

Firm Productivity, Wages, and Sorting*

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Abstract

We study the link between firm productivity and the wages that firms pay. Guided by a model of labor market sorting with large firms, we infer firm productivity by estimating firm-level production functions, recognizing that worker ability and firm productivity may interact at the match level. Using German data, we find that the most productive firms do not pay the highest wages. Worker transitions from high to medium productivity firms are on average associated with wage gains. Productivity sorting, that is, the sorting of high-ability workers into high-productivity firms, is less pronounced than sorting into high-wage firms.

Keywords: Assortative Matching, Labor Market Sorting, Wage Inequality, Job Mobility, Unobserved Heterogeneity, Firm Productivity, Production Function Estimation

JEL Classifications: J24, J31, J40, J62, J64, L25

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1 Introduction

Using the seminal Abowd et al. (1999) (AKM) approach to decompose wages into worker- and firm-specific components, Card et al. (2013) (CHK) and Song et al. (2019) show that increases in wage inequality are largely driven by increases in the sorting of high-wage workers into high-wage firms. Presumably, the wages that these firms pay are related to their productivity. However, the mapping from productivity to wages is far from clear. On the one hand, highly productive firms may share profits with their workers and pay high wages. On the other hand, wages could be lower due to labor market imperfections or compensating differentials. Less productive (perhaps younger) firms, in turn, may have to pay relatively high wages to retain workers or to expand their workforce if workers have valuable outside options.

The primary contribution of this paper is to shed light on the link between firm productivity and the wages that firms pay. To this end, we estimate worker and firm-specific wage components following AKM and firm productivity following Ackerberg et al. (2015) (ACF). We present a parsimonious search-and-matching model to elucidate the minimal set of assumptions necessary to combine the AKM and ACF approaches to study sorting. The model is compatible with both the discrete-time model of dynamically optimizing firms underlying ACF and the log-linear wage equation underlying AKM. It features matching frictions, decreasing returns, multiworker firms, intrafirm bargaining (following Cahuc et al., 2008), as well as firm heterogeneity and worker-firm complementarities.¹

Our empirical analysis proceeds in two steps. First, we estimate an AKM model for the full population of private sector employees in Germany and merge the estimated AKM wage components with detailed German establishment survey data.² Second, we estimate establishment-level production functions and calculate firm productivity using the refined control-function approach proposed by ACF. Based on the model's wage equation, we separate firm productivity from the effect of heterogeneous worker ability on output, using the estimated AKM effects for all workers at the surveyed establishments.

We present empirical results concerning (i) productivity sorting, i.e., the sorting between worker ability (inferred from AKM worker effects) and firm productivity (inferred using ACF); (ii) the link between firm productivity and wages; and (iii) the role of productivity sorting for rising wage inequality.

(i) Productivity sorting is less pronounced than wage sorting; the average rank correlation between wage-based worker types and productivity-based firm types is 0.07, while the correlation between AKM worker and firm fixed effects is 0.27. Moreover, the AKM correlation increases over time, while our measure of productivity sorting remains rel-

¹Theoretical sorting models highlight complementarities in production as the primary determinant of positive sorting (Becker, 1973; Shimer and Smith, 2000).

²We do not observe firms in the legal sense; rather, we observe establishments, i.e., individual production units. We use the terms “firm” and “establishment” interchangeably throughout the paper.

atively flat. Studying the dynamics of productivity sorting reveals that this flat trend is the result of two developments that extend in opposite directions, namely, increasing productivity sorting of medium-ability workers into medium-productivity firms and decreasing productivity sorting of both high-ability workers into high-productivity firms and low-ability workers into low-productivity firms.

(ii) The empirical relationship between wages and firm productivity helps to explain these developments. It follows a hump-shaped pattern. Specifically, the most productive firms pay lower wages than somewhat less productive firms, and this tendency has become more pronounced over time. Consistently, worker transitions from high to medium productivity firms are on average associated with wage gains. Moreover, the most productive firms are not the largest in terms of headcount, capital stock, or their total wage bill, despite their very high labor productivity (value added per worker). Taken together, findings (i) and (ii) suggest that increasing wage sorting could be accompanied by a decrease in the allocative efficiency of the labor market.

The hump-shaped, nonmonotonic relation between wages and productivity we document poses a challenge for widely used structural models of wage dispersion. In the parsimonious model that we use to outline the assumptions underlying our empirical strategy, wages are monotone in productivity. To make progress toward replicating the non-monotonic relation with structural models, we discuss several possible model extensions. We evaluate their potential to break the tight link between wages and productivity and comment on their compatibility with the AKM approach (in the sense of retaining a log-linear wage equation). We consider on-the-job search, general production functions, and multidimensional firm and worker heterogeneity (firms: productivity and non-wage characteristics of jobs (amenities); workers: ability and preferences/networks).

(iii) To understand the role of productivity sorting in rising wage inequality, we decompose wage variance into variance both within and between establishments. The contribution of the between-firm component rose by almost 10% in Germany between 1998 and 2008. This is in line with the findings of Song et al. (2019) for the U.S. Quantitatively, between-firm inequality is comparable in magnitude to the relatively stable within-firm component of wage dispersion. However, this picture changes when we decompose the wage variance using the estimated firm productivity and worker ability types. We find that the share of the overall wage variance explained by the between-firm productivity type variance is low and increases by only approximately 4% over time. Its overall contribution to inequality is dwarfed by the variance shares of the within-firm productivity type and between-worker ability type variances. We conclude that productivity sorting contributes less to rising wage inequality than wage sorting.

Contribution to the Literature

We show how the wage determination mechanism of a search-and-matching model with multiworker firms, intrafirm wage bargaining, decreasing returns, and worker-firm complementarities can be used to facilitate the estimation of unobserved firm productivity. We build on Cahuc et al. (2008), who embed the Stole and Zwiebel (1996) intrafirm bargaining framework into the canonical search-and-matching model. With multiworker firms, our approach differs from existing structural work in the sorting literature that assumes one-worker-one-firm matches. Under this assumption, the focus lies on worker *quality*, while the *quantity* dimension (i.e., the number of workers) is not considered.³

Eeckhout and Kircher (2018) relax the one-worker-one-firm assumption and study both the quantity and the quality dimensions of production. The firm must decide which worker type(s) to hire and, additionally, how many workers of each type to employ. The main finding is that firms optimally hire multiple workers of exactly one type. This result holds for both frictionless matching and competitive search. In this paper, we are interested in the empirically relevant case of firms that are simultaneously matched with multiple worker types. Thus, we focus on a model with random search, nondegenerate matching sets, and a production function that is geared toward our empirical approach; in contrast, Eeckhout and Kircher (2018) study general production functions.

Our findings contribute to the empirical literature on wage dispersion. The aforementioned study of Germany by CHK and that of the U.S. by Song et al. (2019) follow the AKM approach. These studies decompose wage dispersion into the contributions of unobserved worker ability, firm wage premia, and wage sorting in the labor market. Wage sorting measures the extent to which workers who receive high wages are matched with firms that pay high wages. We show that how one measures firm heterogeneity, that is, by the wages that firms pay or by firm productivity, affects this kind of decomposition. In our data, firms with the highest estimated productivity do not pay the highest wages. For this reason, sorting into high-productivity firms is quantitatively less important for rising dispersion than is wage sorting.

The main difference between AKM-inspired analyses of labor market sorting and structural analyses is typically the implied (non)monotonicity of the wage equation. Due to complementarities, wages are not a monotonic function of firm type in many theoretical models (e.g., Gautier and Teulings, 2006; Eeckhout and Kircher, 2011; Lise et al., 2016; Hagedorn et al., 2017; Lopes de Melo, 2018). However, the log-linear AKM wage equation implies that wages are strictly increasing in the firm fixed effect for a given worker. Abowd et al. (2018) show that it is possible to find common ground. These authors formally relate the equilibrium quantities derived from the directed-search sorting model

³Examples are Shimer and Smith (2000), Atakan (2006), Lise et al. (2016), and Lise and Robin (2017). A notable exception is Bagger and Lentz (2019). Here, multiworker firms are present, but production is linear, and firm size is limited by search frictions, not decreasing returns.

of Shimer (2005) to the worker and firm heterogeneity components that can be identified by using the AKM approach. We follow this example and discuss how the wage equation implied by our model maps onto the AKM model and which assumptions are critical for this mapping. Moreover, our empirical results contribute to the discussion about wage (non)monotonicity. We show how wages evolve across firm productivity types and find quantitatively important deviations from monotonicity at high- and low-productivity firms. Across medium-productivity firms, wages largely evolve in a monotonical manner.

The literature has developed multiple ways to capture firm heterogeneity when analyzing labor markets, and the productivity-based firm ranking we propose extends this toolbox. Bagger et al. (2014a) also build on Cahuc et al. (2008) and estimate firm-level production functions with heterogeneous labor inputs to study wage dispersion using Danish data. Hagedorn et al. (2017) rank firms based on the value of a vacancy, which can be identified from wage data in the context of their model. Bartolucci et al. (2018) use balance sheet data for a set of Italian firms and rank them by observed profits. Taber and Vejlin (2020) and Bagger and Lentz (2019) rank firms by the share of workers they poach from other firms. Sorkin (2018) ranks firms by amenities based on a revealed-preference argument. Haltiwanger et al. (2018) and Bertheau et al. (2020) study worker flows using gross output and value added per worker as firm quality measures.

Another related body of literature focuses on rent sharing and imperfect competition in the labor market. Card et al. (2018) show that rent-sharing elasticities are largely driven by worker quality (measured by AKM worker effects). Nevertheless, they find a sizable positive correlation between value added per worker and the AKM firm effect, which might be driven by compensating differentials. Building on this work, Lamadon et al. (2022) estimate a model with compensating differentials, worker preference heterogeneity, and monopsony power on U.S. data. In line with our findings, they show that high-productivity firms pay relatively low wages; the reason is that these firms compensate their workers disproportionately with amenities. We will return to this idea in the discussion of potential model extensions.

2 Model

This section introduces a model of multiworker firms with decreasing returns, intrafirm wage bargaining, worker-firm complementarities, and matching frictions. This parsimonious model is designed to elucidate the minimal set of assumptions necessary to combine AKM and ACF. Specifically, we show how the model's wage equation maps onto the log-linear AKM wage equation. Using this mapping and estimated AKM wage components, we construct model-consistent firm-level labor inputs for production function estimation. We highlight the assumptions under which the model is compatible with the model of dynamically optimizing firms underlying ACF. Model details are relegated to Appendix A.

Consider an economy in which atomistic firms produce a numeraire good using multiple heterogeneous labor inputs. Worker heterogeneity is summarized by $n > 1$ ability types indexed by x . Worker types are time-invariant. Firm productivity, denoted as Ω , may change due to idiosyncratic firm-level shocks that evolve according to a stochastic process characterized by the conditional CDF $G(\Omega'|\Omega)$. Workers and firms meet randomly. Conditional on meeting, a match is not guaranteed because the match surplus may be too low. Appendix A.1 provides the details of the assumed matching mechanism.

To set up the firm's problem, we build on Cahuc et al. (2008), who generalize the canonical search-and-matching model to allow for multiworker firms with heterogeneous labor inputs and decreasing returns, strategic interactions, and intrafirm bargaining in the spirit of Stole and Zwiebel (1996). We add heterogeneous firm productivity and a production structure that is consistent with positive worker-firm sorting. To facilitate a simple and transparent link to the empirical models that we use (i.e., AKM and ACF), we abstract from complementarities between worker types within the same firm.

Furthermore, we make six simplifying assumptions. First, workers and firms are risk neutral. Second, both worker and firm heterogeneity are one-dimensional.⁴ Third, worker ability and firm productivity are known to all market participants and are cardinally measurable. Fourth, infinitely lived workers supply one infinitesimally small unit of labor (no extensive/intensive margin choice) so that labor input is a continuous variable.⁵ Fifth, we abstract from capital inputs when discussing the model.⁶ Sixth, we present the model in discrete time because the ACF approach to production function estimation also relies on a discrete-time model of dynamically optimizing firms. Time indices are omitted.

A general concave, firm-level production function is as follows:

$$Y = F(L, \Omega), \quad (1)$$

where Y is value added and $L = \sum_x xL_x$ is a scalar composite labor input in units of worker ability. It combines all heterogeneous labor inputs L_x , which is the number of type- x workers employed by the firm. Ω is the current productivity of the firm. Our focus lies on worker-firm sorting. Therefore, we assume that output is (log-)supermodular at the match level; i.e., the complementarity between firm productivity Ω and worker ability determines the contribution of every single match to firm-level output. The marginal product of an additional unit of worker ability is firm-specific. We interpret firm productivity as a “nonrival” resource; i.e., we do not consider the span-of-control problem in the optimal allocation of resources to heterogeneous workers (Eeckhout and Kircher, 2018).

⁴For recent explorations of sorting with multidimensional characteristics, see Lindenlaub (2017), Lindenlaub and Postel-Vinay (2020), and Lise and Postel-Vinay (2020). We discuss how allowing for multidimensional firm/worker heterogeneity would affect our empirical approach in Section 7.

⁵We take part-time labor into account when we estimate the production functions in Section 4.2.

⁶Cahuc et al. (2008) explicitly consider a predetermined capital stock; thus, our model could be extended in this direction. ACF assumes a similar capital accumulation mechanism; see Section 4.2.

A simple production structure in line with our assumptions is as follows:

$$F(L, \Omega) = \left(\sum_{x=1}^n (x \times \Omega) L_x \right)^{\beta_l}, \quad (2)$$

where $0 < \beta_l < 1$ is the output elasticity of the composite labor input. This production function is (weakly) log-supermodular at the match level, which is in line with the sufficient conditions for positive assortative matching (PAM) derived by Shimer and Smith (2000).⁷ Under our assumptions, worker ability units are perfect substitutes at the firm level. That is, output depends on Ω and the number of efficiency units of labor employed. The marginal product of an additional unit of type- x labor is as follows:

$$F_x(L, \Omega) = \frac{\partial F(L, \Omega)}{\partial L_x} = x \beta_l \Omega^{\beta_l} L^{\beta_l - 1}. \quad (3)$$

Note that worker ability x scales the marginal product at an (L, Ω) firm. We make use of this property below. Furthermore, the marginal product is increasing in firm productivity and the output elasticity of labor but decreasing in the total composite labor input L due to decreasing returns.

As in Cahuc et al. (2008) and ACF, employment is a state variable. The firm's problem is to optimally choose how many vacancies to post given its expected profits from hiring. We assume that vacancies cannot be targeted to specific worker types and are subject to a flow cost c , which could depend on the firm type. In Appendix A.2, we solve the firm's problem and derive the relevant optimality conditions in the steady state.

The bargained wage satisfies the standard Nash sharing rule. The firm's surplus consists of its marginal profit from hiring an additional worker of type x (equation A.11). Its threat point is to renegotiate wages with all other employees (Stole and Zwiebel, 1996). The worker's surplus is the difference between the option values of employment and unemployment (equations A.14 and A.15, respectively). Wage (re)negotiations happen instantaneously so that firm-level employment remains fixed. In Appendix A.3, we show that the outcome of intrafirm wage bargaining in the model is described by the following differential equation:

$$w(x, L, \Omega) = \alpha \left(F_x(L, \Omega) - L \frac{\partial w(x, L, \Omega)}{\partial L} \right) + (1 - \alpha)(1 - \beta)U(x). \quad (4)$$

This is a discrete-time version of the wage bargaining outcome derived by Cahuc et al. (2008) for the “single labor case”. We can utilize their result due to our assumption of

⁷Shimer and Smith (2000) establish the existence of an equilibrium in this environment. PAM arises with a log-supermodular match-level production function, while log-submodularity leads to negative assortative matching (NAM). In this empirical paper, we do not attempt to generalize the Eeckhout and Kircher (2018) large-firm sorting conditions for random search models.

perfectly substitutable ability units. α is the workers' bargaining power parameter, and β is the common discount factor. The first term in parentheses shows that the wage of a type- x worker at an (L, Ω) firm is a function of the worker-firm-specific marginal product, $F_x(L, \Omega)$. The second term captures the inframarginal effect that hiring the marginal worker has on all other workers' wages. This mirrors the finding that firms can reduce incumbent workers' wages by increasing employment in the presence of decreasing returns.⁸ Absent complementarities between worker types, the inframarginal adjustment reflects decreasing returns only and is unambiguously negative. Finally, the third term in equation (4) captures the worker-specific outside option, which we will return to below. The solution⁹ to the differential equation (4) is as follows:

$$w(x, L, \Omega) = (1 - \alpha)(1 - \beta)U(x) + \int_0^1 z^{\frac{1-\alpha}{\alpha}} F_x(Lz, \Omega) dz, \quad (5)$$

which indicates that the wage depends on the worker's outside option and an integral expression that combines the worker's marginal product and the inframarginal effect. The latter is weighted by the worker's bargaining power and decreases in the distance to the margin. The model's wage equation makes clear that the wages that firms pay are not directly informative about firm productivity.

2.1 From Theory to Estimation

To empirically recover Ω , we propose a two-step approach that combines techniques from the empirical wage dispersion and industrial organization (IO) literatures.

First, we estimate the log-linear AKM model with German matched employer-employee data; see Section 3.3. The estimation yields, for all individual workers and firms in the largest connected set, worker fixed effects, firm fixed effects, and a residual. There is a direct correspondence between the AKM wage components and the wage equation in our model, which we discuss below.

Second, to estimate the production function, we merge rich survey data on input and output for a representative sample of firms with the estimated AKM wage components for their full workforce and use them to predict firm-level labor inputs (wage bills). Due to our model's separability property, the worker fixed effect captures the full effect of worker ability on wages. Thus, this wage component can be used to separate firm productivity from the effect of worker ability on output in the production function. We also include the effects of observable worker characteristics (not included in our model). Moreover, we vary the way in which we include the firm fixed effects, the AKM residuals, or both in the

⁸Cahuc et al. (2008) allows for unrestricted substitutability/complementarity patterns between worker types. In this case, the effects on coworker wages can be either positive or negative. Firms may strategically over/underemploy specific worker types depending on their contribution to the total wage bill. Smith (1999) studies the efficiency of job creation in such a setting.

⁹We follow Stole and Zwiebel (1996) and Cahuc et al. (2008). Details are relegated to Appendix A.3.

labor input. This approach allows us to investigate the extent to which the firm-specific and time-(in)variant wage components matter for the inferred productivity measure.

The model's wage equation maps onto the AKM model as follows. First, consider worker fixed effects. Both the outside option and the integral expression in (5) are functions of worker ability. Their effects on wages are absorbed by the AKM worker fixed effect when both wage components are linear in regard to worker ability. Given our production structure, the integral expression is indeed linear in regard to worker ability x because worker ability merely scales the marginal product of labor. This feature is crucial for the applicability of AKM. Plugging in (3) yields the following:

$$w(x, L, \Omega) = (1 - \alpha)(1 - \beta)U(x) + x \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz, \Omega)}{Lz} dz, \quad (6)$$

where x can be written in front of the integral sign. We show in Appendix A.5 that the worker's outside option in the first term is also linear with respect to x under two additional assumptions. First, the worker's flow value of unemployment, denoted as $b(x)$ in the model, must be proportional to x . This is a standard assumption. Second, matching sets must cover the whole type space; that is, there are no unacceptable combinations of worker and firm types. An implication of this assumption combined with our matching technology is that all worker types are unemployed for the same (expected) duration. In Section 4.1, we verify that this assumption holds in our data (conditional on observables). Another implication is that there is no endogenous sorting in the model, despite the complementarity in the production function.

Second, consider firm fixed effects. How the integral expression in (6) is captured by the AKM regression depends on the underlying data-generating process (DGP). For the sake of argument, suppose that our wage equation describes reality without error. The economy is in a steady state, and firm productivity does not change over time. In this case, (a log-affine transformation of) the firm-specific integral term, which represents the effects of firm productivity and decreasing returns on the wage, is absorbed by the time-invariant firm fixed effect, and the residuals are zero. With a more complex DGP, the value of the integral term changes over time. Suppose productivity Ω follows an AR(1) process as assumed by ACF. In this case, the AKM firm fixed effect captures a time average of the effect of changing productivity on the wage. Thus, the more persistent the productivity process is, the more informative is the time-invariant firm effect. The AKM residual, which is the third component, absorbs fluctuations around the time average of the effect of changing productivity on the wage, which include transitory shocks.

Finally, note that the mapping we describe could be obscured when using real data, for example, due to measurement/sampling error in short panels. For this reason, we assess the robustness of our main results based on both worker and firm subsamples in Section 6.

3 Data

Our analysis combines two data sets provided by the Institute for Employment Research (Institut für Arbeitsmarkt- und Berufsforschung, IAB) of the German Federal Employment Agency (Bundesagentur für Arbeit, BA). The first is the “IAB Employee History File” (BeH), which comprises the universe of employment spells recorded by the German social security system. The second data set is the “IAB Establishment Panel” (EP), which is a representative establishment-level survey that can be linked to the BeH.¹⁰

In this section, we describe the two data sets, explain how we prepare and combine them, and discuss the wage regressions we use to decompose observed wages. These wage components allow us to conduct the model-inspired adjustment of labor inputs at the establishment level, which we then use to estimate productivity. We relegate additional details on sample selection and the imputation procedures to Appendix B.

3.1 Data Sources and Preparation

The BeH employment spell data contain information on worker gender, age, and education¹¹, as well as start and end dates for the spells, total earnings, and occupation/industry codes. The data cover the vast majority of the German workforce. Only civil servants and self-employed workers who do not pay social security contributions are excluded. Each worker and each establishment have a unique identification number, which allows us to follow workers over time and across establishments.

Regarding sample selection, we largely follow CHK and Lochner et al. (2020a). We start with the universe of employment spells observed between 1998 and 2008. There is no exact information on hours worked in the BeH; thus, we restrict our sample to full-time employees (males and females) aged 20 to 60. The age restriction circumvents interference from apprenticeship training and early retirement. In any given year, we define the job with the highest total wage (including bonuses) as each worker’s main job.

The EP is a comprehensive annual survey of establishments, that is, single production units such as factories or branches.¹² This survey provides us with the necessary data to estimate production functions at the establishment level; we observe revenue, intermediate goods purchases (reported as revenue shares), value added (calculated as revenue minus intermediate goods purchases), and net investments in four different categories of capital goods (buildings, production machinery, IT, and transport equipment). We restrict our analysis to EP establishments with nonmissing data on revenue and intermediate goods purchases.¹³ To ensure that our results are not driven by outliers, we drop

¹⁰The EP is a random sample of all establishments in Germany, stratified according to size, industry, and federal state. See Kölling (2000) and Fischer et al. (2009) for a detailed description of the EP data.

