GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 15: NAIVE BAYESIAN CLASSIFICATION

Francesco Mosconi
DAT10 SF // November 19, 2014

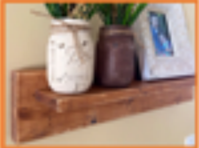
DATA SCIENCE IN THE NEWS

DATA SCIENCE IN THE NEWS

Posts by Robert Hall

Personalized Recommendations at Etsy

Items

Users						...
	Alice	1	1	0	0	
	Bob	0	0	1	1	
	Corey	1	0	1	0	
	...					

An implicit feedback dataset in which a set of users have "favorited" various items, note that we do not observe explicit dislikes, but only the presence or absence of favorites

source: https://codeascraft.com/2014/11/17/personalized-recommendations-at-etsy/

LAST TIME:

I. SUPPORT VECTOR MACHINES
II. MAXIMUM MARGIN HYPERPLANES
III. SLACK VARIABLES
IV. NONLINEAR CLASSIFICATION

QUESTIONS?

AGENDA

I. RECAP OF PROBABILITY
II. NAÏVE BAYESIAN CLASSIFICATION

I. RECAP OF PROBABILITY

Q: What is a probability?

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A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

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The probability of event A is denoted $P(A)$.

Q: What is the set of all possible events called?

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A: This set is called the sample space Ω. Event A is a member of the sample space, as is every other event.

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The probability of the sample space $P(\Omega)$ is 1.

Q: Consider two events A & B. How can we characterize the intersection of these events?

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A: With the joint probability of *A* and B, written *P(AB)*.

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NOTE

This information about B transforms the sample space.

Take a moment to convince yourself of this!

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This is called the conditional probability of A given B, written $P(A|B) = P(AB) / P(B)$.

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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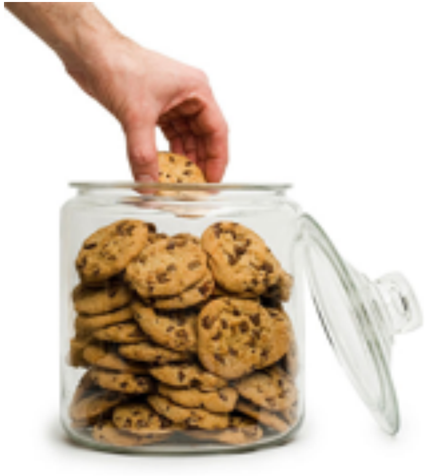
This can be written as $P(A|B) = P(A)$.

Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) / P(B) = P(A) \rightarrow \quad P(AB) = P(A) * P(B)$$

A motivating example: COOKIES!

Bowl 1 contains:
30 vanilla cookies
10 chocolate chip cookies



Bowl 2 contains:
20 vanilla cookies
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Now suppose you choose one of the bowls at random and, without looking, select a cookie at random. The cookie is vanilla. What is the probability that it came from Bowl 1?

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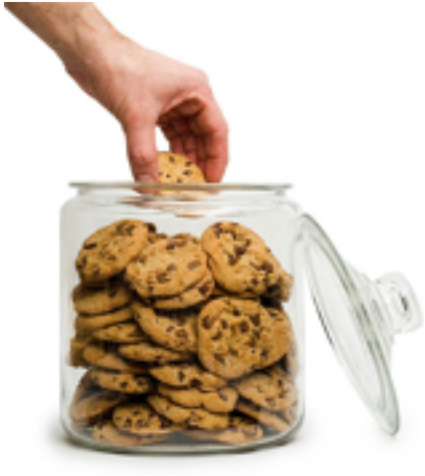

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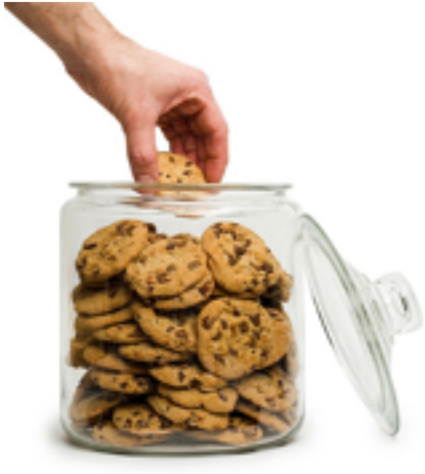

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In other words, we want: P(Bowl 1 | vanilla) This is a conditional probability.

