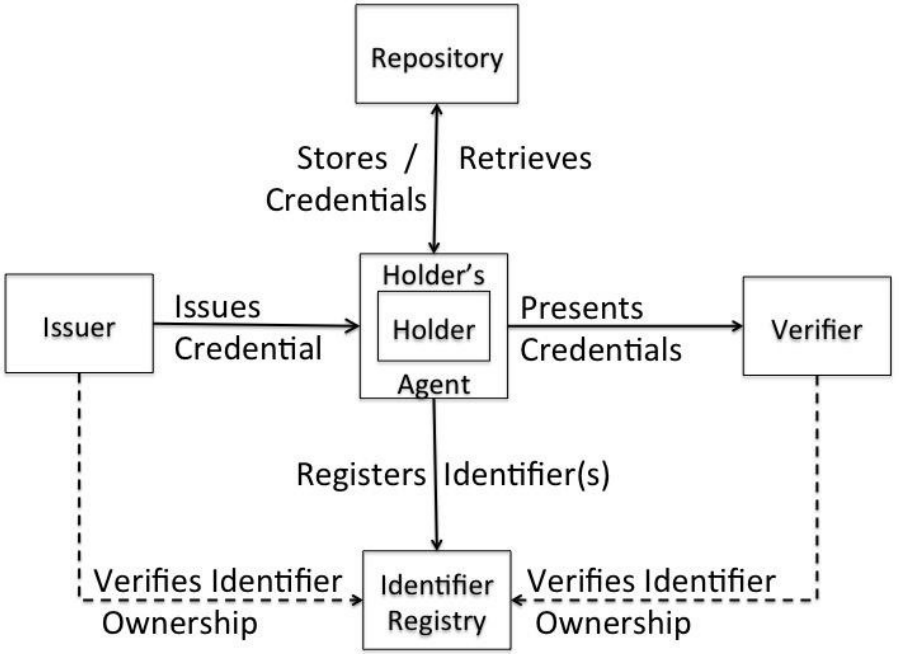
Explorations of Category Theory for Verifiable Credentials

Internet Identity Workshop # 35

Brent Shambaugh

Verifiable Credentials Lifecycle



https://w3c-ccg.github.io/vc-lifecycle/

Identifier Registry contains DIDs and possibly full or a hash representation of DID Documents and/or Schemas?

