(x)}_{\text{no meeting}} + \underbrace{\beta q_u(\theta) \int \frac{g_v(\Omega)}{V} \mu^+(x,L,\Omega) E(x,L,\Omega) \mathrm{d}\Omega}_{\text{successful match}} \\
+ \underbrace{\beta q_u(\theta) U(x) \int \frac{g_v(\Omega)}{V} \mu^-(x,L,\Omega) \mathrm{d}\Omega}_{\text{meet unacceptable firm}},
\end{aligned}
\tag{A.15}
$$

where $b(x)$ is the flow value of unemployment, e.g. the value of increased leisure, home production or unemployment insurance benefits.

Next, we compute the difference $E(x,L,\Omega) - U(x)$ to be plugged into equation (A.13):

$$
E(x,L,\Omega) - U(x) = w(x,L,\Omega) + \beta\delta U(x) + \beta(1-\delta)E(x,L,\Omega) - U(x). \tag{A.16}
$$

After adding and subtracting $\beta U(x)$, this can be rearranged to

$$
E(x,L,\Omega) - U(x) = \frac{w(x,L,\Omega) - (1-\beta)U(x)}{1-\beta(1-\delta)}, \tag{A.17}
$$

which can be combined with (A.13) to get

$$
\frac{\alpha}{1-\alpha} J_x(L,\Omega) = \frac{w(x,L,\Omega) - (1-\beta)U(x)}{1-\beta(1-\delta)}. \tag{A.18}
$$

Finally, substituting marginal profits according to equation (A.11) and rearranging yields the wage bargaining outcome:

$$
w(x,L,\Omega) = \alpha \left(F_x(L,\Omega) - \sum_{k=1}^{n} L_k \frac{\partial w(k,L,\Omega)}{\partial L_x} \right) + (1-\alpha)(1-\beta)U(x). \tag{A.19}
$$

Due to our assumption of perfect substitutability of worker ability units at the firm level, the inframarginal adjustment term $\sum_{k=1}^{n} L_k \frac{\partial w(k,L,\Omega)}{\partial L_x}$ solely reflects decreasing returns and is unambiguously negative. Moreover, it does not vary with x. This yields the following simplified differential equation

$$
w(x,L,\Omega) = \alpha \left(F_x(L,\Omega) - L \frac{\partial w(x,L,\Omega)}{\partial L} \right) + (1-\alpha)(1-\beta)U(x), \tag{A.20}
$$

which we can solve following the the steps for the "single labor case" described in the Appendix of Cahuc et al. (2008). For the single steps and detailed technical assumptions, see their equations (B.1)–(B.6), p 961–962. The solution is

$$
w(x,L,\Omega) = (1-\alpha)(1-\beta)U(x) + \int_0^1 z^{\frac{1-\alpha}{\alpha}} F_x(Lz,\Omega) \mathrm{d}z. \tag{A.21}
$$

A.4 Wages, Labor Demand, and Productivity

It is worth considering how wages vary with firm productivity. Partially differentiating the wage equation (A.21) twice with respect to Ω yields

$$\frac{\partial w(x,L,\Omega)}{\partial \Omega} = \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F_x(Lz,\Omega)}{\Omega} \mathrm{d}z > 0, \tag{A.22}$$

$$\frac{\partial^2 w(x,L,\Omega)}{\partial \Omega^2} = (\beta_l - 1) \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F_x(Lz,\Omega)}{\Omega^2} \mathrm{d}z < 0, \tag{A.23}$$

which implies that wages increase in firm productivity, but at a decreasing rate. This model property is a direct consequence of the assumed production function, which exhibits a complementarity at the match level, see equation (2). Worker ability and firm productivity jointly determine how much one unit of labor of a given type contributes to output, but the total composite labor input is subject to decreasing returns. For this reason, the second derivative of the wage with respect to productivity is negative. The derivatives also show that firm productivity directly affects the MPL, $F_x(L,\Omega)$, and, additionally, scales down the inframarginal wage effect. Intuitively, this can be explained by the presence of worker-firm complementarities: the higher the firm's productivity, the more costly it is to reduce the marginal product of labor in terms of foregone output.

Based on the solution for the wage equation in (A.21), we can also compute the derivative in the last term of (A.12) for the single labor case:

$$F_x(L,\Omega) = w(x,L,\Omega) + \frac{c(\Omega)(1-\beta(1-\delta))}{q_v(\theta)\frac{g_u(x)}{U}\mu(x,L,\Omega)} + (\beta_l - 1)\int_0^1 z^{\frac{1-\alpha}{\alpha}+\beta_l-1} F_x(Lz,\Omega)\mathrm{d}z. \tag{A.24}$$

This expression pins down labor demand because it equalizes the marginal product to the labor cost, which consist of the wage, turnover costs, and the employment effect on the wage. The factor we write in front of the integral is negative for $\beta_l < 1$. This implies that firms can reduce their labor cost by expanding employment in the presence of decreasing returns (overemployment, as in Stole and Zwiebel, 1996; Smith, 1999; Cahuc et al., 2008).

A.5 Linearity of the Wage Equation

Plugging in the worker-firm-specific MPL (3) into (A.21) yields

$$w(x,L,\Omega) = (1-\alpha)(1-\beta)U(x) + x\int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz,\Omega)}{Lz} \mathrm{d}z, \tag{A.25}$$

so the integral expression is only scaled by, and hence linear in, worker ability x.

To establish that the full wage equation is linear in x, the outside option $U(x)$ has to be linear in x, too. Consider the outside option according to equation (A.15). A straightforward assumption to ensure that $U(x)$ is indeed linear in x is that all worker types' matching sets cover the whole type space. In other words, conditional on meeting, there are no unacceptable firms. This claim is easily verified empirically, see Section 5. In the model, this implies that $\mu(x, L, \Omega) = 1$ holds for all potential matches and, thus, the last term of (A.15) is zero. Now rearrange equation (A.14) such that

$$E(x, L, \Omega) = \frac{w(x, L, \Omega) + \beta\delta U(x)}{1 - \beta(1-\delta)}. \tag{A.26}$$

Under our assumption, this can be plugged into equation (A.15) to yield an expression in the wage and the outside option only.

$$U(x) = b(x) + \beta(1 - q_u(\theta))U(x) + \beta q_u(\theta) \int \frac{g_v(\Omega)}{V} \frac{w(x, L, \Omega) + \beta\delta U(x)}{1 - \beta(1-\delta)} \mathrm{d}\Omega. \tag{A.27}$$

Plugging in our solution for the wage, equation (A.25), into this expression and collecting the $U(x)$ terms in front of the integral yields

$$\begin{aligned} U(x) = b(x) + \beta(1 - q_u(\theta))U(x) + \beta q_u(\theta) \frac{1 - \alpha + \beta(\alpha + \delta - 1)}{1 - \beta(1-\delta)} U(x) \int \frac{g_v(\Omega)}{V} \mathrm{d}\Omega \\ + \frac{\beta q_u(\theta)}{1 - \beta(1-\delta)} \int \frac{g_v(\Omega)}{V} x \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz, \Omega)}{Lz} \mathrm{d}z \mathrm{d}\Omega, \end{aligned} \tag{A.28}$$

where $\int \frac{g_v(\Omega)}{V} \mathrm{d}\Omega = 1$. After collecting all $U(x)$ terms on the LHS and dividing, we get the following expression for $U(x)$:

$$U(x) = \frac{(1 - \beta(1-\delta))b(x) + x \frac{\beta q_u(\theta)}{1-\beta} \int \frac{g_v(\Omega)}{V} \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz,\Omega)}{Lz} \mathrm{d}z \mathrm{d}\Omega}{1 - \beta(1 - \delta - \alpha q_u(\theta))}, \tag{A.29}$$

where worker ability x can be written in front of both integral sign. Thus, for $U(x)$ to be linear in x, we additionally have to assume that the workers flow value of unemployment, $b(x)$ is proportional to x (i.e. $b(x) \propto x$), which is a standard assumption used in e.g. Postel-Vinay and Robin (2002).

B Details of Data Preparation

B.1 Wage Imputation

In the BeH data, earnings are right censored at the contribution assessment ceiling ('Beitragsbemessungsgrenze'). This earning limit is given by the statutory pension fund and is adjusted annually due to changes in earnings. First we deflate daily wages by using the CPI. Then, in each year, we identify censored wage observations by comparing wages with the contribution assessment ceiling. We define a wage observation as censored whenever the reported wage is higher than 99% of the censoring threshold.

Following CHK and Dustmann et al. (2009), we fit a series of Tobit regression to impute the right tail of the wage distribution. We estimate Tobit regressions by year, sex, education and age group. In all these regressions we additionally control for the exact age, the mean log wage in other years, the fraction of censored wages in other years, the number of full time employees at the current establishment and its square, an indicator for large firms, the mean years of schooling and the fraction of university graduates at the current establishment, the mean log wage of co-workers and the fraction of co-workers with censored wages, an indicator for individuals observed only one year, an indicator for employees in one-worker establishments, and an indicator for regions. We assume that the error term is normally distributed but each education and age category can have a different variance. For each year, we impute censored wages as the sum of the predicted wage and a random component which is computed based on standard error of the forecast. This component is drawn from separate normal distributions with mean zero and the different variances for each education and age category.

B.2 Education Imputation

The employee education information is reported by employers after every year and whenever a job ends. Its quality may suffer because employers do not face consequences for non- and misreporting. However, the existence of a reporting rule allows for corrections. It prescribes that only the highest educational degree of an employee needs to be reported. Therefore the individual educational attainment should not decline over consecutive job spells. The imputation procedure (IP1) suggested by Fitzenberger et al. (2006) exploits this reporting rule by assuming that there is any over-reporting in the data.

