# Joint Directed Search

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# Introduction

- Labor income risk affects workers' consumption and wealth because of incomplete asset markets.

- Role for optimal UI and tax policy.

- Self-insurance through wealth accumulation and job search (e.g., directed search).

- Households insurance: motive for marriage, joint search, added worker effects.

- Some (but not all) policies condition on marital status (UI system, means testing, joint/individual taxation). Why?

- The literatures on wealth/inequality/policy (macro) and households' joint labor supply decisions (structural labor) are largely disjoint.

# How to make progress?

- Our starting point is Chaumont & Shi (JME, 2022)

  - Directed search equilibrium, on-the-job search, risk-averse workers, assets.

  - Search outcomes affect earnings and wealth accumulation.

  - Wealth and earnings affect search by changing the optimal trade-off between wage and matching probability.

  - Endogenous labor demand, firms condition wage offer on applicant's wealth.

  - Self-insurance through the interaction of wealth and directed search.

  - Limited role of unemployment insurance.

# How to make progress?

- Idea: bring in the household dimension.

  - Labor market status of the spouse affects the job-search strategy (Guler et al., 2012).

  - Firms cater to this by offering wages conditional on type of spouse (and assets).

    - Workers with employed spouse (married women?) behave as if high asset value.
      - $\rightarrow$ searches for high wage offers if employed so less likely to leave through EE.
      - $\rightarrow$ Higher present value of the job for the firm.
      - $\rightarrow$ High tightness and high job-finding rate.
      - $\rightarrow$ Low matching probability for the firm, only enters if wage low.
    - Workers with unemployed spouse (married men?) more likely to leave through EE.
      - $\rightarrow$ Lower present value of the job for the firm.
      - $\rightarrow$ Lower tightness and low job-finding rate, higher matching prob for firm, higher wages.

  - Equilibrium feedback from the firm side seems to reinforce marital wage premia.

  - Endogenous quits? Interaction with wealth? Interaction with joint taxation?

# Model: basic assumptions

- Continuous time, discount rate $\rho$.

- Directed search.

- On-the-job search.

- Singles and couples. Abstract from Marriage and Divorce.

- (Un)employed workers search with intensity $(\lambda_u)\ \lambda_e$.

- If a worker searches in a sub-market with tightness $\theta$, they meet a firm at rate $p(\theta)$ while a vacancy meets a worker at rate $q(\theta)$.

- The target wages in submarkets towards which workers direct their search is $\hat{w}$.

- All matches are equally productive and produce flow output $z$.

- For now there's no ex-ante heterogeneity (e.g., gender).

# Model: risk and insurance

- Workers are risk-averse:
    - Singles consuming $c$ have utility flow $u(c)$.
    - A unitary household with two members each consuming $c$ has flow utility $2u(c/2)$ (Guler et al., 2012).

- UI benefits $b$. Levels or replacement rate. Means testing.

- Hand-to-Mouth Workers: abstract from consumption-savings choice and asset accumulation for now.

- Proportionate tax rate $\tau$ to finance unemployment benefits: $w = (1-\tau)w^*$ where $w^*$ is the gross wage. Wasteful government spending $G$ (introduced to calibrate a sensible $\tau$).

## Model: singles

The HJB equations for singles read:

$$\rho v^u(b) = \max_{\hat{w}} u(b) + p(\theta(\hat{w})) \left( v^e(\hat{w}) - v^u(b) \right),$$

$$\rho v^e(w) = \max_{\hat{w}} u(w) + \lambda_e p(\theta(\hat{w})) \left( v^e(\hat{w}) - v^e(w) \right) + \delta \left( v^u(b) - v^e(w) \right).$$

The corresponding firm-value HJB, given worker's optimal search strategy $\hat{w}$ is straightforward in this case:

$$\rho J(w) = z - w/(1 - \tau) - (\delta + \lambda_e p(\theta(\hat{w}))) J(w) \implies J(w) = \frac{z - \frac{w}{1-\tau}}{\rho + \delta + \lambda_e p(\theta(\hat{w}))}.$$

## Model: couples uu

In case of both members unemployed, the HJB reads:

$$\rho v^{uu}(b,b) = \max_{\hat{w}_1,\hat{w}_2} 2u(b) + p\left(\theta\left(\hat{w}_1,b\right)\right)\left(v^{eu}\left(\hat{w}_1,b\right) - v^{uu}(b,b)\right)$$
$$+ p\left(\theta\left(b,\hat{w}_2\right)\right)\left(v^{eu}\left(\hat{w}_2,b\right) - v^{uu}(b,b)\right).$$

# Model: couples eu

- When only one member is unemployed (without e.g., gender heterogeneity, there is no need to have two separate value functions $v^{eu}$ and $v^{ue}$), we have:

$$\rho v^{eu}(w,b) = \max_{\hat{w}_1,\hat{w}_2} 2u((w+b)/2) + \lambda_e p\left(\theta\left(\hat{w}_1,b\right)\right)\left(v^{eu}\left(\hat{w}_1,b\right) - v^{eu}(w,b)\right)$$
$$+ p\left(\theta\left(b,\hat{w}_2,\right)\right)\left(v^{ee}\left(w,\hat{w}_2\right) - v^{eu}(w,b)\right) + \delta\left(v^{uu}(b,b) - v^{eu}(w,b)\right),$$

- With means testing, benefits may adjusts to spousal's earnings. In that case, $v^{eu}(w,b(w))$ and need to keep track of $b(w)$ as $w$ changes.

