

# Families' Career Investments and Firms' Promotion Decisions

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  3. Does the interplay between families and firms matter for the effects of *policies aimed at reducing gender differences*?
- ▶ To answer these and other questions, we need a model of the marriage market, families' joint labor supply choices, and firms' training and promotion decisions.

# The Link between marriage and labor markets

**Marriage Market**

**Households**

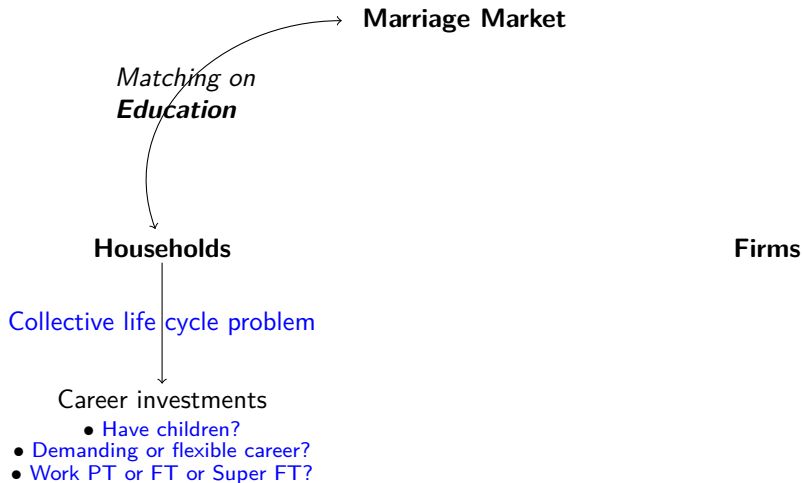
**Firms**



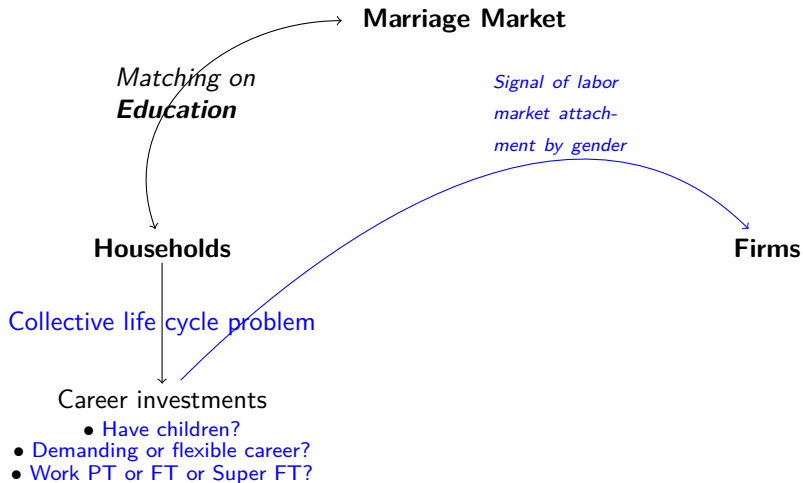
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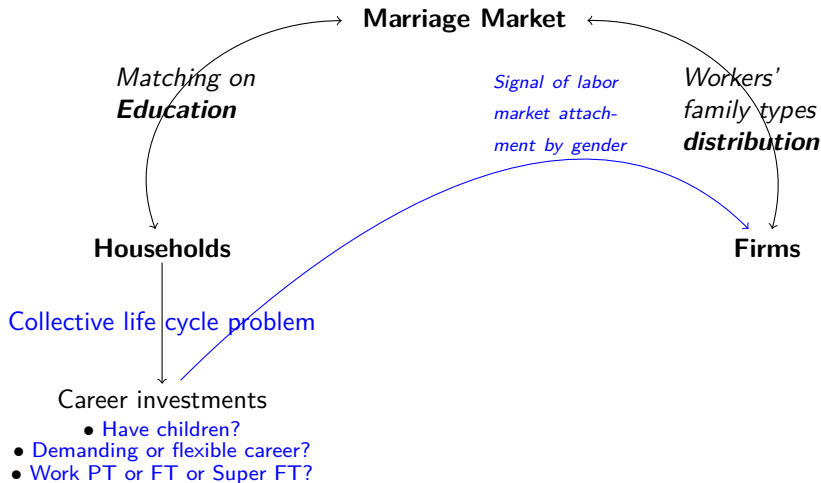
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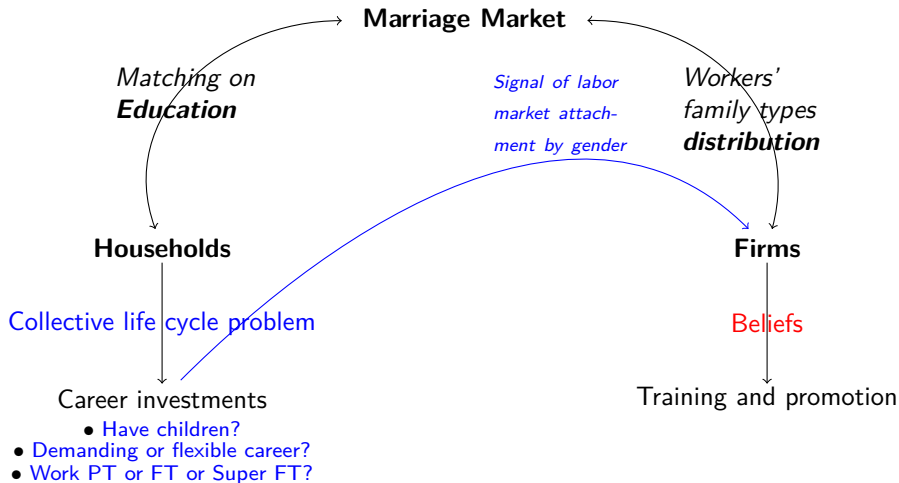
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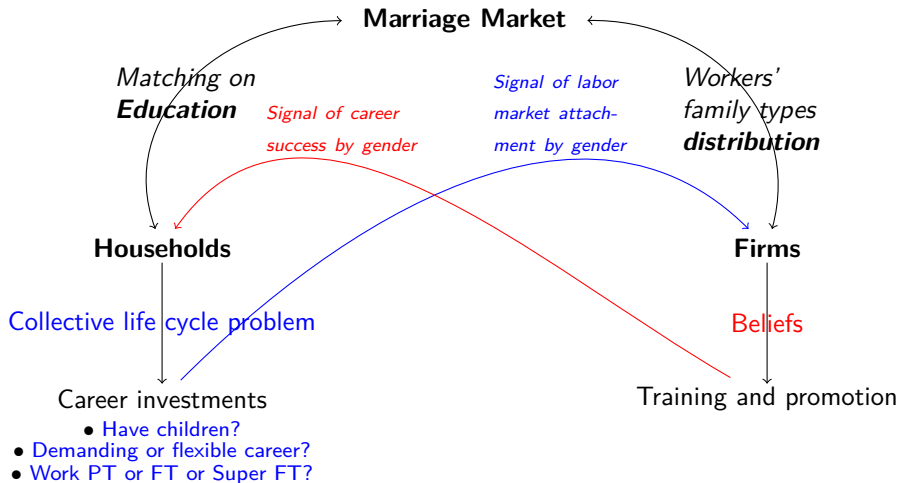
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- initial differences are amplified through the interaction of families and firms.
- 3. Evaluate policies to promote families' and firms' investments in women.
  - ▶ We consider different parental leave policies and a management quota.
  - ▶ Quantify heterogeneous effects by gender, education, and type of couple.
  - ▶ Heterogeneity of couples and the marriage market matter for policy effects.

# Roadmap

Data

Facts

Model

Estimation

Policy Analysis

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Appendix

# Data: Follow families and firms across a cohort's life cycle

- ▶ Danish register data:

- education, family history, and labor market history for the full population.
- labor force survey: detailed work hours (weekly, overtime, evening, weekend).

▶ Details

- ▶ Follow the cohort who graduates from highest degree between 1991 and 1995:

- ▶ their *decisive* domestic partner, and
- ▶ their employers and occupations,
- ▶ from household formation and labor market entry,
- ▶ over their life cycle (approx. 25 years).

- ▶ Dataset of ~120K households and all of their employers.



# Measurement

- ▶ **Ambition** types (AFRSV, 2024),  $\theta_i$ 
  - ▶ For 1800+ **education programs**, compute average starting wages  $w_0$  and 10Y wage growth  $g$  of all program **graduates**.
  - ▶ Categorize programs into 4 groups ranging from low-level, low-growth to high-level, high-growth programs. [▶ Ambition Details](#)
- ▶ Career **ladders**, steep and flat
  - ▶ Compute 10-year wage growth by **firm-occupation pair**.
  - ▶ Group into **steep** and **flat** ladders using cutoff at 80th percentile [▶ Ladder Details](#)
- ▶ Promotion to **manager**
  - ▶ First time in occupational codes for "Management" (combines middle and top management jobs) [▶ Promotion Details](#)
- ▶ Firm's **managerial training** combines
  - ▶ participation in **managerial training programs**, and
  - ▶ **job assignments** that predict subsequent manager promotion. [▶ Training Details](#)
- ▶ **Super-full-time** work (SFT). Captures overtime and lack of work-life balance. [▶ SFT Details](#)

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# Facts: Families' and firms' investments interact

1. Large gender gaps in training and promotion. ▶ Fact 1
2. *Firm-side* investments heterogeneous across workers' family characteristics:
  - ▶ Gender gaps vary by family type; ▶ Fact 2a
  - ▶ are notoriously big when husbands ever become managers. ▶ Fact 2b
  - ▶ Likelihood of receiving investments positively associated with spouse's type, conditional on own type. Labor market controls diminish this effect. ▶ Fact 2c
3. Fertility & spousal time allocation within the household play a key role:
  - ▶ Within-couple gaps in human capital increase upon arrival of children; ▶ Fact 3a
  - ▶ and timing of fertility depends on wives' ambition types. ▶ Fact 3b

→ Facts motivate a model in which family-side and firm-side investments interact.

