GENERAL ASSEMBLY

INTRO TO DATA SCIENCE
LECTURE 5: REGRESSION & REGULARIZATION

Francesco Mosconi
DAT10 SF // October 20, 2014

DATA SCIENCE IN THE NEWS

DATA SCIENCE IN THE NEWS

Will Apple Inc. Sell 63 Million iPhones This Quarter?

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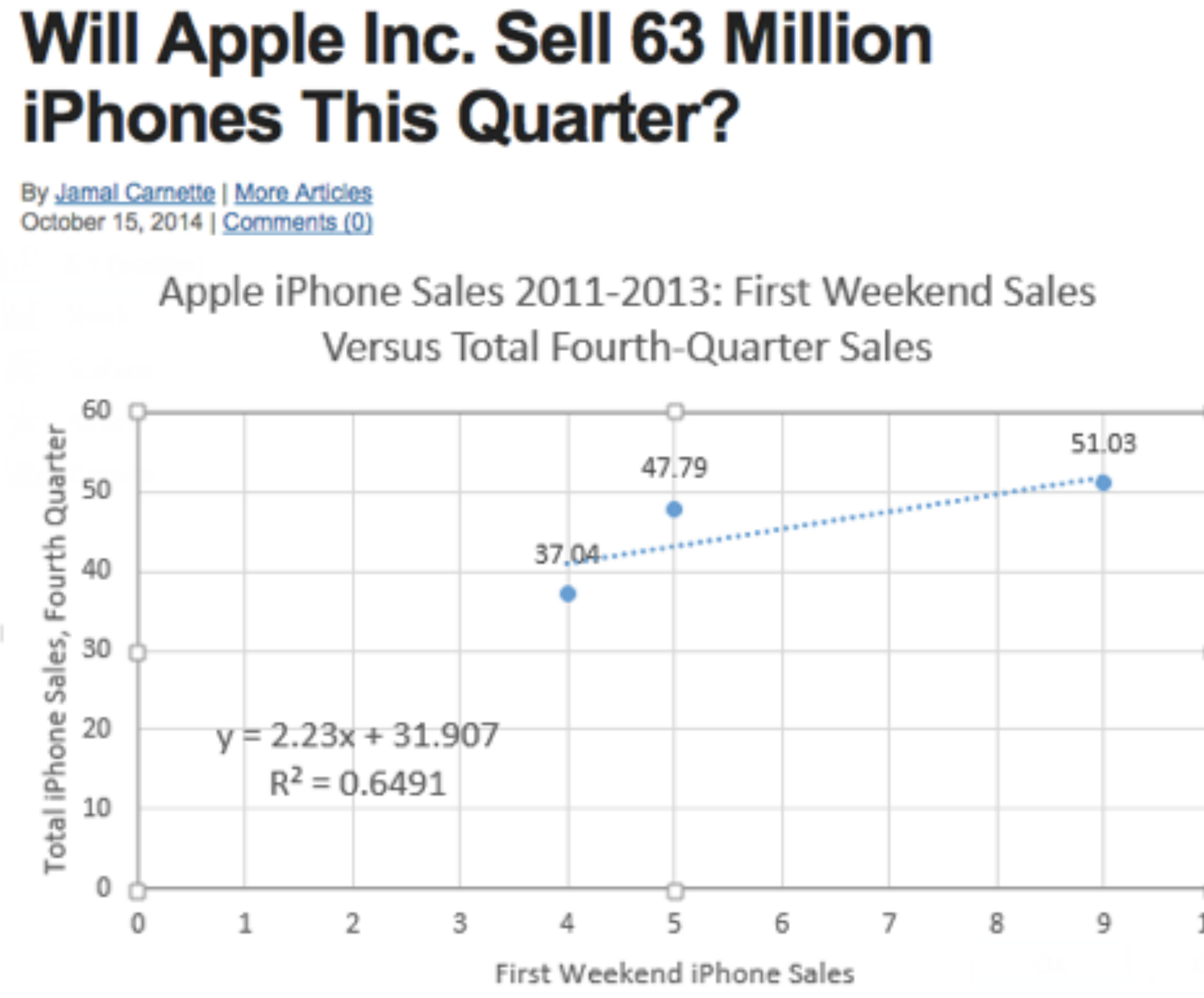
DATA SCIENCE IN THE NEWS