- Simpler but perhaps unrealistic: $b$ is determined at the time of layoff.

$$\rho v^{eu}(w,b) = \max_{\hat{w}_1,\hat{w}_2} \quad 2u((w+b)/2) + \lambda_e p\left(\theta\left(\hat{w}_1,b\right)\right)\left(v^{eu}\left(\hat{w}_1,b\right) - v^{eu}(w,b)\right)$$
$$+ p\left(\theta\left(b,\hat{w}_2,\right)\right)\left(v^{ee}\left(w,\hat{w}_2\right)\right.$$
$$\left. - v^{eu}(w,b)\right) + \delta\left(v^{uu}(b(w),b) - v^{eu}(w,b)\right)$$

## Model: couples ee

For a household with both members employed:

$$\rho v^{ee}(w_1, w_2) = \max_{\hat{w}_1, \hat{w}_2} 2u((w_1 + w_2)/2) + \lambda_e p(\theta(\hat{w}_1, w_2))(v^{ee}(\hat{w}_1, w_2) - v^{ee}(w_1, w_2))$$
$$+ \lambda_e p(\theta(w_1, \hat{w}_2,))(v^{ee}(w_1, \hat{w}_2) - v^{ee}(w_1, w_2)) + \delta(v^{eu}(w_1, b) + v^{eu}(w_2, b) - 2v^{ee}(w_1, w_2)).$$

# Model: value of filled job with couples

- To write down the value of a filled job with a worker from a 2-member household, we need to keep track of what the other worker is doing.
- Convention: the first member of the household is the employee of the firm.

$$(\rho + \delta + \lambda_e p(\hat{w}_1)) J^u(w_1, b) = z - w_1/(1-\tau) + p(\theta(\hat{w}_2))(J^e(w_1, \hat{w}_2) - J^u(w_1, b))$$

$$(\rho + \delta + \lambda_e p(\hat{w}_1)) J^e(w_1, w_2) = z - w_1/(1-\tau) + \lambda_e p(\theta(\hat{w}_2))(J^e(w_1, \hat{w}_2) - J^e(w_1, w_2)).$$

# Model: government budget

- Share of singles $S$. There are $(1-S)/2$ couples (or, equivalently, $1-S$ workers in two-member households).

- The distribution of singles over (gross) wages is $f^{\mathcal{S}}(w^*)$ and the distribution of workers in two-member households over (gross) wages is $f^{\mathcal{C}}(w^*)$.

- There are $u^s$ unemployed singles and $u^c$ unemployed in 2-member households so the overall unemployment rate is $u = u^s + u^c$. The government budget constraint is:

$$bu + G = \tau \left[ S\left(1 - u^s\right) \int f^{\mathcal{S}}\left(w^*\right) w^* dw^* + (1-S)\left(1 - u^c\right) \int f^{\mathcal{C}}\left(w^*\right) w^* dw^* \right].$$

- Closing the model: market tightnesses are pinned down by free-entry given flow cost of maintaining an open vacancy $k$.

# Model: means testing

- Share of singles $S$. There are $(1-S)/2$ couples (or, equivalently, $1-S$ workers in two-member households).

- The distribution of singles over (gross) wages is $f^{\mathcal{S}}(w^*)$ and the distribution of workers in two-member households over (gross) wages is $f^{\mathcal{C}}(w^*)$.

- There are $u^s$ unemployed singles and $u^c$ unemployed in 2-member households so the overall unemployment rate is $u = u^s + u^c$. The government budget constraint is:

$$bu + G = \tau \left[ S(1-u^s) \int f^{\mathcal{S}}(w^*) w^* dw^* + (1-S)(1-u^c) \int f^{\mathcal{C}}(w^*) w^* dw^* \right].$$

- Closing the model: market tightnesses are pinned down by free-entry given flow cost of maintaining an open vacancy $k$.

# Model: wage dispersion

- This class of models struggles to generate wage dispersion (Hornstein et al., 2011).

- One solution is to increase risk aversion (e.g., CRRA utility with relative risk aversion of 5 is not unheard of).

- Adding UI expiry as in Chaumont & Shi and incomplete markets will help as well because of precautionary motive in job search.

- Joint search of couples gives us an additional boost.