¹¹We follow CHK and use five education categories. The details are described in Appendix B.2.

¹²Of the surveyed establishments, 80% report that they are single-establishment firms.

¹³The main reason for missing information is that some firms choose not to report revenue as their

establishments that are found to be below the 1st and above the 99th percentile of the revenue distribution.

We supplement the EP data with covariates from the “Establishment History Panel” (BHP).¹⁴ These include average wages, headcounts, shares of full-time/part-time workers, and worker shares by skill (education) group. Moreover, the BHP provides administrative information on firm age and a consistent industry classification.

3.2 Imputations

We observe nominal gross daily wages, which we deflate using the consumer price index from the German national accounts. One limitation of the wage data is that earnings are tracked only up to a threshold, the social security contribution assessment ceiling (“Beitragsbemessungsgrenze”). We follow Dustmann et al. (2009) and impute the upper tail of the wage distribution by running a series of Tobit regressions, allowing for heterogeneity by fitting the model separately by gender, tenure, education level, and five-year age group; see Appendix B.1. We impute missing and inconsistent education data using the methodology proposed in Fitzenberger et al. (2006); see Appendix B.2.

The EP data contain information on net investments. To estimate the level of the capital stock, we use a perpetual inventory method following Müller (2008). This method approximates the establishment-level capital stock by combining net investments with the average economic lives (depreciation rates, available from the national accounts) of the different types of capital goods we observe.

3.3 Wage Regressions

We estimate an AKM-type wage regression on the largest connected set in the BeH data for our period of analysis, namely, 1998–2008. We include both men and women. The connected set contains more than 233 million person-years, corresponding to 35 million individual workers at 3.3 million establishments; 55% of the observations are classified as movers between establishments. We use the CHK specification; that is, we estimate a log-linear wage equation for worker i who works at firm $j(i, t)$ in year t as follows:

$$w_{it} = \alpha_i + \psi_{j(i,t)} + x'_{it}\beta + \varepsilon_{it}, \quad (7)$$

where w_{it} are log real daily wages, α_i is the worker fixed effect, $\psi_{j(i,t)}$ is the establishment fixed effect, and x'_{it} contains time-varying controls, i.e., an unrestricted set of year dummies and quadratic and cubic terms for age fully interacted with educational attainment.

measure of output. This applies mainly to financial institutions and public sector firms.

¹⁴The BHP covers all establishments with at least one employee subject to social security withholding on a reference date (June 30th). See Spengler (2008) for a detailed description of the BHP data.

ε_{it} is the residual. The regression model is identified for workers who move between establishments.¹⁵ The adjusted R^2 of the regression is 0.92, which is broadly in line with CHK. The correlation between the estimated worker and firm fixed effects, which is often interpreted as a measure of wage sorting in the labor market, is 0.27 in our time interval. This is slightly higher than what CHK report, which is likely due to the longer time period and broader sample we consider.¹⁶ Estimated worker and firm fixed effects, as well as their covariance, could be biased due to limited mobility in the connected set.¹⁷ We test the parametric correction suggested by Andrews et al. (2008) for two subperiods in our data and find that the limited mobility bias is small.¹⁸

Table 1 shows the variance decompositions for multiple groups of workers based on the estimated AKM wage components. Column (a) includes all person-years, (b) all women, (c) all men, and (d) all men in West Germany (the CHK sample). We replicate the well-known finding that the majority of the wage variance is explained by unobserved worker heterogeneity; i.e., the worker fixed effect explains almost half of the observed variation in wages (slightly more for women and slightly less for men). The second-most important source of variation is the firm fixed effects, which explain roughly one-quarter of the wage variance across the four groups. The third largest determinant is the covariance between worker and firm effects, which explains between 12 and 19% of the wage variance.¹⁹ At only 2%, the share of the wage variance explained by the time-varying observable characteristics is almost negligible. The same is true for the covariances of the observable characteristics with the worker and firm effects. Note that time-invariant covariates such as education are absorbed by the worker effect. The residual explains wage variance shares between 7 and 9% across the four BeH samples.

We use the estimated AKM wage effects for men, women, and both movers and stayers from the full BeH sample (column (a) in Table 1) to construct the labor inputs for the production function estimation at the firm level. Based on the discussion in Section 2.1, we construct wage bill measures that include (exponentiated) worker fixed effects, the

¹⁵Following CHK, we impute worker fixed effects and residuals for “stayers” (workers who do not change their employer over the entire sample period).

¹⁶CHK report correlations for shorter time intervals, i.e., 0.17 (1996–2002) and 0.25 (2002–2009). Moreover, CHK includes only men in former West Germany, whereas we include both men and women in reunited Germany from 1998–2008.

¹⁷As shown by Andrews et al. (2008, 2012) and recently revisited by Borovičková and Shimer (2020) and Kline et al. (2020), estimated variances of worker and firm fixed effects are biased upwards, and their covariance is biased downwards.

¹⁸The fact that we estimate the AKM model on the universe of German register data over a ten-year time window and include both men and women mitigates concerns of severe limited mobility bias due to the large number of observed transitions. Lochner et al. (2020b) and Lachowska et al. (2020) confirm that this bias is limited in data sets with a large number of transitions. The Andrews et al. (2008) correction implies that in our setting, the variance in worker effects decreases by 5%/4% and the variance in firm effects decreases by 4%/3% in the 1998–2002/2003–2008 subperiods, respectively. The covariance between the worker and firm effects increases by 7%/5%.

¹⁹Interestingly, women are less positively sorted in terms of wages than men. This is in line with what Card et al. (2016) and Bruns (2019) find using Portuguese and German data, respectively.

Table 1: Wage Variance Decompositions

	(a)		(b)		(c)		(d)	
	Regression (7) BeH, full		Regression (7) BeH, women		Regression (7) BeH, men		Regression (7) BeH, men, West	
Var(w_{it})	0.276	(100%)	0.277	(100%)	0.245	(100%)	0.226	(100%)
Var($\hat{\alpha}_i$)	0.126	(46%)	0.138	(50%)	0.105	(43%)	0.106	(47%)
Var($\hat{\psi}_{j(i,t)}$)	0.068	(25%)	0.076	(27%)	0.061	(25%)	0.049	(22%)
Var($x'_{it}\hat{\beta}$)	0.005	(2%)	0.006	(2%)	0.005	(2%)	0.005	(2%)
$2 \times \text{Cov}(\hat{\alpha}_i, \hat{\psi}_{j(i,t)})$	0.049	(18%)	0.032	(12%)	0.047	(19%)	0.037	(16%)
$2 \times \text{Cov}(\hat{\alpha}_i, x'_{it}\hat{\beta})$	0.004	(0%)	0.001	(0%)	0.005	(2%)	0.006	(3%)
$2 \times \text{Cov}(\hat{\psi}_{j(i,t)}, x'_{it}\hat{\beta})$	0.003	(0%)	0.001	(0%)	0.004	(2%)	0.004	(2%)
Var($\hat{\varepsilon}_{it}$)	0.021	(8%)	0.025	(9%)	0.018	(7%)	0.019	(8%)
Sample mean wage	4.450		4.261		4.553		4.621	
R^2	0.92		0.93		0.91		0.92	
# Observations	233,117,492		82,267,794		150,849,698		123,087,610	

Notes: Variance decompositions for log real daily wages according to regression model (7) for various BeH samples. Mean wages, variances, and covariances are rounded to three decimal places. Source: BeH.

effects of observable characteristics, and, in some specifications, firm fixed effects and residuals to distinguish the different channels through which wages and firm productivity are related according to our theory.

3.4 Final Samples

We use two final samples in the subsequent analysis. We refer to the first sample as *All Matches*. This sample includes matches that formed both before and after 1998. Here, we do not condition on the origin of the match; that is, we do not distinguish between J2J flows and matches formed out of nonemployment (OON). There are 4,695,108 employment spells for 1,344,382 workers employed at 10,004 EP establishments.

We refer to the second sample as *New Matches*. This sample includes matches formed from 1999 onward, and we distinguish between J2J flows and matches formed OON. Thus, we need one initial observation (1998) to determine employment status before the new match. There are 1,656,280 employment spells for 633,831 workers at 9,659 establishments. A total of 64% (1,055,151) of new employment spells are J2J moves from one employer to another, while 36% (601,129) are spells formed OON. Our definition of nonemployment includes marginal employment, unemployment (benefit receipt), and inactivity.²⁰ Thus, OON spells also include young workers who enter the labor market.

²⁰We do not observe UI benefits in the BeH. Thus, unemployment and inactivity are indistinguishable.

In some parts of our empirical analysis, we study changes over time. To this end, we split our sample period (1998–2008) into the subperiods 1998–2002 and 2003–2008.

4 Ranking Workers and Firms

4.1 The Worker Rankings

According to our model, the estimated AKM worker fixed effects fully capture the effect of unobserved worker ability on wages. This result can serve as a “microfoundation” for ranking workers based on these wage effects. Using this ability measure, we rank individual workers and create 50 ability bins of equal size.²¹ Let $\bar{x}(i)$ denote the ability bin to which worker i belongs. In what follows, individual workers in the same bin are considered to have the same ability type.

To show how the bins summarize worker heterogeneity, we decompose the respective variances of observed wages, age, and education into the shares explained within and between the bins. Little within-bin variance implies homogeneity among workers in the respective dimension. The rankings are based on the worker fixed effects, which explain the majority of the wage variation in the data. Thus, the between-bin share of the wage variance is large (70%). For age and education, the picture is quite different. We find that 96% of the age variation and 74% of the education variation are within the bins. Because we control for time-varying age and education effects in the AKM regression, this finding is a direct reflection of the low correlation between the estimated worker effects and the time-varying observables (recall Table 1).

Figure 1 illustrates how log wages, age, and education vary across worker bins. Panel (a) shows that mean wages increase monotonically across worker bins, as expected. In contrast, Panel (b) shows that the mean age across worker bins is relatively flat. Only the highest bin has a somewhat higher mean age. However, due to the large standard deviation, this difference is not significant. Panel (c) shows that the mean education increases above bin 35 but is flat below it. This suggests that highly ranked workers are more likely to have tertiary degrees, while lower-ranked workers tend to have vocational training only. Again, however, the differences are hardly significant. Note that the dispersion of education increases among the highest-ranked workers. It is more common to observe high-rank workers with little education than low-rank workers with tertiary degrees. Finally, Panel (d) shows the duration of nonemployment across worker types. The mean observed nonemployment duration is virtually flat. Recall that we control for the effect of observable characteristics in our AKM estimation. The finding that nonemployment durations do not change with worker type supports the assumption that matching sets cover the whole type space, which we made in Section 2.

²¹There are 702,540 individual workers in each of the 50 bins.

Figure 1: Log Wage, Age, Education, and Nonemployment Duration across Worker Bins



Notes: The figures plot the means \pm one standard deviation for log wages, age, education, and nonemployment duration for every fifth worker bin. The ages of the individual workers in our sample range from 20 to 60. There are 5 education categories, which are defined in Appendix B.2. Data source: BeH.

4.2 Firm Rankings

We rank firms based on their unobserved productivity, which we infer by estimating production functions at the establishment level. We know from the empirical IO literature that this approach is susceptible to two challenges. The first is “transmission bias”, which is an endogeneity problem (Marschak and Andrews, 1944). Input choices, e.g., the demand for labor or intermediate inputs, are likely correlated with firm productivity.²² To address this challenge and estimate an unbiased measure of firm productivity, we rely on the ACF version of the control function approach.²³ The key assumption is that intermediate input demand (the control function) is a strictly increasing function of (scalar and unobserved) firm productivity, which may change over time due to shocks. This assumption implies that one can invert the control function and effectively control for unobserved firm productivity by substituting it out of the production function. Importantly, the ACF model allows the firm’s labor choice to have dynamic implications. This suits our model well, as we discuss below.

The second challenge is that in the presence of heterogeneous worker ability, the

²²This is because inputs are demanded optimally by the firm’s management based on current firm productivity, which is unobserved by the econometrician.

²³This approach was originally developed by Olley and Pakes (1996) and refined by Levinsohn and Petrin (2003) and Wooldridge (2009).

quality of the labor inputs varies across firms (Griliches, 1957). Physical units, e.g., headcounts or hours worked, do not reflect differences in worker ability and imply measurement error. This complicates the separate identification of firm productivity and the output elasticity of labor. The empirical IO literature addresses this challenge by controlling for labor quality differences when estimating the production function (e.g., Fox and Smeets, 2011; Irarrazabal et al., 2013). We go one step further and construct model-based labor input measures that isolate the effect of worker ability on output from other wage components, as discussed in Section 2.1.

Production Function Estimation

We start from the production structure assumed in Section 2, equation 2. We rewrite the equation in terms of the composite labor input in ability units L_{jt} and add capital inputs K_{jt} , as well as indices for individual firms j and time t :

$$Y_{jt} = (\Omega_{jt} L_{jt})^{\beta_l} (\Omega_{jt} K_{jt})^{\beta_k}. \quad (8)$$

β_l and β_k are the output elasticities of labor and capital, respectively. Both inputs are scaled by the firm's current productivity Ω_{jt} . Note that Y_{jt} is value added (revenue minus expenditures on intermediate goods). Intermediate inputs are therefore not part of the production function (8). Without assuming constant returns to scale, by the homogeneity of the general Cobb-Douglas function (8), we obtain the following:

$$Y_{jt} = \Omega_{jt}^{\beta_l + \beta_k} L_{jt}^{\beta_l} K_{jt}^{\beta_k}. \quad (9)$$

The sum of the output elasticities in the exponent on Ω_{jt} is irrelevant for the purpose of ranking the firms. Thus, we define $\omega_{jt} = (\beta_l + \beta_k) \ln \Omega_{jt}$ when taking logs. We estimate the following value-added production function, where lowercase letters indicate logarithms:

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + z'_{jt} \gamma + \epsilon_{jt}. \quad (10)$$

We add a constant β_0 ; the residual ϵ_{jt} , which absorbs transitory shocks; and a vector of additional controls z'_{jt} , including indicators for West German establishments, four firm age categories, the share of part-time workers, and employee representation in management. We also include time and sector fixed effects (32 categories).

We describe the technical details of the ACF estimation procedure in Appendix C. Here, we discuss the key features that make this approach compatible with the model in Section 2. ACF builds on a discrete-time model of dynamically optimizing firms. As in our model, labor input has dynamic implications and therefore enters the firm's dynamic problem as a state variable (cf. equation A.1). We abstract from capital inputs in

Section 2, while ACF assumes that the capital stock is predetermined by the investment $i_{j,t-1}$ made in the previous period:

$$k_{jt} = \kappa(k_{j,t-1}, i_{j,t-1}). \quad (11)$$

Our model builds on Cahuc et al. (2008), who explicitly consider a capital accumulation process similar to that described by equation (11). Thus, our model and the ACF model are also compatible in the capital dimension.

In the ACF model, the demand for labor and intermediate goods may change in response to contemporaneous firm productivity shocks.²⁴ The conditional expectation of firm productivity is modeled using a first-order Markov process:

$$\omega_{jt} = E(\omega_{jt} | \omega_{j,t-1}) + \xi_{jt} = \rho\omega_{j,t-1} + \xi_{jt}, \quad (12)$$

where ξ_{jt} is an innovation that is assumed to be uncorrelated with ω_{jt} and k_{jt} . For the estimation, we follow the ACF model and assume that ω_{jt} follows an AR(1) process with parameter ρ . In our model, both the firm's vacancy-posting decision and its matching set depend on current productivity; see Appendix A.2. Moreover, the firm's dynamic problem involves the expected future evolution of productivity. Recall that we abstract from productivity shocks when deriving the wage equation (4) and establishing its linearity in worker type x . This assumption simplifies the derivation but is not critical for our results. As long as the evolution of firm productivity is independent of the employed worker types, the wage equation will still be (log-)linear in x under our assumed production structure.²⁵

Finally, the ACF approach allows for (exogenous) across-firm differences in labor adjustment costs and wage setting. This flexibility is key to our approach, as it makes the ACF approach a good fit for models with search, matching, and bargaining. Earlier approaches in the empirical IO literature have needed to make more restrictive assumptions about input prices due to the use of unconditional intermediate input demand as the control function.²⁶ Intuitively, this is because a firm's unconditional intermediate input demand likely depends on its labor input and, thus, on its wage conditions or adjustment costs. Conditioning on labor input eliminates this link. In the context of our model, differences in adjustment costs and wage setting across firms arise for two reasons. First, vacancy posting costs can be heterogeneous, and firms have limited control over the hir-

²⁴The firm's information set when making input choices includes all past and present productivity shocks $\{\omega_{j\tau}\}_{\tau=0}^t$, but it does not include future productivity shocks.

²⁵Consider an environment with productivity shocks and, accordingly, changing labor demand and endogenous separations. In this case, the wage equation contains an additional forward-looking component that captures the expected evolution in firm productivity. Worker type x can still be written in front of the integral sign under the mild assumption that x and the firm productivity process are independent.

²⁶As explained in ACF (p. 2431), their approach can handle serially correlated, exogenous, and unobserved shocks to the prices of capital and labor. Levinsohn and Petrin (2003) can only allow for such shocks to the price of capital inputs. Neither approach allows for shocks to the price of intermediates.

Table 2: Production Function Estimation Results

Dependent Variable	Value Added				
	(a)	(b)	(c)	(d)	(e)
<i>Labor input</i>	0.7129 (0.0141)	0.7131 (0.0141)	0.7125 (0.0141)	0.7099 (0.0135)	0.8573 (0.0133)
<i>Capital input</i>	0.1706 (0.0117)	0.1703 (0.0117)	0.1707 (0.0117)	0.1512 (0.0116)	0.1358 (0.0110)
<i>Labor input variable</i>	AKM Predicted Wage Bill				Headcount
<i>Worker fixed effects</i>	Yes	Yes	Yes	Yes	—
<i>Worker observables</i>	Yes	Yes	Yes	Yes	—
<i>Firm fixed effects</i>	No	Yes	No	Yes	—
<i>Residual</i>	Yes	No	No	Yes	—
<i>Firm-level controls</i>	Yes	Yes	Yes	Yes	Yes
<i>Year FEs</i>	Yes	Yes	Yes	Yes	Yes
<i>Sector FEs</i>	Yes	Yes	Yes	Yes	Yes
<i>Variance of $\hat{\omega}_{jt}$</i>	0.074	0.073	0.072	0.064	0.050
<i>Variance of ϵ_{jt}</i>	0.628	0.627	0.626	0.597	0.583
<i>N</i>	38,598	38,598	38,598	38,598	39,808

Notes: Bootstrapped standard errors (500 iterations) are given in parentheses. All estimated coefficients and standard errors are rounded to four decimal places. Data sources: BHP, EP, BeH.

ing process.²⁷ Second, due to decreasing returns to labor and intrafirm bargaining, wages have a firm-size component that varies across firms.²⁸

Estimation Results

Table 2 presents the results of the production function estimation. We show five different specifications in which we vary how we control for worker ability. All specifications include the aforementioned firm-level controls, as well as year and sector fixed effects.

Specifications (a)–(d) use the predicted wage bills based on different combinations of the estimated AKM wage components. We always include worker fixed effects, which, according to our model, capture the effect of worker ability on output, along with the effects of worker observables. In addition, we include the AKM residual in (a), the firm fixed effect in (b), neither the residual nor the firm fixed effect in (c), and both in (d).

²⁷With random search, the time that passes until a suitable worker arrives is stochastic.

²⁸Our model implies a nondegenerate firm-size distribution due to heterogeneous productivity.

The labor input in (d) is thus equal to the total observed wage bill for the firm. For the sake of comparison, we also show the results for a specification that uses the worker headcount as the labor input measure (e).

The first interesting finding is the large difference between the estimation results for (a)–(d) and those for (e). The headcount mismeasures the labor input in the presence of worker heterogeneity; thus, the relatively high estimate of the output elasticity of labor is likely biased upwards in specification (e), while the capital elasticity and the estimated productivity $\hat{\omega}_{jt}$ are biased downwards. Across specifications (a)–(d), the results are very consistent, with an estimated output elasticity of labor of approximately 0.71. Together with the capital elasticity, these estimates imply decreasing returns to scale, in line with our model’s assumption.

The second interesting finding is that the AKM wage components beyond the worker fixed and observable effects have little effect on the estimation results. In light of our model, this is unsurprising. The worker fixed effects capture the effect of worker ability on output; thus, this wage component is a direct measure of the workers’ contribution to the firm’s production. The firm effect, however, does not directly reflect how workers contribute to production. As explained in Section 2.1, it captures the time average of the effect that firm productivity has on wages.

Third, consider the effect of adding the AKM residual. According to our model, the AKM residual absorbs the effect of firm productivity shocks on wages, which have been shown to affect wages (“pass-through”).²⁹ Guiso et al. (2005) show that firms fully insure their workers against transitory productivity shocks but only partially against the more permanent productivity changes captured by $\hat{\omega}_{jt}$. A correlation between the contemporaneous productivity realization and the labor input does not threaten our estimation strategy.³⁰ Notably, adding the residual has little impact on the estimated output elasticities. However, capturing the reaction of wages to shocks in our input measure could still be important. If worker ability and firm productivity are complements in production, the contribution of worker ability to firm output could shrink after a negative productivity shock. For this reason, we select specification (a), which includes the AKM residuals, as our benchmark specification.

Finally, we examine the dispersion and persistence of estimated firm productivity $\hat{\omega}_{jt}$. The estimated variance of 0.074 in our benchmark specification is almost 50% higher than that in the headcount specification (e). It is, however, an order of magnitude smaller than the residual variance. Recall that the residual absorbs transitory shocks, whereas productivity ω_{jt} is the realization of a persistent stochastic process, and the estimated autocorrelation of $\hat{\omega}_{jt}$ in the benchmark specification is indeed high (0.75). Given the link between firm productivity and AKM firm effects that our model suggests, this high

²⁹See also Chan et al. (2021), who study this channel with Danish data and summarize the literature.

³⁰See the ACF moment conditions, which we discuss in Appendix C, equation (C.5).

Table 3: Firm Ranking Correlations

	\bar{v}_j	\bar{v}_j/\bar{n}_j	$\bar{\pi}_j/\bar{n}_j$	\bar{n}_j	\bar{k}_j	\bar{k}_j/\bar{n}_j	Workforce education
Correlation with $\hat{\omega}(j)$	0.43	0.25	0.16	0.40	0.24	-0.03	0.08

Notes: The table shows the correlation of the time-invariant estimated firm ranks, $\hat{\omega}(j)$, with the means of the following firm statistics over time: log value added (\bar{v}_j), log value added per worker (\bar{v}_j/\bar{n}_j), profit per worker ($\bar{\pi}_j/\bar{n}_j$), the log size of the workforce (\bar{n}_j), the log capital stock (\bar{k}_j), the log capital stock per worker (\bar{k}_j/\bar{n}_j), and workforce education as measured by the mean of the workers' education variable within the firm. Data sources: BHP, EP, BeH.

degree of persistence implies that firm fixed effects should be rather stable over time. Recent evidence presented by Lachowska et al. (2020) supports this conjecture.

