Stochastic Mortality in R

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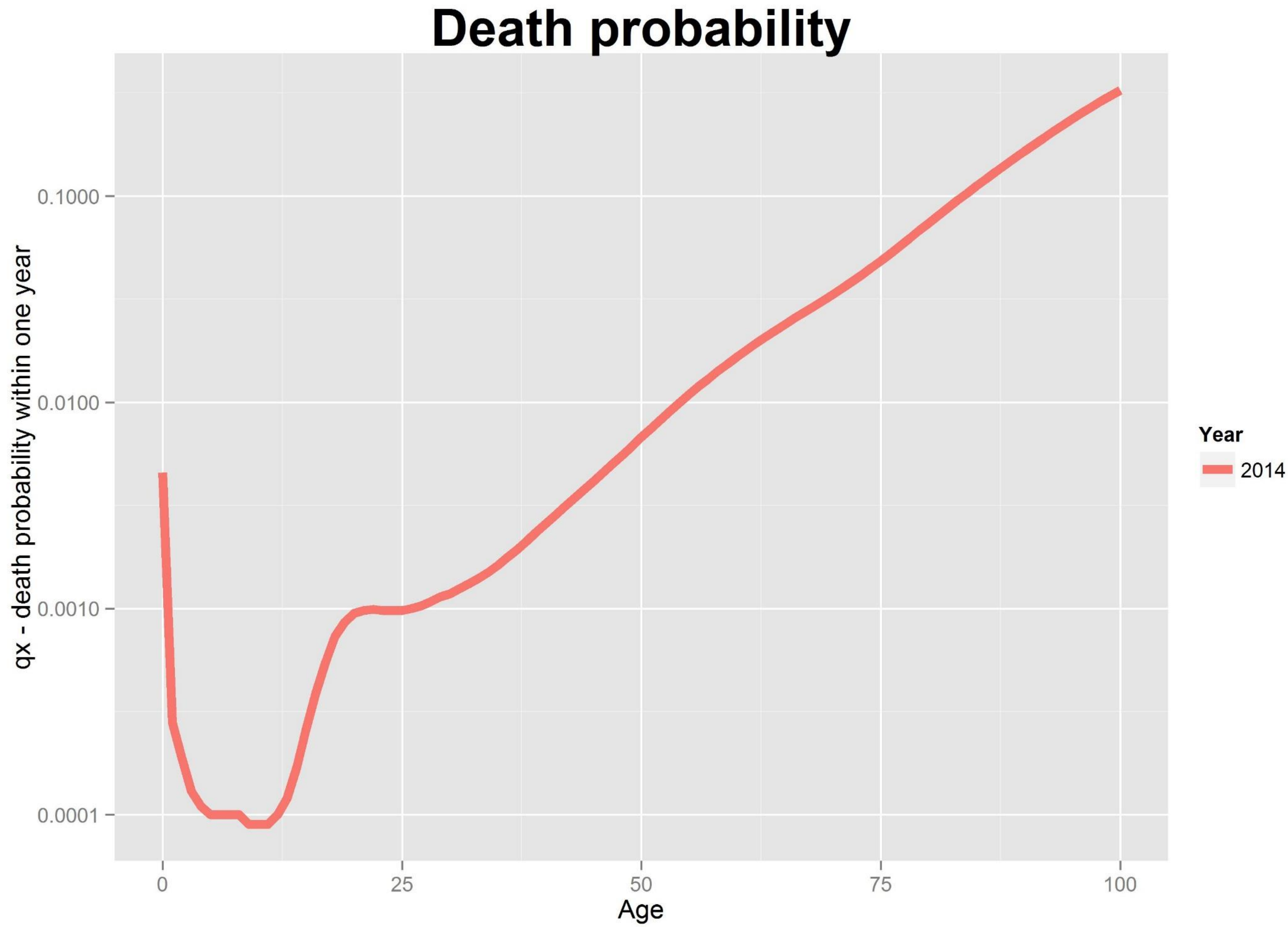

Death probability
qx - death probability within one year
0.1000
0.0100
0.0010
0.0001
0
25
50
75
100
Age
Year
2014

Death probability

Based on data on number of deaths and size of a population we are able to calculate death probability. We will focus on conditional death probability within one year (qx)

Factors that we will take into consideration are as fallow:

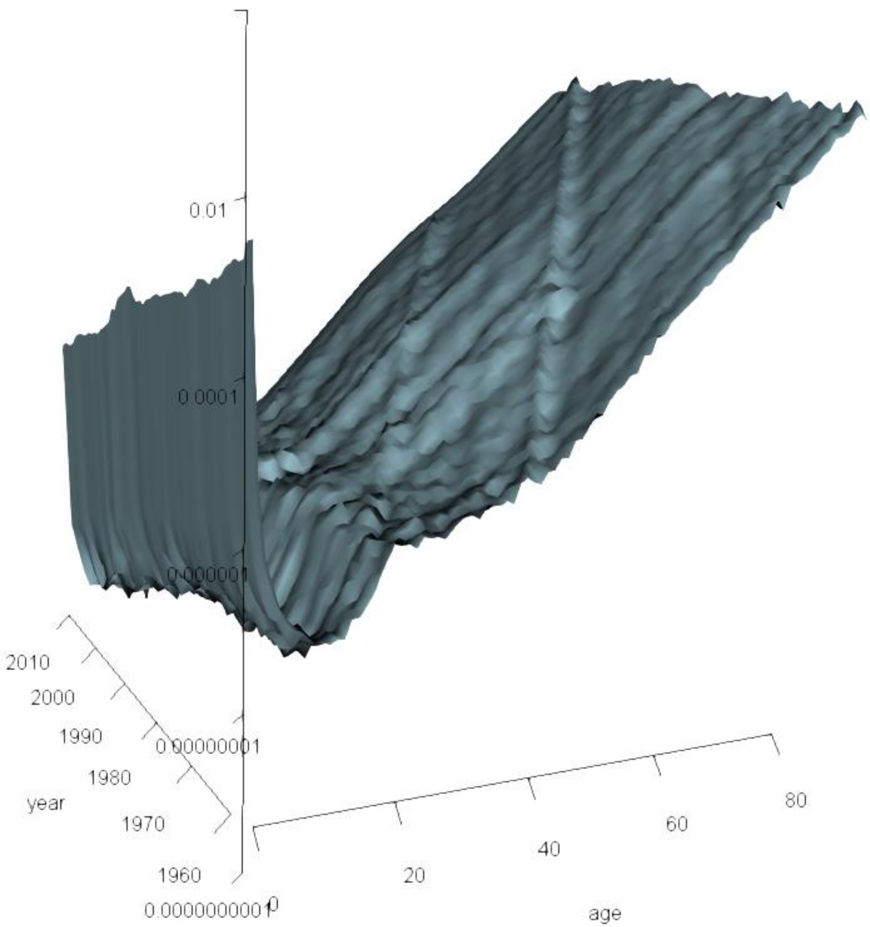
- Gender
- Age
- Calendar year
- Cohort (year of birth)

