Families' Career Investments and Firms' Promotion Decisions

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Nordic Data Meeting 2024
Aarhus University

May 29, 2024

Context

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  2. Households jointly determine career investments of spouses.

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- Importance of managerial positions for both workers and firms.
- Big and persistent gender promotion gap.
- Two key decision margins:
  1. Firms select workers for managerial training and promotions.
  2. Households jointly determine career investments of spouses.
- Our insights:
  1. Families' choices & firms' training and promotion policies interplay.
  2. The marriage market equilibrium connects families and firms.

$\rightarrow$ Gender gaps in career investments and firm's training reinforce each other.

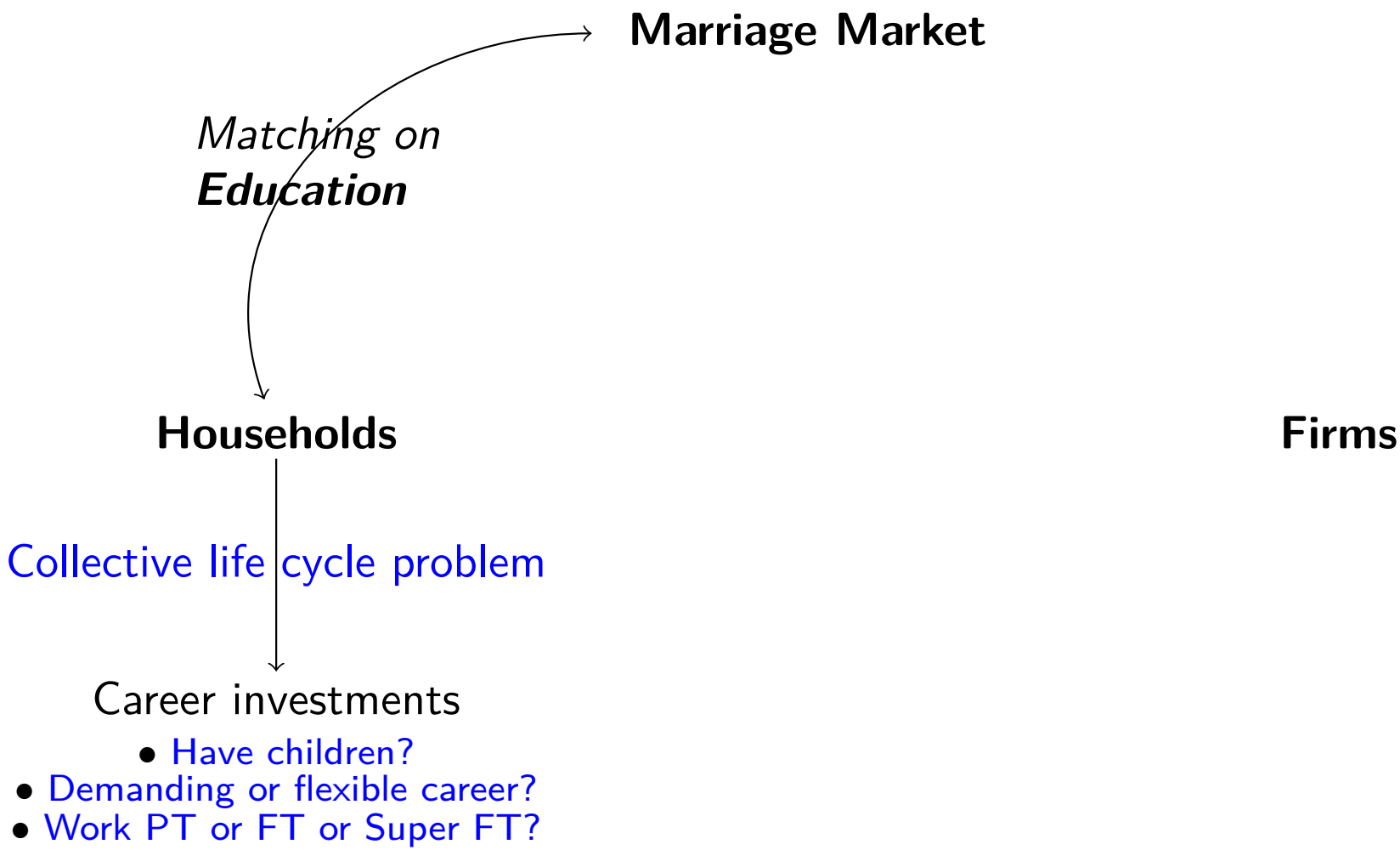
The link between households and firms

Marriage Market

Households **Firms**

The link between households and firms

The link between households and firms



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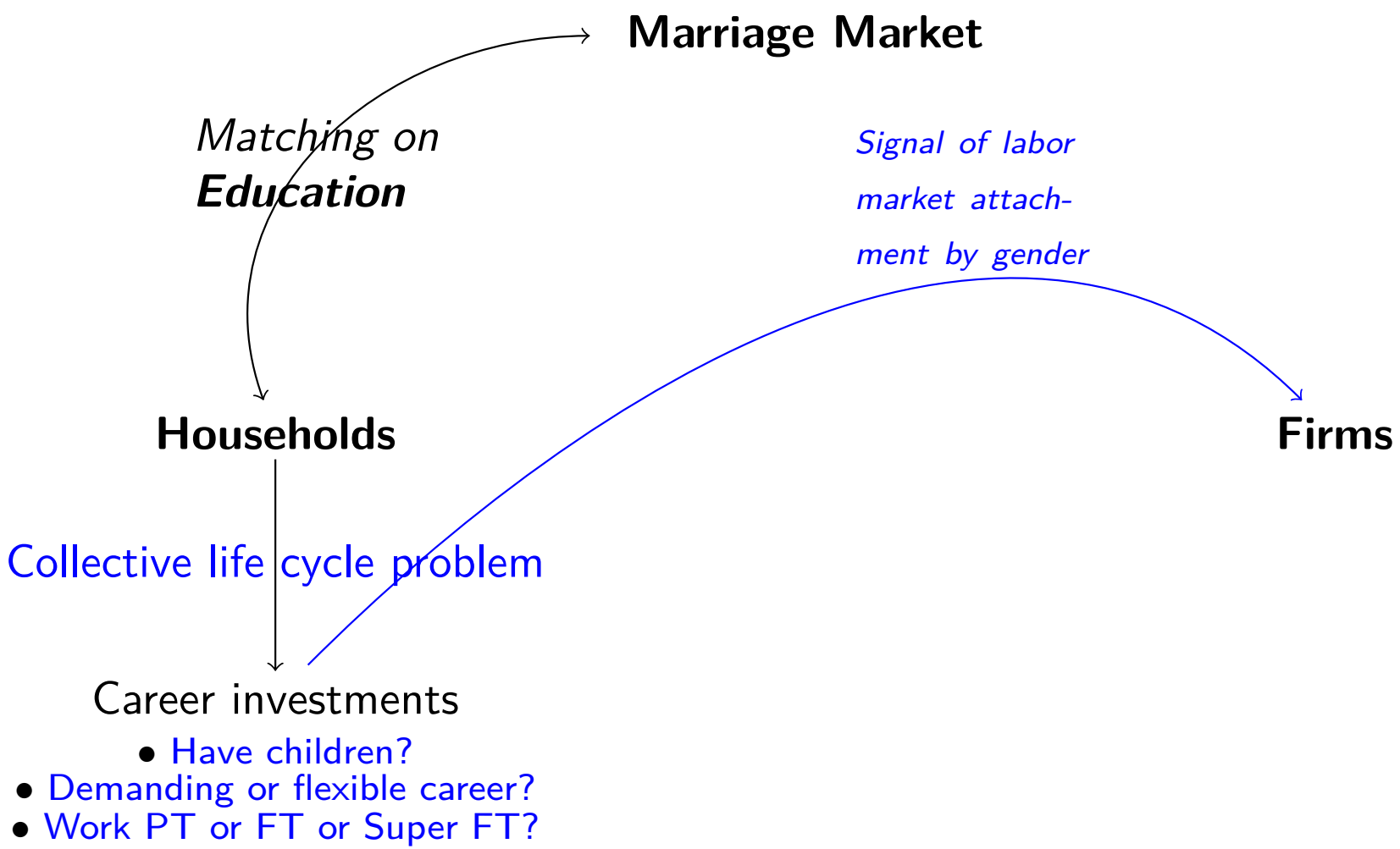

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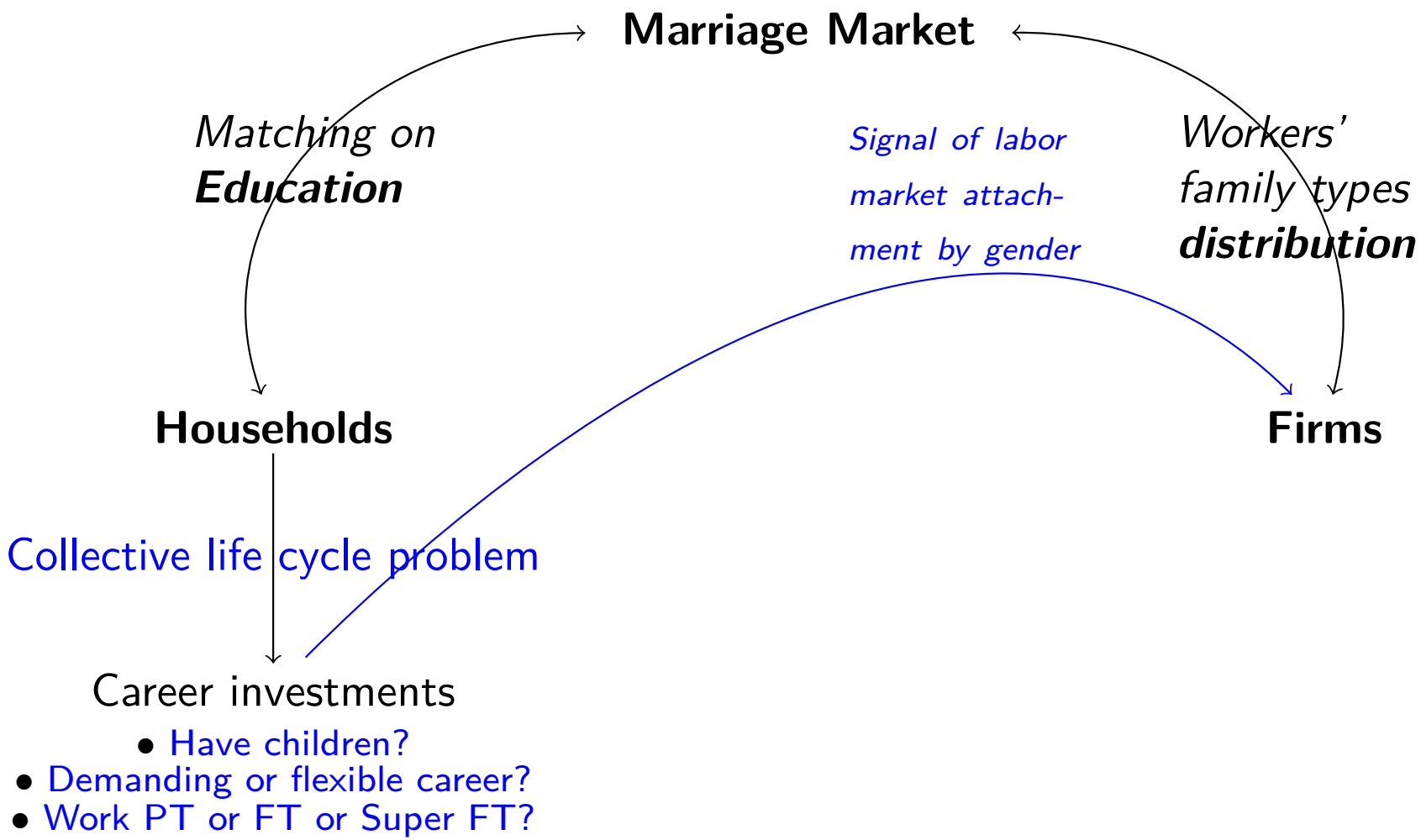

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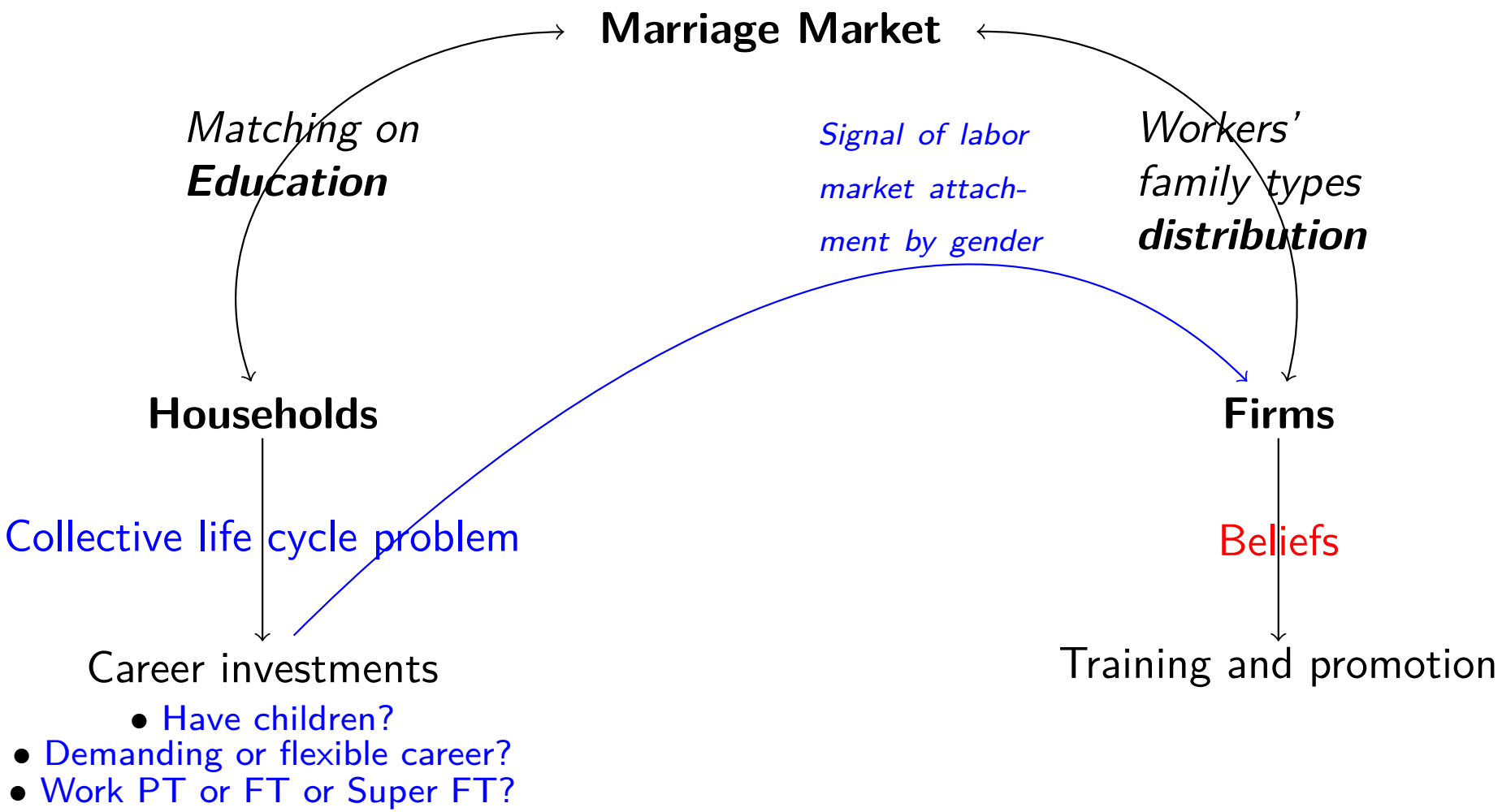