Decentralized Identifier Architecture

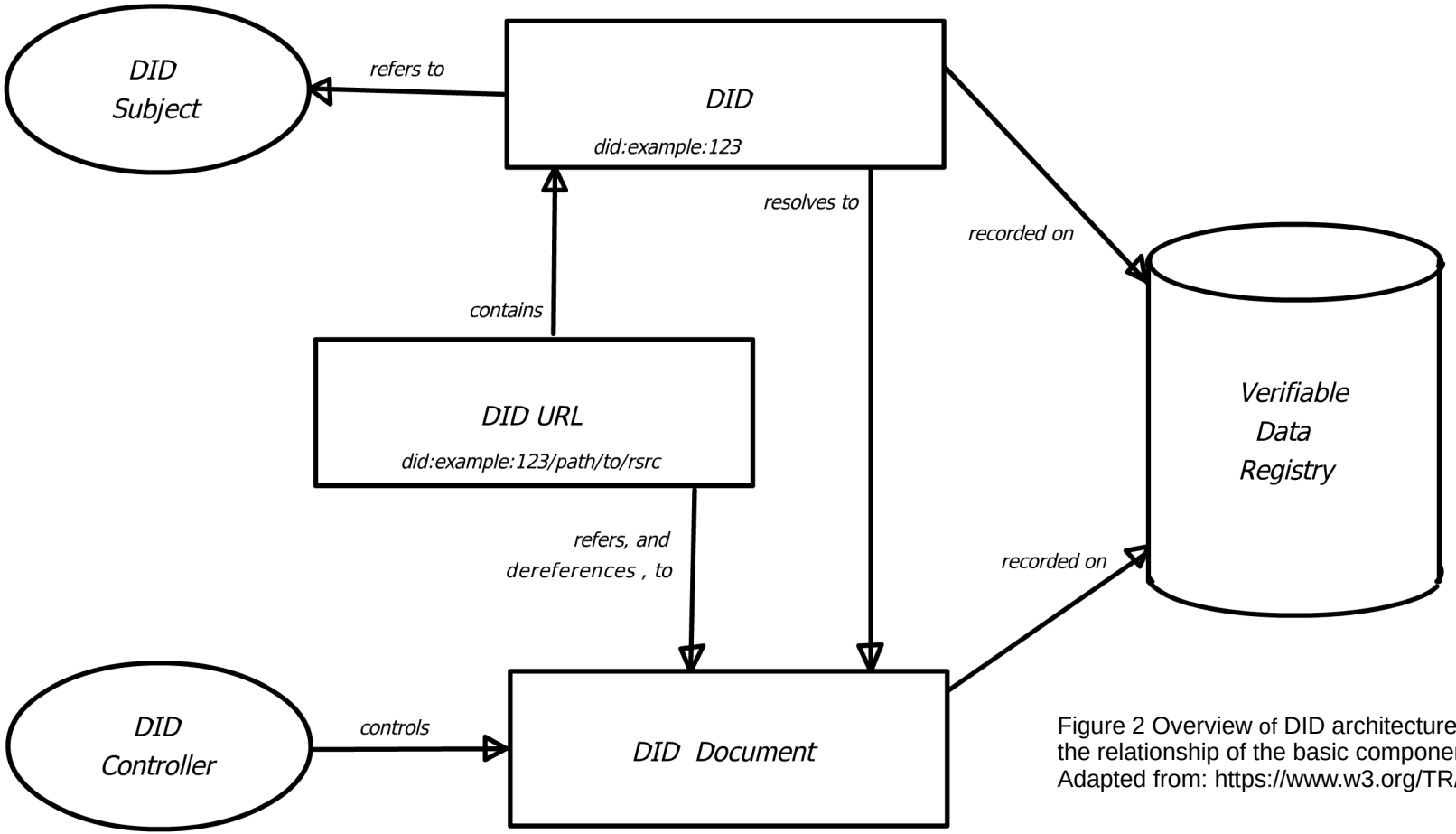
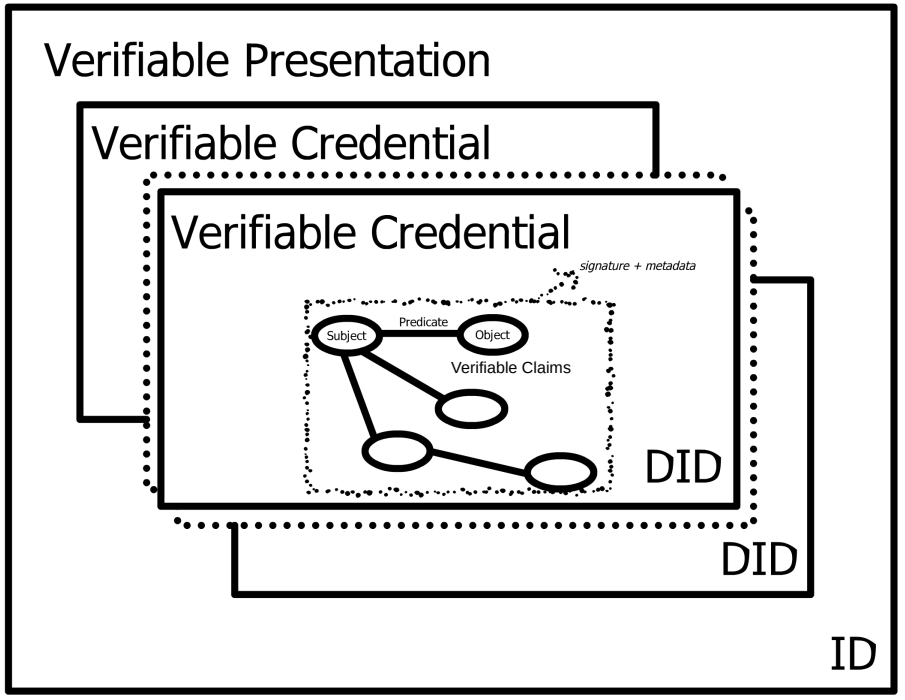
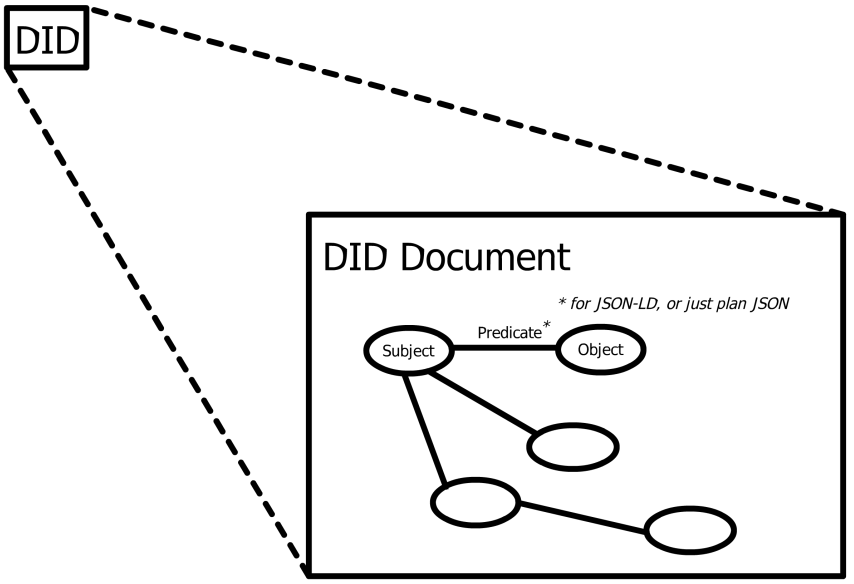


