Spatial Search

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Motivation

- In some locations, it is easier to meet buyers than in others.
  - Fifth Avenue, New York; Bond Street, London
  - Areas that are easily reachable (parking facilities, metro, city center).
  - Top position on a search engine

This paper

- (Random) Search model that takes into account that some locations are better than others.
- Use this framework to answer the following questions:
    - what type of sellers benefit from good locations?
    - how does this translate into spatial sorting?
    - what drives price of a location?
    - social efficiency
    - when is it desirable for urban planner to make locations similar and when different?

Model in a nutshell

- Sellers offer one unit of a product that is characterized by quality and the probability that a buyer likes it.
- Locations differ in how many (effective) buyers there are per seller
- Sellers choose locations in a competitive market
- Buyer-resource constraint
- Each location has a CRS Poisson meeting rate.
- Planner cares about meetings with one or more buyers
- Sellers care about meetings with multiple buyers

Motivation

The White Teeth (toothbrush shop) locates in the center of Amsterdam and not in the village of Edam.

Motivation

In NYC (and not in Pinedale, Wyoming) you find shops that specialize in:

- cashmere sweaters for dogs
- mandolins
- fountain pens
- spy products
- hand-rolled cigars
- African drums
- neon signs
- rubber stamps

Two cases

- Distribution of locations is: (1) exogenous, (2) endogenous.
- What is optimal distribution of locations given the distribution of seller types?
  - total number of meetings is maximized when sellers are located at equidistance
  - equidistance is not always the welfare-maximizing topography.
  - heterogeneous locations results in fewer trades but a larger fraction of high quality trades

Moving to good location versus search intensity

- Differences:
  - If some locations receive more buyer traffic due to a metro stop, others receive less.
  - The price of a location is endogenous in our model.

Random search with locations versus directed search

- Random search is relevant for settings where full ex-ante commitment is not possible.
- In our model, locations act as submarkets
- Directed search also allows sellers to create submarkets
- Differences:
  - higher price for a good location does not go to the buyers
  - high-quality sellers may either overinvest or underinvest in good locations

Literature

Evidence

- **Product market.** Neiman and Vavra (2023) find that niche consumption is largest in areas with many buyers per seller.
- **Labor market.** Gautier and Teulings (2003): locations that can be reached by many workers (based on home-work patterns) hire more extreme skills.

Selling specialized products requires more buyer traffic

- Menzio (2023) considers time variation in search frictions.
- Our work considers spatial variation in search frictions (adds a location choice and sorting dimension to the firm's problem).

Literature

- Spatial sorting and search frictions.
  - Helsey and Strange (1990): better match quality in large cities
  - Gautier and Teulings (2009a) IRS attracts firms that benefit from low search frictions
  - Combes et al. (2008) and Dauth et al. (2022): high-skilled workers sort in dense areas in France and Germany, respectively.
  - Kim (1989): workers specialize more in large markets
  - Gautier et al. (2009): cities are good marriage markets for attractive singles
- Distribution of locations is exogenous

Location choice in urban economics

- Trade-off between positive agglomeration effects and mobility cost.
  - e.g., Ellison and Glaeser (1999), Fujita and Thisse (2002), Ellison, Glaeser and Kerr (2010), Moretti (2012).
- This literature is mostly complementary to this paper
- We have not much to say about the size distribution of cities.
- Our aim is to understand
  - what types of sellers locate in attractive areas
  - why buyer-seller ratios differ accross space
  - whether heterogeneity in location quality is desirable or not.

The model

Agents

- Goods are characterized by their quality z and the corresponding probability $x(z)$ that a buyer likes the good.
- Distribution of z is $F(z)$
- $x(z)$ is weakly decreasing in z
- Expected value of good z to a random buyer, $zx(z)$, is weakly increasing in z.
- Goods with low x (and corresponding high value of z) are niche products.

Search and locations

- Search is random
- Locations are points on a circle with circumference 1.
- Consider $N_s = 6$ sellers and finite number of buyers (N_b).
- Sellers randomly arrive on the circle according to a probability distribution.
- After that, buyers are placed uniformly (normalization) on the circle and go clockwise to the nearest seller.

