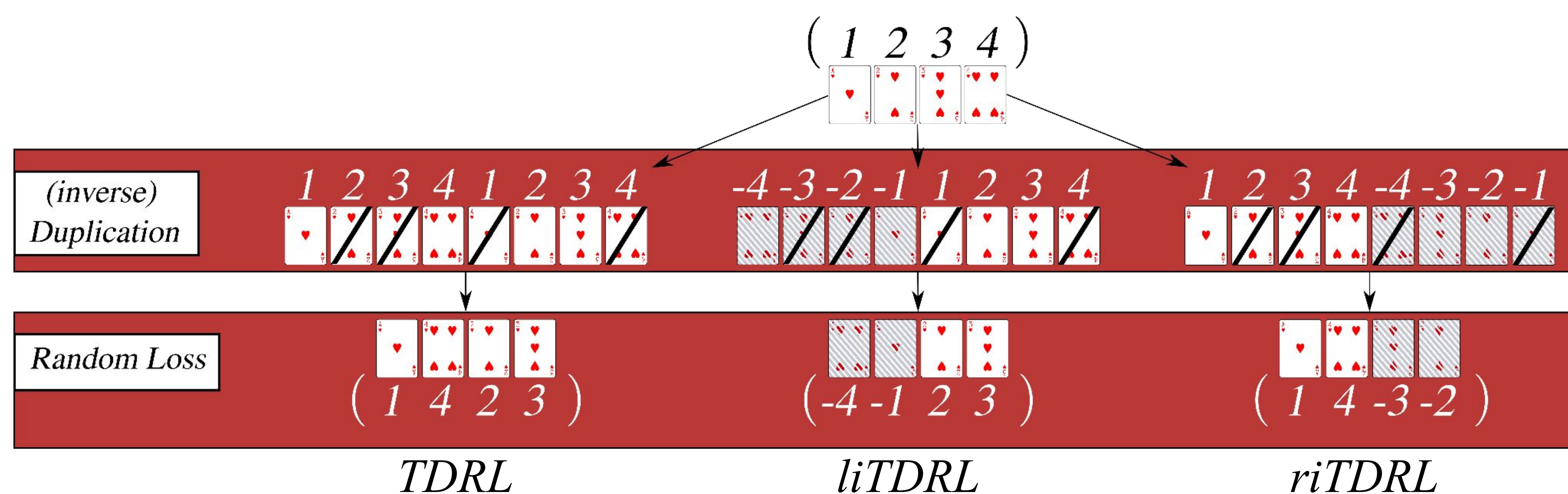# Sorting by TDRL and iTDRL

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What is *Tandem Duplication Random Loss* (TDRL) and *Inverse Tandem Duplication Random Loss?* (iTDRL)


The *Sorting Problem*

Given a set of generators, i.e. sorting operations, and a signed permutation π, the sorting problem asks for a shortest sequence of operations to stepwise transform the identity permutation $\iota = (1\ 2\ \dots\ n)$ into π. The aim of our work is to find the optimal solution to the sorting problem for TDRL and iTDRL.