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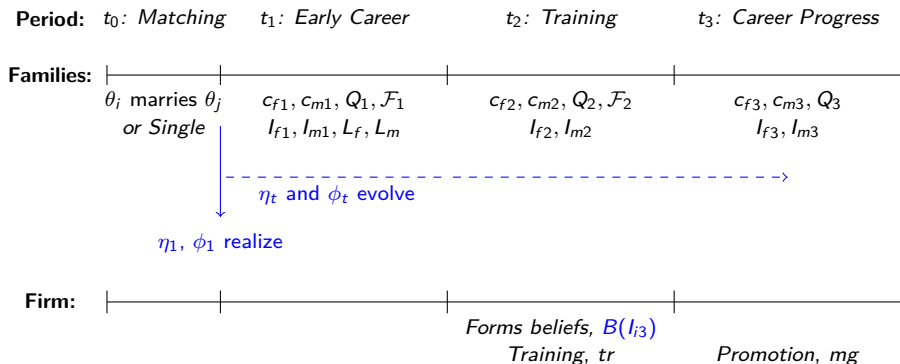
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# The life cycle of individual $\theta_i$ and the representative firm



► Periods in our model correspond to life cycle stages in the data.

► Periods in the data

► Environment

► Market Human Capital

► Family Human Capital

► Worker and Family Types

► Firm problem

► Family problem

► Marriage Market

# Equilibrium

A competitive equilibrium is a set of assignments, prices, and probabilities:

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- ▶ In the labor market:
  - ▶ wage rates,  $w(\eta, L, J)$  and beliefs,  $B(l_3 \mid \omega_2)$ ;
  - ▶ training policy  $tr(\omega_2) \in \{0, 1\}$ ;
  - ▶ promotion policy  $mg(\omega_2 \mid tr(\omega_2), l_3(\omega_2) = 1) \in \{0, 1\}$ ;



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such that:

- ▶ The marriage market is in equilibrium,
- ▶ individuals and households maximize life-time utility,
- ▶ the firm's beliefs are consistent with household behavior,
- ▶ and the firm maximizes expected profits.

# Taking Stock

- ▶ The model gives rise to endogenous gender gaps in training and promotion.
- ▶ Key mechanisms include:
  - ▶ marriage market matching
  - ▶ women's initial advantage at home and
  - ▶ selective leadership training of firms.
- ▶ Families invest less in women  $\leftrightarrow$  firms invest less in women.
- ▶ Small initial differences can get amplified into large gender gaps.
- ▶ How do these forces and mechanisms interact with policies?
- ▶ Can policies that incentivize training women eradicate the “bad” equilibrium?

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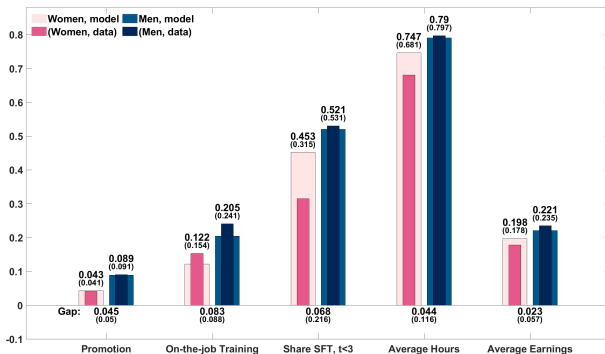
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# Estimation

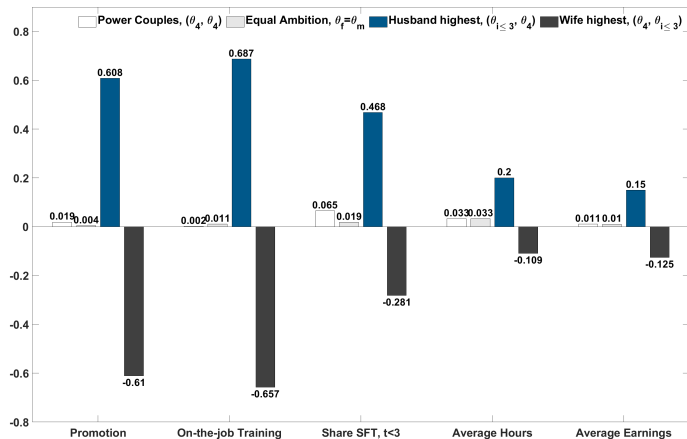
- ▶ We estimate the model using simulated method of moments.
- ▶ The full list of parameters is
  - ▶ production function parameters  $a_{L,j}$  and  $b_{L,j}$
  - ▶ initial level of market human capital by ambition type  $\mu_{\eta,\theta}$  and dispersion  $\sigma_{\eta}$
  - ▶ market human capital accumulation  $\alpha_{L,\theta}$  and depreciation rates  $\delta_L^P$  and  $\delta_L^N$
  - ▶ training skill boost  $\tau$  and quadratic training cost parameter  $c$
  - ▶ initial level of family human capital  $\mu_{\phi}$  and dispersion  $\sigma_{\phi}$
  - ▶ initial advantage of women  $\kappa$  and persistence of family human capital  $\gamma$
  - ▶ utility boost  $\chi^u$  and household cost  $\chi^Q$  from having children
  - ▶ dispersion of marriage market shocks  $\sigma_{\beta}$
- ▶ 38 structural parameters
- ▶ disciplined by 56 moments relating to earnings, labor supply, marriage patterns, fertility, on-the-job training, promotions.

# Implications for gender inequality (untargeted moments)



- ▶ At baseline, both firms and families invest relatively more in men than in women.
- ▶ Firms train 20.5% of men and only 12.2% of women on-the-job.
- ▶ Women 6.8 pp less likely to work SFT than men.
- ▶ Men devoting 4.4% more of their total lifetime to their careers.
- ▶ Implies women are half as likely as men to be promoted to managers.

# The role of the family



- ▶ As in the data, family-side & firm-side investments interact in interesting ways.
- ▶ Gaps in both is highest among people in families in which type-4 men marry down.
- ▶ Still positive among equal-ambition and power couples—matching the data.

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# Policy Analysis: Overview

- ▶ We use the model as a laboratory to evaluate the *equilibrium* effects of policies.
- ▶ Stylized examples of two alternative sets of policies:
  1. Paid and mandatory parental leave for  $\sim 10$  to 12 months.
    - ▶ Part-time equivalent replacement rate for women only
    - ▶ Part-time equivalent replacement for both men and women
    - ▶ Note! A small fraction of a model period (10%).
  2. 50% Quota for female managers (with qualifications!)
    - ▶ Note: “non-patronizing” by design.



# Parental leave only for mothers increases all gaps

**Table:** Gender gaps in counterfactual policies and their % change relative to baseline

|   | Promotion |         | On-the-job Training |         | Fraction sft, $t < 3$ |         | Average hours |         |
|---|-----------|---------|---------------------|---------|-----------------------|---------|---------------|---------|
|   | Value     | %Change | Value               | %Change | Value                 | %Change | Value         | %Change |
| <i>Panel A. Paid parental leave for mothers</i>             |           |         |                     |         |                       |         |               |         |
| <b>Men</b>  | 0.091     | 2.756   | 0.207               | 1.345   | 0.536                 | 2.853   | 0.794         | 0.413   |
| <b>Women</b>  | 0.041     | -5.534  | 0.122               | -0.221  | 0.446                 | -1.347  | 0.745         | -0.204  |
| <b>Gap</b>  | 0.050     | 10.671  | 0.086               | 3.655   | 0.089                 | 30.702  | 0.049         | 10.947  |
| <i>Panel B. Paid parental leave for mothers and fathers</i> |           |         |                     |         |                       |         |               |         |
| <b>Men</b>  | 0.089     | -0.412  | 0.203               | -1.036  | 0.528                 | 1.367   | 0.791         | 0.087   |
| <b>Women</b>  | 0.044     | 0.860   | 0.125               | 2.575   | 0.472                 | 4.313   | 0.752         | 0.704   |
| <b>Gap</b>  | 0.045     | -1.627  | 0.077               | -6.359  | 0.056                 | -18.166 | 0.039         | -10.437 |
| <i>Panel C. Managerial quota</i>                            |           |         |                     |         |                       |         |               |         |
| <b>Men</b>  | 0.067     | -24.384 | 0.207               | 1.108   | 0.523                 | 0.459   | 0.790         | -0.049  |
| <b>Women</b>  | 0.065     | 50.004  | 0.125               | 2.370   | 0.451                 | -0.264  | 0.746         | -0.085  |
| <b>Gap</b>  | 0.002     | -95.419 | 0.082               | -0.754  | 0.072                 | 5.250   | 0.044         | 0.555   |

- ▶ Both families and firms increase investment in men
- ▶ and reduce investment in women.
- ▶ Results in a widening of promotion gap by over 10%.

