An Empirical Framework for Matching with Imperfect Competition

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Motivation - Income Inequality

- **Recent focus on role of market power in wage setting to explain cross-sectional variation in earnings for observationally similar workers.**

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- **Recent focus on role of market power in wage setting to explain cross-sectional variation in earnings for observationally similar workers.**
- In this paper, we build, identify and estimate a static, many-to-one matching model of labor market featuring oligopsony and two-sided heterogeneity
- Key Features:
  - Firms have heterogeneous productivity and non-pecuniary amenities (vertical differentiation)
  - Workers have heterogeneous preferences over firm amenities (horizontal differentiation)
  - Worker-firm-specific match productivity (Roy sorting)
  - Employer concentration and strategic interactions (SI) in labor markets
  - Non-employment

Literature

Two main approaches to modeling labor market competition:

- Monopsonistic competition: e.g. Lamadon, Mogstad, Setzler (2022, LMS), Card, Cardoso, Heining, and Kline (2018, CCHK).
  - Assume "atomistic" firms $\rightarrow$ no role for market concentration to matter.
  - Uniform markdowns.
- Oligopsony: e.g. Berger, Herkenhoff, and Mongey (2022, BHM).
  - Allow for strategic interactions.
  - Restricts worker heterogeneity in skills and preferences.

We build on both approaches, allowing wages to depend on:

- Market concentration and strategic interactions.
- Worker and firm heterogeneity.
- Endogenous markdowns $\rightarrow$ ignoring SI leads to overestimating labor supply elasticities.

more

Our Contribution

1. Tractable characterization of model equilibrium that permits to :
    - show **existence** and **uniqueness** of the equilibrium for a large class of worker preferences and firm production technologies
    - show a rich set of comparative statics
    - analyze how strategic interactions amplify pass-through effects
    - show that globally convergent methods allow to solve for unique equilibrium of model counterfactuals
    - decompose wage inequality into skill, preferences, etc.
2. New measure of market concentration linked to social welfare function.
3. Novel approach to identification of labor supply and demand.
    - Labor supply following Berry (1994)
    - Labor demand following productivity literature (e.g.: Doraszelski and Jaumandreu, 2018)

Model Outline

Workers

- Large population divided into K types, $k \in \{1, ..., K\}$.
  - Population m_k of individuals of type k.
  - k defined as function of underlying characteristics $\overline{X}$ observed by firms (e.g., age, gender, education, ability, non-cognitive skills, etc).
  - Econometrician may observe only a part of k.

Firms

- Finite set of firms $j \in \{1, ..., J\}$.
- Firms differentiate workers at k level.
- Production function: $Y^j = F^j(\ell_{.j})$ where ℓ_{kj} is # of k workers at j.
- Non-pecuniary amenities u_{kj}.
- Choose wages w_{kj} to min costs, taking wages of other firms as given.

Model - Worker Preferences for Firms (ARUM)

- Workers choose job (firm) which offers greatest utility U_{ij}.
  Utility of individual i of type k working at firm j with wage w_{kj} is:

$$U_{ij} = \underbrace{\ln u_{kj} + \beta_{kj} \ln w_{kj}}_{v_{kj}} + \epsilon_{ij}, \; j \in \{1, ..., J\}$$

  - ϵ_{ij}: stochastic payoff unknown to firms

- Utility of non-employment:

$$U_{i0} = \underbrace{\beta_{k0} \ln w_{k0}}_{v_{k0}} + \epsilon_{i0}$$

- Given a population of m_k type-k workers, labor supply is:

$$(\ell_{kj})^s = m_k \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}$$

  where $G_{k\cdot}(v_{k\cdot}) = \mathbb{E}\left[\max_{j \in \mathcal{J} \cup \{0\}} \{v_{kj} + \epsilon_{ij}\}\right]$ is the *social surplus function* for type k.

Wage Setting

- Optimal wage offer to type k worker given by:

$$w_{kj} = \underbrace{\underbrace{\lambda_j}_{mr_j} \underbrace{F_k^j(\ell_{\cdot j})}_{mpl_{kj}}}_{mrpl_{kj}} \times \underbrace{\frac{\mathcal{E}_{kj}(w_{k\cdot})}{1 + \mathcal{E}_{kj}(w_{k\cdot})}}_{md_{kj}}$$

  - λ_j: Marginal Revenue (mr_j) of output at the optimal choice of Y^j.
  - $F_k^j(\ell_{\cdot j}) = \frac{\partial F^j(\ell_{\cdot j})}{\partial \ell_{kj}}$: kj Marginal Product of labor (mpl_{kj}).
  - $mrpl_{kj}$: Marginal Revenue Product of Labor for kj.
  - $\mathcal{E}_{kj} \equiv \frac{\partial \ell_{kj}}{\partial w_{kj}} \frac{w_{kj}}{s_{kj}}$: Elasticity of kj Labor Supply (LS).
  - md_{kj}: markdown for kj.

Model - Equilibrium

Definition: Worker-firm matching function and equilibrium wage equation such that optimality and population constraints hold.

Key Features

- Unrestricted strategic interaction unlike LMS, CCHK, and BHM
- Under fairly general shape conditions on demand/production, we prove **existence** and **uniqueness** of equilibrium.
- $\exists$ globally convergent methods (e.g., Gauss-Jacobi) for recovering unique equilibrium outcome.

▸ Eqm Details ▸ Model: Comparative Statics ▸ Model: Generalized Concentration Index

Empirical Model: Worker Preferences

- Workers Preferences: Nested Logit
- Partition firms into nests ("local markets") indexed by g.
- The ϵ_{ijt} within each nest are correlated, $\sigma_{kg} = \frac{1}{\sqrt{1-corr(\epsilon_{ijt},\epsilon_{ilt})}} \geq 1$
- $\underbrace{s_{kjt}}_{\text{firm j's share of total mkt}} \equiv \underbrace{s_{kjt|g}}_{\text{j's share of mkt g}} \times \underbrace{s_{kgt}}_{\text{mkt g's share of total market}}$
- Labor supply elasticity is

$$\mathcal{E}_{kjt} = \beta_k [\sigma_{kg} + \underbrace{(1-\sigma_{kg}) s_{kjt|g} - s_{kjt}}_{\leq 0}]$$

- Assuming atomistic firms/mkts ($s_{kjt|g} = 0$, $s_{kjt} = 0$) biases $\mathcal{E}_{kjt} \uparrow$

Empirical Model: Firm Technology

- Firm-specific Production Function:

$$F^j(\ell_{\cdot j}) = \left(\sum_k \tilde{\gamma}_{kjt} \ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}}$$

- WLOG can decompose $\tilde{\gamma}_{kjt} = \theta_{jt}\gamma_{kjt}$ with $\sum_k \gamma_{kjt} = 1$

- $\rho_k < 1$ allows workers to be imperfect substitutes.