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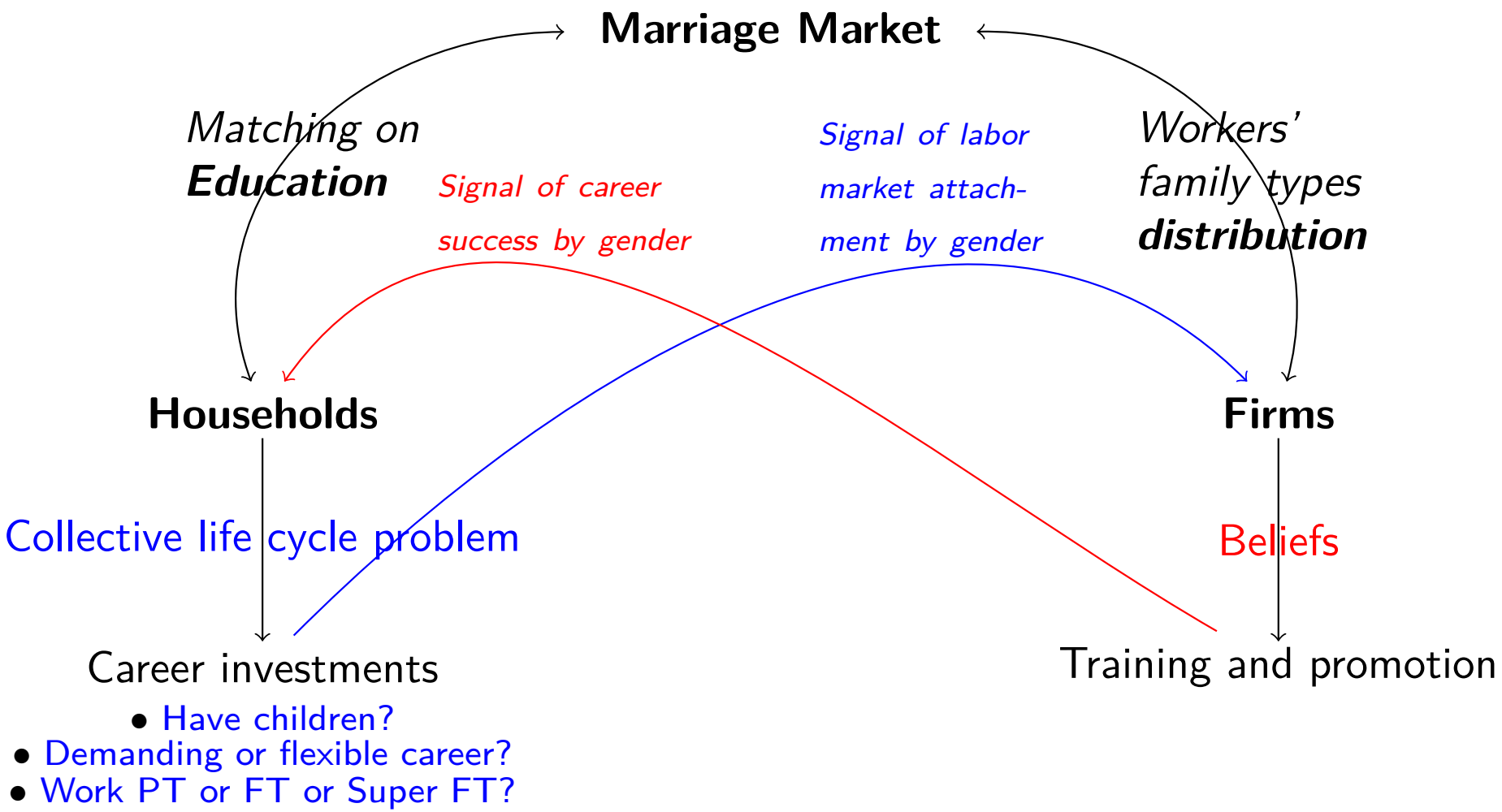

This paper

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2. Specify and estimate quantitative equilibrium model of this interaction.
   - Marriage market: matching based on initial human capital (career ambition).
   - Family: home production, fertility choice, and initial advantage of women.
   - Firm: capacity constraints for trainees and managers and uncertainty about future performance.

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   - Marriage market: matching based on initial human capital (career ambition).
   - Family: home production, fertility choice, and initial advantage of women.
   - Firm: capacity constraints for trainees and managers and uncertainty about future performance.
3. Evaluate policies to promote families' and firms' investments in women.
   - Focus on policies being discussed and implemented in different regions.
   - Quantify heterogeneous effects by gender, education, and type of couple.
   - Families and firms endogenously react to the policy environment.
   - Effects vary with the (degree of sorting in the) marriage market.

Literature Even more literature

Outline

1. **Data & Measurement**
2. Model Environment
3. Estimation & Policy Analysis

Follow families and their firms across cohort's life cycle

- Danish register data:
  - → education, family history, and labor market history for the full population.
  - → labor force survey: detailed work hours (weekly, overtime, evening, weekend).
    ▸ Details
- Follow the cohort who graduates from highest degree between 1991 and 1995:
  - their main partner, and
  - their employers and occupations,
  - from household formation and labor market entry,
  - over their life cycle.
- Dataset of ∼120K households and all of their employers observed for ∼25 years.

Measurement of key variables

- Ambition types (AFRSV, 2024), θ_i
  - For 1800+ education *programs*, compute average starting wages w_0 and 10Y wage growth g of all program graduates.
  - Categorize programs into 4 groups ranging from low-level, low-growth to high-level, high-growth programs. ▸ Ambition Details
- Career ladders, steep and flat
  - Compute 10-year wage growth by firm-occupation pair.
  - Group into steep and flat ladders using cutoff at 80th percentile ▸ Ladder Details
- Promotion to manager
  - First time in occupational codes for "Management" (combines middle and top management jobs) ▸ Promotion Details
- Firm's managerial training combines
  - participation in managerial training programs, and
  - job assignments that predict subsequent manager promotion. ▸ Training Details

Families' and firms' investment interactions are salient

1. Large gender gaps in training and promotion. ▸ Fact 1

2. *Firm-side* investments heterogeneous across workers' family characteristics:
   - Gender gaps vary by family type; ▸ Fact 2a
   - are notoriously big when husbands ever become managers. ▸ Fact 2b
   - Likelihood of receiving investments ↑ in spouse's type conditional on worker type. ▸ Fact 2c

3. Fertility & spousal time allocation within the household play a key role:
   - Within-couple gaps in human capital increase upon arrival of children; ▸ Fact 3a
   - and timing of fertility depends on wives' ambition types. ▸ Fact 3b

→ Motivates a model in which family-side and firm-side investments interact.

Outline

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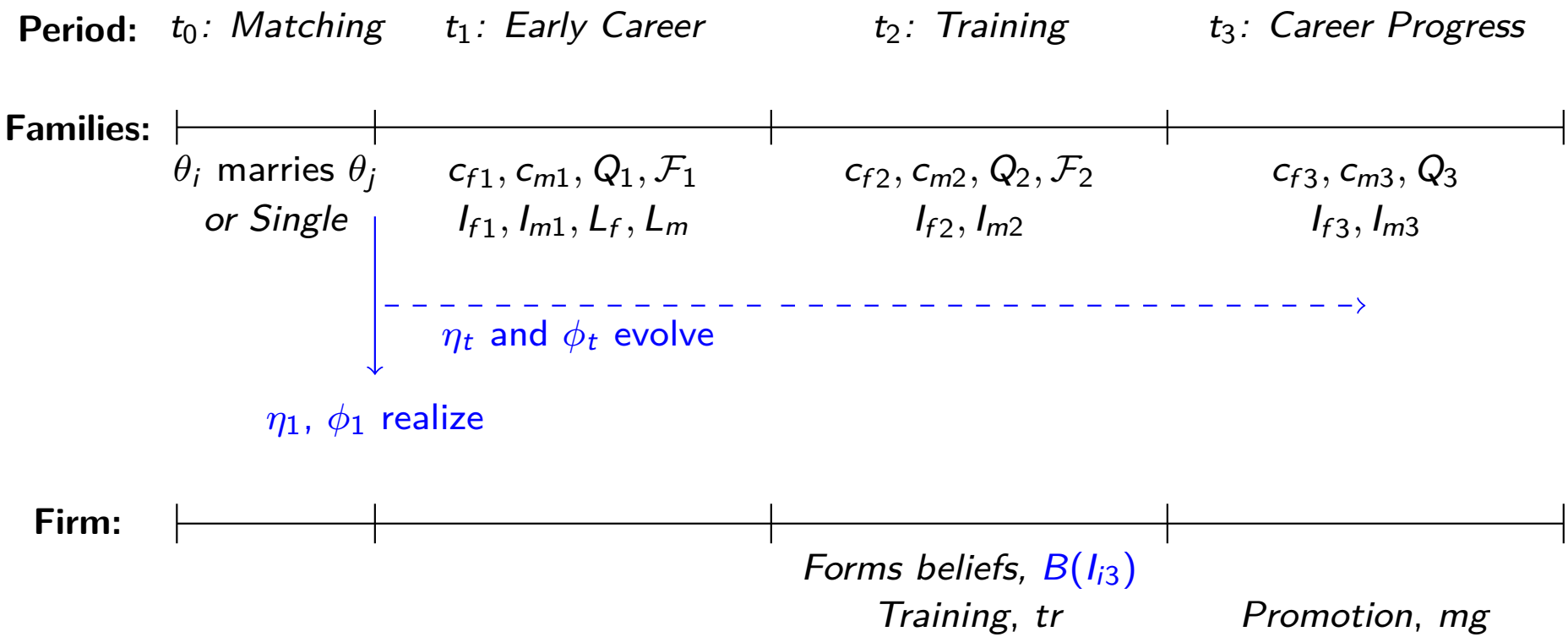
Model Environment: Setup

- Three periods, t.
- Two markets: The marriage market and the labor market.
- A representative firm with:
  - Two ladders (or career paths), $L = \{L_1, L_2\}$
  - Two production technologies, producer or manager, $J = \{p, mg\}$
- Equal mass of women and men, $i = \{m, f\}$, of gender $\mathcal{G} = \{\mathcal{X}, \mathcal{Y}\}$
- Distinguished by their *initial* human capital, θ_i
  - relevant for matching in the marriage market, and
  - sorting into *career paths*.

The life cycle of individual θ_i and the representative firm



- Households choose career ladder L_{it} and labor supply I_{it} for each spouse with market human capital η_{it} and family human capital ϕ_{it}. ▸ Family's problem
- Worker and family types evolve over time. ▸ Details
- Firm's beliefs based on individual employment history and MM matching patterns. ▸ Firm's problem I ▸ Firm's problem II
- Periods in our model correspond to life cycle stages in the data. ▸ Life cycle in the data

Equilibrium

A competitive equilibrium is a set of assignments, prices, and probabilities:

- In the marriage market:
  - An assignment of women's types θ_f to men's type θ_m, $\mu(\theta)$.
- In the household:
  - career trajectories, fertility, and consumption, for all households type (θ_f, θ_m), and
  - distribution of worker types, $\{\omega_{it}\}$.
- In the labor market:
  - wage rates, $w(\eta, L, J)$ and beliefs, $B(I_3 \mid \omega_2)$;
  - training policy $tr(\omega_2) \in \{0, 1\}$;
  - promotion policy $mg(\omega_2 \mid tr(\omega_2), I_3(\omega_2) = 1) \in \{0, 1\}$;

such that:

- The marriage market is in equilibrium, Marriage Market
- individuals and households maximize life-time utility, Families' problem
- the firm's beliefs are consistent with household behavior,
- and the firm maximizes expected profits.

Taking stock

- The model gives rise to endogenous gender gaps in traning and promotion.
- Key mechanisms include:
  - marriage market matching
  - women's initial advantage at home and
  - selective leadership training.
- Families invest less in women $\leftrightarrow$ firms invest less in women.
- Tiny initial differences get amplified into large gender gaps.
- How do these forces and mechanisms interact with policies?
- Can policies that incentivize training eradicate the "bad" equilibrium?

Outline

1. Data & Measurement
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Estimation

- We estimate the model using simulated method of moments.
- The full list of parameters is
  - production parameters $a_{L,J}$ and $b_{L,J}$
  - initial level of market human capital by ambition type $\mu_{\eta,\theta}$ and dispersion σ_η
  - market human capital accumulation $\alpha_{L,\theta}$ and depreciation rates δ_L^P and δ_L^N
  - training skill boost τ and quadratic training cost parameter c
  - initial level of family human capital μ_ϕ and dispersion σ_ϕ
  - biological advantage of women κ and persistence of family human capital γ
  - utility boost χ^u and household cost χ^Q from having children
  - dispersion of marriage market shocks σ_β
- ▸ Prelim Estimates featured marked ambition types and ladders, sensible accumulation and depreciation rates, non-negligible initial biological advantage of women.
- ▸ Good fit to key moments (fertility patterns, marriage patterns, earnings), but room for improvement.

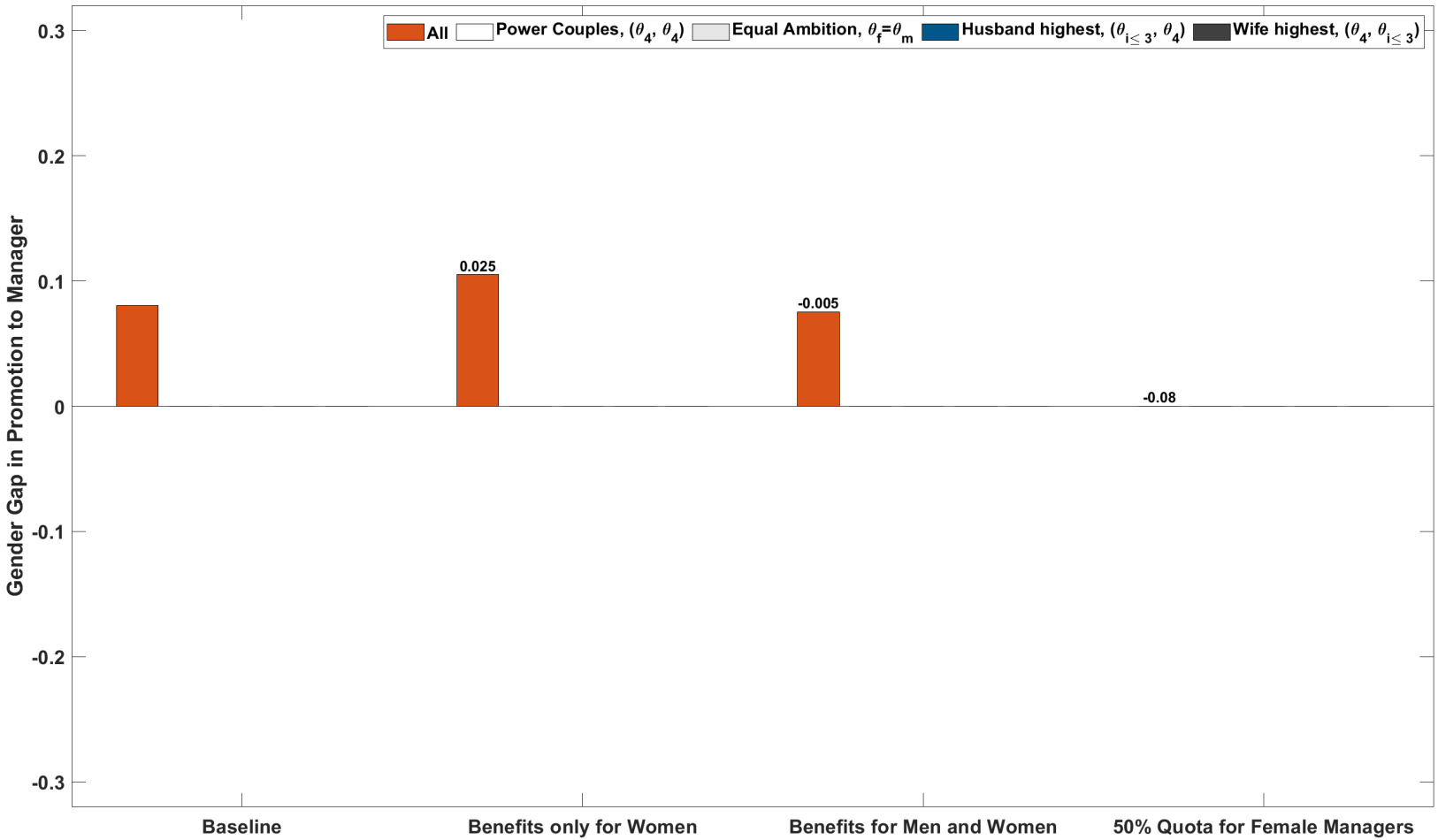
Policy Analysis: Overview

- Model as a laboratory to evaluate the *equilibrium* effects of policies.
- Today: Stylized examples of two alternative sets of policies:

1. Parental leave benefits
   - 100% replacement rate for women only
   - Full earnings replacement for both men and women
2. 50% Quota for female managers (with qualifications!)
   - Note: "non-patronizing" by design.