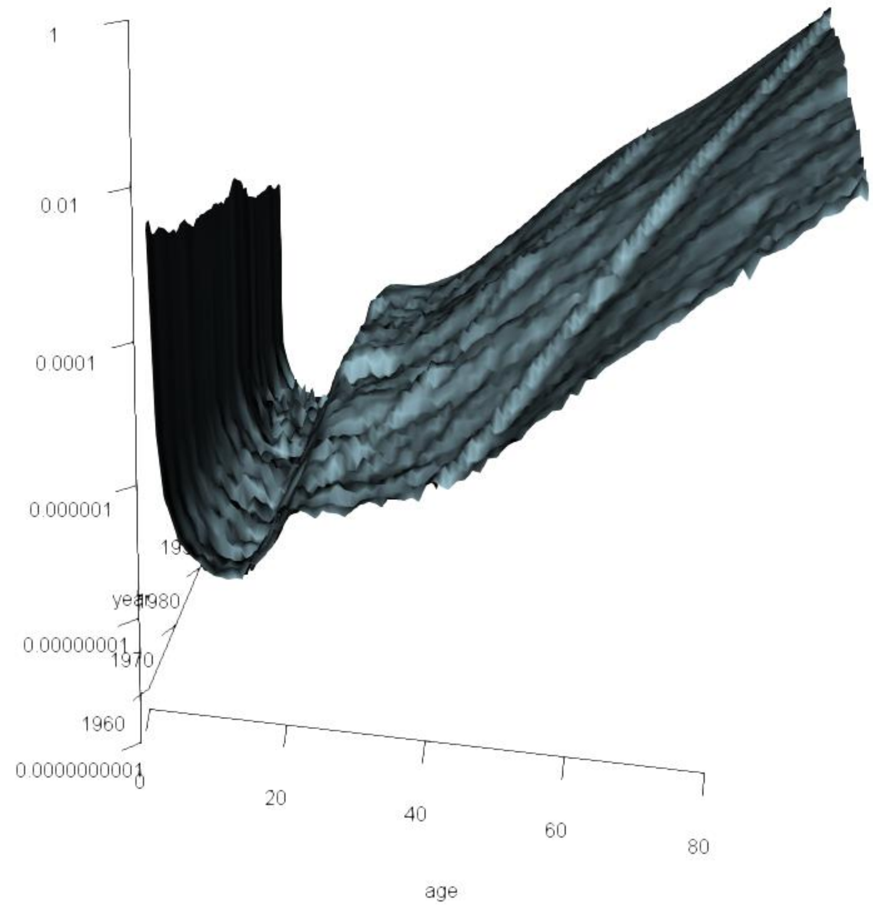
Mortality tables

Framework that is used to capture this conditional death probability is called mortality table.

Age\Year	2011	2012	2013	2014
50	0.81%	0.78%	0.73%	0.68%
51	0.89%	0.86%	0.81%	0.75%
52	0.97%	0.94%	0.89%	0.83%
53	1.07%	1.03%	0.98%	0.91%
54	1.16%	1.12%	1.07%	1.00%
55	1.26%	1.22%	1.17%	1.09%
56	1.36%	1.33%	1.28%	1.20%
57	1.47%	1.44%	1.39%	1.30%
58	1.59%	1.55%	1.51%	1.42%
59	1.71%	1.68%	1.64%	1.54%
60	1.84%	1.81%	1.77%	1.66%

Visualization of mortality tables





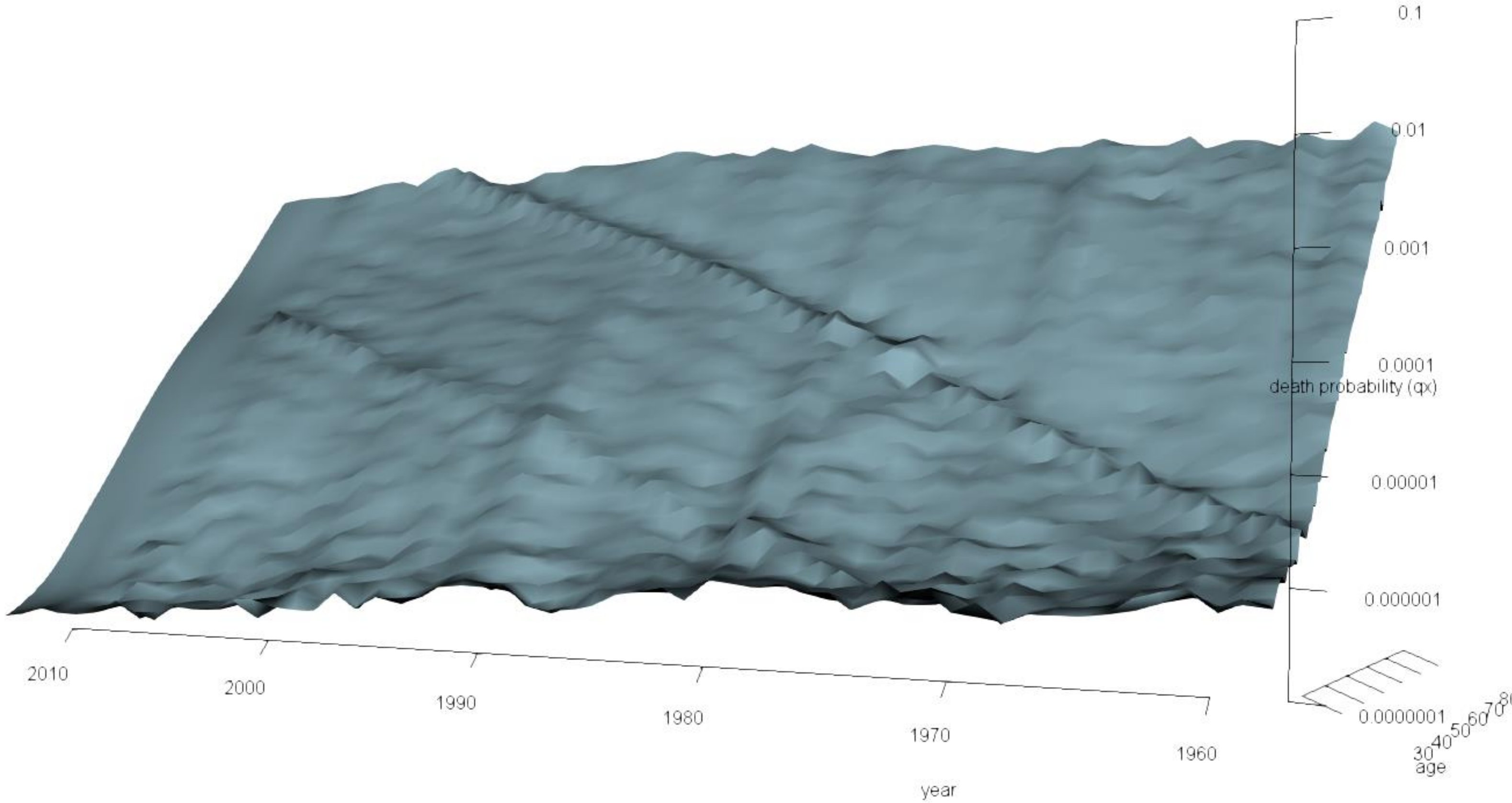
Data restictions

a) Male population of Poland was selected

b) Ages from 25 to 85 were selected:

- Data for ages below 25 express different dependency on age
- Data for ages above 85 is not of sufficent quality

Visualization of mortality tables with restrictions



Data source

Human Mortality Database (1958-2009):

http://www.mortality.org

Central Statistical Office of Poland (1990-2014):

http://stat.gov.pl/en/topics/population/life-expectancy/life-expectancy-in-poland,1,1.html

Stochastic mortality models

Stochastic mortality in the literature is mainly identified with Lee Carter model and its extensions.

As stated in [2] they could be framed into Generalized Age-Period-Cohort stochastic mortality models (GAPC).

Generalized Age-Period-Cohort stochastic mortality models

In their definition they are close to widely used generalized linear models (GLM). As in GLM one needs to assume what distribution modelled risk follow.

For GAPC number of deaths D_{xt} is treated as modelled phenomena.