Maximal Increasing Sign-Consistent Substrings

A (consecutive) substring of a permutation π is called maximal increasing sign-consistent substring (misc-substring) if all its elements appear in ascending order and share the same sign, and it cannot be extended into a longer misc-substring in π. The string that represents all misc-substrings of a permutation is called its misc-encoding.
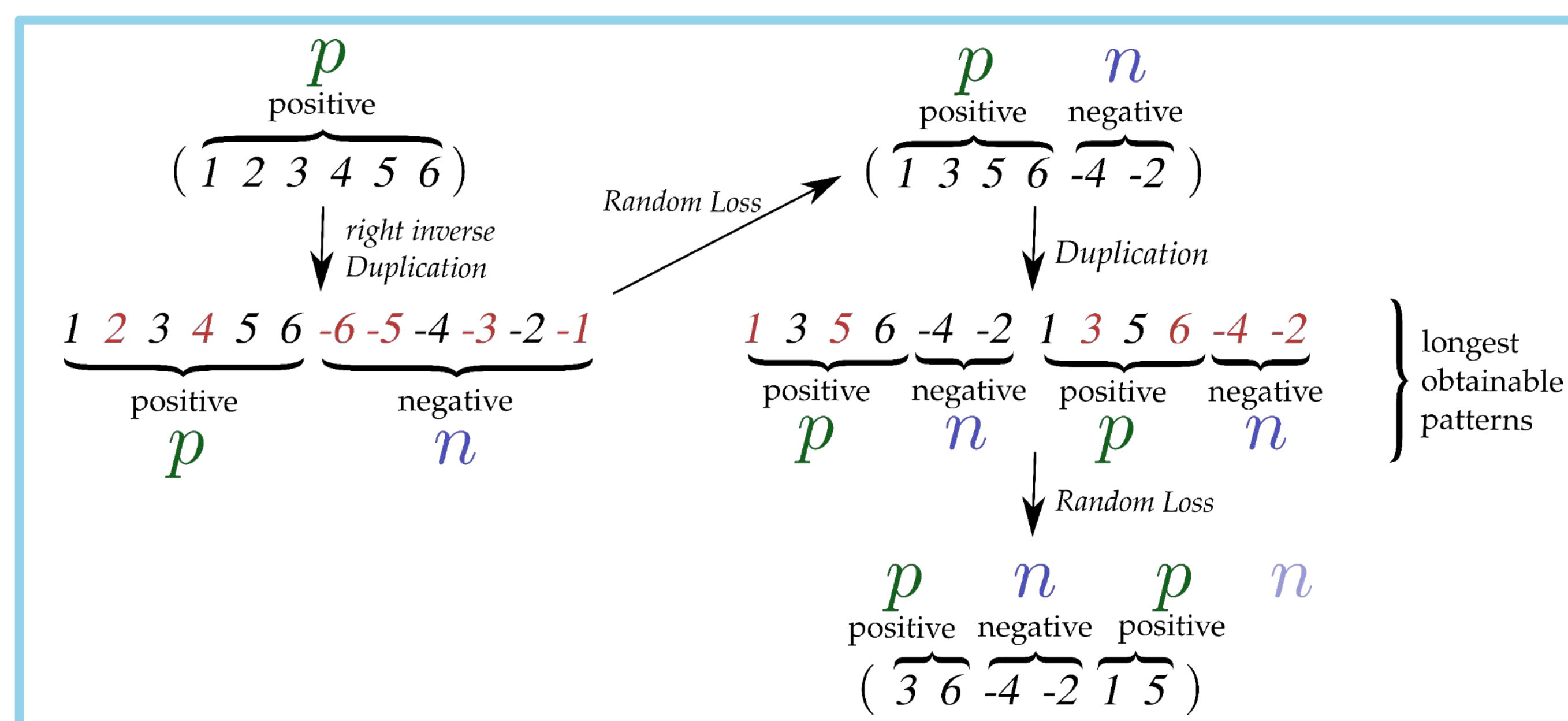

TDRL and *iTDRL* Patterns

TDRL and iTDRL patterns are the longest misc-encodings that can be obtained by applying a series of TDRL/iTDRL operations to the identity permutation. By finding the shortest pattern that the misc-encoding of a permutation π is subsequence of, the optimal order of sorting operations to sort ι into π can be inferred.
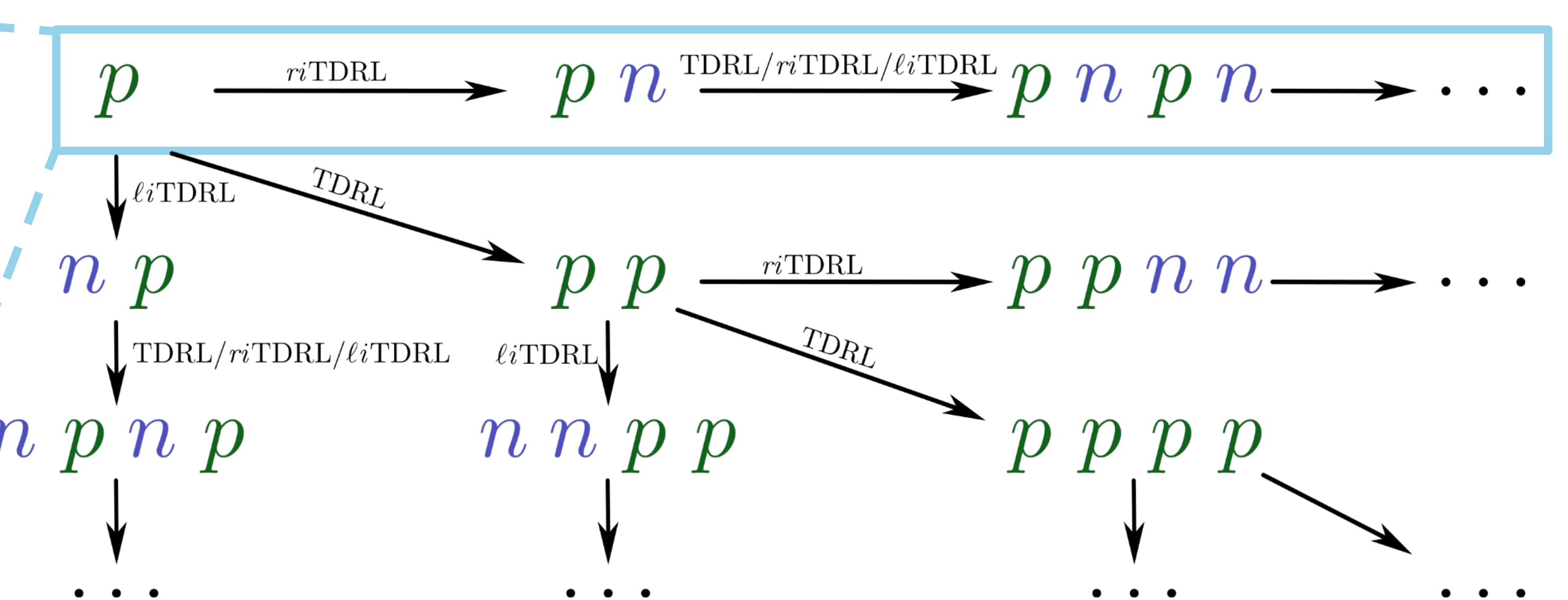
Shuffling Permutations and Sorting by TDRL and iTDRL