Promotion gender gaps at baseline, change with policies

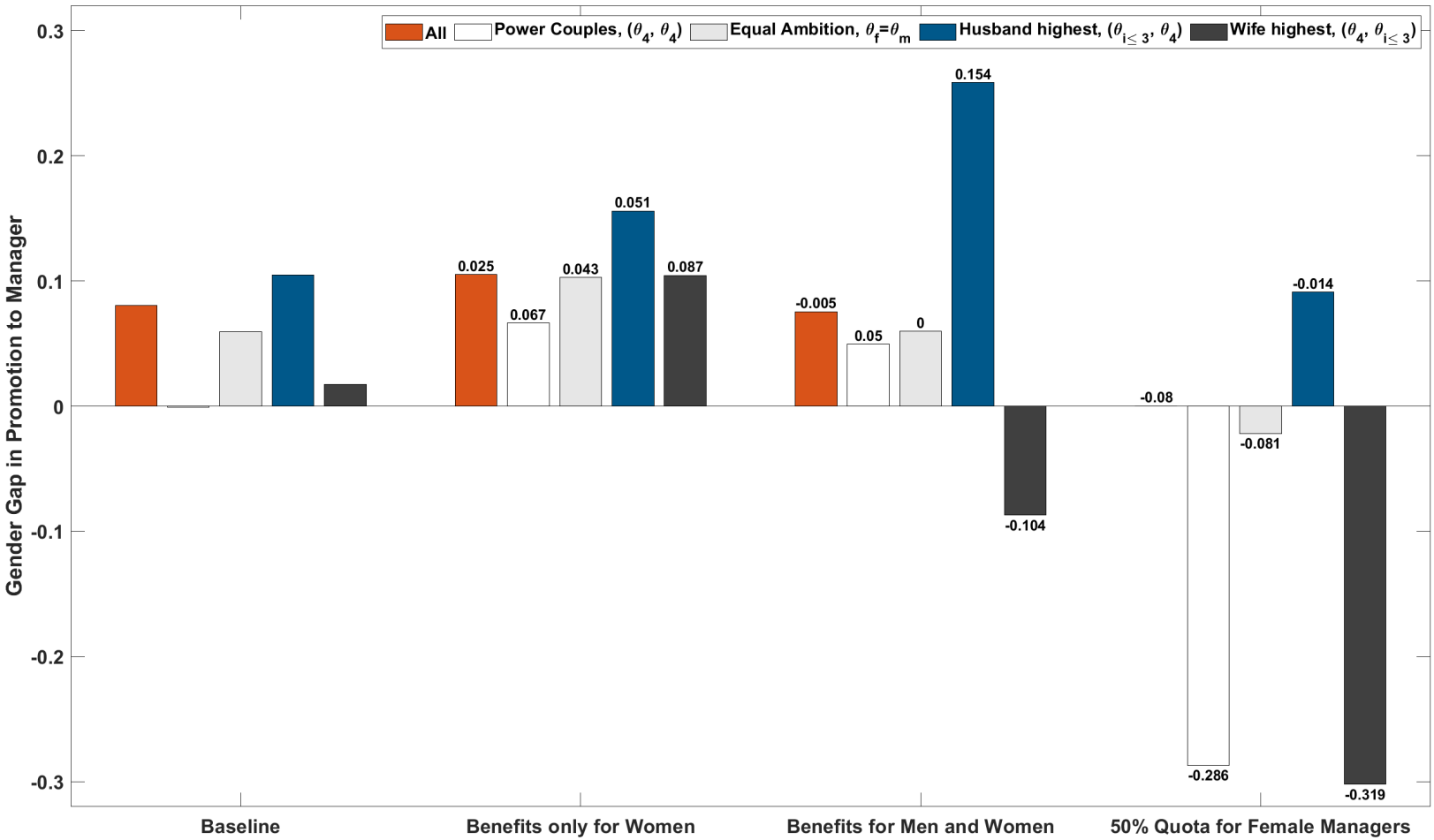
Change relative to baseline above bars



- Baseline estimates imply gender gaps in promotions, matching the data.
- Gap widens if policy targets only women & narrows if targets both spouses.
- Quota eliminates the gap.

Heterogeneous policy effects by types of couple

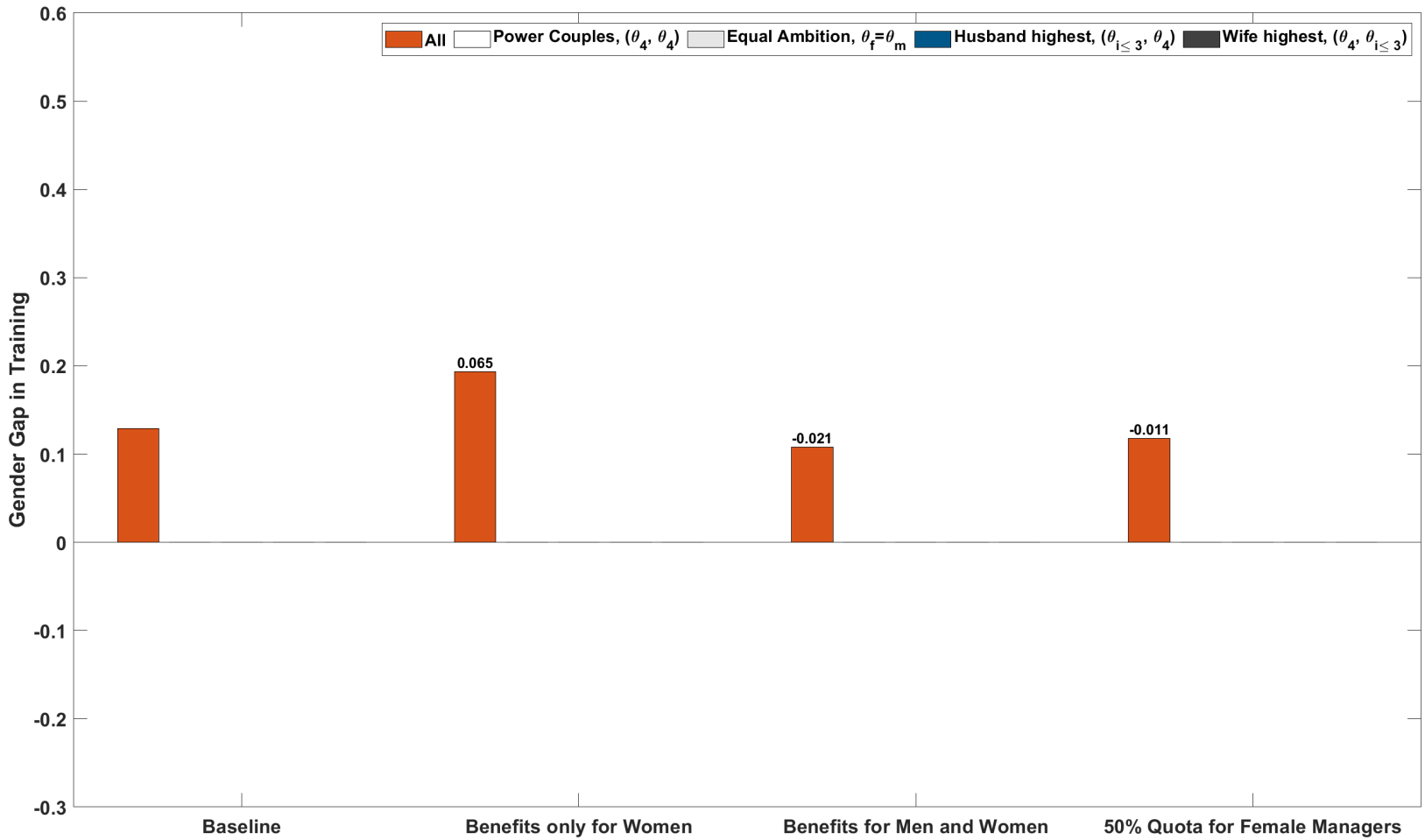
Change relative to baseline above bars



- Benefits only to women can harm women even in *initially* equal households.
- Leave policies for both spouses help women who marry down.
- 50% Quota has the highest impact on high-ambition women. ▸ Household Welfare

Firm-side investments react to the policy environment

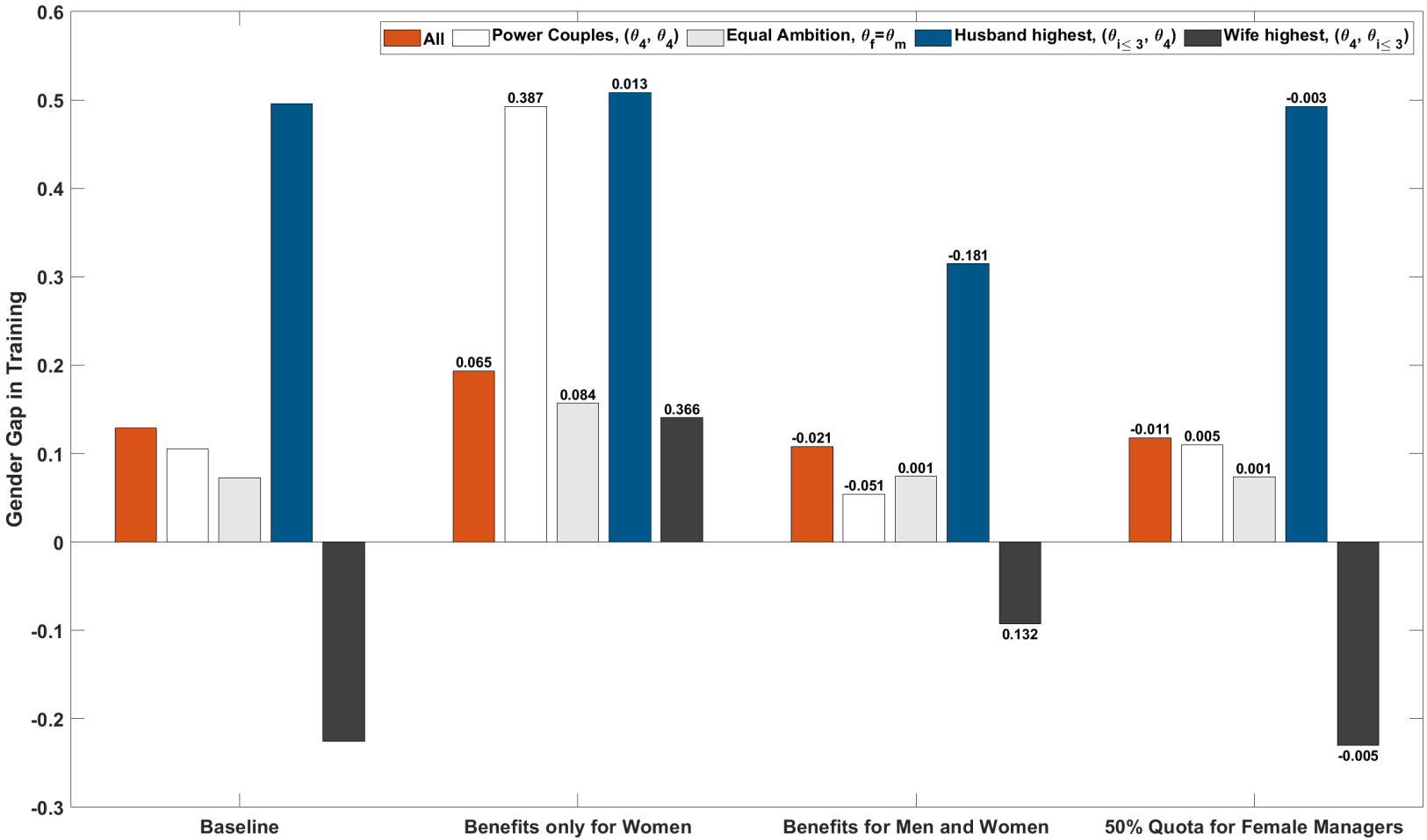
Change relative to baseline above bars



- Positive gaps in training at baseline, matching the data.
- 50% Quota and leave to both spouses reduce the gender gap in training.
- Parental Leave benefits to women only widens the baseline gap.

Firm-side investments reactions vary with couple type

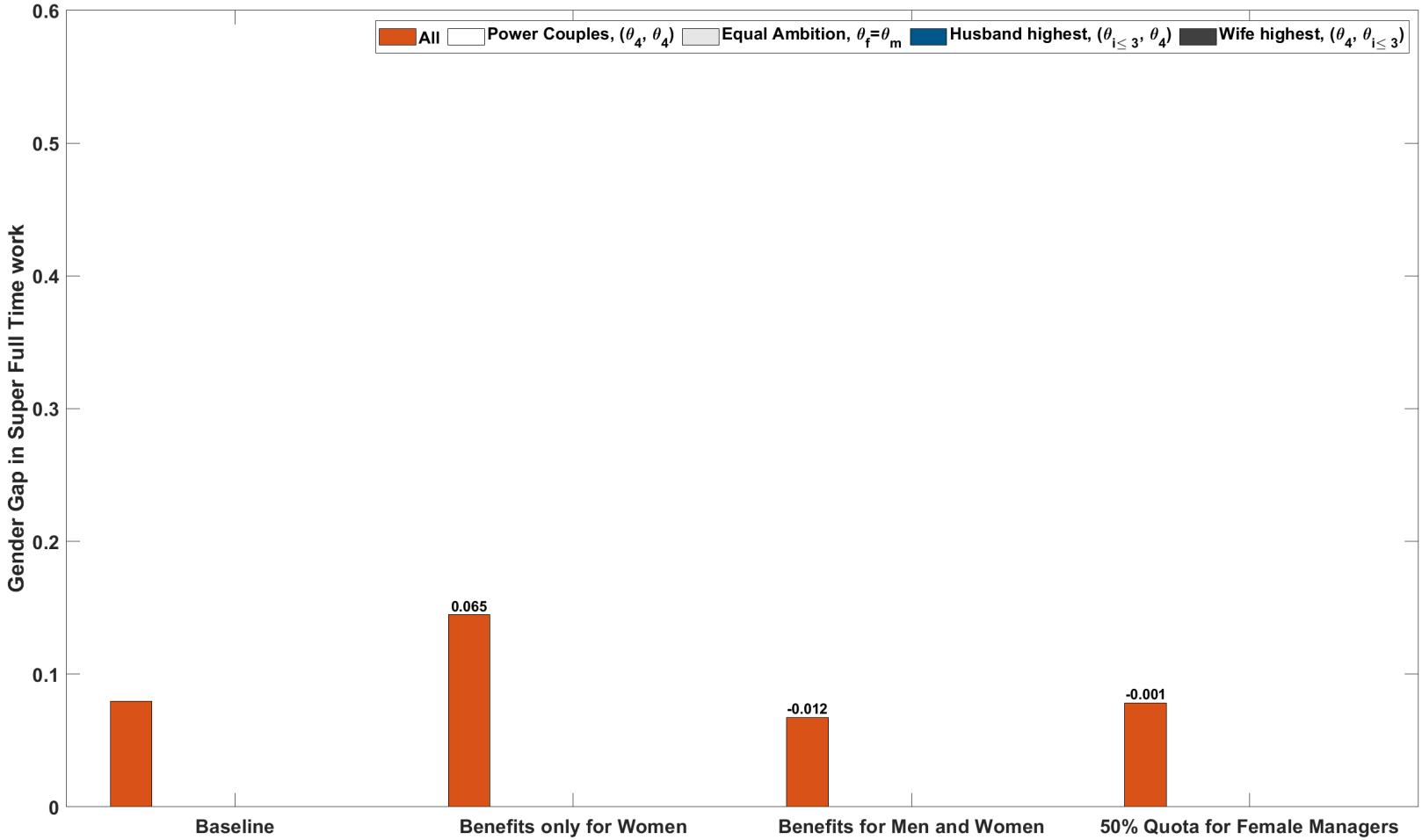
Change relative to baseline above bars



- Gaps in training are highest in couples with husbands more ambitious than their wives.
- Gaps in those couples reduce the most under shared parental leave benefits.
- The quota benefits women in unequal couples the most.

