Labor Market Dynamics with Sorting*

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Abstract

I study a dynamic search-matching model with two-sided heterogeneity, a production complementarity that induces labor market sorting, and aggregate shocks. In response to a positive productivity shock, incentives to sort increase disproportionately. Firms respond by posting additional vacancies, and the strength of the response is increasing in firm productivity. The distribution of unemployment worker types adjusts slowly, which amplifies job creation in the short run. In the long run, falling unemployment curtails the firms' vacancy posting. The model closely matches time-series moments from U.S. labor market data and produces realistic degrees of wage dispersion and labor market sorting.

Keywords: Search, Matching, Sorting, Mismatch, Aggregate Shocks, Worker Heterogeneity, Firm Heterogeneity, Unemployment Dynamics

JEL Classifications: E24, E32, J63, J64

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1 Introduction

To form a match in the labor market, workers and firms have to spend time and resources. Costly job search explains the coexistence of unemployed workers and vacant jobs and is the core of search-matching models of the labor market (specifically, the DMP model, following Diamond, 1982; Mortensen, 1982; Pissarides, 1985). Intuitively, costly job search can be understood as a result of heterogeneity: if workers and firms differ in terms of their skills and productivity, finding the right match is costly. Yet, the textbook DMP model (Pissarides, 2000) exhibits representative workers and firms. Shimer (2005) shows that this model fails to match time-series moments of labor market data because it does not generate sufficient amplification in response to shocks.

Existing solutions to this so-called "unemployment-volatility puzzle" reduce the responsiveness of wages to shocks. This leads to increased vacancy-posting of firms (Hall, 2005; Hall and Milgrom, 2008; Hagedorn and Manovskii, 2008). However, there is little empirical support for rigid wages of new hires (Pissarides, 2009). In this paper, I propose an alternative mechanism to amplify shocks in a search-matching model. The combination of worker and firm heterogeneity, a production complementarity that induces positive sorting, and free entry of firms amplifies job creation because the incentives to sort increase disproportionately in a boom.

I extend a DMP equilibrium search model by two-sided heterogeneity and sorting in the spirit of Shimer and Smith (2000). In the Shimer and Smith (2000) model, the equilibrium allocation exhibits positive assortative matching (PAM) if worker and firm types are complements (as in Becker, 1973). With free entry of firms (as in the DMP model but conditional on type), PAM is no longer guaranteed to arise. The reason is that firms have no incentive to wait for a better match upon meeting a worker. Conversely, workers trade off matching against their positive option value of continued search. Thus, not all meetings lead to matches. In equilibrium, an endogenous and type-specific productivity threshold determines which firms workers are willing to match with.

I link a production function from the sorting literature with a stochastic process for aggregate labor productivity and find that a calibrated version of the model matches time-series (volatility) and cross-sectional moments (wage dispersion, sorting) of U.S. labor market data. In response to a positive shock, the incentives to be optimally matched increase. All firms—and high-type firms in particular—post additional vacancies. This increases the value of continued search for workers. The availability of unemployed high-type workers further amplifies job creation. Thus, in the short-run, workers' and firms' choices mutually reinforce one another.[1] The distribution of unemployed workers adjusts slowly due to search frictions and meetings with unacceptable types. In the long run, the pool of unemployed workers shifts towards low types, which curtails the firms' response.

[1] They are strategic complements (Bulow et al., 1985) because the surplus function is supermodular.

To decide how many vacancies to post, firms solve a dynamic problem. They form expectations about the future match surplus, the set of worker types willing to match with them, and the rates at which they meet these worker types. Thus, firms' vacancy posting decisions are dynamic and depend on the distribution of unemployed worker types, which is part of the model's state space. To efficiently compute the adjustment paths of the match surplus, acceptance sets, the endogenous distributions, and aggregate labor market variables in response to shocks, I follow the perturbation approach (Reiter, 2009). Specifically, I linearize the model around its steady state and define an auxiliary state variable that represents the integral term in the firms job creation condition. Using an efficient external function to compute the log deviation of the intergral term from its steady state value in response to the shock, I can simulate the dynamic sorting model's response to shocks efficiently on a relatively fine grid with 100 discrete worker and firm types. Details are provided in a computational appendix.

Related Literature

In the literature on the macro-dynamics of labor markets, a closely related recent paper is Lise and Robin (2017). These authors build on Postel-Vinay and Robin (2002) and Robin (2011) to combine a model of on-the-job search with heterogeneity, sorting, and aggregate shocks. Using sequential auctions considerably simplifies the computation because the distribution functions drop out of the surplus and, thus, the model's state space. However, the firms' entry decision is static, so vacancy postings immediately adjust in response to shocks and do not depend on the distribution of unemployed worker types.

The contribution of this paper is complementary to Lise and Robin (2017). I abstract from on-the-job search and focus on the role that the distribution of unemployed worker types plays for the firms' dynamic job creation condition. My model is fairly stylized, and the primary goal of this paper is to develop theoretical intuitions for amplification in the sorting model. Future work should combine sorting, on-the-job search, and dynamic firm entry to study the macro-dynamics of labor markets.

In contrast to the canonical sorting model (Becker, 1973; Shimer and Smith, 2000), sorting is not guaranteed to arise in the model considered in this paper although worker and firm types are complements and the production function induces PAM. The reason is that—due to free entry conditional on the firm's type—firms have no option value of continued search and are willing to match with all worker types. The calibrated stationary equilibrium of the model does not feature a strong degree of PAM. This is in line with recent findings in the empirical literature, which documents low to moderate degrees of PAM using German, Swedish, Danish, and US data (Card et al., 2013; Bagger et al., 2013; Lise et al., 2016; Hagedorn et al., 2017; Bonhomme et al., 2019; Bagger and Lentz, 2019; Lochner and Schulz, 2022).

Other related papers in the literature on unemployment and business cycles with worker heterogeneity are Mueller (2017) and Baley et al. (2022). Mueller (2017) documents that the pool of unemployed shifts toward workers with high wages in their previous job in recessions. This observation is compatible with pro-cyclical sorting, i.e., less mismatch in booms. Baley et al. (2022) find evidence for counter-cyclical sorting and explain it with mismatch due to information frictions in a directed search model. As I explain below, the change of the surplus function in response to shocks in my stylized model is compatible with both pro-cyclical (higher incentives to sort in booms) and counter-cyclical (workers are less picky in booms) sorting. Furthermore, this paper is related to earlier work that studies the macro-dynamics of labor markets using models with heterogeneity but no sorting (e.g., Pissarides, 1985; Pries, 2008; Mukoyama, 2019).

The theoretical literature on sorting considers two types of models. Depending on the functional form of the production function, sorting can arise based on comparative advantage or absolute advantage. In comparative-advantage sorting models, the worker/firm type does not matter in itself; only the interaction of the types determines output (e.g., Marimon and Zilibotti, 1999; Gautier et al., 2010; Gautier and Teulings, 2015). In sorting models with absolute advantage (e.g., Shimer and Smith, 2000), high-type workers (firms) produce more than low types, no matter what type of firm (worker) they are matched with. In such a model, the production function implies an unambiguous ranking of worker and firm types. I focus on the hierarchical sorting model because it is empirically more appealing: absolute advantage implies a global ranking of workers and firms, and the data typically used to rank workers and firms in the empirical literature, e.g., education, job tenure, firms size, or value added, are inherently hierarchical.

The remainder of this paper is structured as follows: Section 2 discusses the model and its stationary equilibrium, including comparative statics. Section 3 adds aggregate uncertainty to the model and analyzes the dynamic firm entry problem and (briefly) wage formation. Section 4 discusses the computational strategy and presents results from numerical simulations of the sorting model in comparison to a baseline search and matching model and U.S. labor market data. Section 5 concludes.

2 The Model

2.1 Setup

Time is discrete. Infinitely-lived workers and firms maximize their future discounted income and profit streams, respectively. The common discount factor is β. Workers and firms are characterized by ex-ante heterogeneity of skills and productivity, respectively. Worker skills and firm productivity are assumed to be complements in production.

Table 1: Distribution functions of matched and unmatched worker and firm types

Distribution of	Relation	Aggregate Stock
Active matches	$g_m(x,y)$	$M = \iint g_m(x,y)\ \mathrm{d}x\mathrm{d}y$
Employed workers	$g_e(x) = \int g_m(x,y)\ \mathrm{d}y$	$E = \int g_e(x)\ \mathrm{d}x$
Unemployed workers	$g_u(x) = g_w(x) - g_e(x)$	$U = \int g_u(x)\ \mathrm{d}x$
Producing firms	$g_p(y) = \int g_m(x,y)\ \mathrm{d}x$	$P = \int g_p(y)\ \mathrm{d}y$
Vacant firms	$g_v(y) \rightarrow$ free entry	$V = \int g_v(y)\ \mathrm{d}y$

I focus on one-to-one matches between workers and firms.[2] Thus, I abstract from firm size and coworker complementarities.[3] There is full information about the types and availability of potential matching partners.

Worker and Firm Heterogeneity

A continuum of workers is endowed with heterogeneous skills $x \in [0,1]$ that are distributed according to a probability density function (pdf) $g_w(x)$. Similarly, a continuum of firms is endowed with heterogeneous productivity $y \in [0,1]$ with pdf $g_f(y)$. Both $g_w(x)$ and $g_f(y)$ are exogenous and uniform. $g_m(x,y)$ is the endogenous two-dimensional joint distribution of active (i.e., producing) (x,y) matches. The distributions of employed workers, $g_e(x)$, and producing firms, $g_p(y)$, can be obtained by integrating out the respective dimension of $g_m(x,y)$. The distribution of unemployed worker types, $g_u(x)$, is obtained by subtracting the distribution of employed workers from $g_w(x)$. The distribution of vacancies, $g_v(y)$, is governed by free entry and a cost function, specified below. The distributions of active matches, employed/unemployed workers, and producing/vacant firms are equilibrium objects. They integrate to the stocks of active matches, M, employed workers, E, unemployed workers, U, producing firms, P, and vacancies, V. Table 1 summarizes how the endogenous distributions of (in)active worker and firm types are related to the exogenous densities.

Production

$F(x,y)$ is a non-negative and twice continuously differentiable production function. I assume supermodularity of $F(x,y)$, i.e., the cross-partial derivative, $F_{xy}(x,y)$, is positive. The implied complementarity of worker skills x and firm productivity y is the force towards PAM in the model (Becker, 1973). Supermodularity of $F(x,y)$ is not sufficient for equilibrium existence in this setting (Shimer and Smith, 2000). Both $F(x,y)$ and the

[2]This is a common simplifying assumption that follows Pissarides (1985).