# Some Data

Table: Labor Market Transition Rates of Singles

|            | 93–97 | 98–02 | 03–07 | 08–12 | 13–17 |
|------------|-------|-------|-------|-------|-------|
| T_sie_sie_m | 0.131 | 0.152 | 0.119 | 0.131 | 0.138 |
| T_sie_siu   | 0.094 | 0.069 | 0.074 | 0.073 | 0.055 |
| T_siu_sie   | 0.308 | 0.302 | 0.233 | 0.240 | 0.243 |
| T_sje_sje_f | 0.131 | 0.141 | 0.120 | 0.139 | 0.151 |
| T_sje_sju   | 0.086 | 0.065 | 0.077 | 0.060 | 0.055 |
| T_sju_sje   | 0.168 | 0.215 | 0.178 | 0.211 | 0.220 |

## Table: Labor Market Transition Rates of Couples

|                 | 93–97 | 98–02 | 03–07 | 08–12 | 13–17 |
|-----------------|-------|-------|-------|-------|-------|
| T_mieje_mieje_f | 0.043 | 0.040 | 0.031 | 0.036 | 0.040 |
| T_mieje_mieje_m | 0.035 | 0.034 | 0.027 | 0.030 | 0.034 |
| T_mieje_mieju   | 0.106 | 0.104 | 0.101 | 0.089 | 0.077 |
| T_mieje_miuje   | 0.036 | 0.029 | 0.029 | 0.025 | 0.019 |
| T_mieju_mieje   | 0.107 | 0.142 | 0.135 | 0.147 | 0.179 |
| T_mieju_mieju_m | 0.032 | 0.030 | 0.023 | 0.030 | 0.033 |
| T_mieju_miuju   | 0.043 | 0.041 | 0.032 | 0.029 | 0.020 |
| T_miuje_mieje   | 0.143 | 0.150 | 0.159 | 0.122 | 0.154 |
| T_miuje_miuje_f | 0.021 | 0.024 | 0.018 | 0.031 | 0.030 |
| T_miuje_miuju   | 0.118 | 0.090 | 0.091 | 0.078 | 0.053 |
| T_miuju_mieje   | 0.026 | 0.023 | 0.014 | 0.026 | 0.138 |
| T_miuju_mieju   | 0.093 | 0.129 | 0.101 | 0.121 | 0.208 |
| T_miuju_miuje   | 0.052 | 0.053 | 0.052 | 0.057 | 0.045 |

**Table:** Wage Distribution Moments by Marital Status, Men

|            | 93–97  | 98–02  | 03–07  | 08–12  | 13–17  |
|------------|--------|--------|--------|--------|--------|
| w_p10_m    | 7.663  | 7.845  | 7.498  | 6.981  | 7.319  |
| w_p50_m    | 15.586 | 16.747 | 18.098 | 16.781 | 16.826 |
| w_p90_m    | 26.995 | 31.273 | 35.963 | 33.888 | 34.752 |
| w_p10_m_se | 5.248  | 4.926  | 4.285  | 4.230  | 4.063  |
| w_p50_m_se | 13.382 | 13.986 | 13.456 | 13.047 | 12.326 |
| w_p90_m_se | 23.115 | 27.047 | 27.077 | 27.224 | 26.696 |
| w_p10_m_mel| 8.378  | 9.196  | 9.704  | 8.672  | 9.094  |
| w_p50_m_mel| 16.293 | 17.740 | 19.826 | 18.198 | 18.332 |
| w_p90_m_mel| 28.022 | 32.519 | 38.446 | 36.022 | 36.766 |

Table: Wage Distribution Moments by Marital Status, Women

|              | 93–97  | 98–02  | 03–07  | 08–12  | 13–17  |
|--------------|--------|--------|--------|--------|--------|
| w_p10_f      | 5.558  | 5.042  | 5.358  | 5.224  | 5.630  |
| w_p50_f      | 12.405 | 13.241 | 14.168 | 13.331 | 13.425 |
| w_p90_f      | 20.685 | 23.355 | 26.122 | 25.262 | 25.853 |
| w_p10_f_se   | 4.917  | 4.535  | 4.139  | 4.552  | 4.917  |
| w_p50_f_se   | 12.241 | 12.596 | 12.592 | 12.388 | 12.495 |
| w_p90_f_se   | 20.294 | 22.668 | 23.941 | 23.555 | 24.135 |
| w_p10_f_mle  | 5.779  | 5.335  | 6.088  | 5.790  | 6.189  |
| w_p50_f_mle  | 12.466 | 13.543 | 14.799 | 14.000 | 14.034 |
| w_p90_f_mle  | 20.807 | 23.725 | 27.065 | 26.306 | 26.922 |

# Conclusions

- None so far.

**Thank you for your attention.**

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# Some Literature

- Unemployment, especially male unemployment, is associated with an increase in the divorce rate (e.g. Jensen and Smith, 1990; Hansen, 2005; Amato and Beattie, 2011).

- Marriage/divorce rates negatively correlated with unemployment over the business cycle (e.g. Schaller, 2013; González-Val and Marcén, 2017a/b).

- Does female labor market participation decrease or increase marital stability? (Newman and Olivetti, 2018 vs. Folke and Rickne, 2020).

- Marriage market matching models (with and without frictions, TU/NTU): Becker (1973/74), Burdett & Coles (1997), Shimer & Smith (2000), Jacquemet & Robin (2012), Choo & Siow (2006), Choo (2015), Chiappori et al. (2015).

- Joint search: Guler et al. (2012), Pilossoph & Wee (2021), Fang & Shephard (2019).

- Most related: Goussé et al. (2017), Greenwood et al. (2016), Flabbi & Flinn (2015)

Go back