The original education variable distinguishes the following four different educational degrees: high school, vocational training, technical college and university. By imputing following the IP1 procedure we extrapolate both back and forwards and do some additional adjustments using individual information on age and occupational status. As a result we get six education categories which can be ranked in increasing order. We drop the remaining 2% of observations for which education cannot be imputed.

C Production Function Estimation Details

We start from the production function to be estimated, which corresponds to equation (10). Lower-case letters indicate logs, and here we ignore firm-level controls.

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + \epsilon_{jt}, \tag{A.30}$$

Following ACF, the control function we use is the demand for intermediate inputs *conditional on labor*[47] and capital:

$$m_{jt} = f_t(l_{jt}, k_{jt}, \omega_{jt}). \tag{A.31}$$

A natural interpretation of this demand function is that, conditional on both labor and capital, more productive firms use more intermediate goods in production. Intermediate inputs m_{jt} are chosen either simultaneously with the labor input l_{jt} or afterwards, reflecting that labor is a dynamic input.

Under the assumption of strict monotonicity in ω_{jt}, we can invert equation (A.31) to get $\omega_{jt} = f_t^{-1}(l_{jt}, k_{jt}, m_{jt})$, and substitute ω_{jt} out of (A.30):

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + f_t^{-1}(l_{jt}, k_{jt}, m_{jt}) + \epsilon_{jt} = \Phi_t(l_{jt}, k_{jt}, m_{jt}) + \epsilon_{jt}. \tag{A.32}$$

Following ACF, we adopt a two-stage procedure. First, value added is regressed on a polynomial approximation of $\Phi_t(l_{jt}, k_{jt}, m_{jt})$. This does not identify any of the parameters but delivers an estimate $\hat{\Phi}_t(l_{jt}, k_{jt}, m_{jt})$ and separates productivity ω_{jt} from transitory shocks absorbed by ϵ_{jt}.

To derive the second stage estimation equation, we use the assumption that firm productivity follows an AR(1) process (equation 12). Thus, the conditional expectation of ω_{jt} formed at $t-1$ is $E\left[\omega_{jt}|\omega_{j,t-1}\right] = \rho\omega_{j,t-1}$ and the productivity innovation is denoted ξ_{jt}. Additionally, the following holds: $\omega_{j,t-1} = y_{j,t-1} - \beta_0 - \beta_l l_{j,t-1} - \beta_k k_{j,t-1} - \epsilon_{j,t-1}$ (from A.30) and $\epsilon_{j,t-1} = y_{j,t-1} - \hat{\Phi}_{t-1}(l_{j,t-1}, k_{j,t-1}, m_{j,t-1})$ (from A.32, with the first stage estimate plugged in). Using these expressions along with the conditional expectation of firm productivity in equation (A.30) yields:

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \rho(\hat{\Phi}_{t-1}(.) - \beta_0 - \beta_l l_{jt-1} - \beta_k k_{jt-1}) + \xi_{jt} + \epsilon_{jt}, \tag{A.33}$$

where $\xi_{jt} + \epsilon_{jt}$ is a composite error term. Following ACF, we use GMM and four (uncon-

[47] ACF use of the conditional (on labor) input demand function to address a problem of functional dependence and improve identification of the labor input parameter relative to Olley and Pakes (1996) and Levinsohn and Petrin (2003).

ditional) second stage moment conditions to identify the four parameters $(\beta_0, \beta_l, \beta_k, \rho)$:

$$E\left[\left(\underbrace{y_{jt} - \beta_0 - \beta_l l_{jt} - \beta_k k_{jt} - \rho(\hat{\Phi}_{t-1}(.) - \beta_0 - \beta_l l_{jt-1} - \beta_k k_{jt-1})}_{\xi_{jt}+\epsilon_{jt}}\right) \otimes \begin{pmatrix} 1 \\ k_{jt} \\ \hat{\Phi}_{t-1}(.) \\ l_{j,t-1} \end{pmatrix}\right] = 0 \tag{A.34}$$

The first three moment conditions are standard in the literature and identical to Levinsohn and Petrin (2003): (i) The composite error term $\xi_{jt} + \epsilon_{jt}$ is zero in expectation. (ii) The capital stock is predetermined, recall equation (11), and therefore uncorrelated with the error term. (iii) The polynomial representation of the control function, estimated in the first stage and evaluated at $t-1$, $\hat{\Phi}_{t-1}$, is also uncorrelated with the composite error term because the intermediate input choice of the last period should be uncorrelated with the current productivity innovation and transitory shock.

The fourth moment condition is specific to the ACF approach and reflects that labor is a dynamic input. Only $l_{j,t-1}$ needs to be uncorrelated with the error term, while l_{jt} is allowed to correlate freely with ξ_{jt}. In other words, labor is chosen in response to the current firm productivity realization, subject to across-firm differences in adjustment costs and wage setting (firm-specific, serially correlated, unobserved shocks to the price of labor). The conditional (on l_{jt}) intermediate input demand does not depend on these costs/shocks. ACF do not allow for shocks to the price of intermediate inputs (due to the scalar unobservable assumption, which is required for the inversion of $f_t(l_{jt}, k_{jt}, \omega_{jt})$).[48] As ACF summarize in their discussion, Olley and Pakes (1996) cannot allow for any input price shocks, Levinsohn and Petrin (2003) can allow for shocks relating to k_{jt} but not to l_{jt} and m_{jt}, and ACF can allow for shocks relating to k_{jt} and l_{jt}, but not to m_{jt}.

Finally, note that not including m_{jt} in the production function implies that we estimate a value-added production function (value-added is simply defined as revenues minus the costs of intermediate inputs). One structural interpretation of this is that the production function is Leontief in the intermediate input, and this intermediate input needs to be proportional to output. As explained in ACF, their approach is designed for value-added production functions and is not suitable to identify the parameters of gross output production functions without further assumptions (see also Bond and Söderbom (2005) and Gandhi et al. (2020)).

[48] Arguably, intermediate inputs are often commodities implying only little price variation across firms.

D Additional Results

D.1 Rank Correlations

Table D.1: Spearman rank correlation coefficients and numbers of observations for different time intervals and samples

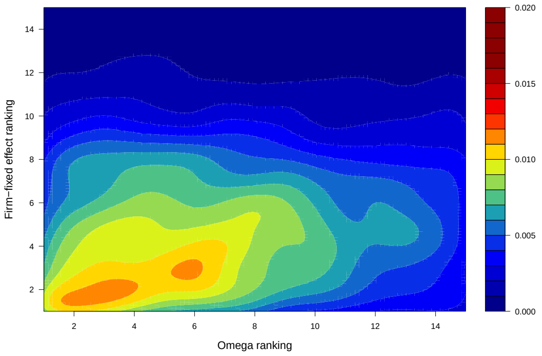
	All Matches	New Matches	Out of Nonemp.	Job-to-Job
1998-2008	0.065 (4,695,108)	0.124 (1,656,280)	0.132 (601,954)	0.110 (1,082,460)
1998-2002	0.055 (2,182,011)	0.139 (474,341)	0.141 (174,310)	0.120 (305,339)
2003-2008	0.074 (2,513,097)	0.118 (1,181,939)	0.129 (427,644)	0.107 (777,121)
1998	0.013 (311,861)	–	–	–
1999	0.046 (338,125)	0.140 (35,865)	0.133 (15,094)	0.129 (20,771)
2000	0.048 (493,323)	0.107 (107,740)	0.108 (41,731)	0.090 (66,009)
2001	0.073 (536,559)	0.148 (158,351)	0.142 (56,627)	0.137 (101,724)
2002	0.077 (502,143)	0.152 (172,385)	0.165 (60,777)	0.134 (111,608)
2003	0.080 (470,279)	0.146 (180,623)	0.156 (64,926)	0.131 (115,697)
2004	0.065 (458,467)	0.114 (191,207)	0.129 (67,932)	0.100 (123,275)
2005	0.081 (428,122)	0.129 (197,755)	0.136 (69,890)	0.118 (127,865)
2006	0.101 (415,153)	0.146 (206,984)	0.139 (74,340)	0.142 (132,644)
2007	0.051 (391,535)	0.084 (209,567)	0.096 (77,811)	0.074 (131,756)
2008	0.063 (349,541)	0.094 (195,803)	0.119 (72,001)	0.076 (123,802)

Notes: In all cells, we test the null hypothesis that worker and firm bins are statistically independent. All rank correlation coefficients are different from 0 at 1% level of significance. Rounded to 3 decimal places. Numbers of observations (matches according to the respective definition) are reported in brackets.