- $\alpha_{jt} > 0$ governs returns to scale/output elasticities.

- Common approach restricts $\rho_k = 1$, $\alpha_{jt} = \alpha$, and imposes multiplicative structure for γ_{kjt} (i.e.: $\gamma_{kj} = \gamma_k \gamma_j$).

- Parameters of interest:

$$\phi_{kj} = (\underbrace{\theta_{jt}, \alpha_{jt}, \gamma_{kjt}, \rho_k}_{\text{Demand}}, \underbrace{\beta_k, \sigma_{kg}, u_{kjt}}_{\text{Supply}}),$$

Identification: Labor Supply

- Wages are not only endogenous to amenities (LMS) **but also** to other wages and their relationship to own wage (a competition index)

- Introduce quasi-labor supply function following Berry (1994):

$$\log \frac{s_{kjt}}{s_{k0t}} = \beta_k \log \frac{w_{kjt}}{w_{k0t}} + (1 - 1/\sigma_{kg}) s_{kjt|g} + \log u_{kjt}$$

- **Identification challenge:** wages and inside share are correlated with worker-firm amenities u_{kjt}

- Take long Δs of the quasi-labor supply function and use short Δs in $[R_{jt}, s_{kjt|g}, s_{-kjt|g}]$ as *IV*s for the long Δs in wages and inside share

- Valid if productivity shocks (proxied by short Δs in revenue) are persistent and amenity shocks (or their component correlated to productivity) are transitory (similar to LMS)

- Once we recover β_k and σ_{kg}, we can recover labor supply elasticities $\mathcal{E}_{kjt}$ and infer u_{kjt} from quasi-labor supply function

Identification: Labor Demand

- Recall $F^j(\ell_{\cdot j}) = \left(\sum_{k \in C_j} \tilde{\gamma}_{kjt} \ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}}$

- Define $\tilde{w}_{kjt} \equiv \frac{\mathcal{E}_{kjt}+1}{\mathcal{E}_{kjt}} w_{kjt}$ ($= MRPL_{kjt}$ from FOC)

- Stacking the firm FOC for types k and h and taking logs:

$$\log \frac{\tilde{w}_{kjt}}{\tilde{w}_{hjt}} = (\rho_k - 1) \log \ell_{kjt} - (\rho_h - 1) \log \ell_{hjt} + \log \frac{\rho_k}{\rho_h} + \log \frac{\tilde{\gamma}_{kjt}}{\tilde{\gamma}_{hjt}}$$

- **Identification challenge:** labor productivities $\tilde{\gamma}_{\cdot jt}$ correlated with labor demand

- Solution: impose standard timing restrictions on labor productivities

$$\tilde{\gamma}_{kjt} = \overline{z}_{kt} z_{kjt} \quad \text{where} \quad \log z_{kjt} = \delta \log z_{kjt-1} + \overline{\eta}_k + \underbrace{\eta_{kjt}}_{iid}$$

Identification: Labor Demand

- Substitution yields:

$$\log \frac{\tilde{w}_{kjt}}{\tilde{w}_{hjt}} = c_{kht} + (\rho_k - 1)\log \ell_{kjt} - (\rho_h - 1)\log \ell_{hjt} + \delta \log \frac{\tilde{w}_{kjt-1}}{\tilde{w}_{hjt-1}} - \delta(\rho_k - 1)\log \ell_{kjt-1} + \delta(\rho_h - 1)\log \ell_{hjt-1} + \eta_{khjt}$$

- $\eta_{khjt} \equiv \eta_{kjt} - \eta_{hjt}$ correlated with ℓ_{kjt} and ℓ_{hjt} but uncorrelated with lagged inputs, wages and revenues

- Identify ρ_k, ρ_h and δ using lagged inputs, wages and revenues as IVs

- Finally can back out α_{jt}, γ_{jkt}, and θ_{jt} (so $\tilde{\gamma}_{kjt}$) using firm's FOC for wages Details

Empirical Application

- Matched employee-employer data from Denmark (2001 to 2019)
  - Focus on **full-time workers in private sector** excl. finance (31.8% female v. 48.7% of all workers in all sectors)
  - Partition workers into observable types: age-education-sex cells
  - Labor market = Denmark $\times$ Year for given worker type
  - Local market (nest) = Commuting Zone $\times$ Industry
  - **Note:** estimation dataset selects long-lived large establishments due to the estimation strategy/data requirements Firm Selection
- Estimate empirical model to recover:
  - Labor supply/demand/substitution elasticities
  - Markdowns and job amenities
  - Labor productivities
  - Market concentration indices GCI
- Counterfactuals [in progress]

Data Summary

	k-group (k)	share of worker-obs.	avg. earnings (in 2022 DKK)	share of establishments
1	Female, 26-35, no college	0.046	358,580	0.1771
2	Female, 26-35, college	0.033	457,272	0.0919
3	Male, 26-35, no college	0.118	435,593	0.3645
4	Male, 26-35, college	0.052	545,413	0.1373
5	Female, 36-50, no college	0.110	404,992	0.2977
6	Female, 36-50, college	0.052	562,668	0.1224
7	Male, 36-50, no college	0.238	497,331	0.4991
8	Male, 36-50, college	0.095	740,494	0.2073
9	Female, 51-60, no college	0.059	401,460	0.1915
10	Female, 51-60, college	0.018	547,066	0.054
11	Male, 51-60, no college	0.139	484,611	0.3365
12	Male, 51-60, college	0.040	753,551	0.1178
Number of worker-observations				12,742,746
Number of unique establishments				238,299
Number of establishment $\times$ k-group $\times$ year obs.				4,054,235
Number of local markets (16 CZ $\times$ 16 Ind)				256
Firms that appear at least once in the estimation dataset				3,069,490
Observations in the estimation dataset				1,101,541
Number of unique establishments				63,525