Search and locations

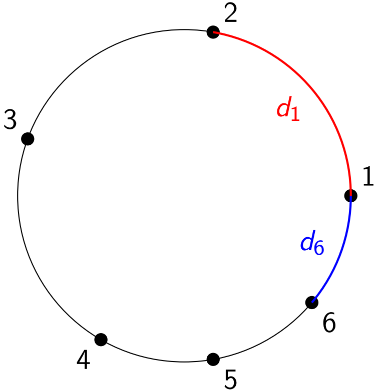


Figure: Finite number of sellers, $d_1 = 2d_6 = 2/9$.

Search and locations

- Quality of a seller's location depends on the arc distance to counterclockwise neighbor, d_i.
- 3 sellers have good spots ($d_i = 2/9$), 3 have bad spots ($d_i = 1/9$)
- Probability that a given buyer visits seller i is d_i, so probability that seller i meets n buyers is given by

$$\binom{N_b}{n} d_i^n \left(1 - d_i\right)^{N_b - n}. \tag{1}$$

Continuum of buyers and sellers

- $N_b \to \infty$ and $N_b/N_s \to \lambda$.
- Expected buyer-seller ratio at location i is $N_b d_i \to \lambda s_i$ where $s_i = d_i N_s$.
- Probability that seller i meets n buyers converges to a Poisson with mean λs_i
- In good locations, λs_i is large
- Advantage of using s_i rather than d_i is that its mean is $\sum_{i=1}^{N_s} s_i/N_s = \sum_{i=1}^{N_s} d_i = 1$, while $d_i \to 0$
- Assumption that buyers arrive uniformly is normalization since only $L(s)$ matters.

Search and locations

- Location-type distribution: $L(s)$ with pdf $s\,dL(s)$.
- In the example, Pr(buyer meets seller in location $s = 4/3$) is $4/3 \cdot 1/2 = 2/3$ and for $s = 2/3$, it is $2/3 \cdot 1/2 = 1/3$.
- Special cases: sellers are placed
  - **equidistant**. $L(s)$ is degenerate at $s = 1$, $P_n(\lambda)$ is Poisson with mean λ.
  - **uniformly**: $L(s) = 1 - e^{-s}$ (exponential) where $s \geq 0$, and $P_n(\lambda)$ is geometric.

Market for locations and payoffs

- Sellers can trade their initial location in a competitive market at **location price** $r(s)$.
- Products are traded by a second-price auction
- Expected **surplus** by an (s, z) seller

$$S(s,z) = z \sum_{n=1}^{\infty} e^{-\lambda s} \frac{(\lambda s)^n}{n!} \left[1 - (1 - x(z))^n\right] = z \left(1 - e^{-\lambda s x(z)}\right),$$

- Expected **payoff seller**

$$\pi(s,z) = z \left(1 - e^{-\lambda s x} - \lambda s x e^{-\lambda s x}\right) = z \mathcal{P}(\lambda x s),$$

- Expected **payoff of a buyer** who meets this seller is

$$z x e^{-\lambda s x},$$

Sellers' location choice

- Maximize expected payoff from choosing location type s,

$$\Pi(z) \equiv \max_s \pi(s, z) - r(s) + R,$$

- R is average location price
- Earnings from location trade irrelevant for location choice.
- First-order condition implies

$$r'(s) = \lambda^2 z x^2 s e^{-\lambda x s},$$

where $z = z^*(s), x = x(z^*(s))$

Buyer entry and equilibrium

- Free entry of buyers at cost K.

$$K = \int_s z^*(s)x^*(s)e^{-\lambda s x^*(s)} s \, dL(s)$$

Definition

An equilibrium is an assignment of sellers to locations, a price schedule for locations $r(s)$ and a measure of buyers λ such that

1. sellers maximize profits
2. the market for locations clears
3. free entry of buyers

Exogenous distribution of locations

Homogeneous products (same z)

- Consider a vegan restaurant in a particular location
  - on average 2 (effective) buyers per seat who have a desire for vegan food for whom this restaurant is nearest
  - meeting rate does not depend on how many people do not like vegan food
- Probability to meet one or more buyers (before locations realized)

$$m(\lambda) \equiv 1 - P_0(\lambda) = \int_s \left(1 - e^{-\lambda s}\right) dL(s).$$