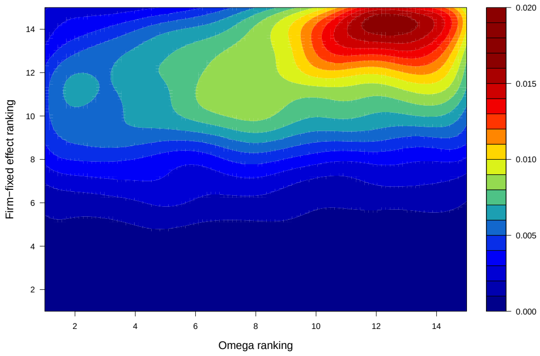
D.2 Ranking Comparisons

Figure D.1: Comparison of Productivity-based and fixed-effect-based Firm Ranking by Wages, Age, and Size

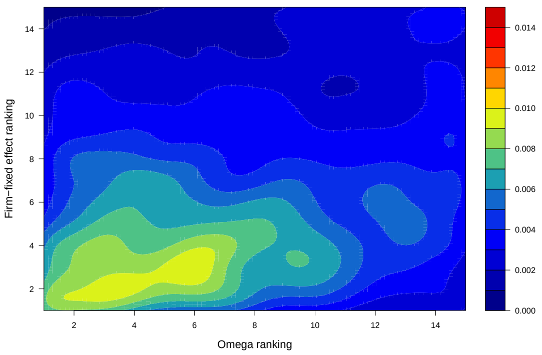
(a) Low-wage firms



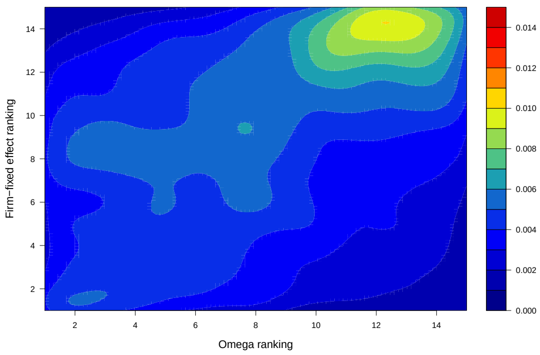
(b) High-wage firms



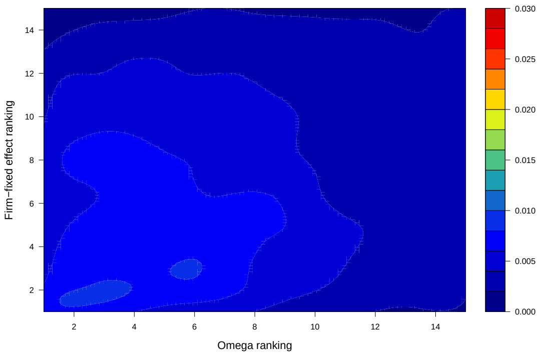
(c) Young firms



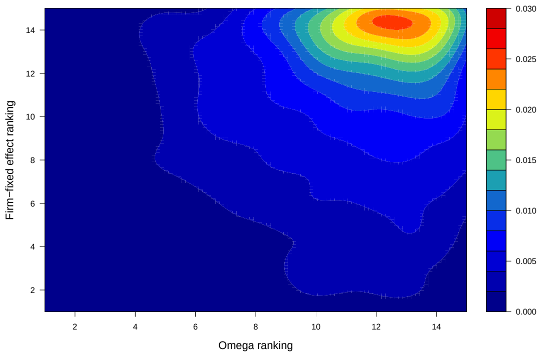
(d) Old firms



(e) Small firms



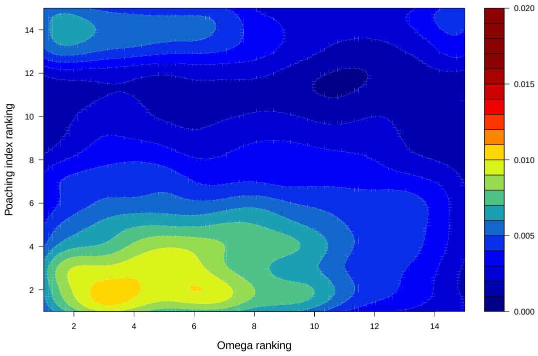
(f) Large firms



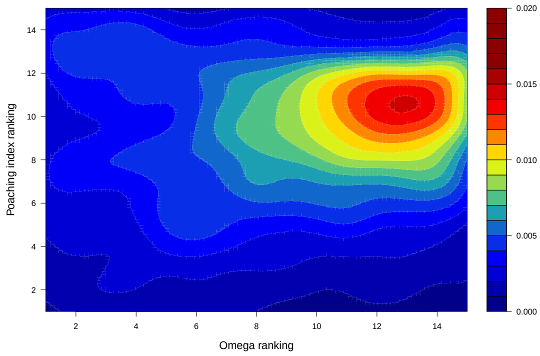
Notes: The six plots depict contours of the joint empirical distributions of firm years across combinations between the omega ranking (15 bins) and AKM firm-effect ranks (15 bins). In Panels (a) and (b), high-wage firms pay more than the grand mean of all firm-level mean wages and low-wage firms pay less. In Panels (c) and (d), the age of young firms is less than 15 years, old firms are 15 years and older. In Panels (e) and (f), small firms have less than 100 employees, large firms have more. Data sources: BHP, EP, BeH.

Figure D.2: Comparison of Productivity-based Firm Ranking and poaching-index-based Firm Ranking by Wages, Age, and Size

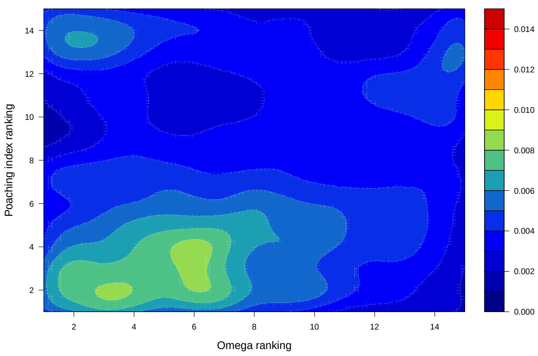
(a) Low-wage firms



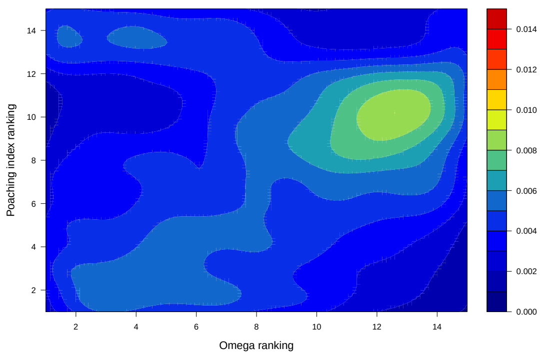
(b) High-wage firms



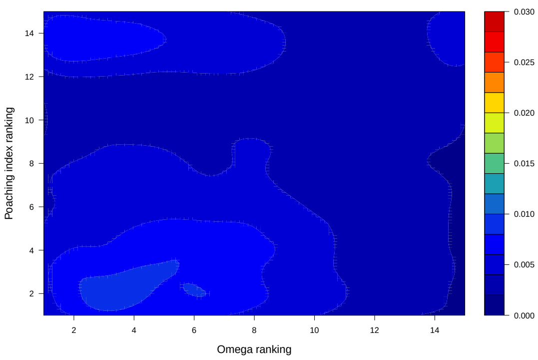
(c) Young firms



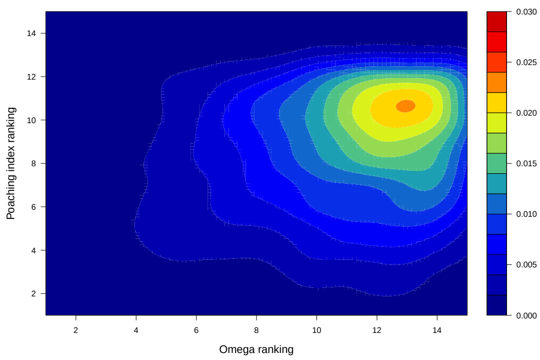
(d) Old firms



(e) Small firms



(f) Large firms

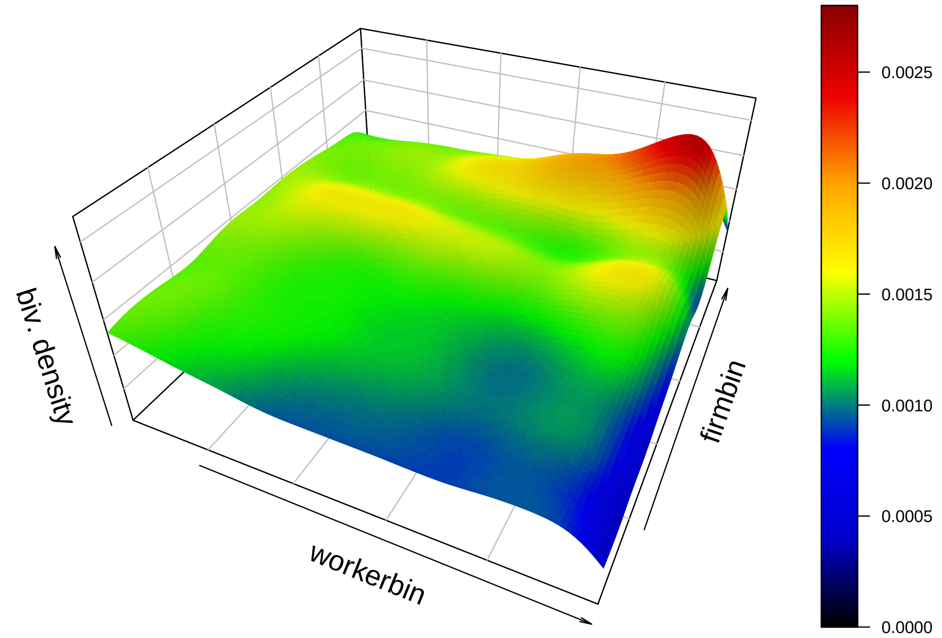


Notes: The six plots depict contours of the joint empirical distributions of firm years across combinations between the omega ranking (15 bins) and poaching index ranks (15 bins). In Panels (a) and (b), high-wage firms pay more than the grand mean of all firm-level mean wages and low-wage firms pay less. In Panels (c) and (d), the age of young firms is less than 15 years, old firms are 15 years and older. In Panels (e) and (f), small firms have less than 100 employees, large firms have more. Data sources: BHP, EP, BeH.