Source: http://www.fool.com/investing/general/2014/10/15/will-apple

DATA SCIENCE IN THE NEWS

Source: https://www.cryptocoinsnews.com/coming-bitcoin-trading-machine-overlords/

DATA SCIENCE IN THE NEWS

Bayesian regression and Bitcoin

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***Abstract*—In this paper, we discuss the method of Bayesian regression and its efficacy for predicting price variation of Bitcoin, a recently popularized virtual, cryptographic currency. Bayesian regression refers to utilizing empirical data as proxy to perform Bayesian inference. We utilize Bayesian regression for the so-called "latent source model". The Bayesian regression for "latent source model" was introduced and discussed by Chen, Nikolov and Shah [1] and Bresler, Chen and Shah [2] for the purpose of binary classification. They established theoretical as well as empirical efficacy of the method for the setting of binary classification.**

In this paper, instead we utilize it for predicting real-valued quantity, the price of Bitcoin. Based on this price prediction method, we devise a simple strategy for trading Bitcoin. The strategy is able

In the classical setting, d is assumed fixed and $n \gg d$ which leads to justification of such an estimator being highly effective. In various modern applications, $n \asymp d$ or even $n \ll d$ is more realistic and thus leaving highly under-determined problem for estimating θ^*. Under reasonable assumption such as 'sparsity' of θ^*, i.e. $\|\theta^*\|_0 \ll d$, where $\|\theta^*\|_0 = |\{i : \theta_i^* \neq 0\}|$, the regularized least-square estimation (also known as Lasso [4]) turns out to be the right solution: for appropriate choice of $\lambda > 0$,

$$\hat{\theta}_{LASSO} \in \arg\min_{\theta \in \mathbb{R}^d} \sum_{i=1}^{n} (y_i - x_i^T \theta)^2 + \lambda \|\theta\|_1. \quad (2)$$

At this stage, it is worth pointing out that the above framework, with different functional forms, has

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LAST TIME:

- INTRO TO ML
- KNN CLASSIFICATION
- INTRO TO MATPLOTLIB FOR VISUALIZATION

QUESTIONS?

AGENDA

I. LINEAR REGRESSION (INCL. MULTIPLE REGRESSION)
II. POLYNOMIAL REGRESSION
III. REGULARIZATION

LAB:
IV. IMPLEMENTING MULTIPLE REGRESSION & POLYNOMIAL REGRESSION IN PYTHON

I. LINEAR REGRESSION

	Continuous	Categorical
Supervised	???	???
Unsupervised	???	???

	Continuous	Categorical
Supervised	regression	classification
Unsupervised	dimension reduction	clustering

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$$y = \alpha + \beta x + \varepsilon$$

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α = intercept (where the line crosses the y-axis)

β = regression coefficient (the model “parameter”)

ε = residual (the prediction error)