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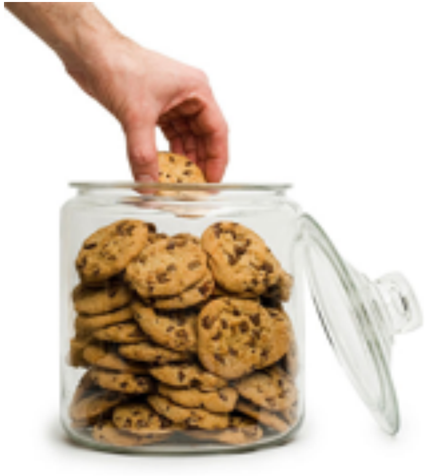

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How can we compute this?

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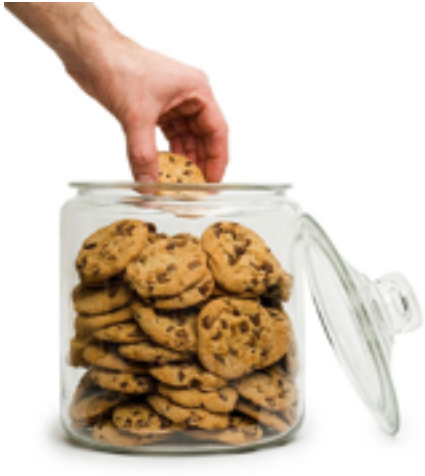

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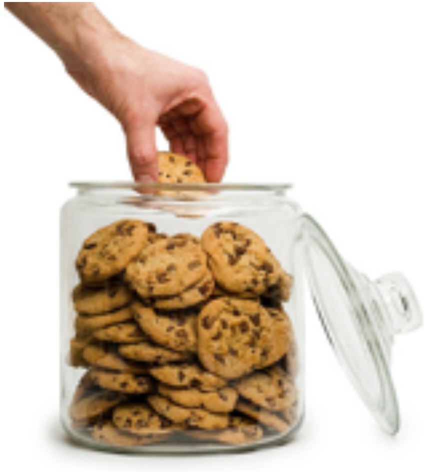

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What about P(vanilla | Bowl1) ? That's easy! P(vanilla | Bowl1) = 30/40 = 3/4

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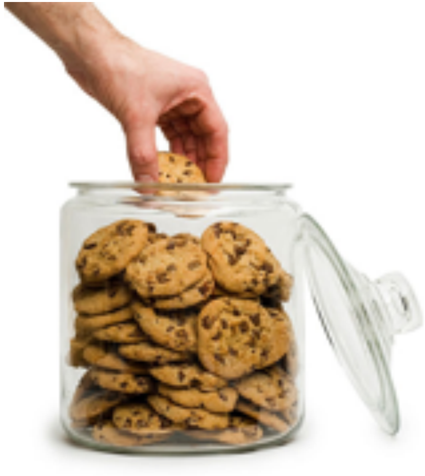

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In other words, we want: P(Bowl 1 | vanilla)

But P(Bowl1 | vanilla) is NOT equal to P(vanilla | Bowl1) = 3/4

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→ $P(A|B) = P(B|A) * P(A) / P(B)$ by rearranging last step

This result is called Bayes' theorem.

$$P(A|B) = P(A) * P(B|A) / P(B)$$

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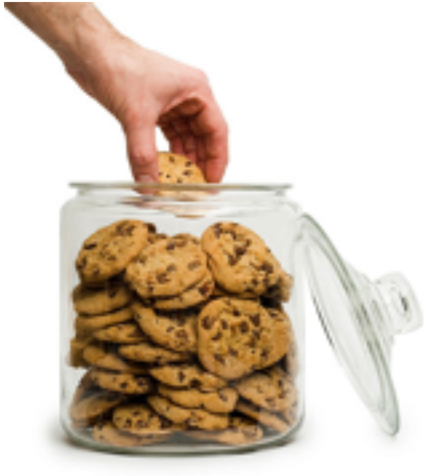

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P(A|B) = P(A) * P(B|A) / P(B)

What is P(A)?

What is P(B)?

What is P(B|A)?