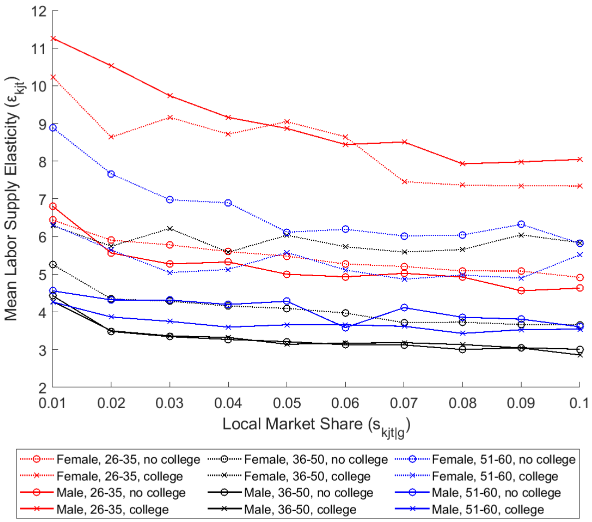
Labor Supply Elasticities and Markdowns

		Labor Supply Elasticities		Markdowns	
	k-group (k)	IV	OLS	IV	OLS
1	Female, 26-35, no college	6.221	-0.010	0.857	-0.010
2	Female, 26-35, college	9.061	-0.489	0.889	-1.144
3	Male, 26-35, no college	6.606	1.724	0.858	0.619
4	Male, 26-35, college	10.747	1.535	0.900	0.591
5	Female, 36-50, no college	5.096	1.121	0.824	0.519
6	Female, 36-50, college	6.141	0.249	0.849	0.197
7	Male, 36-50, no college	4.325	1.392	0.800	0.574
8	Male, 36-50, college	4.100	0.369	0.793	0.265
9	Female, 51-60, no college	8.426	1.695	0.871	0.616
10	Female, 51-60, college	5.755	0.956	0.837	0.479
11	Male, 51-60, no college	4.508	1.561	0.788	0.598
12	Male, 51-60, college	4.070	0.657	0.787	0.388
	Overall	5.750	1.112	0.829	0.434

Estimated labor supply elasticities, $\mathcal{E}_{kjt}$, and markdowns, md_{kjt}, from the labor supply model. Mean of the pooled (over time) distribution of establishment-level labor supply elasticities and markdowns for each k-group and overall. IV and OLS estimates.

▸ Labor Supply Parameters

Labor Supply Elasticities by Local Market Share



Job Amenities

Examine correlation of job amenities and job characteristics by running:

$$\log u_{kjt} = \text{kgroup} + \text{cz} + \text{ind} + \text{year} + X_{jt}\beta + \epsilon_{kjt}$$

where X_{jt} includes logs of establishment/firm size, wage bill, revenue

High Amenity:

- Large Cities (Copenhagen, Aarhus, Aalborg)
- Manufacturing, Retail, Knowledge-Based Services
- Large establishments

Low Amenity

- Rural locations (south-west Jutland and Zeeland)
- Mining and energy-water provision, Accommodation/Food Services, Arts/Entertainment
- High wage and high revenue firms

Labor Demand and Production Function

Key findings from estimation of production function:

- **Significant heterogeneity in returns to scale (α_{jt})**
  - Mean: 0.214, Median: 0.181
  - Mean is driven by long right tail α_{jt} Distribution
- **Labor types are close substitutes**
  - Estimate $\rho_k \approx 1$ for all k Production Function Parameters
  - Implies high elasticity of substitution EoS
- **Labor demand is heterogeneous and highly elastic**
  - Median labor demand elasticities range from -3 (middle-aged uneducated men) to -12 (educated women). Details
- **Labor match productivity varies across workers and firms**
  - Productivity is highly persistent, $\delta = 0.81$
  - γ_{kjt} higher for college workers (especially older workers)
  - γ_{kjt} increasing, then decreasing, in age. Details

Conclusion

Takeaways from modeling strategic interactions in labor markets:

- Tractable equilibrium characterization of model with two-sided heterogeneity and oligopsony in labor markets
- New model-based measure of concentration with link to welfare ▸ GCI
- Novel strategy for identifying labor supply/demand in presence of strategic interactions
- Estimation of heterogeneity in supply/demand elasticities, labor productivity and amenities

Next steps:

- Counterfactuals for decomposing wage inequality:
  - Roy Sorting
  - Compensating differentials
  - Market concentration / Strategic interaction
- Counterfactuals for merger analysis in labor markets, etc.

Appendix

Literature

- **Many-to-one matching models**
    - Kelso and Crawford (1982), Azevedo (2014)
- **Imperfect Competition in labor market:**
    - "Monopsonistic competition" (Atomistic firms): Burdett and Mortensen (1998), Taber and Vejlin (2016), **Card, Cardoso, Heining and Kline (2018, CCHK)**, Kline, Petkova, Williams, and Zidar (2019), **Lamadon, Mogstad and Setzler (2019, LMS)**, Kroft, Luo, Mogstad and Setzler (2021), Yeh, Macaluso and Hershbein (2021)
        - Allow for two-sided heterogeneity but assume "atomistic" firms $\rightarrow$ no role for market concentration, elasticities and markdowns are biased
    - "Oligopsony": Manning (2003, 2006), MacKenzie (2018) Jarosch, Nimczik and Sorkin (2019), **Berger, Herkenhoff and Mongey (2019, BHM)**
        - Model a limited form of strategic interactions and restrict worker heterogeneity in skills and preferences $\rightarrow$ no role for sorting
    - Concentration, outside options and wages: Caldwell and Harmon (2019), Benmelech, Bergman and Kim (2020), Rinz (2020), Azar, Marinescu, and Steinbaum (2020), Caldwell and Danieli (2022)

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Comparative statics: Pass-through

The equilibrium effect of productivity shocks on wages is positive:

- Firm wages increasing in productivity of own and other firms
- SI amplify the pass-through effect of a firm-specific productivity shock on equilibrium wages

The equilibrium effect of amenity shocks on wages is ambiguous:

- Increase in firm's amenities directly *lowers* the wages at that firm,
- But *increases* other firms' wages,
- This will lead to price competition for which the final sign depends on the primitives of the model

With SI, can no longer use pass-through to infer monopsony power Back

Sample Selection

Sample Selection

	step	observations
1	All salaried jobs in Denmark between 2001 and 2019	76,869,608
2	Keep jobs held by workers in relevant k-groups	50,263,511
3	Keep jobs with market information (primary jobs)	32,486,151
4	Drop workers in small commuting zones	32,106,644
5	Drop jobs with no earnings or hours	32,094,227
6	Drop public sector jobs	20,719,775
7	Drop financial sector jobs	19,538,794
8	Keep full-time, highest-paying jobs	12,742,746
9	Collapse at the establishment-year-kgroup level	4,487,628
	Number of unique establishments	259,195
10	Firms that appear at least once in the estimation dataset	3,069,490
	Observations in the estimation dataset	1,101,541
	Number of unique establishments	63,525