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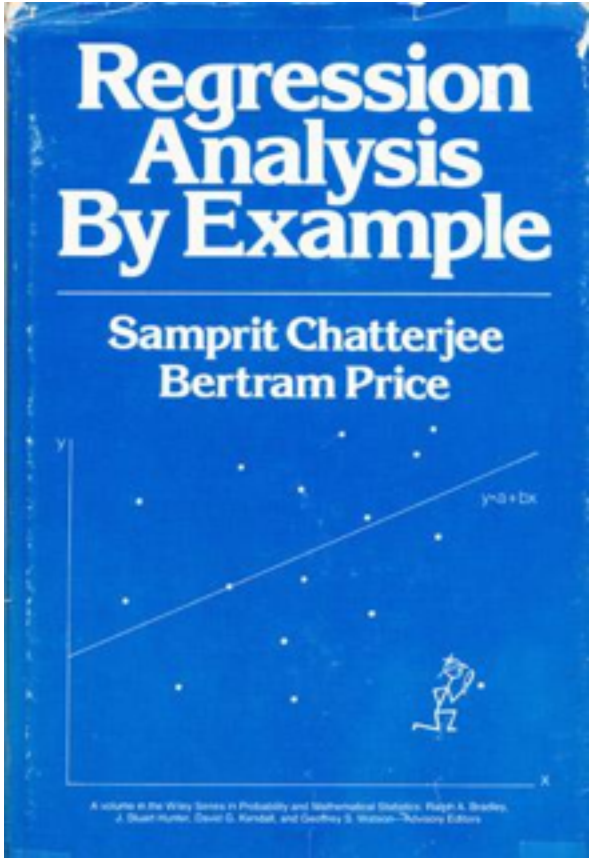
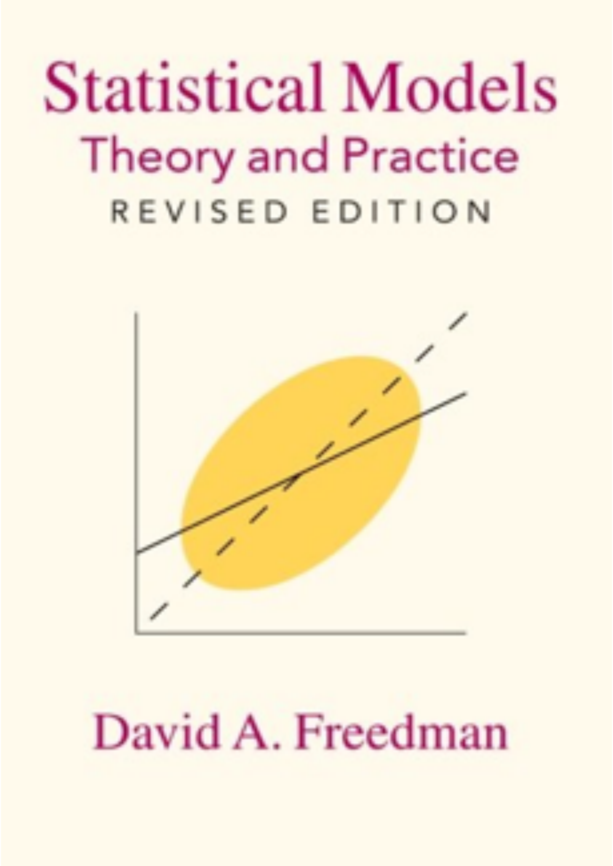
$$y = \alpha + \beta_1 x_1 + \dots + \beta_n x_n + \varepsilon$$

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

The math is not very important for our purposes, but you should check it out if you get serious about solving regression problems.


Regression Analysis By Example
Samprit Chatterjee
Bertram Price
y
y=a+bx
x


Statistical Models
Theory and Practice
REVISED EDITION
David A. Freedman

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But again, if you get serious about regression, you should learn how this works!

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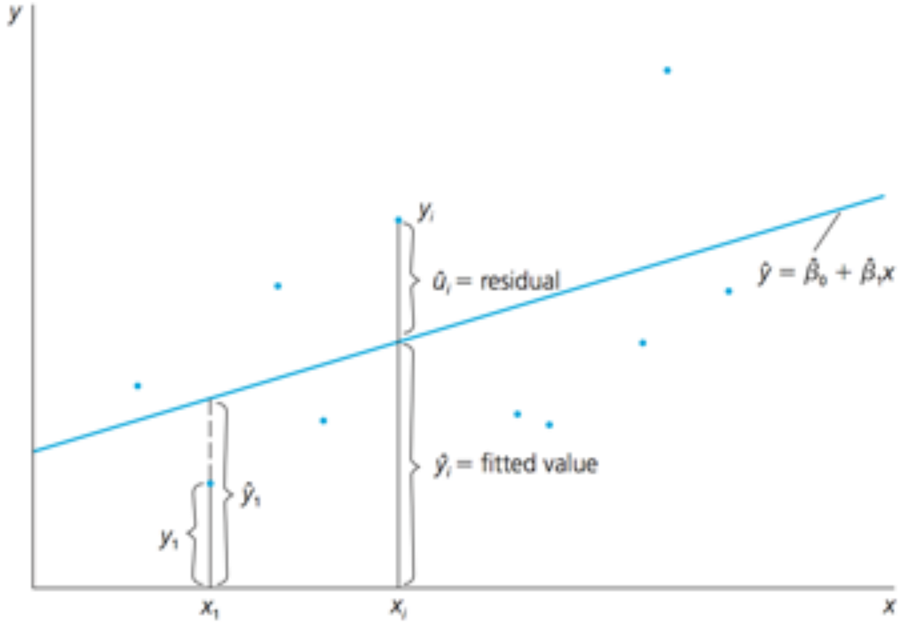
A: In theory, minimize the sum of the squared residuals (OLS).