We want: P(Bowl 1 | vanilla)

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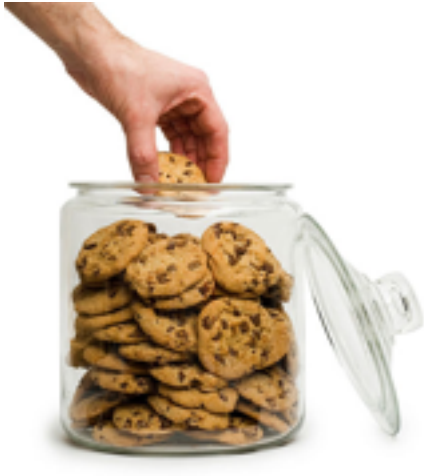

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$$P(A|B) = P(A) * P(B|A) / P(B)$$

$P(A) = 0.5$

$P(B) = 50 / 80 = 5/8$

$P(B|A) = 30/40 = 3/4$

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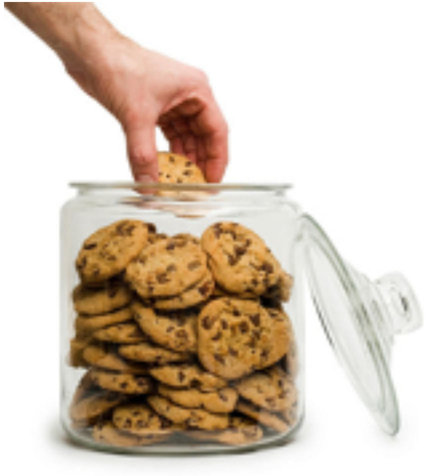

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P(A|B) = P(A) * P(B|A) / P(B) = 0.5 * 6/8 / 5/8 = **3/5**

P(A) = 0.5

P(B) = 50 / 80 = 5/8

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This result is called Bayes' theorem. Here it is again:

$$P(A|B) = P(B|A) * P(A) / P(B)$$

Some facts:
- This is a simple algebraic relationship using elementary definitions.

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Some facts:
- This is a simple algebraic relationship using elementary definitions.
- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.
- It's a very powerful computational tool.

Briefly, the two interpretations can be described as follows:

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The frequentist interpretation regards an event's probability as its limiting frequency across a very large number of trials.

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The frequentist interpretation regards an event's probability as its limiting frequency across a very large number of trials.

The Bayesian interpretation regards an event's probability as a "degree of belief," which can apply even to events that have not yet occurred.

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This a good direction to head if you like math and/or if you're interested in learning about cutting-edge data science techniques.

II. NAÏVE BAYESIAN CLASSIFICATION

Suppose we have a dataset with features x_1, …, x_n and a class label C. What can we say about classification using Bayes' theorem?

Suppose we have a dataset with features $x_1, \ldots, x_n$ and a class label C. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

source: Data Analysis with Open Source Tools, by Philipp K. Janert. O'Reilly Media, 2011.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the likelihood function. It represents the joint probability of observing features $\{x_i\}$ given that that record belongs to class *C*.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The **likelihood** of seeing that evidence if your hypothesis is correct.

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We can observe the value of the likelihood function from the training data.

This term is the prior probability of *C*. It represents the probability of a record belonging to class *C* before the data is taken into account.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The **prior**

This term is the prior probability of *C*. It represents the probability of a record belonging to class *C* before the data is taken into account.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The value of the prior is also observed from the data.

This term is the normalizing constant. It doesn't depend on *C*, and is generally ignored until the end of the computation.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The probability of the data under any hypothesis.

THE NORMALIZATION CONSTANT

This term is the normalizing constant. It doesn't depend on *C*, and is generally ignored until the end of the computation.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The normalizing constant doesn't tell us much.

This term is the posterior probability of *C*. It represents the probability of a record belonging to class *C* after the data is taken into account.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

In other words, the probability of the hypothesis **after** seeing the evidence.

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The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to update our beliefs about the distribution of *C* using the data ("evidence") at our disposal.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Methods	Predictions
“classical” (frequentist)	point estimates
Bayesian	distributions

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

Remember the likelihood function?

$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\})|C)$$

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$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\})|C)$$

Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

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$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

Q: So what can we do about it?
A: Make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

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This "naïve" assumption simplifies the likelihood function to make it tractable.