[3]Herkenhoff et al. (2018) and Freund (2022) are two recent studies that focus on coworker complementarities.

derivatives $F_x(x,y)$ and $F_{xy}(x,y)$ need to be log-supermodular.[4] The following parametric form fulfills these conditions (Shimer and Smith, 2000; Teulings and Gautier, 2004):

$$F(x,y) = \mathrm{e}^{xy}. \tag{1}$$

To study the amplification of shocks, I introduce labor productivity z, which augments output multiplicatively:

$$F(x,y,z) = z\ \mathrm{e}^{xy}. \tag{2}$$

Labor productivity z is a time-varying technology parameter that is common to all matches. The strength of the production complementarity increases in labor productivity, i.e., both the first derivatives and the cross-parital derivative of (2) scale with z.[5] This implies that, given worker skills, the marginal gain from working at a better firm increases in labor productivity. Similarly, given firm productivity, the marginal gain from hiring a better worker increases in labor productivity. Thus, the incentive to be optimally matched in the sense of the sorting model increases with labor productivity.

For the description of the model, z is assumed to be fixed and therefore omitted.

Matching and Separations

Only unemployed workers search for jobs, and search is random. Meetings are governed by a Cobb-Douglas function with constant returns to scale, $m(U,V) = \vartheta U^{\xi} V^{1-\xi}$, where ξ $(1-\xi)$ is the elasticity of new matches with respect to unemployment (vacancies).[6] ϑ represents the efficiency of the meeting process. The Poisson meeting rates are functions of aggregate labor market tightness $\theta = {}^{V}\!/_{U}$. $q_v(\theta) = {}^{M(U,V)}\!/_{V}$ is the rate at which vacant firms meet unemployed workers and, correspondingly, $q_u(\theta) = {}^{M(U,V)}\!/_{U}$ is the rate at which unemployed workers meet vacancies. $q_v(\theta)$ is decreasing, and $q_u(\theta)$ increasing in θ.

Not all meetings lead to matches in a sorting model. If the option value of continued search is higher than the discounted income stream from matching for at least one party, the surplus will be negative and the match not consummated.

Matches end for two reasons. First, idiosyncratic separation shocks arrive at rate δ. Second, matches may end endogenously when labor productivity z changes. The reason is that the surplus of marginally profitable matches may become negative in response to an aggregate produtivity shock.

[4] For any $x' > x$ and $y' > y$, $F(x',y')F(x,y) \geq F(x',y)F(x,y')$, $F_x(x',y')F_x(x,y) \geq F_x(x',y)F_x(x,y')$, and $F_{xy}(x',y')F_{xy}(x,y) \geq F_{xy}(x',y)F_{xy}(x,y')$. These conditions also imply that the acceptance strategies can be summarized by nonempty, closed, and convex matching sets (Shimer and Smith, 2000).

[5] That is, $\frac{\partial F(x,y,z)}{\partial x} = F_x(x,y,z) = z\ x\ \mathrm{e}^{xy}$ and $\frac{\partial F_x(x,y,z)}{\partial y} = F_{xy}(x,y,z) = z\ \mathrm{e}^{xy}\ (x\ y+1)$.

[6] Note that using a linear search technology with heterogeneous workers and firms implies congestion effects between different worker and job types. I stick to the Cobb-Douglas matching function for simplicity and comparability to other studies. A quadratic search technology, as used in Shimer and Smith (2000), eliminates this congestion externality. Nöldeke and Tröger (2009) extend Shimer and Smith (2000) to models with linear search technologies.

Surplus

The willingness to match is captured by the match surplus for any (x, y) combination. It is defined as the sum of the differences between the values of being matched and unmatched for both workers and firms:

$$\mathcal{S}(x,y) = \mathcal{E}(x,y) - \mathcal{U}(x) + \mathcal{P}(x,y) - \mathcal{V}(y), \tag{3}$$

where $\mathcal{E}(x,y)$ is the value of an employed worker, $\mathcal{U}(x)$ is the value of unemployment, $\mathcal{P}(x,y)$ is the value of a producing firm, and $\mathcal{V}(y)$ is the value of a vacancy. All value functions are spelled out below. When the surplus is positive, both parties agree to form a match. When the surplus is negative, both parties prefer to continue their search. The decisions to match (or not) are always mutually consistent. The indicator function $\mu(x,y)$ denotes whether match between a worker with skills x and a firm with productivity y is acceptable. This is the case if and only if the match surplus is positive:

$$\mu(x,y) = \begin{cases} 1 & \text{if} \quad \mathcal{S}(x,y) > 0 \\ 0 & \text{if} \quad \mathcal{S}(x,y) < 0. \end{cases} \tag{4}$$

Bargaining

Nash bargaining determines how the match surplus is shared between workers and firms. The respective surplus shares are equal to the difference of the matched values and the outside options, which are the threat point in the bargaining game. The workers' bargaining power parameter is $\alpha \in (0, 1)$.

$$\alpha\mathcal{S}(x,y) = \mathcal{E}(x,y) - \mathcal{U}(x) \tag{5}$$

$$(1-\alpha)\mathcal{S}(x,y) = \mathcal{P}(x,y) - \mathcal{V}(y) \tag{6}$$

This sharing rule also determines the wage. However, wages play no allocative role in the model, and specifying them is therefore not necessary to characterize the equilibrium. I relegate the derivation of the wage equation to Appendix C and briefly discuss the dynamic properties of wages in Section 3.

Value Functions

The value of employment of a worker with skills x at a firm with productivity y is

$$\mathcal{E}(x,y) = W(x,y) + \underbrace{\beta\delta\mathcal{U}(x)}_{\text{separation}} + \underbrace{\beta(1-\delta)\max\{\mathcal{E}(x,y), \mathcal{U}(x)\}}_{\text{continued employment}}, \tag{7}$$

which consists of the payment flow of the match-specific wage, $W(x,y)$, and the discounted value of two contingencies in the next period. With probability δ, the match is hit by a separation shock and the worker receives the value of unemployment $\mathcal{U}(x)$. With probability $(1-\delta)$, the match survives and the worker receives the maximum of the values of unemployment and employment at firm y. The max operator captures endogenous separations when labor productivity is stochastic.

The value of unemployment for a worker with skills x is

$$\begin{aligned}\mathcal{U}(x) = b(x) + \underbrace{\beta(1-q_u(\theta))\mathcal{U}(x)}_{\text{no meeting}} + \underbrace{\beta q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\mu^+(x,y)\mathcal{E}(x,y)\mathrm{d}y}_{\text{successful match}} \\ + \underbrace{\beta q_u(\theta)\mathcal{U}(x)\int_0^1 \frac{g_v(y)}{V}\mu^-(x,y)\mathrm{d}y}_{\text{meet unacceptable firm}}\,.\end{aligned} \tag{8}$$

Unemployed workers receive the type-specific value of home production $b(x)$ with $\frac{\partial b(x)}{\partial x} > 0$. In the following period, the unemployed worker does not meet any firm and remains unemployed with probability $1-q_u(\theta)$. With probability $q_u(\theta)$, a meeting occurs. The firm's productivity y can either be acceptable for the worker $(\mu^+(x,y) : \mu(x,y) = 1)$ or not $(\mu^-(x,y) : \mu(x,y) = 0)$.

Equation (8) separates the option value of meeting a firm into two parts: (i) meetings that lead to the formation of an employment relationship (successful match). Here, $\mu(x,y) = 1$ and $g_v(y)/V$ represents the probability of meeting an acceptable firm among all vacancies. (ii) Meetings that do not lead to a match (meet unacceptable firm). Here, $\mu(x,y) = 0$ and $g_v(y)/V$ represents the probability of meeting unacceptable firms. Note that the integration includes all firm productivity levels in both terms.

The flow payoff of a matched firm is the match-specific output $F(x,y)$ minus the wage, $W(x,y)$. In the next period, the match breaks up with probability δ or continues with probability $(1-\delta)$. Similar to the value of employment (7), the max operator is included to capture endogenous separations in response to labor productivity shocks.

$$\mathcal{P}(x,y) = F(x,y) - W(x,y) + \underbrace{\beta\delta\mathcal{V}(y)}_{\text{separation}} + \underbrace{\beta(1-\delta)\max\{\mathcal{P}(x,y),\mathcal{V}(y)\}}_{\text{continued production}} \tag{9}$$

Finally, the value of a vacancy is defined by

$$\begin{aligned}\mathcal{V}(y) = -c(g_v(y)) + \underbrace{\beta(1-q_v(\theta))\mathcal{V}(y)}_{\text{no meeting}} + \underbrace{\beta q_v(\theta)\int_0^1 \frac{g_u(x)}{U}\mu^+(x,y)\mathcal{P}(x,y)\mathrm{d}x}_{\text{successful match}} \\ + \underbrace{\beta q_v(\theta)\mathcal{V}(y)\int_0^1 \frac{g_u(x)}{U}\mu^-(x,y)\mathrm{d}x}_{\text{meet unacceptable worker}}\,.\end{aligned} \tag{10}$$

The value of a productivity-y vacancy depends on a flow cost of holding it open, $c(g_v(y))$ where $c(.)$ is monotonic function and $g_v(y)$ the measure of vacancies posted, and the option value of meeting workers with different skill levels. The firm either meets no worker, a suitable worker, or an unsuitable worker. Again, $\mu^+(x,y)$ ($\mu^-(x,y)$) indicates that a match will (not) be formed and ${g_u(x)}/{U}$ represents the probability of meeting an unemployed worker with skills x.

The value functions (7)–(10) can be simplified by adding and subtracting on both sides either $\mathcal{U}(x)$ or $\mathcal{V}(y)$. The surplus sharing rules (5) and (6) can then be plugged in and the surplus function along with a max operator appears under the integral signs.[7]

$$\mathcal{E}(x,y) = W(x,y) + \beta\left(\mathcal{U}(x) + \alpha(1-\delta)\max\{\mathcal{S}(x,y),0\}\right), \tag{11}$$

$$\mathcal{U}(x) = b(x) + \beta\left(\mathcal{U}(x) + \alpha q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\max\{\mathcal{S}(x,y),0\}\mathrm{d}y\right), \tag{12}$$

$$\mathcal{P}(x,y) = F(x,y) - W(x,y) + \beta\left(\mathcal{V}(y) + (1-\alpha)(1-\delta)\max\{\mathcal{S}(x,y),0\}\right), \tag{13}$$

$$\mathcal{V}(y) = -c(g_v(y)) + \beta\left(\mathcal{V}(y) + (1-\alpha)q_v(\theta)\int_0^1 \frac{g_u(x)}{U}\max\{\mathcal{S}(x,y),0\}\mathrm{d}x\right). \tag{14}$$

2.2 Stationary Equilibrium

Characterization

The stationary equilibrium of the sorting model is characterized by the surplus $\mathcal{S}(x,y)$ with free entry imposed, which pins down the acceptance strategy $\mu(x,y)$, and the distributions of unemployment workers and vacancies across skills and productivities, respectively. These equilibrium objects are jointly determined by the surplus value function, a steady state flow equation, and the job-creation condition.