- Seller trades when meeting at least one effective buyer (who likes the product),

$$1 - \sum_{n=0}^{\infty} P_n(\lambda)(1-x)^n = m(\lambda x),$$

Examples

- $m(\lambda x)$ is probability that a seller meets at least one (effective) buyer
- Ex 1. If $L(s)$ is degenerate at 1, then $m(\lambda x) = 1 - e^{-\lambda x}$ (urn-ball)
- *Ex* 2. If $L(s)$ is discrete with $\mathbb{P}(s = s_i) = \ell_i$, then

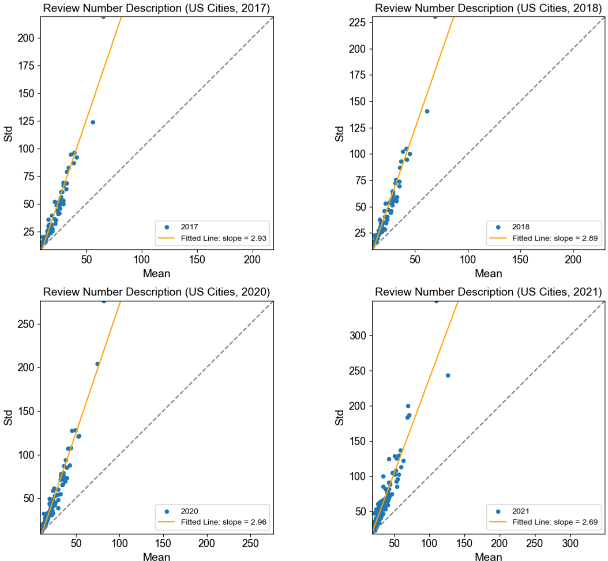
$$m(\lambda x) = \sum_{i=1}^{I} \ell_i \left(1 - e^{-\lambda s_i x}\right).$$

- *Ex* 3. If $L(s)$ is exponential with $L(s) = 1 - e^{-s}$ where $s \geq 0$, then

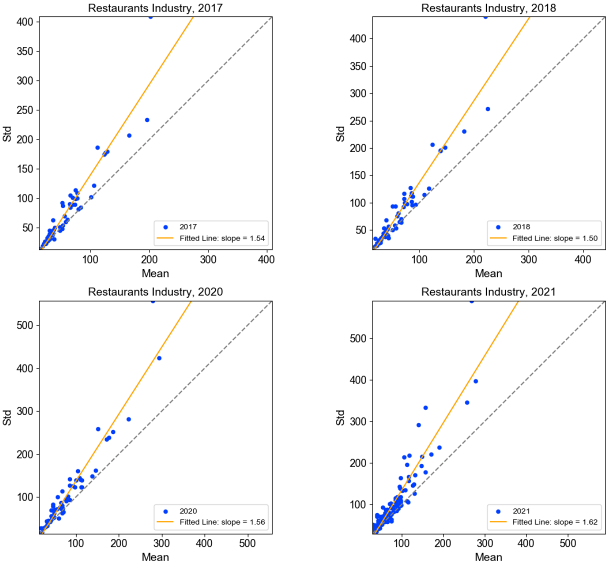
$$m(\lambda x) = \int_0^\infty \left(1 - e^{-\lambda s x}\right) dL(s) = 1 - \frac{1}{1 + \lambda x} \quad \text{(geometric)}$$

- *Ex* 4. If $L(s)$ is a Gamma distribution then $m(\lambda x)$ is negative binomial

Evidence (Yelps) negative binomial



Evidence (Yelps) negative binomial



Planner's problem

- Sellers are identical, so no spatial sorting problem
- Planner selects the measure of buyers λ that maximizes total net surplus (per seller) $Y(\lambda) - \lambda K$, where $Y(\lambda)$ equals

$$Y(\lambda) = \int_s z \left(1 - e^{-\lambda s x(z)}\right) dL(s) = zm(\lambda x(z)),$$

- First-order condition is both necessary and sufficient:

$$K = Y'(\lambda) = \int_s z s x(z) e^{-\lambda s x(z)} \, dL(s).$$

Decentralized equilibrium

- Free entry condition buyers

$$K = \int_s zx(z)e^{-\lambda s x(z)} s \, dL(s),$$

- Entry is efficient
  - No search externalities
  - Second-price auction guarantees that buyers receive their marginal contribution to surplus.
  - Marginal contribution is one if the buyer is pivotal for the trade (no other effective buyers) and zero otherwise.