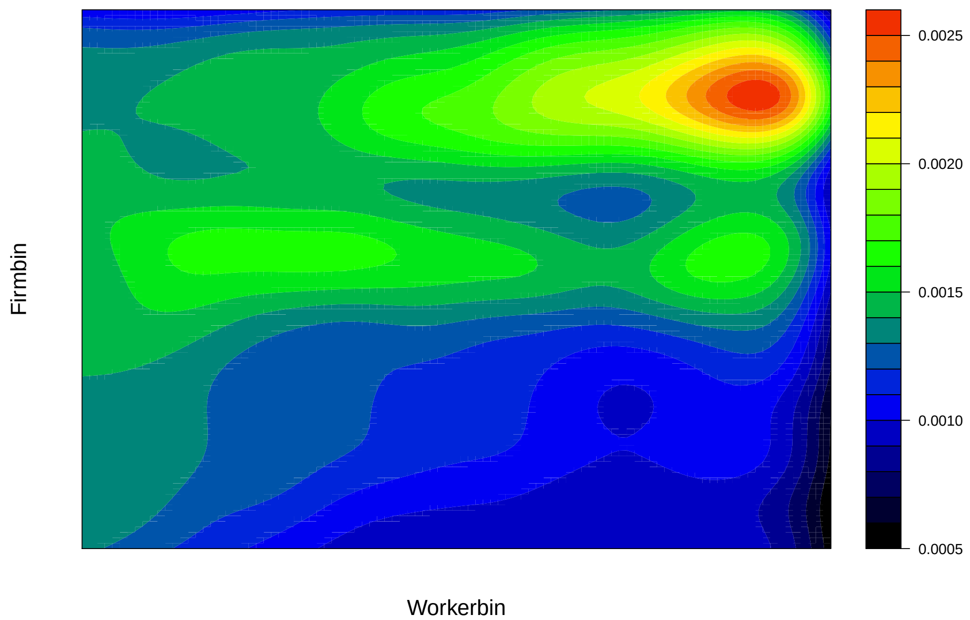
D.3 Joint Distribution of Matches

Figure D.3: Joint Distribution for all Worker-Firm Type Combinations (1998–2008)

(a) Joint Density



(b) Projection on the $(\bar{x}, \bar{\omega})$ plane

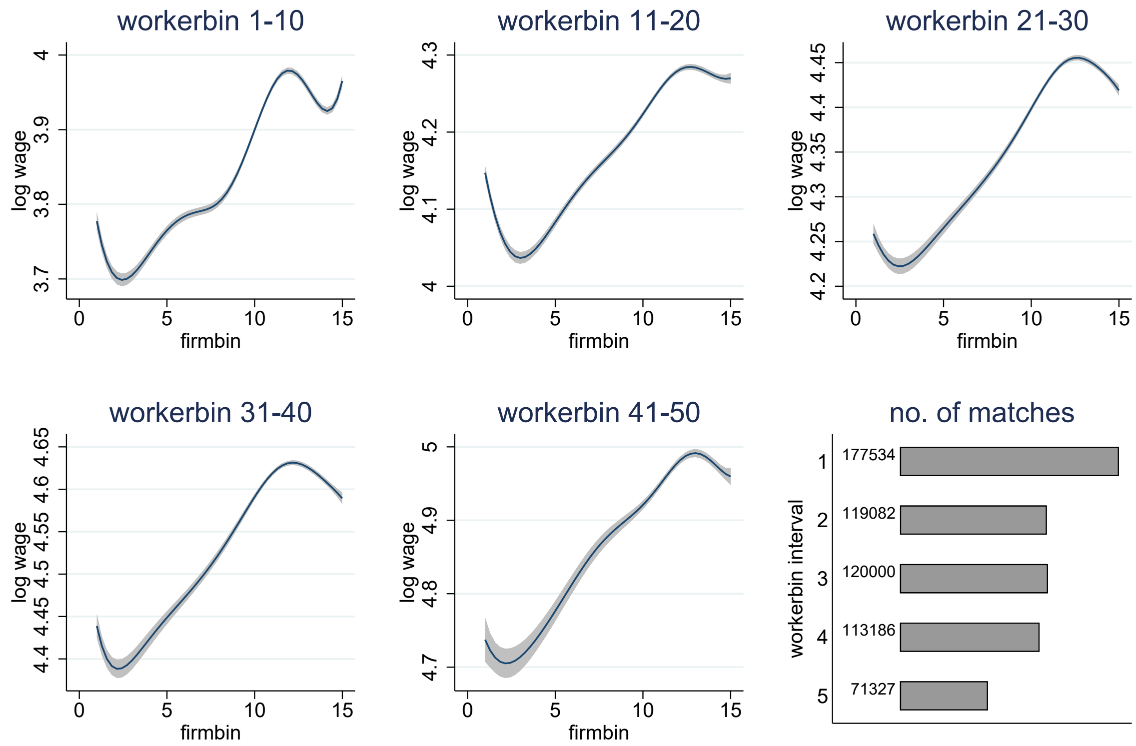


Notes: The plots shows estimate joint kernel density of matches and it projection on the $(\bar{x}, \bar{\omega}$ plane for all combinations of worker and firm types in the sample of all matches on a grid with dimensions 50×15 (#worker types $\times$ #firm types). Data sources: BHP, EP, BeH.