Since number of deaths is an integer one could use:

a) Binomial distribution (with logit link function)
b) Poisson distribution (log link function)

Stochastic mortality models

Linear formula that is then imputed into link function in its general form:

$$\eta_{xt} = \alpha_x + \sum_{i=1}^{N} B_x^{(i)} \kappa_t^{(i)} + B_x^{(0)} \gamma_{t-x}$$

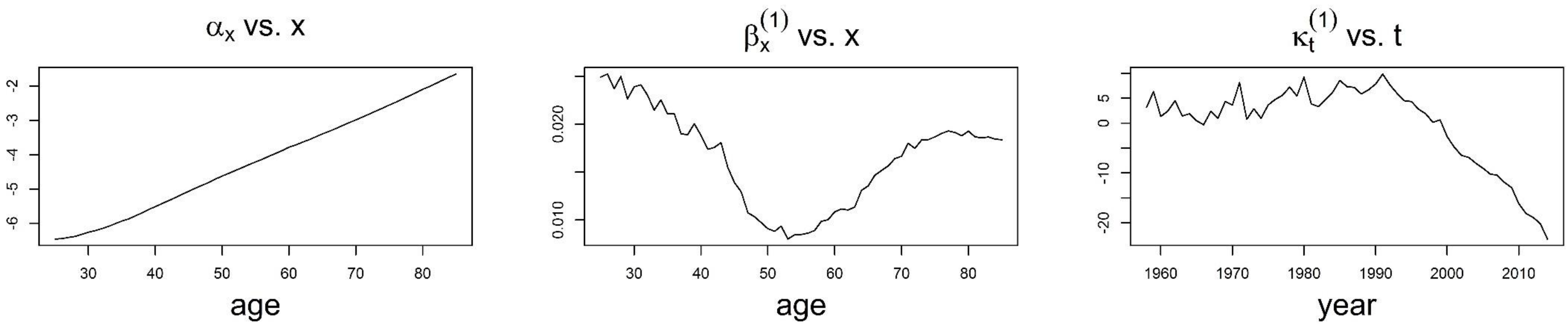
- $\alpha_x$stands for static age function capturing the general shape of mortality by age
- N ≥ 0 is an integer indicating the number of age-period terms describing the mortality trends, with each time index $\boldsymbol{\kappa_t^{(i)}}$, i = 1,…,N, contributing in specifying the mortality trend and $\boldsymbol{B_x^{(i)}}$ modulating its effect across ages.
- The term γ_{t-x} accounts for the cohort effect with $B_x^{(0)}$modulating its effect across ages.

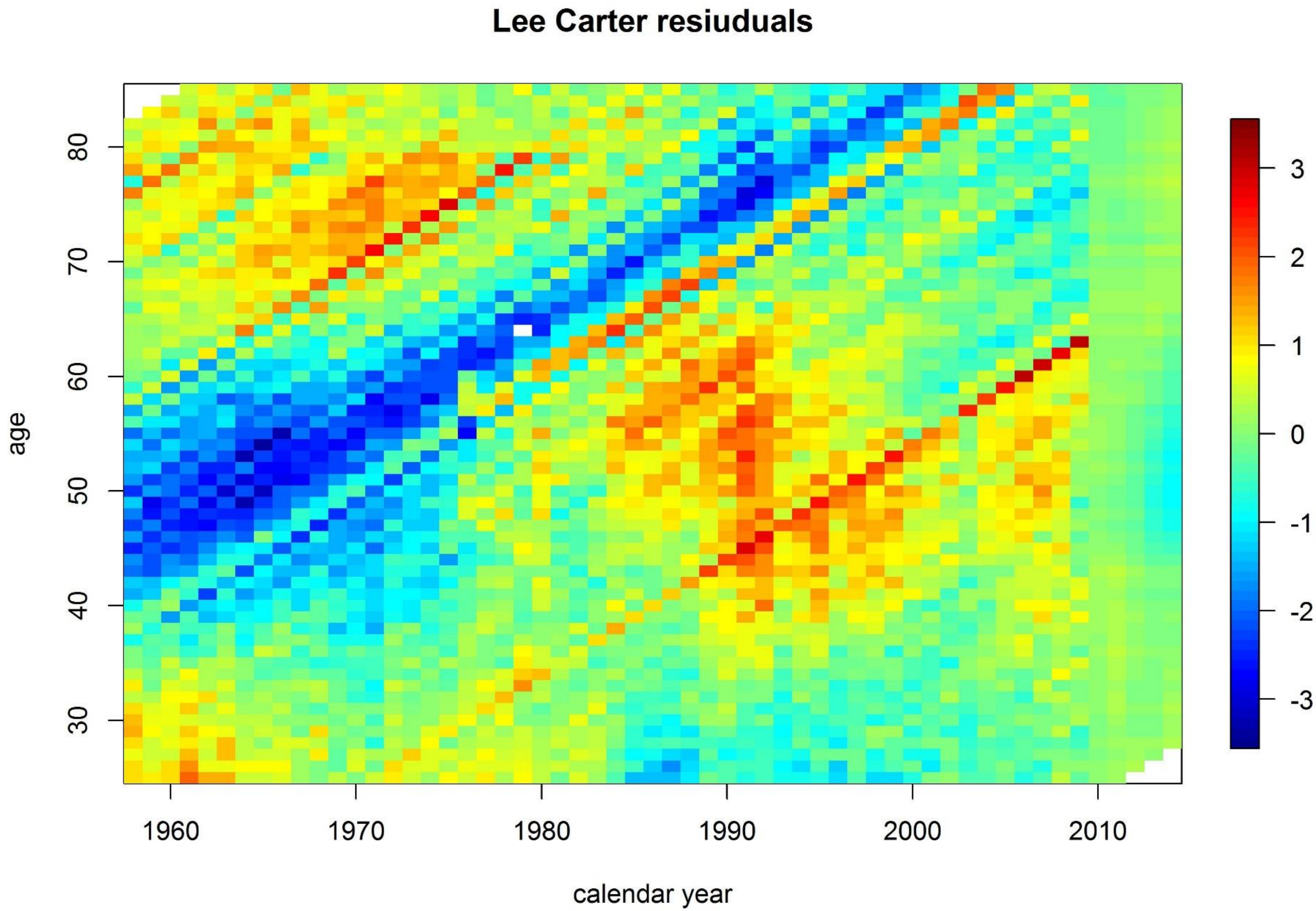
Stochastic mortality models – Lee Carter

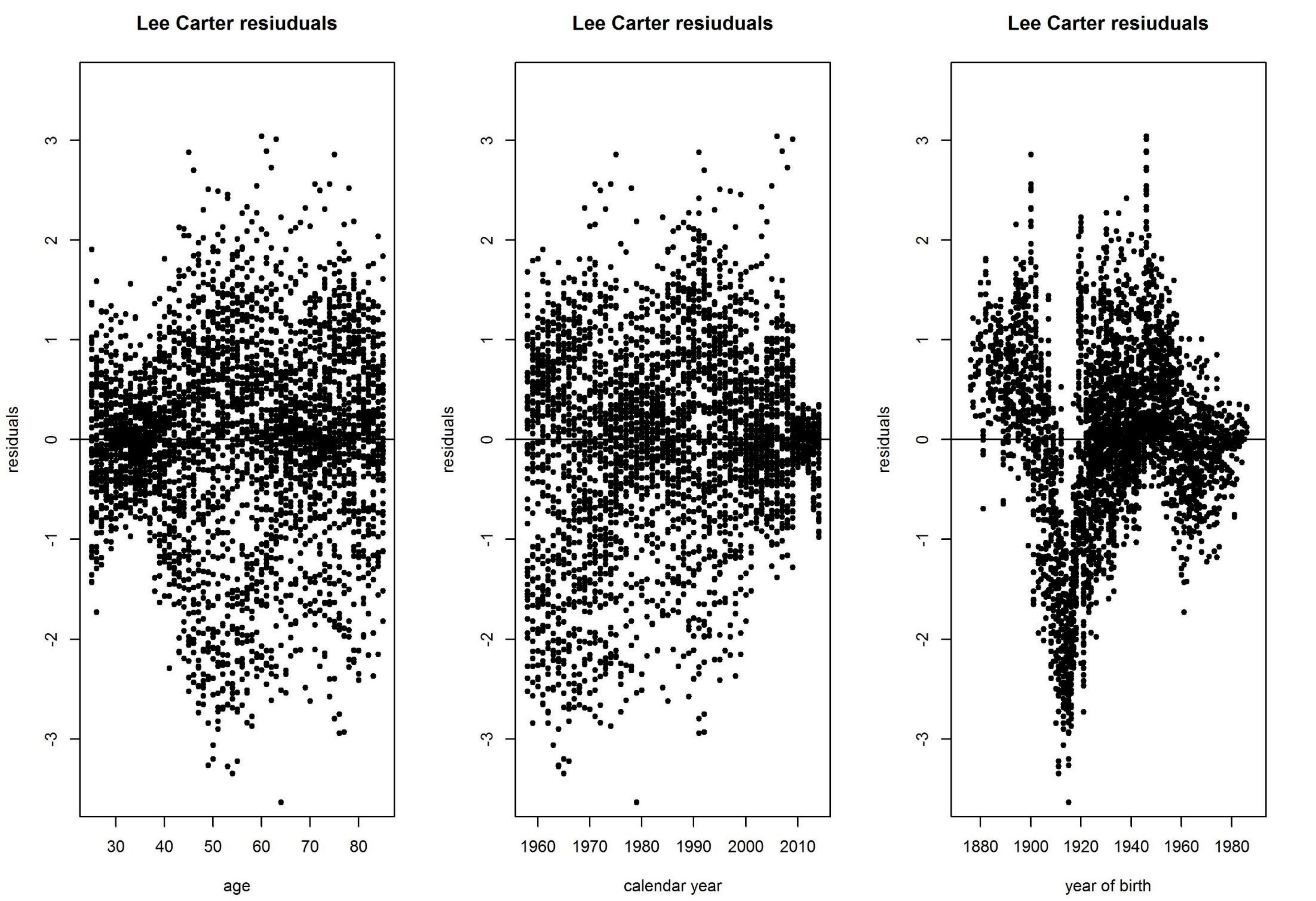
It is good to start with simplest and most popular model in this class defined under same notation. It could provide intuitive understanding of PLAT model for those already familiar with Lee-Carter.