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i.$$

$$\hat{u}_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i.$$

$$\sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2,$$



II: POLYNOMIAL REGRESSION

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Q: This represents a nonlinear relationship. Is it still a linear model?
A: Yes, because it's linear in the β's!

"Although polynomial regression fits a nonlinear model to the data, as a statistical estimation problem it is linear, in the sense that the regression function E(y|x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regression." -- Wikipedia

Polynomial regression allows us to fit very complex curves to data.

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But there is one problem with the model we've written down so far.

Q: Does anyone know what it is?

A: This model violates one of the assumptions of linear regression!

This model displays multicollinearity, which means the predictor variables are highly correlated with each other.

$y = \alpha + \beta_1 x + \beta_2 x^2 + \ldots + \beta_n x^n + \varepsilon$

Multicollinearity causes the linear regression model to break down, because it can’t tell the predictor variables apart.

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OPTIONAL NOTE

These polynomial functions form an *orthogonal basis* of the function space.

So far, we’ve seen how polynomial regression allows us to fit complex nonlinear relationships, and even to avoid multicollinearity (by using basis functions).

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Q: Can a regression model be too complex?

III: REGULARIZATION

Recall our earlier discussion of overfitting.

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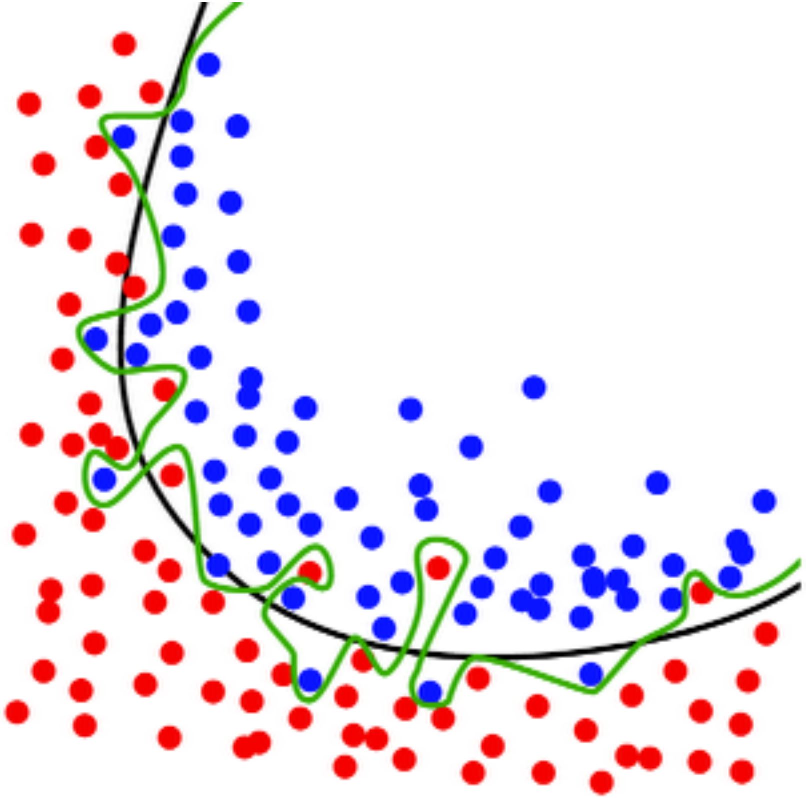
In other words, an overfit model matches the noise in the dataset instead of the signal.

OVERFITTING EXAMPLE (CLASSIFICATION)