Surplus under Free Entry

The surplus under free entry is obtained by plugging equations (11)–(13) into (3) and setting $\mathcal{V}(y) = 0$:

$$\begin{aligned}\mathcal{S}(x,y) = {} & F(x,y) + \beta(1-\delta)\max\{\mathcal{S}(x,y),0\} \\ & - \left(b(x) + \beta\alpha q_u(\theta)\int_0^1 \frac{g_v(y)}{V}\max\{\mathcal{S}(x,y),0\}\ \mathrm{d}y\right).\end{aligned} \tag{15}$$

The match surplus depends positively on output and the future discounted surplus (first two terms on the RHS). It depends negatively on the type-dependent outside option of the worker, which has two parts: (i) the type-specific value of home production $b(x)$ (third term on the RHS); (ii) the option value of continued search (fourth term on the

[7] Note that $\int_0^1 \frac{g_v(y)}{V}\max\{\mathcal{S}(x,y),0\}\mathrm{d}y$ is equivalent to $\int_0^1 \frac{g_v(y)}{V}\mu(x,y)\mathcal{S}(x,y)\mathrm{d}y$.

RHS), which depends on the worker's bargaining power α, the meeting rate $q_u(\theta)$, and the integrated surplus, weighted with the respective probabilities of meeting a vacancy of a productivity-y firm, ${g_v(y)}/{V}$. Note that the worker's outside option depends on an integral over the endogenous distribution function $g_v(y)$ but not directly on the firm type because y is integrated out.[8] Firm productivity y directly affects the surplus only via the production function $F(x, y)$.

Free entry in a sorting model has two important implications. First, there is no option value of continued search for the firm. Conditional on entering, firms are willing to match with every worker they meet. However, production is not always high enough to compensate the worker for his outside option. Thus, workers of a given skill level choose an endogenous productivity threshold (as in, e.g., Pissarides, 1985) and match with all firms above this threshold.[9] Across worker types, this productivity threshold corresponds to the acceptance strategy summarized in $\mu(x, y)$.

Second, PAM does not necessarily arise although the production function is log-supermodular. In Shimer and Smith (2000), sorting arises because there is an option value of continued search for both the worker and the firm. With free entry conditional on firms' productivity type, sorting is not guaranteed to arise because the option value of continued search for firms becomes zero.[10]

Steady-State Flows

The steady-state flow condition of the model pins down the distribution of unemployment workers across types, $g_u(x)$:

$$\delta g_m(x, y) = g_u(x)\ q_u(\theta)\ \frac{g_v(y)}{V}\ \mu(x, y). \tag{16}$$

The LHS of equation (16) captures exogenous separations, i.e., the flow from active matches into unemployment for all (x, y) combinations. The RHS captures the workers' flow into employment, which depends on the meeting rate, $q_u(\theta)$, the probability of meeting a specific firm type y, ${g_v(y)}/{V}$, and the acceptance strategy $\mu(x, y)$.

After integrating out the firm dimension and rearranging, we get the following ex-

[8] This is an important property because it implies that the distribution of vacancies is part of the model's state space. In Lise and Robin (2017), who develop a sorting model with on-the-job search and sequential auctions, the workers outside option of continued search does not reduce the surplus because the worker can move from job to job.

[9] Compared to Pissarides (1985), there are two differences: (i) the acceptance threshold varies across heterogeneous worker types; (ii) as we shall see below, the acceptance threshold falls in response to positive shocks (due to the incentive to sort), while it increases in Pissarides (1985).

[10] Hagedorn et al. (2017) sustain the Shimer and Smith (2000) sorting result by adding an ex-ante entry stage in which firms pay a cost to learn about their productivity type. In that case, the distribution of firm productivity remains exogenously fixed, while it is endogenous in this paper. The calibrated model produces a small degree of PAM, which is in line with empirical evidence, see Section 4.2.

pression for $g_u(x)$:

$$g_u(x) = \frac{\delta g_w(x)}{\delta + q_u(\theta) \int_0^1 \frac{g_v(y)}{V} \mu(x,y) \mathrm{d}y}. \tag{17}$$

The integral term in the denominator reflects that not all meetings lead to matches. Consequently, the density of unemployed workers may differ across workers with different skill levels. In fact, steady-state unemployment is higher in the sorting model than in the textbook model (Pissarides, 2000) for identical meeting and separation rates if there exists at least one (x, y) combination with negative surplus.[11]

For workers with a relatively low acceptance threshold in terms of firm-productivity y (and with sufficiently many vacancies), the density of unemployed workers is going to be low. In turn, for picky workers, i.e., high-skill workers that produce a large surplus with high-productivity firms, unemployment can be relatively high. Unemployed high-type workers, in turn, encourage job creation at high-productivity firms.

I study the properties of the stationary equilibrium for a calibrated version of the sorting model in the next subsection. In line with the considerations here, the unemployment rate will be relatively high for picky high-type workers in the calibrated steady state. I discuss the calibration in Section 4.2.

Job Creation

The job creation condition, which follows from (14) under free entry ($\mathcal{V}(y) = 0$), determines how many vacancies firms post depending on their productivity y and the availability of unemployed worker with different skill levels:

$$c(g_v(y)) = \beta(1-\alpha) q_v(\theta) \int_0^1 \frac{g_u(x)}{U} \mu(x,y) \mathcal{S}(x,y) \mathrm{d}x. \tag{18}$$

Given $g_u(x)$, $\mu(x,y)$, $\mathcal{S}(x,y)$, and the monotonic cost function c, equation (18) can be solved for the density of vacancies at firms with productivity y. Firms post vacancies until the discounted value on the RHS is lower than the cost implied by $c(g_v(y))$. The RHS depends on meeting probabilities with workers of all skill levels ($q_v(\theta)$ and ${}^{g_u(x)}/_{U}$), their acceptance strategies ($\mu(x,y)$), and the firm's surplus share ($(1-\alpha)\mathcal{S}(x,y)$).

In the sorting model, the surplus is especially high for matches between high-productivity firms and high-skill workers. For this reason, the availability of high-skill workers is an important factor in the firm's job creation decision, and this importance is increasing in firm productivity. Suppose there are relatively few unemployed high-skill workers. In that case, the high surplus that a high productivity firm could produce with a high-skill worker receives a relatively low weight in the integral expression (${}^{g_u(x)}/_{U}$ is relatively low

[11] To see this, integrate out the worker dimension x in (17). The resulting expression $U = {}^{\delta}/_{\left(\delta + q_u(\theta) \iint \frac{g_v(y)}{V} \mu(x,y) \mathrm{d}y \mathrm{d}x\right)}$ is equal to the textbook "Beveridge Curve" equation if the integral is 1, cf. equation (1.5) in Pissarides (2000).

for high x), and vacancy posting is diminished. If, however, high-skill workers are relatively picky and, thus, more likely to be unemployed than low-skill workers, then job creation is amplified because high-productivity firms are more likely to meet high-skill workers ($g_u(x)/U$ is relatively high for high x).

In line with these considerations, I show below that the measure of vacancies posted is indeed increasing in firm productivity in the calibrated model (see Section 4.2). To ensure that the distribution of vacancies is non-degenerate in equilibrium, I assume that $c(.)$ is convex.[12] Once $g_u(x)$ and $g_v(y)$ are known, integrating over worker and firm types yields the aggregate stocks of unemployed workers and vacancies, which in turn determine labor market tightness θ and the meeting rates $q_u(\theta)$ and $q_v(\theta)$.

There are important differences in the timing of adjustments to the three components of the job creation condition in response to a shock. These differences matter for the firms' vacancy posting responses in the dynamic model. In response to a positive productivity shock, output and surplus increase instantaneously and firms post more vacancies. The distribution of unemployed workers across skill types $g_u(x)$ and aggregate unemployment U, however, adjust slowly because search frictions and meetings with unacceptable types hinder match formation. As time progresses, the change in $g_u(x)$ curtails the firms' incentives to post vacancies. I study this mechanism in detail in Section 2.3.

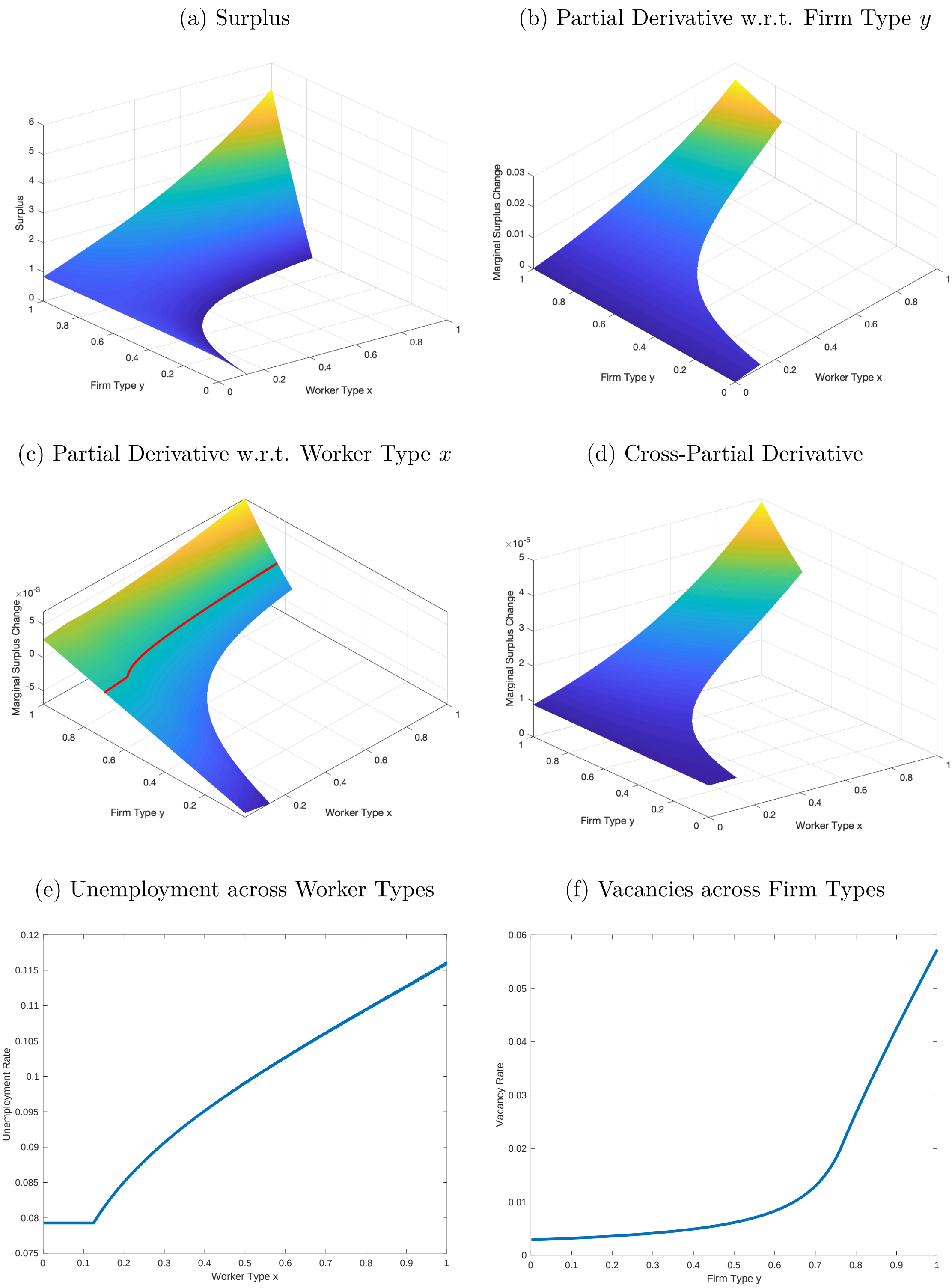
Properties of the Stationary Equilibrium

To study the properties of the sorting model's stationary equilibrium, I solve the model for 1000 discrete worker and firm types using the computational approach described in Section 4.1 and Appendix A.1. The calibration of the parameter values is discussed in Section 4.2.

