Project Report

Heat equation: It is a 2nd order differential equation that describes how temperature of a material varies in space with time. The aim is not to solve it analytically but numerically. First, we discretise the surface into nodes. For each node, we write the differential equation governing its temperature and convert it into an IVP.

Task one: You are given a one-dimensional rod whose temperature is set to 1 at time t=0. The temperature of the first node is made to vary sinusoidally with time. Additionally, d phi/d x for last node is given 0. The task is to determine how the temperature of other nodes is varying with time.

Methodology used:

Let us assume for simplicity that we have 10 nodes. The temperature of the first node is already specified in the problem statement. Thus, the task is to find temperature of 9 successive nodes.

Firstly, semi-discretisation is used in over to write one differential equation for node number 2 to 9. Ghost-node technique is used to write discretised differential equation for node 10. Thus, we obtain 9 IVP's with 9 variables. These were solved on MATLAB.

In the first iteration, the so obtained system of IVP's was solved using Euler forward method.

In the second iteration, the system was solved using unconditional theta scheme. In a nutshell, A*Y(t+1) = B(t) was solved to successively obtain Y(t+1), i.e. next state temperatures of 9 nodes.

In the third iteration, the code was simplified by introduction of new_phi variable. In this new code, the temperature of previous states was no longer stored.

Task two: One-dimensional rod is extrapolated to a two-dimensional heat plate. The temperature of one edge of the plate is varying sinusoidally with time. The temperature of other three edges is set to 1. The task is to determine how the temperature of nodes other than those at boundary is varying with time.

Methodology used:

Let us say a is the number of (unknown) nodes along the length of the plate and b along the breadth. Therefore, the number of nodes whose temperature need to be found out is a*b. For each node, we have a corresponding heat equation. This makes it a system of IVP's with number of variables (temperature of nodes) equal to a*b. The partial differential equation at each node is firstly discretised, then theta scheme is used to convert it to an equation containing (t+1) level variables and (t) level variables. These equations are properly written down in matrix format as A*Y(t+1) = B(t) and hence Y(t+1) is found. Here, Y(t+1) denotes a column matrix containing temperature of all nodes.

Structure of code:

The user needs to provide with the length of the square plate, delta x, delta y, delta t and theta. The coefficient matrix A was typed on MATLAB with the help of loops and then the system of linear equations was solved using MATLAB functions.

In the first iteration, k (diffusion coefficient) was kept constant.

In the second, it was made to vary linearly with x.

In the third one, it was made to vary arbitrarily with x and y. In fact, the user gets to decide what will be the value of k at each node.

Next, the final temperature of the nodes was plotted using a surf plot on MATLAB. Finally, a simulation of heat map was made, which showed how the temperature of nodes changed at each instant of time.

Task three: The governing equation of heat flow is changed. The set up remains the same. The objective again is to determine how the temperature of nodes other than those at boundary is varying with time.

Methodology: After semi-discretizing, we get a system of non-linear equations to solve. These equations in theory, need to be solved iteratively by the used of numerical methods (newton Raphson method). In MATLAB, function called fsolve greatly simplifies the work. In a nutshell, these non-linear equations were fed in fsolve fuction with the help of for loops and temperatures after each time step were obtained.

In the first iteration iteration, k was made to vary arbitrarily in space.

Next, k of each node was made to vary as a function of current temperature of that node.

Heat map simulation was created to observe the variation of temperature and flow of heat.