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Establishment Characteristics by Commuting Zone

commuting zone	n. establishments per local market (avg. over years) mean	st. dev.	establishment size (n. of workers) mean	st. dev.	establishment revenue (thousands of USD) mean	st. dev.
1. North and East Zealand (Copenhagen)	2,214	2,302	9.464	47.563	5,217	59,088
2. West and South Zealand (Slagelse)	273	296	6.120	32.348	3,599	46,472
3. West and South Zealand (Køge)	300	346	6.124	20.539	3,333	19,655
4. West and South Zealand (Nykøbing Falster)	117	120	5.625	15.201	2,851	12,032
7. Fyn (Odense)	479	505	7.785	25.773	4,545	26,329
8. Fyn (Svendborg)	79	83	5.266	10.771	2,776	8,437
9. South Jutland (Sønderborg)	153	168	8.671	50.425	5,083	22,917
10. South Jutland (Ribe)	57	62	5.768	18.300	4,013	17,634
11. South Jutland (Kolding)	251	289	7.754	20.269	4,735	16,300
12. Mid-South Jutland (Vejle)	379	434	8.282	47.698	5,367	44,826
13. South-West Jutland (Esbjerg)	288	301	7.474	24.683	5,339	42,731
14. West Jutland (Herning)	260	285	7.486	23.830	4,362	18,486
15. North-West Jutland (Thisted)	62	63	6.689	22.267	3,737	13,857
16. East Jutland (Aarhus)	806	877	7.990	27.837	4,596	37,670
17. Mid-North Jutland (Viborg)	215	222	7.349	49.867	3,823	22,287
19. North Jutland (Aalborg)	448	446	7.327	23.713	4,556	47,503
20. North Jutland (Frederikshavn)	172	179	6.025	18.502	3,531	14,180
All of Denmark	385	819	8.064	37.002	4,665	43,897

Number of establishments in the local markets (commuting zone $\times$ industry) in the commuting zone, average over the years. Average establishment size (in number of workers of all types) and revenue per commuting zone, average over the years. Commuting zones from Eckert, Hejlesen and Walsh (2021).

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Establishment Characteristics by Industry

	n. establishments per local market (avg. over years)		establishment size (n. of workers)		establishment revenue (thousands of USD)	
commuting zone	mean	st. dev.	mean	st. dev.	mean	st. dev.
A. Agriculture, forestry, and fishery	309	147	2.448	4.409	1,662	2,678
B. Mining and quarrying	12	8	15.342	71.899	37,222	283,213
C. Manufacturing	588	525	20.072	78.832	11,523	57,847
D. Electricity, gas, steam etc.	27	17	16.582	52.103	32,114	253,964
E. Water supply, sewerage etc.	49	40	11.068	22.613	4,126	9,651
F. Construction	758	807	5.633	16.489	2,432	8,558
G. Wholesale and retail trade	1,725	2,175	5.819	16.923	6,323	29,710
H. Transportation	352	398	11.677	51.127	6,868	122,580
I. Accommodation and food services	287	482	3.498	9.671	1,421	4,909
J. Information and communication	309	698	11.573	53.543	4,634	16,620
K. Financial and insurance	202	281	15.941	72.875	170	4,603
L. Real estate	258	436	3.708	9.556	1,117	3,888
M. Knowledge-based services	550	999	7.957	33.071	2,623	12,952
N. Travel agent, cleaning etc.	284	458	7.336	22.805	2,877	9,222
R. Arts, entertainment, recreation	149	197	6.078	15.149	1,010	21,641
S. Other services	309	386	4.740	14.807	390	2,184
All industries	385	819	8.064	37.002	4,665	43,897

Number of establishments in the local markets (commuting zone $\times$ industry) by industry, average over the years. Average establishment size (in number of workers of all types) and revenue by industry, average over the years. 5-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors.

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Existence and Uniqueness

A unique equilibrium exists if the following conditions hold:

1. Independence of ϵ_{ij} from v_{kj} and general conditions on smoothness
2. Holding other factors constant, supply system satisfies:

$$\frac{\partial s_{kj}}{\partial w_{kl}} \begin{cases} \geq 0, & \text{if } l = j \\ \leq 0, & \text{if } l \in \mathcal{J}_0 \setminus \{j\} \end{cases}$$

$$\zeta_{kjl} \equiv \frac{w_{kl}}{\mathcal{E}_{kj}} \frac{\partial \mathcal{E}_{kj}}{\partial w_{kl}} \begin{cases} \leq 0, & \text{if } l = j \\ \geq 0, & \text{if } l \in \mathcal{J}_0 \setminus \{j\} \end{cases}$$

$\Longrightarrow$ Sign restrictions on "cross-wage super elasticities"

$\Longrightarrow$ Trivially holds in the Nested Logit Economy

3. Sufficient but Not necessary condition is an additive separable production function.

$$F^j(\ell_{\cdot j}) = \sum_{k \in \mathcal{K}} h_k(\ell_{kj})$$

where $h'_k(x) \geq 0$ and $h''_k(x) \leq 0$.

- Uniq' may hold for certain level of non-separability/complementarity
- We derive a test for it

Model - Societal Welfare

- The societal welfare function:

$$\mathcal{W}(\Xi, \lambda, \mathcal{R}) = \sum_{k=1}^{K} m_k G_{k\cdot}(\widetilde{v}_{k\cdot})$$

$$\widetilde{v}_{kj} \equiv \begin{cases} v_{kj} + \ln(1 + \phi(s, w; \lambda, \mathcal{R})), & \text{if } (k, j) \in \mathcal{R} \\ v_{kj}, & \text{if } (k, j) \notin \mathcal{R}. \end{cases}$$

where $\widetilde{v}_{kj}$ is utility inclusive of redistributed profits ▸ function

- **Duality result:** $G_{k\cdot}(\widetilde{v}_{k\cdot}) = \sum_{j=0}^{J} \widetilde{v}_{kj} s_{kj} - G^*_{k\cdot}(s_{k\cdot})$ where $G^*_{k\cdot}(s_{k\cdot})$ is the convex conjugate (Galichon and Salanié, 2020)
- $-G^*_{k\cdot}(s_{k\cdot})$ is an entropy index, a measure of dispersion of the firm-specific idiosyncratic shocks

Model - Societal Welfare and Market Concentration

- We have market concentration when we have a market with few firms with a large share of the market
- Define generalized exponential concentration index $GCI \equiv e^{G^*_{k\cdot}(s_{k\cdot})}$

$$\mathcal{W}(\Xi, \lambda, \mathcal{R}) = \overbrace{\left[\sum_{(k,j)\in\mathcal{K}\times\mathcal{J}_0} m_k v_{kj} s_{kj} + \ln[1 + \phi(s, w; \lambda, \mathcal{R})] \sum_{(k,j)\in R} m_k \beta_{kj} s_{kj} \right]}^{deterministic\ \ gains} \underbrace{- \sum_{k=1}^{K} m_k \ln GCI(s_{k\cdot})}_{concentration\ \ index}$$

- **Intuition:** Negative link between societal welfare and market concentration. The more firms in a labor market, the more firm-specific preference shocks the worker draws, the more likely it is that one of the draws is high.