Panel (a) of Figure 1 plots the surplus plane across all (x, y)-type combinations. Negative values are omitted. Towards the top, the surplus reflects the properties of the production function $F(x, y)$, which increases in both the worker and firm type. However, the surplus decreases towards the lower right corner. The intersection of the surplus plane with zero pins down the workers' acceptance strategy (firm-productivity thresholds). No matches are formed between high-type workers and low-type firms because output is too low relative to the workers' outside option. In turn, the workers' outside option increases in x due to both the increasing value of home production $b(x)$ and the increasing value of continued search. Based on the calibration, the most skilled workers match only with, roughly, the top quarter of firm types. In contrast, high-type firms are willing to match with low-type workers because, conditional on entering, their option value of continued search is zero.

[12] Due to the assumed production function, high-type firms produce more than low-type firms irrespective of the worker type they are matched with. The convexity helps to discipline the vacancy posting of highly productive firms.

Figure 1: Stationary Equilibrium



(a) Surplus

(b) Partial Derivative w.r.t. Firm Type y

(c) Partial Derivative w.r.t. Worker Type x

(d) Cross-Partial Derivative

(e) Unemployment across Worker Types

(f) Vacancies across Firm Types

Consider how the derivatives of the surplus function (15) depend on the worker and the firm type. Due to free entry, the surplus is affected by firm productivity exclusively through the production function $F(x, y)$. Thus, the partial derivative of the surplus function with respect to the firm type y is positive everywhere and increasing in x, see Panel (b). The sign of the partial derivative with respect to the worker type x, however, depends on the combination of types, see Panel (c). Here, the red line marks the intersection with the zero plane. The derivative is positive when the firm type is relatively high. In this case, the increase of output in x dominates the increase of the outside option in x due to the high firm type. Conversely, if the firm type is relatively low, the slope of the surplus in the direction of the worker type becomes flatter and eventually even negative for very low firm types.

The cross-partial derivative of the surplus function is both positive and increasing in x and y, see Panel (d). The derivative of the outside option drops out because firm productivity is integrated out. Thus, the surplus function inherits the supermodularity property from the production function $F(x, y)$. The cross-partial derivative is also increasing towards the top, which suggests that the supermodularity of the surplus is particularly relevant for matches between high-type workers and high-type firms.

In game theory, supermodularity of the payoff function implies that players' choices are strategic complements, i.e., they mutually reinforce one another (Bulow et al., 1985). In the sorting model, workers and firms are the players but the arguments of the surplus function are not choices but immutable types. Still, the analogy is helpful to clarify why workers' and firms' decisions mutually reinforce one another and, thus, contribute to the amplification of shocks in the sorting model. Due to the random search assumption, firms and workers cannot control which type of worker or firm they meet. But firms generate more meetings by posting vacancies and can in this way increase their chances of meeting the right worker type. The more vacancies firms post, the more picky workers are due to the increased option value of continued search. In turn, picky workers provide incentives for firms to post even more vacancies because picky workers are more likely to be unemployed and, thus, available for firms to match with. This is the mutual reinforcement. Strategic complementarity is a well-known property of sorting models that follows from the supermodularity assumption these models typically rest upon.[13]

To show how workers' acceptance strategies and firms' vacancy posting interact in shaping the distributions of unmatched types, I plot the unemployment rate across worker types in Panel (e) and the measure of vacancies across firm types in Panel (f). The unemployment rate increases in the worker type. In the calibrated model, the unemployment rate per worker type is below 8% for low-type workers who are willing to match with all firm types. For high-type workers, however, the unemployment rate extends to 12%.[14]

[13] See Shimer and Wu (2021) for a recent application.

[14] An important question is whether the pickiness of high-type workers is driven by the utility flow

The measure of vacancies in Panel (f) also increases in the firm type. As explained above, it is more profitable for high-type firms to post vacancies for two reasons: (i) the surplus and, thus, the incentives to sort are particularly high at the top; (ii) high-type workers have a valuable outside option, are more likely to be unemployed, and are therefore available to match with high-type firms. In other words, as the firm type increases in Panel (f), firms "get access to" workers who are willing to match with them and relatively likely to be unemployed. Low-type firms match only with the 10–20% of worker types at the bottom, while high-type firms in the top 20% can match with every worker they meet.

Taken together, the acceptance strategy of high-type workers, who are willing to wait for high-surplus matches, and the job creation decisions of high-type firms, which have particularly strong incentives to post vacancies when high-type workers are available, mutually reinforce one another, and this mechanism allows the sorting model to amplify the response to productivity shocks.

2.3 Comparative Statics

To explore the quantitative implications of a change in labor productivity, I analyze how the model responds to a positive productivity shock. In a two-step comparative-statics exercise, I first increase the labor productivity parameter z by 5% and keep the distribution of unemployed worker types $g_u(x)$ fixed. This highlights the short-run response in the model because surplus and vacancy posting increase instantaneously but forming new matches takes time. Second, I let the $g_u(x)$ distribution adjust to the new steady state level to show how the vacancy-posting response of firms is curtailed in the long run due to fewer available high-type workers.

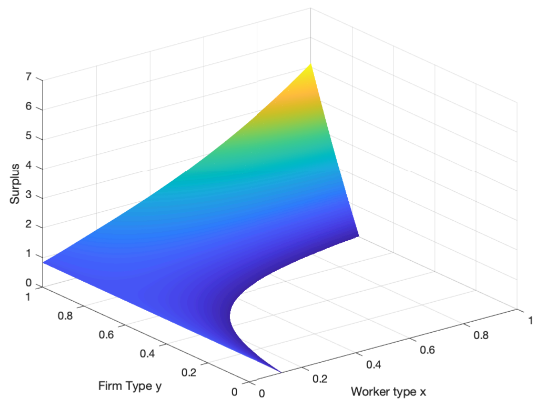
In Panels (a) and (b) of Figure 2, I compare the surplus with $z = 1$ and with $z = 1.05$ and $g_u(x)$ fixed (short run). As anticipated based on the production function, the change of surplus gets bigger toward the top. For low-type workers, who are willing to match with all firm types, the surplus increases by roughly 0.25. For picky high-type workers, however, the surplus change is more than five times bigger, it extends to 1.3 at the very top. The reason is that the strength of the production complementarity in $F(x, y, z)$, which the surplus reflects, increases in the worker and the firm type, recall Figure 1d. The fact that the incentive to be optimally matched increases with labor productivity induces pro-cyclical sorting, consistent with empirical findings by Mueller (2017).[15]

from home production during unemployment, which we allow to be increasing in x, or the option value of continued search, which is related to sorting. To investigate this, I recompute the stationary equilibrium with a constant b and find that the properties of the equilibrium do not change much, see Appendix Figure A.3. The matching cutoff hardly differs between the constant-b and increasing-b case, so the surplus is not affected much by the specification of b, although unemployment is higher for all worker types when b increases with the type x.

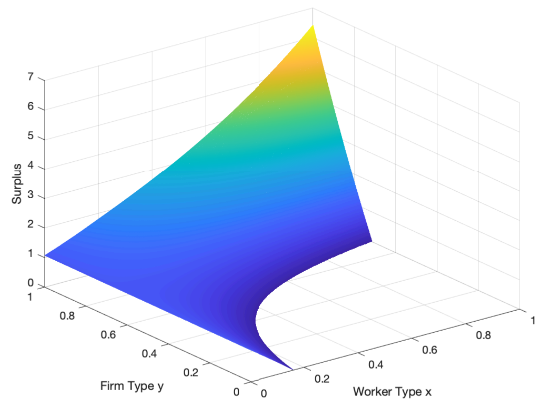
[15] Mueller (2017) documents that in recessions the pool of unemployed workers shifts toward workers

Figure 2: Comparative Statics of a Change in z

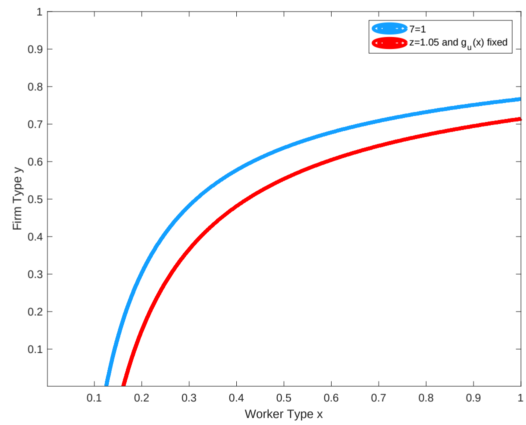
(a) Surplus with z=1



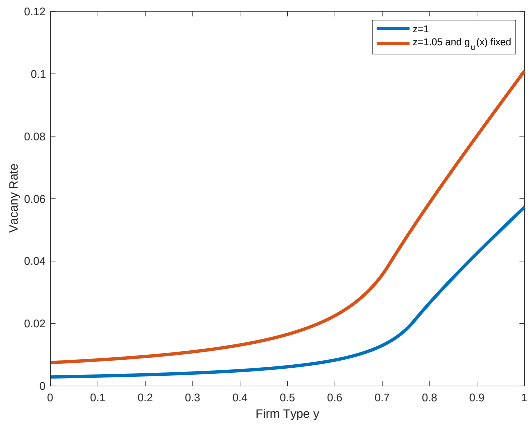
(b) Short run: Surplus with z=1.05, $g_u(x)$ fixed



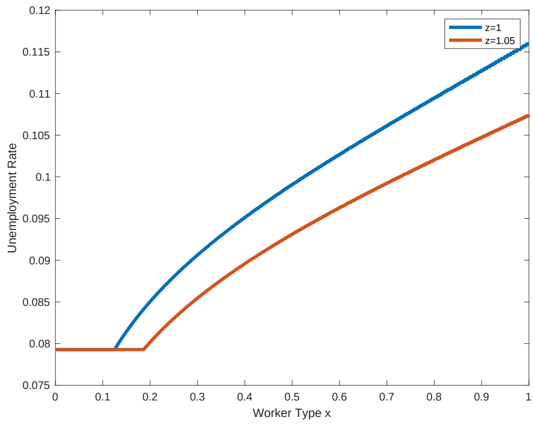
(c) Short run: Matching Cutoff



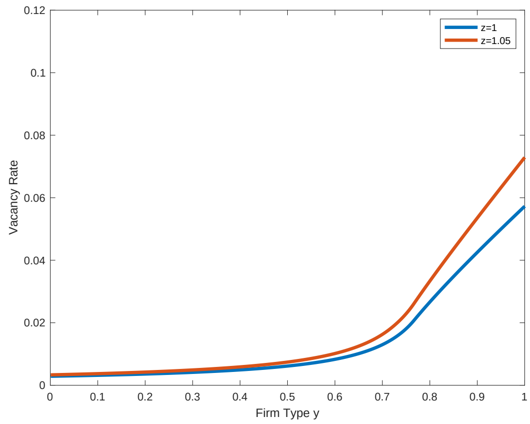
(d) Short run: Vacancies across Firm Types



(e) Long run: Unemp. across Worker Types



(f) Long run: Vacancies across Firm Types



with high wages in their previous job. This is consistent with more sorting (and less mismatch) in booms.

