Explorations of Category Theory for Verifiable Credentials

Internet Identity Workshop # 35

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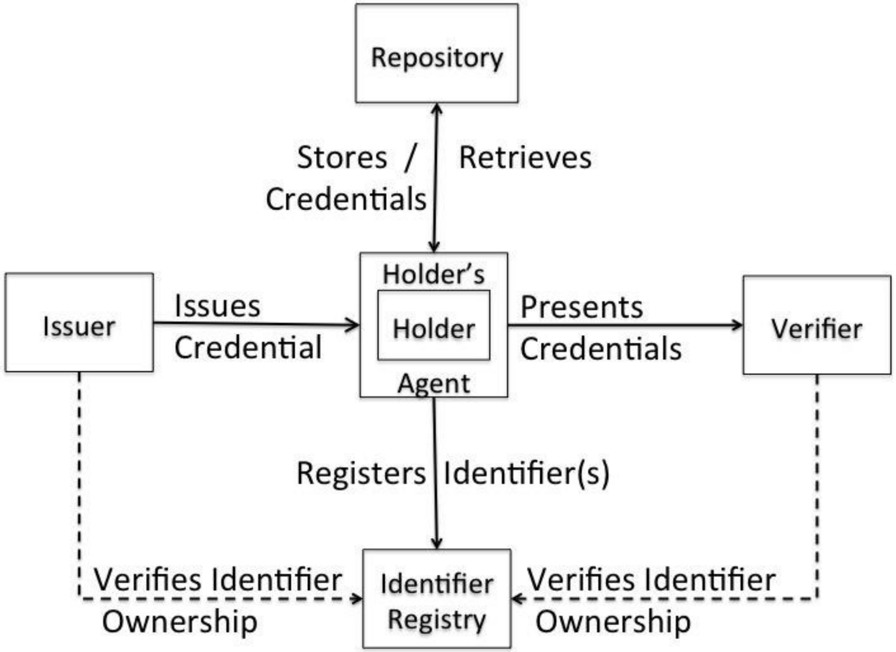
Verifiable Credentials Lifecycle



https://w3c-ccg.github.io/vc-lifecycle/

Identifier Registry contains DIDs and possibly full or a hash representation of DID Documents and/or Schemas?