Family-side investments react to the policy environment

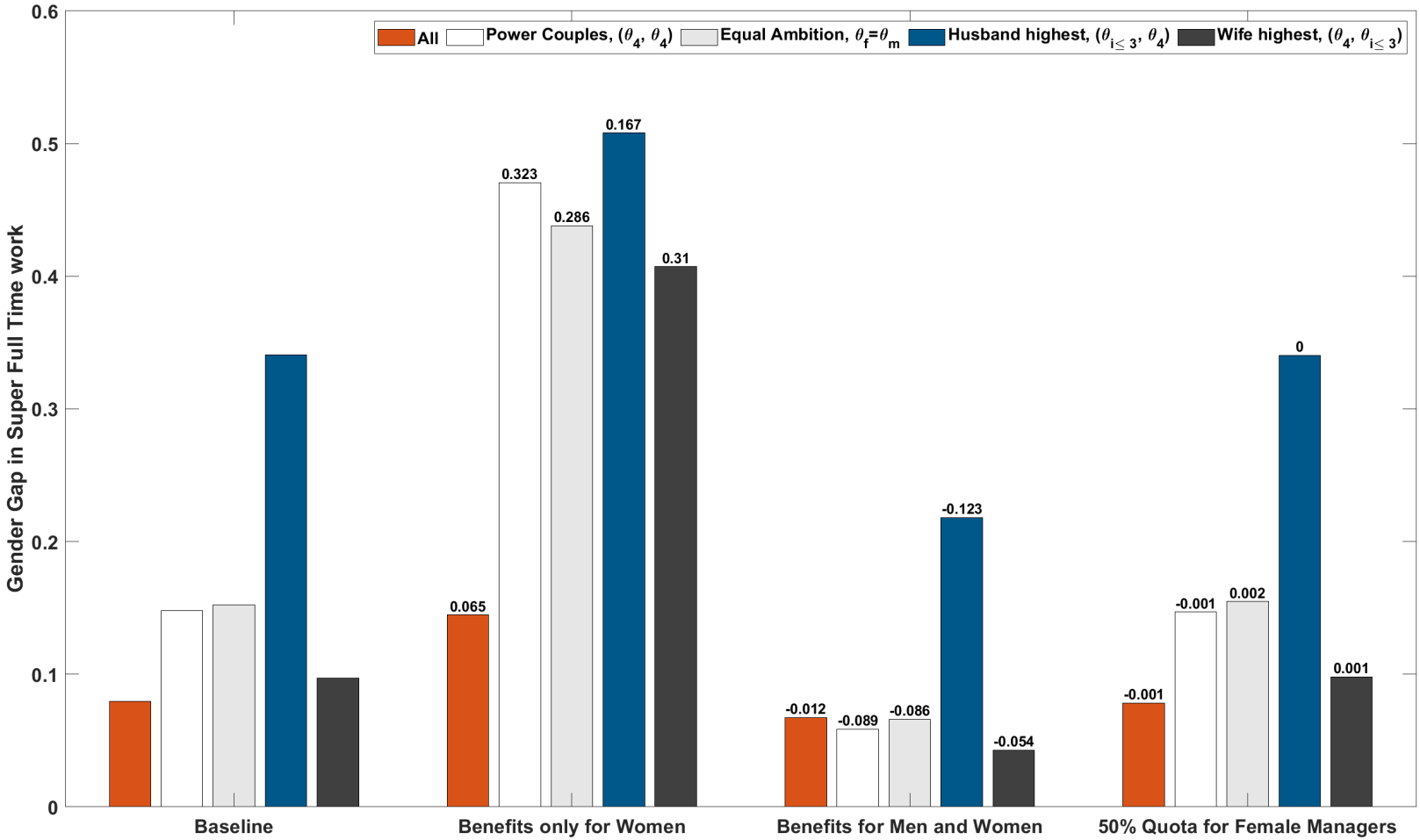
Change relative to baseline above bars



- 50% Quota and leave to both spouses reduce household specialization.
- Parental leave benefits to women only widens the baseline gap.
- The Quota affects labor supply less than it affects training

Family-side investments reactions vary with couple type

Change relative to baseline above bars



- Benefits only to women incentivize all types of couples to reduce wives' hours of work.
- Shared parental leave benefits reduces specialization in all types of couples.
- 50% Quota reduces specialization in power couples the most.

Conclusion

- Previously undocumented facts on heterogeneity in firm-side investments by households types.
- Rich Danish data $\rightarrow$ follow households and their employers over life cycle.
- Build an equilibrium model in which who marries whom affects the link between workers' investments and firms' investments.
  - lifecycle collective household model with fertility and
  - career progression within the firm.
- Preliminary policy analysis suggests that blanket policies conceal important heterogeneous effects.
  - Overall, paid leave to both spouses can reduce gender gaps in promotions, but some groups may lose.
  - Management quota shifts the focus of households on career investments for ambitious women, but to different extent.
- Highlights importance of considering interactions with the MM.

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We combine three strands in a unified framework

- Build on literature on the career cost of workers' choices.
  - Adda, Dustmann, Stevens (2017); Kleven, Landais, Søgaard (2019); Angelov, Johansson, Lindahl (2016); Goldin (2014); Cortes & Pan (2019);

- We incorporate Marriage Market (MM) and Firm-side investments.

Back

We combine three strands in a unified framework

- MM: workers' investments depend on who they marry.
  - Chiappori, Costa-Dias, Meghir (2018); Gayle & Shephard (2019); Reynoso (2024); Calvo (2022).

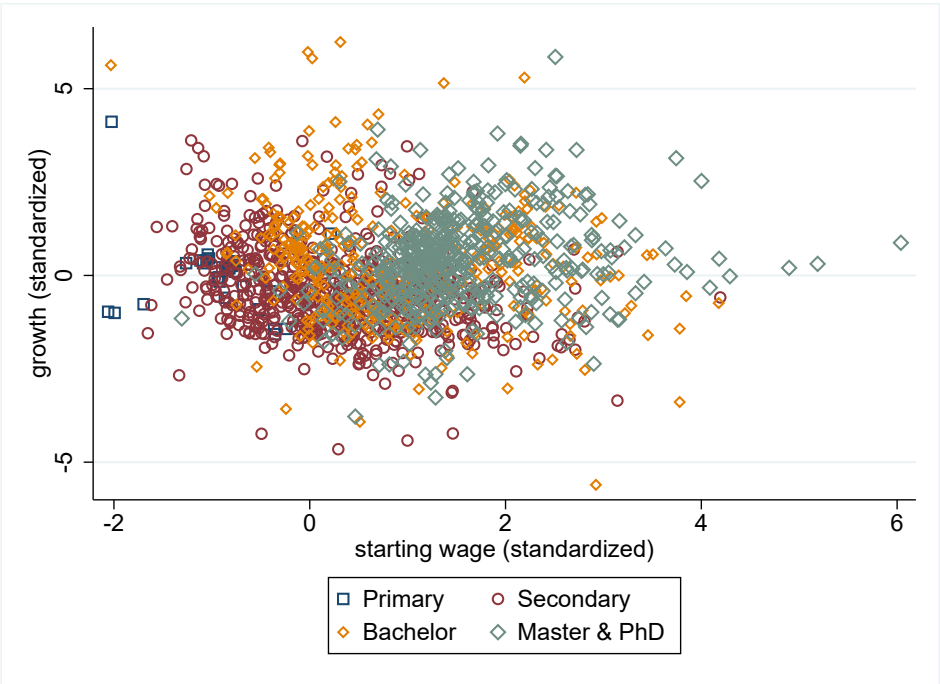
- Firms have limited manager slots and invest in more attractive workers.
  - Training matters: Blundell, Costa-Dias, Goll, Meghir (2021)
  - job assignment/ promotions: Friedrich (2020), Gibbons and Waldman (1999).
  - firms expectations about workers' performance: Gayle & Golan (2012).

- Extend literature on Marriage and Labor Markets interactions
  - Dynamic framework with fertility, on-the-job training, managerial promotions.
  - Calvo, Lindenlaub, Reynoso (fothcoming); Holzner & Schulz (2023), Philossoph & Wee (2023), AFRSV (2024).

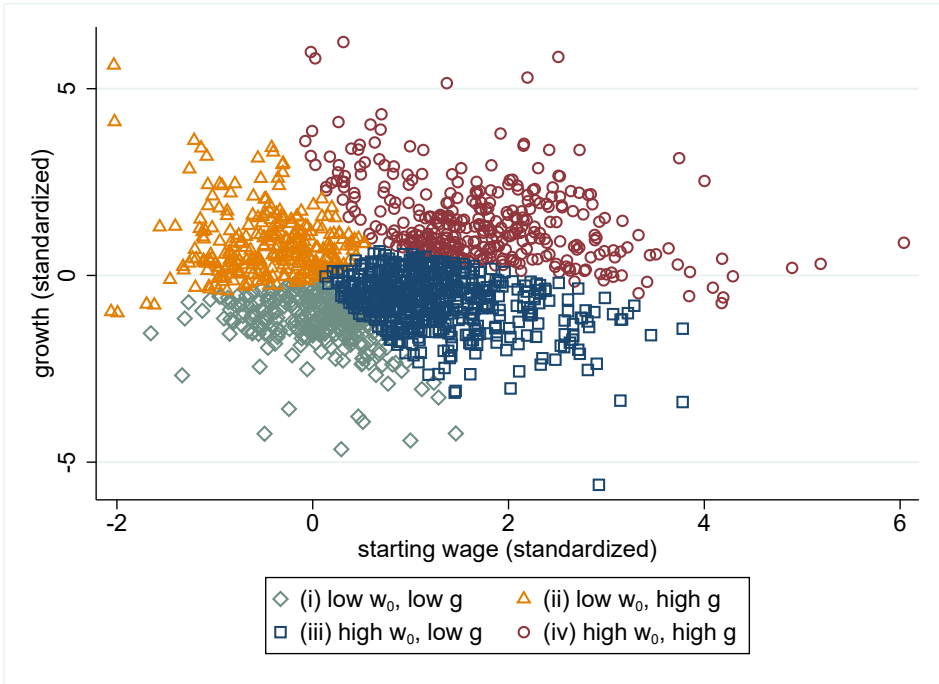
Back

Ambition types (AFRSV, 2024), θ_i

Educational level, all programs



Educational ambition, all programs

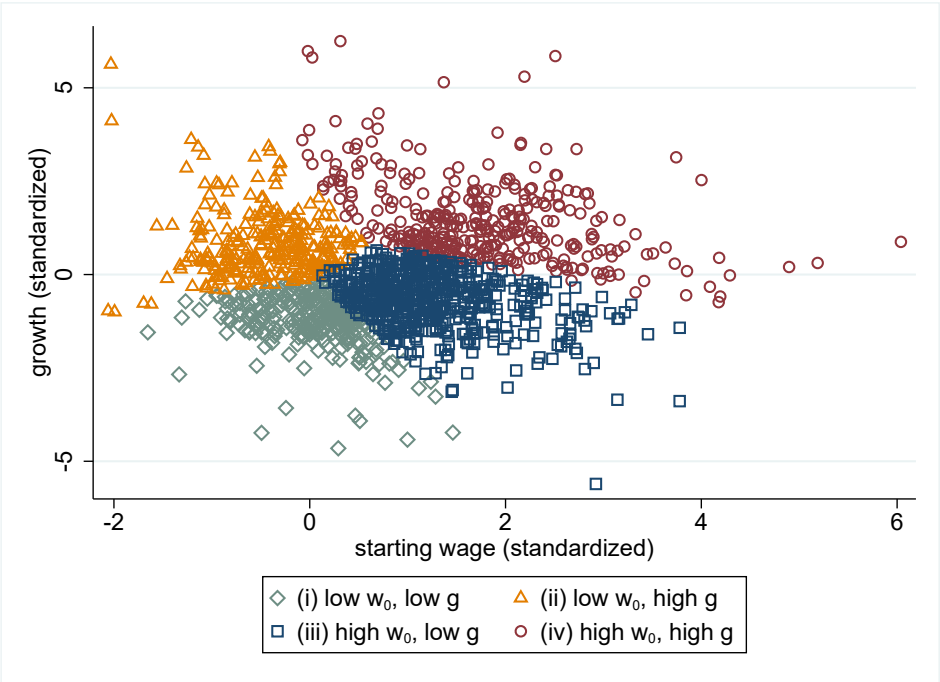


- Programs matter for marital sorting (Wiswal and Zafar, 2021),
- and differ in initial conditions and long-term outcomes (Altonji, Kahn, Speer, 2014, 2016; Kirkeboen, Leuven, Mogstad, 2016)

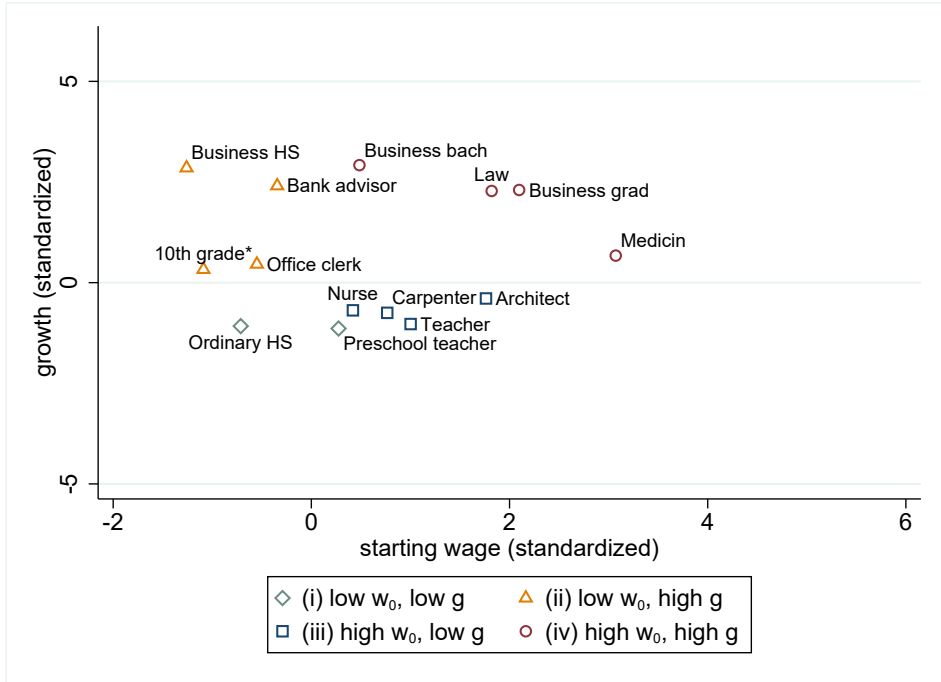
Back

Classification of most frequent programs (AFRSV, 2023)