Lee-Carter model is defined as:

$$\eta_{xt} = \alpha_x + B_x^{(1)} \kappa_t^{(1)}$$



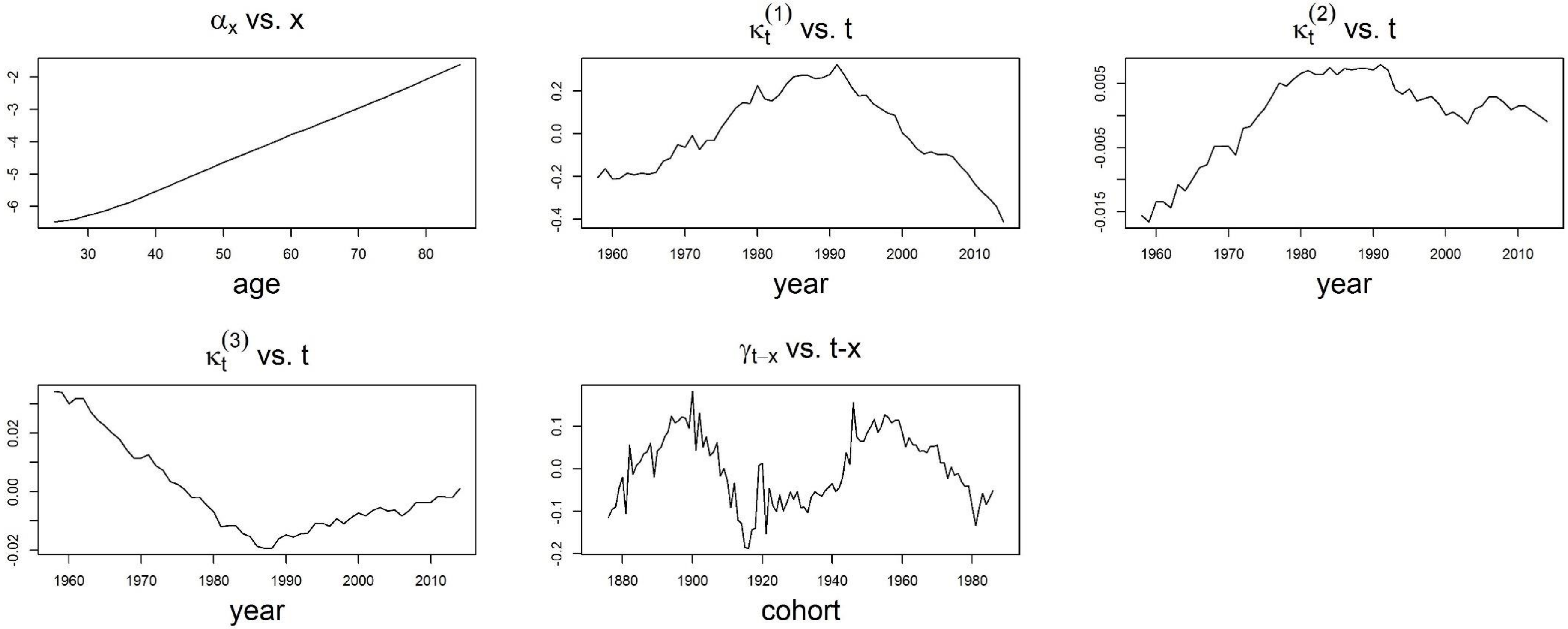

Lee Carter resiuduals
age
80
70
60
50
40
30
1960
1970
1980
1990
2000
2010
calendar year
3
2
1
0
-1
-2
-3