Decentralized Identifier Architecture

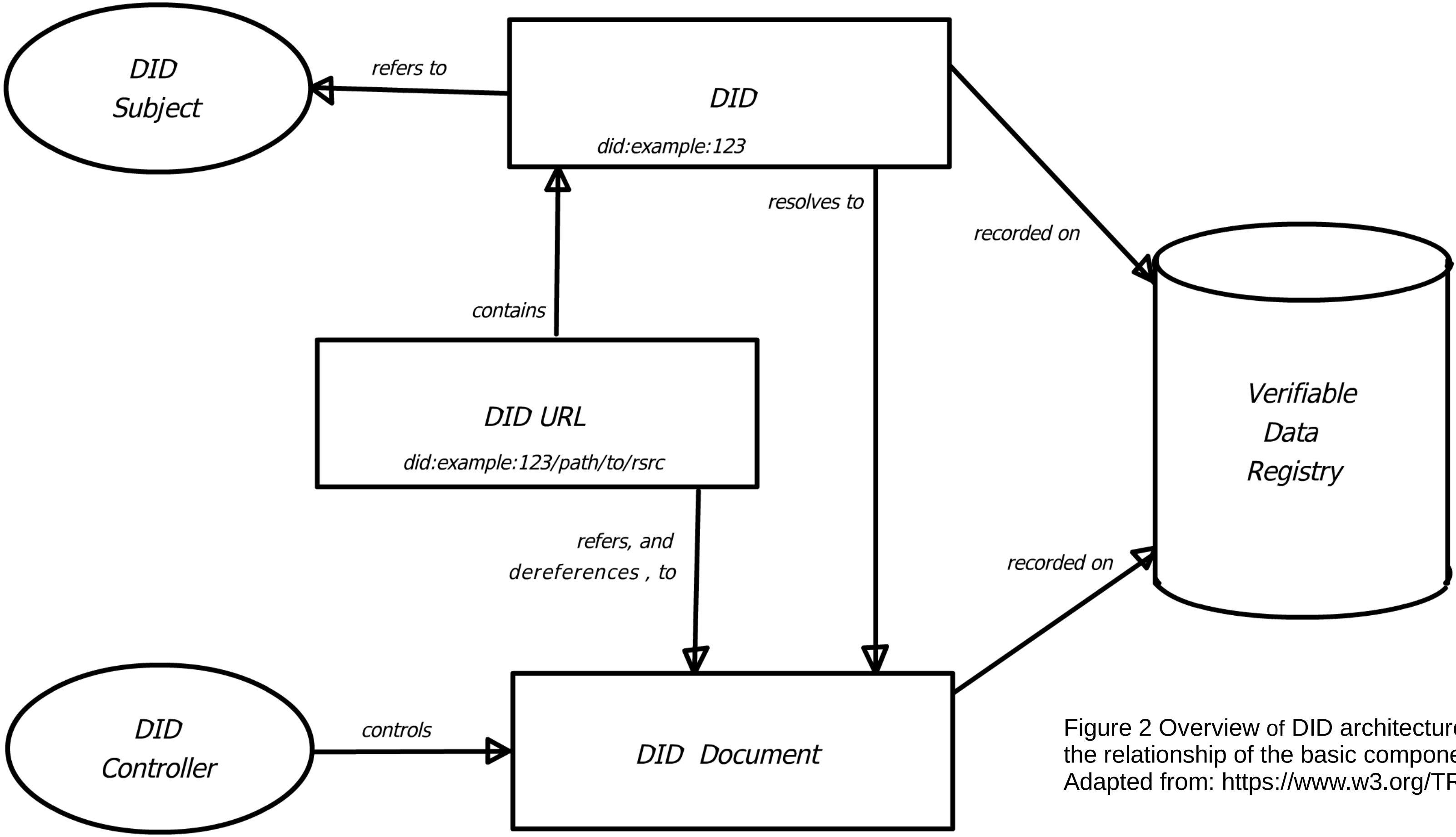
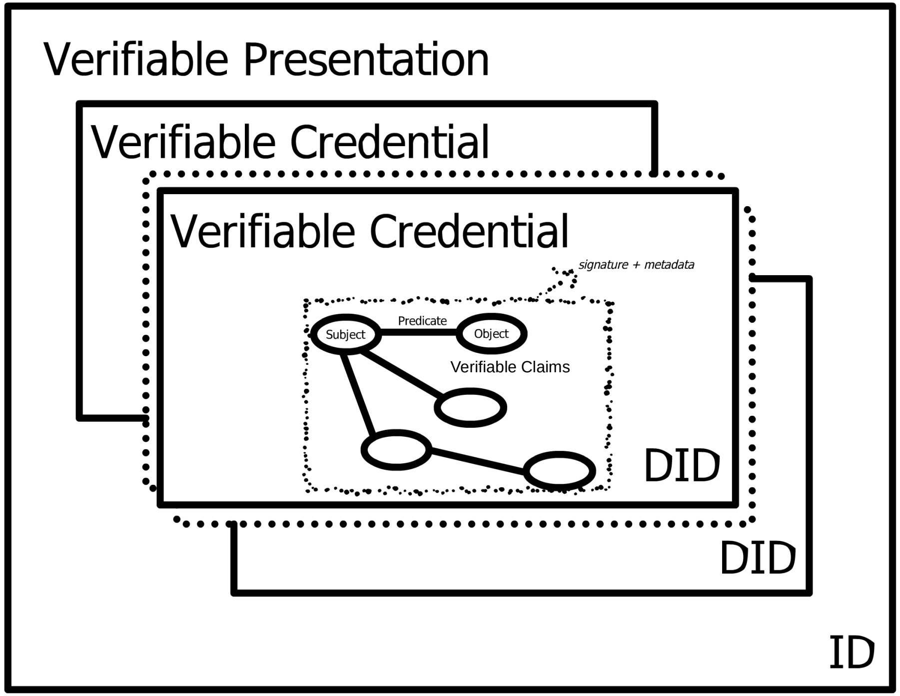
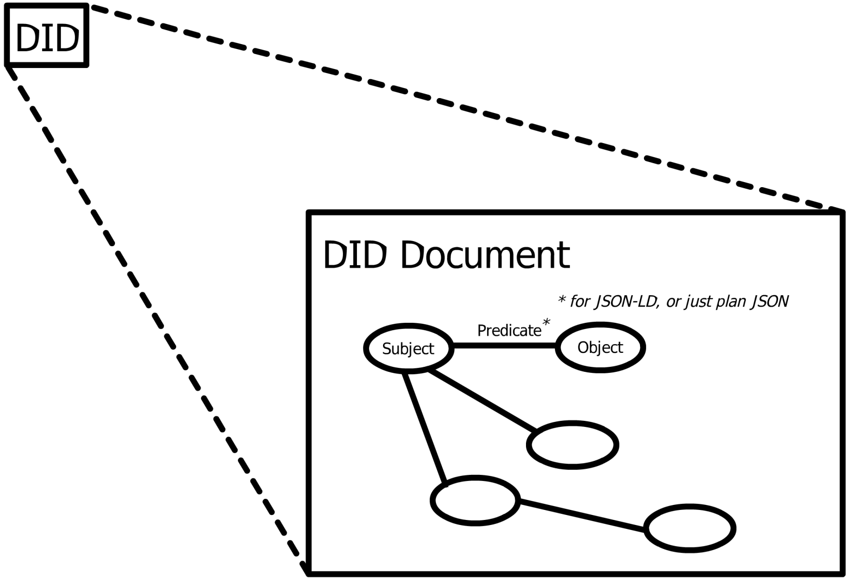