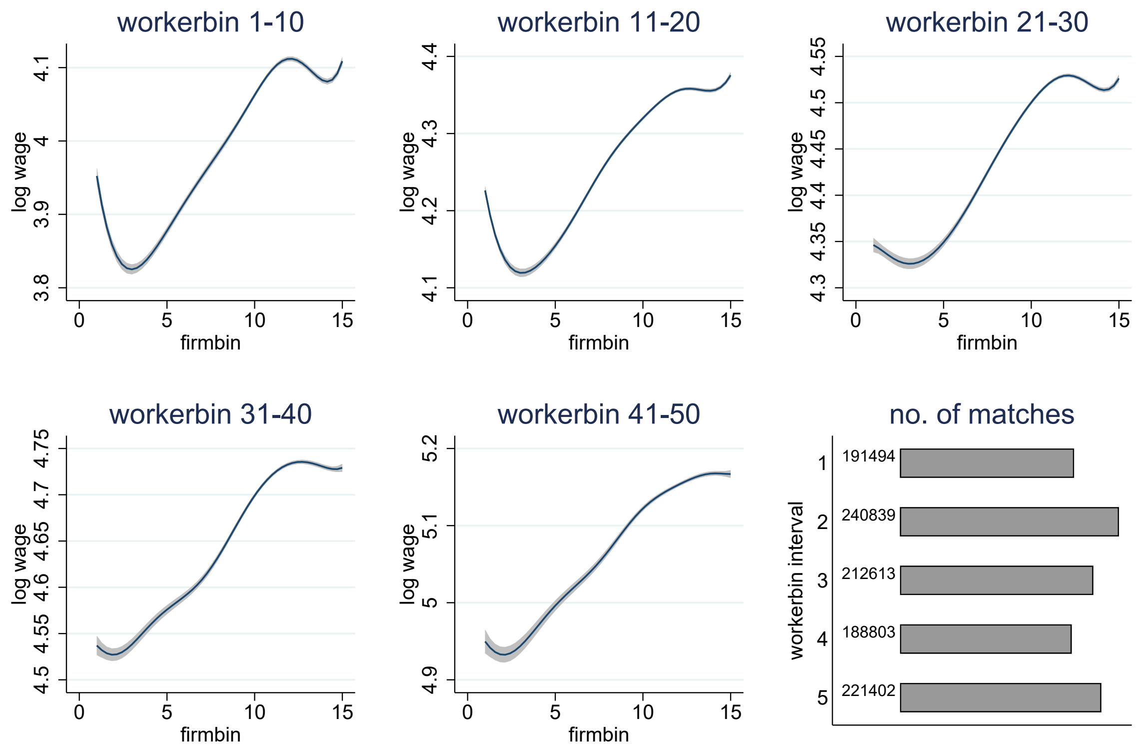
D.4 Wages and Transitions

Figure D.4: Wage-Productivity Profiles, new Matches

(a) Out of non-employment



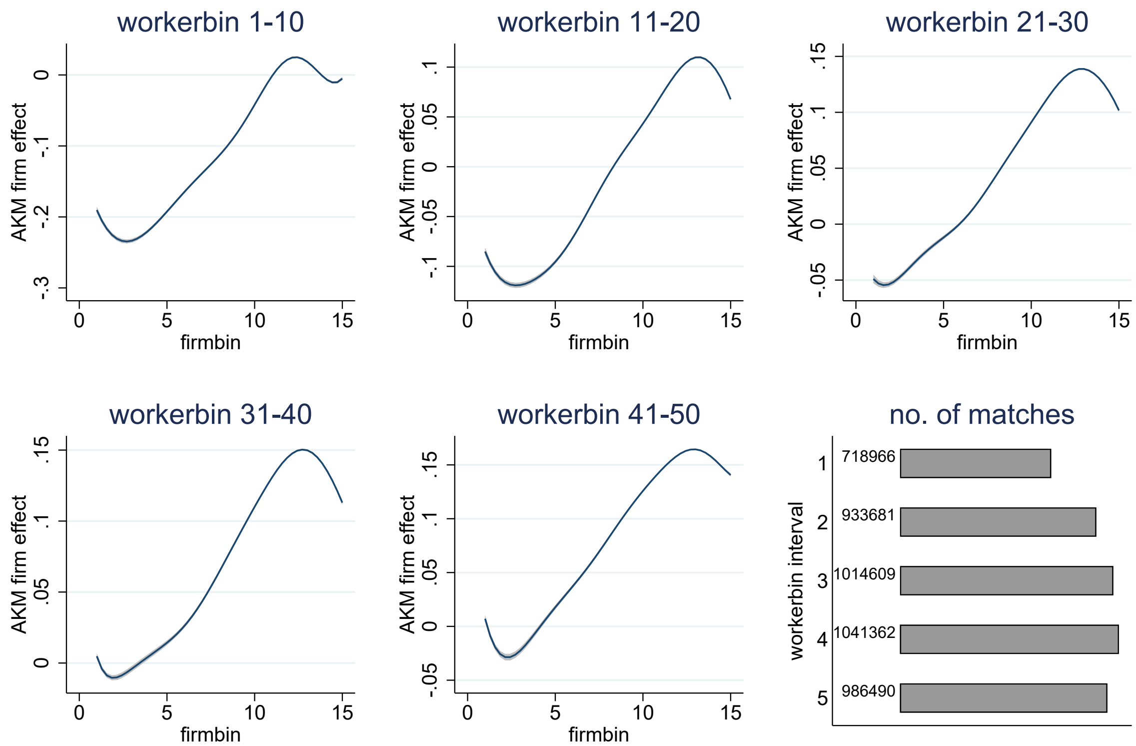
(b) Job-to-Job



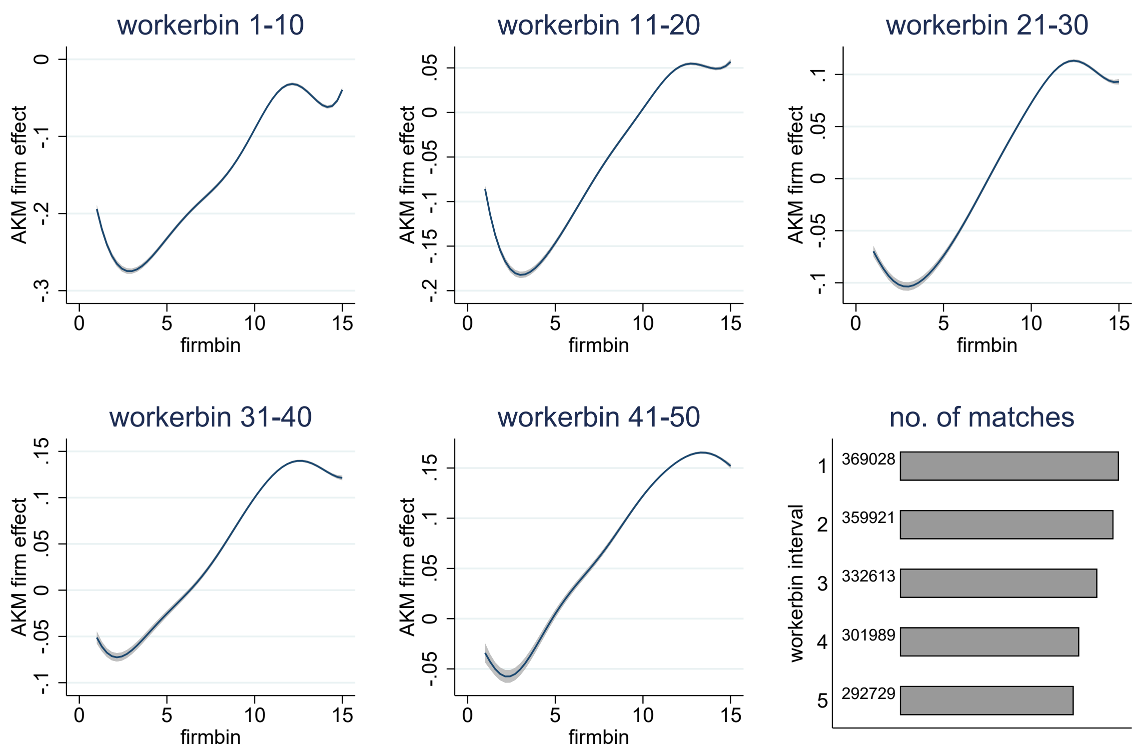
Notes: Plots show estimated wage-productivity profiles across firm bins for new matches out of non-employment and job-to-job. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.5: Wage-Productivity Profiles, Firm-Fixed Effects only

(a) All Matches



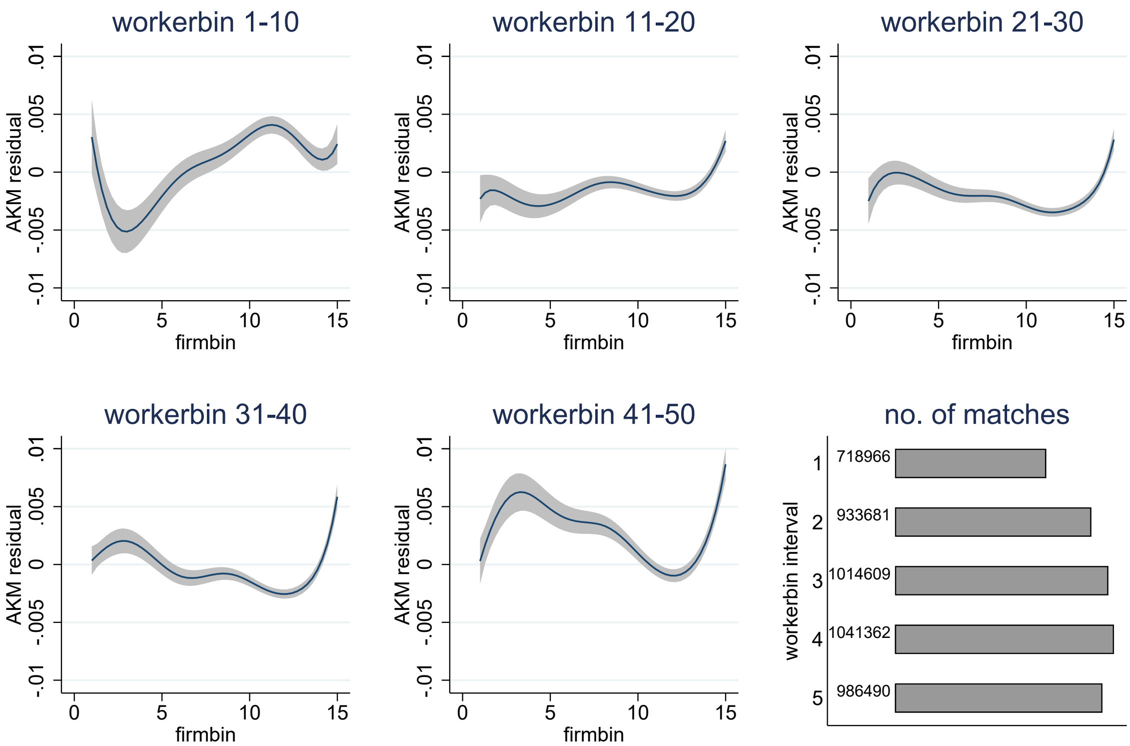
(b) New Matches



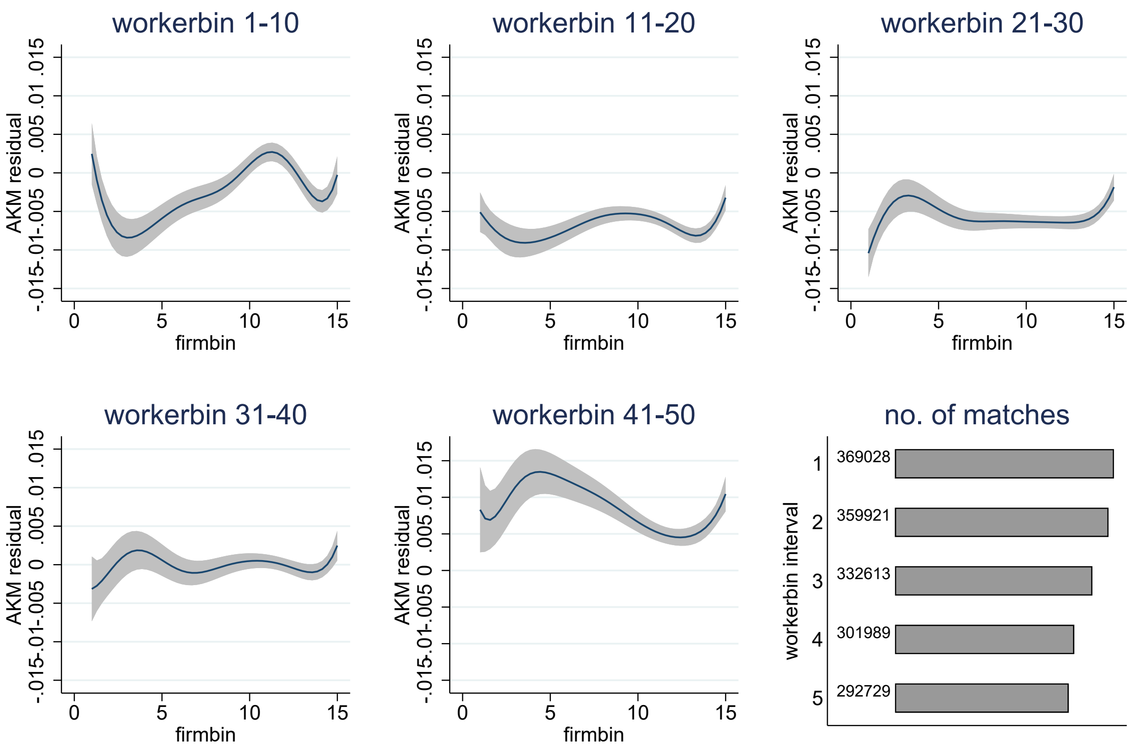
Notes: Plots show estimated wage-productivity profiles across firm bins for all matches and new matches when using AKM firm-fixed effects as the wage variable. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.6: Wage-Productivity Profiles, Residuals only