Figure 2 Overview of DID architecture and the relationship of the basic components. Adapted from: https://www.w3.org/TR/did-core/

Data Models



Inspired by:
https://www.w3.org/TR/vc-data-model/,
https://identity.foundation/presentation-exchange/spec/v2.0.0/



Inspired by:
https://www.w3.org/TR/did-core/

Elliptic Curves

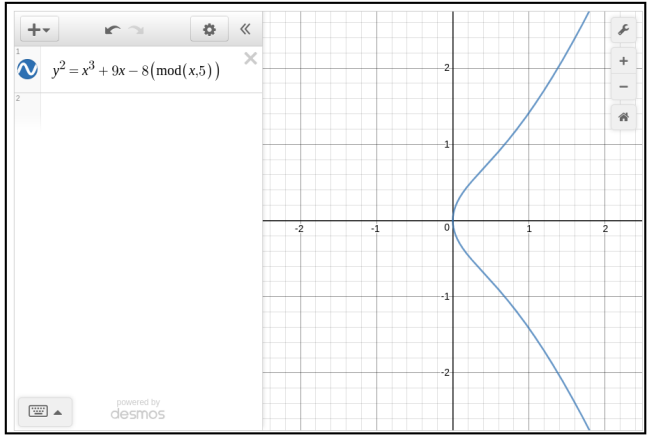
$Ax^3+Bx^2y+Cxy^2+Dy^3+Ex^2+Fxy+Gy^2+Hx+Iy+J=0$ General Elliptic Curve[1]

$y^2=x^3+ax+b$ Weierstrauss Form
{used for secp256k1, secp256r1, secp384rl, secp521r1}[2,3]