Firm Rankings

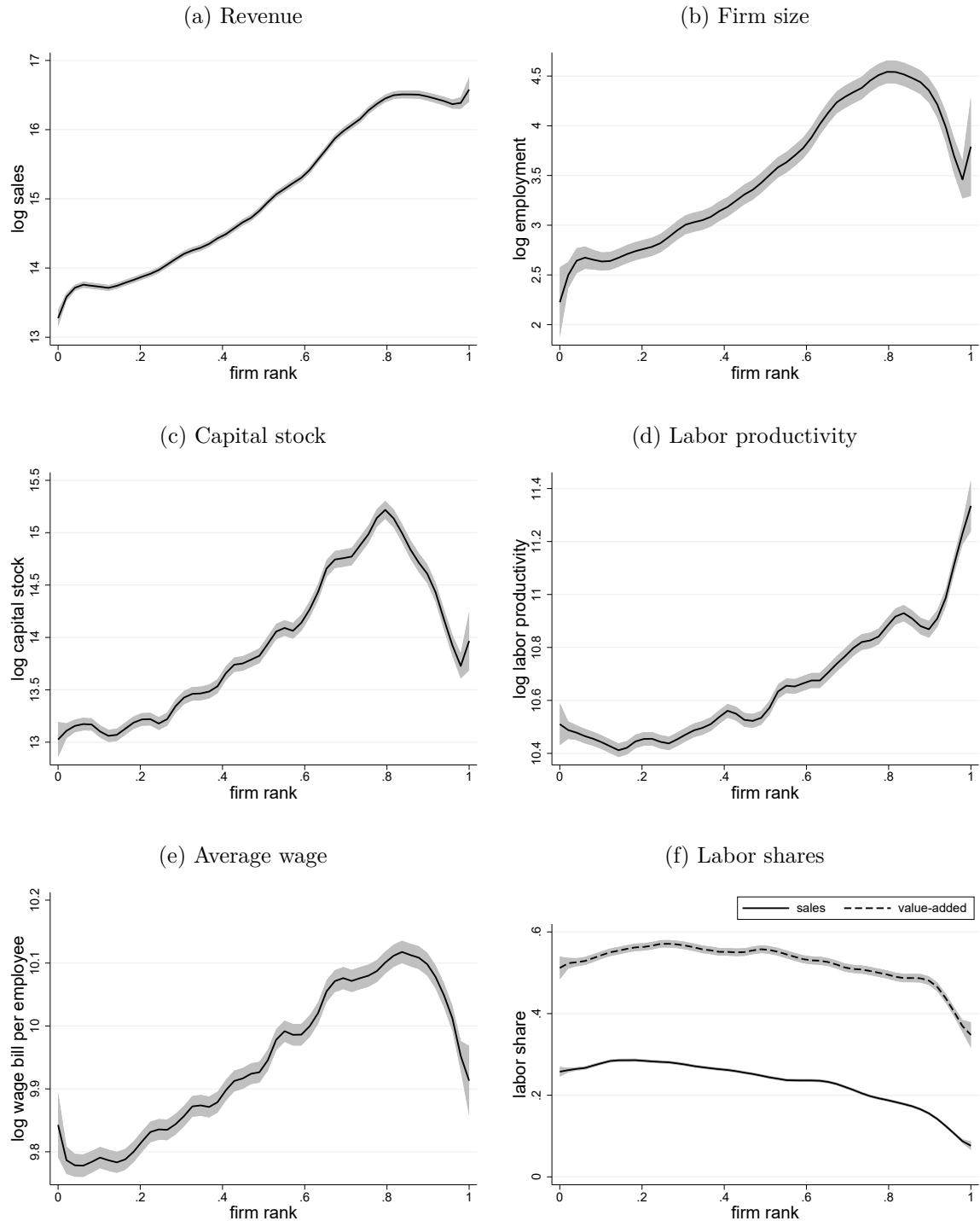
Given the high degree of persistence in the estimated productivity, we rank firms by the firm-level mean. The estimated rank of firm j is denoted $\hat{\omega}(j)$. The use of the mean is in line with the empirical literature on wage inequality and labor market sorting, which typically assumes the permanency of worker and firm types. The fixed effects-based worker types are also time-invariant.

Table 3 shows the correlation of estimated firm ranks with several firm-level statistics. The ranks are moderately positively correlated with firm size in terms of the employee headcount (0.40), value added (0.43), value added per worker (0.25), capital stock (0.24), and profit (0.16). The ranks are virtually uncorrelated with capital per worker (-0.03). These correlations suggest that the largest firms in terms of output, workforce, and assets in our sample are not the most productive firms. The correlation of firm rank with the share of highly educated workers is only 0.08. Thus, highly productive firms are not necessarily characterized by a highly educated workforce.

Figure 2 shows the estimated kernel densities for six firm performance measures across the firm ranks. Revenue (measured by log sales, Panel (a)) mostly increases in rank but becomes relatively flat at the top of the rankings. The most productive firms do not have greater revenue than firms around the 80th percentile of the productivity distribution. In Panel (b), the relation between firm rank and log employment (measured by the headcount) is shown. This size measure increases until approximately the 80th percentile and falls steeply thereafter. The most productive firms are comparable to the median-productivity firms in terms of size. In line with the correlations reported above, they are not the largest in terms of employment. The same holds for the capital stock (Panel (c)).

Panel (d) shows that log labor productivity (value added per worker) is relatively flat in the bottom half of the distribution but increases substantially for the most productive firms. The labor productivity of the firms at the top of the distribution is approximately twice as high as that of the firms at the bottom. The log wage bill per employee, that

Figure 2: Firm Performance Measures by Estimated Firm Rank



Notes: Estimated univariate kernel densities for selected firm performance measures across estimated firm ranks normalized to be between zero and one. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is 0.05 for (a)–(e) and 0.04 for (f). The qualitative findings are robust to the bandwidth choice. The 95% confidence bands are shown in gray. Data sources: BHP, EP, BeH.

is, the average wage, is shown in Panel (e). Among the firms above the 80th percentile, average wages fall. Notably, the largest firms in terms of employee headcount are also situated near the 80th percentile; therefore, average wages are closely related to size but less related to productivity. Interestingly, the least productive firms pay higher average wages than the other firms located below the 25th percentile.

Finally, Panel (f) shows labor shares, which are computed using both revenue and value added. Both labor share measures exhibit a hump shape but clearly fall in estimated firm rank. The labor share in terms of revenue falls from just below 30% for the least productive firms to less than 10% for the most productive firms. For the value added measure, the labor share falls from 50–60% to approximately 35%. Thus, the average worker obtains a significantly lower share of the output at high-productivity firms.³¹

Binning the Firms

In the next step, we group all the individual firms into 15 productivity bins of equal size.³² Let $\bar{\omega}(j)$ denote the bin to which firm j belongs. In what follows, individual firms in the same bin are considered to be of the same type. For the worker bins described above, we decompose the variance of some key variables.

The firm bins exhibit a high within-bin wage variance (94% of the total variance). This is a reflection of the fact that we control for heterogeneous worker ability when estimating firm productivity. It also implies that our firms cannot be categorized into high-wage and low-wage firms—all firm types pay dispersed wages to their workers, depending on those workers’ ability and contributions to output.

There is also within-bin heterogeneity in terms of output (value added and value added per worker) and size (employee headcount and capital stock). The majority of the variance is always within bins, which suggests that binning does not merely group firms of similar size or production capacity. We also check for relations between the productivity bins and industry or the prevalence of collective bargaining agreements and employee representation but find none.

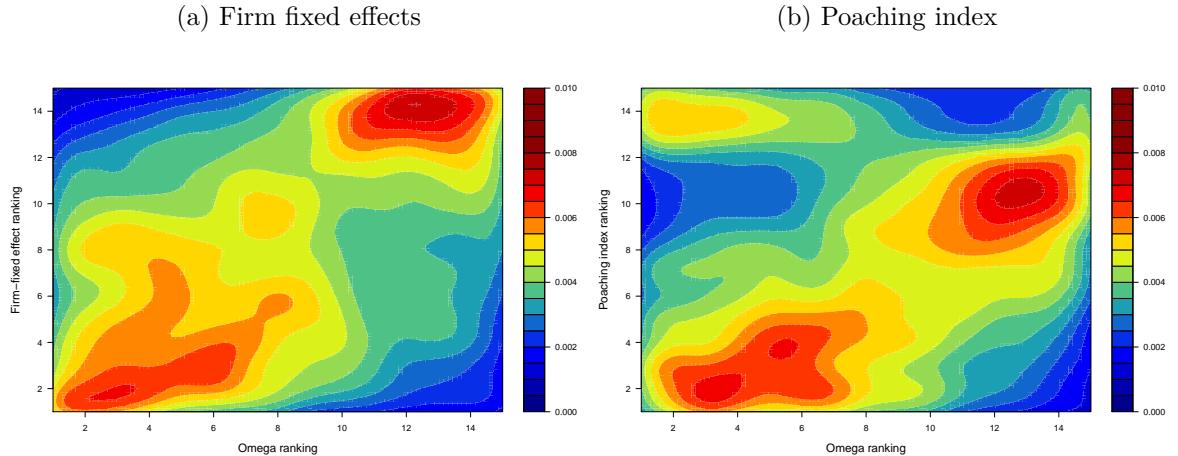
Comparison with Alternative Rankings

Finally, we compare our productivity ranking with the results of other ranking techniques used in the literature on wage dispersion and labor market sorting. We create two alternative firm rankings. The first is based on the AKM firm fixed effects and the second on

³¹These low labor shares are interesting given the falling aggregate labor share in many developed economies. In fact, our firm ranking appears to mirror an explanation emphasized in Autor et al. (2017, 2020), i.e., the emergence of so-called “superstar” firms.

³²The EP sample used to estimate firm productivity contains 10,026 individual establishments, so there are 668 establishments in each bin.

Figure 3: Comparison with Alternative Firm Rankings



Notes: The two plots depict the contours of the joint empirical distribution of firm-years across combinations of the ω ranking (15 bins) with the AKM firm-effects ranking (15 bins, Panel (a)) and the poaching index ranking (15 bins, Panel (b)). Data sources: BHP, EP, BeH.

the poaching index used in Bagger and Lentz (2019) and Taber and Vejlin (2020).³³ Correlations with the productivity ranking are positive, i.e., 0.280 and 0.119. To graphically analyze how the rankings are related, we create two sets of 15 firm bins based on both the firm-effects ranking and the poaching ranking. This approach allows us to compute the empirical distribution of firm-years across the different firm bin combinations. We observe how firms with a given productivity rank are distributed across the firm fixed-effect and poaching rank bins. Figure 3 plots the contours of these empirical distributions.

In Panel (a), the AKM firm-effects bins are depicted on the vertical axis, and the ω ranking bins are depicted on the horizontal axis. The mass of observations is concentrated along the diagonal, which is in line with the positive correlation reported above. In the upper-right quadrant of the plot, the observations are highly concentrated somewhat above the diagonal, thereby reaffirming our observation that the highest-paying firms (here in terms of the AKM wage premia) are located below the top of the productivity distribution. In the lower-left quadrant, the observations are more dispersed. It is not uncommon to observe firm-years in which the estimated AKM wage premium is around the median but estimated productivity is very low and vice versa. Here, the disagreement between the two rankings is large. Figure D.1 in the Appendix shows that the observations in the lower-left quadrant are mainly young and small firms. The high-wage firms in the upper-right quadrant are older and larger firms. Panels (a) and (b) in Figure D.1 show that low-wage (high-wage) firms have on average a low (high) AKM rank; however, there

³³The poaching index is based on the idea that high-paying firms poach workers from other firms rather than hiring unemployed workers. We compute this index by comparing the annual number of workers hired directly from other firms to the number of all hires at the firm level and then rank firms based on the firm-level mean of the time-varying poaching index. We use the Administrative Wage and Labor Market Flow Panel (AWFP); see Stüber and Seth (2019). In this data set, the aggregated establishment-level worker flows needed to compute the poaching index are readily available.

is a sizable overlap between the two groups in terms of productivity.

In Panel (b), the mass of observations lies below the diagonal; that is, a firm's poaching rank tends to be lower than its productivity rank. Many high-productivity firms have high poaching ranks, but they are almost never located at the top. Additionally, it is not uncommon even for medium-productivity firms to hire mainly OON, as evidenced by the high density of low-poaching rank firms that extends far to the right. Interestingly, the firm years with the highest poaching index are clustered in the upper-left quadrant. Apparently, some low-productivity firms very actively poach workers from other firms. Figure D.2 in the Appendix shows that many of the firms at the top of the poaching index distribution are small and young. One possible explanation is that these firms attempt to grow quickly by poaching workers from other firms. The larger and older firms, which also pay the highest wages, are concentrated in the upper-middle of the poaching rank distribution. They hire a nonnegligible number of their employees OON.

In summary, the comparison of the different rankings shows that firm ranks based on firm wage premia and observed worker mobility are systematically different from our productivity-based firm ranking.

5 Productivity Sorting

To analyze the allocation of workers to firms and measure the degree of productivity sorting, we merge the two data sets containing our estimated worker and firm rankings. In this step, we lose all employment spells at firms that are not part of the EP sample. For this reason, we rebin workers and firms in the merged sample. The number of workers per bin decreases to 26,888. The number of firms per bin is almost unchanged at 667.³⁴ For all results below, we rely on the two samples defined in Section 3.4: (i) all the existing matches and (ii) new matches, which are formed either through J2J flows or OON.

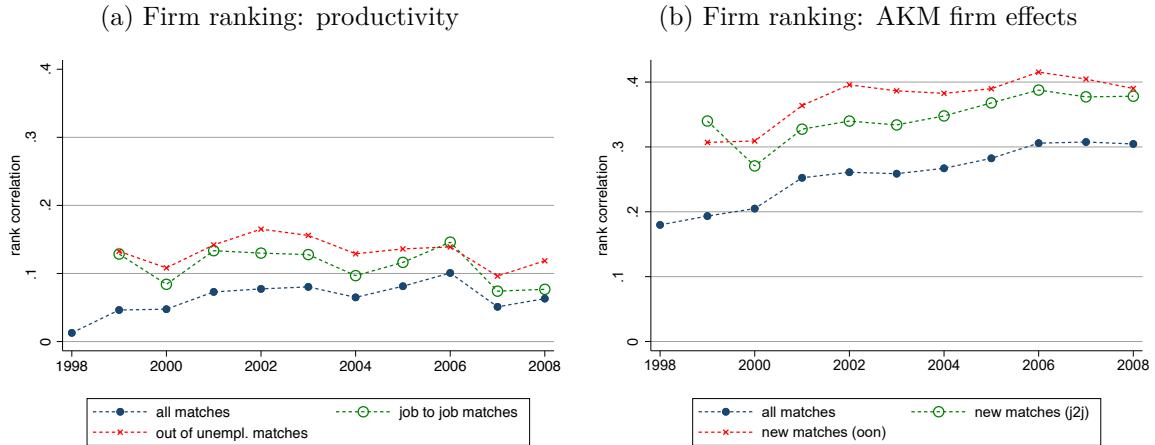
5.1 Rank Correlations

We use rank correlation coefficients (Spearman's ρ) to study the association between the estimated worker and firm ranks in the data. The rank correlation is a simple yet informative summary statistic of how workers are allocated to firms. For all matches in all years, we find a significantly positive but relatively low rank correlation of 0.07. For new matches, this correlation is 0.12. The correlation for J2J moves (0.11) is somewhat lower than the correlation for new matches OON (0.13).³⁵

³⁴We are unable to merge the employment spell information for 22 out of 10,026 unique establishments.

³⁵For clarity, the rank correlations presented here are calculated based on the worker and firm bins. Correlations based on worker and firm ranks (not reported) differ only marginally. Table D.1 provides an overview of all estimated rank correlations for different samples and time periods.

Figure 4: Rank Correlation Coefficients over Time (1998–2008)



Notes: Panel (a) shows the Spearman rank correlation coefficients based on our productivity-based (ω) firm ranking. Panel (b) ranks firms using the AKM firm fixed effects. Both plots use the worker ability ranking introduced in Section 4.1. Data sources: BHP, EP, BeH.

The positive rank correlation we find is an indication of PAM based on worker ability and firm productivity. However, the implied degree of productivity sorting is low, both in an absolute sense and compared to earlier studies of wage sorting in German data. CHK and Hagedorn et al. (2017) find correlations of 0.17–0.25 and 0.71, respectively.³⁶ The key difference between our approach and that of earlier studies is that we rank firms based on their estimated productivity, while both CHK and Hagedorn et al. (2017) rely on wages and observed worker mobility to rank firms.

Next, we consider how the estimated rank correlations change over time to clarify the time dynamics in productivity sorting. To this end, we present the rank correlation coefficients for all samples and all years in Figure 4. First, in Panel (a), the estimated rank correlations are calculated using the productivity-based firm ranking. The blue line depicts the correlations for all matches, while the red and green lines depict the correlations for new matches OON and for J2J switches, respectively. For all matches, the degree of sorting first increases over time and then is relatively stable just below 0.1, with a drop in 2007. For both types of new matches, the correlation levels are higher but also fall near the end. The degree of sorting for new matches OON decreases steadily after 2002. For J2J switches, the time dynamics closely mirror those for all matches. This outcome reflects the fact that approximately 65% of all new matches are J2J switches (see also Table D.1). Thus, new matches drive up the rank correlation for all matches in the beginning of the sample period; however, once their correlation decreases, the correlation for all matches also levels off.

Panel (b) of Figure 4 facilitates the comparison with wage-based approaches. Here,

³⁶CHK use the log-linear AKM model and interpret the correlation between estimated worker and firm effects in the data as a measure of sorting. Hagedorn et al. (2017) study sorting through the lens of a structural model with worker and firm heterogeneity.

we compute the same rank correlation coefficients using the same worker ability rankings but using firm rankings that are based on the AKM firm fixed effects. Notably, this wage-based firm ranking implies higher rank correlations (more positive sorting) and a significant increase over time. The correlation for all matches reaches 0.3 near the end of our sample period. These numbers are close in magnitude to the findings of CHK. For new matches, the correlations are even higher, just below 0.4. New matches OON exhibit the highest rank correlations with both the AKM and the productivity-based firm rankings. We delve more deeply into the sources of these changing dynamics for new and all matches in the next subsection.

5.2 Distributional Dynamics

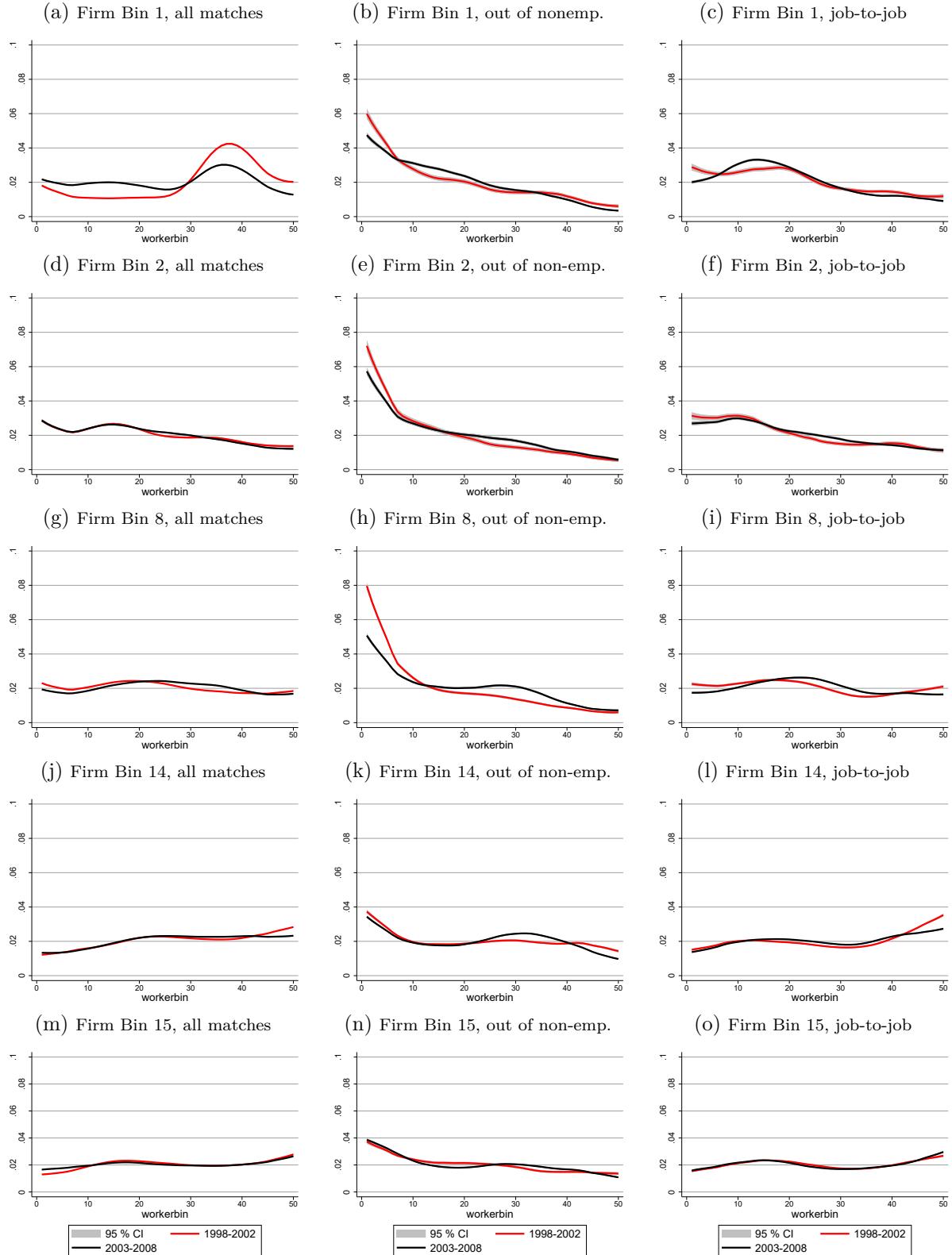
Rank correlations are only a summary statistic for the allocation of workers to firms. To more carefully investigate which worker-firm-type combinations contribute to changes in productivity sorting, we study which worker types the different firm types hire and how this has changed over time. This approach allows us to precisely track which worker types became “more sorted” and which became “less sorted”. To this end, we estimate univariate density functions for the employed worker types. Figure 5 presents these densities for low-productivity firms (bins 1–2), medium-productivity firms (bin 8), and high-productivity firms (bins 14–15).³⁷ We compare the first half of our sample, 1998–2002 (red line), to the second half, 2003–2008 (black line), and show the estimated densities for all matches, new matches OON, and J2J flows, along with the 95% confidence intervals.