(a) All Matches



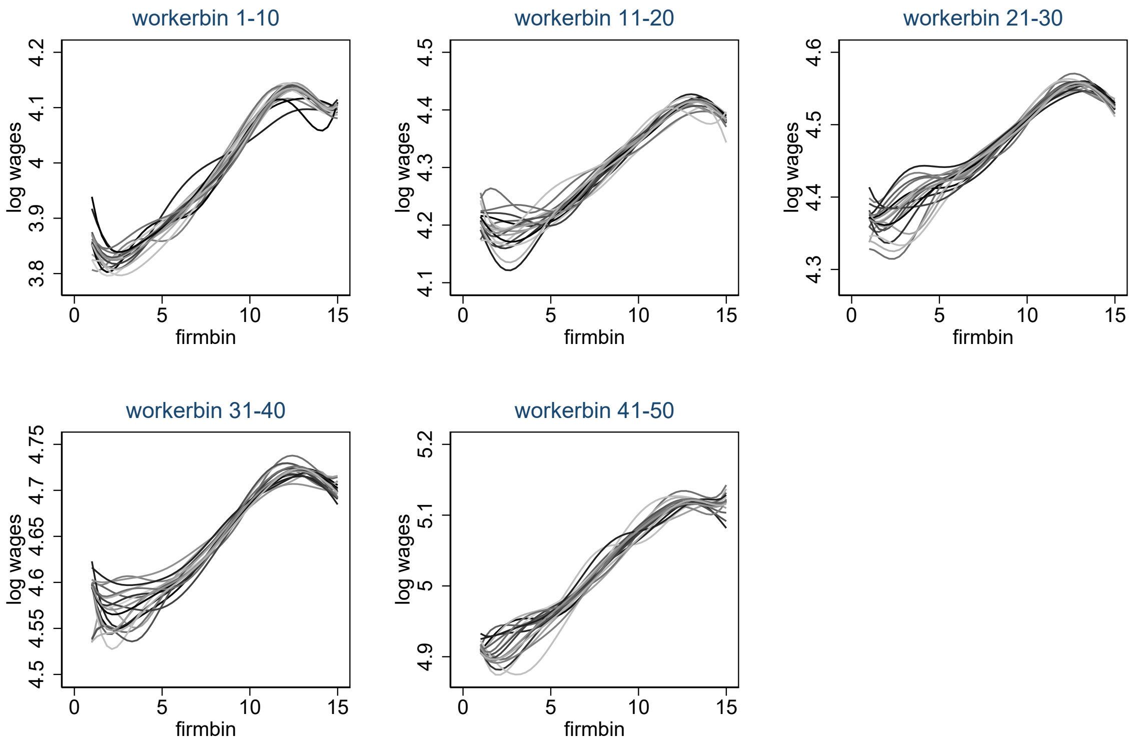
(b) New Matches



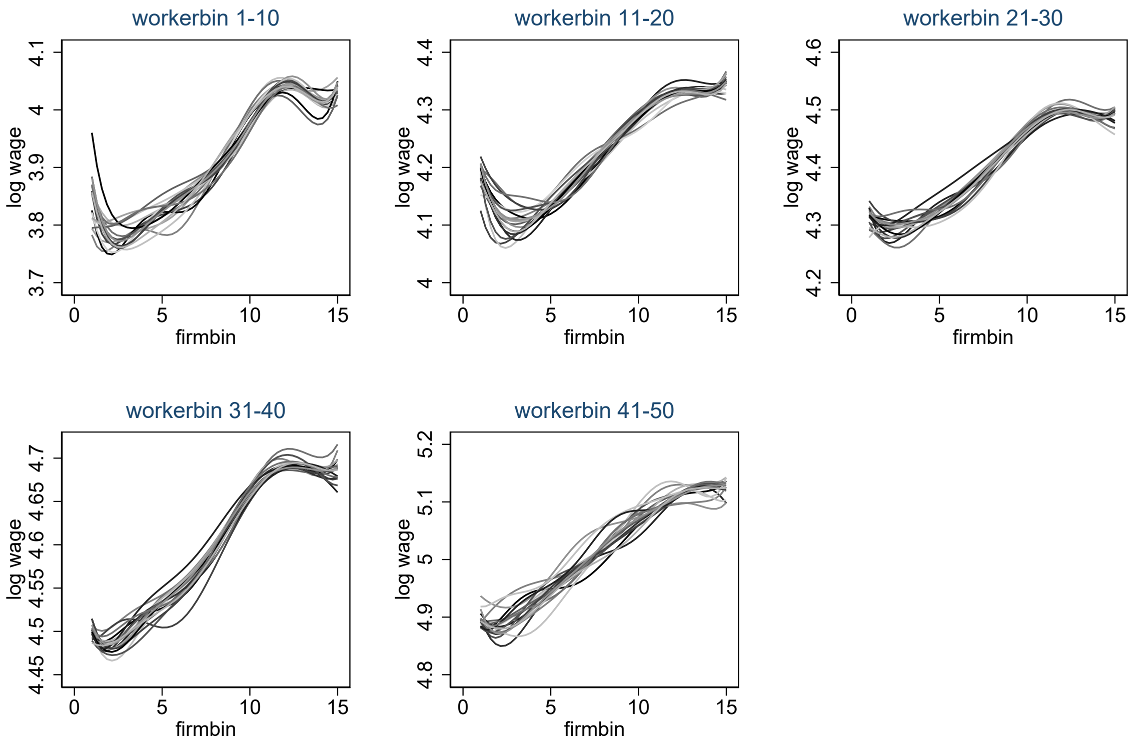
Notes: Plots show estimated wage-productivity profiles across firm bins for all matches and new matches when using AKM residuals as the wage variable. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.7: Wage-Productivity Profiles, Sub-Samples

(a) All Matches



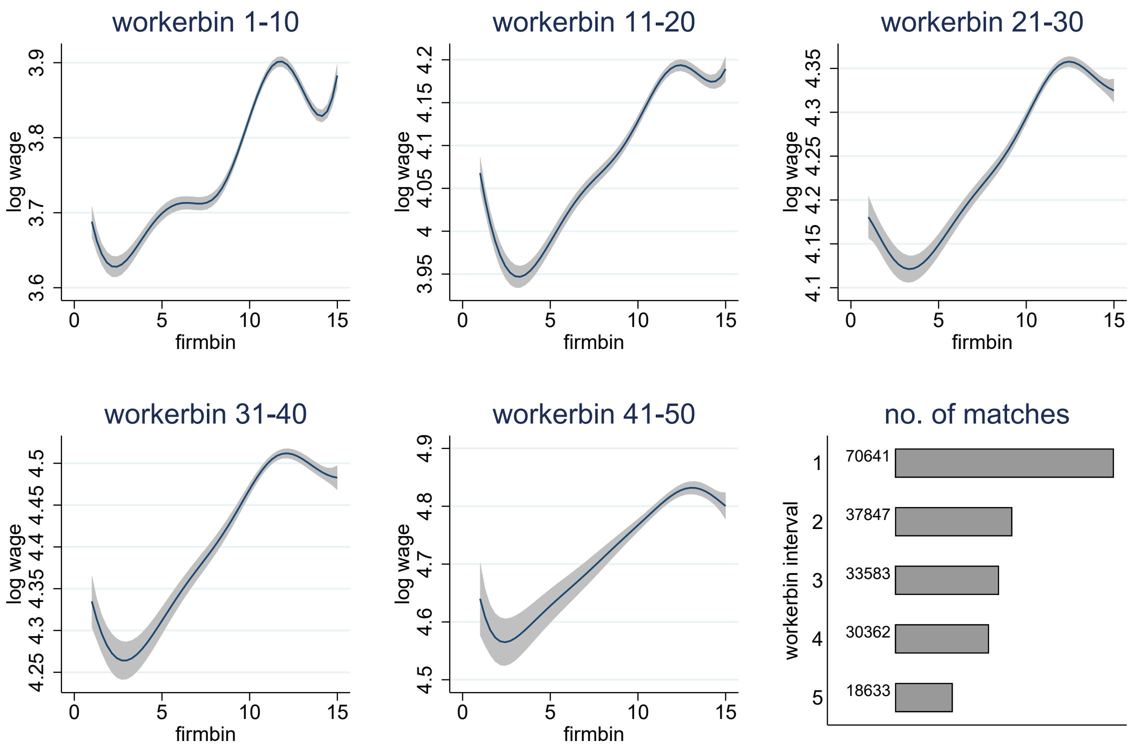
(b) New Matches



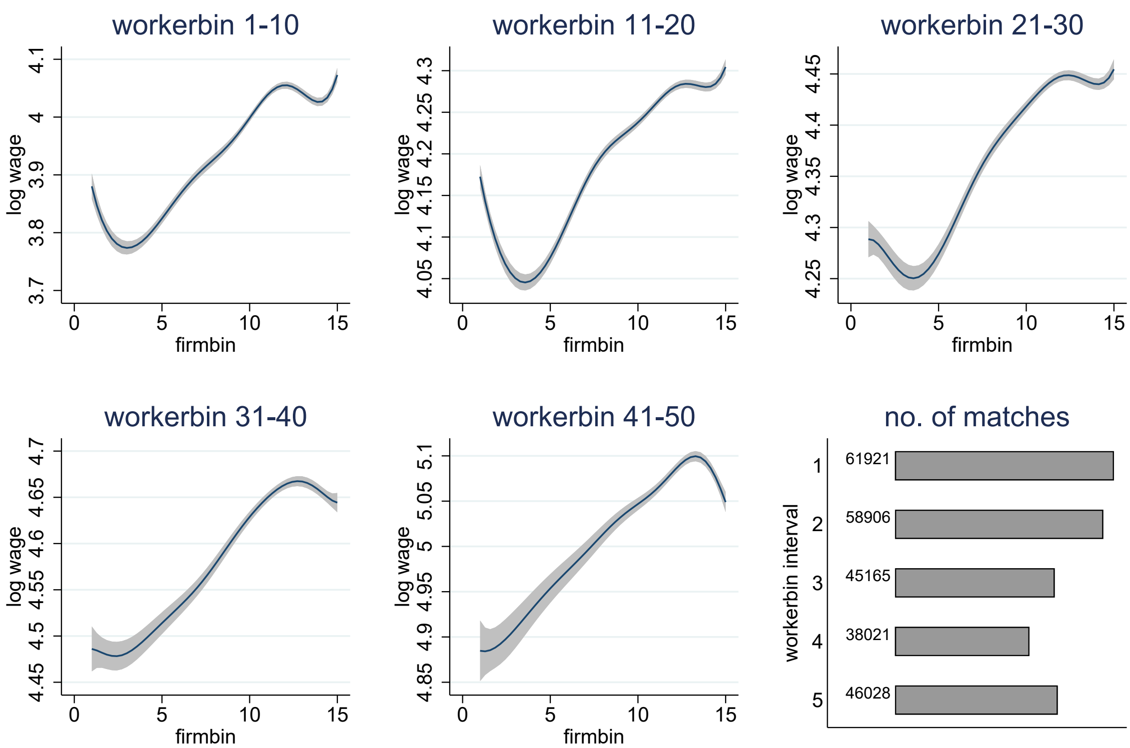
Notes: Plots show estimated wage-productivity profiles across firm bins constructed from 20 random sub-samples (with replacement, clustered by firm-years) from the original sample (size=N). The sub-sample size M randomly varies between $0.5 * N < M \leq N$. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.8: Wage-Productivity Profiles, new Matches, first Match-Year only