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- ▶ Families increase labor supply but more so for women.
- ▶ Firms train more women and less men.
- ▶ Higher investments in women relative to leave for mothers-only.
- ▶ Helps women in equal couples, harms women who marry-up

▶ By couple

# Managerial quotas induce more equal firm-side investments

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- ▶ Promotion gap almost disappears by design.
- ▶ Firms respond by training more workers, especially women.
- ▶ Families increase investments in men who face more competition.
- ▶ Helps especially women in power couples, and women who marry-up

▶ By couple

# Fixing marriage choices amplifies gender inequalities

Table: Policy impacts at *baseline marriage market equilibrium*

|   | Promotion |         | On-the-job Training |         | Share SFT, $t < 3$ |         | Average Hours |         |
|---|-----------|---------|---------------------|---------|--------------------|---------|---------------|---------|
|   | Value     | %Change | Value               | %Change | Value              | %Change | Value         | %Change |
| <i>Panel A. Paid parental leave for mothers</i>             |           |         |                     |         |                    |         |               |         |
| <b>Men</b>  | 0.092     | 3.884   | 0.206               | 0.591   | 0.533              | 2.397   | 0.793         | 0.334   |
| <b>Women</b>  | 0.040     | -7.453  | 0.120               | -1.843  | 0.441              | -2.579  | 0.743         | -0.520  |
| <b>Gap</b>  | 0.052     | 14.710  | 0.086               | 4.179   | 0.092              | 35.388  | 0.050         | 14.894  |
| <i>Panel B. Paid parental leave for mothers and fathers</i> |           |         |                     |         |                    |         |               |         |
| <b>Men</b>  | 0.089     | 0.064   | 0.202               | -1.406  | 0.515              | -1.114  | 0.788         | -0.341  |
| <b>Women</b>  | 0.043     | -0.077  | 0.125               | 2.460   | 0.451              | -0.425  | 0.745         | -0.161  |
| <b>Gap</b>  | 0.046     | 0.198   | 0.077               | -7.104  | 0.064              | -5.688  | 0.042         | -3.412  |
| <i>Panel C. Managerial quota</i>                            |           |         |                     |         |                    |         |               |         |
| <b>Men</b>  | 0.067     | -24.812 | 0.207               | 0.954   | 0.523              | 0.483   | 0.790         | -0.092  |
| <b>Women</b>  | 0.065     | 50.403  | 0.126               | 2.939   | 0.449              | -0.729  | 0.746         | -0.140  |
| <b>Gap</b>  | 0.002     | -96.636 | 0.081               | -1.971  | 0.074              | 8.518   | 0.044         | 0.729   |

- ▶ Fixing the marriage market amplifies family-side investment gaps,
- ▶ Increment in gender gaps bigger and declines smaller.
- ▶ Partner choice is an important mechanism for the impact of policies.

# The welfare effects of narrowing gender gaps

- ▶ Aggregate welfare decreases under the leave policies.
  - ▶ Additional income during leave does not compensate for the career costs of leave-induced skill depreciation.
  - ▶ Both men and women have lower average earnings under both leave scenarios.
  - ▶ The additional leave for mothers reduces average household utility by 1.052% and leave for both parents reduces welfare by 3.387%.
- ▶ Aggregate welfare increased marginally under managerial quota (0.011%).
  - ▶ Firms must increase their efforts to find suitable women for management and respond by training both women and men to a larger extent.
  - ▶ Substantial heterogeneity in welfare gains/losses across couple types.
  - ▶ Policy disincentivizes career investments of highly ambitious women who are protected by the quota.
  - ▶ Reduced welfare for households of highly ambitious men and less ambitious women.
- ▶ **Changes in the marital sorting** (long-run) can **mitigate** negative welfare effects because fewer negatively affected couples form under this policy.

# Roadmap

Data

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Conclusion

Appendix

# Conclusion

- ▶ We document new facts on heterogeneity in firm-side investments by households types.
- ▶ Rich Danish data → follow households and their employers over life cycle.
- ▶ Build an equilibrium model in which who marries whom affects the link between workers' investments and firms' investments.
  - ▶ lifecycle collective household model with fertility and
  - ▶ career progression within the firm.
- ▶ The MM is important for the transmission of policies.
  - ▶ Overall, paid leave to both spouses can reduce gender gaps in promotions, but some groups may lose.
  - ▶ Management quota shifts the focus of households on career investments for ambitious women.
  - ▶ Blanket policies do not take into account important heterogeneity.
- ▶ Highlights importance of considering interactions with the MM.

# Families' Career Investments and Firms' Promotion Decisions

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SaM 2025

May 8, 2025



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Appendix

# Ambition types (AFRSV, 2024), $\theta_i$

Educational level, all programs



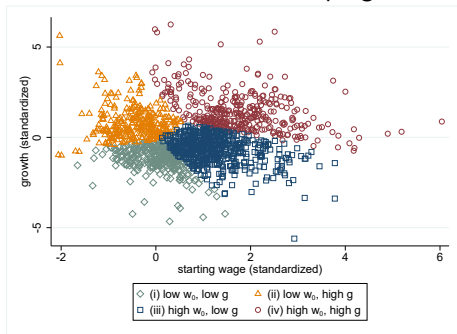
Educational ambition, all programs



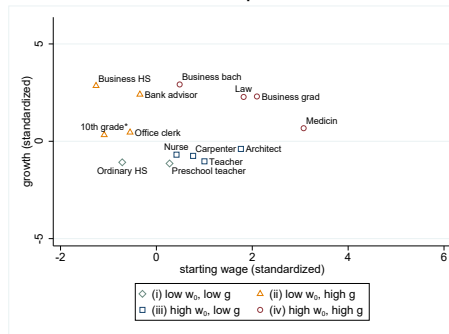
- ▶ Programs matter for marital sorting (Wiswal and Zafar, 2021),
- ▶ and differ in initial conditions and long-term outcomes (Altonji, Kahn, Speer, 2014, 2016; Kirkeboen, Leuven, Mogstad, 2016)

# Classification of most frequent programs (AFRSV, 2023)

## Educational ambition, all programs

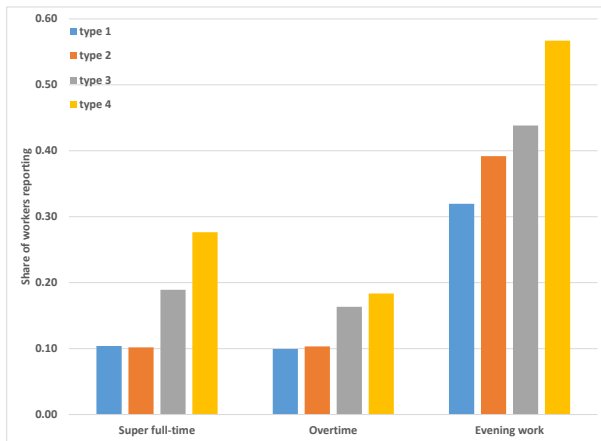


## Examples



- ▶ Our method groups programs based on labor market starting conditions and progress.
- ▶ Successful measure to differentiate tertiary degrees
  - ▶ e.g.: Architecture  $\neq$  Business; Nurse  $\neq$  Doctor.

# Ambition types and hours worked



- Higher ambition types work longer and more irregular hours, often requiring working at home and in the evenings.

# Career ladders

- ▶ We aim to measure career path choices based on occupational choice and firm at labor market entry
- ▶ We distinguish *steep* and *flat* career ladder for tractability
  - ▶ Steep is defined as the top 20% occupation-firm pairs with highest hourly wage growth over first 10 years.
  - ▶ Calculate average growth based on coworkers.
  - ▶ Coarsen comparison group if necessary to avoid small-cells issues.
- ▶ Ambition type is about earnings potential ex ante, ladder choice is about the career path that individuals enter in the labor market.
  - Law graduate decides to work at a private law firm or in public sector administration.

# Promotion to manager

- ▶ Managers are workers with occupations coded "1: Management Work."
  - ▶ Categories include "Top management," "Management within administration," "Management within production," and "Management within services."
  - ▶ Examples: Manager in production company (132100), manager of internal IT (133020), top manager in public company (111200) etc.
- ▶ Promotion means transition to these occupation codes for the first time.
- ▶ Significant pay increase
  - ▶ Managers make 40% more than non managers, conditional on training.

# On-the-job training

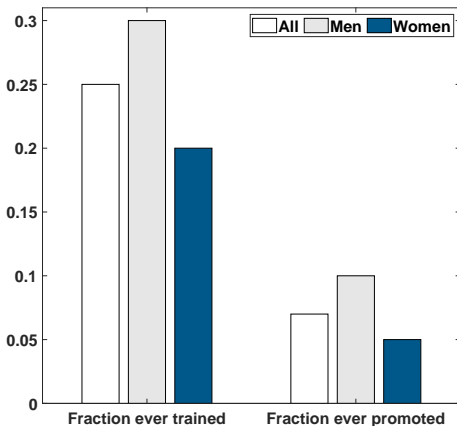
- ▶ Direct measure of firm investment: data on individuals' participation in **management training programs** (paid by firm while on the job).
  - 45% of program participants are subsequently promoted to managers.
  - 10% of all managers previously received this management training.
- ▶ Another type of firm investment: **lateral moves across occupations** within a firm
  - Returns to specialization vs preparing for management
- ▶ Predictive model to identify workers who receive training based on both sources:
  - classifies 85.75% of individuals correctly (managers with training and non-managers without training).
  - **12.92%** of trainees are subsequently promoted to managers.
  - **50.3%** of all managers previously received training.