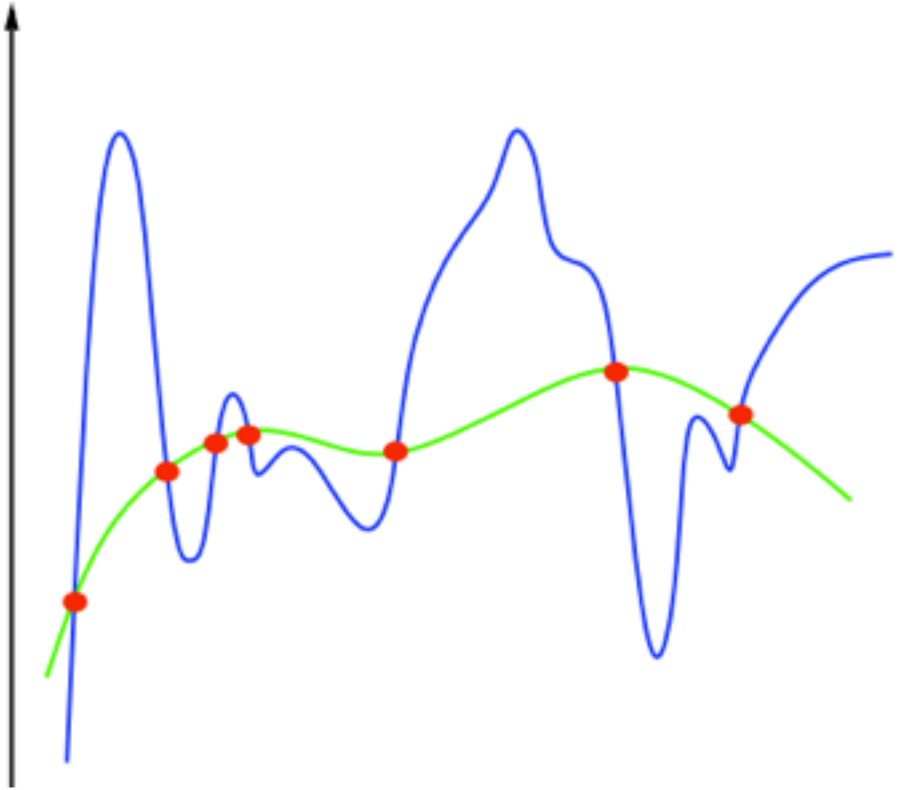
source: http://upload.wikimedia.org/wikipedia/commons/1/19/Overfitting.svg

The same thing can happen in regression.

It’s possible to design a regression model that matches the noise in the data instead of the signal.

This happens when our model becomes too complex for the data to support.

OVERFITTING EXAMPLE (REGRESSION)



source: http://www.mit.edu/~9.520/spring12/slides/class02/class02.pdf

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Ex 1: $\Sigma\, |\beta_i|$

Ex 2: $\Sigma\, \beta_i^2$

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A: One method is to define complexity as a function of the size of the coefficients.

Ex 1: $\Sigma\,|\beta_i|$ this is called the L1-norm

Ex 2: $\Sigma\,\beta_i^2$ this is called the L2-norm

These measures of complexity lead to the following regularization techniques:

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Regularization refers to the method of preventing overfitting by explicitly controlling model complexity.

These regularization problems can also be expressed as:

L1 regularization: $min(\|y - x\beta\|^2 + \lambda\|\beta\|)$

L2 regularization: $min(\|y - x\beta\|^2 + \lambda\|\beta\|^2)$

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L1 regularization (Lasso): $min(\|y - x\beta\|^2 + \lambda\|x\|)$

L2 regularization (Ridge): $min(\|y - x\beta\|^2 + \lambda\|x\|^2)$

This (Lagrangian) formulation reflects the fact that there is a cost associated with regularization.

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Variance refers to predictions that are generally inaccurate.