Figure 2 Overview of DID architecture and the relationship of the basic components. Adapted from: https://www.w3.org/TR/did-core/

Data Models





Inspired by:
https://www.w3.org/TR/did-core/

Inspired by:
https://www.w3.org/TR/vc-data-model/,
https://identity.foundation/presentation-exchange/spec/v2.0.0/

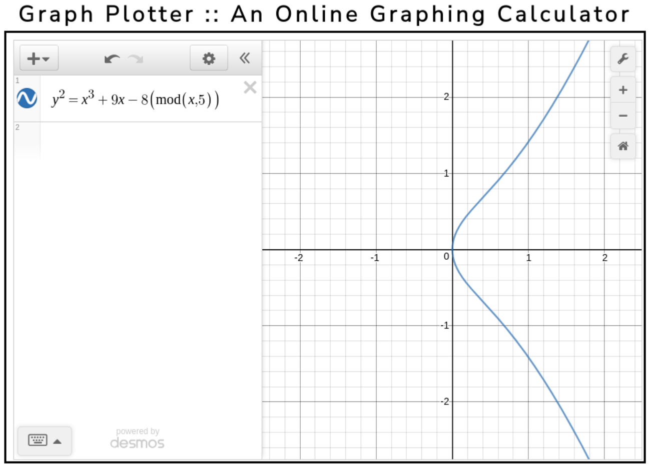
Elliptic Curves

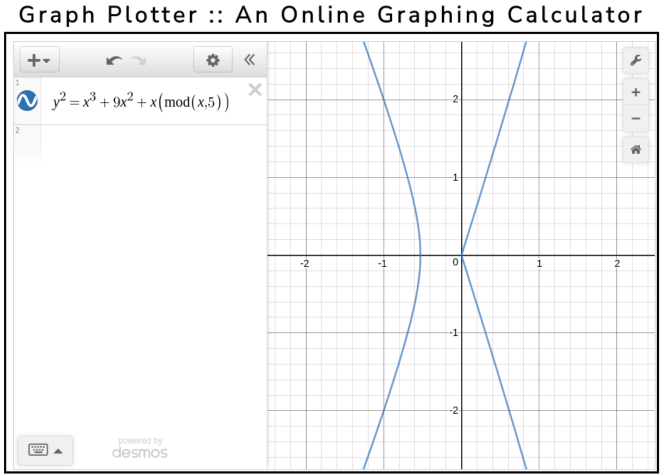
$Ax^3+Bx^2y+Cxy^2+Dy^3+Ex^2+Fxy+Gy^2+Hx+Iy+J=0$ General Elliptic Curve[1]