# Super-full-time work

- ▶ We distinguish between four labor supply states at the yearly level: nonparticipation, part-time, full-time, and super-full-time.
- ▶ Nonparticipation refers to not being employed by the end of November in a given year.
- ▶ Those who are employed by the end of November in a given year are characterized as either part-time or full-time depending on the hours worked per week, e.g., less or more than 32 hours.
- ▶ The Danish standard full-time working week has 37 hours.
- ▶ The **super-full-time** status is constructed based on information from the Danish LFS. We classify a worker as working super-full-time in the following cases:
  1. reports working 38 hours or more per week
  2. reports working 37 hours and, additionally, irregular hours, e.g., in the evening, weekends, etc.



# Gender gaps in training and promotion

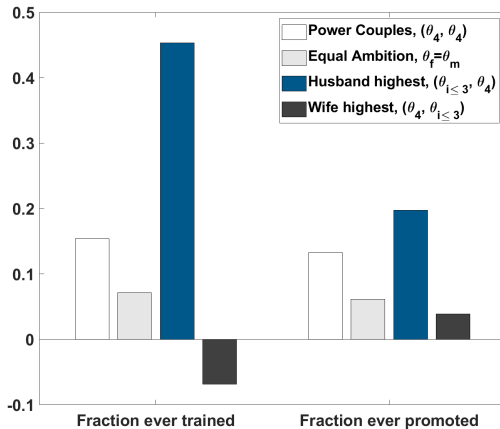


- ▶ Gender gap in training is 33%, increases to 50% at the promotion stage.
- ▶ Big and significant even with firm-occupation fixed effects.

▶ Regression

# Gender gaps in firm-side investments vary with worker's family type

Difference in men's and women's outcome



- ▶ Interestingly, gaps positive within power couples.
- ▶ Depend on own and spousal type.

## Firm-side investments vary with worker's family type

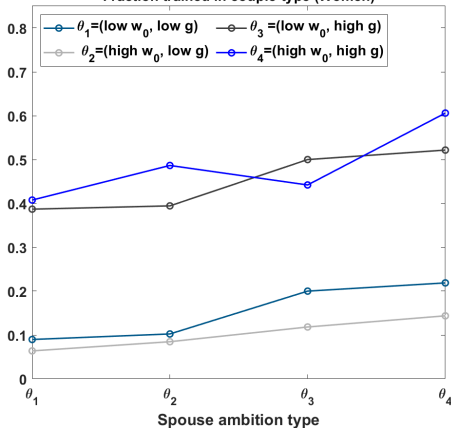
$$O_{ilt} = \beta_0 + \beta_1 \cdot \text{female}_i + \beta_2 \cdot \text{high-ambition}_i + \beta_3 \cdot \text{high-ambition}_i \cdot \text{female}_i \\ + \beta_4 \cdot \text{high-ambition spouse}_i + \beta_5 \cdot \text{high-ambition spouse}_i \cdot \text{female}_i + X'_{ilt} \gamma + \epsilon_{ilt}$$

|   | (1)                   | (2)                   | (3)                   | (4)                   |
|---|-----------------------|-----------------------|-----------------------|-----------------------|
|   | Training              |                       | Manager               |                       |
| female  | -0.0287***<br>(0.002) | -0.0150***<br>(0.003) | -0.0090***<br>(0.001) | -0.0043***<br>(0.001) |
| high-ambition                                 | 0.4302***<br>(0.005)  | 0.2992***<br>(0.005)  | 0.0475***<br>(0.002)  | 0.0401***<br>(0.002)  |
| high-ambition * female                        | -0.0738***<br>(0.007) | -0.0628***<br>(0.007) | -0.0150***<br>(0.002) | -0.0140***<br>(0.002) |
| high-ambition spouse                          | 0.1318***<br>(0.007)  | 0.0824***<br>(0.007)  | 0.0386***<br>(0.003)  | 0.0322***<br>(0.003)  |
| high-ambition spouse * female                 | -0.0652***<br>(0.008) | -0.0372***<br>(0.008) | -0.0326***<br>(0.003) | -0.0256***<br>(0.003) |
| FE for Firm-Ladder, Age, LS History           | No                    | Yes                   | No                    | Yes                   |
| Observations                                  | 1,860,063             | 1,827,942             | 1,860,063             | 1,827,942             |
| R-squared                                     | 0.199                 | 0.428                 | 0.020                 | 0.245                 |
| Total Effect, high-ambition spouse for female | 0.0666                | 0.0453                | 0.00603               | 0.00660               |
| P-Value                                       | <0.0001               | <0.0001               | <0.0001               | <0.0001               |

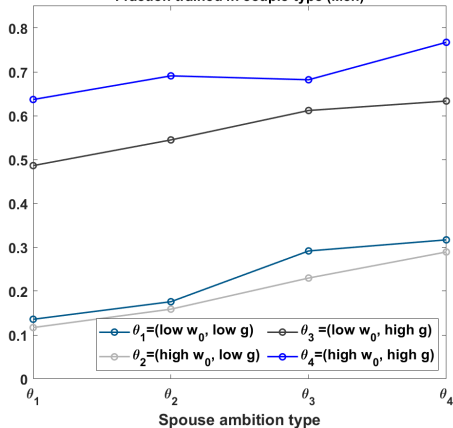
- Gender gaps widen for ambitious women and for women with ambitious spouses.
- The role of the spouse declines conditional on labor market choices.

# Firm-side investments vary with worker's family type

Fraction trained in couple-type (Women)



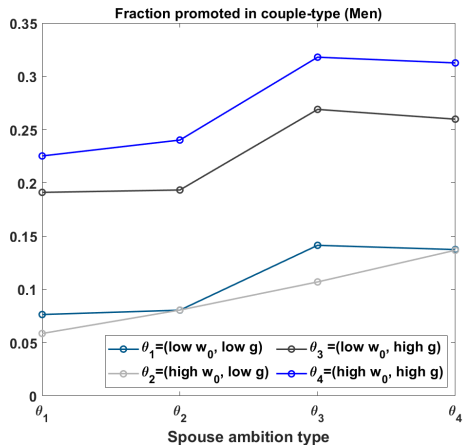
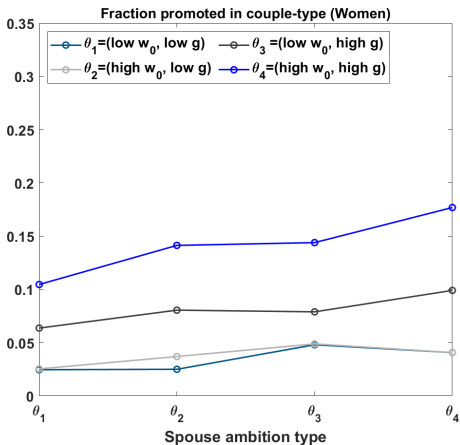
Fraction trained in couple-type (Men)



► Probability of receiving training  $\uparrow$  with own and spouse's ambition

► but more so for men. [► Back](#)

# Firm-side investments vary with worker's family type



► Probability of reaching managerial position  $\uparrow$  with spouse's ambition

► but more so for men. [► Back](#)

# Gender gaps in training and promotion

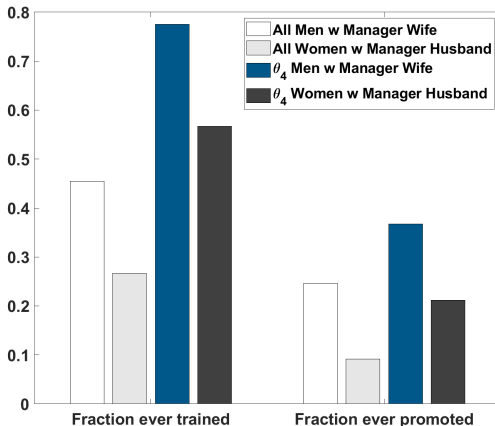
$$O_{ilt} = \beta_0 + \beta_1 \cdot \text{female}_i + \delta_l + \delta_\theta + \delta_{\{l_i\}_t} + \epsilon$$

|                    | (1)                   | (2)                   | (3)                   | (4)                   | (5)                   | (6)                   |
|--------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
|                    |                       | Training              |                       |                       | Manager Promotion     |                       |
| Female             | -0.0881***<br>(0.003) | -0.0639***<br>(0.003) | -0.0290***<br>(0.003) | -0.0196***<br>(0.001) | -0.0197***<br>(0.001) | -0.0092***<br>(0.001) |
| Firm-Ladder FE     | No                    | Yes                   | Yes                   | No                    | Yes                   | Yes                   |
| Worker Ambition FE | No                    | No                    | Yes                   | No                    | No                    | Yes                   |
| Worker Exp FE      | No                    | No                    | Yes                   | No                    | No                    | Yes                   |
| Observations       | 1,860,063             | 1,860,063             | 1,827,942             | 1,860,063             | 1,860,063             | 1,827,942             |
| R-squared          | 0.011                 | 0.359                 | 0.430                 | 0.003                 | 0.213                 | 0.245                 |

- ▶ Gender gaps big and significant even with firms and ladders.
- ▶ Decline conditional on labor market choices.

