$$x = \frac{-100 r_3 \pm \sqrt{100^2 r_3^2 - 4(-2r_3)(-2r_1)}}{2(-2r_3)} \; ; \quad \begin{cases} x = 13,82 \\ x = 36.18 \end{cases} \quad (r_1 = 2.5 \, , \, r_3 = 0.005)$$

We have real solutions if:

$$100^2 r_3^2 - 16 r_3 r_1 \geq 0$$

$$100^2 r_3^2 - 16 r_3 r_1 = 0 \rightsquigarrow r_3 \left(100^2 - 16 r_1\right) = 0 \quad \begin{cases} r_1 = 0 \\ r_3 = \frac{16 r_1}{100^2} \end{cases}$$

$+ \quad 0 \quad - \quad \frac{16 r_1}{100^2} \quad +$

$$\Rightarrow r_3 \geq \frac{16 r_1}{100^2} \; *$$

Steady states:

$A_1 = 25, \quad A_2 = 13.82 \, , \quad A_3 = 36.18$

- Stability analysis:

$$-\frac{2k_3}{v^2} \langle n \rangle^3 + \frac{3k_3}{v^2} N \langle n \rangle^2 - \left(2k_1 + \frac{k_3}{v^2} N^2\right) \langle n \rangle + k_1 N = 0$$

$$N = 50 \, , \quad k_1 = 2.5 \, , \quad \frac{k_2}{v} = 2 \, , \quad \frac{k_3}{v^2} = 0.05$$

$$-2\left(0.005\right) \langle n \rangle^3 + 3 \cdot 0.005 \cdot 50 \, \langle n \rangle^2 - \left(2 \cdot 2.5 + 0.005 \cdot 50^2\right) \langle n \rangle + 2.5 \cdot 50 = 0$$

$$f(\langle n \rangle) := -0.01 \langle n \rangle^3 + 0.75 \langle n \rangle^2 - (17.5) \langle n \rangle + 125 = 0$$

$$\frac{\partial f}{\partial \langle n \rangle} = -3 \, 0.01 \langle n \rangle^2 + 2 \cdot 0.75 \langle n \rangle - 17.5 = -0.03 \langle n \rangle^2 + 1.5 \langle n \rangle - 17.5$$

- Subs: $\langle n \rangle = 25$ in $\frac{\partial f}{\partial \langle n \rangle}$: $-0.03 (25)^2 + 1.5 \cdot 25 - 17.5 = 1.25 > 0$

  $\Rightarrow A_1 = 25$ <u>unestable</u>

- Subs $\langle n \rangle = 13.82$, in $\frac{\partial f}{\partial \langle n \rangle}$: $-0.03 (13.82)^2 + 1.5 \cdot 13.82 - 17.5 = -2.5 < 0$

  $A_2 = 13.82$ <u>stable</u>

- Subs: $\langle n \rangle = 36.2$ in $\frac{\partial f}{\partial \langle n \rangle}$: $-0.03 (36.2)^2 + 1.5 \cdot 36.2 - 17.5 = -2.51 < 0$

  $A_3 = 36.18$ <u>stable</u>