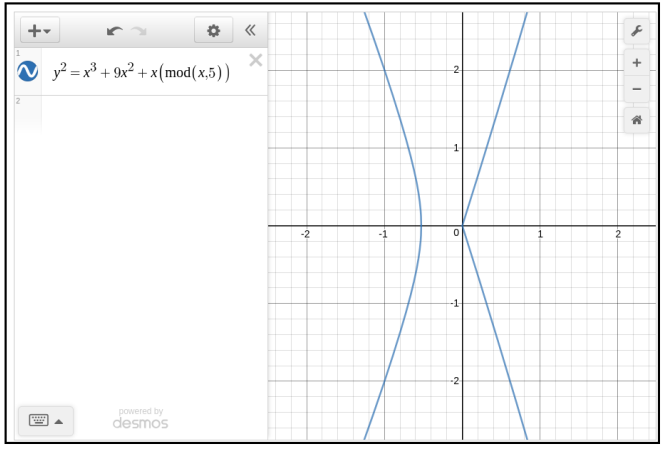
$y^2=x^3+ax^2+x$ Montgomery Form {used for ed25519}[3]

a and b are large integer constants in sources 2 and 3

Graph Plotter :: An Online Graphing Calculator



Graph Plotter :: An Online Graphing Calculator



1. https://mathworld.wolfram.com/EllipticCurve.html
2. http://www.secg.org/sec2-v2.pdf , pg. 9 - 12
3. https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-186-draft.pdf, pg. 38

Graph Plotter Link:
https://www.transum.org/Maths/Activity/Graph/Desmos.asp

Defintion of a Group

A group must have the properties:

Closure: For any **a** and **b**, ***a * b*** is also in the group

Associativity: For any **a**,**b**,**c** in a group, ***a * (b * c) = (a * b) * c***

Identity Element: For any **a** in the group ***a * 1 = a***

Inverse Element: For any **a** in the group, there is an $\mathbf{a^{-1}}$ as well, such that $\mathbf{a * a^{-1} = 1}$

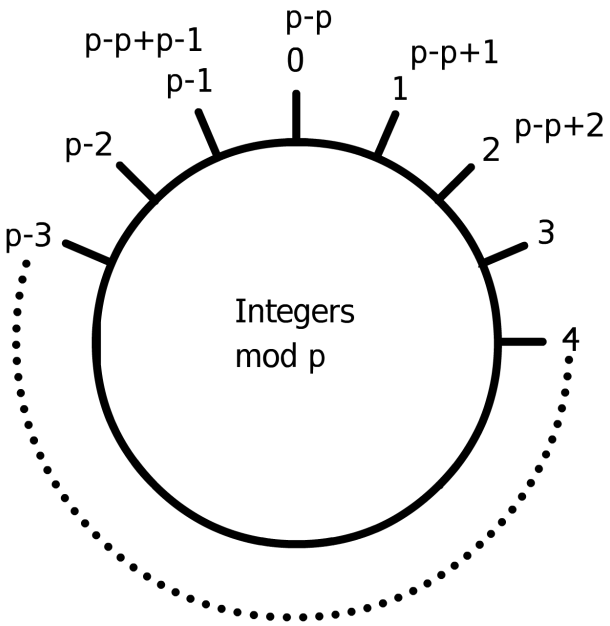
Quoting, page 92, Real World Cryptography, David Wong, Manning Publications

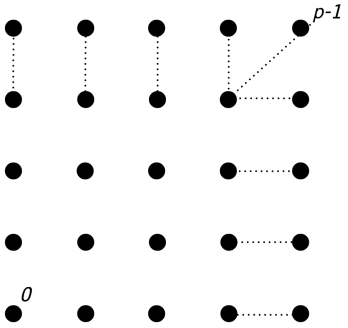
Groups in ECC

$y^2 = x^3 + ax + b \pmod p$

$y^2 = x^3 + ax^2 + x \pmod p$

F_p p is a large number in a finite integer Field





The curves only have integers as points

The points on the curve can form a cyclic group

The total number of points on the curve is called the order, and this is a prime number.