Model - Societal Welfare and Market Concentration

- GCI is a natural measure of concentration in our model
- Unlike Herfindahl-Hirschman Index (HHI), it has some desirable properties (Maasoumi and Slottje 2003)
  - Direct link with the Societal Welfare
    - $GCI \uparrow \Rightarrow \mathcal{W}(\Xi, \lambda, \mathcal{R}) \downarrow$
  - Features additive decomposability (Nested Logit case)
    - Concentration is sum of "between group" component and "within group" component
    - Useful for dealing with heterogeneity in firms, regions, market areas
    - Policy maker can identify impacts of policy on any desired group of firms and overall concentration

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Generalized Concentration Index Across k-groups

In the Nested Logit economy, the HHI is $\sum_j s_{kj}^2$ and the GCI is

$$\color{red}{GCI(s_{k\cdot})} \equiv \left[\Pi_{g=0}^{G} \left(\underbrace{\exp\left\{ \sum_{j \in N_g} s_{kj|g} \ln s_{kj|g} \right\}}_{\text{within group concentration index}} \right)^{\frac{s_{kg}}{\sigma_{kg}}} \right] \times \underbrace{\left[\exp\left\{ \sum_{g=0}^{G} s_{kg} \ln s_{kg} \right\} \right]}_{\text{between group concentration index}}$$

		GCI					
	k-group (k)	Overall	Within-Group	Between-Group	Mean Local WCI	Mean Local HHI	Overall HHI
10	Female, 51-60, college	**0.1131**	0.6934	0.1622	0.5581	0.1701	**0.0002**
9	Female, 51-60, no college	**0.0960**	0.6861	0.1393	0.3521	0.0995	**0.0001**
1	Female, 26-35, no college	**0.0913**	0.5955	0.1510	0.3496	0.1276	**0.0000**
2	Female, 26-35, college	**0.0716**	0.6048	0.1179	0.5725	0.1410	**0.0003**
5	Female, 36-50, no college	**0.0448**	0.5563	0.0804	0.2655	0.0921	**0.0001**
6	Female, 36-50, college	**0.0422**	0.5319	0.0789	0.4230	0.1384	**0.0005**
12	Male, 51-60, college	**0.0294**	0.5275	0.0554	0.4374	0.1267	**0.0003**
11	Male, 51-60, no college	**0.0279**	0.5430	0.0511	0.2561	0.0697	**0.0001**
4	Male, 26-35, college	**0.0248**	0.4957	0.0499	0.3912	0.1506	**0.0004**
3	Male, 26-35, no college	**0.0235**	0.4925	0.0475	0.2563	0.0941	**0.0001**
7	Male, 36-50, no college	**0.0187**	0.4776	0.0391	0.2097	0.0643	**0.0001**
8	Male, 36-50, college	**0.0173**	0.4290	0.0402	0.3297	0.1241	**0.0005**

Generalized Concentration Index Across k-groups

In the Nested Logit economy, the HHI is $\sum_j s_{kj}^2$ and the GCI is

$$GCI(s_{k\cdot}) \equiv \left[\Pi_{g=0}^{G}\left(\underbrace{\exp\left\{\sum_{j\in N_g} s_{kj|g}\ln s_{kj|g}\right\}}_{\text{within group concentration index}}\right)^{\frac{s_{kg}}{\sigma_{kg}}}\right] \times \underbrace{\left[\exp\left\{\sum_{g=0}^{G} s_{kg}\ln s_{kg}\right\}\right]}_{\text{between group concentration index}}$$

		GCI					
	k-group (k)	Overall	Within-Group	Between-Group	Mean Local WCI	Mean Local HHI	Overall HHI
10	Female, 51-60, college	0.1131	0.6934	0.1622	**0.5581**	**0.1701**	0.0002
9	Female, 51-60, no college	0.0960	0.6861	0.1393	**0.3521**	**0.0995**	0.0001
1	Female, 26-35, no college	0.0913	0.5955	0.1510	**0.3496**	**0.1276**	0.0000
2	Female, 26-35, college	0.0716	0.6048	0.1179	**0.5725**	**0.1410**	0.0003
5	Female, 36-50, no college	0.0448	0.5563	0.0804	**0.2655**	**0.0921**	0.0001
6	Female, 36-50, college	0.0422	0.5319	0.0789	**0.4230**	**0.1384**	0.0005
12	Male, 51-60, college	0.0294	0.5275	0.0554	**0.4374**	**0.1267**	0.0003
11	Male, 51-60, no college	0.0279	0.5430	0.0511	**0.2561**	**0.0697**	0.0001
4	Male, 26-35, college	0.0248	0.4957	0.0499	**0.3912**	**0.1506**	0.0004
3	Male, 26-35, no college	0.0235	0.4925	0.0475	**0.2563**	**0.0941**	0.0001
7	Male, 36-50, no college	0.0187	0.4776	0.0391	**0.2097**	**0.0643**	0.0001
8	Male, 36-50, college	0.0173	0.4290	0.0402	**0.3297**	**0.1241**	0.0005

Generalized Concentration Index Across k-groups

In the Nested Logit economy, the HHI is $\sum_j s_{kj}^2$ and the GCI is

$$GCI(s_{k\cdot}) \equiv \left[\Pi_{g=0}^{G} \left(\underbrace{\exp\left\{ \sum_{j\in N_g} s_{kj|g} \ln s_{kj|g} \right\}}_{\text{within group concentration index}} \right)^{\frac{s_{kg}}{\sigma_{kg}}} \right] \times \underbrace{\left[\exp\left\{ \sum_{g=0}^{G} s_{kg} \ln s_{kg} \right\} \right]}_{\text{between group concentration index}}$$

	k-group (k)	GCI Overall	GCI Within-Group	GCI Between-Group	Mean Local WCI	Mean Local HHI	Overall HHI
10	Female, 51-60, college	**0.1131**	**0.6934**	**0.1622**	0.5581	0.1701	0.0002
9	Female, 51-60, no college	**0.0960**	**0.6861**	**0.1393**	0.3521	0.0995	0.0001
1	Female, 26-35, no college	**0.0913**	**0.5955**	**0.1510**	0.3496	0.1276	0.0000
2	Female, 26-35, college	**0.0716**	**0.6048**	**0.1179**	0.5725	0.1410	0.0003
5	Female, 36-50, no college	**0.0448**	**0.5563**	**0.0804**	0.2655	0.0921	0.0001
6	Female, 36-50, college	**0.0422**	**0.5319**	**0.0789**	0.4230	0.1384	0.0005
12	Male, 51-60, college	**0.0294**	**0.5275**	**0.0554**	0.4374	0.1267	0.0003
11	Male, 51-60, no college	**0.0279**	**0.5430**	**0.0511**	0.2561	0.0697	0.0001
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8	Male, 36-50, college	**0.0173**	**0.4290**	**0.0402**	0.3297	0.1241	0.0005