$y^2=x^3+ax+b$ Weierstrauss Form
{used for secp256k1, secp256r1, secp384rl, secp521r1}[2,3]

$y^2=x^3+ax^2+x$ Montgomery Form {used for ed25519}[3]

a and b are large integer constants in sources 2 and 3





1. https://mathworld.wolfram.com/EllipticCurve.html

2. http://www.secg.org/sec2-v2.pdf , pg. 9 - 12

3. https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-186-draft.pdf, pg. 38

Graph Plotter Link:
https://www.transum.org/Maths/Activity/Graph/Desmos.asp

Defintion of a Group

A group must have the properties:

Closure: For any **a** and **b**, ***a * b*** is also in the group

Associativity: For any **a**,**b**,**c** in a group, ***a * (b * c) = (a * b) * c***

Identity Element: For any **a** in the group ***a * 1 = a***

Inverse Element: For any **a** in the group, there is an $\mathbf{a^{-1}}$ as well, such that $\mathbf{a * a^{-1} = 1}$

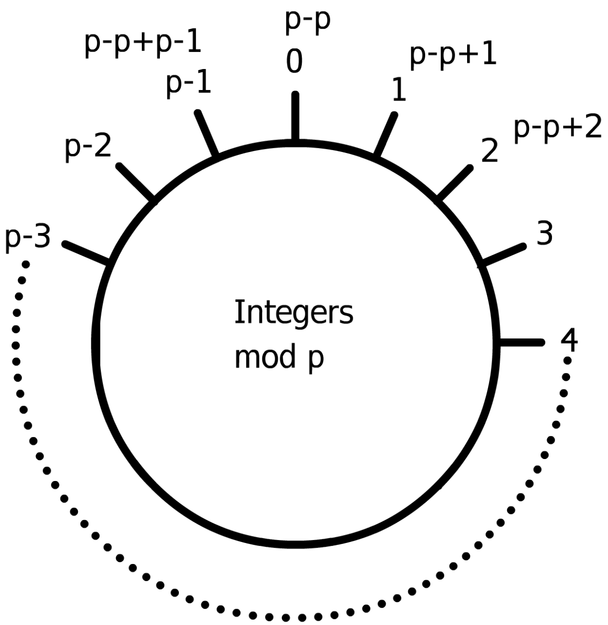
Quoting, page 92, Real World Cryptography, David Wong, Manning Publications

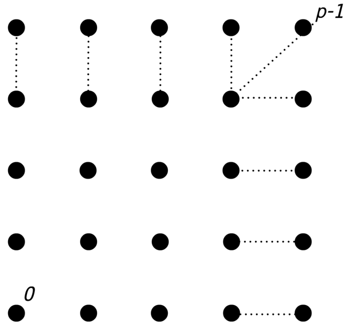
Groups in ECC

$y^2 = x^3 + ax + b \,(mod\ p)$

$y^2 = x^3 + ax^2 + x \,(mod\ p)$

F_p p is a large number in a finite integer Field





The curves only have integers as points

The points on the curve can form a cyclic group

The total number of points on the curve is called the order, and this is a prime number.

“That is, it is a set of invertible elements with a single associative binary operation, and it contains an element g such that every other element of the group may be obtained by repeatedly applying the group operation to g or its inverse. Each element can be written as an integer power of g in multiplicative notation, or as an integer multiple of g in additive notation. This element g is called a generator of the group.”
https://en.wikipedia.org/wiki/Cyclic_group