(a) Out of non-employment



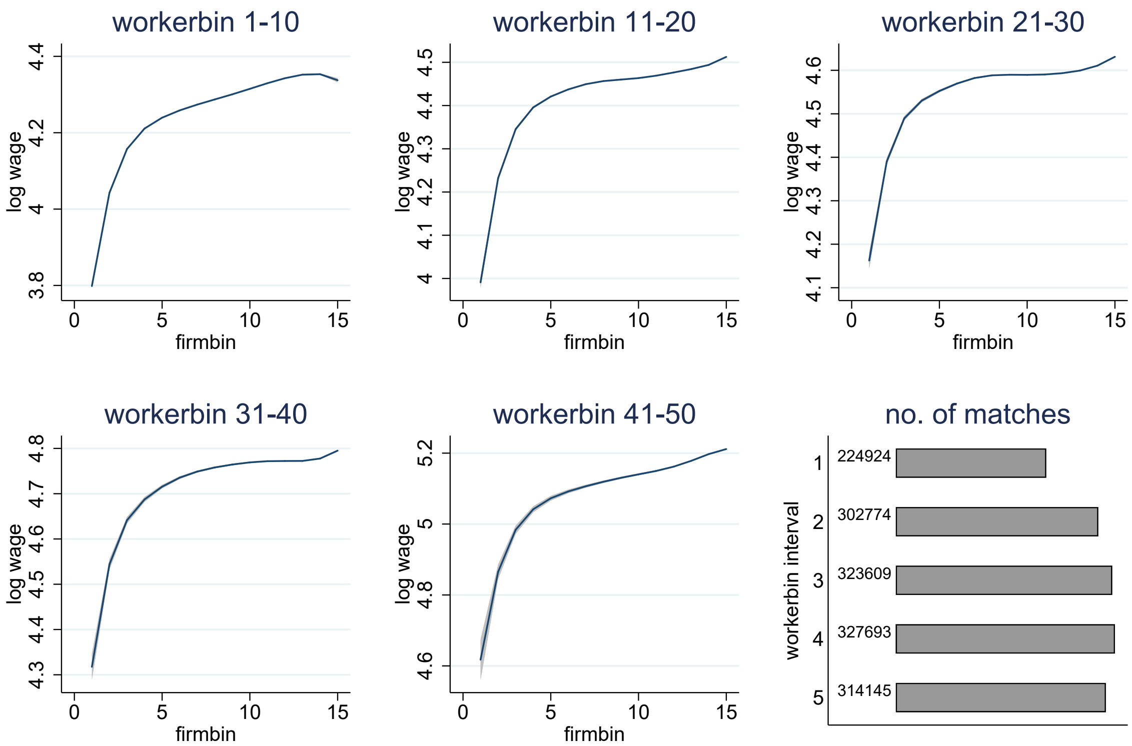
(b) Job-to-Job



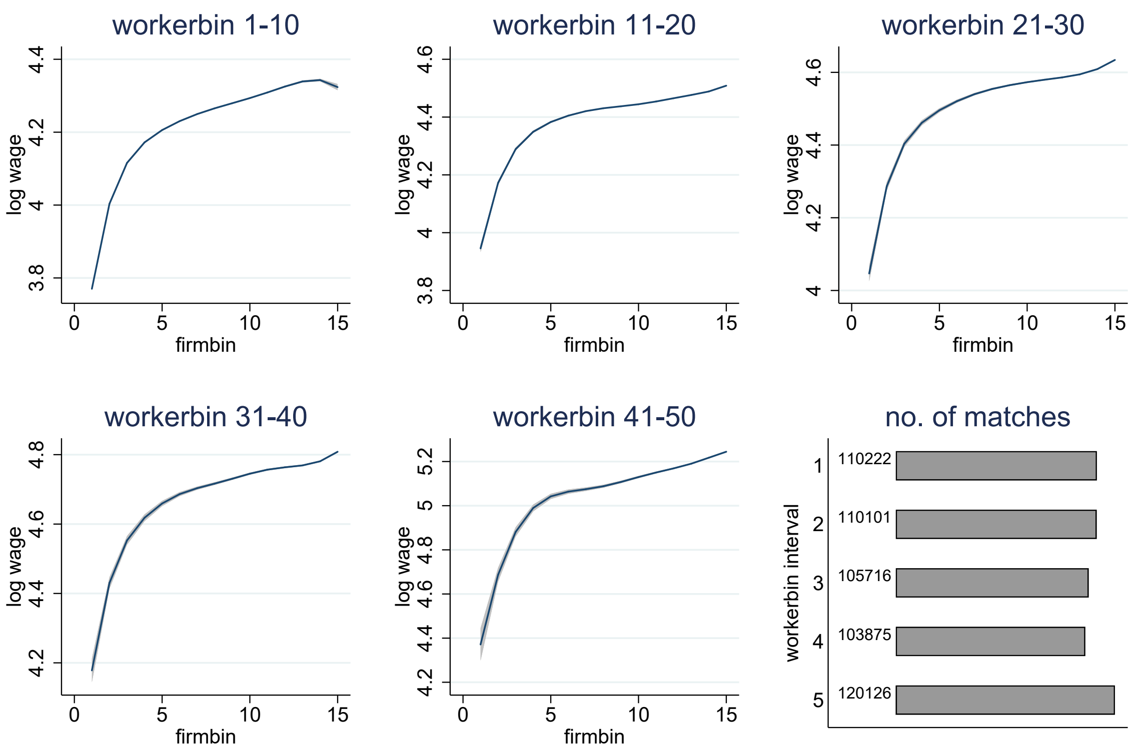
Notes: Plots show estimated wage-productivity profiles across firm bins constructed using only the first yearly wage observation for all matches and new matches to remove tenure effects. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.9: Wage Profiles with AKM-based Firm Types