“That is, it is a set of invertible elements with a single associative binary operation, and it contains an element g such that every other element of the group may be obtained by repeatedly applying the group operation to g or its inverse. Each element can be written as an integer power of g in multiplicative notation, or as an integer multiple of g in additive notation. This element g is called a generator of the group.”
https://en.wikipedia.org/wiki/Cyclic_group

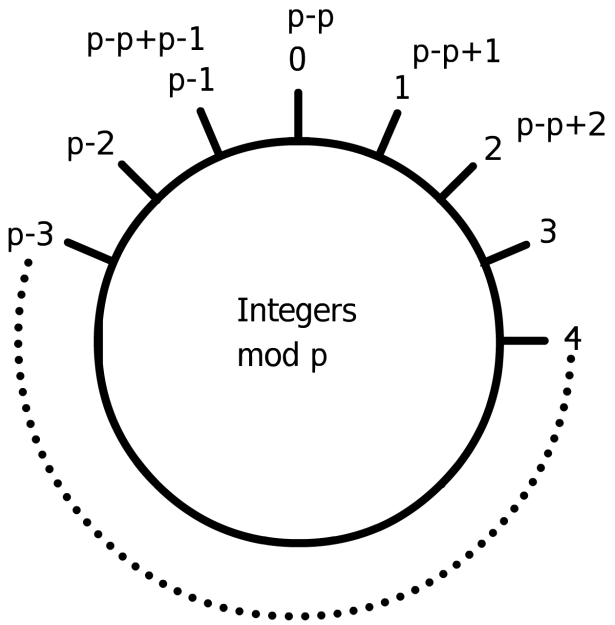
Groups in ECC

"An elliptic curve over a finite field can form a finite cyclic algebraic group [that is an order n that is prime][2], which consists of all points on the curve."

https://cryptobook.nakov.com/asymmetric-key-ciphers/elliptic-curve-cryptography-ecc#order-and-cofactor-of-elliptic-curve

Groups in ECC

F_p p is a large prime number in a finite Field



https://cryptobook.nakov.com/asymmetric-key-ciphers/elliptic-curve-cryptography-ecc

Cryptographic Signatures: ECDSA

To generate a signature $\{ r, s \}$:

$P = k * G$

$r = P_x \qquad s = k^{-1}(hash(m) + d_v * P_x) \bmod p$

k is a random secret number used once in the range [0...p-1]

P_x is the x-coordinate of P

p is the order of the subgroup of the points generated by G

d_v is the private signing key

m is the message

G is the generator point

Signature is not deterministic due the random number k

Validate the signature:

$s_m = s^{-1} \bmod p$ is the modular inverse of s

$R' = (hash(m) * s_m) * G + (r * s_m) * d_p$

if $R'_x = P_x$ the signature is valid

d_p is the public key

Real World Cryptography, David Wong, Manning, pg. 143 - 144

https://cryptobook.nakov.com/digital-signatures/ecdsa-sign-verify-messages

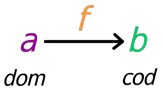
https://learn.saylor.org/mod/book/view.php?id=36341&chapterid=18920

Cryptographic Signatures: EdDSA

Definition of a Category

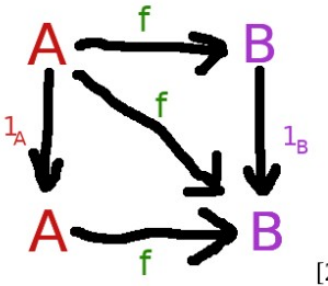
A category consists of:

- a collection of objects
- a collection of arrows



Identity:

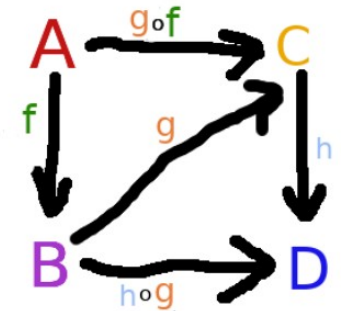
f o 1a = f = 1b o f



[2]

Associativity:

If morphism A → B is f, B → C is g , C → D is h then A → D is (h o g) o f = h o (g o f) = h o g o f