Price of locations

- $\mathcal{P}(\lambda x(z)s) \equiv \Pr(\geq 2$ effective buyers$)$
- When sellers are identical,

$$r(s) = z\mathcal{P}(\lambda x(z)s).$$

- $r(s)$ has a logistic (S) shape
  - Starting from worst location, adding buyers generates little value because $\Pr(\geq 2$ effective buyers$)$ is close to 0.
  - If we further increase s, better locations help to substantially increase $\Pr(\geq 2$ effective buyers$)$
  - When $\Pr(\geq 2$ effective buyers$)$ is close to 1, better locations add little value

Maximum number of matches

- For a given $L(s)$, $m(\lambda)$ can be thought of as a mixture of urn-ball processes.
- Invariant meeting technologies can be written as

$$m(\lambda) = \int_s \left(1 - e^{-\lambda s}\right) \, dL(s).$$

- Aggregate matching efficiency can be improved by making locations more equal (Jensen's inequality).

Heterogeneous sellers

Spatial Sorting

- Seller type distribution: $F(z)$.
- Assume $zx(z)$ is weakly increasing in z.
- Since surplus $S(s,z) = z\left(1 - e^{-\lambda s x(z)}\right)$ is supermodular in (s,z), the planner's solution is characterized by PAM
- Suppose F and L are continuous. Optimal assignment is

$$1 - F(z) = 1 - L(s^*(z)),$$

where $s^*(z)$ is the optimal location s for seller type z.

Surplus

- Given optimal location of sellers, expected total surplus is

  $Y(\lambda) - \lambda K$, where

  $$Y(\lambda) = \int_s z^*(s) \left(1 - e^{-\lambda s x^*(s)}\right) dL(s),$$

  $x^*(s) \equiv x(z^*(s))$ and $z^*(s)$ is the inverse of $s^*(z)$.

- Planner
  - instructs PAM between sellers and locations
  - optimal buyer entry

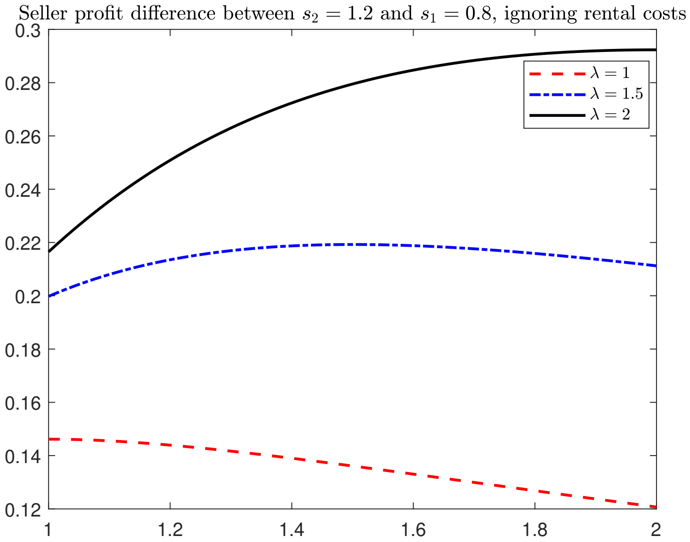
Decentralized equilibrium

Proposition

The decentralized equilibrium is constrained efficient if and only if $zx'(z)/x(z) \geq -1/2$ for any z.

- Understanding the inefficiency.
    - Inefficiency occurs if $x(z)$ decreases fast with z
    - A higher s increases planner's surplus $S(s,z)$ because it increases Pr(seller meets ≥ 1 effective buyer(s))
    - A higher s increases firm's profit $\pi(s,z)$ because it increases Pr(seller meets ≥ 2 effective buyers) ▸ Shape
    - Investment of a seller in a good location is partly a rent-seeking activity.