The change to the workers' acceptance threshold, summarized by the match indicator function $\mu(x, y, z)$, is depicted in Panel (c). It shifts toward the lower-right corner, which means that more matches between high-type workers and low-type firms become acceptable.[16] This is the net effect of two counteracting forces. First, due to increased output, more matches can compensate workers for their outside option. Workers are less picky and the firm productivity threshold decreases (the surplus shifts up for all (x, y)-combinations). Second, the outside option of workers increases because the incentives to sort rise and firms post more vacancies, especially at the top (see Panel (d)). This makes workers more picky and tends to increase the firm productivity threshold. This mechanism reinforces vacancy posting (strategic complementarity). The net effect is a downward shift of the firm-productivity threshold, and this is in line with counter-cyclical sorting, i.e., less positive sorting in booms and pro-cyclical mismatch. Baley et al. (2022) present empirical evidence for counter-cyclical sorting based on U.S. data.

Whether sorting is pro or counter-cyclical is ultimately an empirical question. Baley et al. (2022) and Mueller (2017) use different models and measurement strategies. The adjustment mechanism in response to higher productivity in the stylized model discussed here is compatible with both pro and counter-cyclical sorting.

Panel (d) shows how the density of vacancies across firm types changes with higher labor productivity in the short run. That is, the distribution of unemployed worker types is held fixed. High-type firms have strong incentives to post additional vacancies because the surplus and the incentives to sort are disproportionately higher and relatively many unemployed high-type workers are available (recall Figure 1e). Consequently, the effect of higher labor productivity on vacancy posting is moderate for low-type firms but strongly increasing in firm productivity. For example, for firms with the highest productivity the absolute increase in the measure of vacancies posted increases roughly four times more than for productivity 0.5.

Panel (e) and (f) show the response to higher productivity in the long run, i.e., when the distribution of umemployed workers $g_u(x)$ has adjusted to higher output. In line with the lower firm-productivity threshold, the unemployment rate decreases most at the top (about one percentage point), see Panel (e).[17] The unemployment rate does not change for workers who are willing to match with all firm types (flat density). However, the number of worker types that are not picky at all increases with higher labor productivity. In the long run, the measures of vacancies posted by firm type go back almost entirely to their initial level, see Panel (f). Vacancy posting remains elevated at the top where higher output has the strongest impact on the surplus, but for low-type firms there is almost no effect once the distribution of unemployed worker types has adjusted.

[16] The additional surplus covered by the extended acceptance set is relatively small, so this adjustment is not quantitatively important for the response to shocks.

[17] This is again qualitatively consistent with the empirical findings of Mueller (2017).

Taken together, the comparative statics exercises show that one can expect a significant short-run amplification of aggregate productivity shocks in a dynamic sorting model. Over time, as unemployment falls (especially for high-type workers), firms' vacancy posting decreases but remains elevated at the top.

3 The Dynamic Sorting Model

In the dynamic sorting model, z is stochastic. Let z_t denote the realization of aggregate labor productivity z in period t. I assume that z follows a Markov process, which is specified and calibrated in Section 4.2. To simplify notation, let z' be next period's realization $(t+1)$. Time subscripts are omitted in the following to avoid notational clutter. I define the state of the system to be $\Omega(g_m(x,y,z),z)$. This state contains the exogenous state variable z and the endogenous state, which can be summarized in the distribution of active matches $g_m(x,y,z)$. All endogenous objects depending on the state have Ω as an argument in the following.

The timing of the model is as follows: a period begins when the state of aggregate labor productivity z is revealed. Workers form their acceptance strategies, and firms decide about job creation based on the exogenous state z and the primitives of the model. Both endogenous and exogenous separations take place, and new matches are formed. Workers and firms separated in the same period do not start their search until the next period. Finally, production commences and wages are paid.

Based on these assumptions, the dynamic surplus value function becomes

$$\begin{aligned}\mathcal{S}(x,y,\Omega) &= F(x,y,z) + \beta\mathbb{E}\left[(1-\delta)\max\{\mathcal{S}(x,y,\Omega'),0\}\right] \\ &\quad - \beta\mathbb{E}\left[b(x) - \alpha q_u(\theta(\Omega'))\int_0^1 \frac{g_v(y,\Omega')}{V(\Omega')}\mu(x,y,\Omega')\mathcal{S}(x,y,\Omega')\,\mathrm{d}y\right].\end{aligned} \tag{19}$$

$F(x,y,z)$ depends on the exogenous state only. Home production $b(x)$ does not depend on z. $\mathbb{E}$ is the expectation operator regarding the future aggregate state. It includes all information available in the period the expectation is formed.

3.1 Job Creation

The firms' dynamic job creation problem is pivotal for the amplification result. A firm of type y will post vacancies as long as the expected discounted value of production is at least as high as the flow cost implied by $c(.)$:

$$c(g_v(y,\Omega)) = \beta(1-\alpha)\mathbb{E}\left[q_v(\theta(\Omega'))\int_0^1 \frac{g_u(x,\Omega')}{U(\Omega')}\mu(x,y,\Omega')\mathcal{S}(x,y,\Omega')\,\mathrm{d}x\right]. \tag{20}$$

Thus, job creation depends on an expectation over the meeting rate and the firm-specific integral in (20). It includes the match indicator function $\mu(x, y, \Omega')$, the surplus function $\mathcal{S}(x, y, \Omega')$, and the probabilities of meeting specific worker types, $\frac{g_u(x,\Omega')}{U(\Omega')}$. In contrast to the standard model and Lise and Robin (2017), this is a dynamic problem because the firm forms an expectation over the slow-moving distribution of unemployed worker types.[18]

As argued above, all endogenous objects on the RHS change in response to shocks but the adjustment of $\frac{g_u(x,\Omega')}{U(\Omega')}$, and hence $\theta(\Omega')$, is delayed by search frictions (as in, e.g., Pissarides, 1985) and meetings with unacceptable types (due workers' incentives to sort). In contrast, the surplus and workers' acceptance decisions change instantaneously. Thus, firms immediately post more vacancies, especially at the top where the surplus increases disproportionately. Moreover, as discussed in Section 2.2, high-type workers have higher unemployment rates in the sorting model which provides further incentives for high-type firms to post vacancies. This effect is most pronounced in the short run due to the delayed adjustment of unemployment.

3.2 Wages

I derive the wage equation of the dynamic sorting model in Appendix C.

$$W(x, y, \Omega) = \alpha \left(F(x, y, z) + c(y)\mathbb{E}\left[\frac{\int_0^1 g_v(y, \Omega')\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y}{\int_0^1 g_u(x, \Omega')\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}x} \right] \right) + (1 - \alpha)b(x). \tag{21}$$

An interesting property is that the match-specific wage $W(x, y, \Omega)$ does not depend on aggregate labor market tightness. Instead, the ratio of the firm's integral term (the denominator of the expectation) and the worker's integral term (the numerator of the expectation) determines the wage. This feature can disconnect match-specific wages from fluctuations in aggregate tightness in the sorting model. Intuitively, what matters for the wage bargain is not the stocks of vacancies and unemployed workers but the alternative matching opportunities of both parties that are summarized by the integral terms.

4 Numerical Simulations and Empirical Analysis

I study the model's ability to match aggregate time-series moments from U.S. labor market data. To this end, I run numerical simulations of a calibrated model in a stochastic environment. Aggregate shocks to labor productivity z drive the business cycle. I compare the responses of the dynamic sorting model to Shimer (2005).

[18]The static job creation condition in Pissarides (2000) is $c = \beta(1-\alpha)\mathbb{E}[q_v(\theta(z'))\mathcal{S}(z')]$. Here, vacancy posting costs are a constant and without heterogeneity the firm does not consider different worker types or type-specific meeting probabilities.

4.1 Computation

I first discuss how the stationary equilibrium considered in Section 2.2 is computed. Second, I explain how i analyze this heterogeneous-agent model with aggregate shocks out of steady state. Details are relegated to Appendices A.1–A.3.

Stationary Equilibrium

I approximate the stationary equilibrium of the sorting model using value function iteration on a discrete grid. After initialization, the first step is to compute the fixed point of (15). In practice, it is necessary to allow for some smoothing of the acceptance strategy $\mu(x, y)$ along the cutoff to ensure convergence. I allow for mixed strategy solutions close to the cutoff, following Hagedorn et al. (2017). In the second step, knowing $\mathcal{S}(x, y)$ and $\mu(x, y)$, I use the steady-state flow condition (16) to solve for the endogenous distribution of unemployed worker types, $g_u(x)$. The third and final step is to calculate the measure of vacancies $g_v(y)$ along with the stocks U, V, and the meeting rates. I solve (18) numerically and integrate over $g_u(x)$ and $g_v(y)$ to compute the stocks of unemployment workers and vacancies.

The solution algorithm alternates between computing the fixed point of the surplus function for all (x, y) combinations and updating the distributions until convergence is achieved.[19] Due to vectorization and fast solvers, the stationary equilibrium can be computed in a matter of seconds for a grid of 100×100 types. Performance is critical for computing the model's response to shocks in Section 4. Note that uniqueness is not guaranteed in the Shimer and Smith (2000) class of models. I verify that the surplus function is contracting in the parameter space I consider by repeatedly solving the model for different initial conditions.[20]

Out-of-Steady-State Dynamics

The dynamic sorting model has a complex state space Ω, which consists of the exogenous state z (labor productivity) and the endogenous state $g_m(x, y, z)$ (joint distribution of active matches). All other endogenous objects—the surplus, the matching sets, the distributions of unmatched worker and firm types, the stocks U and V, the arrival rates $q_u(\theta)$ and $q_v(\theta)$, and aggregate labor market tightness θ—follow from the state of the system Ω. Keeping track of Ω's evolution is computationally challenging because it contains endogenous distributions and is thus infinite-dimensional.

I use perturbation following Reiter (2009) and log-linearize the model around its steady state. I simulate the model for all x and y combinations separately in Dynare

[19] That is, until the absolute difference of the surplus between two iterations is less than 10^{-6}.