Educational ambition, all programs



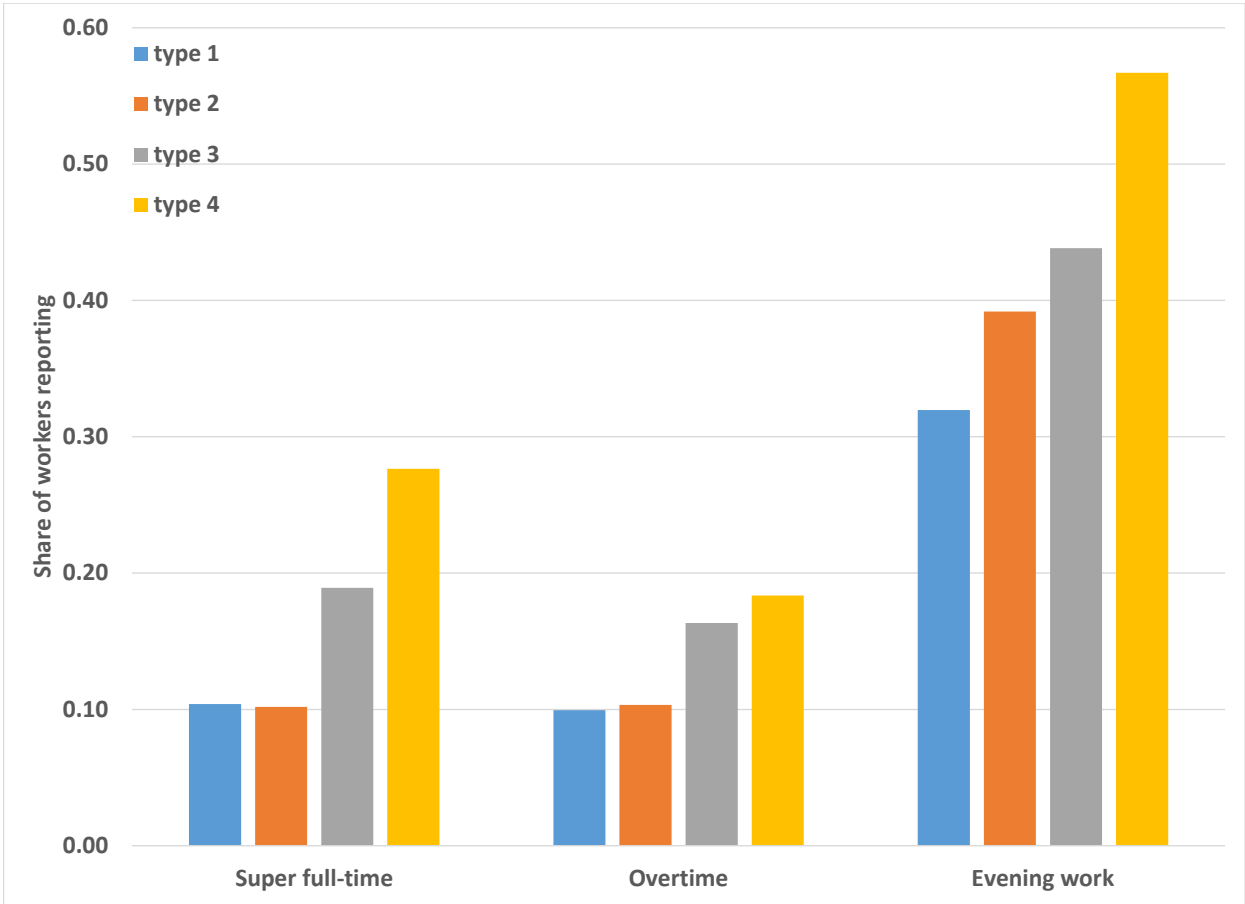
Examples



- Our method groups programs based on labor market starting conditions and progress.
- Successful measure to differentiate tertiary degrees
  - e.g.: Architecture $\neq$ Business; Nurse $\neq$ Doctor.

Back

Ambition types and hours worked



- Higher ambition types work longer and more irregular hours, often requiring working at home and in the evenings.

Back

Career ladders

- We aim to measure career path choices based on occupational choice and firm at labor market entry

- We distinguish *steep* and *flat* career ladder for tractability
  - Steep is defined as the top 20% occupation-firm pairs with highest hourly wage growth over first 10 years.
  - Calculate average growth based on coworkers.
  - Coarsen comparison group if necessary to avoid small-cells issues.

- Ambition type is about earnings potential ex ante, ladder choice is about the career path that individuals enter in the labor market.

  → Law graduate decides to work at a private law firm or in public sector administration.

Back

Promotion to manager

- Managers are workers with occupations coded “1: Management Work.”
  - Categories include “Top management,” ”Management within administration,” ”Management within production,” and ”Management within services.”
  - Examples: Manager in production company (132100), manager of internal IT (133020), top manager in public company (111200) etc.
- Promotion means transition to these occupation codes for the first time.
- Significant pay increase
  - Managers make 40% more than non managers, conditional on training.

Back

On-the-job training

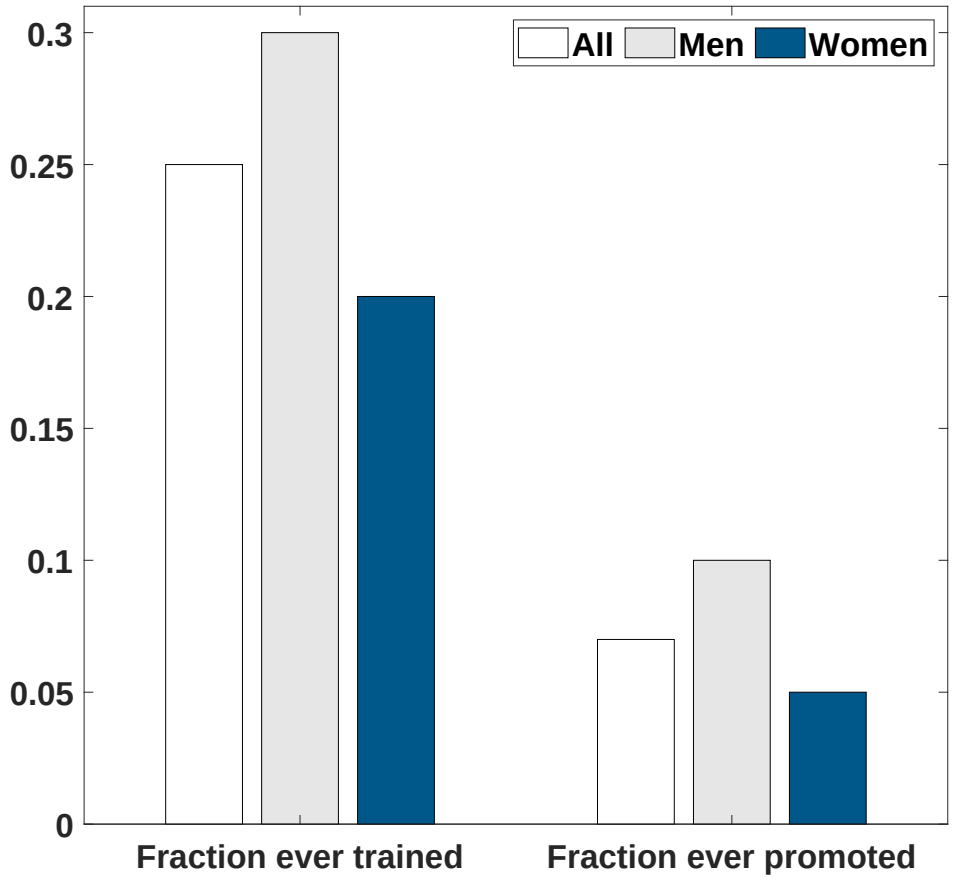
- Direct measure of firm investment: data on individuals' participation in management training programs (paid by firm while on the job).
  - → 45% of program participants are subsequently promoted to managers.
  - → 10% of all managers previously received this management training.

- Another type of firm investment: lateral moves across occupations within a firm
  - → Returns to specialization vs preparing for management

- Predictive model to identify workers who receive training based on both sources:
  - → classifies 85.75% of individuals correctly (managers with training and non-managers without training).
  - → 12.92% of trainees are subsequently promoted to managers.
  - → 50.3% of all managers previously received training.

Back

Gender gaps in training and promotion

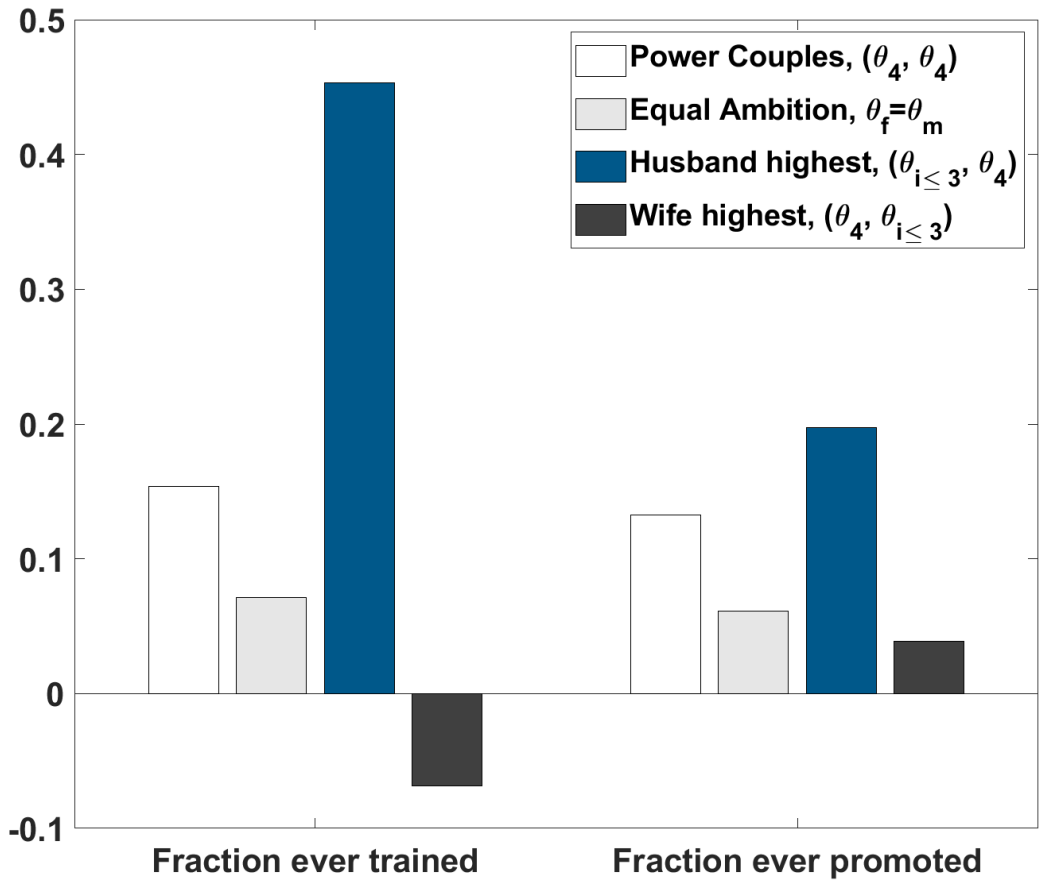


- Gender gap in training is 33%, increases to 50% at the promotion stage.
- Big and significant even with firm-occupation fixed effects. ▸ Regression

▸ Back

Gender gaps in firm-side investments vary with worker's family type

Difference in men's and women's outcome



- Interestingly, gaps positive within power couples.
- Depend on own and spousal type. Back

Firm-side investments vary with worker's family type

$$mg_{ijt} = \beta_0 + \beta_1 \cdot F_i + \beta_2 \cdot \theta_{i,\geq 3} + \beta_3 \cdot \theta_{i,\geq 3} \cdot F_i + \beta_3 \cdot \theta_{j,\geq 3} + \beta_4 \cdot \theta_{j,\geq 3} \cdot F_i + X'\gamma + \epsilon$$

	(1)	(2)	(3)	(4)
	Training		Manager Promotion	
female	-0.0322***		-0.0049***	
	(0.003)		(0.001)	
high-ambition	0.2392***		0.0295***	
	(0.003)		(0.001)	
high-ambition * female	-0.0469***		-0.0153***	
	(0.005)		(0.001)	
high-ambition spouse	0.0521***		0.0201***	
	(0.004)		(0.001)	
high-ambition spouse * female	-0.0305***		-0.0181***	
	(0.005)		(0.002)	
Control for LS Choices	No		No	
Observations	2,311,023		2,311,023	

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

- Gender gaps widen for ambitious women and for women with ambitious spouses.
- The role of the spouse declines conditional on labor market choices.

Back

Firm-side investments vary with worker's family type

$$mg_{ijt} = \beta_0 + \beta_1 \cdot F_i + \beta_2 \cdot \theta_{i,\geq 3} + \beta_3 \cdot \theta_{i,\geq 3} \cdot F_i + \beta_3 \cdot \theta_{j,\geq 3} + \beta_4 \cdot \theta_{j,\geq 3} \cdot F_i + X'\gamma + \epsilon$$

	(1)	(2)	(3)	(4)
	Training		Manager Promotion	
female	-0.0322***	-0.0266***	-0.0049***	-0.0035***
	(0.003)	(0.003)	(0.001)	(0.001)
high-ambition	0.2392***	0.1897***	0.0295***	0.0285***
	(0.003)	(0.004)	(0.001)	(0.001)
high-ambition * female	-0.0469***	-0.0426***	-0.0153***	-0.0140***
	(0.005)	(0.004)	(0.001)	(0.001)
high-ambition spouse	0.0521***	0.0261***	0.0201***	0.0152***
	(0.004)	(0.004)	(0.001)	(0.001)
high-ambition spouse * female	-0.0305***	-0.0114**	-0.0181***	-0.0094***
	(0.005)	(0.005)	(0.002)	(0.002)
Control for LS Choices	No	Yes	No	Yes
Observations	2,340,453	2,304,425	2,340,453	2,304,425

Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

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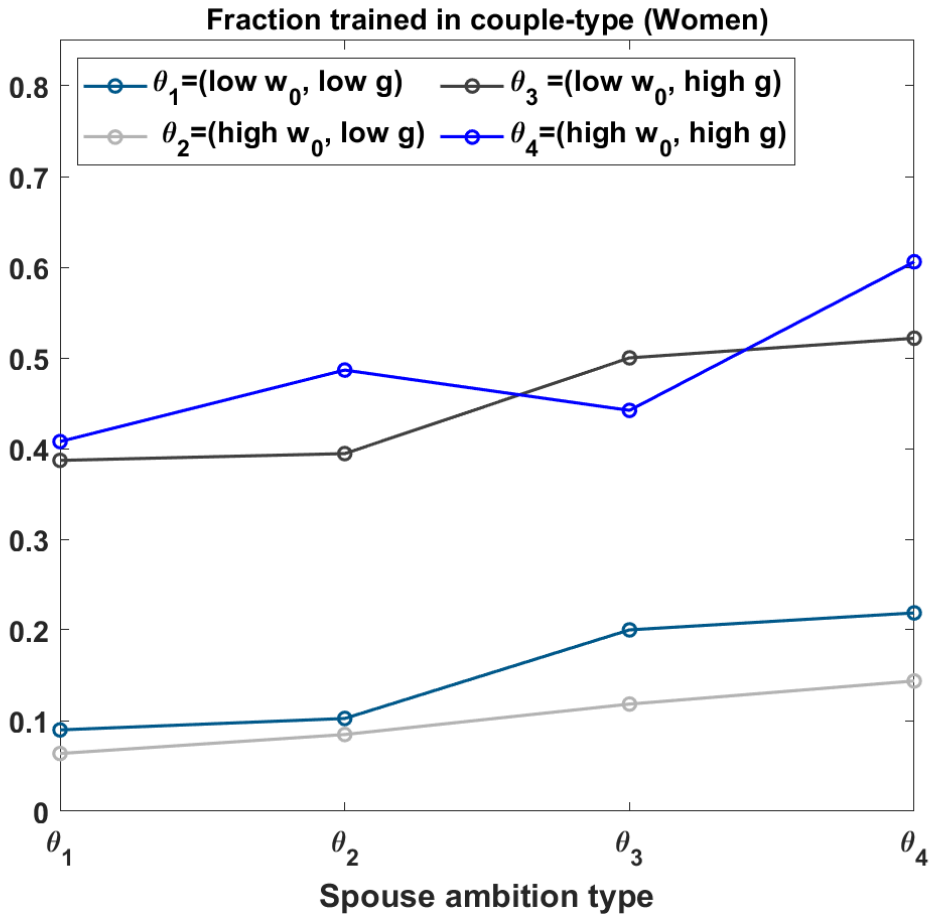
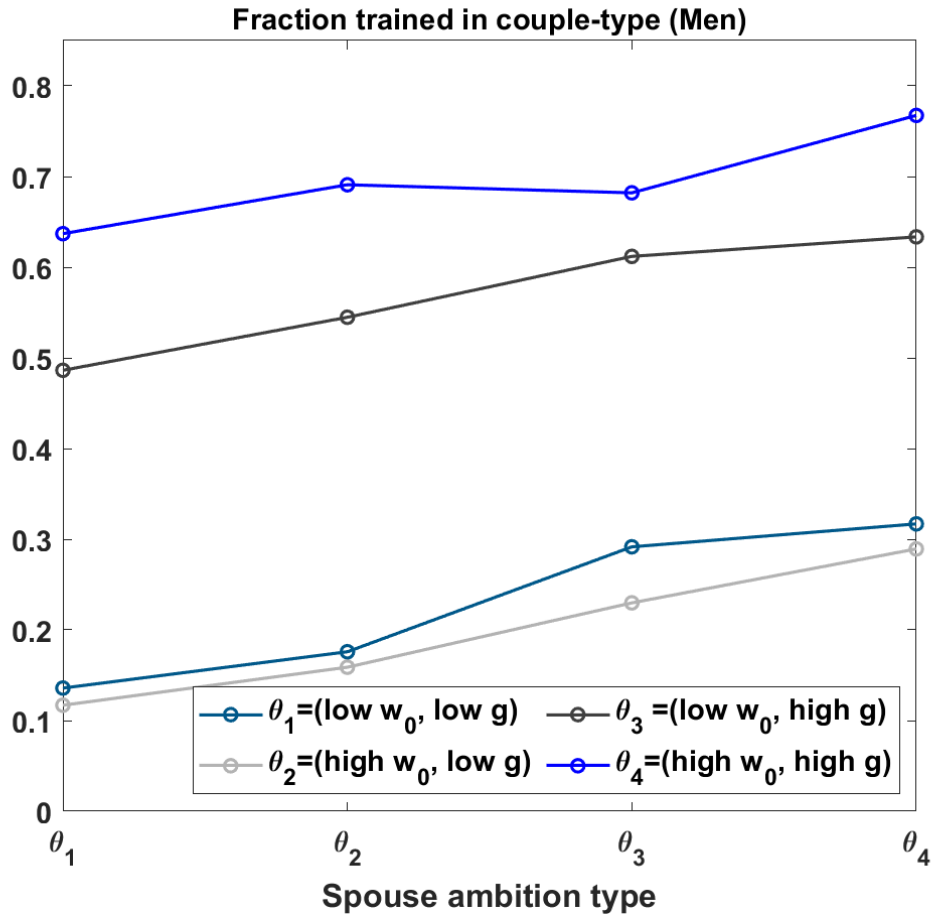
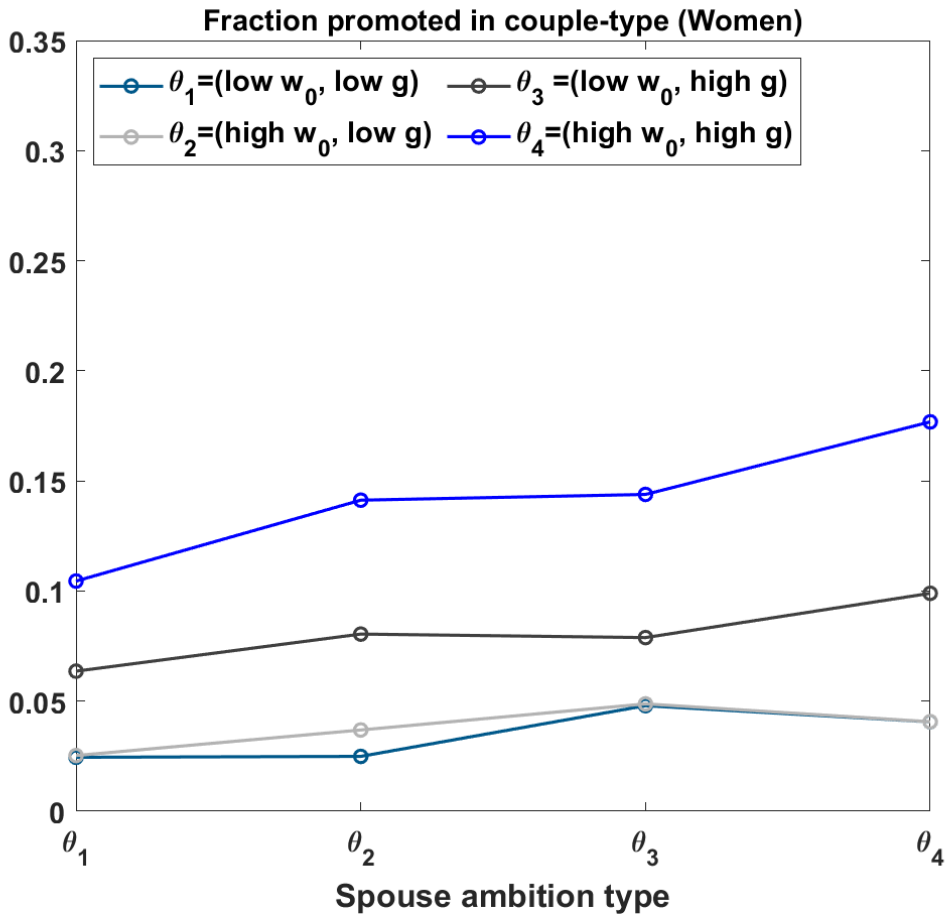
Firm-side investments vary with worker's family type

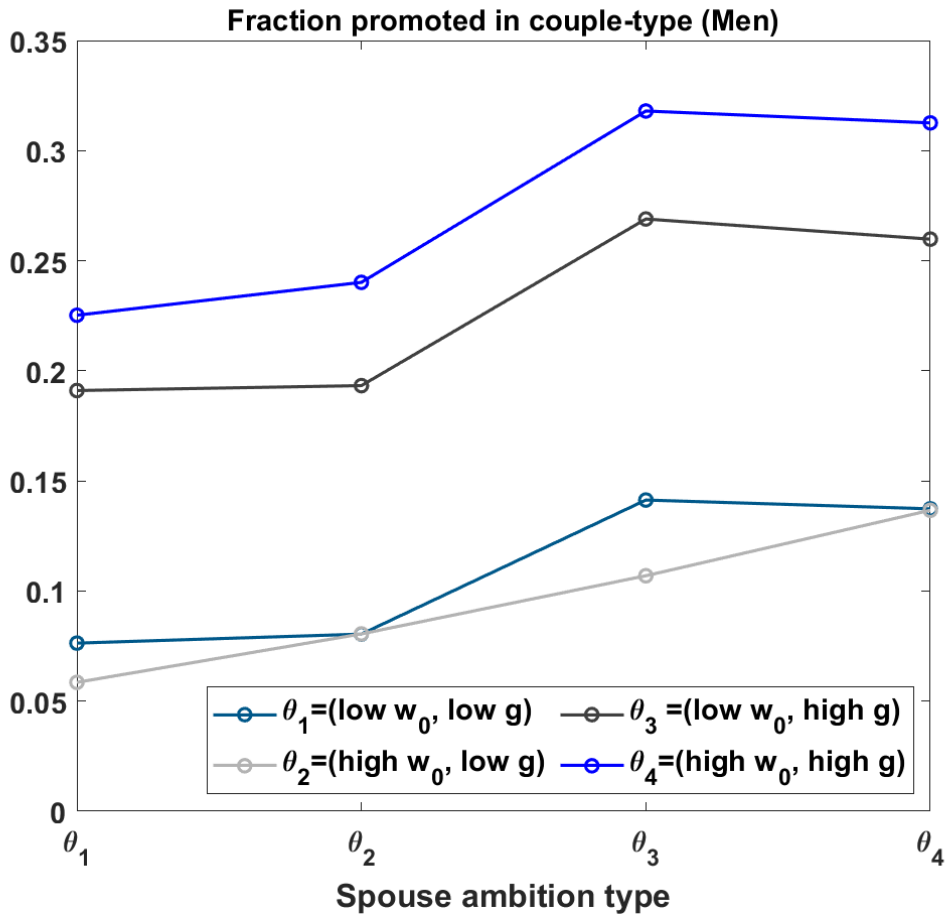




- Probability of receiving training ↑ with own and spouse's ambition
  - but more so for men. Back

Firm-side investments vary with worker's family type





- Probability of reaching managerial position ↑ with spouse's ambition
  - but more so for men. Back

Gender gaps in training and promotion

$$mg_{ift} = \beta_0 + \beta_1 \cdot F_i + \delta_{f,L} + \theta_i + \delta_{\{l_i\}_t} + \epsilon$$

	(1)	(2)	(3)	(4)	(5)	(6)
		Training			Manager Promotion	
female	-0.0819***	-0.0579***	-0.0285***	-0.0192***	-0.0190***	-0.0096***
	(0.002)	(0.003)	(0.003)	(0.001)	(0.001)	(0.001)
Firm-Ladder FE	No	Yes	Yes	No	Yes	Yes
Worker Ambition FE	No	No	Yes	No	No	Yes
Worker Exp FE	No	No	Yes	No	No	Yes
Observations	2,340,453	2,340,453	2,304,425	2,340,453	2,340,453	2,304,425
R-squared	0.010	0.347	0.412	0.003	0.199	0.231

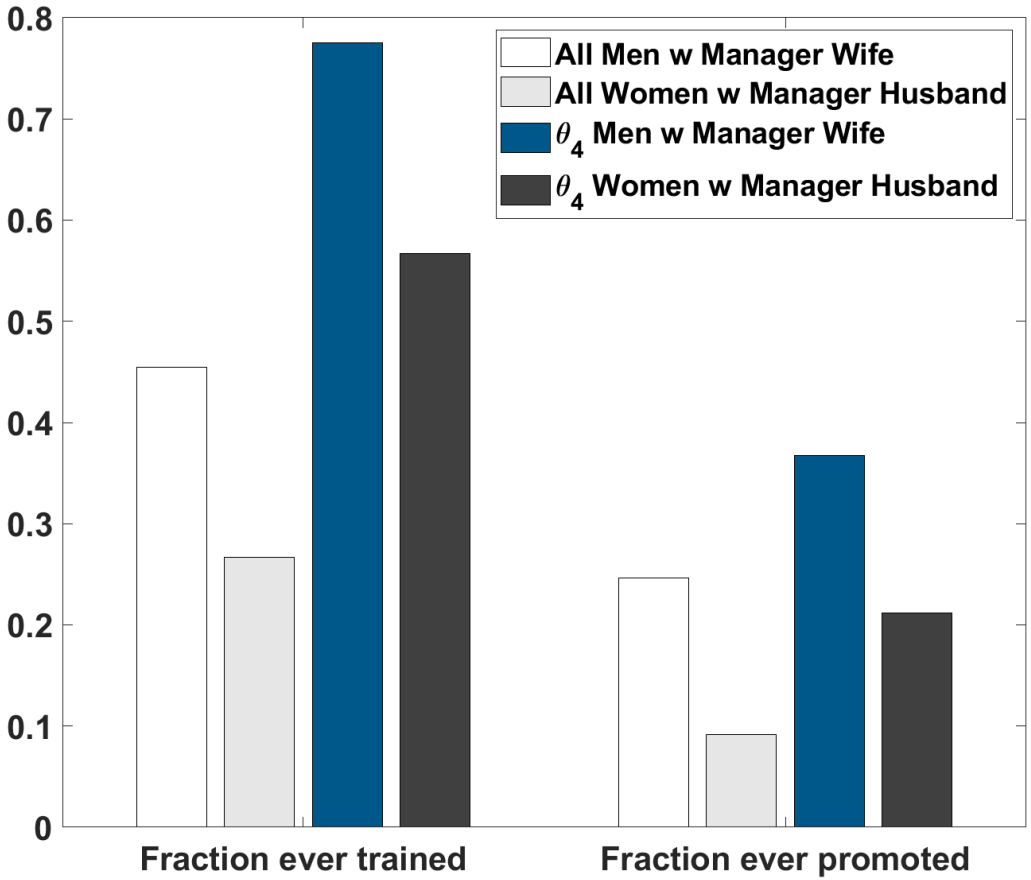
Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

- Gender gaps big and significant even with firms and ladders.
- Decline conditional on labor market choices.

Back

When the husband is a manager

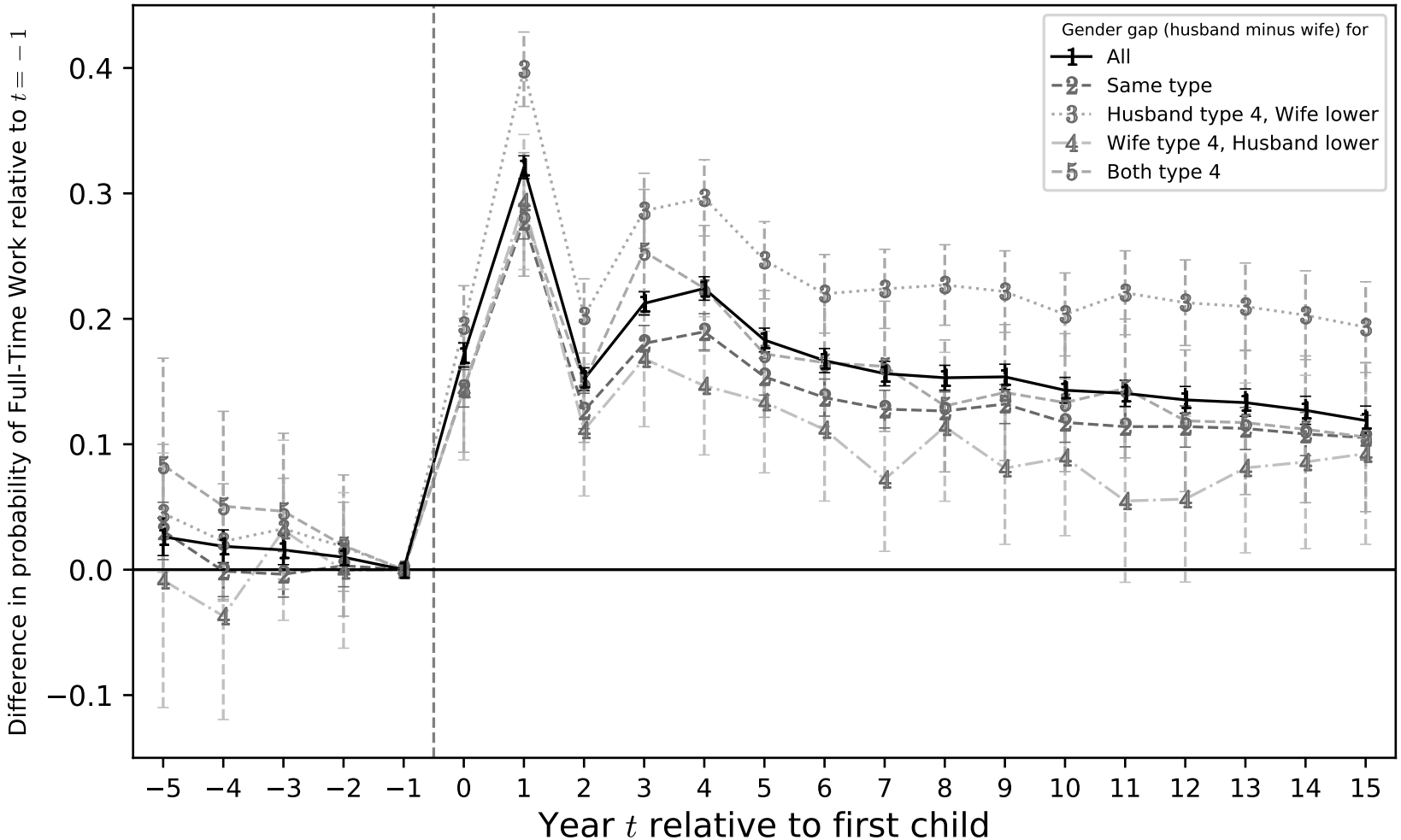
Difference in men's and women's outcome



- Most ambitious women who marry a manager 42.51% less likely to become managers than men married to managers.
  - Might be behind the higher gaps among power couples.