Low-type firms (bins 1–2) hire mainly low-ability workers OON. This is where the density of new matches is highest, and it clearly falls in the worker type. This finding is consistent with positive productivity sorting. However, we also find that the average quality of new hires in low-productivity firms has increased over time. In Panels (b), (c), (e), and (f), the densities for the second half of our sample (black densities) are significantly higher for medium-ability workers hired both OON and due to J2J switches than those for the red densities. Moreover, the densities for low-ability workers are shown to decrease. Regarding the sample of all matches, the densities of worker types hired by bin 1 firms (Panel (a)) have become more uniform over time. That is, in addition to making higher quality new matches, these firms also have sizable outflows of high-ability workers. Taken together, the changes in low-productivity firms confirm the aggregate trend toward less productivity sorting.

For medium-type firms (bin 8), we observe a strong increase in the hiring of medium-ability workers both through poaching and OON; see Panels (h)/(i). For very productive firms (bin 14), the increase in the hiring of medium-ability workers is also present but less

³⁷The estimated univariate densities for the remaining firm bins are available upon request. Appendix Figure D.3 shows the full joint distribution of matches.

Figure 5: Changes in the Worker Type Distributions within Different Firm Bins



Notes: Estimated univariate kernel densities for all new matches conditional on worker bin, time, and match type. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is calculated with Silverman's rule of thumb. Pointwise confidence intervals are calculated using a quantile of the standard normal distribution. Data sources: BHP, EP, BeH.

pronounced. This increase is paired with a notable decrease in the hiring of high-ability workers for new matches made both through poaching and OON; see Panels (k)/(l). In sum, the plot for all matches suggests a significant reduction in hiring for high-ability workers and an increase for medium-ability workers, although that increase is smaller than that found for medium-productivity firms. For the most productive firms in bin 15, the aforementioned developments are, for the most part, muted; see Panels (n)/(o). We even observe a small but significant increase in new matches with high-ability workers through poaching. The density for all matches does not change for worker types beyond bin 10 (Panel (m)), which indicates that the allocation is quite stable at the very top.

This distributional analysis reveals why productivity sorting is relatively stable over time. High-ability workers have, to some extent, been replaced by medium-ability workers. Moreover, low-productivity firms have reduced their hiring of low-ability worker types and increased the quality of their average worker. The stable rank correlation is thus the result of two opposing trends, namely, reduced productivity sorting at the bottom and the top (fewer low-low and high-high matches) and increased sorting in the middle (more medium-medium matches). In the final section, we analyze how these changes in allocation are related to the wages that workers earn at different firms.

6 Wages and Inequality

First, we investigate how wages vary by worker and firm type. Second, we check whether observed worker transitions are directed toward higher wages and/or higher firm productivity. Third, we decompose wage inequality using productivity-based firm types and ability-based worker types to assess the role of productivity sorting in wage inequality.

6.1 Wage Variation across Worker and Firm Bins

Our approach of controlling for worker ability when estimating firm productivity allows us to test for wage monotonicity by examining how the wages of different worker types vary across firm types. In Figure 6, we plot the resulting wage-productivity profiles for five groups that include ten worker bins each.³⁸ All the matches are shown in Panel (a), and new matches are shown in Panel (b).³⁹ We observe quantitatively important deviations from monotonicity in both samples. The wage-productivity profiles exhibit a characteristic S-shape; i.e., low-productivity firms pay relatively high wages. The lowest wages are paid by firms in bins 3–4. Wages then increase monotonically until firm bins

³⁸The plots are based on a series of kernel-weighted local polynomial regressions.

³⁹We relegate plots for matches OON and J2J moves to Appendix Figure D.4. The results we discuss herein do not depend on this distinction.

11–12 and decrease thereafter.⁴⁰ Moreover, the wage drop at the most productive firms decreases in worker ability. The drop is most pronounced for low- and medium-ability workers⁴¹ but is almost nonexistent for high-ability workers. For new matches in worker bins 41–50, the wage merely levels off.

This non-monotonic, hump-shaped relation between wages and firm productivity poses a challenge for empirical models of wage dispersion. From a theoretical perspective, complementarities may imply that wages are a hump-shaped function of the firm type (e.g., Gautier and Teulings, 2006; Eeckhout and Kircher, 2011). However, many structural models cannot generate a hump-shaped relation between wages and firm productivity. Consider two widely-used models of on-the-job search (OJS). The most productive firm pays the highest wage in the wage-posting model of Burdett and Mortensen (1998). In the sequential auction model of Postel-Vinay and Robin (2002), wages decline in the poaching firm’s productivity. This is consistent with the lower inflection point at firm bins 3–4. However, the negative wage effect of the poaching firm’s productivity is minimized at the most productive firms, which implies a U-shaped relation between wages and productivity, not a hump-shaped one. We further discuss OJS in Section 7.

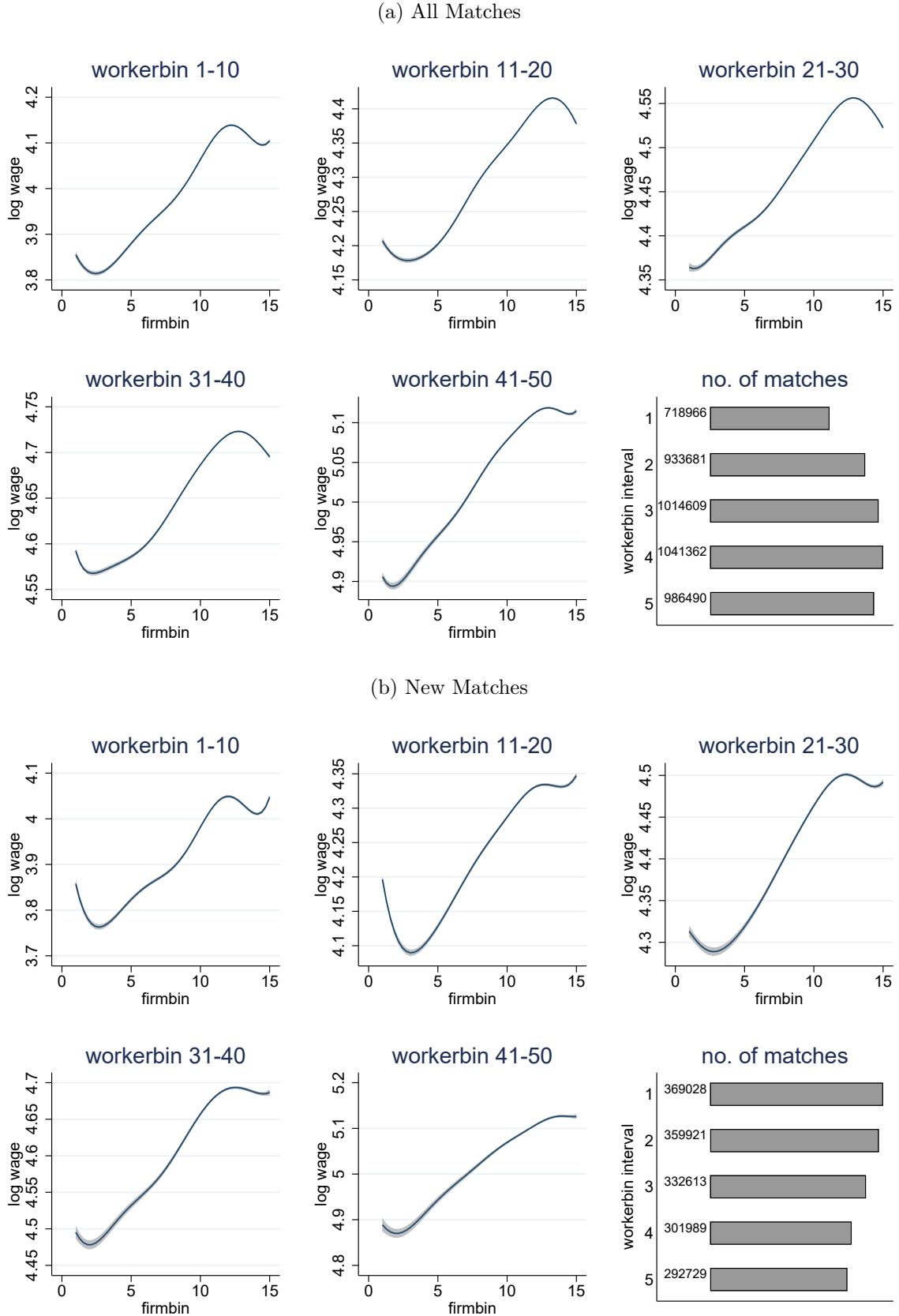
In the model that we use to motivate our empirical strategy, wages are monotonic in firm productivity. As we show in Appendix A.4, they increase in firm productivity at a decreasing rate. To generate a hump shape, the model needs a mechanism that allows the wage to be locally decreasing in firm productivity. To explore how such a mechanism could arise in an extended model, we take a closer look at the two key features of the empirical wage–productivity profiles: the observed wage drop at the most productive firms and the fact that this drop decreases in worker ability. To this end, we again rely on the estimated AKM wage components. Appendix Figures D.5 and D.6 reproduce Figure 6 using, respectively, firm fixed effects and AKM residuals instead of the log wage on the vertical axis.

Looking at firm fixed effects (Figure D.5), we observe a hump shape at the top across all worker types, including high-ability workers. This suggests that the wage decrease at the most productive firms is driven by firm-specific factors. According to our model, the firm fixed effect captures a time average of the effect of firm productivity on wages. In an extended model with multidimensional firm heterogeneity, the firm effect would also absorb the wage effect of other firm characteristics, e.g., non-wage characteristics of jobs (amenities), which have the potential to reduce wages. We discuss how adding multidimensional heterogeneity would change our model and empirical approach in Section 7.

⁴⁰The maximum around firm bin 12 can be related to the firm performance measures shown in Figure 2, Section 4.2. The firms in bin 12 (around the 80th percentile of the productivity distribution) pay the highest average wages and are the largest firms in terms of headcount. The firms above are smaller, pay less on average, and exhibit greater labor productivity as well as lower labor shares.

⁴¹In the sample of all matches, the average wage loss from being employed in a bin 15 firm instead of a bin 12 firm for a bin 11–20 worker in terms of (deflated) log daily wages is approximately 4%, which translates into approximately 1,177 euros on an annual basis.

Figure 6: Wage-Productivity Profiles



Notes: The plots show the estimated wage-productivity profiles across firm bins for all matches (a) and new matches (b). The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. The 95% confidence bands are shown in gray. Data sources: BHP, EP, BeH.

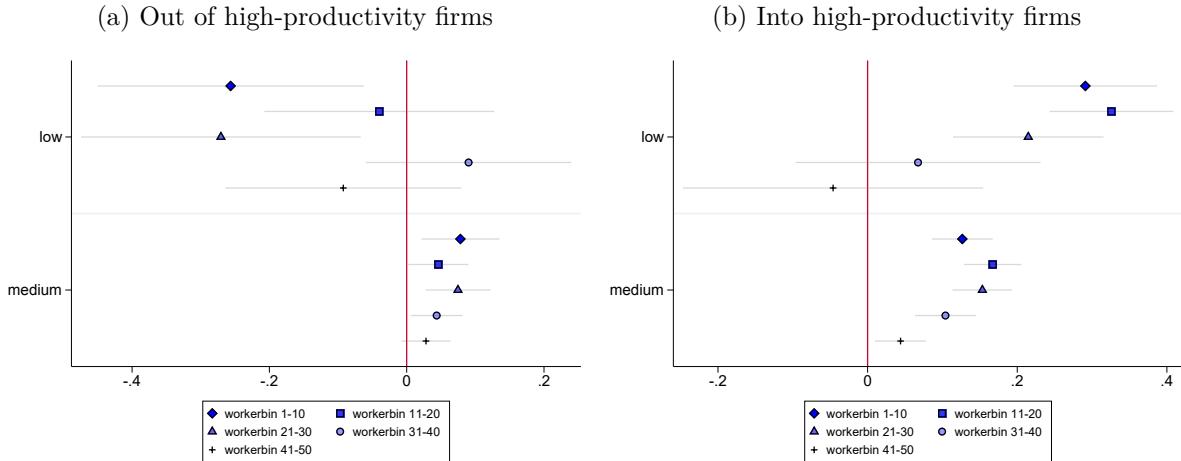
Looking at AKM residuals (Figure D.6), we observe large positive values for the matches between the most productive firms and high-ability workers. For these worker types, the positive wage residuals compensate the relatively small firm-specific pay components at high-productivity firms, so the hump shape is less pronounced. According to our model, the residual absorbs the effect of transitory productivity shocks on wages. The quantitative importance of this channel might be higher at the most productive firms, and it might also covary with worker ability. Moreover, the positive residuals could indicate that production complementarities are more complex than in our simple model. For example, high-ability workers might increase the marginal product of other worker types, which could, in turn, add to their wage. Cahuc et al. (2008) study such mechanisms in detail but they are absent in our simple model due to the assumptions we make to nest AKM. In Section 7, we explore how more complex production functions loosen the link between wages and productivity, and how they would affect our empirical approach.

Finally, we check the robustness of the empirical wage-productivity profiles. First, concerns about their validity might be related to potential sampling error in the data used to rank workers and firms. On the worker side, we use the universe of employment spells available through the German social security registers. We assess the wage-productivity profiles' sensitivity to including fewer years in the sample used to rank workers. Specifically, we rerank workers based on AKM worker effects from a shorter panel (2003–2008). The S-shape is robust; see Figure D.7. On the firm side, we rely on EP (firm survey) data, which might be prone to sampling error. To alleviate this concern, we re-estimate the wage-productivity profiles using 20 random subsamples (drawn with replacement and clustered at the firm level). Again, the S-shape is robust; see Figure D.8. Second, we confirm in Figure D.9 that the nonmonotonicities are not related to tenure effects. We re-estimate the wage–productivity profiles using the first match-year only. If anything, the wage drop at the top is more pronounced in this case. Additionally, Figure D.10 shows that firm types based on AKM firm fixed effects do not reveal any nonmonotonicities, as one would expect.

6.2 Worker Transitions

The nonmonotonicities documented in Figure 6 reflect the mean wage differences across firm types for a given set of worker types. The next step is to check whether observed worker transitions between firm bins are consistent with these wage–productivity profiles; i.e., do workers transition toward higher wages, even if this implies switching to a less-productive employer? In this case, transitions out of the most productive firms should lead to wage gains, at least for a subset of destination firm bins. Similarly, when moving up the productivity ladder, wage gains should be decreasing in the origin firm bin and might even be negative for transitions to the most productive firms.

Figure 7: Wage Changes for Observed Transitions



Notes: The plots show estimated coefficients and 95% confidence intervals (robust standard errors) from a linear regression of individual-level wage differences among transitioning workers on dummies for origin and destination firm bins. The sample consists of new matches (J2J switches, no intermittent nonemployment spell) for five groups of worker types. The depicted coefficients are for transitions out of (Panel (a)) and into (Panel (b)) high-productivity firms (bins 13–15). The vertical axis captures the destination/origin firm bin groups: low (bins 1–3) and medium (bins 4–12). Data sources: BHP, EP, BeH.

Figure 7 shows that wage changes for observed J2J transitions largely support these conjectures. We regress log-wage differences for the same five groups of worker bins on a set of origin and destination firm bin dummies.⁴² To simplify the graphical illustration, we also group firm bins into high (13–15), medium (4–12), and low (1–3) bins. Panel (a) shows the estimated wage changes for transitions out of high-productivity firms. Low- and medium-type workers experience significant wage gains of 7.8% (bins 1–10), 4.6% (bins 11–20), 7.5% (bins 21–30), and 4.4% (bins 31–40) when they transition from high-productivity to medium-productivity firms. For the highest-ability worker types, the wage differences are not significantly different from zero, which is consistent with the muted wage drop for these worker types. Wage differences for transitions from high-productivity to low-productivity firms are quite noisy and in most cases not significantly different from zero. Overall, we find that transitions down the firm productivity ladder lead to wage gains for many worker types and that even transitions into low-productivity firms do not always come with a negative wage change. These findings suggest that job mobility and sorting are guided by wages and not by firm productivity. This also explains the trend of decreasing productivity sorting at the top (see Section 5.2).

Panel (b) presents estimated wage changes for transitions into high-productivity firms in a similar manner. As conjectured, we find significant positive wage changes for most worker types, and they decrease in the origin firm type. For workers in bins 31–50, we cannot reject the hypothesis that transitioning from a low- to a high-productivity firm

⁴²We measure the difference between the wage during the last spell in the pretransition firm and the wage during the first spell in the posttransition firm.

yields no wage gain, which is in line with the S-shaped wage-productivity profiles. Low- to medium-ability workers (1–30) can expect significant wage gains of more than 20% when moving from a low-productivity to a high-productivity firm. When the origin firm is of medium productivity, wage gains are between 4.4% and 16.7% and decrease in worker ability. Based on the wage drops at the top in Figure 6, one might expect that some of these transitions lead to wage cuts. This is not the case, which reaffirms that workers tend to move toward higher wages.

In summary, wage changes for observed transitions align well with the patterns displayed in Figure 6. Workers select jobs to maximize their wages, even when this implies a downward move along the firm productivity ladder. Upward transitions also tend to yield positive wage effects. We do not observe workers accepting wage cuts to move to the most productive firms. We again test the robustness of our findings by re-estimating the wage changes using 20 random subsamples (drawn with replacement and clustered at the firm level). All findings are robust; see Figure D.11.

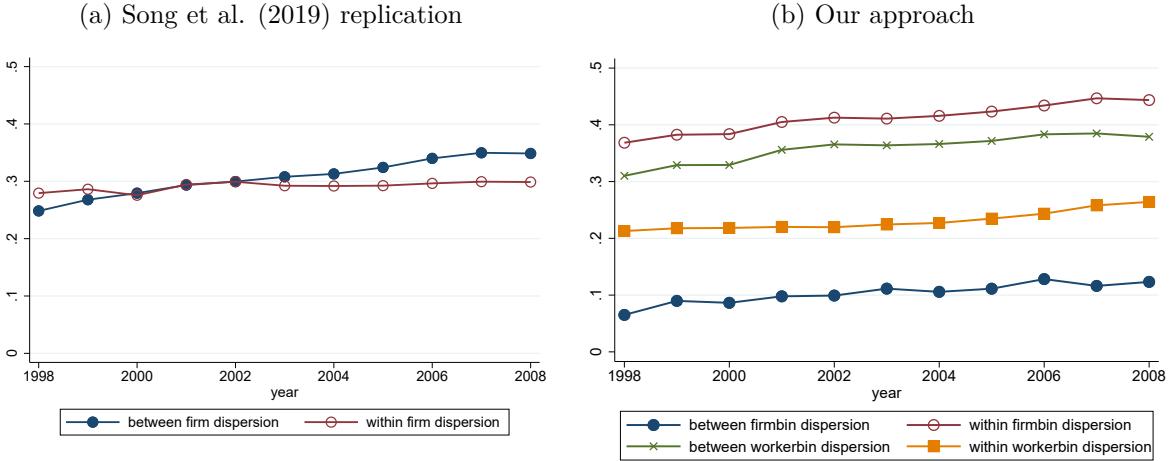
6.3 Wage Inequality

Song et al. (2019) argue that two-thirds of the increase in wage inequality in the U.S. from 1978 to 2013 can be attributed to increasing pay differences between firms. Using the AKM approach, they show that increasing wage sorting (high-wage workers into high-wage firms) and increasing segregation of workers contribute roughly equally to firms' rising contribution to wage inequality. Panel (a) of Figure 8 reveals that this trend is also present in the German data. We use establishment identifiers to decompose the variance in wages into the respective shares explained within and between establishments. Between 1998 and 2008, wage inequality grew because the between-firm component (blue line) grew by approximately 10%. The within-establishment contribution (red line) is of similar magnitude but stable over time.

In Panel (b), we decompose wage inequality based on our productivity-based firm and ability-based worker types. The wage variance between the firm bins (blue line) does increase but only by approximately 3%. It contributes little to overall inequality. The wage variance within firm bins (red line) is quantitatively more important. Worker segregation, that is, wage differences between worker bins (green line), is also a major contributing factor. Both components increase over time at a higher rate than the between-firm bin inequality. The contribution of within-worker bin inequality (orange line) is moderately large and increasing but only in the second half of our sample. We conclude that increasing pay differences between firm productivity types are likely not the main driving force behind increasing wage inequality in Germany.

To scrutinize the increasing between-worker bin and between-firm bin components, Figure D.12 shows how the wage profiles depicted in Figure 6 change between the two

Figure 8: Decomposition of Wage Dispersion over Time



Notes: Panel (a) shows yearly decompositions of the wage dispersion using establishment identifiers. Panel (b) shows yearly decompositions of the wage dispersion into the respective contributions of within- and between-estimated worker bin variance and within- and between-estimated firm bin variance. Data sources: BHP, EP, BeH.

subperiods considered earlier (1998–2002 and 2003–2008). We observe large differences in wage growth across worker types. These differences correspond to increasing worker segregation, that is, the increase in the between-worker variance component in Figure 8b. For low-type workers (bins 1–10), wages are shown to decrease in matches with all firm types above the very bottom and by more than 10% (all matches, firm bins 2–4). For the next set of worker bins (11–20), wages are shown to shrink at low-productivity firms but be relatively stable at more productive firms. For high-type workers (bins 41–50), wages are shown to increase in matches with almost all firm types but most notably with firms at the top, i.e., by more than 7% (all matches, firm bin 15). For medium-type workers (bins 21–40), wages are shown to increase at more-productive firms and decrease at less-productive firms. Notably, the nonmonotonic wage humps at the most productive firms become more pronounced over time. This can explain the decreasing sorting of high-ability workers into high-productivity firms. Moreover, the wage-productivity profiles become steeper. This implies an increase in between-firm wage inequality, which is consistent with Figure 8b.

7 Extensions

In this section, we discuss possible model extensions from two perspectives. The first is their compatibility with AKM. The second is their potential to explain nonmonotonic wage-productivity profiles. We consider OJS, general production functions, and multidimensional firm and worker heterogeneity.