[20] This approach is similar to Shimer and Smith (2000), Lise and Robin (2017), and Hagedorn et al. (2017), among others. I would like to thank Robert Shimer for sharing the code used to produce the numerical results in Shimer and Smith (2000).

(Adjemian et al., 2022) and aggregate in the end. Using Dynare is very helpful for the underlying standard computations, e.g., checking stability of the system and computing policy functions. Furthermore, it is easy to nest simpler models, which I use to reproduce the results of Shimer (2005) below.

Importantly, some model objects are high dimensional because they depend on distributions across the typespace. Consider the firm's integral term in equation (20), which includes the endogenous distribution of unemployed worker types. Computing how this object changes in response to changing labor productivity is the key challenge. To overcome it, I use an external function in Dynare to call the solution algorithm described in Appendix A.1 from within the model block and calculate the integral's log-deviation from its steady state value for the current realization of labor productivity. This procedure is conceptually similar to calculating the numerical derivative of the integral in (20) with respect to labor productivity for a specific firm type.[21]

In summary, I define auxiliary state variables for the integral terms and calculate their adjustment manually but efficiently outside of Dynare. This allows me to keep track of the evolution of the integral terms when labor productivity is stochastic. In the present model, this concerns in particular the RHS of equation (20), which determines vacancy posting.[22] The key to success for this method is a fast and reliable solver to calculate the high-dimensional objects' adjustment to shocks. Using the algorithm sketched in Appendix A, simulating data from the dynamic sorting model is still time-intensive because I loop over all combinations of worker and firm types. It takes several days on a desktop computer to complete the simulations.

Log-linearizations around the steady state are error prone in nonlinear systems. Fortunately, the calibrated labor productivity process used in the context of U.S. labor market dynamics is not very volatile. The standard deviation of z is small, so the model always remains relatively close to the steady state. To check the reliability of my computational approach, I plug simulated data back into the Bellman equations to see whether the simulated data solve them. The mean computational error I make is quite small at 3.8%. This value is very close to the error one makes when solving simple dynamic search and matching models using log-linearization and perturbation, so my method of dealing with the additional complexity of the sorting model's state space does not appear to significantly increase computational errors.[23] Figures A.1 and A.2 in the Appendix show the distribution of computational errors and their positive correlation of the errors with z.

[21] In related work, Boppart et al. (2018) show how to use impulse response functions (IRFs) to efficiently calculate numerical derivatives of complicated functions in heterogeneous-agent models with aggregate shocks.

[22] In an earlier version of this paper that focused on wage dynamics in addition to firm entry, I additionally defined auxiliary state variables for the two integral terms that show up in the wage equation (21).

[23] Petrosky-Nadeau and Zhang (2016) show that solving the representative agent search and matching model in Hagedorn and Manovskii (2008) via log-linearization and perturbation creates a mean computational error of 3.75%. I find a mean error of 3.84% with slightly more dispersion.

4.2 Calibration

As in Shimer (2005), a time period is set to be one quarter. Table 2 shows the calibration of the model based on the U.S. labor market data used for the simulation exercise and for the analysis of the stationary equilibrium in Section 2.2. To ensure comparability with the results in Shimer (2005), identical parameter values are used whenever possible. A value of 0.1 for the separation rate translates into an average employment spell of about 2.5 years during the relevant period (1951–2003). The quarterly discount rate is set to 0.012, representing an annual interest rate of roughly 5%. The discount factor (as it appears in the model equations) is thus $\beta = {}^{1}/_{1.012} \approx 0.99$. The matching function elasticity is set to 0.72, in line with Shimer (2005), which is within the empirically supported range (Petrongolo and Pissarides, 2001). I set the bargaining parameter equal to the matching function elasticity, that is, I follow the Hosios (1990) condition for socially efficient vacancy posting in the decentralized equilibrium.[24]

Several parameters need to be recalibrated in the sorting model. I allow the value of nonmarket activity $b(x)$ to increase in the worker type. I calibrate it to be $0.223 \times \max_x F(x, y)$. This implies that home production $b(x)$ has a mean of 40% of the output a worker of type x can produce in his optimal match. This assumption is a natural extension of Shimer (2005), who assumes a constant b of 0.4 when output is normalized to 1. The efficiency parameter of the aggregate matching function (ϑ) needs to be increased in the sorting model to take into account that not all meetings result in matches. A value of 2 implies, along with the other parameter values, that the *net* job finding rate, that is, the rate of matches that are formed after a meeting, is close to the value Shimer (2005) constructs from the data, which is 1.355 (quarterly). The calibrated economy has a steady-state unemployment rate of about 7.8%.

The convex vacancy posting cost function takes the following form:

$$c(g_v(y)) = \frac{c_0}{1 + c_1} g_v(y)^{1+c_1}.$$

c_0 and c_1 are set to the values shown in Table 2 to target a steady-state aggregate labor market tightness of 1.[25]

The stochastic labor productivity process z is normalized to 1 in steady state and calibrated to resemble empirical labor productivity in the U.S. over the relevant period of time. I follow Hagedorn and Manovskii (2008) and set it up as a first-order autoregressive process:

$$z_t = \rho\, z_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2). \tag{22}$$

$\rho \in (0, 1)$ captures the degree of first-order autocorrelation of the AR(1) process. In-

[24] I do not adapt the Hosios condition for the sorting model, i.e., this parametrization does not necessarily imply that vacancy posting is socially efficient.

[25] In practice, I set c_0 to the value estimated in Lise and Robin (2017) and adjust c_1 to target $\theta = 1$.

Table 2: Parameter values for the quarterly calibration of the search and matching model for the U.S. labor market (1951–2003)

Parameter	Symbol	Value	Source
Discount factor	β	0.99	Shimer (2005)
Separation rate	δ	0.1	Shimer (2005)
Workers' bargaining power	α	0.72	Shimer (2005)
Matching function elasticity	ξ	0.72	Shimer (2005)
Matching function constant	ϑ	2	Calibrated to match
Value of nonmarket activity	$b(x)$	$0.223 \times \max_x F(x, y)$	steady-state unemp.
Vacancy posting costs	c_0	0.03	Calibrated to match
	c_1	0.4	steady-state θ.
First order autocorrelation	ρ	0.765	Hagedorn
Standard deviation	σ_ϵ	0.013	& Manovskii (2008)

novations are drawn from a Gaussian distribution with mean 0 and standard deviation σ_ϵ. Both parameters are set to match quarterly U.S. labor productivity.[26] All values in Table 2 are based on quarterly data. Shimer's (2005) simulation results as well as my own results are reported as deviations from a HP trend, which is conventional in the literature.[27]

Using the calibration in Table 2, the model produces wage dispersion and labor market sorting of magnitudes that are comparable to benchmarks reported in the literature. The mean-min ratio is 1.35, which is close to the empirically-supported range of 1.5–2 identified in Hornstein et al. (2011) and much higher than the value that can be generated with the canonical search-matching model.[28] Spearman's rank correlation coefficient, a measure of sorting, is 0.095. That is, the extent of positive sorting is not high despite the production complementarity. Mismatch is substantial. This is in line with the finding that sorting breaks down in a Shimer and Smith (2000) model with free entry conditional on firm type because firms have no incentive to wait for better matches. A small degree of PAM is also in line with the empirical literature. For the U.S., Lise et al. (2016) report

[26] Shimer (2005), Hornstein et al. (2005), and Hagedorn and Manovskii (2008) report the parameter values necessary to represent U.S. labor productivity "as seasonally adjusted quarterly real average output per person in the non-farm business sector constructed by the BLS" (Hagedorn and Manovskii (2008), p. 1694).

[27] The Hodrick-Prescott (HP) filter is a technique for decomposing the trend and the cyclical component of a time series (Hodrick and Prescott, 1997). Shimer (2005) sets the smoothing parameter of the filter to $\lambda = 10^5$ instead of to the more common value of $\lambda = 1600$ for quarterly data. This makes the cyclical component less volatile and more persistent. I use the same value as Shimer to generate comparable moments. Hornstein et al. (2005) point out that a more volatile trend, using the common smoothing parameter $\lambda = 1600$ for quarterly data, "has almost no effect on the relative volatilities" (p. 23).

[28] The canonical model generates a mean-min ratio of at most 1.05 according to Hornstein et al. (2011).

Table 3: Actual and simulated standard deviations of labor market variables

	Standard deviations	U	V	θ	$q_u(\theta)$	z	$F(x,y,z)$
1.	U.S. data	0.190	0.202	0.382	0.118	0.02	-
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010	0.02	-
3.	No sorting, no heterogeneity	0.009	0.026	0.035	0.010	0.02	-
4.	Sorting, hierarchical model	0.102	0.277	0.380	0.168	0.02	0.06

Note: Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28, 39. Calculated based on quarterly U.S. data, 1951–2003. Rows 3 & 4: Standard deviations of simulated data from my model with and without sorting. All moments come from HP-filtered data with $\lambda = 10^5$.

a small degree of PAM.[29] For Germany, Lochner and Schulz (2022) measure sorting by correlating firm types based on estimated firm productivity and worker types based on estimated worker ability. They find a correlation of 0.07.

4.3 The Amplification Effect of Sorting

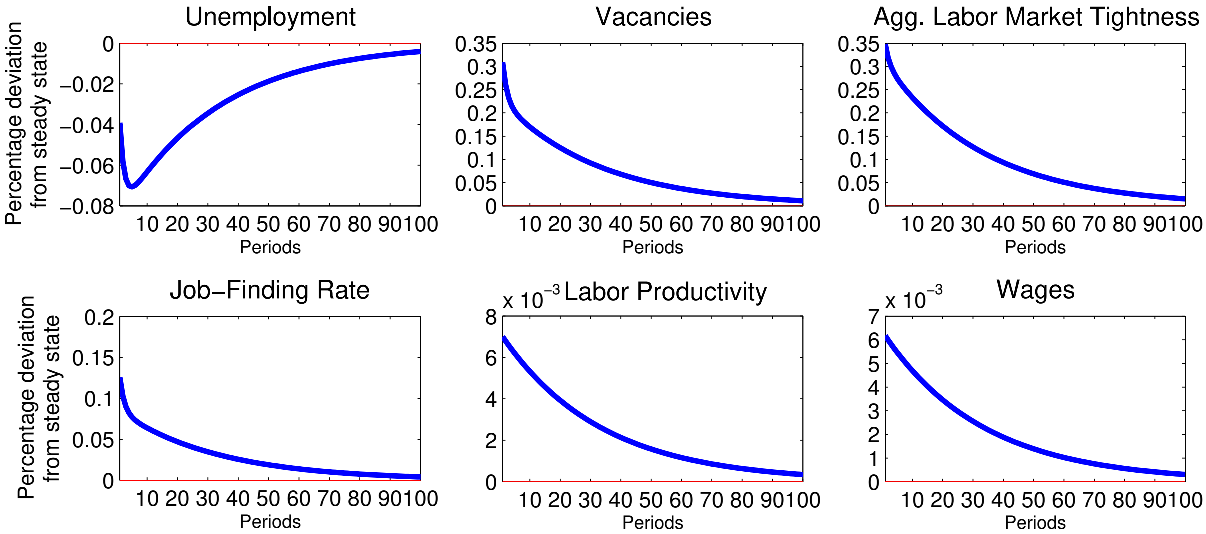
The Shimer Puzzle revolves around the canonical search-matching model's ability (or lack thereof) to explain the volatility of the unemployment rate, the vacancy rate, aggregate labor market tightness, and the job-finding rate over the business cycle. The search-matching model with sorting produces sufficient amplification in response to shocks and can match the data. Second moments of simulated time series data are of the same order of magnitude as the volatility observed in U.S. labor market data for the relevant period of time. In particular, the simulated standard deviations of unemployment, vacancies, labor market tightness, and the job-finding rate are much closer to empirical second moments than simulated data from standard search and matching models. Table 3 compares my results to those of Shimer (2005) and the empirical data moments.