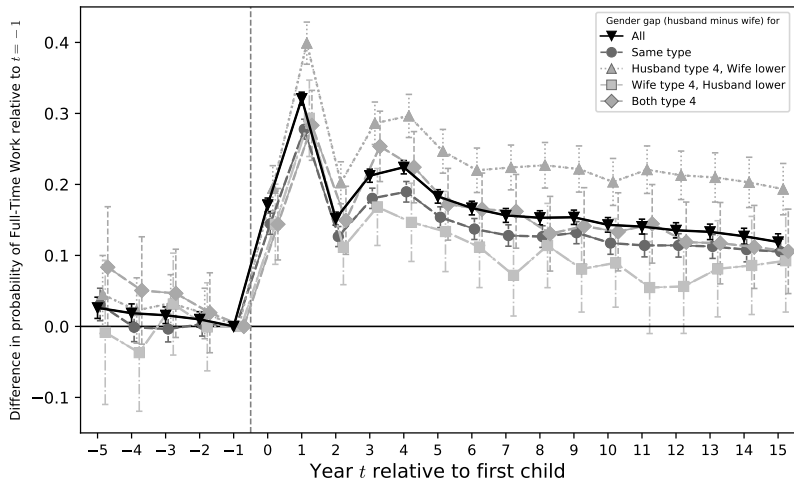
# When the husband is a manager

Difference in men's and women's outcome



- ▶ Most ambitious women who marry a manager 42.51% less likely to become managers than men married to managers.
  - ▶ Might be behind the higher gaps among power couples.

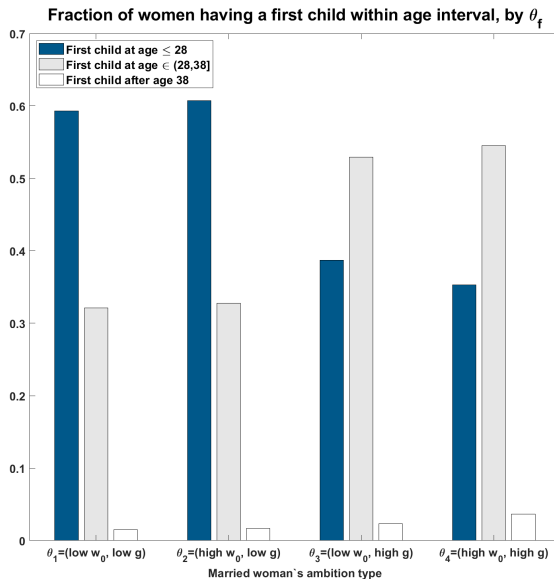
# Time allocation after arrival of children



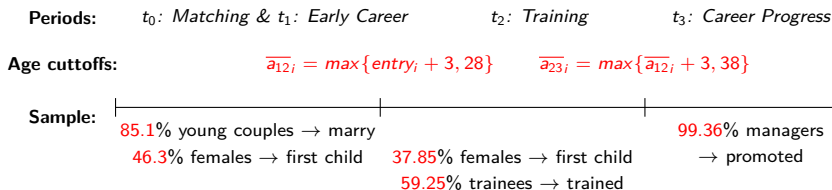
- ▶ Large and persistent child penalty even for women with highest ambition.
- ▶ Women married to more ambitious spouse show a more persistent penalty.



# More ambitious women delay fertility significantly more



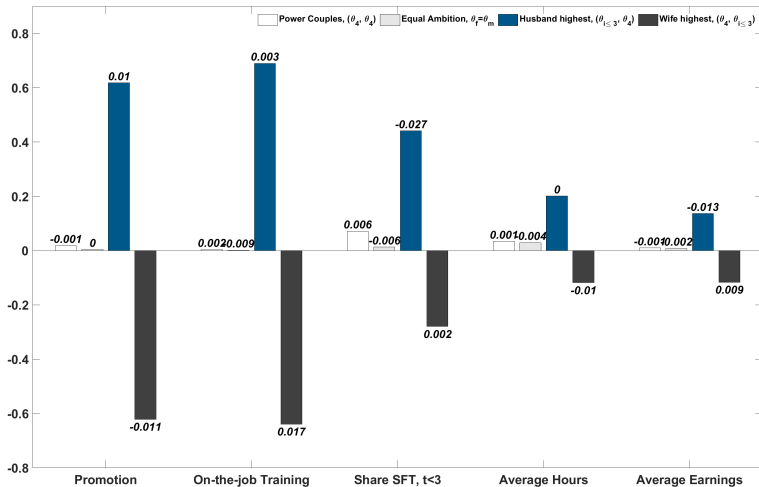
# Model Periods and the life cycle of individuals in the data



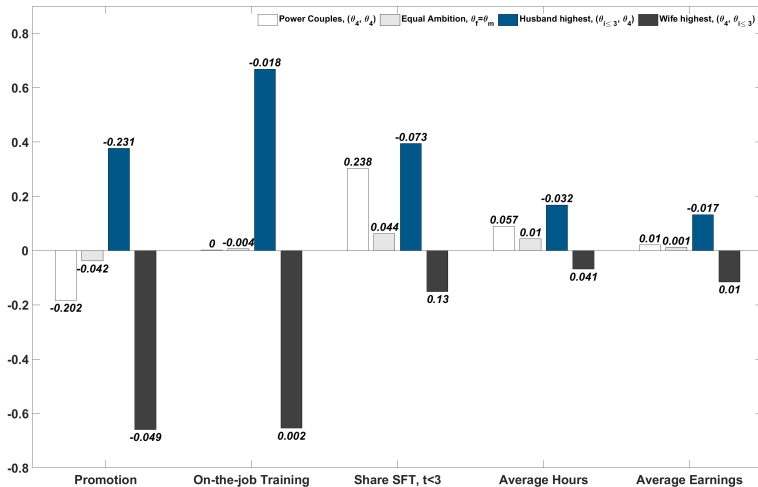
# Endogenous gender gaps in promotion

- ▶ Initial women's advantage at home imply women tend to stay at home more.
- ▶ Firms tend to see women as workers with lower market human capital,  $\eta$ .
- ▶ Firms tend to expect women to work less in  $t = 3$ .
- ▶ Training is offered relatively more to men.
- ▶ Expecting this bias, families tend to invest even more in husbands.
- ▶ In equilibrium, gender gaps in training and promotion arise.
- ▶ How do these forces and mechanisms interact with policies?

# Parental leave for both mothers & fathers by type of couple



# Managerial quotas by type of couple



# Hours Worked

- ▶ Detailed responses on hours worked from labor force survey (9.4% of individuals)
- ▶ "Ever managers" have much higher and irregular working hours:
  - ▶ Higher share working "super full-time" (more than 37 hours per week) and reporting overtime work, especially in the mid-career phase.
  - ▶ Higher share working usually or sometimes in the evening (excl. shift work) and on the weekend in mid and late career.
- ▶ Higher and more irregular hours worked on the steep than the flat ladder:
  - ▶ 1.5 hours more per week on average, 19% report working "super full-time" (vs 11% on flat ladder).
  - ▶ 5pp higher shares of overtime work and evening work, respectively.

# Moments I: Earnings Process (EP)

| Label | Description   | Model | Data |
|-------|---|-------|------|
| EP1   | Mean earnings of $\theta_i = 1$ in $L_1$ and $t = 1$  | 0.15  | 0.16 |
| EP2   | Mean earnings of $\theta_i = 2$ in $L_1$ and $t = 1$  | 0.16  | 0.18 |
| EP3   | Mean earnings of $\theta_i = 3$ in $L_1$ and $t = 1$  | 0.15  | 0.16 |
| EP4   | Mean earnings of $\theta_i = 4$ in $L_1$ and $t = 1$  | 0.18  | 0.19 |
| EP5   | Mean earnings of $\theta_i = 1$ in $L_2$ and $t = 1$  | 0.23  | 0.16 |
| EP6   | Mean earnings of $\theta_i = 2$ in $L_2$ and $t = 1$  | 0.25  | 0.18 |
| EP7   | Mean earnings of $\theta_i = 3$ in $L_2$ and $t = 1$  | 0.16  | 0.18 |
| EP8   | Mean earnings of $\theta_i = 4$ in $L_2$ and $t = 1$  | 0.21  | 0.20 |
| EP9   | Earnings growth of FT workers in $L_1$                | 0.05  | 0.06 |
| EP10  | Earnings growth of FT workers in $L_2$                | 0.07  | 0.13 |
| EP11  | Earnings growth of $\theta_i = 1$ FT workers in $L_1$ | 0.02  | 0.05 |
| EP12  | Earnings growth of $\theta_i = 2$ FT workers in $L_1$ | 0.04  | 0.05 |
| EP13  | Earnings growth of $\theta_i = 3$ FT workers in $L_1$ | 0.09  | 0.09 |
| EP14  | Earnings growth of $\theta_i = 4$ FT workers in $L_1$ | 0.12  | 0.13 |
| EP15  | Earnings growth of $\theta_i = 1$ FT workers in $L_2$ | 0.03  | 0.07 |
| EP16  | Earnings growth of $\theta_i = 2$ FT workers in $L_2$ | 0.06  | 0.08 |
| EP17  | Earnings growth of $\theta_i = 3$ FT workers in $L_2$ | 0.13  | 0.13 |
| EP18  | Earnings growth of $\theta_i = 4$ FT workers in $L_2$ | 0.12  | 0.17 |
| EP19  | Earnings difference FT vs NT at $t = 2$ in $L_1$      | 0.06  | 0.08 |
| EP20  | Earnings difference FT vs NT at $t = 2$ in $L_1$      | 0.10  | 0.11 |
| EP21  | Earnings growth without training                      | 0.07  | 0.05 |
| EP22  | Earnings growth with training                         | 0.16  | 0.09 |
| EP23  | Mean earnings of $\theta_i = 3$ managers in $L_1$     | 0.54  | 0.42 |
| EP24  | Mean earnings of $\theta_i = 4$ managers in $L_1$     | 0.57  | 0.56 |
| EP25  | Mean earnings of $\theta_i = 3$ managers in $L_2$     | 0.59  | 0.62 |
| EP26  | Mean earnings of $\theta_i = 4$ managers in $L_2$     | 0.64  | 0.74 |
| EP27  | Earnings difference SFT vs FT at $t = 2$ in $L_1$     | 0.07  | 0.04 |
| EP28  | Earnings difference SFT vs FT at $t = 2$ in $L_2$     | 0.09  | 0.06 |