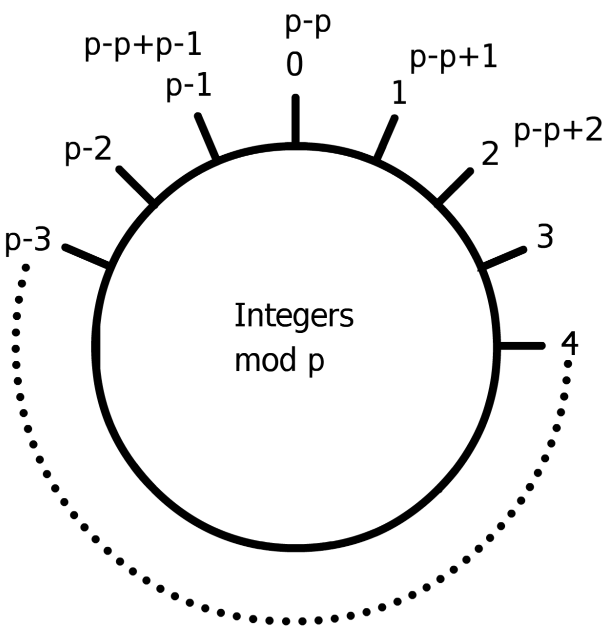
Groups in ECC

"An elliptic curve over a finite field can form a finite cyclic algebraic group [that is an order n that is prime][2], which consists of all points on the curve."

https://cryptobook.nakov.com/asymmetric-key-ciphers/elliptic-curve-cryptography-ecc#order-and-cofactor-of-elliptic-curve

Groups in ECC

F_p p is a large prime number in a finite Field



https://cryptobook.nakov.com/asymmetric-key-ciphers/elliptic-curve-cryptography-ecc

Cryptographic Signatures: ECDSA

To generate a signature $\{ r, s \}$:

$P = k * G$

$r = P_x$ $\quad s = k^{-1}(hash(m) + d_v * P_x) \, mod \; p$

k is a random secret number used once in the range [0...p-1]

P_x is the x-coordinate of P

p is the order of the subgroup of the points generated by G

d_v is the private signing key

m is the message

G is the generator point

Signature is not deterministic due the random number k

Validate the signature:

$s_m = s^{-1} \, mod \; p$ is the modular inverse of s

$R' = (hash(m) * s_m) * G + (r * s_m) * d_p$

if $R'_x = P_x$ the signature is valid

d_p is the public key

Real World Cryptography, David Wong, Manning, pg. 143 - 144

https://cryptobook.nakov.com/digital-signatures/ecdsa-sign-verify-messages

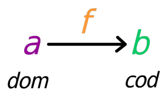
https://learn.saylor.org/mod/book/view.php?id=36341&chapterid=18920

Cryptographic Signatures: EdDSA

Definition of a Category

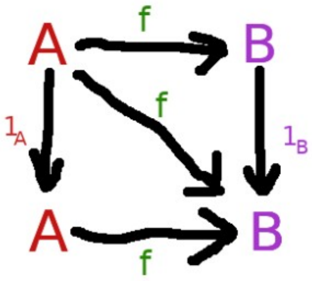
A category consists of:

- a collection of objects
- a collection of arrows



Identity:

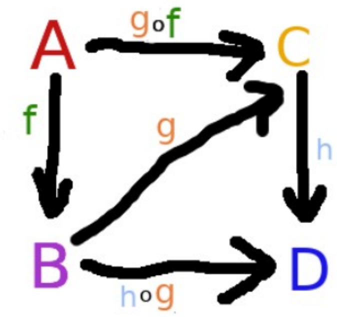
f o 1a = f = 1b o f



[2]

Associativity:

If morphism A → B is f, B → C is g , C → D is h then A → D is (h o g) o f = h o (g o f) = h o g o f

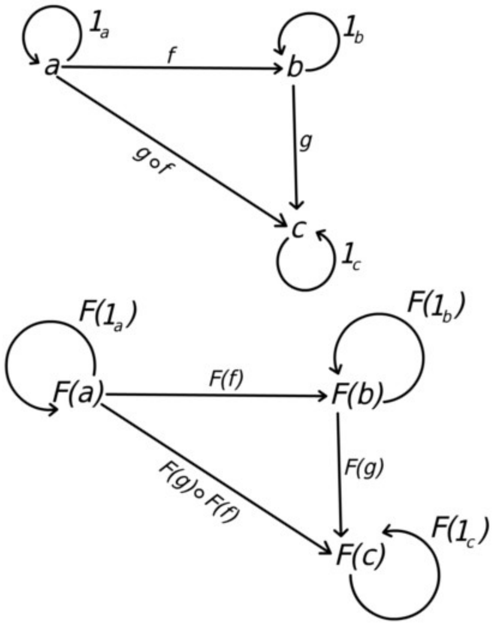


[2]