The idea of the sorting algorithm is to stepwise transform π into ι such that after each transformation, the misc-encoding of the transformed π is subsequence of a shorter pattern. This is done such that after each transformation, a TDRL/iTDRL can be derived which is exactly the inverse to the transformation performed. The inverse transformations (and consequently the sorting TDRL/iTDRL) can be computed efficiently by finding the shortest pattern the misc-encoding of π is a subsequence of.
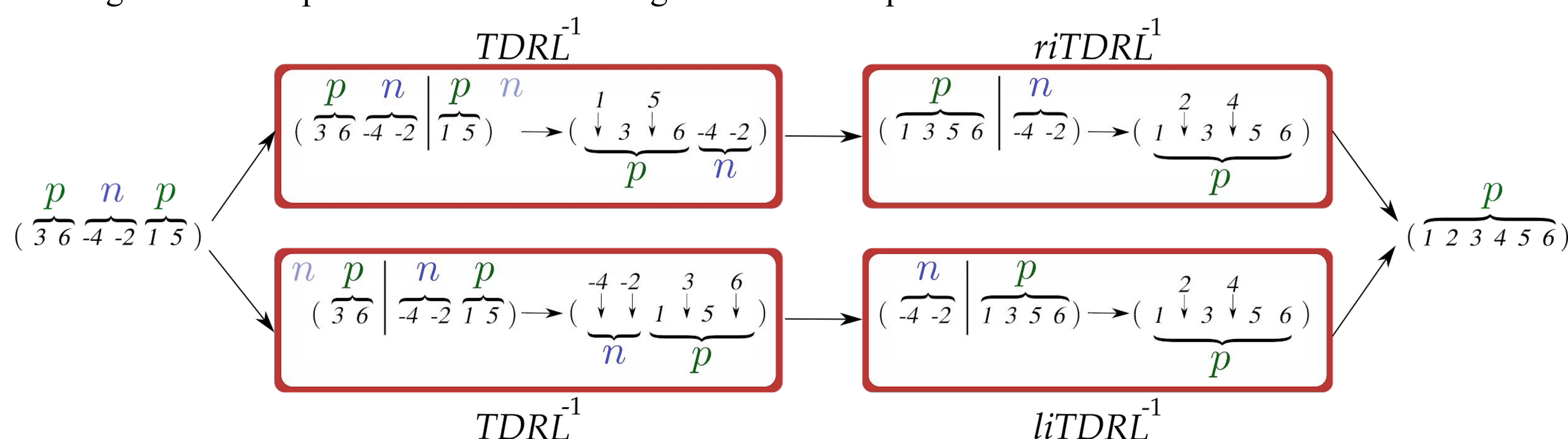
Conclusion

We introduced a simple signature called **misc-encoding** in order to solve the **sorting problem** for **TDRL** and **iTDRL** in $\mathcal{O}(n\ log\ n)$ time. An implementation that follows the concepts outlined here can be found on GitHub by scanning the QR code below.