Model - Profit Redistribution

Total firm profits are redistributed to an exogenous group $\mathcal{R}$:

$$\sum_{j=1}^{J} \underbrace{\left(\lambda_j F^j(\ell_{\cdot j}) - \sum_{k=1}^{K} w_{kj}\ell_{kj} \right)}_{\pi_j} = \sum_{(k,j)\in\mathcal{R}} \phi(s, w; \lambda, \mathcal{R}) w_{kj}\ell_{kj}$$

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Labor Supply Parameter Estimates Across k-groups

		IV			OLS		
		β_k	σ_{kg}		β_k	σ_{kg}	
	k-group (k)		CZ 1 (CPH)	Avg. across CZ		CZ 1 (CPH)	Avg. across CZ
1	Female, 26-35, no college	1.7008	3.9655	3.2278	-0.0019	5.5485	4.3425
		[1.3857; 2.0786]	[3.0143; 4.4446]		[-0.0199; 0.0129]	[4.3590; 5.7150]	
2	Female, 26-35, college	1.9224	5.6982	2.8030	-0.0994	6.3524	3.4048
		[1.3154; 2.5072]	[3.1683; 7.3241]		[-0.1238; -0.0717]	[4.6266; 6.3820]	
3	Male, 26-35, no college	1.3915	5.6536	3.7996	0.3211	6.5604	4.2404
		[1.3771; 1.5974]	[4.0430; 5.8627]		[0.3084; 0.3252]	[5.1794; 6.1460]	
4	Male, 26-35, college	2.2248	3.9258	3.9227	0.3227	5.0573	3.4226
		[1.8226; 2.6060]	[2.7578; 4.6116]		[0.3055; 0.3407]	[3.8569; 5.0560]	
5	Female, 36-50, no college	1.0778	6.1691	3.9135	0.2257	6.3466	3.9910
		[0.9966; 1.3178]	[4.3853; 6.5157]		[0.2162; 0.2293]	[4.7693; 5.9782]	
6	Female, 36-50, college	1.5397	4.4633	3.7759	0.0000	4.8126	3.6567
		[1.2340; 2.1316]	[2.9458; 5.0524]		[0.0493; 0.0788]	[3.5650; 4.5589]	
7	Male, 36-50, no college	0.8738	6.5446	3.9297	0.2722	6.3509	4.2412
		[0.9171; 1.0248]	[4.5864; 6.0434]		[0.2628; 0.2735]	[4.6799; 5.4609]	
8	Male, 36-50, college	1.0803	4.4032	3.0397	0.0978	4.5304	2.8523
		[0.9420; 1.3421]	[3.1971; 4.2648]		[0.0870; 0.1056]	[3.4419; 4.1216]	
9	Female, 51-60, no college	1.0730	7.5244	6.0719	0.2820	7.3534	4.7559
		[0.8017; 1.3607]	[5.5800; 9.4166]		[0.2750; 0.2920]	[6.1462; 7.8470]	
10	Female, 51-60, college	1.0397	6.5277	4.7269	0.2248	5.3630	3.1703
		[0.6634; 1.4159]	[4.5384; 9.8245]		[0.2090; 0.2457]	[3.8532; 6.2031]	
11	Male, 51-60, no college	0.7374	7.4153	5.6223	0.2707	7.6109	4.7654
		[0.7235; 0.9123]	[5.1050; 7.5457]		[0.2584; 0.2718]	[5.8271; 7.0742]	
12	Male, 51-60, college	0.9378	4.0508	3.8517	0.1571	4.5812	3.2370
		[0.7156; 1.2335]	[2.8674; 4.2529]		[0.1497; 0.1702]	[3.4596; 4.5567]	

The first column are the point estimates for β_k (and 95% confidence intervals in parenthesis). The second column shows estimates for the σ_{kg} for the Copenhagen commuting zone. The third column shows the average σ_{kg} estimate across commuting zones.

Labor Supply Elasticity by Commuting Zone

	Labor Supply Elasticities		Markdowns	
	IV	OLS	IV	OLS
1. North and East Zealand (Copenhagen)	6.625	1.288	0.863	0.416
2. West and South Zealand (Slagelse)	4.554	0.909	0.802	0.425
3. West and South Zealand (Køge)	3.542	0.672	0.745	0.365
4. West and South Zealand (Nykøbing Falster)	3.647	0.918	0.780	0.433
7. Fyn (Odense)	4.959	1.042	0.813	0.442
8. Fyn (Svendborg)	2.873	0.715	0.720	0.374
9. South Jutland (Sønderborg)	4.432	1.075	0.823	0.448
10. South Jutland (Ribe)	2.006	0.383	0.646	0.261
11. South Jutland (Kolding)	8.599	1.102	0.871	0.447
12. Mid-South Jutland (Vejle)	4.885	0.911	0.802	0.421
13. South-West Jutland (Esbjerg)	3.510	0.760	0.754	0.389
14. West Jutland (Herning)	4.294	0.858	0.795	0.409
15. North-West Jutland (Thisted)	3.435	0.632	0.759	0.355
16. East Jutland (Aarhus)	7.148	1.270	0.858	0.463
17. Mid-North Jutland (Viborg)	3.093	0.641	0.746	0.356
19. North Jutland (Aalborg)	6.730	1.359	0.851	0.466

Estimated labor supply elasticities and markdowns from the labor supply model. Average of the pooled (over time) distribution of establishment-level estimates for each commuting zone.

Source: Administrative registers, Statistics Denmark. Commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2021). We drop six small island and merge Aalborg and Frederikshavn. The names of the commuting zones reflect the main geographic areas in Denmark and the major city/town.