The first two rows of Table 3 show the well-known unemployment-volatility puzzle emphasized by Shimer (2005). The standard deviations of unemployment, U, vacancies, V, labor market tightness, θ, and the job-finding rate, $q_u(\theta)$, in simulated time series data miss the empirical standard deviations by a factor of 10 to 20. I first replicate Shimer's results by simulating a model from without worker/firm heterogeneity and sorting. That is, I set output to 1 and use the same calibration as Shimer (2005). Vacancy posting costs c and the value of home production b are constants in this case. Reassuringly, the results in the third row of Table 3 are nearly identical to Shimer's results.

My main result is reported in the fourth row of Table 3. The second moments of simulated time series data from the model with sorting are much closer to the data than are those from the standard model without sorting. The standard deviation of the HP-

[29] Lise et al. (2016) make a parametric assumption about the production function (CES) and estimate the elasticity of substitution, which is not directly comparable to the correlation reported here.

Figure 3: Impulse Response Functions

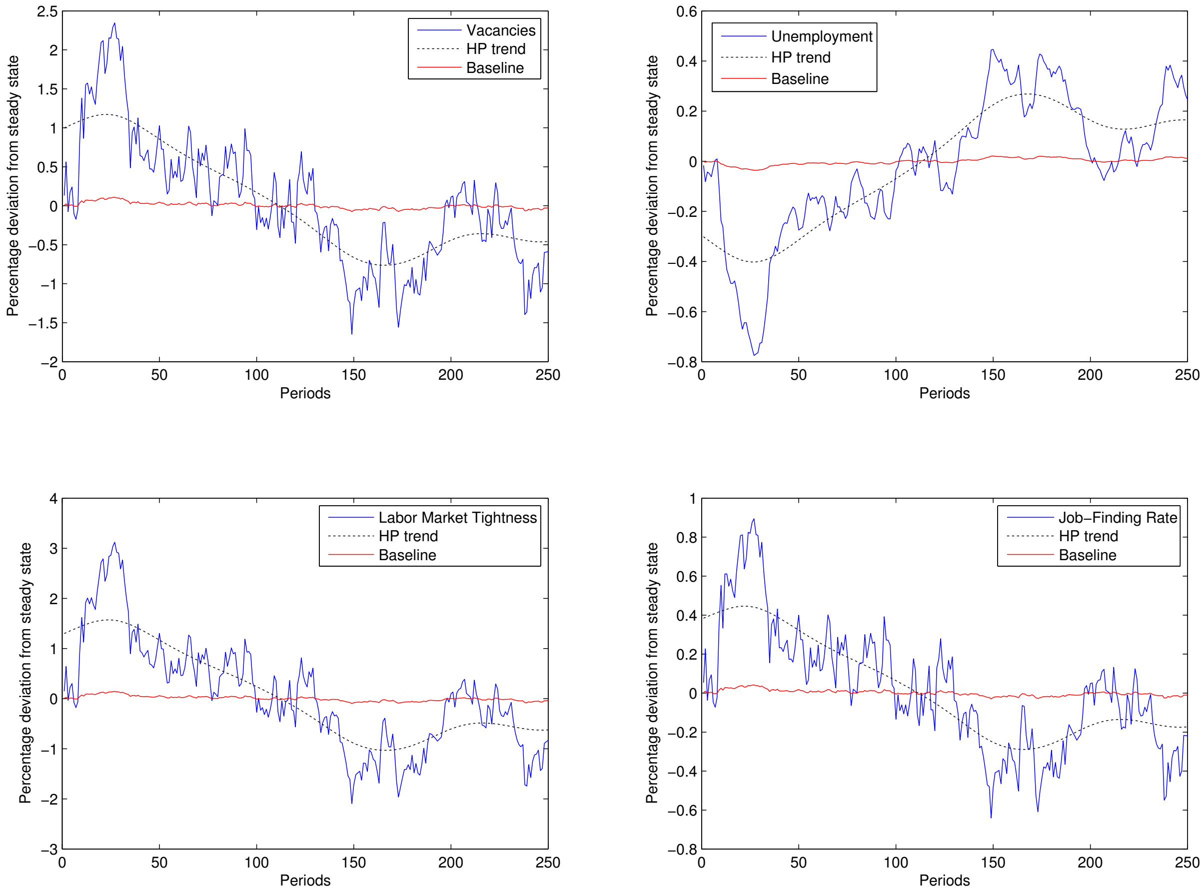


filtered time series of labor market tightness (0.380) is very close to the data (0.382). The standard deviations of vacancies (V) and the job-finding rate (q_t^u) are also much higher than in the baseline model; they even overshoot their empirical counterparts to some extent. This is evidence for strong amplification through firms' vacancy posting, as conjectured in Section 2.3. The standard deviation of unemployment (U) is amplified as well but remains somewhat lower than the empirical value. This is unsurprising in a sorting model. As shown in Section 2.3, the response of the unemployment rate differs across worker types. For high-type workers, unemployment falls disproportionately in response to a positive shock because their outside option changes the most. In contrast, low-type workers are willing to match with all firm types, so the outside-option effect is muted. To improve upon the sorting model's ability to fully match the dynamics of the unemployment rate, one promising approach would be to drop the assumption that the underlying exogenous distribution of worker types, $g_w(x)$ is uniform.

Finally, note also that overall match-specific output, $F(x, y, z)$), fluctuates more in the sorting model than z. This is due to the fact that, as explained in Section 2.3, the production function implies that a higher z increases the incentive to sort (procyclical sorting). It becomes relatively more valuable to be optimally matched as z increases, so $F(x, y, z)$ is more volatile in response to shocks than z alone.

To illustrate the dynamics of the sorting model, Figure 3 shows impulse response functions of six key variables: unemployment, vacancies, aggregate labor market tightness, and the job-finding rate, as well as the autoregressive labor productivity process and wages. In response to a positive shock to labor productivity, unemployment falls by about 6 percentage points initially and shows a hump-shaped return to steady state. Vacancy posting, job finding, and aggregate tightness of the labor market show strong positive reactions directly after the shock. All impulse responses show a high degree of

Figure 4: Simulated Time Series



persistence as well as realistic correlations and cyclical properties. For instance, unemployment and vacancies move in opposite directions in response to the shock and are thus highly negatively correlated (Beveridge Curve). Note also the less-than-proportional adjustment of wages in response to the labor productivity shock: the initial adjustment of wages is roughly 85% of the jump in labor productivity in the depicted example, so in contrast to the textbook model, wages do not increase one-to-one with labor productivity. This is due to the fact that wages are partly shielded from aggregate fluctuations in this model, recall Section 3.2.

Finally, to compare the extent of amplification in the standard and the sorting model, Figure 4 contains simulated time-series data over 250 periods for vacancies, unemployment, labor market tightness, and the job-finding rate. The red time series (baseline) are simulated using the (no sorting, no heterogeneity) model, see row 3 of Table 3 above. The blue time series, simulated with the sorting model, exhibit large and persistent swings in response to shocks. In comparison, the fluctuations generated by the baseline model are hardly visible. Note both simulations are based on a similar series of productivity shocks.

5 Conclusions

This paper studies the amplification of aggregate productivity shocks in a search-matching model with two-sided heterogeneity, sorting, and free entry of firms. The model combines key elements of the DMP equilibrium search model (Pissarides, 2000) with the production complementarities and sorting à la Becker (1973); Shimer and Smith (2000).

A first interesting finding is that the stationary equilibrium of the model does not necessarily produce positive sorting although worker skills, and firm productivity are complements in production. The reason is that firms enter conditional on their productivity. They have no incentive to wait for a better match once they meet a worker because their outside option is zero with free entry. The stationary equilibrium of the model exhibits a small degree of positive sorting, which is in line with recent empirical evidence (e.g., Lise et al., 2016; Lochner and Schulz, 2022).

The main result is that the model amplifies aggregate productivity shocks. The volatility of simulated time series data is of the same order of magnitude as the volatility of unemployment, vacancies, and labor market tightness in U.S. data. Thus, sorting provides a new way to solve the "unemployment-volatility puzzle" (Shimer, 2005). The reason for amplification is that the surplus is supermodular. The incentives to sort increase disproportionately after a positive shock. Firms' vacancy posting and workers' acceptance choices reinforce one another because both parties want to form high-surplus matches. At the same time, unemployment adjusts slowly due to search frictions and meetings with unacceptable types. This leads to a strong vacancy-posting response of firms in the short run, particularly for those with high productivity. In the long run, the adjustment of the distribution of unemployed worker types curtails the firms' vacancy-posting response.

The paper seeks to develop theoretical intuitions, and the model is therefore fairly stylized. In future research, many simplifying assumptions like the absence of on-the-job search and firm size should be relaxed. Furthermore, the model's ability to match cross-sectional facts in addition to time-series facts needs to be scrutinized, e.g., the composition of the pool of unemployed workers over the business cycle and the cyclicality of sorting.

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A Computational Appendix

In the following, I describe the computational procedure used to solve the model and simulate its response to productivity shocks. I also consider the magnitude of computational errors. I use Matlab with Dynare (Adjemian et al., 2022).

A.1 Stationary Equilibrium

Step 1: this describes the Matlab code used to calculate the stationary equilibrium.

1. Setup
   - Set parameter values and density of the grid.
   - Compute the value of output for all (x, y)-combinations based on equation (1).
2. Initialization
   - Set seed for random number generator.
   - Set options for solver of firm's job creation condition (18) (Levenberg–Marquardt algorithm).
   - Create pre-allocations for all matrices and vectors (for speed).
   - Set initial values for all densities listed in Table 1 as well as U and V by integrating the respective densities.
3. Surplus Solution Algorithm: while differences between last and current surplus for all (x, y)-combinations are bigger than some threshold:
   - Update workers' outside option, see equation (15).
   - Update surplus according to (15).
   - Compute acceptance sets by checking where surplus is positive.
   - Smooth along the acceptance cutoff using "mixed strategies" based on random numbers. The goal is to have no discrete jump from 0 to 1 to allow the algorithm to converge more easily.
   - Update distribution of unemployed worker types $g_u(x)$ and aggregate U based on equation (17).
   - Update firms' integral term in the job creation condition, see equation (18).
   - Solve job creation condition (18) for $g_v(y)$ and calculate V and tightness θ.
   - Repeat until differences between iterations of the surplus become smaller than the threshold.