Uses of a Category

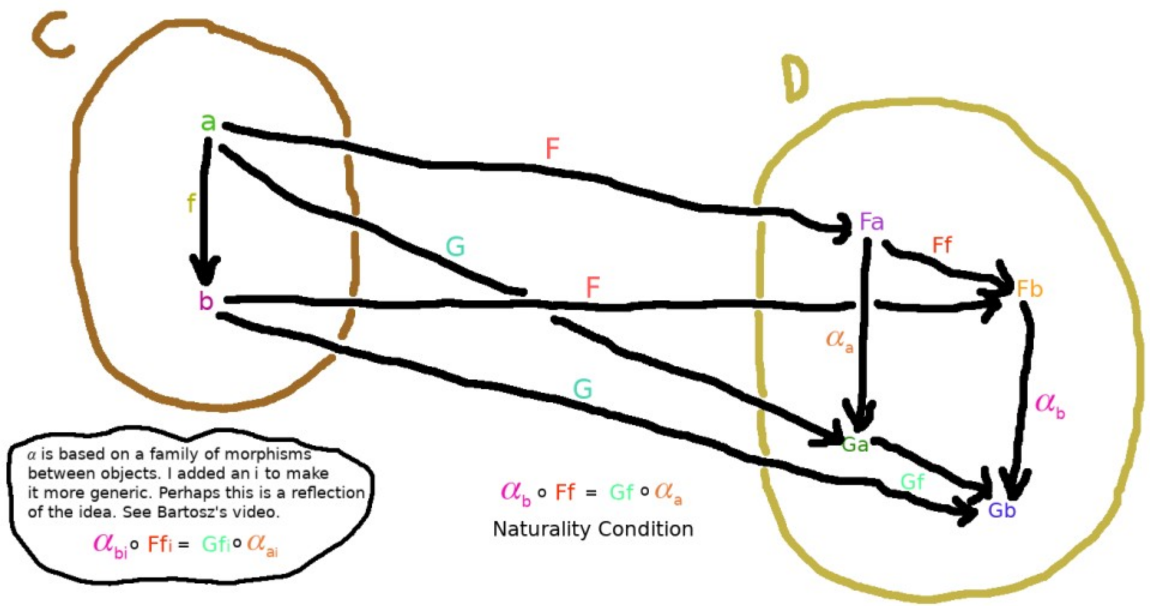
Definition of a Functor:

'A functor is a transformation from one category to another that "preserves" the categorical structure of its source'
pg. 194, The Categorical Analysis of Logic
-Goldblatt



Natural Transformations Condsider Functors to be Objects
pg. 198, Goldblatt

Definition of a Natural Transformation:



Groups as Categories

"In particular, a group is a category with one object, in which every arrow is an iso. If G and H are groups, regarded as categories, then we can consider arbitrary functors between them $f : G \to H$. It is obvious that a functor between groups is exactly the same thing as a group homomorphism."
pg. 72, chap 4, Category Theory, Steve Adowey

Syntactic and Semantic Mappings

- Use RDF serializations like JSON-LD, JSON-Schema
- Cryptographic proof of data:
  https://www.w3.org/TR/vc-data-integrity/
  → Binary or RDF Canonicalization
  https://w3c-ccg.github.io/rdf-dataset-canonicalization/spec/index.html

Burak Sedar's comments in e-mail about interop.

Solutions Out in the Wild

- What does FQL, CQL, and Hydra Do?
- What does Project Cambria Do?
- What does Layered Schema Architecture Do?
- What does Overlay Schema Architecture?
- Benjamin Braatz Thesis