On-the-job Search

The parsimonious model described in Section 2 assumes that only unemployed workers search for jobs. An important question is whether our approach of combining AKM and ACF is still applicable if the DGP features OJS. With OJS, the worker's wage could depend on previous employment and, thus, contain a history component. In this case, worker types could only be identified from wages observed directly after unemployment. To circumvent this issue when applying OJS models to the data, it is common to use functional form assumptions that separate worker ability from labor market history. For example, Postel-Vinay and Robin (2002) show that the wage is log-linear in the worker type in the sequential auction model if the utility function is of the CRRA type. Cahuc et al. (2006) and Bagger et al. (2014b) use similar assumptions.

We make a comparable functional form assumption in our model to obtain a log-linear wage equation. To separate the worker type from the integral term in equation (6), we assume that worker ability units are perfectly substitutable at the firm level. This implies that the marginal product of labor scales with the worker type x . The close relatedness leads us to suspect that a version of our model with OJS and sequential auctions can, for specific production functions, retain a wage equation that is log-linear in the worker type. In this case, concerns regarding wage histories do not apply, and AKM can be used to rank workers in the presence of OJS.⁴³

The second question is whether an OJS version of our model could generate the non-monotonic, hump-shaped wage-productivity profiles documented in this paper. Taking the sequential auction model as the point of departure, note that the productivities of both the poaching firm and the previous employer affect the wage. On the one hand, the wage increases in the productivity of the previous employer. On the other hand, it decreases in the productivity of the poaching firm due to the option value of future wage gains, which entices workers to join productive firms at relatively low wages. Thus, the overall effect of firm productivity on wages is non-linear. However, the magnitude of the second effect decreases in firm productivity because it becomes increasingly unlikely to meet an even more productive firm as workers move up the productivity ladder. This implies a U-shaped relation between firm productivity and wages, which is not in line with the hump-shaped relation we document based on our estimated firm productivity ranks. In summary, while canonical OJS models can yield a log-linear (in the worker type) wage equation, the implied relation between wages and firm productivity is at odds with the non-monotonic, hump-shaped wage-productivity profiles we find.

⁴³DiAddario et al. (2021) show that an augmented AKM model with origin- and destination-firm effects resembles the wage equation of sequential auction models for specific utility functions. They find a small covariance between the worker fixed effect and the origin-firm effect in Italian data, which supports the separability of worker ability and labor market history.

General Production Functions

Our model follows Cahuc et al. (2008) but adds heterogeneity in firm productivity and worker-firm complementarities. In the production function (equation 2), firm productivity augments the effect of worker ability at the match level. Thus, the positive effect of higher productivity on output is dampened by decreasing returns to labor. This implies, as shown in Appendix A.4, that wages increase in firm productivity at a decreasing rate. As a consequence, although wages flatten out toward the top of the productivity distribution, the simple production function used to motivate our empirical strategy is not in line with a hump-shaped relation between productivity and wages.

Cahuc et al. (2008), who embed the Stole and Zwiebel (1996) intrafirm bargaining framework into the canonical search-and-matching model, explore production functions that are more general than the simple case we use to nest AKM. They show that complementarities among worker types can yield diverse wage effects. The reason is that firms exploit decreasing returns and strategically overemploy workers to reduce their marginal product and pay lower wages. With complementarities among worker types, employing another worker of a given type can increase the wage of other worker types whose marginal products increase as a result of the new hire.⁴⁴ This mechanism could overcompensate the negative wage effect of decreasing returns and interact with firm productivity, heterogeneous worker bargaining powers, or the firm's capital stock.

Such mechanisms seem flexible enough to generate a non-monotonic relation between productivity and wages and, additionally, explain why the hump shape is least pronounced for high-ability workers. With a more complex production function, the wage equation would most likely be nonlinear in the worker type, and AKM would no longer be nested; however, more flexible methods to rank workers (and firms) based on unobserved heterogeneity are available in the literature. For example, Bonhomme et al. (2019) explicitly allow for non-linear wage equations. For future research, it seems promising to explore how their method could be combined with ACF-style production functions estimations.

Multidimensional Heterogeneity

First, we discuss multidimensional *firm* heterogeneity. Specifically, we consider heterogeneous amenities in addition to productivity differences. We also touch upon *worker* heterogeneity in preferences for amenities or outside options, e.g., due to networks.

Amenities, i.e., positively-valued nonwage characteristics of jobs are a prominent example of firm characteristics that give rise to wage differences between equally skilled workers (Rosen, 1986; Mortensen, 2003). For amenities to explain the relatively low wages at the most productive firms, these firms would have to have a higher level of

⁴⁴Considering complementarities among worker types provides a link to the growing literature on coworker sorting (e.g., Mas and Moretti, 2009; Cornelissen et al., 2017; Herkenhoff et al., 2018; Jarosch et al., 2021).

amenities than somewhat less productive firms. To test this hypothesis, one needs a firm-level measure of amenities.⁴⁵

The EP survey, which is our source of firm data, contains a number of questions whose answers are indicative of the prevalence of amenities. Thus, we use them to construct a firm-level amenity index.⁴⁶ We regress this index on productivity ranks, controlling for industry (2-digit) and firm size (headcount). The most productive firms indeed exhibit the highest level of amenities; see Figure D.13. A specific amenity that can be related to firm productivity is job security. High-productivity firms might be relatively unlikely to lay off workers, and this perceived job security could entice workers to accept lower wages (as in Jarosch, 2021). Using the EP data, we compute the rate of employer-initiated separations. The most productive firms have the lowest firing rates, see Figure D.14.⁴⁷

Based on both the amenity index and the measure of job security, we find evidence that high-productivity firms exhibit a high level of amenities. This can explain our findings. High-productivity firms with many amenities pay lower wages than some less productive firms with fewer amenities. However, less productive firms pay low wages despite offering few amenities, which suggests that wages depend on both firm productivity and amenities (and worker ability). To set up a model with two-dimensional firm heterogeneity, important issues include how the level of amenities is determined, how it influences wages, and how these mechanisms impact the applicability of the empirical methods we use.

The provision of amenities might be part of the firm's pay policy (as in Sorkin, 2018). Firms could, for example, invest into amenities after a favorable productivity shock. This would be consistent with the positive correlation between amenities and productivity reported above. Whether AKM can be applied in this setting again depends on the production function. With homogeneous worker preferences for amenities and assuming that amenities do not directly affect the marginal product of labor, nothing changes. The wage effect of amenities would be absorbed by the firm-fixed effect. With heterogeneous preferences for amenities⁴⁸, i.e., two-dimensional worker heterogeneity, the composition of the workforce may depend on amenities (as in Lamadon et al., 2022).⁴⁹ If the composition

⁴⁵Such data are scarce. Thus, most papers rely on “revealed preference”-type arguments to study amenities (e.g., Sorkin, 2018; Taber and Vejlin, 2020).

⁴⁶The index is based on (up to) 15 survey questions that relate to the flexibility of working hours, working time accounts, parental benefits, support for young females, equal opportunity agreements, equity participation, and the prevalence of on-the-job training. We define an indicator variable that is 1 if the respective amenity exists and 0 otherwise. For each establishment year, we sum up and divide by the total number of indicators for that year. The mean (variance) of the index is 0.27 (0.08).

⁴⁷This is consistent with the findings of Jarosch (2021), who studies worker mobility and the consequences of job loss using German Social Security data. In his model, jobs differ in terms of productivity and job security (separation rates).

⁴⁸Taber and Vejlin (2020) find that heterogeneous preferences for amenities are a quantitatively important source of wage variation in Danish data.

⁴⁹Preference heterogeneity is a key feature of Lamadon et al. (2022), who combine amenities with monopsonistic wage-setting in a model of imperfect competition. Consistent with our findings, these authors find that high-productivity firms pay relatively low wages.

of the workforce does not matter, as in our assumed case of perfectly substitutable ability units, then nothing changes. However, if heterogeneous preferences for amenities are combined with richer complementarities in the production function (see the previous subsection), then the marginal product of a given worker type could change with amenities because they might attract other workers that make the former worker more productive. This case would again call for a more flexible approach to rank workers. With similar implications, one could allow for heterogeneity in the workers' outside options, e.g., due to coworker networks or referrals (Dustmann et al., 2015; Caldwell and Harmon, 2019).⁵⁰

For the firm side, the key issue is whether amenities change the productivity ranking. Even if the composition of the workforce is affected by amenities, taking into account ability heterogeneity when constructing labor inputs incorporates this effect. Beyond this, amenities do not affect output. They are not an input to the production process. Thus, adding amenities as a second dimension of firm heterogeneity to the model underlying the productivity estimation procedure should not alter the implied firm ranking. Reassuringly, we can empirically verify that this is the case.⁵¹

8 Conclusions

In this paper, we present a parsimonious search-and-matching model that elucidates the minimal set of assumptions necessary to combine AKM and ACF. We show that the model is compatible with both the discrete-time model of dynamically optimizing firms underlying ACF and the log-linear wage equation underlying AKM. Following Cahuc et al. (2008), the model features matching frictions, decreasing returns, multiworker firms, and intrafirm bargaining. To study sorting, we add firm heterogeneity and worker-firm complementarities to this framework.

Based on estimated worker ability (inferred from AKM worker effects) and firm productivity (inferred using ACF), we study productivity sorting, wages across firm productivity types, and inequality trends in the German labor market. Our analysis reveals a number of novel empirical facts. Productivity sorting is positive, low, and relatively stable over time. At the most productive firms, sorting decreases as high-ability workers become more likely to work at somewhat less productive firms that pay higher wages. We show that worker transitions are on average associated with higher wages, including when workers move down the firm productivity ladder. As we show, this also implies that

⁵⁰This type of heterogeneity could also explain wage differences conditional on firm productivity, and, thus, influence how pronounced the hump shape is. For our empirical strategy, considering differences in workers' outside options would be comparable to allowing for heterogeneous amenity preferences. As long as one is willing to assume that such differences do not affect the workers' contribution to output, nothing changes. However, with richer complementarities in the production function and an influence of networks on the firms' hiring decisions, one needs a more flexible approach to rank workers.

⁵¹We add the amenity index as a control variable in the production function estimation. The firm ranking is virtually unaffected by this addition. These results are available upon request.

the sorting into high-productivity firms is quantitatively less important for rising wage inequality than wage sorting.

Our results imply that wages are not globally monotonically increasing in firm productivity. This finding poses a challenge for widely-used empirical models of wage dispersion. Moreover, if workers move away from high-productivity firms to increase their wages, a side effect of increasing wage sorting could be decreasing allocative efficiency and lower aggregate output. Thus, studying productivity sorting is a useful complement to the well-established wage-based analysis of sorting.

We discuss several model extensions in regard to their potential to match the facts we document and their compatibility with the AKM approach. Canonical OJS models are compatible with log-linear (in the worker type) wage equations but are incompatible with the hump-shaped relation between wages and productivity. With more general production functions, our framework could generate the hump shape, but AKM would no longer be nested. Models with multidimensional firm and/or worker heterogeneity can generate a hump shape with relatively simple production functions and thus remain compatible with AKM.

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Online Appendix

(not for publication)

A Model Details

A.1 Matching Technology

Due to random search, firms cannot target their vacancies to specific worker types. They post vacancies v subject to a productivity-dependent cost $c(\Omega)$. Meetings are generated by a Cobb-Douglas matching function with constant returns to scale (Pissarides, 2000; Petrongolo and Pissarides, 2001). Without loss of generality, let worker ability x and firm productivity Ω be distributed uniformly on $[0, 1]$. Meeting rates are functions of aggregate labor market tightness, $\theta = V/U$, where $V = \int g_v(\Omega) d\Omega$ and $U = \int g_u(x) dx$ are the aggregated numbers of vacancies and unemployed workers, respectively. $q_v(\theta)$ is the rate at which firms meet workers and $q_u(\theta)$ is the rate at which unemployed workers meet vacancies.

$g_v(\Omega)$ is the PDF of vacancies at type Ω firms and $g_u(x)$ is the PDF of unemployed workers of type x . In this environment, a match is not guaranteed conditional on meeting. Suppose a type x worker meets a productivity Ω firm. Both parties may prefer to continue searching in case the match surplus $S(x, L, \Omega)$, defined below, is negative. Note that the surplus depends on the worker type, the firm type, and the total composite labor input. In steady state, existing matches can only end at the exogenous rate δ . Endogenous separations may happen out of steady state in case a shock to firm productivity reduces the surplus with specific worker types below zero.

A.2 The Firm's Problem

In the outlined environment, the profit flow of a firm with productivity Ω solves the following Bellman equation. The firm's problem is to maximize output given the current composite labor input and productivity less the total wage bill and hiring costs. Current employment is a state variable. The firm controls future discounted profits by posting costly vacancies, given its expected evolution of productivity:

$$\Pi(L, \Omega) = \max_v \left\{ F(L, \Omega) - \sum_{x=1}^n w(x, L, \Omega) L_x - vc(\Omega) + \beta \int \Pi(L', \Omega') dG(\Omega' | \Omega) \right\}. \quad (\text{A.1})$$

This profit flow is maximized subject to n constraints that capture the evolution of employment for every worker type x at the firm:

$$L'_x = (1 - \delta)L_x + vq_v(\theta) \frac{g_u(x)}{U} \mu(x, L, \Omega) \quad \forall x. \quad (\text{A.2})$$

$g_u(x)/U$ is the probability that conditional on meeting the worker is of type x . The indicator function $\mu(x, L, \Omega)$ returns the value one if a match of a type x worker and productivity Ω firm with composite labor input L has a strictly positive surplus and zero otherwise. In case $\mu(x, L, \Omega) = 0$ no additional type x workers are hired.

The match surplus is defined as

$$S(x, L, \Omega) = J_x(L, \Omega) + E(x, \Omega) - U(x), \quad (\text{A.3})$$

and depends on three option value equations defined below. Thus, the indicator $\mu(x, L, \Omega)$ is defined as

$$\mu(x, \Omega) = \begin{cases} 1 & \text{if } S(x, L, \Omega) > 0 \\ 0 & \text{if } S(x, L, \Omega) \leq 0. \end{cases} \quad (\text{A.4})$$

Below, we indicate $\mu(x, L, \Omega) = 1$ ($\mu(x, L, \Omega_j) = 0$) by writing $\mu^+(x, L, \Omega)$ ($\mu^-(x, L, \Omega)$).

Optimality Conditions

We closely follow Cahuc et al. (2008) and define the marginal value of an additional worker of type x at a firm with productivity Ω and workforce L as

$$J_x(L, \Omega) = \frac{\partial \Pi(L, \Omega)}{\partial L_x}. \quad (\text{A.5})$$

The marginal product of type x labor (MPL) at a productivity- Ω firm is

$$F_x(L, \Omega) = \frac{\partial F(L, \Omega)}{\partial L_x}. \quad (\text{A.6})$$

The FOC of the maximization problem (A.1) with respect to v is

$$0 = -c(\Omega) + q_v(\theta) \frac{g_u(x)}{U} \mu(x, L, \Omega) J_x(L', \Omega'). \quad (\text{A.7})$$

The envelope theorem implies

$$J_x(L, \Omega) = \frac{\partial F(L, \Omega)}{\partial L_x} - \sum_{k=1}^n L_k \frac{\partial w_k(L, \Omega)}{\partial L_x} - w(x, L, \Omega) + \beta(1 - \delta) J_x(L', \Omega'). \quad (\text{A.8})$$

Assuming a steady state where $L' = L$ and $\Omega' = \Omega$, (A.7) can be rewritten as

$$J_x(L, \Omega) = \frac{c(\Omega)}{q_v(\theta) \frac{g_u(x)}{U} \mu^+(x, L, \Omega)}, \quad (\text{A.9})$$

so for every worker type within the firm's matching set ($\mu(x, L, \Omega) = 1$), the marginal profit is equal to the expected recruitment cost at the optimal level of employment. In

case a type x worker is not part of the firm's matching set, $\mu(x, L, \Omega) = 0$, marginal profits are undefined. Integrating the worker type out of (A.9) yields the firm's expected marginal profit of posting a vacancy:

$$J(L, \Omega) = \frac{c(\Omega)}{q_v(\theta) \int \frac{g_u(x)}{U} \mu^+(x, L, \Omega) dx}. \quad (\text{A.10})$$

Applying the steady state assumption to (A.8) yields

$$J_x(L, \Omega) = \frac{F_x(L, \Omega) - w(x, L, \Omega) - \sum_{k=1}^n L_k \frac{\partial w(k, L, \Omega)}{\partial L_x}}{1 - \beta(1 - \delta)}, \quad (\text{A.11})$$

so the marginal profit can also be expressed as the discounted marginal product, net of the individual wage and net of the effect of the marginal hire on the total wage bill.

Equating (A.9) and (A.11), one gets

$$F_x(L, \Omega) = w(x, L, \Omega) + \frac{c(\Omega)(1 - \beta(1 - \delta))}{q_v(\theta) \frac{g_u(x)}{U} \mu(x, L, \Omega)} + \sum_{k=1}^n L_k \frac{\partial w(k, L, \Omega)}{\partial L_x}, \quad (\text{A.12})$$

so the MPL of worker type x at a (L, Ω) firm equals the wage plus expected turnover costs and the marginal effect of the additional worker on the total wage bill.

A.3 Wage Determination

To derive the wage equation, we rely on the Nash sharing rule

$$\frac{\alpha}{1 - \alpha} J_x(L, \Omega) = E(x, L, \Omega) - U(x), \quad (\text{A.13})$$

where $\alpha \in (0, 1)$ is the workers' common bargaining parameter. The RHS captures the worker's surplus of working at a firm with productivity Ω and workforce L relative to the worker's outside option, the value of unemployment, $U(x)$. The firm's surplus consists of the marginal profits of hiring an additional worker of type x , $J_x(L, \Omega)$, as defined above. Its threat point is to fire the worker and renegotiate wages with all other employees (Stole and Zwiebel, 1996). Following Cahuc et al. (2008), we assume that wages are continuously and instantaneously (re)negotiated, so L is fixed during (re)negotiations.

In steady state, the worker's value of employment is

$$E(x, L, \Omega) = w(x, L, \Omega) + \underbrace{\beta \delta U(x)}_{\text{separation}} + \underbrace{\beta(1 - \delta) E(x, L, \Omega)}_{\text{continued employment}}. \quad (\text{A.14})$$

The value of unemployment is

$$U(x) = b(x) + \underbrace{\beta(1 - q_u(\theta))U(x)}_{\text{no meeting}} + \underbrace{\beta q_u(\theta) \int \frac{g_v(\Omega)}{V} \mu^+(x, L, \Omega) E(x, L, \Omega) d\Omega}_{\text{successful match}} \\ + \underbrace{\beta q_u(\theta) U(x) \int \frac{g_v(\Omega)}{V} \mu^-(x, L, \Omega) d\Omega}_{\text{meet unacceptable firm}}, \quad (\text{A.15})$$

where $b(x)$ is the flow value of unemployment, e.g. the value of increased leisure, home production or unemployment insurance benefits.

Next, we compute the difference $E(x, L, \Omega) - U(x)$ to be plugged into equation (A.13):

$$E(x, L, \Omega) - U(x) = w(x, L, \Omega) + \beta \delta U(x) + \beta(1 - \delta) E(x, L, \Omega) - U(x). \quad (\text{A.16})$$

After adding and subtracting $\beta U(x)$, this can be rearranged to

$$E(x, L, \Omega) - U(x) = \frac{w(x, L, \Omega) - (1 - \beta)U(x)}{1 - \beta(1 - \delta)}, \quad (\text{A.17})$$

which can be combined with (A.13) to get

$$\frac{\alpha}{1 - \alpha} J_x(L, \Omega) = \frac{w(x, L, \Omega) - (1 - \beta)U(x)}{1 - \beta(1 - \delta)}. \quad (\text{A.18})$$

Finally, substituting marginal profits according to equation (A.11) and rearranging yields the wage bargaining outcome:

$$w(x, L, \Omega) = \alpha \left(F_x(L, \Omega) - \sum_{k=1}^n L_k \frac{\partial w(k, L, \Omega)}{\partial L_x} \right) + (1 - \alpha)(1 - \beta)U(x). \quad (\text{A.19})$$

Due to our assumption of perfect substitutability of worker ability units at the firm level, the inframarginal adjustment term $\sum_{k=1}^n L_k \frac{\partial w(k, L, \Omega)}{\partial L_x}$ solely reflects decreasing returns and is unambiguously negative. Moreover, it does not vary with x . This yields the following simplified differential equation

$$w(x, L, \Omega) = \alpha \left(F_x(L, \Omega) - L \frac{\partial w(x, L, \Omega)}{\partial L} \right) + (1 - \alpha)(1 - \beta)U(x), \quad (\text{A.20})$$

which we can solve following the steps for the “single labor case” described in the Appendix of Cahuc et al. (2008). For the single steps and detailed technical assumptions, see their equations (B.1)–(B.6), p 961–962. The solution is

$$w(x, L, \Omega) = (1 - \alpha)(1 - \beta)U(x) + \int_0^1 z^{\frac{1-\alpha}{\alpha}} F_x(Lz, \Omega) dz. \quad (\text{A.21})$$

A.4 Wages, Labor Demand, and Productivity

It is worth considering how wages vary with firm productivity. Partially differentiating the wage equation (A.21) twice with respect to Ω yields

$$\frac{\partial w(x, L, \Omega)}{\partial \Omega} = \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F_x(Lz, \Omega)}{\Omega} dz > 0, \quad (\text{A.22})$$

$$\frac{\partial^2 w(x, L, \Omega)}{\partial \Omega^2} = (\beta_l - 1) \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F_x(Lz, \Omega)}{\Omega^2} dz < 0, \quad (\text{A.23})$$

which implies that wages increase in firm productivity, but at a decreasing rate. This model property is a direct consequence of the assumed production function, which exhibits a complementarity at the match level, see equation (2). Worker ability and firm productivity jointly determine how much one unit of labor of a given type contributes to output, but the total composite labor input is subject to decreasing returns. In other words, the positive effect of higher productivity on output is dampened by decreasing returns to labor, and this effect carries over into wages. For this reason, the second derivative of the wage with respect to productivity is negative.