## Moments II: Marriage Patterns (MM)

| Label | Description                                  | Model | Data |
|-------|--|-------|------|
| MM1   | Fraction $\theta_f = \theta_m$               | 0.39  | 0.42 |
| MM2   | Fraction ( $\theta_f = 4, \theta_m = 4$ )    | 0.08  | 0.04 |
| MM3   | Fraction ( $\theta_f \leq 3, \theta_m = 4$ ) | 0.10  | 0.12 |
| MM4   | Fraction ( $\theta_f = 4, \theta_m \leq 3$ ) | 0.03  | 0.03 |
| MM5   | Fraction single men $\theta_m = 1$           | 0.37  | 0.41 |
| MM6   | Fraction single men $\theta_m = 2$           | 0.37  | 0.27 |
| MM7   | Fraction single men $\theta_m = 3$           | 0.40  | 0.36 |
| MM8   | Fraction single men $\theta_m = 4$           | 0.26  | 0.27 |

► Go back



## Moments III: Fertility Patterns (FP)

| Label | Description   | Model | Data |
|-------|---|-------|------|
| FP1   | Fraction $\theta_f = 1$ having first child in $t = 1$ | 0.38  | 0.57 |
| FP2   | Fraction $\theta_f = 4$ having first child in $t = 1$ | 0.36  | 0.31 |
| FP3   | Fraction $\theta_f = 1$ having first child in $t = 2$ | 0.31  | 0.37 |
| FP4   | Fraction $\theta_f = 4$ having first child in $t = 2$ | 0.64  | 0.60 |

► Go back

## Moments IV: Labor Supply (LS)

| Label | Description  | Model | Data |
|-------|--|-------|------|
| LS1   | Participation rate of men $\theta_m = 1$               | 0.74  | 0.85 |
| LS2   | Participation rate of men $\theta_m = 2$               | 0.81  | 0.91 |
| LS3   | Participation rate of men $\theta_m = 3$               | 0.76  | 0.91 |
| LS4   | Participation rate of men $\theta_m = 4$               | 0.96  | 0.98 |
| LS5   | Variance of men's labor supply                         | 0.15  | 0.10 |
| LS6   | Participation gap in homogamous couples                | 0.05  | 0.04 |
| LS7   | Women's probability of re-entry ( $t = 1$ to $t = 2$ ) | 0.61  | 0.49 |
| LS8   | Women's probability of re-entry ( $t = 2$ to $t = 3$ ) | 0.38  | 0.37 |
| LS9   | Share working PT in $t = 1$ and $t = 2$ in $L_1$       | 0.09  | 0.04 |
| LS10  | Share working PT in $t = 1$ and $t = 2$ in $L_2$       | 0.11  | 0.02 |

► Go back

## Moments V: Firm's Investments (FI)

| Label | Description                     | Model | Data |
|-------|---------------------------------|-------|------|
| FI1   | Share of men trained in $L_1$   | 0.16  | 0.19 |
| FI2   | Share of women trained in $L_1$ | 0.09  | 0.11 |
| FI3   | Share of men trained in $L_2$   | 0.32  | 0.47 |
| FI4   | Share of women trained in $L_2$ | 0.21  | 0.35 |
| FI5   | Promotion gender gap in $L_1$   | 0.03  | 0.03 |
| FI6   | Promotion gender gap in $L_2$   | 0.07  | 0.09 |

► Go back

# We combine three strands in a unified framework

- ▶ Build on literature on the career cost of workers' choices.
  - ▶ Adda, Dustmann, Stevens (2017)
  - ▶ Kleven, Landaais, Sjøgaard (2019)
  - ▶ Angelov, Johansson, Lindahl (2016);
  - ▶ Goldin (2014)
  - ▶ Cortes & Pan (2019)
- ▶ This literature usually takes couples as given or abstracts from marital status.
- ▶ We incorporate **Marriage Market (MM)** and **Firm-side investments**.

▶ Go back

# We combine three strands in a unified framework

- ▶ MM: education-based marriage market sorting, workers' post-marriage human capital investments depend on the partner type.
  - ▶ Chiappori, Costa-Dias, Meghir (2018); Gayle & Shephard (2019); Calvo (2022); Reynoso (2024), Almar, Friedrich, Reynoso, Schulz & Vejlin (AFRSV, 2024).
- ▶ Firms have limited manager slots and invest in more attractive workers.
  - ▶ Training matters: Blundell, Costa-Dias, Goll, Meghir (2021)
  - ▶ job assignment/ promotions: Friedrich (2020), Gibbons and Waldman (1999).
  - ▶ firms expectations about workers' performance: Gayle & Golan (2012).
- ▶ We extend the literature on marriage and labor market interactions, household specialization, and joint labour supply decisions of couples
  - ▶ Philosoph & Wee (2021), Holzner & Schulz (2023), Calvo, Lindenlaub, Reynoso (2024); Foerster, Obermeier & Schulz (2024).
  - ▶ First dynamic framework with marriage formation, sorting, fertility, on-the-job training, and managerial promotions (we make the firm side explicit).
  - ▶ We abstract from savings, search frictions, divorce, job-to-job mobility, firm heterogeneity (representative firm). Focus on *information frictions*.

# Our framework offers a fresh approach to policy evaluation

- ▶ Gender gaps in reaching managerial positions are important and persistent.
  - ▶ Bronson & Skogman Thoursie (2021); Hampole, Truffa, & Wong (2023); Gayle, Golan & Miller (2012).
- ▶ We add to the literature on how different policies affect women's careers,
  - ▶ Parental leave policies
    - Das and Polachek (2015); Thomas (2021); Xiao (2021); Bailey, Byker, Patel, Ramnath (2024), Corekcioglu, Francesconi, Kunze (2024).
  - ▶ Diversity, Equity, and Inclusion efforts by firms
    - Bertrand, Black, Jensen, Lleras-Muney (2019).

by accounting for *equilibrium* policy impacts

- ▶ families' and firms' endogenously react to the policy environment, and
- ▶ effects may vary with degree of marriage market sorting.

# Environment I: General

- ▶ Three periods,  $t$ .
- ▶ Two markets: The marriage market and the labor market.
- ▶ A representative **firm** with:
  - ▶ Two ladders (or career paths),  $L = \{L_1, L_2\}$
  - ▶ Two production technologies, producer or manager,  $J = \{p, mg\}$
- ▶ Equal mass of **women and men**,  $i = \{m, f\}$ , of gender  $\mathcal{G} = \{\mathcal{X}, \mathcal{Y}\}$
- ▶ Distinguished by their *initial* human capital,  $\theta_i$ 
  - ▶ relevant for matching in the marriage market, and
  - ▶ sorting into *career paths*.

## Environment II: The Family

- ▶ Flow individual utility:  $u_{it} = c_{it} Q_t \chi_{(children)}^u$
- ▶ Labor supply choices:  $I_i = \{N, P, F, S\} = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ , no leisure.
- ▶ Ladder choices:  $L_i = \{L_1, L_2\}$ .
- ▶ The public good produced with private goods and time:
$$Q_t = c_{Qt} + \phi_{ft}(1 - I_{ft}) + \phi_{mt}(1 - I_{mt}) - \chi_{(children)}^Q$$
- ▶ Complementarity between time and goods: Welfare cost if both spouses choose super-full-time.
- ▶  $\phi_i$ : spouse  $i$ 's *family* human capital.
  - ▶ Depreciates over time.



# Environment III: The Representative Firm

- ▶ Two jobs  $J$  on each ladder  $L$ , producer and manager.
- ▶ *Output per unit of time* in job  $J$  and ladder  $L$  is parameterized as follows:

$$y_{L,J}(\eta_{it}) = a_{L,J} + b_{L,J}\eta_{it}.$$

- ▶ *Manager promotion* requires *leadership training* and *super-full-time work*.
  - ▶ Time use trade-off for families.
  - ▶ Firms are selective due to *capacity constraints*: Convex cost of training and fixed slots for managers.