A.2 Model with Shocks

Step 2: This describes the Matlab/Dynare code used to simulate the sorting model with shocks. It makes use of the solution algorithm described in Step 1 to compute numerical derivatives in external functions. This is described further in Section A.3

1. Run Step 1.

2. Set number of periods for stochastic simulations.

3. Create pre-allocations for all matrices and vectors (for speed).

4. For all combinations of discrete worker types x and firm types y run in Dynare:

    - Set seed for random number generator.
    - Declare endogenous variables (E, U, V, $m(U, V)$, θ, $q_u(\theta)$, z, $F(x, y, z)$).
    - Declare exogenous variable (the shock).
    - Declare parameters.
    - Load and set parameter values from current Matlab workspace.
    - Declare external functions to keep track of integral terms.
    - Declare model equations in log deviations from steady state (8 equations for 8 endogenous variables).
        (a) Accounting identify for employment and unemployment.
        (b) Law of motion for employment including external function for endogenous separations.
        (c) Individual labor productivity.
        (d) Aggregate labor productivity.
        (e) Meeting function.
        (f) Job-creation condition including external function for integral expression.
        (g) Labor market tightness.
        (h) Job-finding rate.
    - Computational checks.
    - Declaration of the stochastic shock.
    - Initialize stochastic simulations.
    - Store simulated time series and HP-filtering.

5. Aggregation based on stored simulated time series for all (x, y) combinations.

6. Make Table 3, Figure 3, Figure 4, etc.

A.3 External Functions

Step 4 in A.2 is run for all x and y combinations separately. The challenge is that some model objects depend on distributions over the whole typespace, e.g., the firm's integral term in equation (18). To address this challenge, I use an external function in Dynare to call the solution algorithm described in A.1 from within the model block. This allows me to keep track of the evolution of the integral terms and the adjustment of the distribution functions when labor productivity is stochastic.

1. External function is called by Dynare to compute the integral expression's deviation from steady state. Current value of labor productivity (in log deviations from steady state) is handed over.
2. Calculate the actual value of labor productivity using the steady state value.
3. Use the surplus solution algorithm in A.1 to update the value of the firm's integral term based on the current value of labor productivity.
4. Calculate log deviation from the steady state value of the integral.
5. Hand solution over to Dynare.

A.4 Computational Errors

To check the accuracy of the computational method described in Section 4.1, I plug simulated data from the dynamic sorting model back into the Bellman equations of the model. Ideally, the simulated data would solve these equations exactly. However, I make heavy use of discretization and approximation techniques, so it is reasonable to expect some imprecision. For convenience, I use the wage equation for this test because it contains both the firms' and the workers' integral terms:

$$W(x,y,\Omega) - \alpha \left(F(x,y,z) + c(g_v(y,\Omega))\mathbb{E}\left[\frac{\int_0^1 g_v(y,\Omega')\max\{\mathcal{S}(x,y,\Omega'),0\}\ \mathrm{d}y}{\int_0^1 g_u(x,\Omega')\max\{\mathcal{S}(x,y,\Omega'),0\}\ \mathrm{d}x}\right]\right) \\ - (1-\alpha)b(x) \stackrel{?}{=} 0.$$

Solving the dynamic sorting model by log-linearization and perturbation results in a mean computational error of 3.84%. The 2.5th, 50th, and 97.5th percentiles of the distribution are −15.0%, 4.67%, and 17.6%, respectively. This distribution is slightly left skewed due to the fact that the model's response to shocks is not symmetric around the steady state, for example, because of endogenous separations, which only happen after negative shock. Figures A.1 and A.2 show a histogram of the computational errors and a scatter plot that shows the positive correlation of the errors with z.

A recent paper by Petrosky-Nadeau and Zhang (2016) can serve as a benchmark for the size and distributions of the errors. The authors show that solving a representative agent search and matching model via log-linearization and perturbation—they use Hagedorn and Manovskii (2008) as an example—creates a mean computational error of 3.75% with the 2.5th, 50th, and 97.5th percentiles of the distribution being -11.1%, -3.66%, and 8.76%, respectively. I conclude from this that the errors resulting from the computational approach in this paper lead to errors of an expectable magnitude, even though the errors I find are slightly more dispersed than what Petrosky-Nadeau and Zhang (2016) find for a representative agent search and matching model.

Figure A.1: Histogram of Computational Errors

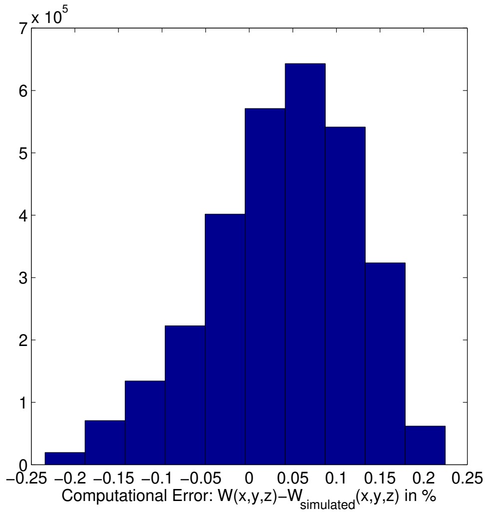
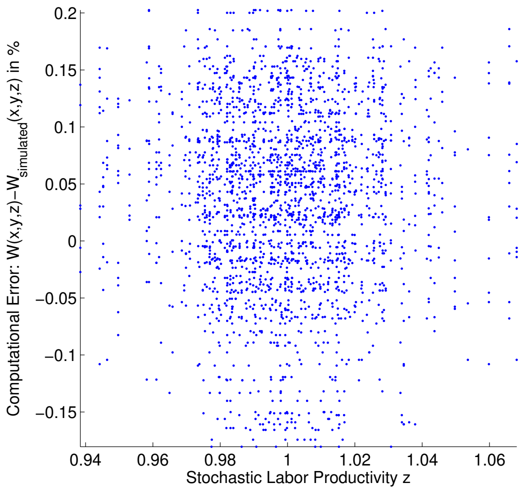


Figure A.2: Scatter Plot of Computational Errors and Stochastic Labor Productivity z

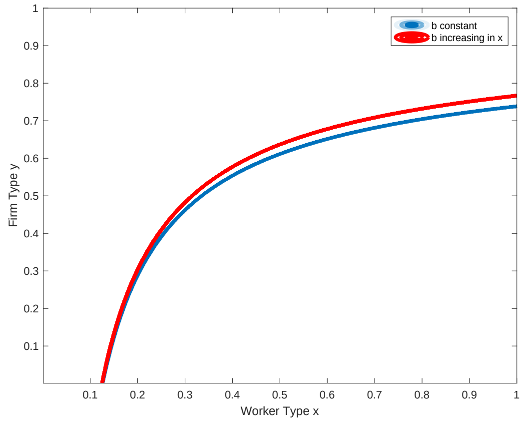


B Additional Results

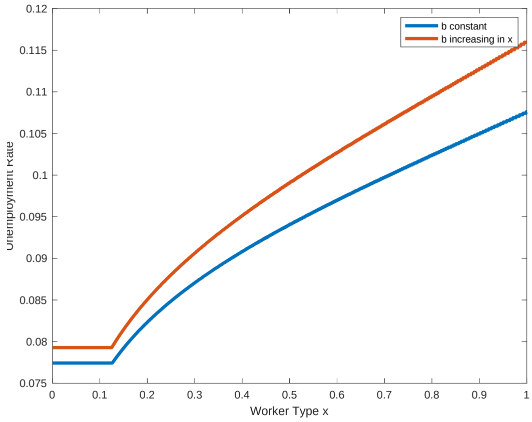
B.1 Stationary Equilibrium with constant b

Figure A.3: Stationary Equilibrium with constant b

(a) Matching Cutoffs



(b) Unemployment across Worker Types



C Wage Formation

To derive match-specific wages in the sorting model, I start with the Nash bargaining solution and impose free entry ($\mathcal{V}(y) = 0$):

$$\mathcal{E}(x, y, \Omega) - \mathcal{U}(x, \Omega) = \frac{\alpha}{1-\alpha}\mathcal{P}(x, y, \Omega). \tag{23}$$

Plugging in the value functions and maximizing the Nash product yields an expression for the match-specific wage in the dynamic model, $W(x, y, \Omega)$:

$$\begin{aligned} W(x, y, \Omega) &= \alpha F(x, y, z) + (1-\alpha)b(x) \\ &\quad + (1-\alpha)\beta\alpha\mathbb{E}\left[q_u(\theta(\Omega'))\int_0^1 \frac{g_v(y, \Omega')}{V(\Omega')}\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y\right]. \end{aligned} \tag{24}$$

As in the standard model, the wage is a convex combination of match-specific output, $F(x, y, z)$, and the worker's outside option. In the sorting model, the integral term deserves attention: the outside option depends on the expected value of the surplus with all other potential employers in the matching set, weighted by the distribution. Thus, the higher the surplus and the higher the probability of meeting other firm types that the worker is willing to match with, the higher the outside option and the bargained wage. After factoring out α, Equation (20) can be used to bring the firm's integral term into the denominator of the expectation:

$$\begin{aligned} W(x, y, \Omega) = \alpha &\left(F(x, y, z) + c(g_v(y, \Omega))\mathbb{E}\left[\theta(\Omega')\frac{\int_0^1 \frac{g_v(y,\Omega')}{V(\Omega')}\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y}{\int_0^1 \frac{g_u(x,\Omega')}{U(\Omega')}\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}x}\right]\right) \\ &+ (1-\alpha)b(x). \end{aligned} \tag{25}$$

Now, the same logic applies from the firm's perspective: the expected value of the surplus, the matching set, and the distribution of other unemployed worker types influence the negotiated wage negatively through the denominator: the more workers are available for the firm to match with and the higher the surplus, the lower is bargained wage with a specufic worker type x. Note that aggregate labor market tightness $\theta(\Omega')$ in front of the quotient cancels out with ${}^1\!/_{V(\Omega')}$ in the numerator and ${}^1\!/_{U(\Omega')}$ in the denominator:

$$W(x, y, \Omega) = \alpha\left(F(x, y, z) + c(y)\mathbb{E}\left[\frac{\int_0^1 g_v(y, \Omega')\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}y}{\int_0^1 g_u(x, \Omega')\mu(x, y, \Omega')\mathcal{S}(x, y, \Omega')\ \mathrm{d}x}\right]\right) + (1-\alpha)b(x), \tag{26}$$

so the match-specific wage $W(x, y, \Omega)$ does not depend on aggregate labor market tightness in the sorting model. The ratio of the two integral term determines how changes in the aggregate state affect the wage.