Based on the solution for the wage equation in (A.21), we can also compute the derivative in the last term of (A.12) for the single labor case:

$$F_x(L, \Omega) = w(x, L, \Omega) + \frac{c(\Omega)(1 - \beta(1 - \delta))}{q_v(\theta) \frac{g_u(x)}{U} \mu(x, L, \Omega)} + (\beta_l - 1) \int_0^1 z^{\frac{1-\alpha}{\alpha} + \beta_l - 1} F_x(Lz, \Omega) dz. \quad (\text{A.24})$$

This expression pins down labor demand because it equalizes the marginal product to the labor cost, which consist of the wage, turnover costs, and the employment effect on the wage. The factor we write in front of the integral is negative for $\beta_l < 1$. This implies that firms can reduce their labor cost by expanding employment in the presence of decreasing returns (overemployment, as in Stole and Zwiebel, 1996; Smith, 1999; Cahuc et al., 2008).

A.5 Linearity of the Wage Equation

Plugging in the worker-firm-specific MPL (3) into (A.21) yields

$$w(x, L, \Omega) = (1 - \alpha)(1 - \beta)U(x) + x \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz, \Omega)}{Lz} dz, \quad (\text{A.25})$$

so the integral expression is only scaled by, and hence linear in, worker ability x .

To establish that the full wage equation is linear in x , the outside option $U(x)$ has to be linear in x , too. Consider the outside option according to equation (A.15). A straightforward assumption to ensure that $U(x)$ is indeed linear in x is that all worker types' matching sets cover the whole type space. In other words, conditional on meeting, there are no unacceptable firms. This claim is easily verified empirically, see Section 5.

In the model, this implies that $\mu(x, L, \Omega) = 1$ holds for all potential matches and, thus, the last term of (A.15) is zero. Now rearrange equation (A.14) such that

$$E(x, L, \Omega) = \frac{w(x, L, \Omega) + \beta\delta U(x)}{1 - \beta(1 - \delta)}. \quad (\text{A.26})$$

Under our assumption, this can be plugged into equation (A.15) to yield an expression in the wage and the outside option only.

$$U(x) = b(x) + \beta(1 - q_u(\theta))U(x) + \beta q_u(\theta) \int \frac{g_v(\Omega)}{V} \frac{w(x, L, \Omega) + \beta\delta U(x)}{1 - \beta(1 - \delta)} d\Omega. \quad (\text{A.27})$$

Plugging in our solution for the wage, equation (A.25), into this expression and collecting the $U(x)$ terms in front of the integral yields

$$\begin{aligned} U(x) &= b(x) + \beta(1 - q_u(\theta))U(x) + \beta q_u(\theta) \frac{1 - \alpha + \beta(\alpha + \delta - 1)}{1 - \beta(1 - \delta)} U(x) \int \frac{g_v(\Omega)}{V} d\Omega \\ &\quad + \frac{\beta q_u(\theta)}{1 - \beta(1 - \delta)} \int \frac{g_v(\Omega)}{V} x \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz, \Omega)}{Lz} dz d\Omega, \end{aligned} \quad (\text{A.28})$$

where $\int \frac{g_v(\Omega)}{V} d\Omega = 1$. After collecting all $U(x)$ terms on the LHS and dividing, we get the following expression for $U(x)$:

$$U(x) = \frac{(1 - \beta(1 - \delta))b(x) + x \frac{\beta q_u(\theta)}{1 - \beta} \int \frac{g_v(\Omega)}{V} \int_0^1 z^{\frac{1-\alpha}{\alpha}} \beta_l \frac{F(Lz, \Omega)}{Lz} dz d\Omega}{1 - \beta(1 - \delta - \alpha q_u(\theta))}, \quad (\text{A.29})$$

where worker ability x can be written in front of both integral sign. Thus, for $U(x)$ to be linear in x , we additionally have to assume that the workers flow value of unemployment, $b(x)$ is proportional to x (i.e. $b(x) \propto x$), which is a standard assumption used in, e.g., Postel-Vinay and Robin (2002).

B Details of Data Preparation

B.1 Wage Imputation

In the BeH data, earnings are right censored at the contribution assessment ceiling ('Beitragsbemessungsgrenze'). This earning limit is given by the statutory pension fund and is adjusted annually due to changes in earnings. First we deflate daily wages by using the CPI. Then, in each year, we identify censored wage observations by comparing wages with the contribution assessment ceiling. We define a wage observation as censored whenever the reported wage is higher than 99% of the censoring threshold.

Following CHK and Dustmann et al. (2009), we fit a series of Tobit regression to impute the right tail of the wage distribution. We estimate Tobit regressions by year,

sex, education and age group. In all these regressions we additionally control for the exact age, the mean log wage in other years, the fraction of censored wages in other years, the number of full time employees at the current establishment and its square, an indicator for large firms, the mean years of schooling and the fraction of university graduates at the current establishment, the mean log wage of co-workers and the fraction of co-workers with censored wages, an indicator for individuals observed only one year, an indicator for employees in one-worker establishments, and an indicator for regions. We assume that the error term is normally distributed but each education and age category can have a different variance. For each year, we impute censored wages as the sum of the predicted wage and a random component which is computed based on standard error of the forecast. This component is drawn from separate normal distributions with mean zero and the different variances for each education and age category.

B.2 Education Imputation

The employee education information is reported by employers after every year and whenever a job ends. Its quality may suffer because employers do not face consequences for non- and misreporting. However, the existence of a reporting rule allows for corrections. It prescribes that only the highest educational degree of an employee needs to be reported. Therefore the individual educational attainment should not decline over consecutive job spells. We follow the imputation procedure suggested by Fitzenberger et al. (2006) which exploits this reporting rule by assuming that there is any over-reporting in the data.

The original education variable in the BeH distinguishes six categories, plus an additional category for missing information. Following CHK, we convert these into the following categories: (0) missing; (1) primary/lower secondary or intermediate school leaving certificate, or equivalent, with no vocational training; (2) primary/lower secondary or intermediate school leaving certificate, or equivalent, with a vocational training; (3) upper secondary school certificate with or without a vocational training; (4) some university degree. Within each job we assign the modal education category observed for an individual in the years he/she is at the same job.

C Production Function Estimation Details

We start from the production function to be estimated, which corresponds to equation (10). Lower-case letters indicate logs, and here we ignore firm-level controls.

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + \epsilon_{jt}, \quad (\text{C.1})$$

Following ACF, the control function we use is the demand for intermediate inputs *conditional on labor*⁵² and capital:

$$m_{jt} = f_t(l_{jt}, k_{jt}, \omega_{jt}). \quad (\text{C.2})$$

A natural interpretation of this demand function is that, conditional on both labor and capital, more productive firms use more intermediate goods in production. Intermediate inputs m_{jt} are chosen either simultaneously with the labor input l_{jt} or afterwards, reflecting that labor is a dynamic input.

Under the assumption of strict monotonicity in ω_{jt} , we can invert equation (C.2) to get $\omega_{jt} = f_t^{-1}(l_{jt}, k_{jt}, m_{jt})$, and substitute ω_{jt} out of (C.1):

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + f_t^{-1}(l_{jt}, k_{jt}, m_{jt}) + \epsilon_{jt} = \Phi_t(l_{jt}, k_{jt}, m_{jt}) + \epsilon_{jt}. \quad (\text{C.3})$$

Following ACF, we adopt a two-stage procedure. First, value added is regressed on a polynomial approximation of $\Phi_t(l_{jt}, k_{jt}, m_{jt})$. This does not identify any of the parameters but delivers an estimate $\hat{\Phi}_t(l_{jt}, k_{jt}, m_{jt})$ and separates productivity ω_{jt} from transitory shocks absorbed by ϵ_{jt} .

To derive the second stage estimation equation, we use the assumption that firm productivity follows an AR(1) process (equation 12). Thus, the conditional expectation of ω_{jt} formed at $t - 1$ is $E[\omega_{jt} | \omega_{j,t-1}] = \rho\omega_{j,t-1}$ and the productivity innovation is denoted ξ_{jt} . Additionally, the following holds: $\omega_{j,t-1} = y_{j,t-1} - \beta_0 - \beta_l l_{j,t-1} - \beta_k k_{j,t-1} - \epsilon_{j,t-1}$ (from C.1) and $\epsilon_{j,t-1} = y_{j,t-1} - \hat{\Phi}_{t-1}(l_{j,t-1}, k_{j,t-1}, m_{j,t-1})$ (from C.3, with the first stage estimate plugged in). Using these expressions along with the conditional expectation of firm productivity in equation (C.1) yields:

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_k k_{jt} + \rho(\hat{\Phi}_{t-1}(\cdot) - \beta_0 - \beta_l l_{j,t-1} - \beta_k k_{j,t-1}) + \xi_{jt} + \epsilon_{jt}, \quad (\text{C.4})$$

where $\xi_{jt} + \epsilon_{jt}$ is a composite error term. Following ACF, we use GMM and four (unconditional) second stage moment conditions to identify the four parameters $(\beta_0, \beta_l, \beta_k, \rho)$:

$$E \left[\left(\underbrace{y_{jt} - \beta_0 - \beta_l l_{jt} - \beta_k k_{jt} - \rho(\hat{\Phi}_{t-1}(\cdot) - \beta_0 - \beta_l l_{j,t-1} - \beta_k k_{j,t-1})}_{\xi_{jt} + \epsilon_{jt}} \right) \otimes \begin{pmatrix} 1 \\ k_{jt} \\ \hat{\Phi}_{t-1}(\cdot) \\ l_{j,t-1} \end{pmatrix} \right] = 0 \quad (\text{C.5})$$

⁵²ACF use of the conditional (on labor) input demand function to address a problem of functional dependence and improve identification of the labor input parameter relative to Olley and Pakes (1996) and Levinsohn and Petrin (2003).

The first three moment conditions are standard in the literature and identical to Levinsohn and Petrin (2003): (i) The composite error term $\xi_{jt} + \epsilon_{jt}$ is zero in expectation. (ii) The capital stock is predetermined, recall equation (11), and therefore uncorrelated with the error term. (iii) The polynomial representation of the control function, estimated in the first stage and evaluated at $t - 1$, $\hat{\Phi}_{t-1}$, is also uncorrelated with the composite error term because the intermediate input choice of the last period should be uncorrelated with the current productivity innovation and transitory shock.

The fourth moment condition is specific to the ACF approach and reflects that labor is a dynamic input. Only $l_{j,t-1}$ needs to be uncorrelated with the error term, while l_{jt} is allowed to correlate freely with ξ_{jt} . In other words, labor is chosen in response to the current firm productivity realization, subject to across-firm differences in adjustment costs and wage setting (firm-specific, serially correlated, unobserved shocks to the price of labor). The conditional (on l_{jt}) intermediate input demand does not depend on these costs/shocks. ACF do not allow for shocks to the price of intermediate inputs (due to the scalar unobservable assumption, which is required for the inversion of $f_t(l_{jt}, k_{jt}, \omega_{jt})$).⁵³ As ACF summarize in their discussion, Olley and Pakes (1996) cannot allow for any input price shocks, Levinsohn and Petrin (2003) can allow for shocks relating to k_{jt} but not to l_{jt} and m_{jt} , and ACF can allow for shocks relating to k_{jt} and l_{jt} , but not to m_{jt} .

Finally, note that not including m_{jt} in the production function implies that we estimate a value-added production function (value-added is simply defined as revenues minus the costs of intermediate inputs). One structural interpretation of this is that the production function is Leontief in the intermediate input, and this intermediate input needs to be proportional to output. As explained in ACF, their approach is designed for value-added production functions and is not suitable to identify the parameters of gross output production functions without further assumptions (see also Bond and Söderbom (2005) and Gandhi et al. (2020)).

⁵³Arguably, intermediate inputs are often commodities implying only little price variation across firms.

D Additional Results

D.1 Rank Correlations

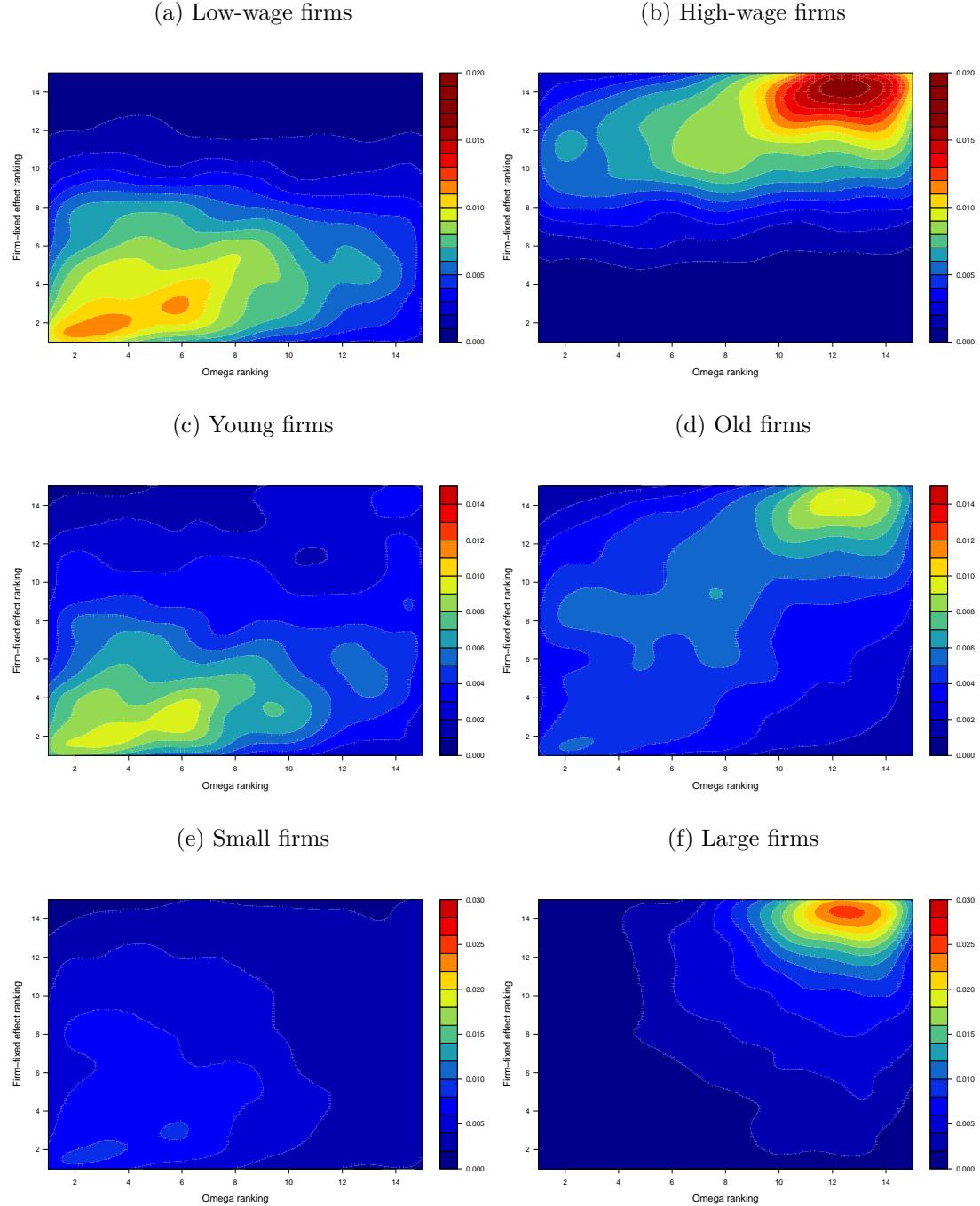
Table D.1: Spearman rank correlation coefficients and numbers of observations for different time intervals and samples

	All Matches	New Matches	Out of Nonemp.	Job-to-Job
1998-2008	0.065 (4,695,108)	0.124 (1,656,280)	0.132 (601,954)	0.110 (1,082,460)
1998-2002	0.055 (2,182,011)	0.139 (474,341)	0.141 (174,310)	0.120 (305,339)
2003-2008	0.074 (2,513,097)	0.118 (1,181,939)	0.129 (427,644)	0.107 (777,121)
1998	0.013 (311,861)	–	–	–
1999	0.046 (338,125)	0.140 (35,865)	0.133 (15,094)	0.129 (20,771)
2000	0.048 (493,323)	0.107 (107,740)	0.108 (41,731)	0.090 (66,009)
2001	0.073 (536,559)	0.148 (158,351)	0.142 (56,627)	0.137 (101,724)
2002	0.077 (502,143)	0.152 (172,385)	0.165 (60,777)	0.134 (111,608)
2003	0.080 (470,279)	0.146 (180,623)	0.156 (64,926)	0.131 (115,697)
2004	0.065 (458,467)	0.114 (191,207)	0.129 (67,932)	0.100 (123,275)
2005	0.081 (428,122)	0.129 (197,755)	0.136 (69,890)	0.118 (127,865)
2006	0.101 (415,153)	0.146 (206,984)	0.139 (74,340)	0.142 (132,644)
2007	0.051 (391,535)	0.084 (209,567)	0.096 (77,811)	0.074 (131,756)
2008	0.063 (349,541)	0.094 (195,803)	0.119 (72,001)	0.076 (123,802)

Notes: In all cells, we test the null hypothesis that worker and firm bins are statistically independent. All rank correlation coefficients are different from 0 at 1% level of significance. Rounded to 3 decimal places. Numbers of observations (matches according to the respective definition) are reported in brackets. Data sources: BHP, EP, BeH.

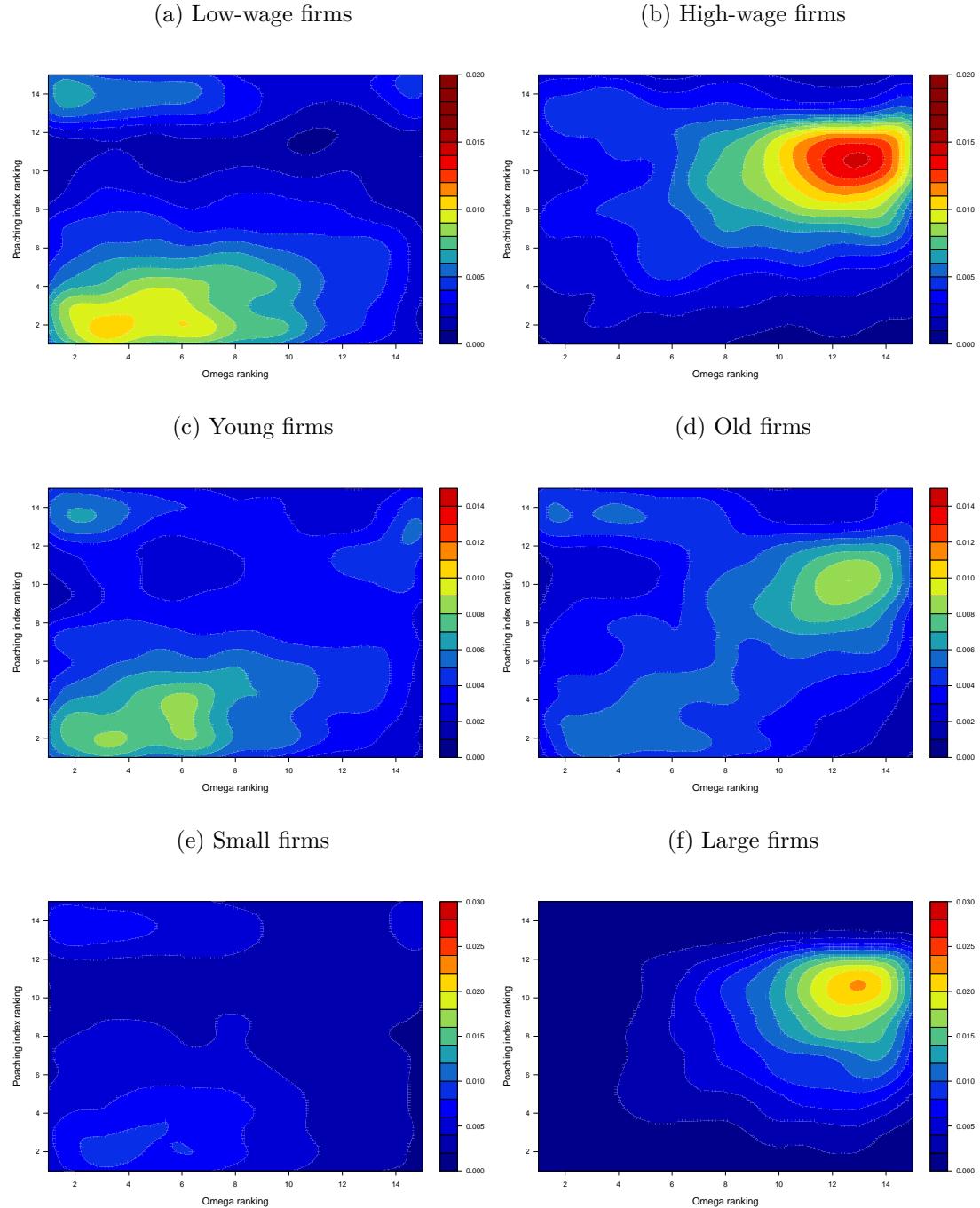
D.2 Ranking Comparisons

Figure D.1: Comparison of Productivity-based and fixed-effect-based Firm Ranking by Wages, Age, and Size



Notes: The six plots depict contours of the joint empirical distributions of firm years across combinations between the omega ranking (15 bins) and AKM firm-effect ranks (15 bins). In Panels (a) and (b), high-wage firms pay more than the grand mean of all firm-level mean wages and low-wage firms pay less. In Panels (c) and (d), the age of young firms is less than 15 years, old firms are 15 years and older. In Panels (e) and (f), small firms have less than 100 employees, large firms have more. Data sources: BHP, EP, BeH.