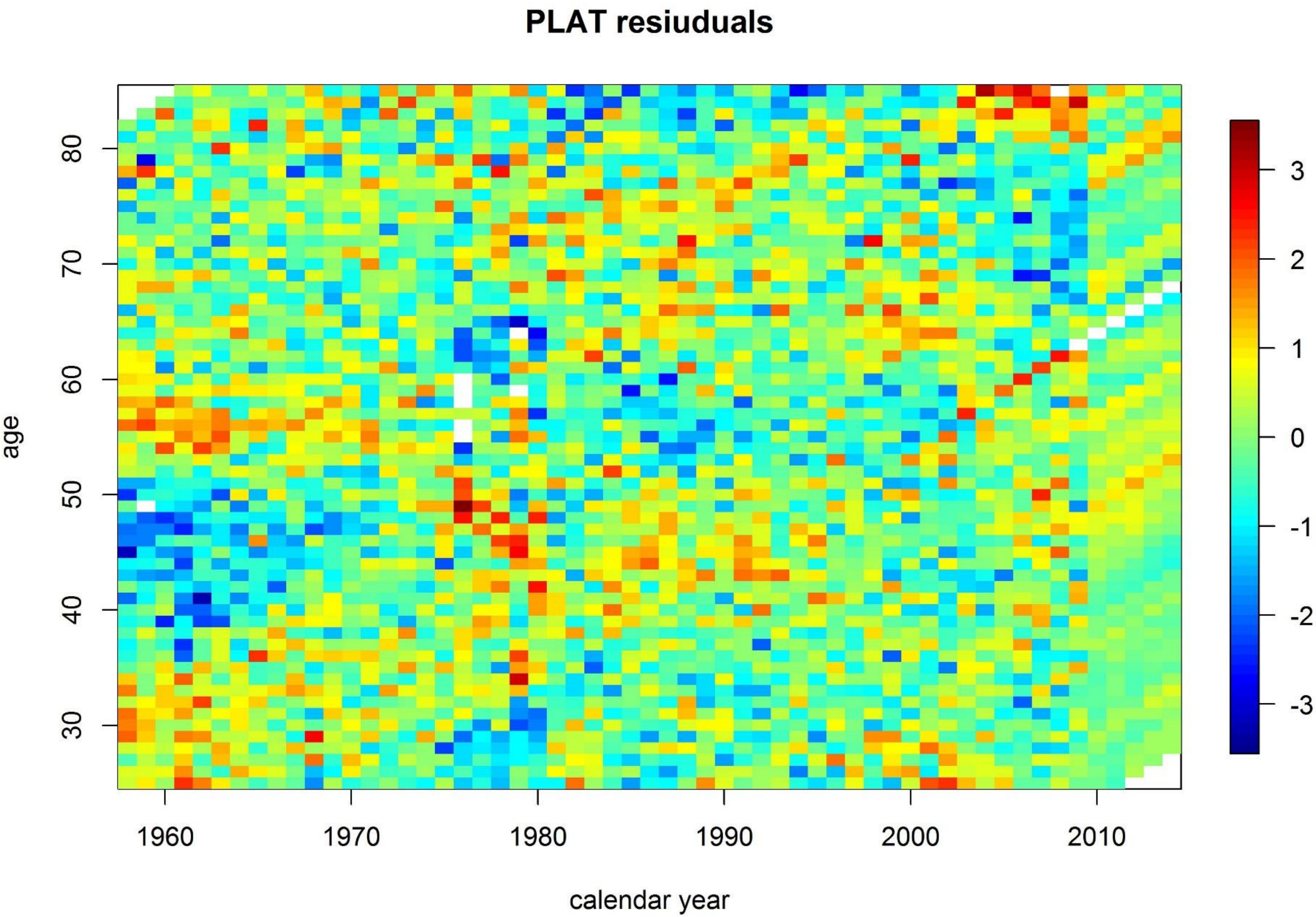

Lee Carter resiuduals
Lee Carter resiuduals
Lee Carter resiuduals
residuals
residuals
residuals
-3
-2
-1
0
1
2
3
30
40
50
60
70
80
age
1960
1970
1980
1990
2000
2010
calendar year
1880
1900
1920
1940
1960
1980
year of birth

Stochastic mortality models – PLAT

PLAT model is much more complex and defined as:

$$\eta_{xt} = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + (\bar{x} - x)^{+}\kappa_t^{(3)} + \gamma_{t-x}$$




PLAT resiuduals
age
calendar year
80
70
60
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1960
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1990
2000
2010
3
2
1
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-1
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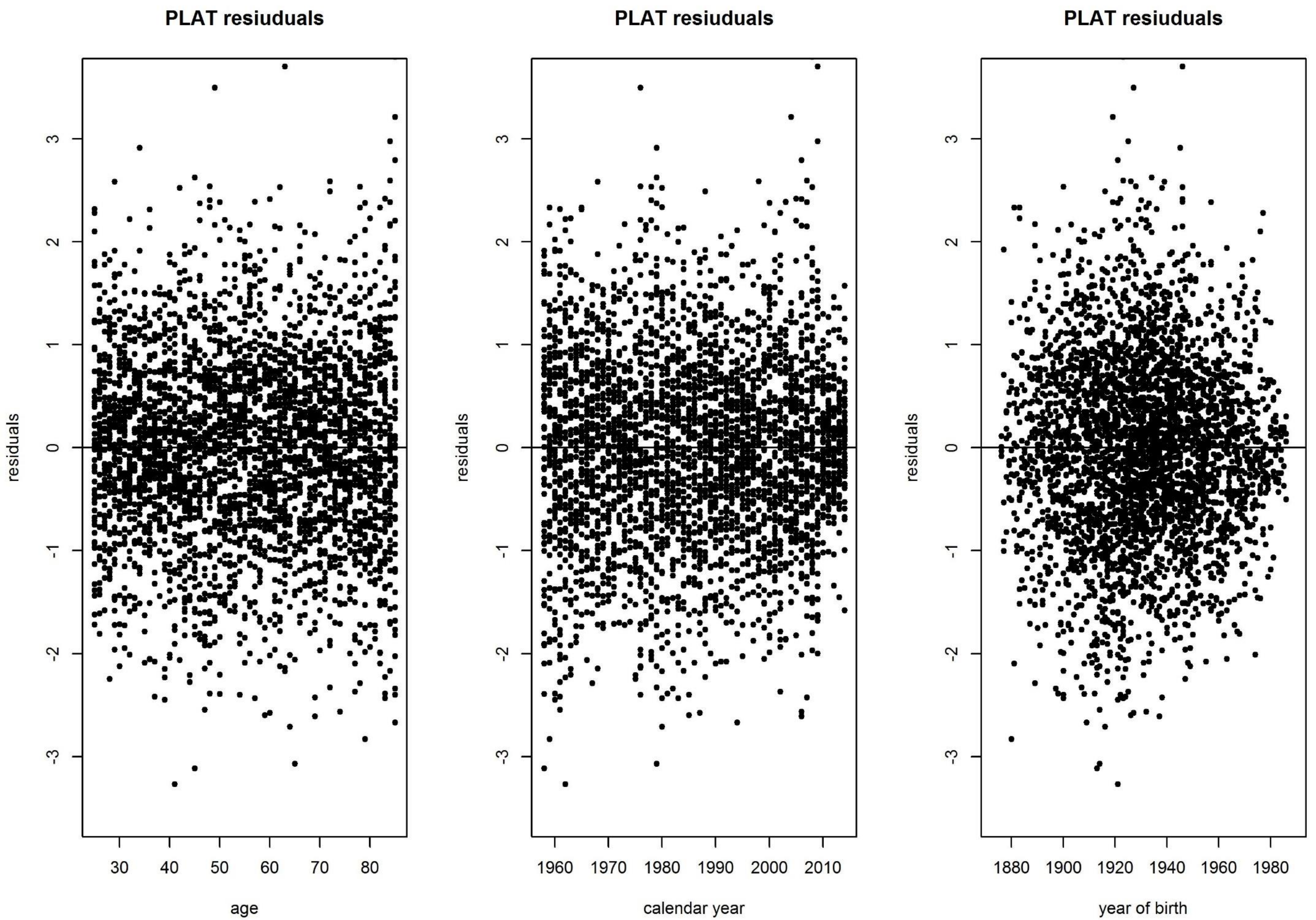
-3



PLAT resiuduals
PLAT resiuduals
PLAT resiuduals
residuals
residuals
residuals
3
2
1
0
-1
-2
-3
30
40
50
60
70
80
age
1960
1970
1980
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2010
calendar year
1880
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year of birth

Stochastic mortality models

In general steps that needs to be applied when working with those models are as follow:

- Fit model to data
- Assess goodness of fit
- Perform forecast (it could be perceives just as forecast of kappa and gamma parameters development over time)

Forecast

Stochastic paths are generated by analytical transformation of fixed and forecasted parameters. Whole mortality projection could be reduced to the issue of forecast of kappa and gamma parameters.

It is assumed that number of deaths D_{xt} follow binomial distribution

$$D_{xt} \sim Binomial(E_{xt}^0, q_{xt})$$

Under that assumption q_{xt} could be calculated from analytical formula (logit link):

$$q_{xt} = \frac{e^{\eta_{xt}}}{(1 + e^{\eta_{xt}})}$$

Where $\eta_{xt} = \alpha_x + \kappa_t^{(1)} + (\bar{x} - x)\kappa_t^{(2)} + (\bar{x} - x)^+\kappa_t^{(3)} + \gamma_{t-x}$ for PLAT model