BIAS AND VARIANCE

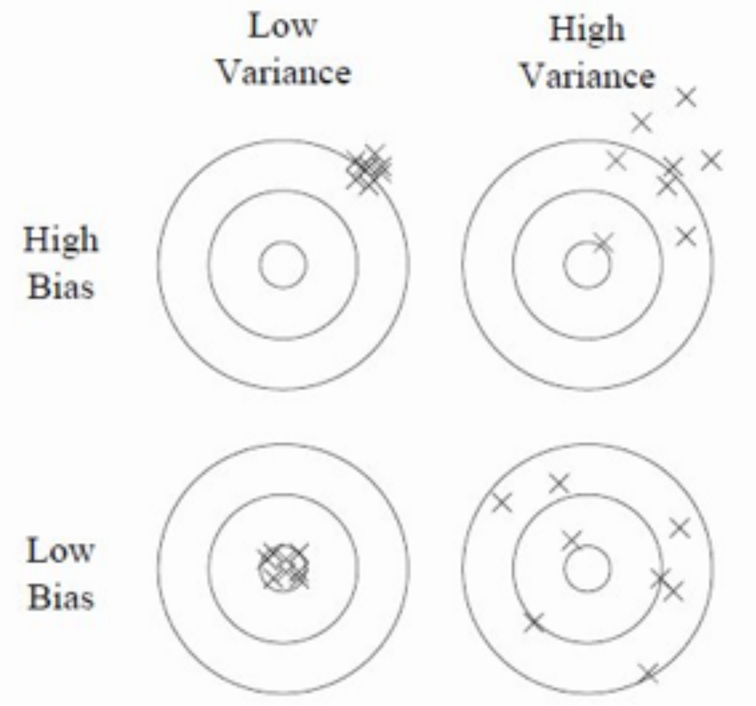


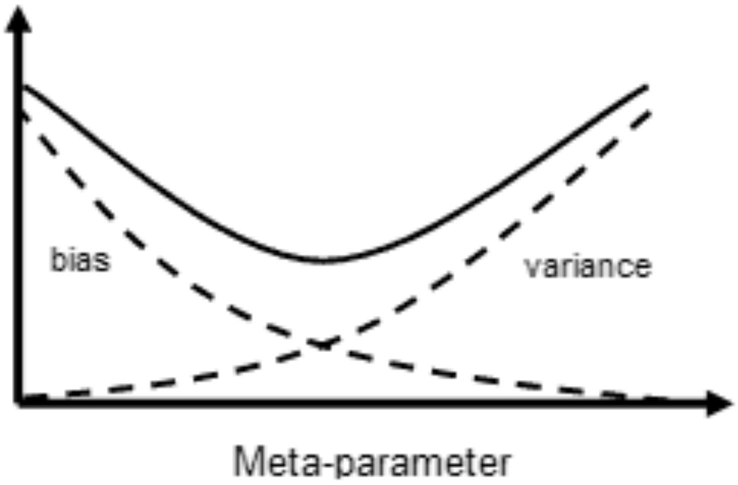
Figure 1: Bias and variance in dart-throwing.

source: http://homes.cs.washington.edu/~pedrod/papers/cacm12.pdf

Q: What are bias and variance?
A: Bias refers to predictions that are systematically inaccurate.
Variance refers to predictions that are generally inaccurate.

It turns out (after some math) that the generalization error in our model can be decomposed into a bias component and variance component.

This is another example of the bias-variance tradeoff.



source: http://www.isu.edu/chem/images/kalivasmeta.gif

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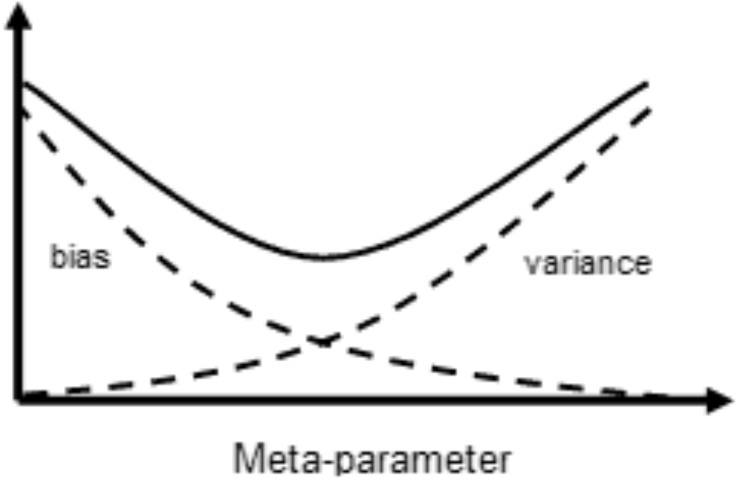

NOTE

The "meta-parameter" here is the lambda we saw above.

A more typical term is "hyperparameter".

source: http://www.isu.edu/chem/images/kalivasmeta.gif

This tradeoff is regulated by a hyperparameter λ, which we've already seen:

L1 regularization: $y = \Sigma\, \beta_i x_i + \varepsilon \quad st. \quad \Sigma\, |\beta_i| < \lambda$

L2 regularization: $y = \Sigma\, \beta_i x_i + \varepsilon \quad st. \quad \Sigma\, \beta_i^2 < \lambda$

So regularization represents a method to trade away some variance for a little bias in our model, thus achieving a better overall fit.

LAB: POLYNOMIAL REGRESSION & REGULARIZATION