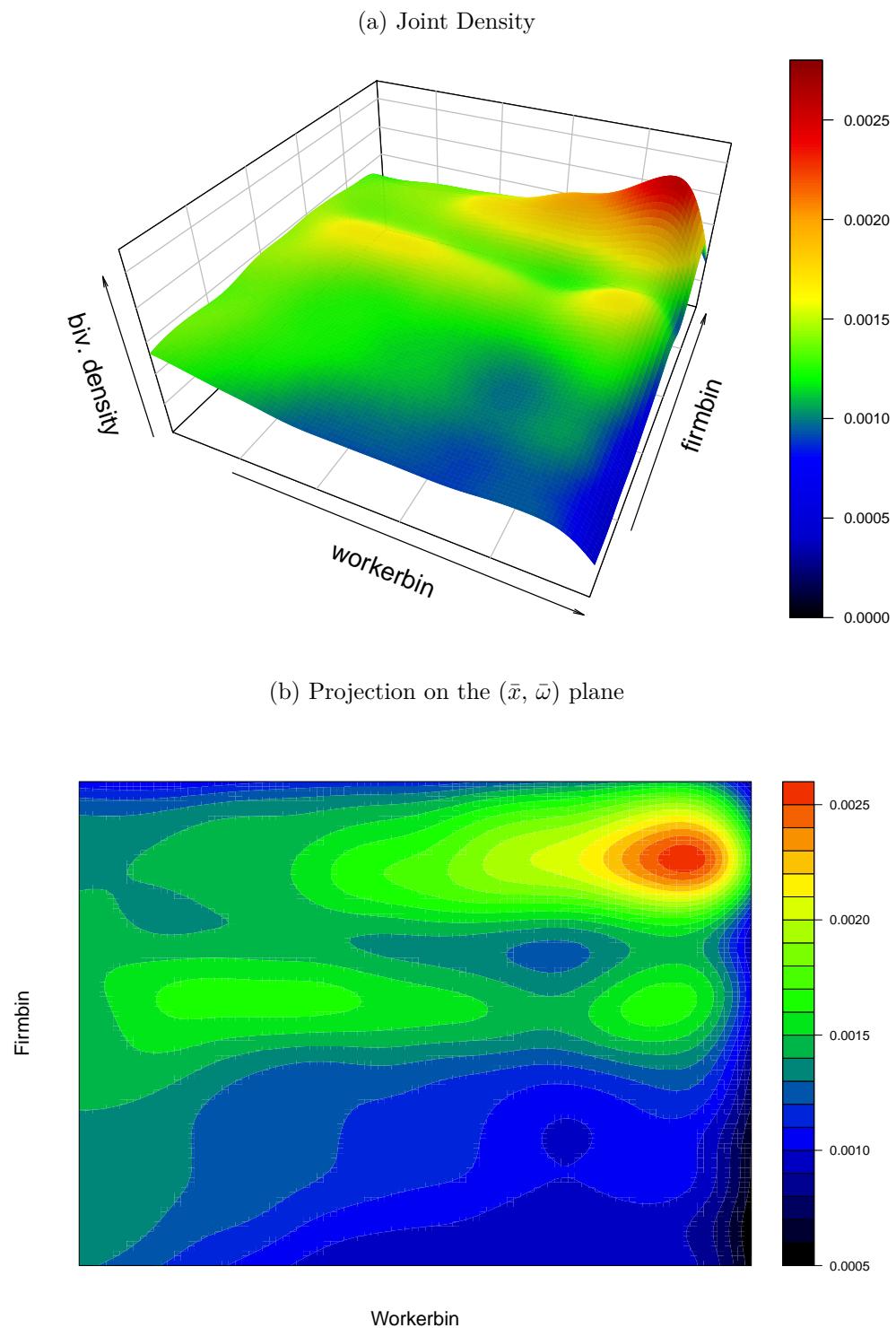
Figure D.2: Comparison of Productivity-based Firm Ranking and poaching-index-based Firm Ranking by Wages, Age, and Size



Notes: The six plots depict contours of the joint empirical distributions of firm years across combinations between the omega ranking (15 bins) and poaching index ranks (15 bins). In Panels (a) and (b), high-wage firms pay more than the grand mean of all firm-level mean wages and low-wage firms pay less. In Panels (c) and (d), the age of young firms is less than 15 years, old firms are 15 years and older. In Panels (e) and (f), small firms have less than 100 employees, large firms have more. Data sources: BHP, EP, BeH.

D.3 Joint Distribution of Matches

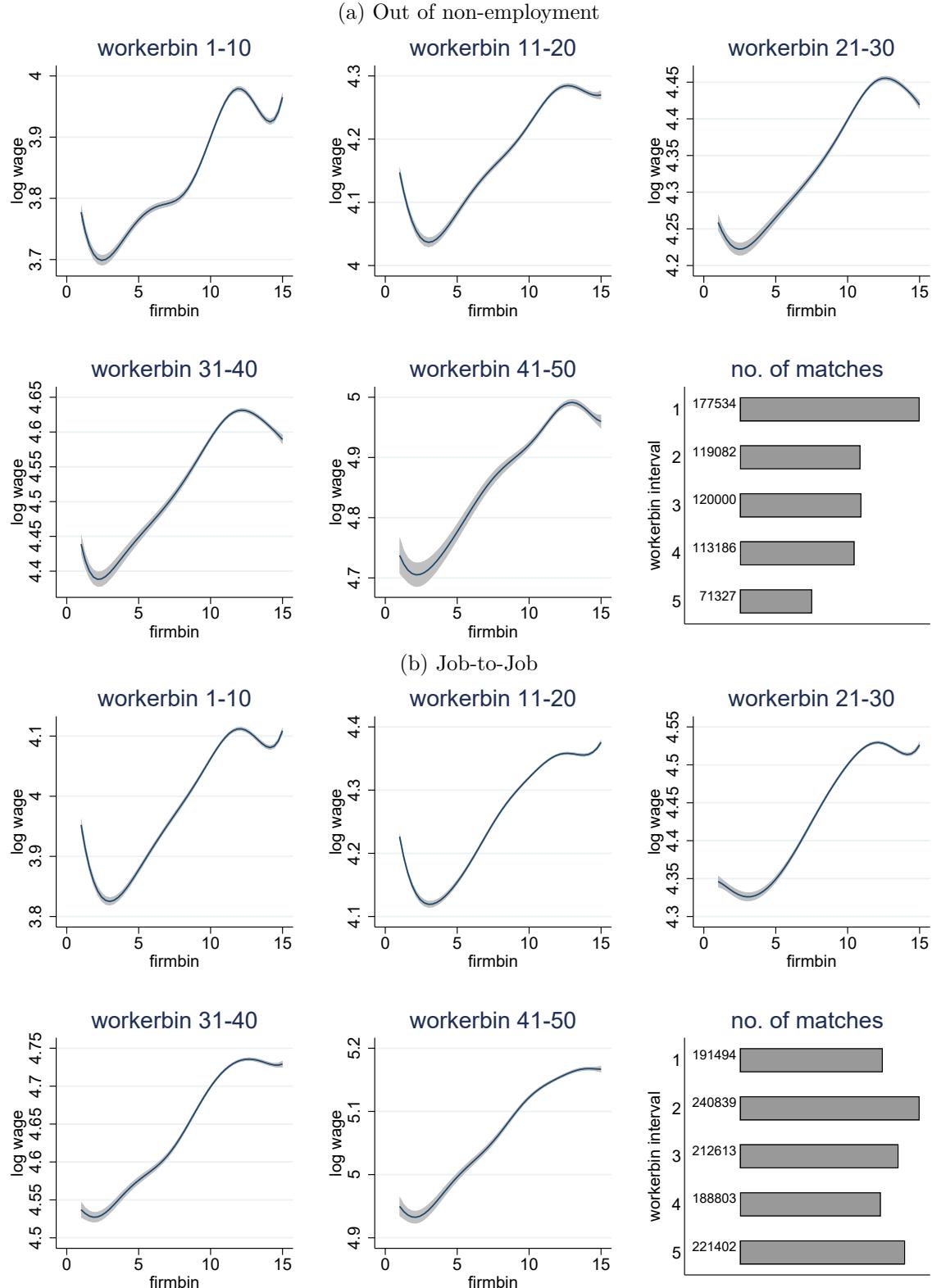
Figure D.3: Joint Distribution for all Worker-Firm Type Combinations (1998–2008)



Notes: The plots shows estimate joint kernel density of matches and it projection on the $(\bar{x}, \bar{\omega})$ plane for all combinations of worker and firm types in the sample of all matches on a grid with dimensions 50×15 (#worker types \times #firm types). Data sources: BHP, EP, BeH.

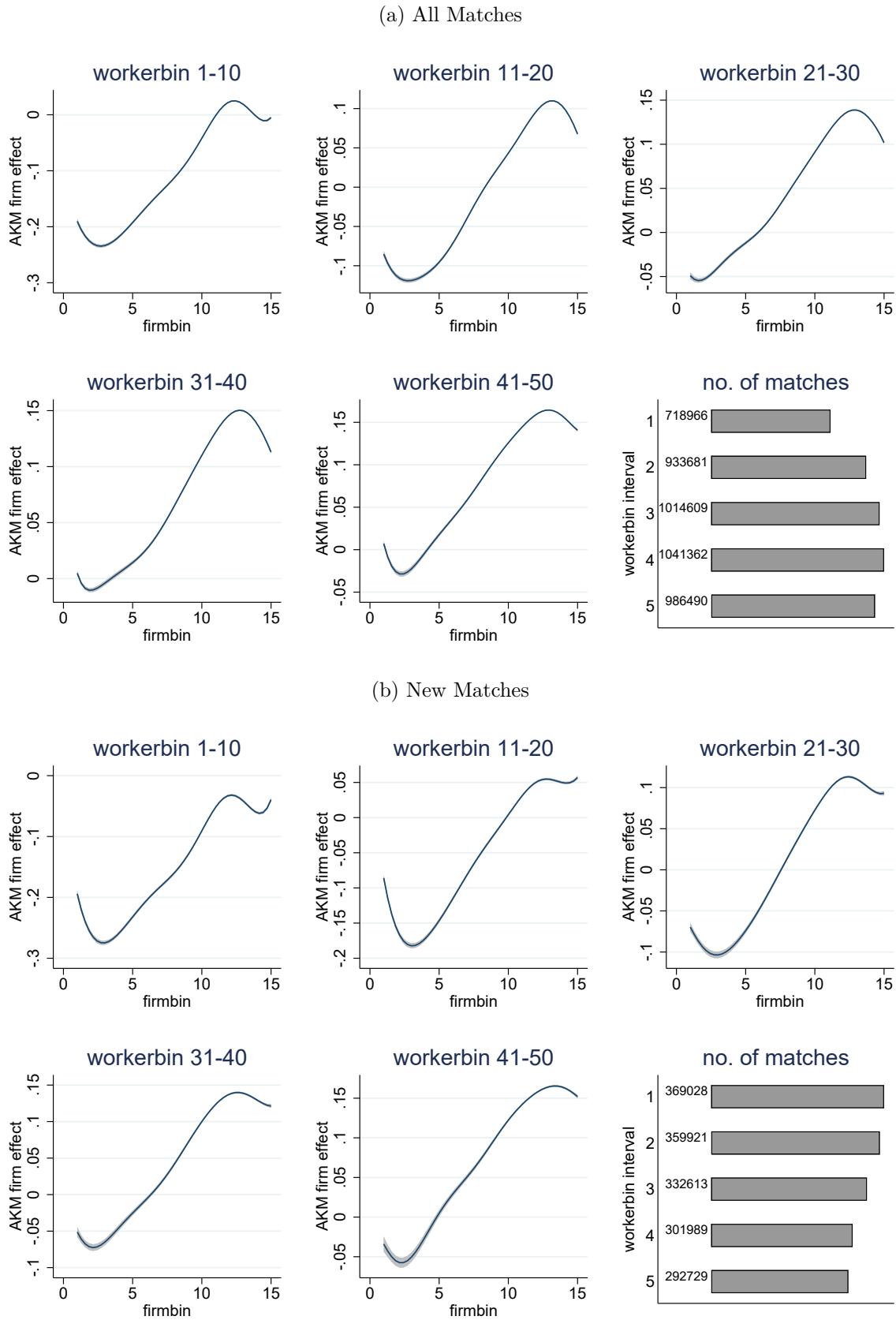
D.4 Wages and Transitions

Figure D.4: Wage-Productivity Profiles, new Matches



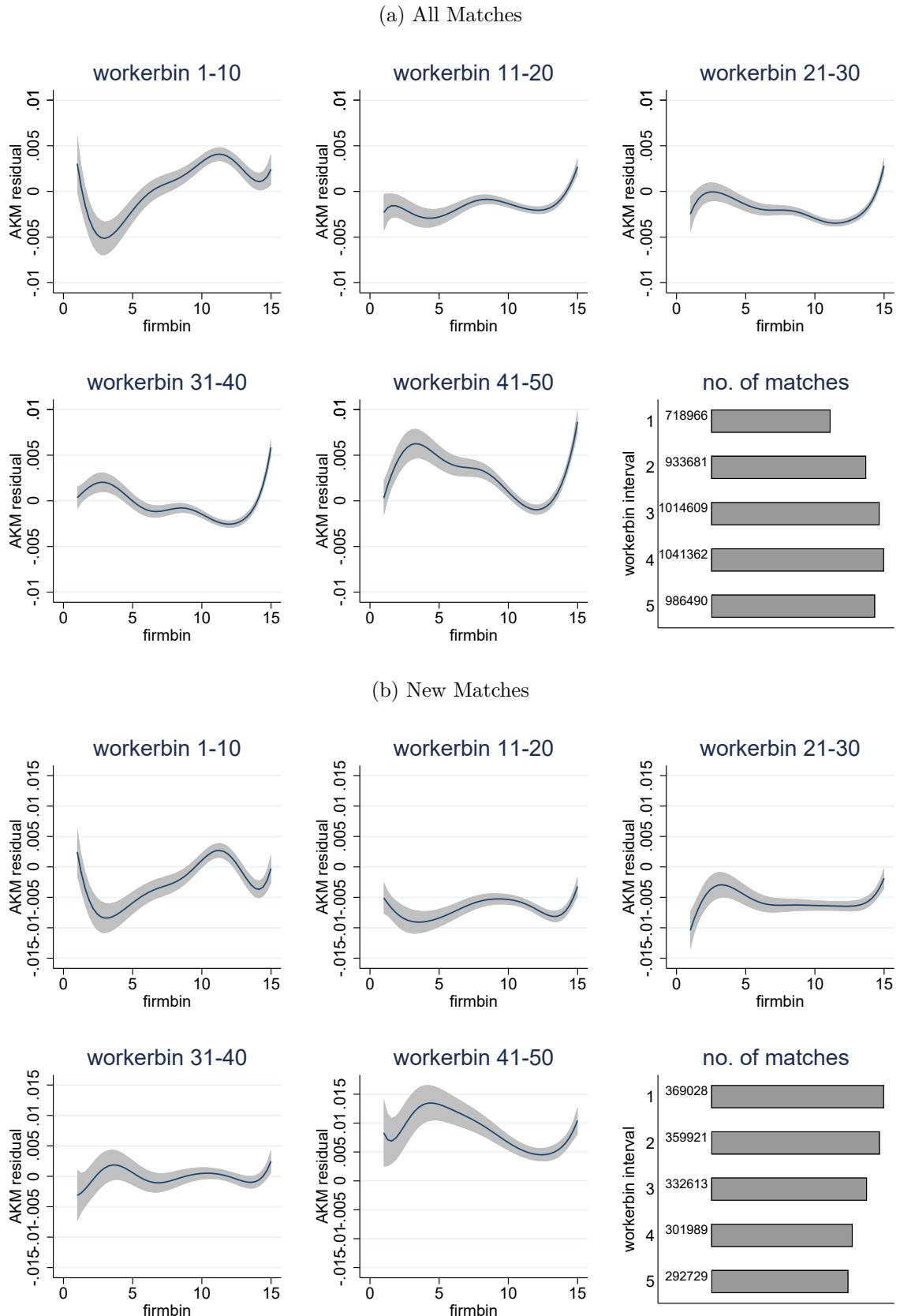
Notes: Plots show estimated wage-productivity profiles across firm bins for new matches out of non-employment and job-to-job. The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.5: Wage-Productivity Profiles, Firm-Fixed Effects only



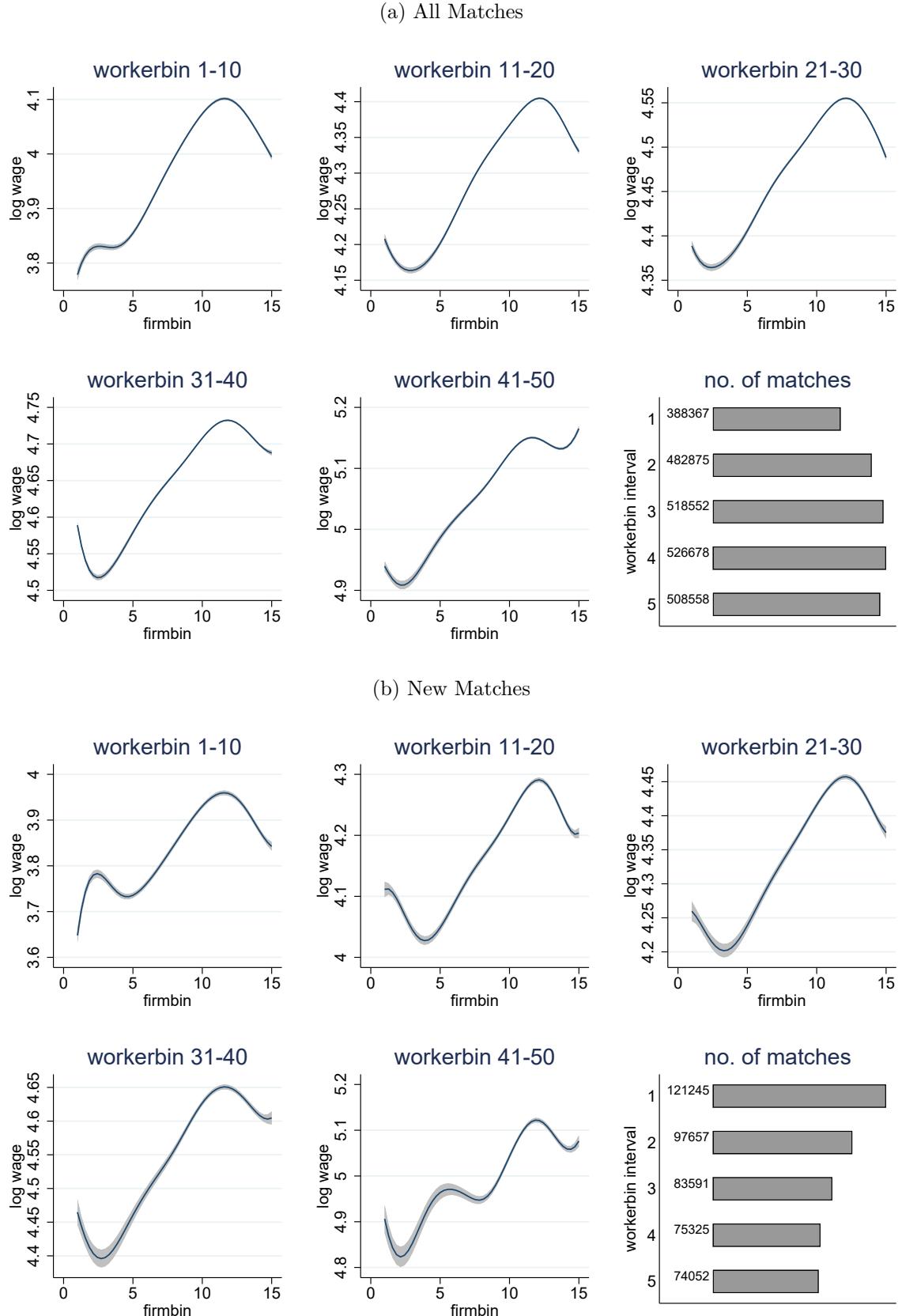
Notes: Plots show estimated wage-productivity profiles across firm bins for all matches and new matches when using AKM firm-fixed effects as the wage variable. The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.6: Wage-Productivity Profiles, Residuals only



Notes: Plots show estimated wage-productivity profiles across firm bins for all matches and new matches when using AKM residuals as the wage variable. The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

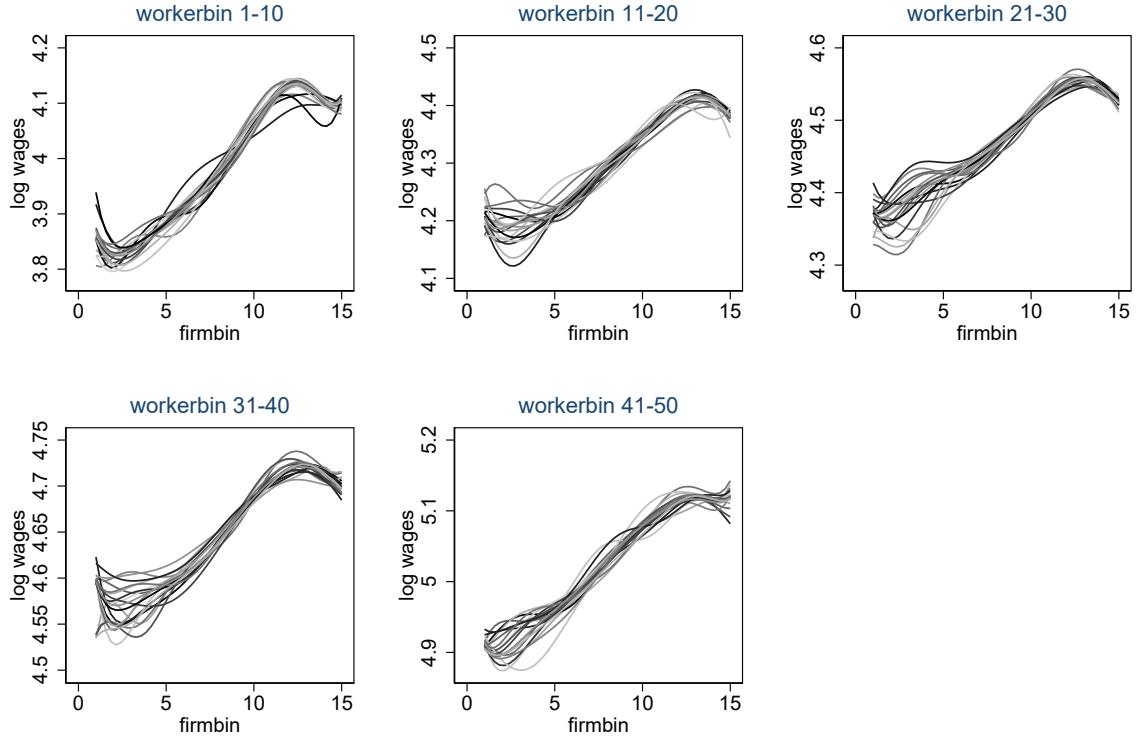
Figure D.7: Wage-Productivity Profiles, Worker Subsamples



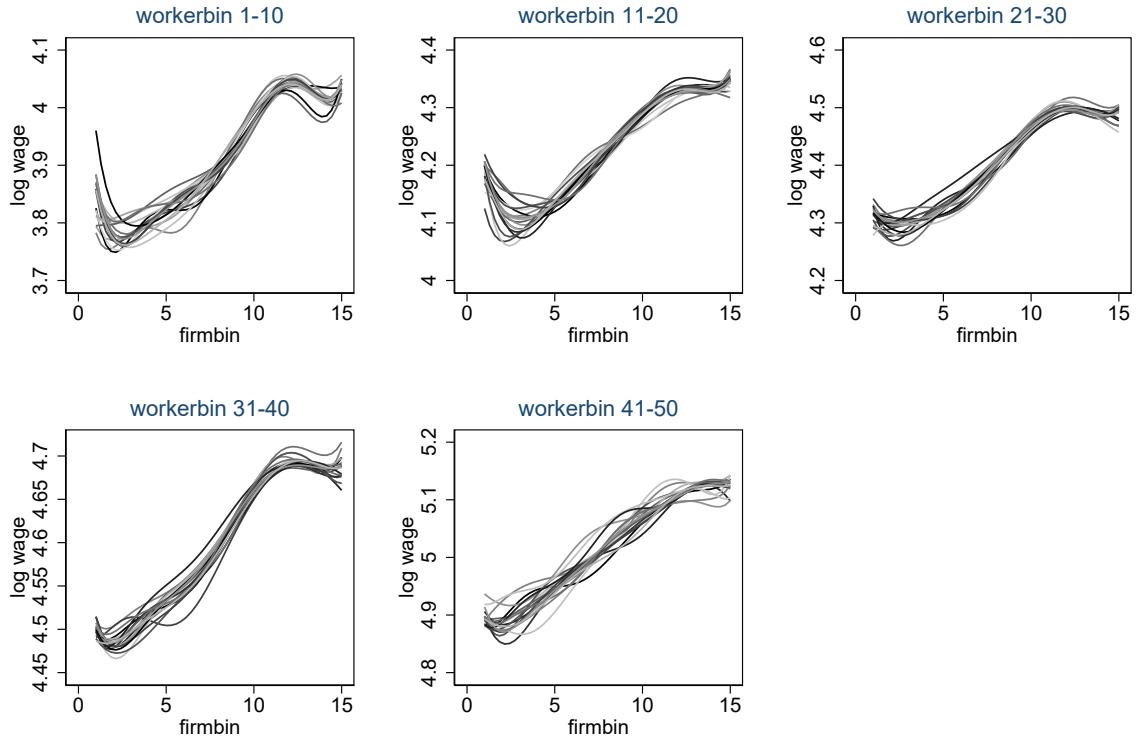
Notes: The plots show the estimated wage-productivity profiles across firm bins for all matches (a) and new matches (b). We use an alternative worker ranking based on AKM worker effects from a shorter panel (2003–2008). The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. The 95% confidence bands are in gray. Data sources: BHP, EP, BeH.