Illustrating inefficient sorting



Example: vertical quality ($x = 1$), z differs across sellers

- Consider two location distributions: $L_0(s) = 1 - e^{-s}$ and $L_1(s)$ is degenerate at $s = 1$.
- $F(z) = 1 - (1/z)^\alpha$ with $\alpha > 1$
- Increasing α, while adjusting z_0 to keep $\overline{z}$ fixed, reduces quality dispersion.
- Surplus is increasing in quality dispersion
- Increasing location quality dispersion:
  - If λ is small ($< 1/\alpha$), location dispersion is desirable
  - If λ is large, location dispersion is not desirable

Implications

- When the location distribution is dispersed, sellers with high z are willing to pay a high price for good locations.
- This can also benefit low quality sellers, since they all own a location.
- With increasing and convex production cost, it is possible that high quality sellers do not enter when locations are similar ($L_1(s)$) but they do when they are dispersed ($L_0(s)$).
- When all locations are identical, offering a high-quality product is too risky.

Endogenous distribution of locations

Endogenous topography

- Planner's problem is given by

$$\max_{s(z)} \quad Y = \int_{z_0}^{\infty} z\left(1 - e^{-\lambda s(z) x(z)}\right) dF(z)$$
$$\text{s.t.} \quad \int_{z_0}^{\infty} s(z) dF(z) = 1$$

- Solving the above equation yields that $s_p(z)$ is increasing so PAM between sellers and locations continues to hold.

Decentralized market (heterogeneous sellers)

- Sellers form a coalition, as in a real estate investment trust (REIT).
- First choose seller-optimal distribution of locations, then redistribute competitive rents back in a lump sum way to sellers.
- After purchase of locations, random meetings and second-price auction.

Decentralized Market

- Coalition optimally assigns arc length $s_c(z)$

$$\mathcal{L} = \int_z z\widehat{\mathcal{P}}(\lambda x(z)s(z))dF(z) + \zeta\lambda\left(1 - \int_z s(z)dF(z)\right)$$

where ζ is modified multiplier (multiplied by λ) and $\widehat{\mathcal{P}}(\cdot)$ is concave hull of $\mathcal{P}$.

- Resource constraint: arc length 1.

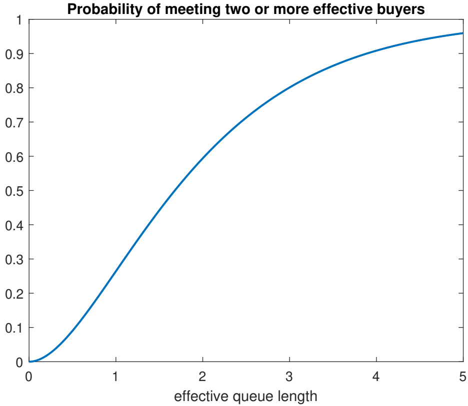
Decentralized Market, heterogeneous sellers

- Coalition can affect λ by making some sellers inactive
- λ will be such that PAM occurs
- Two sources of inefficiencies
  - Coalition would like to exclude some sellers to take advantage of increasing returns to scale region when the effective queue length is too small. ▸ Figure
  - The coalition assigns a longer arc length to high-z sellers and a shorter arc length to low-z sellers, while the total arc length is fixed.

Discussion

- New framework to think about spatial sorting
- High quality and niece sellers sort into locations with many buyers-per seller
- Inefficiencies arise because for sellers *two or more buyers* are valuable whereas for planner *one buyer* is also valuable.
- $x(z)$ was probability that a buyer likes a good. What if it is probability that buyer can afford a good?
- Then, good locations may attract high-quality products that few buyers can afford. Different policy implications.

Probability two or more effective buyer visists



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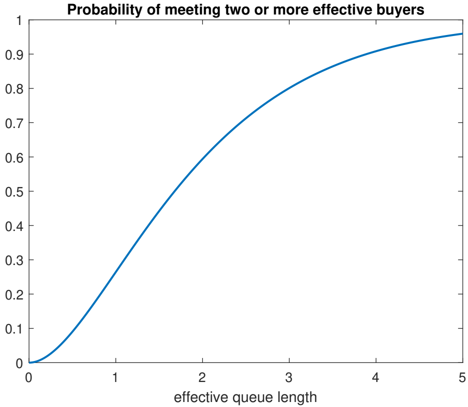
Probability two or more effective buyer visists



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