Back

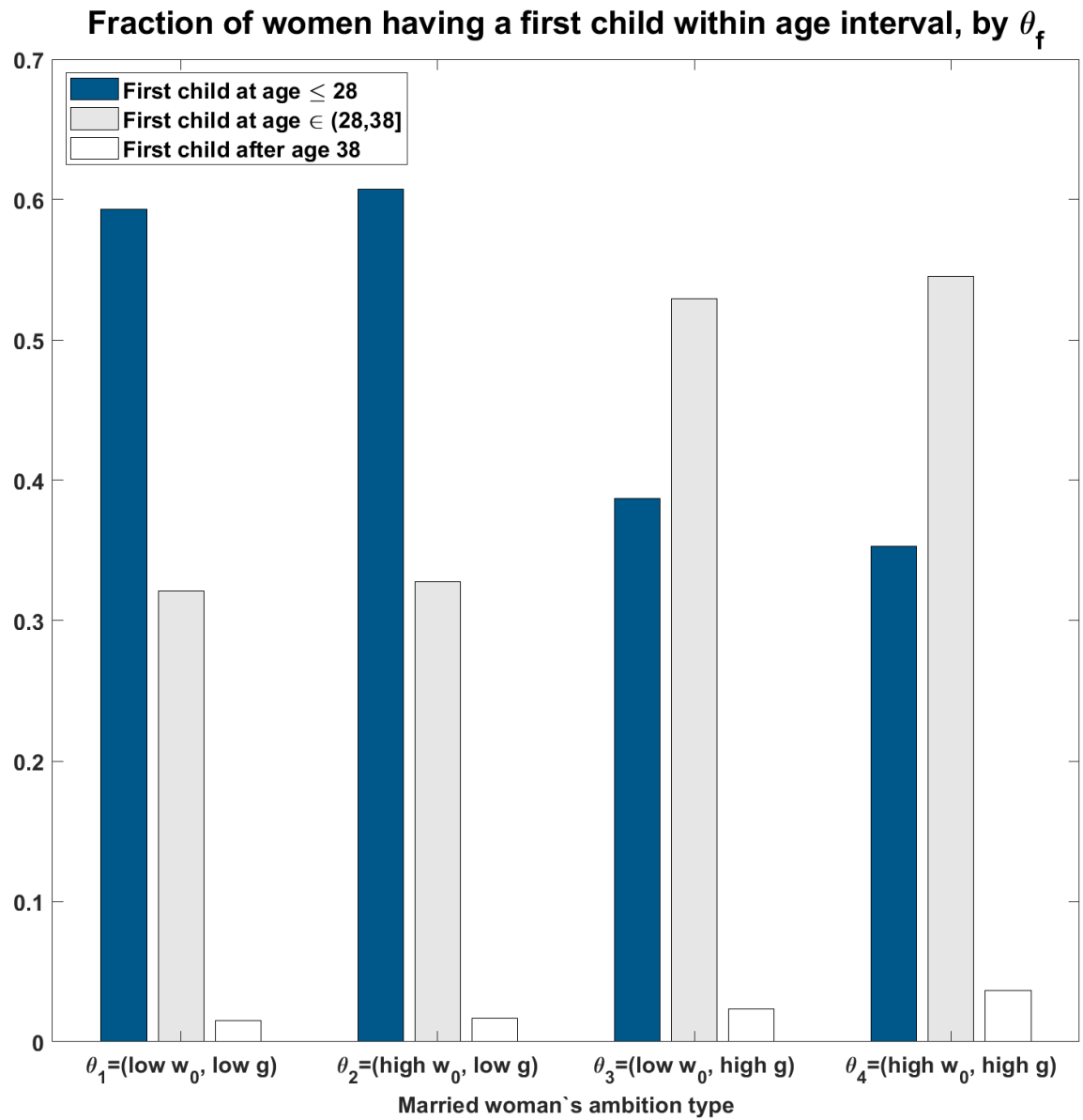
Time allocation after arrival of children



- Large and persistent child penalty even for women with highest ambition.
- Women married to more ambitious spouse show a more persistent penalty.

Back

More ambitious women delay fertility significantly more



Back

Model Periods and the life cycle of individuals in the data

Periods: t_0: *Matching* & t_1: *Early Career* | t_2: *Training* | t_3: *Career Progress*

Age cuttoffs: $\overline{a_{12}}_i = max\{entry_i + 3, 28\}$ $\overline{a_{23}}_i = max\{\overline{a_{12}}_i + 3, 38\}$

Sample:

85.1% young couples → marry
46.3% females → first child

37.85% females → first child
59.25% trainees → trained

99.36% managers
→ promoted

▸ Back

Endogenous gender gaps in promotion

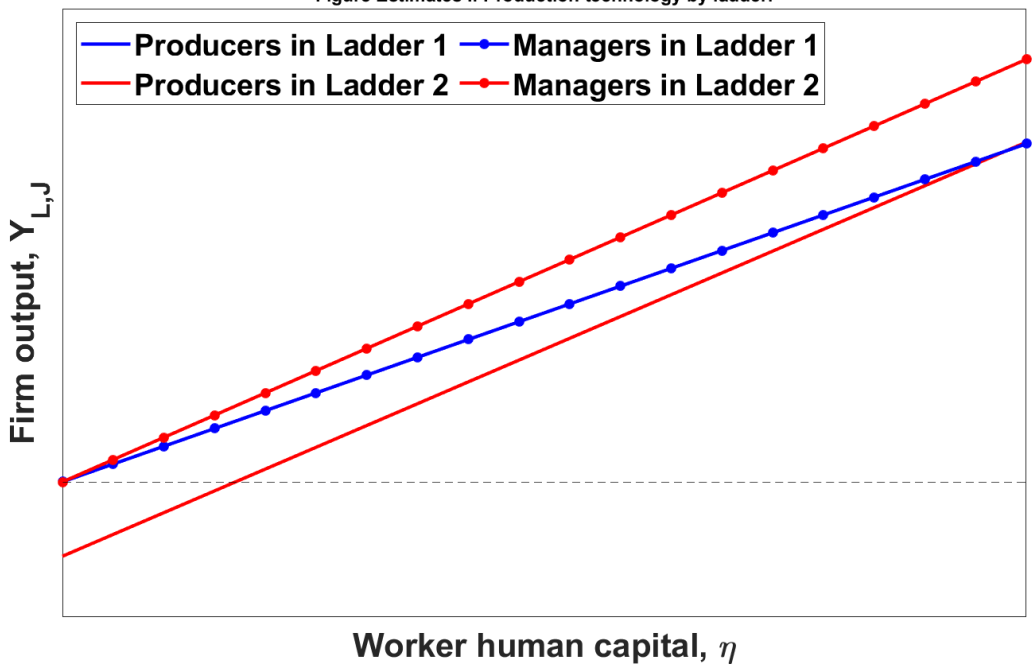
- Initial women's advantage at home imply women tend to stay at home more.
- Firms tend to see women as workers with lower market human capital, η.
- Firms tend to expect women to work less in $t = 3$.
- Training is offered relatively more to men.
- Expecting this bias, families tend to invest even more in husbands.
- In equilibrium, gender gaps in training and promotion arise.
- How do these forces and mechanisms interact with policies?

Estimation

- We estimate the model using simulated method of moments.
- Targeted moments include
  - share of singles and 4 household types of interest (power couples, equal couples, asymmetric couples with one spouse of type 4)
  - participation rates by gender and ambition type, variance in male labor supply
  - participation gap within couples
  - autocorrelation of time at home
  - initial earnings levels by ladder and ambition types
  - differences in earnings growth across ladders
  - differences in earnings growth as a function of training for full-time workers
  - differences in earnings as a function of labor supply choices (spells of part-time work or non-participation, vs. full-time work) by ladder
  - differences in earnings for trained workers who are promoted to managers or not
  - gender promotion gaps by ladder
  - share of women with first child by period and ambition type Back

Preliminary Estimates I: Production technology by ladder

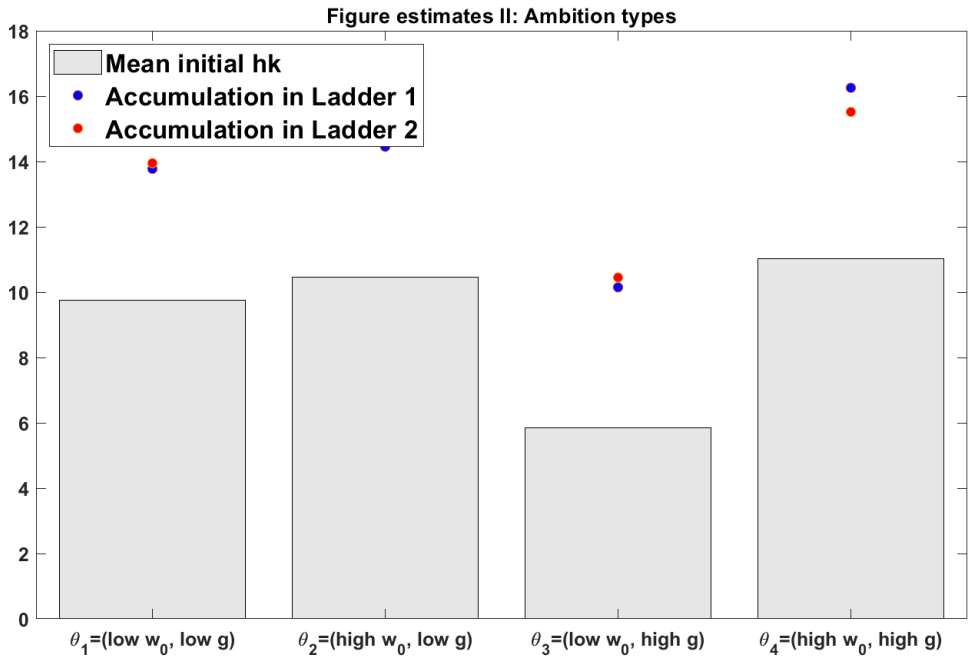
Figure Estimates I: Production technology by ladder.



- Ladder 2 is steeper: marginal productivity of skills is higher in both positions
- At low skill levels, producers have a comparative advantage in L_1.
- The productivity of skills is higher in the managerial position in both ladders.

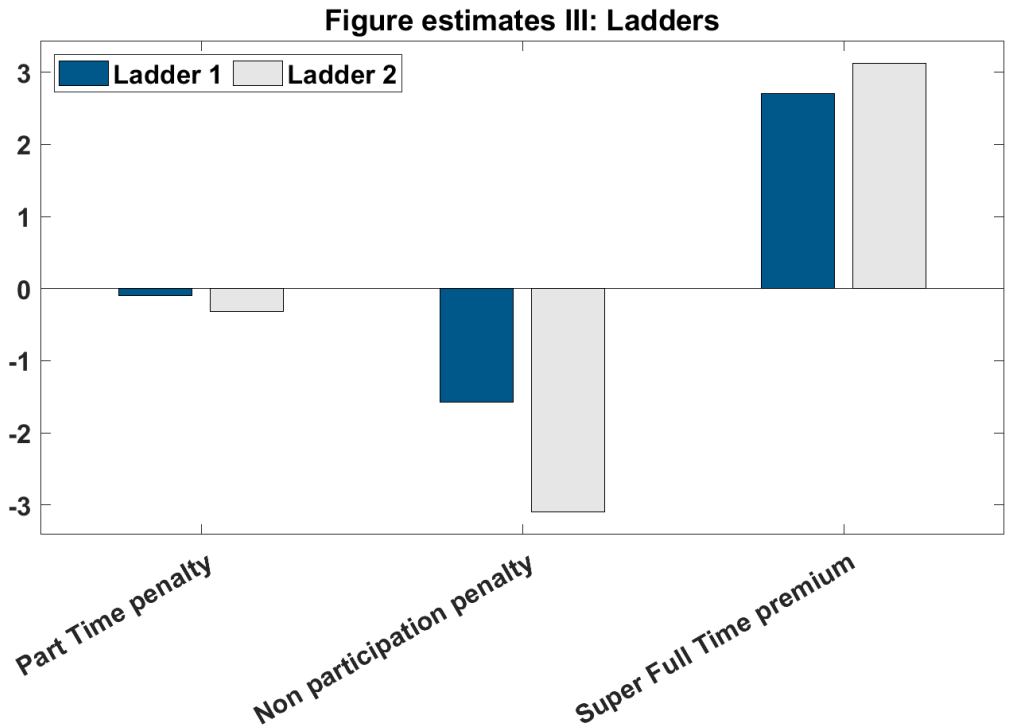
Back

Estimates II: Model produces well-defined ambition types



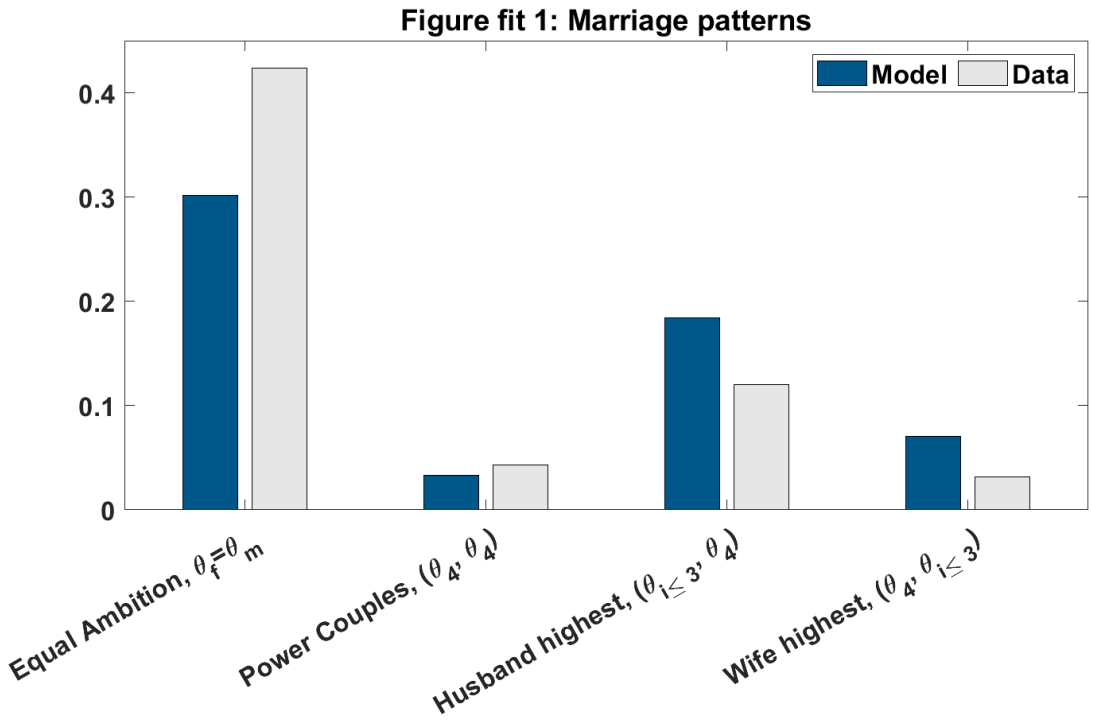
- θ_2 & θ_4 higher average starting human capital; θ_3 & θ_4, higher average growth.
- Ladder 2 is steeper: higher reward for FT work, on average. Back

Estimates III: Depreciation by ladder



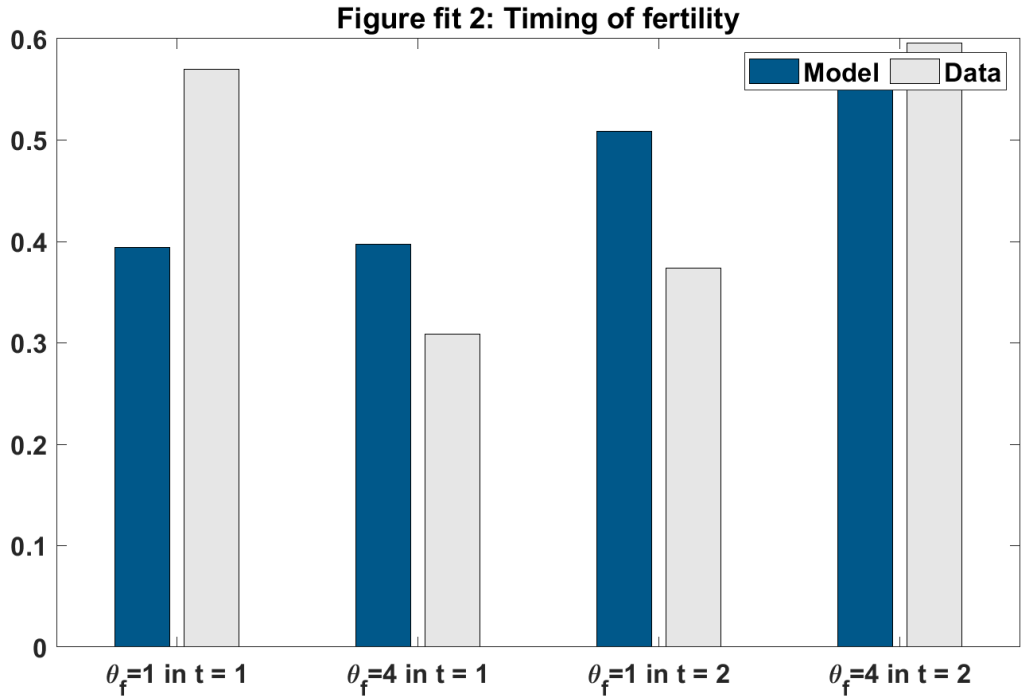
- Ladder 2 is steeper: harder to climb back after a reduction in labor supply.
- Non participation penalty is stronger in both ladders. Back