Figure D.8: Wage-Productivity Profiles, Firm Subsamples

(a) All Matches

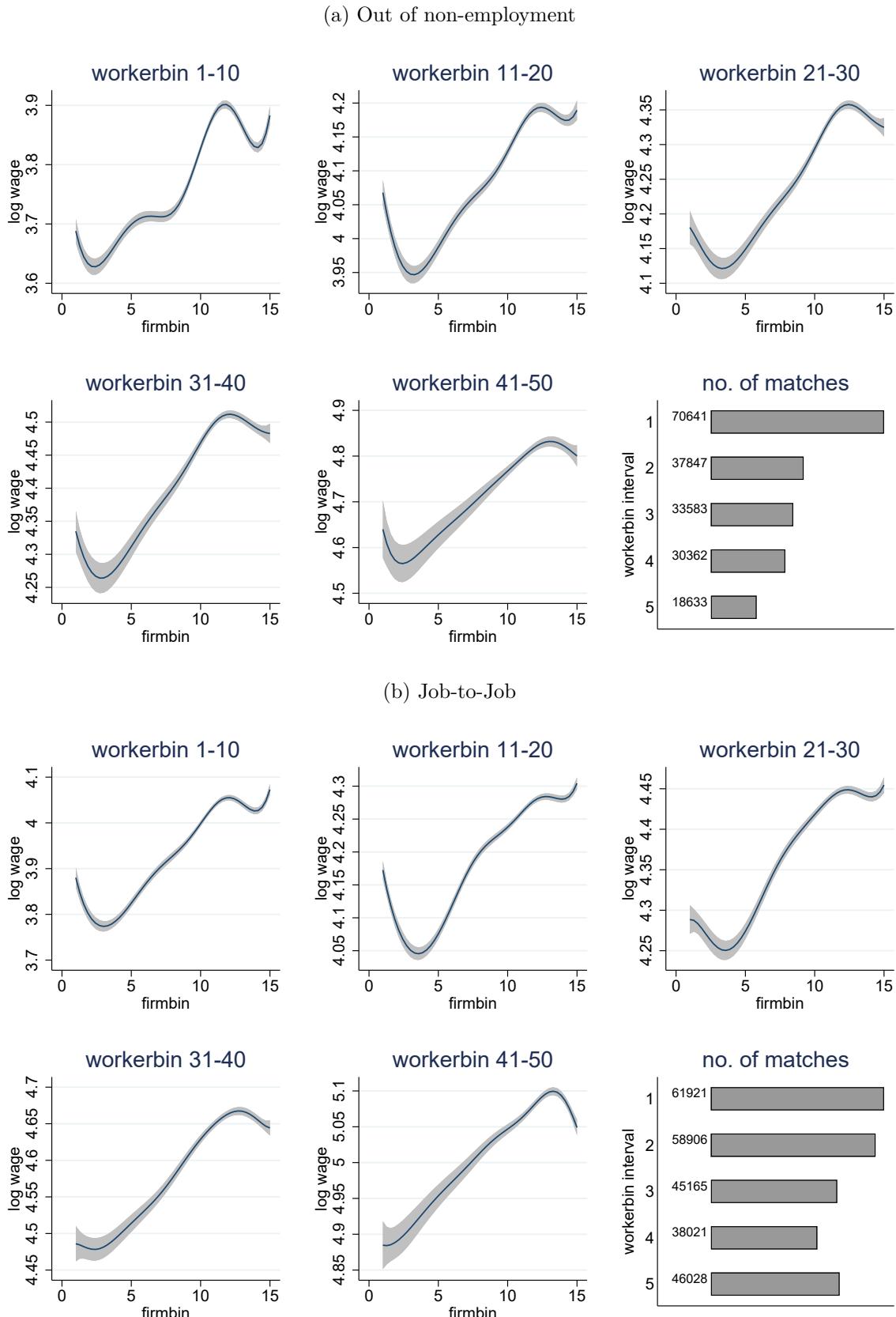


(b) New Matches



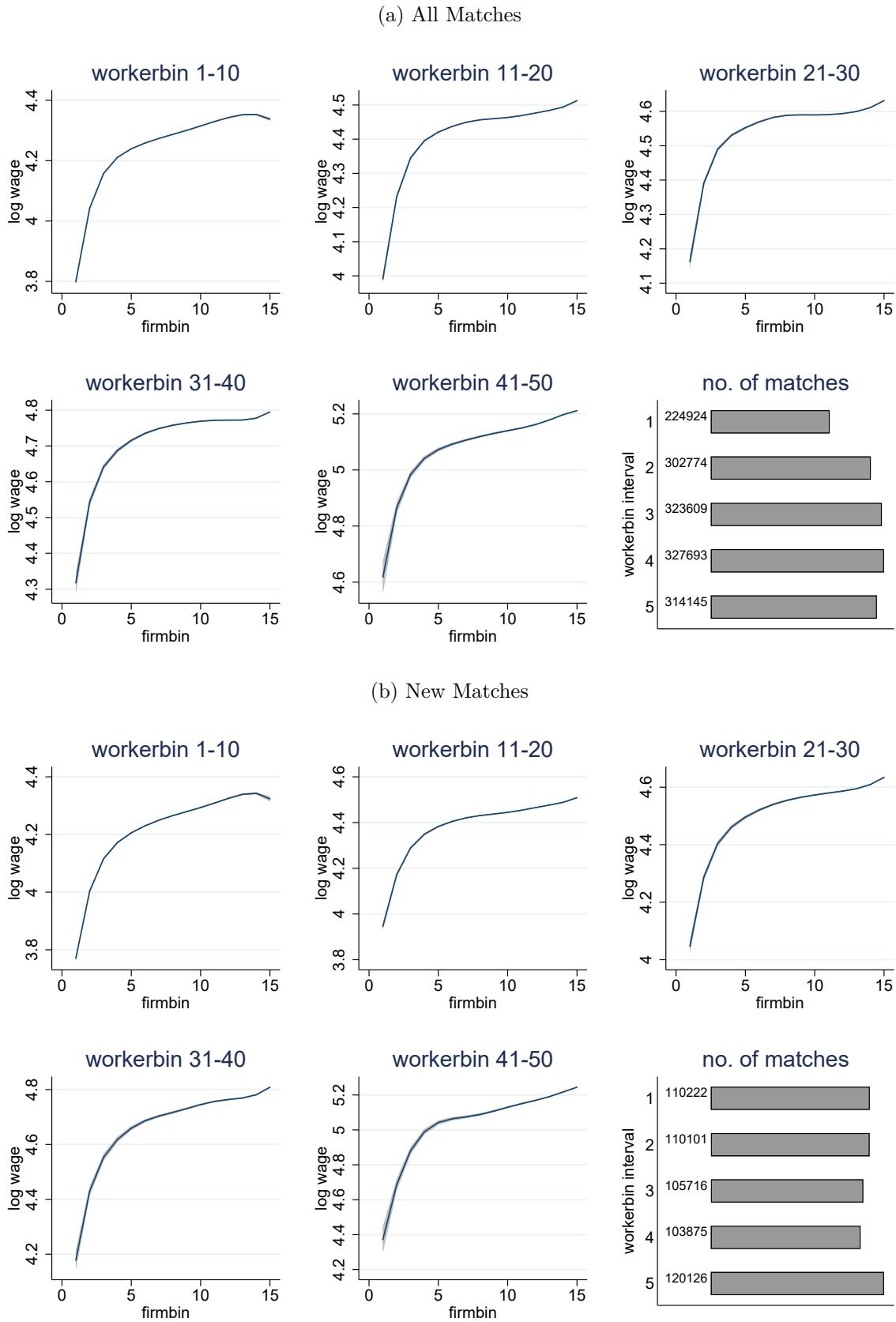
Notes: Plots show estimated wage-productivity profiles across firm bins constructed from 20 random sub-samples (with replacement, clustered by firm-years) from the original sample (size= N). The sub-sample size M randomly varies between $0.5 * N < M \leq N$. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.9: Wage-Productivity Profiles, new Matches, first Match-Year only



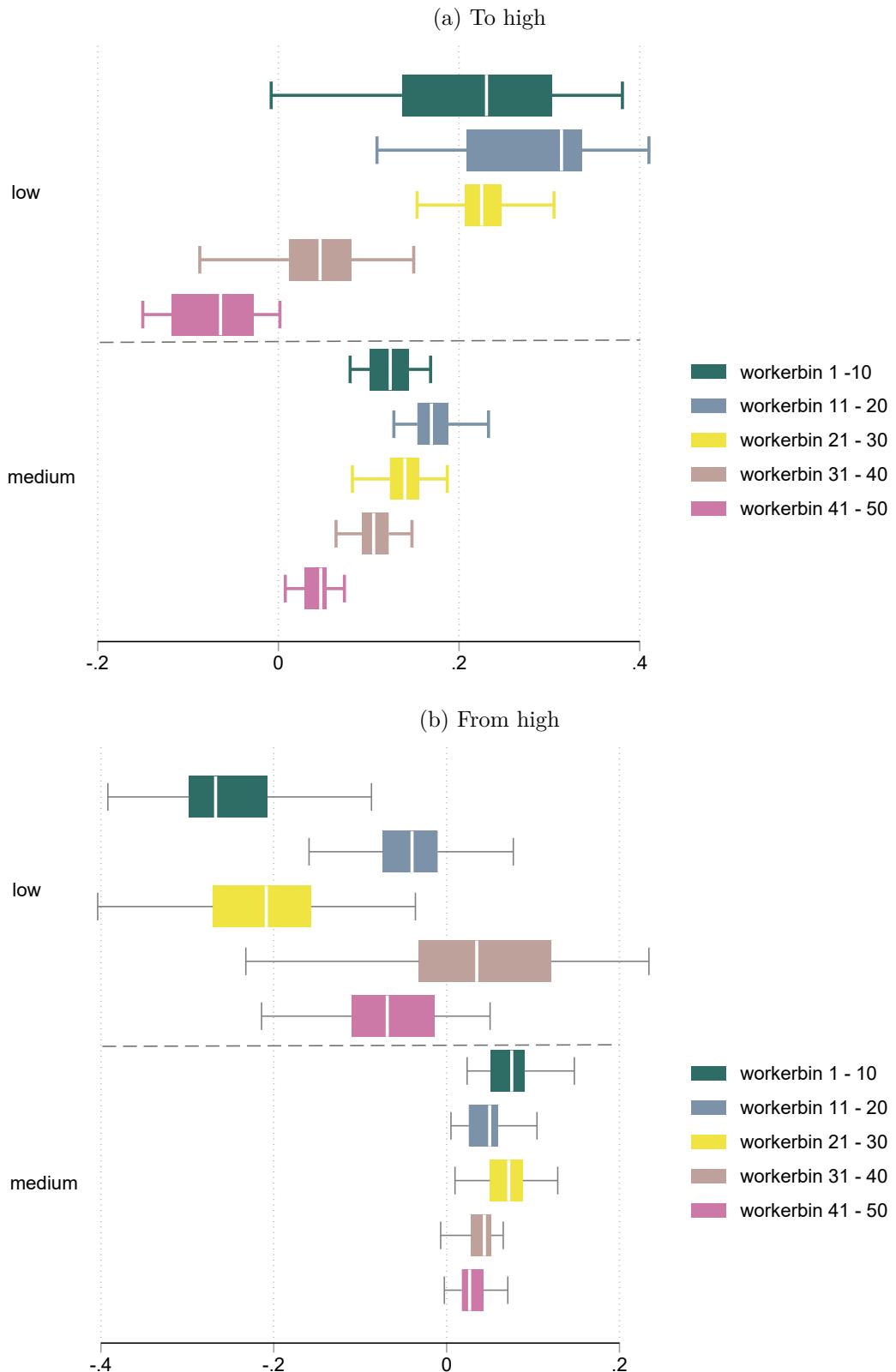
Notes: Plots show estimated wage-productivity profiles across firm bins constructed using only the first yearly wage observation for all matches and new matches to remove tenure effects. The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.10: Wage Profiles with AKM-based Firm Types



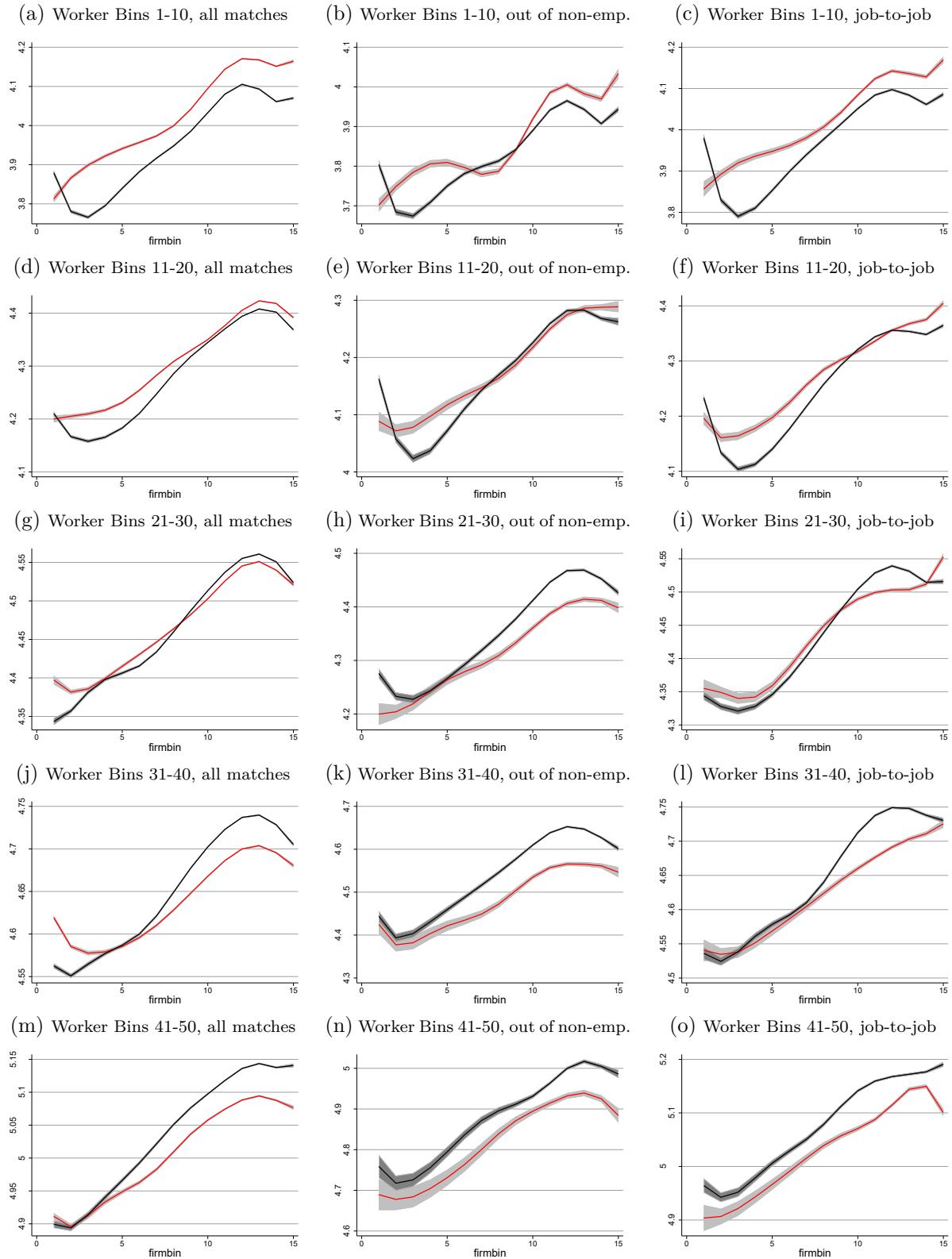
Notes: Plots show estimated wage profiles across firm bins constructed using AKM firm effects for all matches and new matches. The kernel is estimated using a Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.11: Wage Changes for Observed Transitions, Firm Subsamples



Notes: The plots show estimated coefficients and 95% confidence intervals (robust standard errors) from a linear regression of individual-level wage differences of transitioning workers on dummies for origin and destination firm bins using 20 random sub-samples (with replacement, clustered by firm-years) from the original sample (size= N). The sub-sample size M randomly varies between $0.5 * N < M \leq N$. The sub-samples consists of new matches (job-to-job switches, no intermittent non-employment spell) for five groups of worker types. The depicted coefficients are for transitions out of (Panel (a)) and into (Panel (b)) high-productivity firms (bins 13–15). The vertical axes capture destination/origin firm bin groups: low (bins 1–3) and center (bins 4–12). Data sources: BHP, EP, BeH.

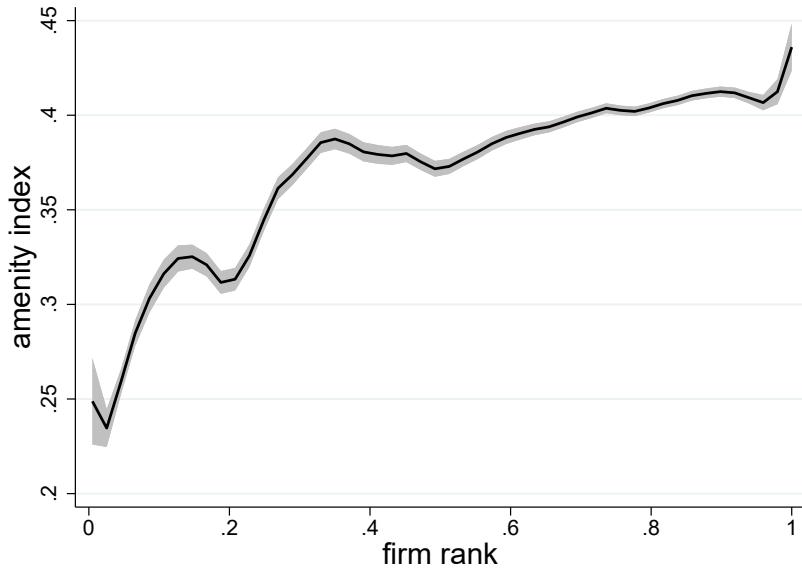
Figure D.12: Changes of Mean Wages across Worker and Firm Types: 1998-2002 (red) vs. 2003-2008 (black)



Notes: Plots show estimated wage profiles for two time periods for grouped worker bins across firm bins for all matches, new matches out of non-employment, and job-to-job moves. The kernel is estimated using an Gaussian kernel function. The bandwidth is 2. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

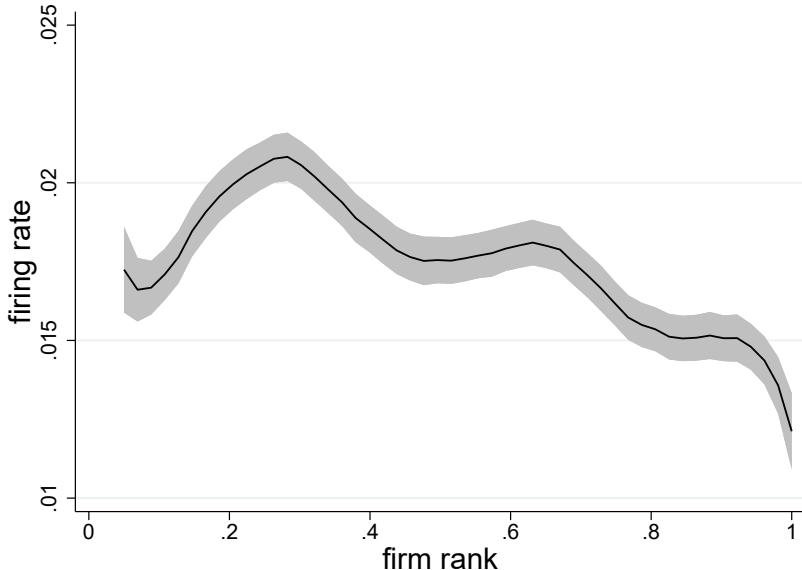
D.5 Extensions

Figure D.13: Amenities



Notes: Estimated univariate kernel density of the amenity index across estimated firm ranks, normalized to be between zero and one. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is 0.05. 95% confidence bands in gray. Data sources: BHP, EP, BeH.

Figure D.14: Job security



Notes: Estimated univariate kernel densities for the semi-annual rate of employer initiated separations across estimated firm ranks, normalized to be between zero and one. The kernel is estimated using an Epanechnikov kernel function. The bandwidth is 0.1. 95% confidence bands in gray. Data sources: BHP, EP, BeH.