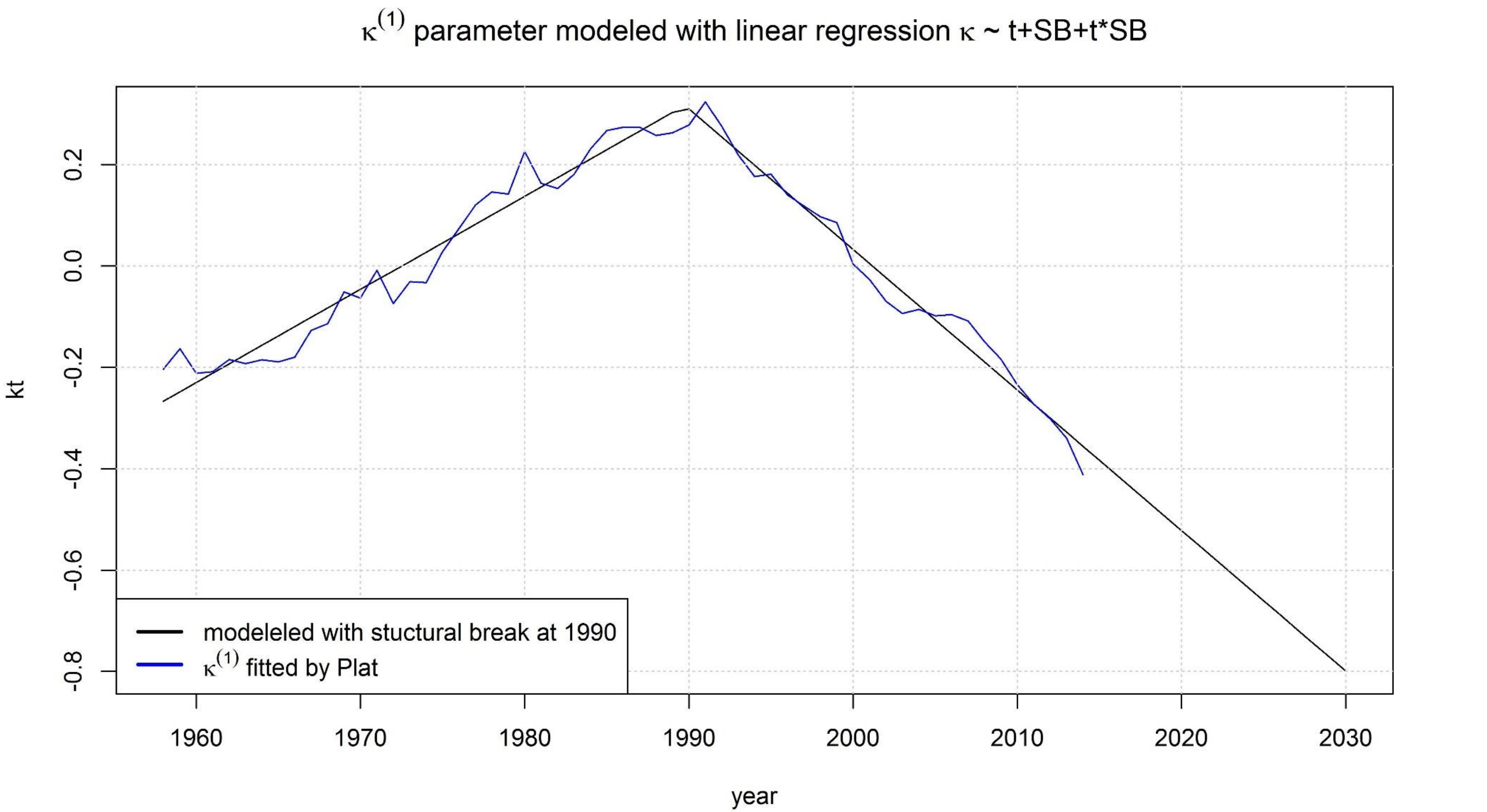
Parameters that needs to be forecasted: $\kappa_t^{(1)}$,$\kappa_t^{(2)}$,$\kappa_t^{(3)}$,γ_{t-x}

Kappa parameter projection

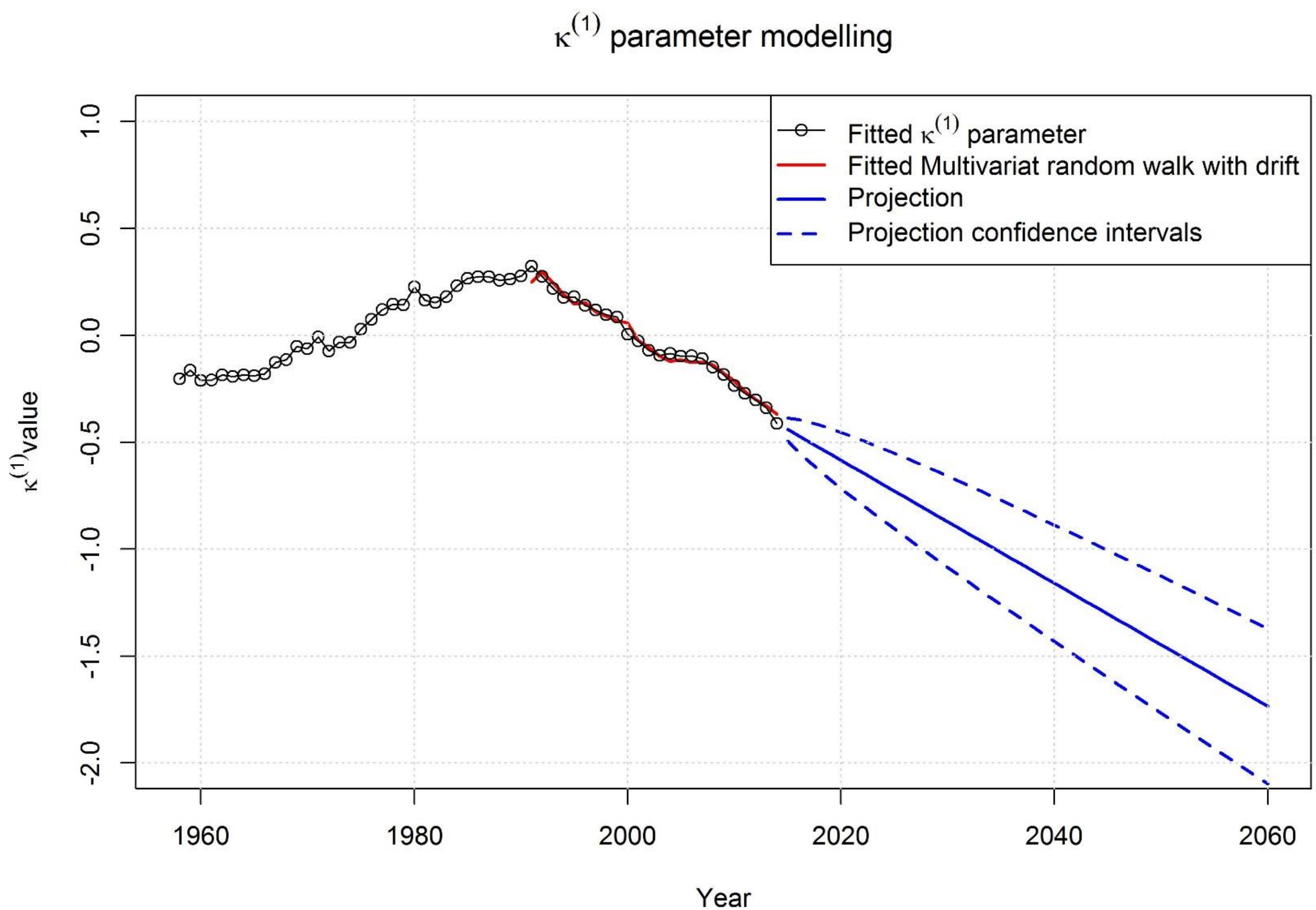
Multivariate random walk with drift is applied (it is most popular approach for kappa parameters in literature).

Also an structural break has been identified. Mathematical framework for performing structural change tests is provided within strucchange package.