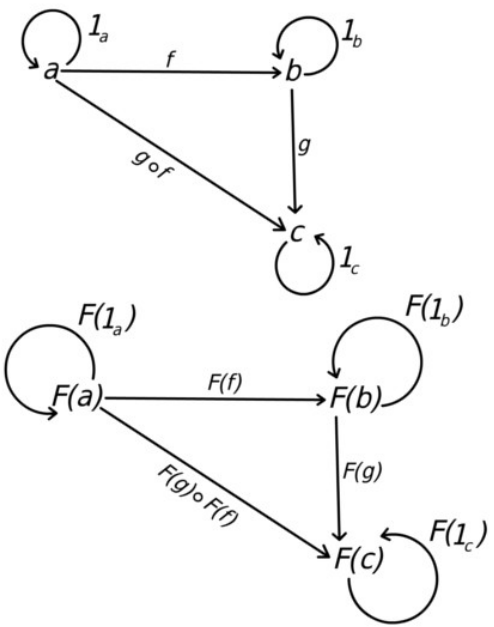


[2]

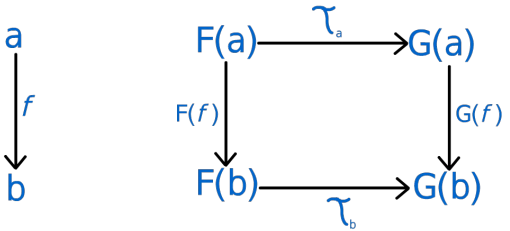
Uses of a Category

Definition of a Functor:

'A functor is a transformation from one category to another that "preserves" the categorical structure of its source'
pg. 194, The Categorical Analysis of Logic
-Goldblatt

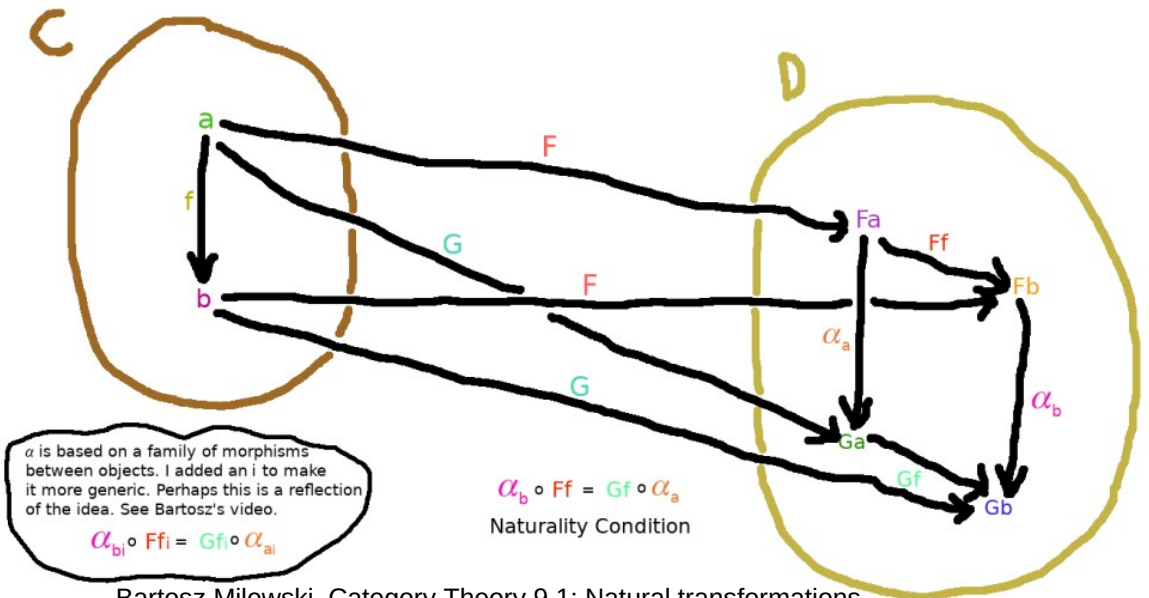


Natural Transformations Consider Functors to be Objects
pg. 198, Goldblatt



pg. 199, Goldblatt

Definition of a Natural Transformation:



Bartosz Milewski, Category Theory 9.1: Natural transformations
https://www.youtube.com/watch?v=2LJC-XD5Ffo

Groups as Categories

“In particular, a group is a category with one object, in which every arrow is an iso. If G and H are groups, regarded as categories, then we can consider arbitrary functors between them f : G → H. It is obvious that a functor between groups is exactly the same thing as a group homomorphism.”
pg. 72, chap 4, Category Theory, Steve Adowey

Syntactic and Semantic Mappings

- Use RDF serializations like JSON-LD, JSON-Schema
  SON Schema, Schema.org, JSON-LD: What's the Difference?
  https://dashjoin.medium.com/json-schema-schema-org-json-ld-whats-the-difference-e30d7315686a
- Cryptographic proof of data:
  https://www.w3.org/TR/vc-data-integrity/
  →Binary or RDF Canonicalization
  https://w3c-ccg.github.io/rdf-dataset-canonicalization/spec/index.html

  Burak Sedar's comments in e-mail about interop. In his opinion Indenity does not consider semantics per se, but "LSA is a framework that aims semantic interoperability, so it offers some interesting ways to pass data through lenses. We do it by converting everything to labeled property graphs, and modifying graphs using openCypher in memory."
  To him, working with RDF only ensures structural interoperabilty, but he went a level beyond and considered semantic interoperability. See the blog post: https://cloudprivacylabs.com/blog/post-20220831/ and the paper: A Demonstration of Layered Schema Architecture as a Semantic Harmonization Tool, Burak Serdar, Cloud Privacy Labs https://ceur-ws.org/Vol-3249/paper3-OSS.pdf.

Solutions Out in the Wild

- What does FQL, CQL, and Hydra Do?
  CQL supersedes FQL. Open source at: https://www.categoricaldata.net/, https://github.com/CategoricalData
  ”Open-source CQL and its integrated development environment (IDE) performs data-related tasks — such as querying, combining, migrating, and evolving databases — using category theory, a branch of mathematics that has revolutionized several areas of computer science. “

  https://github.com/CategoricalData/hydra (Ryan Wisenesky and Josh Shinavier)
  Hydra is based on Dragon. https://eng.uber.com/dragon-schema-integration-at-uber-scale/ [Dragon: Schema Integration at Uber Scale, Uber Engineering] --→ Provides Mappings between Data Models
  and based on Algebraic Property Graphs in Shinavier et al., https://arxiv.org/abs/1909.04881
  ”Observing that algebraic data types are a common foundation of most of the enterprise schema languages we deal with in practice, for graph data or otherwise, we introduce a type theory for algebraic property graphs wherein the types denote both algebraic data types in the sense of functional programming and join-union E/R diagrams in the sense of database theory. We also provide theoretical foundations for graph transformation along schema mappings with by-construction guarantees of semantic consistency. “

Solutions Out in the Wild (Cont.)

- What does Project Cambria Do? https://fission.codes/blog/project-cambria-overview/ (Focuses on how to read data in distributed systems, rather than using for writes to distributed systems. Converts: JSON, JSON PATCH, JSON Schema)

- What does Layered Schema Architecture Do? https://layeredschemas.org/, https://github.com/cloudprivacylabs/lsa
- What does Overlay Schema Architecture? https://oca.colossi.network/guide/introduction.html, Introduction to Overlays Capture Architecture (https://www.youtube.com/watch?v=D_Bqy4Mc514) . It is an archtitecture, not an ontology. It defines rules on how the data should be captured. Works as a data transformation pipeline.
  → Overlays Capture Architecture, a global solution for data capture and semantic harmonization. Uses a broadly defined schema base (seemingly loosely defined like Hydra), and then builds cryptographicly linked layers on top
- Benjamin Braatz Thesis (suggested by Ryan Wisenesky)
  ""While more sophisticated languages for the definition of ontologies have been developed, e. g., the Web Ontology Language (OWL), presented in [MH04] and related documents, we will only deal with RDF and its schema definition language RDF Schema in this thesis."