(a) All Matches



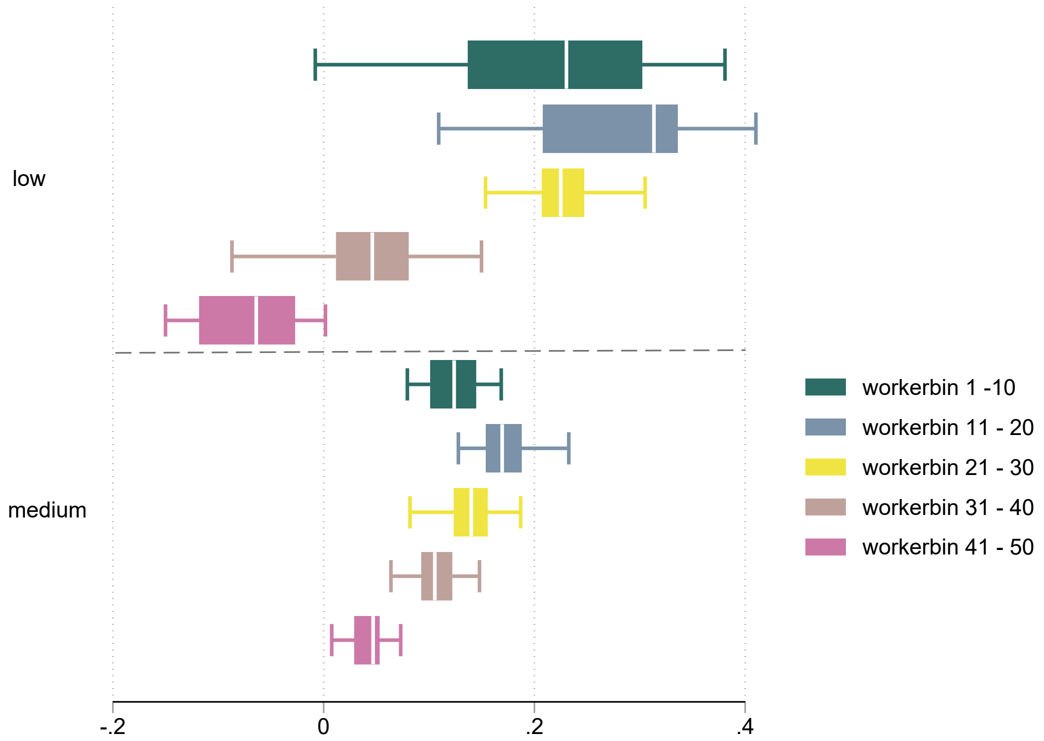
(b) New Matches



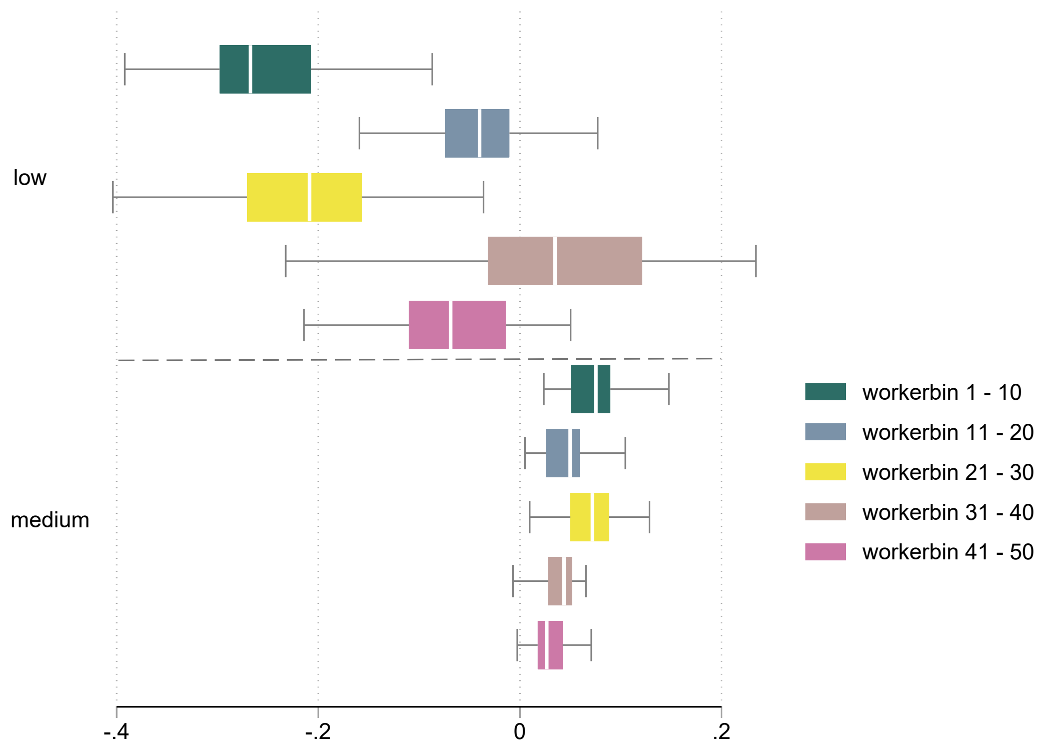
Notes: Plots show estimated wage profiles across firm bins constructed using AKM firm effects for all matches and new matches. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.10: Wage Changes for Observed Transitions, Sub-Samples

(a) To high



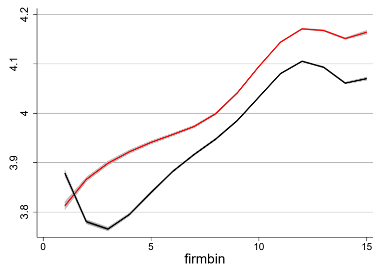
(b) From high



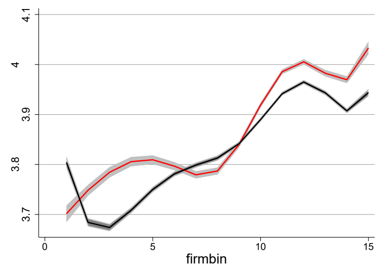
Notes: The plots show estimated coefficients and 95% confidence intervals (robust standard errors) from a linear regression of individual-level wage differences of transitioning workers on dummies for origin and destination firm bins using 20 random sub-samples (with replacement, clustered by firm-years) from the original sample (size=N). The sub-sample size M randomly varies between $0.5 * N < M \leq N$. The sub-samples consists of new matches (job-to-job switches, no intermittent non-employment spell) for five groups of worker types. The depicted coefficients are for transitions out of (Panel (a)) and into (Panel (b)) high-productivity firms (bins 13–15). The vertical axes capture destination/origin firm bin groups: low (bins 1–3) and center (bins 4–12). Data sources: BHP, EP, BeH.

Figure D.11: Changes of Mean Wages across Worker and Firm Types: 1998-2002 (red) vs. 2003-2008 (black)

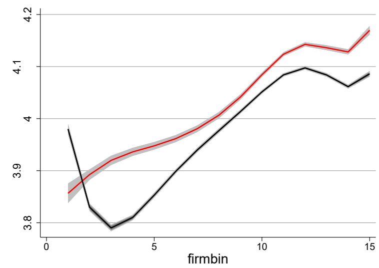
(a) Worker Bins 1-10, all matches



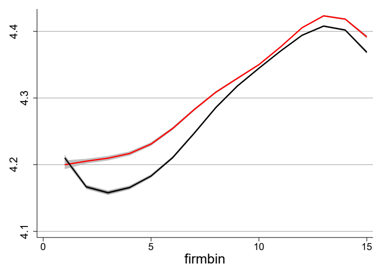
(b) Worker Bins 1-10, out of non-emp.



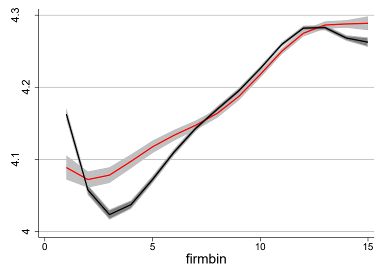
(c) Worker Bins 1-10, job-to-job



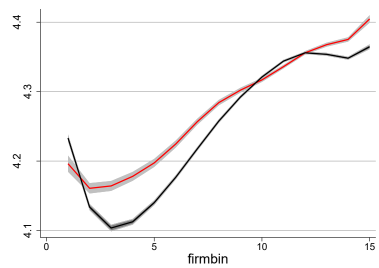
(d) Worker Bins 11-20, all matches



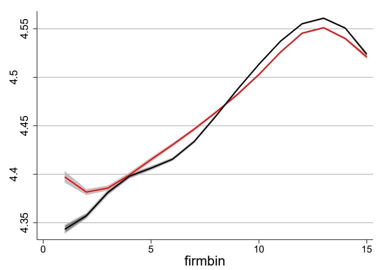
(e) Worker Bins 11-20, out of non-emp.



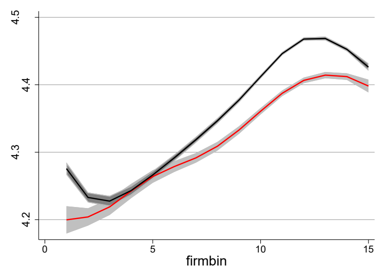
(f) Worker Bins 11-20, job-to-job



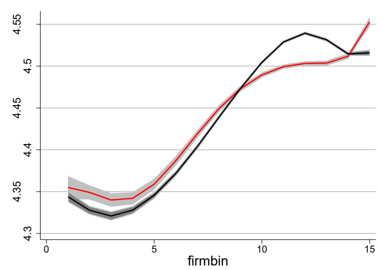
(g) Worker Bins 21-30, all matches



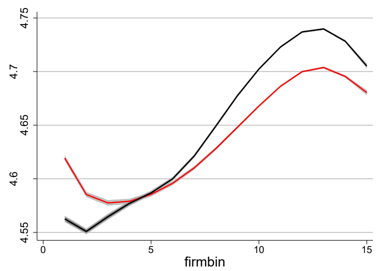
(h) Worker Bins 21-30, out of non-emp.



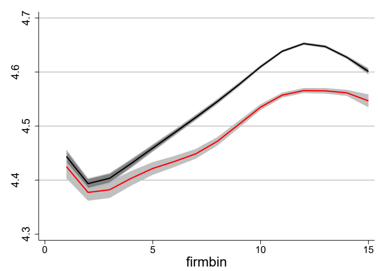
(i) Worker Bins 21-30, job-to-job



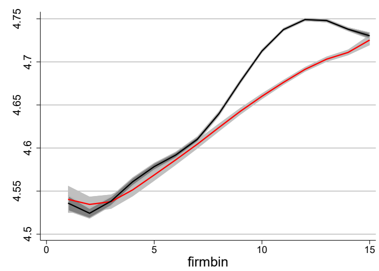
(j) Worker Bins 31-40, all matches



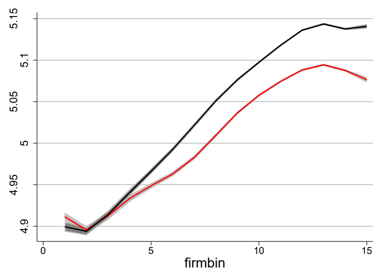
(k) Worker Bins 31-40, out of non-emp.



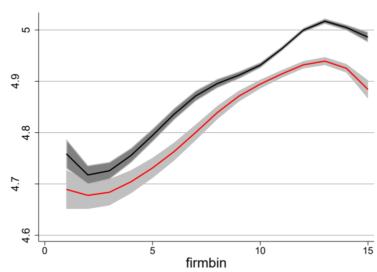
(l) Worker Bins 31-40, job-to-job



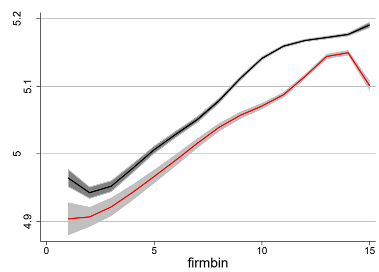
(m) Worker Bins 41-50, all matches



(n) Worker Bins 41-50, out of non-emp.



(o) Worker Bins 41-50, job-to-job



Notes: Plots show estimated wage profiles for two time periods for grouped worker bins across firm bins for all matches, new matches out of non-employment, and job-to-job moves. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.