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Labor Supply Elasticity by Industry

	Labor Supply Elasticities		Markdowns	
	IV	OLS	IV	OLS
A. Agriculture, forestry, and fishery	5.197	1.087	0.810	0.451
B. Mining and quarrying	3.878	0.911	0.756	0.406
C. Manufacturing	5.562	1.074	0.820	0.433
D. Electricity, gas, steam etc.	4.853	0.930	0.795	0.406
E. Water supply, sewerage etc.	5.043	1.121	0.807	0.462
F. Construction	5.372	1.270	0.818	0.510
G. Wholesale and retail trade	5.826	1.139	0.832	0.434
H. Transportation	5.562	1.215	0.824	0.472
I. Accommodation and food services	6.103	1.123	0.841	0.399
J. Information and communication	6.493	1.067	0.847	0.368
L. Real estate	6.069	1.021	0.834	0.363
M. Knowledge-based services	6.372	0.879	0.841	0.298
N. Travel agent, cleaning etc.	6.032	1.127	0.838	0.405
R. Arts, entertainment, recreation	6.032	1.044	0.837	0.376
S. Other services	5.788	1.103	0.832	0.419

Estimated labor supply elasticities and markdowns from the labor supply model. Average of the pooled (over time) distribution of establishment-level estimates for each commuting zone.

Source: Administrative registers, Statistics Denmark. 5-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors.

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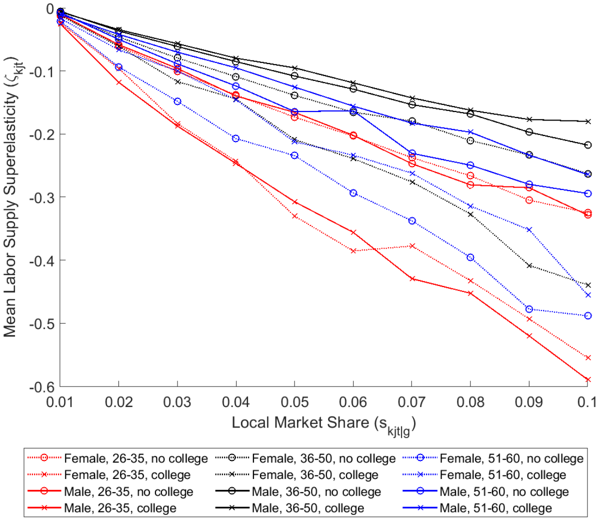
Labor Supply Super Elasticities by Local Market Share



Identification: Additional Production Parameters

- Firm FOC implies $\gamma_{hjt} = A_{khjt}\gamma_{kjt}$ where $A_{khjt} \equiv \frac{\tilde{w}_{kjt}^{-1}\ell_{kjt}^{\rho_k - 1}\rho_k}{\tilde{w}_{hjt}^{-1}\ell_{hjt}^{\rho_h - 1}\rho_h}$
- Use this and $\sum_k \gamma_{kjt} = 1$ to get $\gamma_{kjt} = \frac{1}{\sum_h A_{khjt}}$ for all k
- Define x_{kjt} as revenue share of wages paid to labor type k by firm j
- Under the assumption that $\lambda_{jt} = P_{jt} = 1$ (perfect competition):

$$\alpha_{jt} = x_{ijt}\frac{\mathcal{E}_{kjt} + 1}{\mathcal{E}_{kjt}}\frac{\sum_k \gamma_{kjt}\ell_{kjt}^{\rho_k}}{\gamma_{kjt}\rho_k\ell_{kjt}^{\rho_k}}$$

- Finally, using the production function:

$$\theta_{jt} = \frac{R_{jt}^{\frac{1}{\alpha_{jt}}}}{\sum_{k\in C_j}\gamma_{kjt}\ell_{kjt}^{\rho_k}}$$

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Identification: Multi-equation GMM Approach

- $K+1$ parameters to estimate ($\rho_k \, \forall \, k$ and δ), $K \times (K-1)/2$ equations
- Since not all firms employ every labor type, any subset of equations will somewhat arbitrarily ignore the contribution of some firms.
- No base type of labor employed by all firms (all the labor ratio equations could be cast in terms of that type).
- Solution: use all $K*(K-1)/2$ equations in a multi-equation GMM estimator
- Non-linear system – not feasible
- Linear system with cross-equation parameter restrictions and compound parameters (such as $\delta(\rho_k - 1)$): $2K+1$ parameters to estimate, need $2K+1$ instruments

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Substitution Parameter Estimates Across k-groups

	k-group	IV $\rho_k - 1$	IV $\delta(\rho_k - 1)$	IV δ	IV ρ	OLS ρ
1	Female, 26-35, no college	0.005	0.005	0.806	1.005	0.985
		[-0.0035; 0.0116]	[-0.0015; 0.0104]	[0.8040; 0.8085]	[0.9965; 1.0116]	
2	Female, 26-35, college	0.029	0.028		1.029	0.985
		[0.0191; 0.0382]	[0.0191; 0.0369]		[1.0191; 1.0382]	
3	Male, 26-35, no college	0.007	0.006		1.007	0.987
		[0.0001; 0.0135]	[0.0002; 0.0120]		[1.0001; 1.0135]	
4	Male, 26-35, college	0.028	0.029		1.028	0.981
		[0.0156; 0.0363]	[0.0170; 0.0368]		[1.0156; 1.0363]	
5	Female, 36-50, no college	0.016	0.016		1.016	0.978
		[0.0063; 0.0260]	[0.0068; 0.0252]		[1.0063; 1.0260]	
6	Female, 36-50, college	0.002	-0.004		1.002	0.992
		[-0.0114; 0.0201]	[-0.0178; 0.0119]		[0.9886; 1.0201]	
7	Male, 36-50, no college	-0.024	-0.022		0.976	0.977
		[-0.0333; -0.0151]	[-0.0302; -0.0134]		[0.9667; 0.9849]	
8	Male, 36-50, college	-0.065	-0.067		0.935	0.999
		[-0.0832; -0.0505]	[-0.0841; -0.0527]		[0.9168; 0.9495]	
9	Female, 51-60, no college	0.003	0.002		1.003	0.990
		[-0.0094; 0.0159]	[-0.0097; 0.0134]		[0.9906; 1.0159]	
10	Female, 51-60, college	-0.027	-0.034		0.973	1.017
		[-0.0538; 0.0022]	[-0.0603; -0.0042]		[0.9462; 1.0022]	
11	Male, 51-60, no college	-0.016	-0.013		0.984	0.985
		[-0.0276; -0.0053]	[-0.0245; -0.0028]		[0.9724; 0.9947]	
12	Male, 51-60, college	-0.036	-0.041		0.964	1.026
		[-0.0526; -0.0071]	[-0.0578; -0.0137]		[0.9474; 0.9929]	

Parameter estimates for ρ_k and δ (95% confidence intervals in brackets), IV and OLS.