# Market Human Capital Evolves over Time

- ▶ Initial human capital depends on ambition type:

$$\eta_1(\theta) \sim F(\mu_\theta, \sigma) \quad \forall \theta \in \Theta$$

- ▶ *Beginning-of- $t$*   $\eta_t$  depends on past LS, ambition type, and ladder:

# Market Human Capital Evolves over Time

- ▶ Initial human capital depends on ambition type:

$$\eta_1(\theta) \sim F(\mu_\theta, \sigma) \quad \forall \theta \in \Theta$$

- ▶ *Beginning-of- $t$*   $\eta_t$  depends on past LS, ambition type, and ladder:

$$\eta_{it} = [\eta_{t-1} + \alpha_{L,\theta}]$$

# Market Human Capital Evolves over Time

- ▶ Initial human capital depends on ambition type:

$$\eta_1(\theta) \sim F(\mu_\theta, \sigma) \quad \forall \theta \in \Theta$$

- ▶ *Beginning-of- $t$*   $\eta_t$  depends on past LS, ambition type, and ladder:

$$\eta_{it} = [\eta_{t-1} + \alpha_{L,\theta} + \delta_{L,\theta}^S \mathbb{1}_{\{I_{t-1}=S\}}]$$

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- ▶  $\tau$ : Training boost in human capital reaped at the beginning of  $t = 3$ :

$$\tau \begin{cases} = 1 & \text{if } t = \{0, 1, 2\} \\ > 1 & \text{if } t = 3 \text{ \& } tr = 1 \end{cases}$$

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$$\eta_{it} = [\eta_{t-1} + \alpha_{L,\theta} + \delta_{L,\theta}^S \mathbb{1}_{\{l_{t-1}=S\}} - \delta_{L,\theta}^P \mathbb{1}_{\{l_{t-1}=PT\}} - \delta_{L,\theta}^N \mathbb{1}_{\{l_{t-1}=NP\}}] \tau$$

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$$\tau \begin{cases} = 1 & \text{if } t = \{0, 1, 2\} \\ > 1 & \text{if } t = 3 \text{ \& } tr = 1 \end{cases}$$

- ▶ Evolves due to family's and firm's investments:

- ▶ Returns to experience; skill-depreciation when out-of-work.
- ▶ L choice matters:  $L_1$  rewards experience less but penalizes time-out-of-work less.
- ▶ Training boosts human capital evolution.



# Family Human Capital Evolves over Time

- ▶ Initial random family shock common to both spouses.

- ▶ Women may have an initial advantage:

$$\phi_{i1} = \begin{cases} \bar{\phi}^{\kappa} & \text{if } i = f \\ \bar{\phi} & \text{if } i = m \end{cases}$$

- ▶ Skills at home depreciate (relative to private goods):

$$\phi_{it} = \phi_{it-1}^{\gamma}$$

where  $\gamma > 0$ .

- ▶ The potential initial advantage of women persists over time.
- ▶ Absolute difference between men and women gets smaller due to depreciation.

# Worker Type vs. Family Type at a given time $t$

- ▶ A *worker type* consists of their gender, *ambition type*, *initial hk*, *ladder*, *LS history*, and *training*:

$$\omega_{it} = (\mathcal{G}_i, \theta_i, \eta_{1i}, L_i, \{l_{ir}\}_{r=1}^t, tr_i) \in \Omega_t$$

- ▶ A *family type* consists of *own worker type*, *own family hk*, *spouse's worker type*, and *spouse's family hk*:

$$\varphi_{it} = (\underbrace{\mathcal{G}_i, \theta_i, \eta_{1i}, L_i, \{l_{ir}\}_{r=1}^t, tr_i}_{\omega_{it}}, \underbrace{\mathcal{G}_j, \theta_j, \eta_{1j}, L_j, \{l_{jr}\}_{r=1}^t, tr_j, \phi_{jt}}_{\omega_{jt}})$$

# Firms

- ▶ At  $t = 2$ , the firm takes as given:
  - ▶ Matching in the MM,  $\mu(\theta)$ ;
  - ▶ Distribution of worker types decided by families,  $\{\omega_{i2}\}$ ;
- ▶ Forms beliefs about  $I_{i3} \mid \omega_{i2}$  and profits with and without training
  - ▶ Over unknown *family type* and *family shock*.
- ▶ Chooses fraction of  $N(\omega)$  trained,  $tr(\omega)$ , and promoted,  $mg(\omega)$ ;
- ▶ to maximize expected profits from training.

▶ Go back

# Firm's training and promotion policies I

- ▶ The firm maximizes profits by choosing optimal training and promotion policies on each ladder.
- ▶ The firm takes as given any equilibrium distribution of households from the marriage market,  $\Gamma(\theta_f, \theta_m) : \Theta_0 \times \Theta_0 \setminus (\emptyset, \emptyset) \rightarrow (0, 1)$ .

$$\max_{\{tr(\omega), mg(\omega)\}} \Pi_{tr} = \sum_{\omega \in \Omega_2} tr(\omega) [mg(\omega)E[\pi_{mg}(\omega)] + (1 - mg(\omega))E[\pi_p(\omega)]] \cdot N(\omega) - C(N_{tr})$$

subject to the size of the training program, and the capacity constraint for managers,

$$N_{tr} = \sum_{\omega \in \Omega_2} tr(\omega) \cdot N(\omega)$$
$$N_{mg} \geq \sum_{\omega \in \Omega_2} tr(\omega) \cdot mg(\omega) \cdot B_3(SFT \mid tr, \omega) \cdot N(\omega)$$

## Firm's training and promotion policies II

- ▶ where  $E[\pi_J(\omega)]$  denote expected profits in future assignment  $J$ .
- ▶  $B_3(SFT|tr, \omega)$  denotes firm beliefs about period-3 labor supply.
- ▶ The firm forms these beliefs by anticipating optimal future choices of different family types that are consistent with an individual's observed worker type  $\omega_{i2}$ .

$$B_3(\tilde{l} \mid \omega_{i2}) \equiv B(l_{i3}(\omega_{i2}) = \tilde{l} \mid tr_i, \omega_{i2}) = \sum_{h \text{ s.t. } \omega_{i2} \in \arg \max(HP_h)} \frac{\Gamma(\theta_f, \theta_m)}{\Gamma_{\omega_{i2}}} E_{\varphi(f, m)} \left[ \tilde{l} \in \arg \max(HP_i) \mid tr_i \right].$$

- ▶ and  $\Gamma_{\omega_{i2}} = \sum_h \Gamma(\theta_f, \theta_m) \mathbb{1}\{\omega_{i2} \in \arg \max(HP_h)\}$  is the total share of households consistent with the worker's observed type  $\omega_{i2}$ .

# Families' problem

- ▶ Households  $(\theta_f, \theta_m)$  that formed in the MM take as given:
  - ▶ Matching in the MM,  $\mu(\theta)$ , and women's utility prices,  $\overline{U}_x^{\theta_f \theta_m}$ ;
  - ▶ wage rates,  $w(\omega_i)$ ;
  - ▶ firm's training and promotion policies,  $tr(\omega_2) \in \{0, 1\}$  and  $mg(\omega_3/tr) \in \{0, 1\}$
- ▶ choose a contingent contract of career trajectories, fertility, and consumption,

$$x(\varphi) = \underbrace{\left\{ L_f(\varphi_t), L_m(\varphi_t), \mathcal{F}_t(\varphi_t), I_{ft}(\varphi_t), I_{mt}(\varphi_t), c_{ft}(\varphi_t), c_{mt}(\varphi_t), c_{Qt}(\varphi_t) \right\}}_{x_t(\varphi_t)} \Bigg\}_{t=1}^3$$

so as to solve their collective life cycle problem

$$\begin{aligned} \overline{U}_y^{\theta_f \theta_m} &= \max_{x(\varphi)} E_0 \sum_{t=1}^{T=3} \delta^{t-1} \left\{ u_m(x_t(\varphi_t)) \right\} \\ \text{s.t.} \quad & E_0 \sum_{t=1}^{T=3} \delta^{t-1} \left\{ u_f(x_t(\varphi_t)) \right\} \geq \overline{U}_x^{\theta_f \theta_m} \\ & \forall \varphi_t, t > 0 : \quad c_{ft} + c_{mt} + c_{Qt} = w_{ft}(\omega_{ft}) I_{ft} + w_{mt}(\omega_{mt}) I_{mt} \end{aligned}$$

- ▶ Optimal household behavior determines distribution of worker types,  $\{\omega_{it}\}$ .

# Marriage Market

- ▶ Potential partners in the MM take as given:
  - ▶ Idiosyncratic taste shocks,  $\beta^{\theta_i \theta_j}$
  - ▶ wage rates,  $w(\omega_i)$ ;
  - ▶ firm's training and promotion policies,  $tr(\omega_2) \in \{0, 1\}$  and  $mg(\omega_3/tr) \in \{0, 1\}$ 
    - ▶ Anticipate  $\overline{U}_y^{\theta_f \theta_m}(\overline{U}_x^{\theta_f \theta_m}) \rightarrow$  value of any potential household
- ▶ Male  $\theta_m$  partner-choice problem is to choose the type  $\theta_f \cup \emptyset$  that maximizes:

$$\max \left\{ \underbrace{\overline{U}_y^{\emptyset \theta_m} + \beta_m^{\emptyset \theta_m}}_{single}, \underbrace{\{\overline{U}_y^{\theta_f \theta_m} + \beta_m^{\theta_f \theta_m}\}}_{marry \theta_f} \right\}$$

- ▶ Competitive equilibrium in the MM pins down outputs:
  - ▶ MM matching function  $\mu(\theta) \rightarrow$  who marries whom,
  - ▶ Indirect Expected Utilities  $(\overline{U}_x^{\theta_f \theta_m}, \overline{U}_y^{\theta_f \theta_m}) \rightarrow$  why.