Our model replicates targeted Marriage patterns



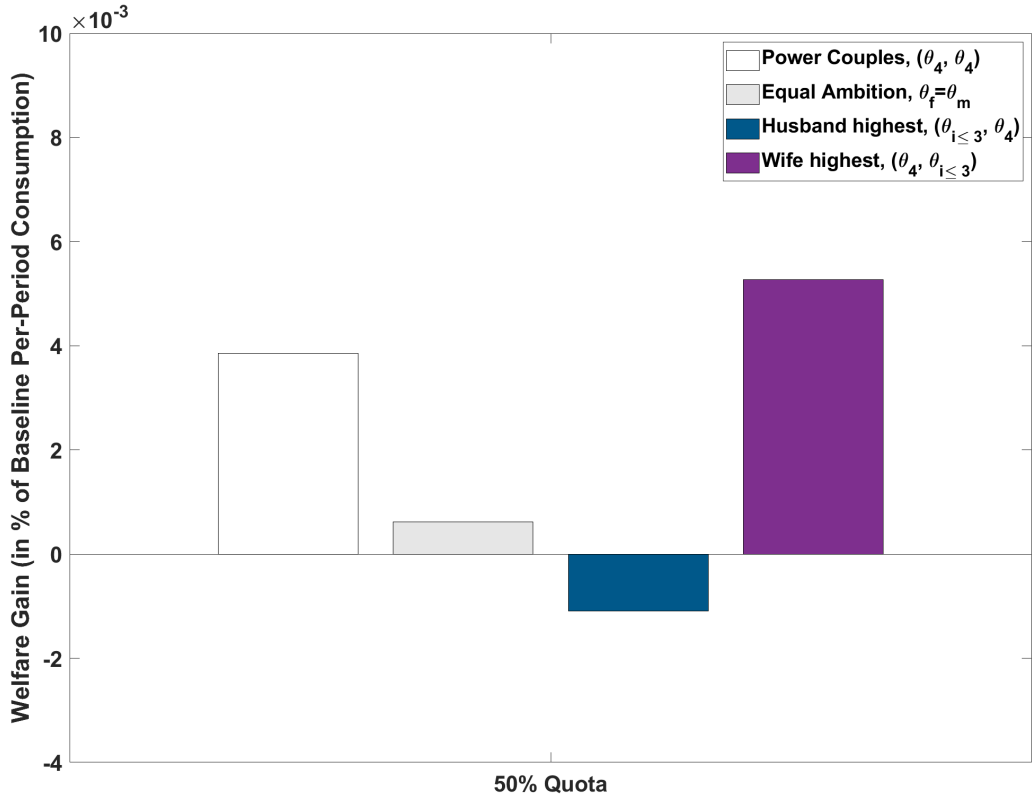
▸ Back

And the U-shaped targeted fertility patterns



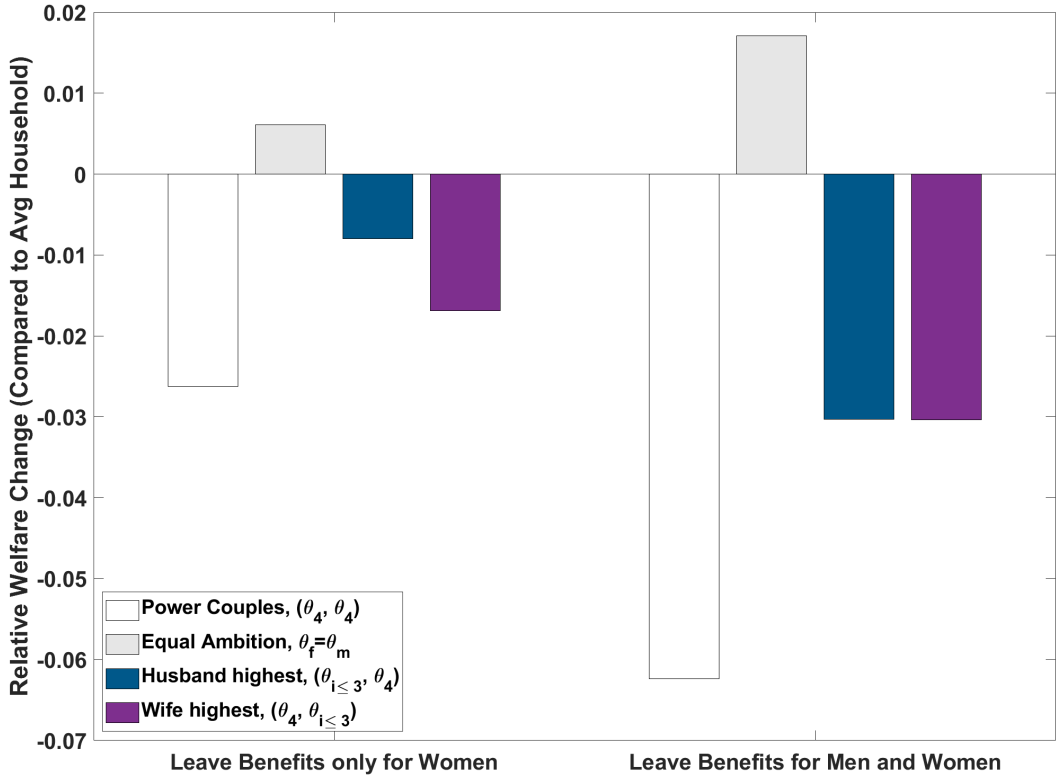
- Women type θ_4 more likely to delay fertility.
- Women type θ_1 more likely to have children early in their careers. Back

Welfare Effects: Management Quota



- As expected, households with highly ambitious women benefit the most from a quota for female managers.
- Households where the husband is ambition type 4 and the wife has lower type lose from the quota. Back

Welfare Effects: Parental Leave Policies



- Households with ambitious women benefit the least from parental leave only for women, consistent with higher opportunity cost of leave.
- Equal couples (except power couples!) benefit more than average when both spouses are eligible for leave, consistent with leave takeup by the spouse with comparative advantage at home. Back

Hours Worked

- Detailed responses on hours worked from labor force survey (9.4% of individuals)
- "Ever managers" have much higher and irregular working hours:
  - Higher share working "super full-time" (more than 37 hours per week) and reporting overtime work, especially in the mid-career phase.
  - Higher share working usually or sometimes in the evening (excl. shift work) and on the weekend in mid and late career.
- Higher and more irregular hours worked on the steep than the flat ladder:
  - 1.5 hours more per week on average, 19% report working "super full-time" (vs 11% on flat ladder).
  - 5pp higher shares of overtime work and evening work, respectively.

Back

Environment II: The Family

- Flow individual utility: $u_{it} = c_{it} Q_t \chi^u_{(children)}$
- Labor supply choices: $l_i = \{N, P, F, S\} = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$, no leisure.
- Ladder choices: $L_i = \{L_1, L_2\}$.
- The public good produced with private goods and time:

$$Q_t = c_{Qt} + \phi_{ft}(1 - l_{ft}) + \phi_{mt}(1 - l_{mt}) - \chi^Q_{(children)}$$

  - Complementarity between time and goods: Welfare cost if both spouses choose super-full-time.

- ϕ_i: spouse i's *family* human capital.
  - Depreciates over time.

Back

Worker Type vs. Family Type at a given time t

- A *worker type* consists of their gender, *ambition type*, *initial hk*, *ladder*, *LS history*, and *training*:

$$\omega_{it} = (\mathcal{G}_i, \theta_i, \eta_{1i}, L_i, \{I_{ir}\}_{r=1}^{t}, tr_i) \quad \in \Omega_t$$

- A *family type* consists of *own worker type*, *own family hk*, *spouse's worker type*, and *spouse's family hk*:

$$\varphi_{it} = (\underbrace{\mathcal{G}_i, \theta_i, \eta_{1i}, L_i, \{I_{ir}\}_{r=1}^{t}, tr_i}_{\omega_{it}}, \phi_{it}, \underbrace{\mathcal{G}_j, \theta_j, \eta_{1j}, L_j, \{I_{jr}\}_{r=1}^{t}, tr_j}_{\omega_{jt}}, \phi_{jt})$$

- Market human capital η_{it} evolves over time depending on LS, ambition type, and ladder. Boosted by training. Details
- ϕ_{it} is initialized with common family shock. Initially, women have an advantage but skills at home depreciate over time (relative to private goods). Details

Go back

Environment III: The Representative Firm

- Two jobs J on each ladder L, producer and manager.
- Output *per unit of time* in job J and ladder L is parameterized as follows:

$$y_{L,J}(\eta_{it}) = a_{L,J} + b_{L,J}\eta_{it},$$

  where η_{it} is market human capital.
- Manager promotion requires leadership training and super-full-time work.
  - Time use trade-off for families.
  - Firms extremely selective due to capacity constraints: Convex cost of training and fixed slots for managers.

Back

Firm's training and promotion problem

- At $t = 2$ the firm takes as given:
  - Matching in the MM, $\mu(\theta)$;
  - Distribution of worker types decided by families, $\{\omega_{i2}\}$;
- Forms beliefs about $l_{i3} \mid \omega_{i2}$ and profits with and without training
  - Over unknown *family type* and *family shock*.
- Chooses fraction of $N(\omega)$ trained, $tr(\omega)$, and promoted, $mg(\omega)$;
- to maximize expected profits from training.
- Expectation about working super full time or less, depends on the family type.

Go back

Firm's training and promotion problem

$$\max_{\{tr(\omega),mg(\omega)\}} \Pi_{tr} = \sum_{\omega\in\Omega_2} tr(\omega)\Big[mg(\omega)E[\pi_{mg}(\omega)]+(1-mg(\omega))E[\pi_p(\omega)]\Big]\cdot N(\omega)-M\cdot C(N_{tr}/M)$$

subject to the size of the training program, and the capacity constraint for managers,

$$N_{tr} = \sum_{\omega\in\Omega_2} tr(\omega)\cdot N(\omega)$$

$$M \geq \sum_{\omega\in\Omega_2} tr(\omega)\cdot mg(\omega)\cdot Pr(SFT \mid tr,\ \omega)\cdot N(\omega)$$

- Optimal firm behavior determines training and promotion policies, $tr(\omega_2)$ and $mg(\omega_2 \mid tr(\omega_2), I_3(\omega_2)=1)$;
- wage rates, $w(\omega_i)$, and beliefs $B(I_3 \mid \omega_2)$.
- Expectation about working super full time or less, depends on the family type.

Back

Marriage Market

- Potential partners in the MM take as given:
  - Idiosyncratic taste shocks, $\beta^{\theta_i \theta_j}$
  - wage rates, $w(\omega_i)$;
  - firm's training and promotion policies, $tr(\omega_2) \in \{0,1\}$ and $mg(\omega_3/tr) \in \{0,1\}$
    - Anticipate $\overline{U}_{\mathcal{Y}}^{\theta_f \theta_m}(\overline{U}_{\mathcal{X}}^{\theta_f \theta_m}) \quad \rightarrow \quad$ value of any potential household

- Male θ_m partner-choice problem is to choose the type $\theta_f \cup \emptyset$ that maximizes:

$$\max\left\{ \underbrace{\overline{U}_{\mathcal{Y}}^{\emptyset \theta_m} + \beta_m^{\emptyset \theta_m}}_{single}, \underbrace{\{\overline{U}_{\mathcal{Y}}^{\theta_f \theta_m} + \beta_m^{\theta_f \theta_m}\}}_{marry\ \theta_f} \right\}$$

- Competitive equilibrium in the MM pins down outputs:
  - MM matching function $\mu(\theta) \quad \rightarrow \quad$ who marries whom,
  - Indirect Expected Utilities $(\overline{U}_{\mathcal{X}}^{\theta_f \theta_m}, \overline{U}_{\mathcal{Y}}^{\theta_f \theta_m}) \quad \rightarrow \quad$ why.

Back

Market Human Capital Evolves over Time

- Initial human capital depends on ambition type:

$$\eta_1(\theta) \sim F(\mu_\theta, \sigma) \quad \forall \theta \in \Theta$$

- *Beginning-of-t* η_t depends on past LS, ambition type, and ladder:

$$\eta_{it} = [\eta_{t-1} + \alpha_{L,\theta} + \delta^S_{L,\theta} \mathbb{1}_{\{I_{t-1}=S\}} - \delta^P_{L,\theta} \mathbb{1}_{\{I_{t-1}=PT\}} - \delta^N_{L,\theta} \mathbb{1}_{\{I_{t-1}=NP\}}]\tau$$

- τ: Training boost in human capital reaped at the beginning of $t = 3$:

$$\tau \begin{cases} = 1 & \text{if } t = \{0, 1, 2\} \\ > 1 & \text{if } t = 3 \ \& \ tr = 1 \end{cases}$$

- Evolves due to family's and firm's investments:
  - Returns to experience; skill-depreciation when out-of-work.
  - L choice matters: L_1 rewards experience less but penalizes time-out-of-work less.
  - Training boosts human capital.

Back

Family Human Capital Evolves over Time

- Initial random family shock common to both spouses.
- Women have an initial advantage:

$$\phi_{i1} = \begin{cases} \bar{\phi}\kappa & \text{if } i = f \\ \bar{\phi} & \text{if } i = m \end{cases}$$

- Skills at home depreciate (relative to private goods):

$$\phi_{it} = \phi_{it-1}\gamma$$

  where $\gamma > 0$.

- Initial advantage of women persists over time.
- Marriage Market matching also matters...

Back

Families' problem

- Households (θ_f, θ_m) that formed in the MM take as given:
  - Matching in the MM, $\mu(\theta)$, and women's utility prices, $\overline{U}_{\mathcal{X}}^{\theta_f \theta_m}$;
  - wage rates, $w(\omega_i)$;
  - firm's training and promotion policies, $tr(\omega_2) \in \{0,1\}$ and $mg(\omega_3/tr) \in \{0,1\}$
- choose a contingent contract of career trajectories, fertility, and consumption,

$$x(\varphi) = \Big\{ \underbrace{L_f(\varphi_t), L_m(\varphi_t), \mathcal{F}_t(\varphi_t), l_{ft}(\varphi_t), l_{mt}(\varphi_t), c_{ft}(\varphi_t), c_{mt}(\varphi_t), c_{Qt}(\varphi_t)}_{x_t(\varphi_t)} \Big\}_{t=1}^{3}$$

so as to solve their collective life cycle problem

$$\begin{aligned} \overline{U}_{\mathcal{Y}}^{\theta_f \theta_m} = \max_{x(\varphi)} \quad & E_0 \sum_{t=1}^{T=3} \delta^{t-1} \Big\{ u_m(x_t(\varphi_t)) \Big\} \\ s.t. \quad & E_0 \sum_{t=1}^{T=3} \delta^{t-1} \Big\{ u_f(x_t(\varphi_t)) \Big\} \geq \overline{U}_{\mathcal{X}}^{\theta_f \theta_m} \\ \forall \varphi_t, t > 0 : \quad & c_{ft} + c_{mt} + c_{Qt} = w_{ft}(\omega_{ft}) l_{ft} + w_{mt}(\omega_{mt}) l_{mt} \end{aligned}$$

- Optimal household behavior determines distribution of worker types, $\{\omega_{it}\}$.