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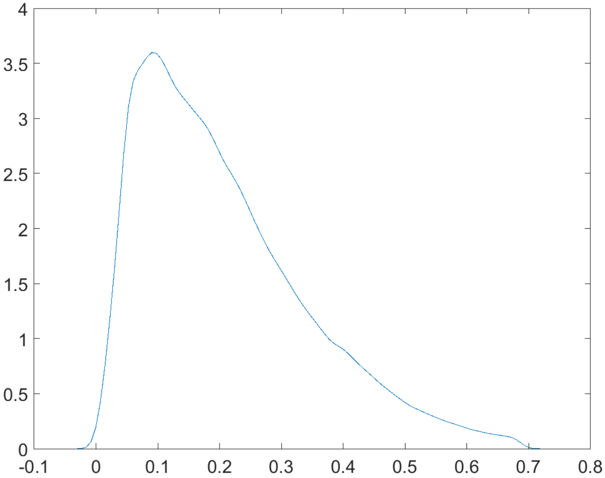
Distribution of Returns to Scale (α_{jt})



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Labor Demand Elasticity (η^d_{kjt}) Estimates Across k-groups

	k-group	mean	median
1	Female, 26-35, no college	-27.070	-9.859
2	Female, 26-35, college	21.871	-8.528
3	Male, 26-35, no college	9.423	-5.557
4	Male, 26-35, college	-60.934	-9.597
5	Female, 36-50, no college	-9.958	-7.228
6	Female, 36-50, college	-28.406	-12.042
7	Male, 36-50, no college	-4.003	-2.961
8	Male, 36-50, college	-4.884	-4.326
9	Female, 51-60, no college	-24.150	-10.801
10	Female, 51-60, college	-13.663	-12.035
11	Male, 51-60, no college	-6.225	-4.537
12	Male, 51-60, college	-8.461	-7.265
	Overall	-10.130	-5.317
	α_{jt}	**0.214**	**0.181**

Mean and Median moments of the estimated distribution of the firm-level labor demand elasticities, η^d_{jkt}, and of α_{jt}.

$$\eta^d_{kjt} = \frac{F^j_k}{\ell_{kj} F^j_{kk}} = \left[\left(\alpha_{jt} - 1\right) \rho_k \frac{\gamma_{kjt} \ell^{\rho_k}_{kjt}}{\sum_k \gamma_{kjt} \ell^{\rho_k}_{kjt}} + (\rho_k - 1) \right]^{-1}$$

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Within-firm Heterogeneity in γ_{kjt} by k-group

$$\gamma_{kjt} = \textbf{k-group} + \text{firm} \times \text{year} + \epsilon_{kjt} \quad \text{(OLS Regression)}$$

k-groups (reference: Male, 36-50, college)	
Female, 26-35, no college	-0.141***
Female, 26-35, college	-0.133***
Male, 26-35, no college	-0.110***
Male, 26-35, college	-0.112***
Female, 36-50, no college	-0.119***
Female, 36-50, college	-0.096***
Male, 36-50, no college	-0.046***
Female, 51-60, no college	-0.126***
Female, 51-60, college	-0.084***
Male, 51-60, no college	-0.057***
Male, 51-60, college	-0.016***
Observations	2,212,859
R^2	0.866

- γ_{kjt} higher for college workers – specially older workers.
- γ_{kjt} increasing, then decreasing, in age.
- γ_{kjt} higher for men than women (selection into public sector?)

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Generalized Morishima Elasticity of Substitution (EoS)

- Equal to standard (Allen-Uzawa) EoS in 2-factor case
- Well-defined for non-homogeneous functions with >2 factors
- Asymmetric in >2 factor case and depends on input levels

kgroup (k)		1	2	3	4	5	6	7	8	9	10	11	12
Female, 26-35, no college	**1**	0.000	-42.408	-161.292	-213.633	-78.236	509.590	53.027	199.254	-188.191	2305.295	275.522	259.186
Female, 26-35, college	**2**	-167.702	0.000	-116.351	-36.067	-68.926	-738.942	-30.376	25.459	-137.810	38.114	-24.768	13.799
Male, 26-35, no college	**3**	-183.081	-44.982	0.000	-37.974	-62.162	-470.888	22.544	-48.417	-188.822	12.312	32.264	26.610
Male, 26-35, college	**4**	135.456	-34.794	-123.300	0.000	-63.261	777.898	37.310	16.324	203.975	672.550	170.036	86.509
Female, 36-50, no college	**5**	-156.040	-34.324	-129.924	-34.621	0.000	-446.429	-13.551	18.938	-143.926	26.302	-12.732	23.749
Female, 36-50, college	**6**	-625.230	1.886	-95.268	-347.174	19.897	0.000	-230.629	25.288	-469.772	3160.167	238.521	364.677
Male, 36-50, no college	**7**	53.561	-47.594	-92.527	7.871	-88.013	304.341	0.000	16.831	156.177	14.468	59.949	23.035
Male, 36-50, college	**8**	191.783	-26.738	-79.587	-2.763	-59.367	689.618	43.243	0.000	285.119	178.059	96.934	34.477
Female, 51-60, no college	**9**	-335.283	-64.344	-205.839	-283.844	-92.330	55.278	199.206	239.285	0.000	2410.664	312.512	242.736
Female, 51-60, college	**10**	726.857	-31.966	-289.987	106.195	-73.053	2680.956	41.468	-16.329	594.126	0.000	110.431	17.422
Male, 51-60, no college	**11**	185.110	-41.939	-130.870	28.557	-120.615	430.314	51.008	15.262	172.789	-129.088	0.000	16.049
Male, 51-60, college	**12**	388.163	-46.231	-142.917	42.493	-68.685	1608.536	40.619	6.247	222.158	107.447	78.366	0.000

Each cell is the **mean** Generalized Morishima elasticity of substitution calculated across all firms which employ both types of labor.

▸ Formula ▸ back

Generalized Morishima Elasticity of Substitution

Definition: Proportionate change in two input factors resulting from a change in their marginal rates of substitution *holding all other rates of substitution and output fixed.*

$$GMEOS_{khjt} = \frac{F^j_{ht}}{\ell_{kjt}} \frac{H_{khjt}}{H_{jt}} - \frac{F^j_{ht}}{\ell_{hjt}} \frac{H_{hhjt}}{H_{jt}}$$

where $F^j_{kt} = \frac{\partial F^j}{\partial \ell_{kjt}}$, H_{jt} is the bordered Hessian for F^j, and $H_k hjt$ is the cofactor of the k, h term in H_{jt}.

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