# Firm's Production Function Paramters

| Symbol       | Description                    | Par.       | s.e.  | Sensitivity Moments |      |      |
|--------------|--------------------------------|------------|-------|---------------------|------|------|
| $b_{L_1,p}$  | Slope (producers) in $L_1$     | 0.020      | 0.000 | EP1                 | MM4  | MM3  |
| $a_{L_1,mg}$ | Intercept (managers) in $L_1$  | 0.110      | 0.019 | EP11                | EP24 | EP19 |
| $b_{L_1,mg}$ | Slope (managers) in $L_1$      | 0.022      | 0.000 | EP4                 | EP19 | EP17 |
| $a_{L_2,p}$  | Intercept (producers) in $L_2$ | -0.070     | 0.001 | EP13                | MM4  | EP11 |
| $b_{L_2,p}$  | Slope (producers) in $L_2$     | 0.025      | 0.000 | MM4                 | EP1  | MM3  |
| $a_{L_2,mg}$ | Intercept (managers) in $L_2$  | 0.000      | 0.051 | EP26                | EP22 | EP19 |
| $b_{L_2,mg}$ | Slope (managers) in $L_2$      | 0.025      | 0.000 | EP3                 | LS9  | EP22 |
| $\zeta$      | Cost of training               | 5.0018e-07 | 0.000 | MM4                 | LS9  | EP17 |

► Moments and Fit I (EP)

► Moments and Fit II (MM)

► Moments and Fit III (FP)

► Moments and Fit IV (LS)

► Moments and Fit V (FI)

► Go back



# Market Human Capital Parameters

| Symbol           | Description                               | Par.   | s.e.  | Sensitivity Moments |      |      |
|------------------|---|--------|-------|---------------------|------|------|
| $\mu_1^\eta$     | Mean of initial hk draw of $\theta_1$     | 9.909  | 0.041 | LS1                 | EP1  | LS3  |
| $\mu_2^\eta$     | Mean of initial hk draw of $\theta_2$     | 10.504 | 0.046 | EP11                | EP6  | LS9  |
| $\mu_3^\eta$     | Mean of initial hk draw of $\theta_3$     | 9.360  | 0.190 | EP4                 | EP7  | EP17 |
| $\mu_4^\eta$     | Mean of initial hk draw of $\theta_4$     | 11.200 | 0.088 | MM4                 | MM1  | EP1  |
| $\sigma^\eta$    | Variance of initial hk draw               | 4.145  | 0.203 | EP1                 | EP17 | MM4  |
| $\alpha_{L_1,1}$ | Accumulation rate in $L_1$ for $\theta_1$ | 1.199  | 0.071 | EP11                | EP1  | EP21 |
| $\alpha_{L_1,2}$ | Accumulation rate in $L_1$ for $\theta_2$ | 2.271  | 0.163 | EP6                 | EP16 | EP24 |
| $\alpha_{L_1,3}$ | Accumulation rate in $L_1$ for $\theta_3$ | 4.709  | 0.216 | EP17                | EP13 | EP25 |
| $\alpha_{L_1,4}$ | Accumulation rate in $L_1$ for $\theta_4$ | 5.446  | 0.119 | EP19                | EP18 | MM1  |
| $\alpha_{L_2,1}$ | Accumulation rate in $L_2$ for $\theta_1$ | 1.511  | 0.166 | EP15                | EP21 | EP10 |
| $\alpha_{L_2,2}$ | Accumulation rate in $L_2$ for $\theta_2$ | 2.383  | 0.035 | LS10                | MM1  | EP3  |
| $\alpha_{L_2,3}$ | Accumulation rate in $L_2$ for $\theta_3$ | 4.714  | 0.189 | EP17                | EP13 | MM3  |
| $\alpha_{L_2,4}$ | Accumulation rate in $L_2$ for $\theta_4$ | 5.215  | 0.088 | EP19                | EP18 | MM1  |
| $\delta_{L_1}^P$ | PT Depreciation rate in $L_1$             | 0.200  | 0.070 | LS9                 | EP28 | FI2  |
| $\delta_{L_2}^P$ | PT Depreciation rate in $L_2$             | 0.290  | 0.034 | EP11                | EP1  | EP21 |
| $\delta_{L_1}^N$ | NT Depreciation rate in $L_1$             | 1.588  | 0.026 | MM4                 | MM3  | EP4  |
| $\delta_{L_2}^N$ | NT Depreciation rate in $L_2$             | 1.402  | 0.279 | EP11                | EP1  | EP24 |
| $\delta_{L_1}^S$ | Skill boost from working SFT in $L_1$     | 0.002  | 0.074 | FI1                 | MM4  | LS9  |
| $\delta_{L_2}^S$ | Skill boost from working SFT in $L_2$     | 0.002  | 0.001 | FI6                 | EP24 | EP19 |
| $\tau$           | Skill boost from training                 | 1.235  | 0.021 | MM4                 | EP5  | EP22 |

► Moments and Fit I (EP)

► Moments and Fit II (MM)

► Moments and Fit III (FP)

► Moments and Fit IV (LS)

► Moments and Fit V (FI)

# Marriage Market and Family Parameters

| Symbol                                  | Description                                       | Par.  | s.e.  | Sensitivity Moments |      |      |
|---|---|-------|-------|---------------------|------|------|
| <i>Marriage and Fertility Processes</i> |   |       |       |                     |      |      |
| $\sigma_\beta$                          | MM preference shock (scale)                       | 0.003 | 0.000 | EP6                 | EP27 | EP18 |
| $\chi_{1,2}^\emptyset$                  | Value of singlehood for $\theta_1$ and $\theta_2$ | 1.351 | 0.002 | EP11                | EP27 | EP18 |
| $\chi_{3,4}^\emptyset$                  | Value of singlehood for $\theta_3$ and $\theta_4$ | 1.441 | 0.017 | MM4                 | EP4  | MM3  |
| $\chi^u$                                | Utility boost with children                       | 1.169 | 0.003 | FP1                 | MM3  | FP4  |
| $\chi^Q$                                | Min HP with children                              | 0.037 | 0.000 | FP1                 | FP3  | EP6  |
| $\chi^S$                                | Penalty both SFT                                  | 0.087 | 0.034 | LS9                 | FI5  | MM1  |
| <i>Family Human Capital</i>             |   |       |       |                     |      |      |
| $\mu^\phi$                              | Mean of initial family shock                      | 0.162 | 0.001 | EP7                 | EP17 | EP1  |
| $\sigma^\phi$                           | Variance of initial family shock                  | 0.030 | 0.002 | EP11                | EP1  | EP21 |
| $\kappa$                                | Biological advantage of women                     | 1.161 | 0.026 | EP11                | EP20 | EP6  |
| $\gamma$                                | Depreciation rate                                 | 0.681 | 0.025 | EP1                 | EP28 | EP11 |

► Moments and Fit I (EP)

► Moments and Fit II (MM)

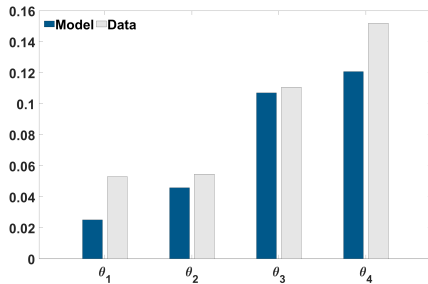
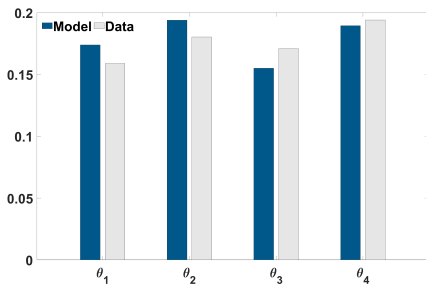
► Moments and Fit III (FP)

► Moments and Fit IV (LS)

► Moments and Fit V (FI)

# Our estimated model captures ambition types very well

Figure: Initial earnings (left) & earnings growth (right) by ambition type



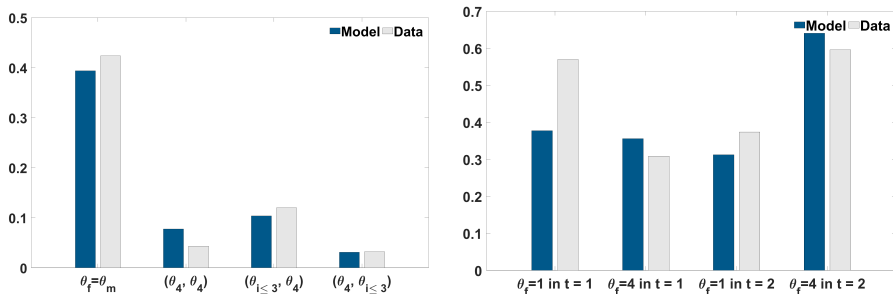
► Go back

## and ladders

- ▶ Productivity of skills in  $J = \{producer, manager\}$  higher in ladder  $L_2$ .
- ▶ Within ladder, managers productivity increase more with skills
- ▶  $L_2$  features:
  - ▶ higher gains in human capital due to full-time work,
  - ▶ a bigger loss of human capital when working part-time, and
  - ▶ slightly bigger super-full-time premium.
- ▶ Therefore we interpret  $L_2$  as the *steep* ladder.

# Our model replicates marriage and fertility patterns

Figure: Frequency of couple type  $(\theta_f, \theta_m)$  (left) & Timing of fertility (right)



# We replicate well gender gaps

- ▶ Initial advantage is estimated at  $\kappa = 1.1613$ .
- ▶ Implies a small initial gender gap in participation
  - ▶ 0.05 (model)—matching the observed 0.04.
- ▶ Initial differences amplify over the life cycle:

Figure: Firm-side investments