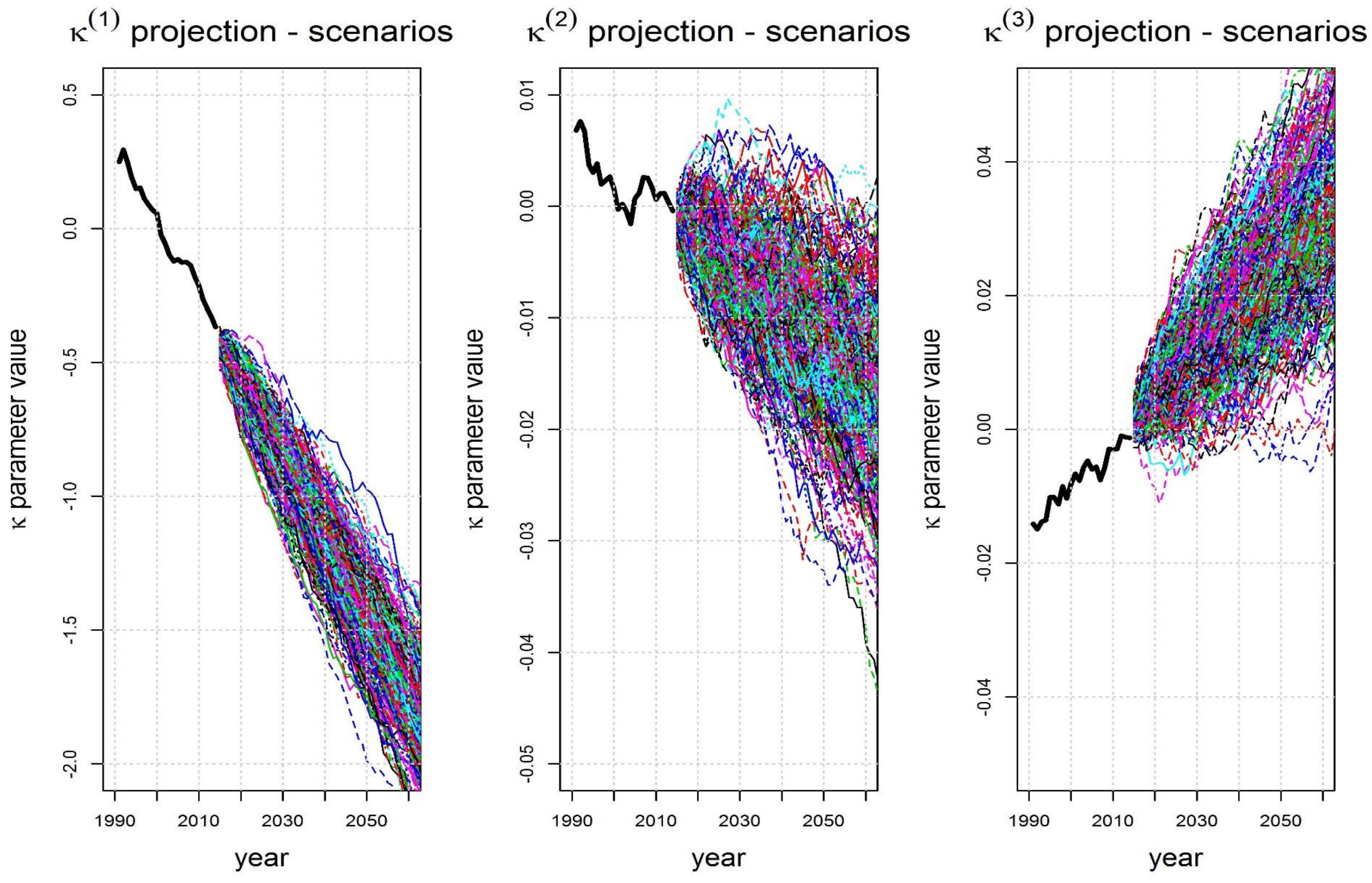
Structural break in 1990



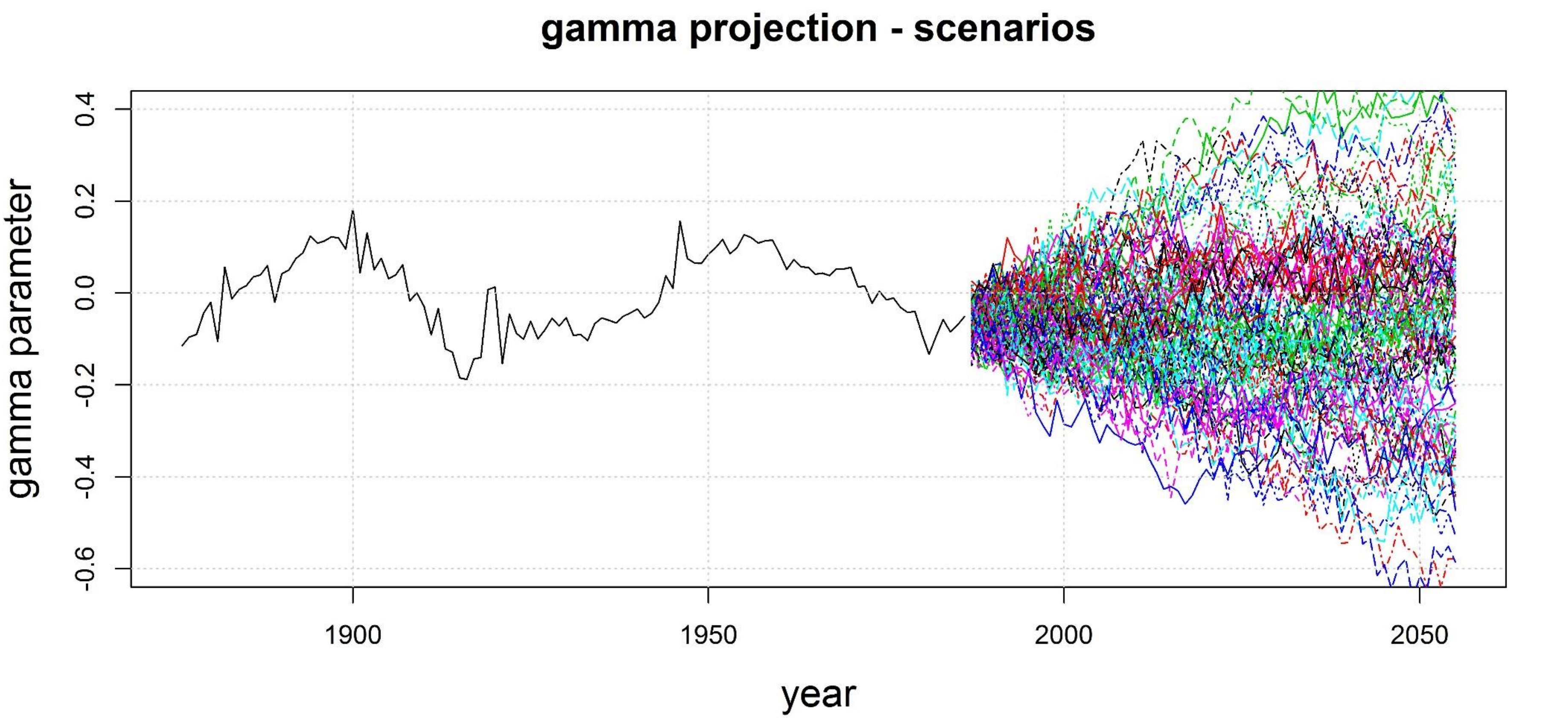
Structural break in 1990



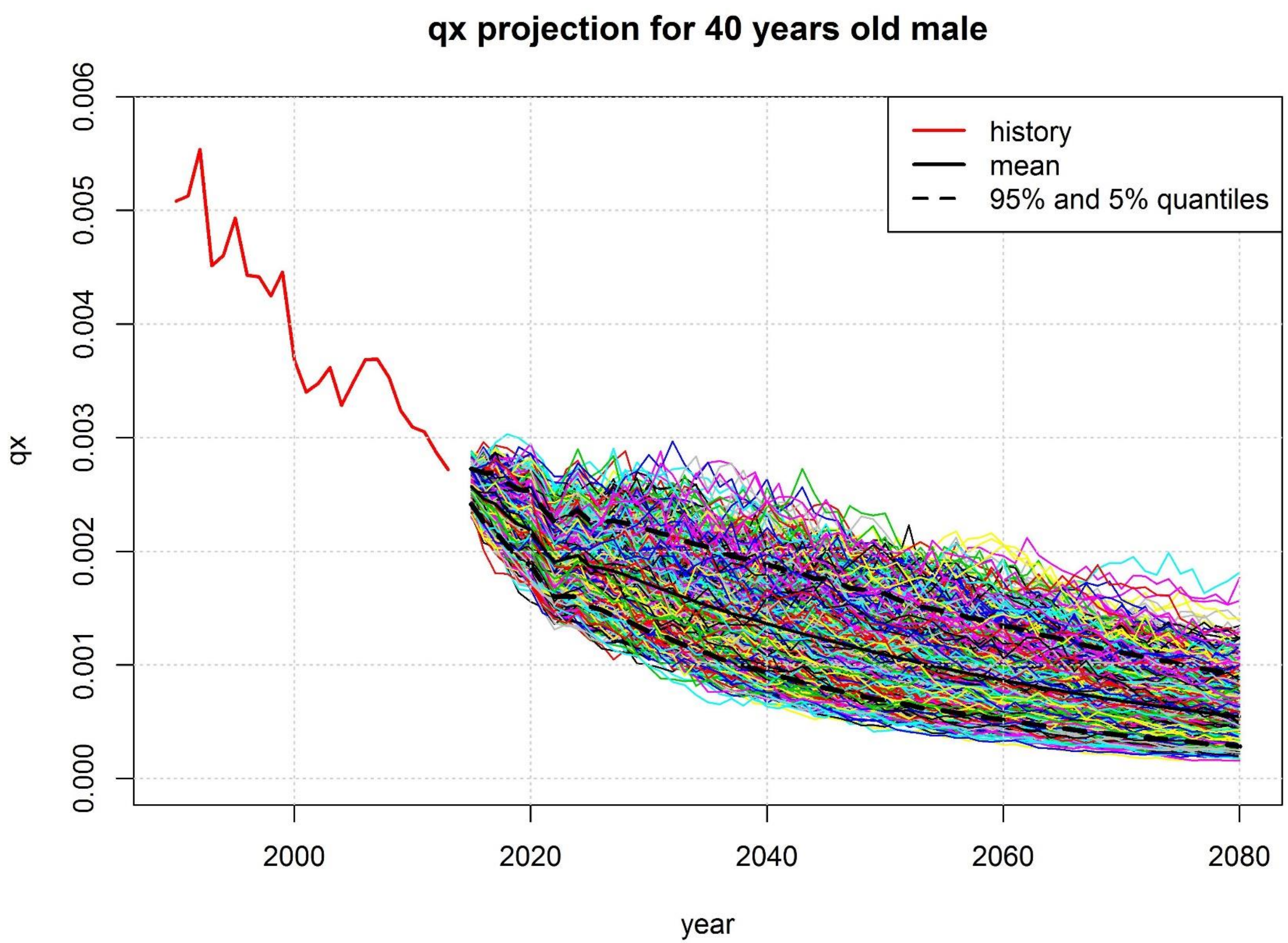
Kappa parameters projection



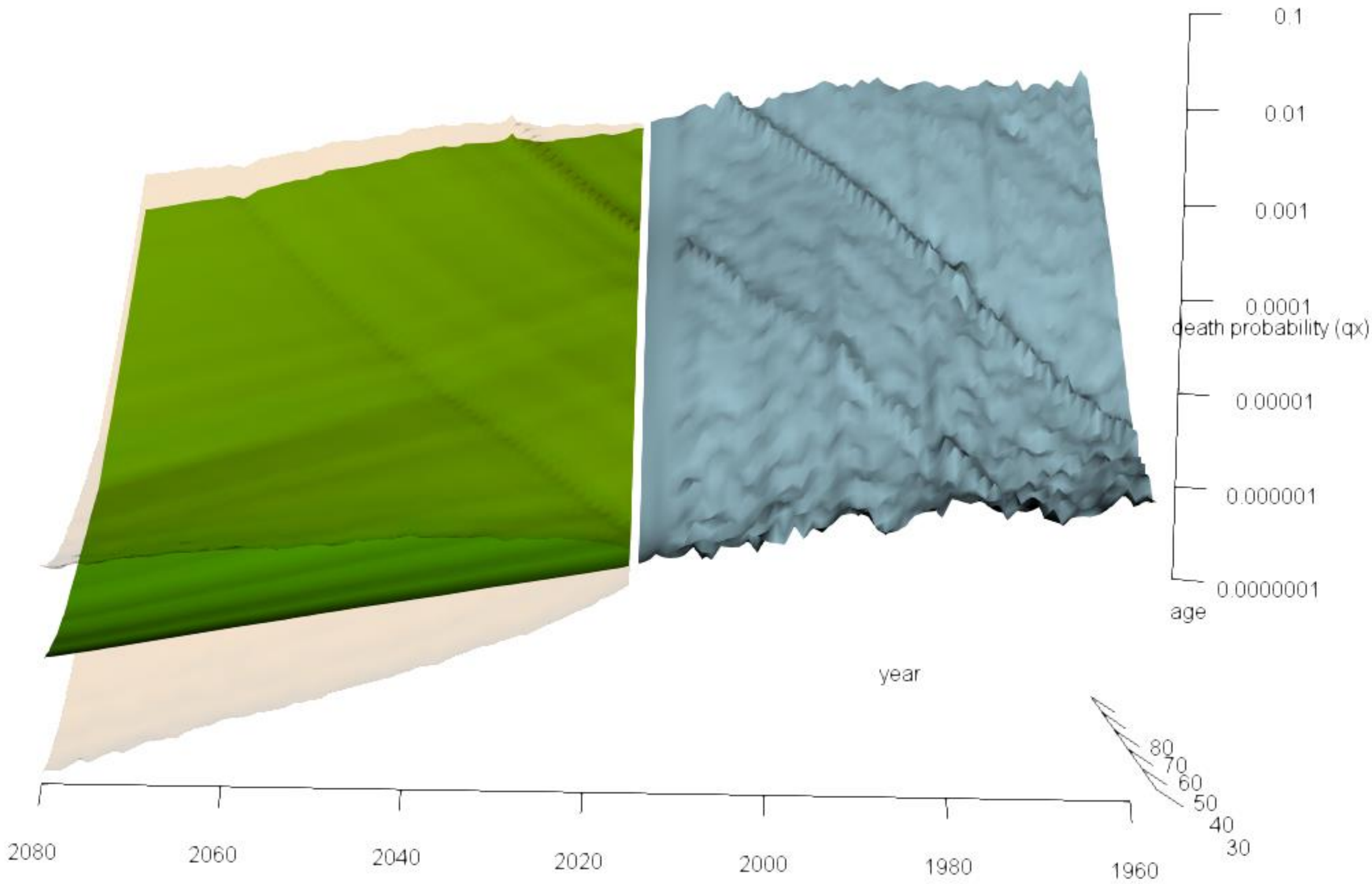
Gamma parameter projection



Projection – single age perspective



Projection – whole surface perspective



References

1. StMoMo: Stochastic Mortality Modelling, 2015 https://cran.r-project.org/web/packages/StMoMo/index.html
2. Andrés M. Villegas, Vladimir Kaishev, Pietro Millossovich, 2015, StMoMo: An R Package for Stochastic Mortality Modelling https://cran.r-project.org/web/packages/StMoMo/vignettes/StMoMoVignette.pdf
3. Richard Plat, 2009, On Stochastic Mortality Modelling
4. Colin O'Hare, Youwei Li, 2014,Identifying Structural Breaks In Stochastic Mortality Models
5. Edviges Coelho, Luis C. Nunes, 2009, Forecasting Mortality in the Event of a Structural Change
6. Heather Booth, Leonie Tickle, 2008, Mortality modelling and forecasting: A review of methods
7. Zeileis A., Leisch F., Hornik K., Kleiber C.,2002, strucchange: An R Package for Testing for Structural Change in Linear Regression Models

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Questions and answers