TIES483 Nonlinear Optimization

Decision-making under Uncertainty (part1)

spring 2022

Babooshka Shavazipour, PhD
Babooshka.b.shavazipour@jyu.fi

Contents

Learning outcomes

- To understand the importance of handling uncertainty in real-world problems
- To understand different types of uncertainty in (MO)OPs
- To understand how to deal with uncertainty in (MO)OPs
- Example approaches

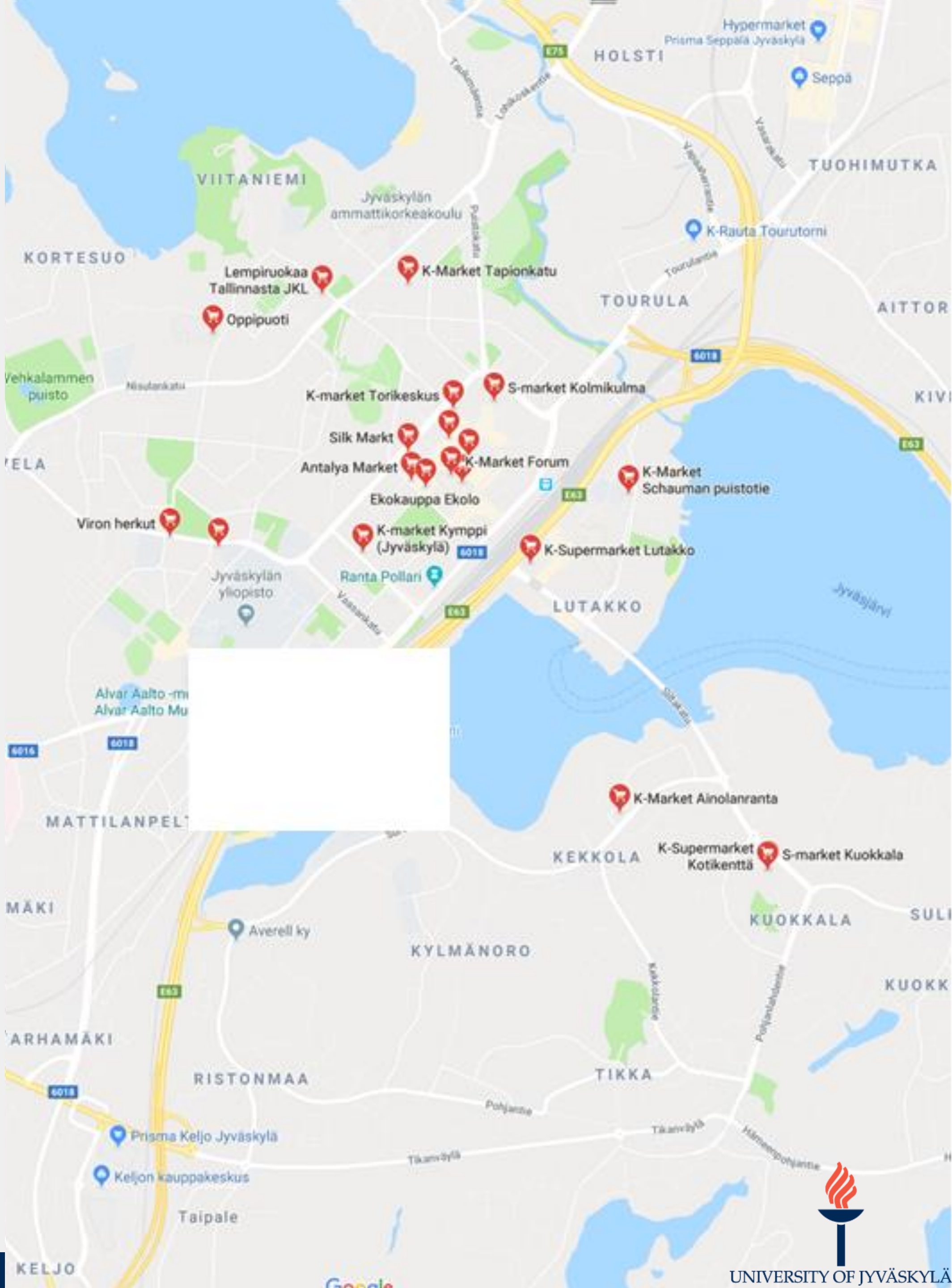
Decision-making under uncertainty

- So far, we assumed that all the required information are certainty known.
- i.e. all the parameters, objectives, constraints are certain without any variation.
- However, most of the decisions in real-life problems need to be made in the absence of complete knowledge about the consequences of the decision.
- The presence of uncertainty brings more complexity to the problem.

Uncertainty:

General definitions (Metaphorical map)

- If a complete detailed map were reached; i.e. we know everything, with absolute certainty
- Consequences of all alternatives could be seen and compared → Choose the most preferred one.
- However, sometimes, the decisions need to be made when the complete, detailed map is unavailable.
- **Lack of any details in the map** could be viewed as *uncertainty*.



Different definitions of uncertainty

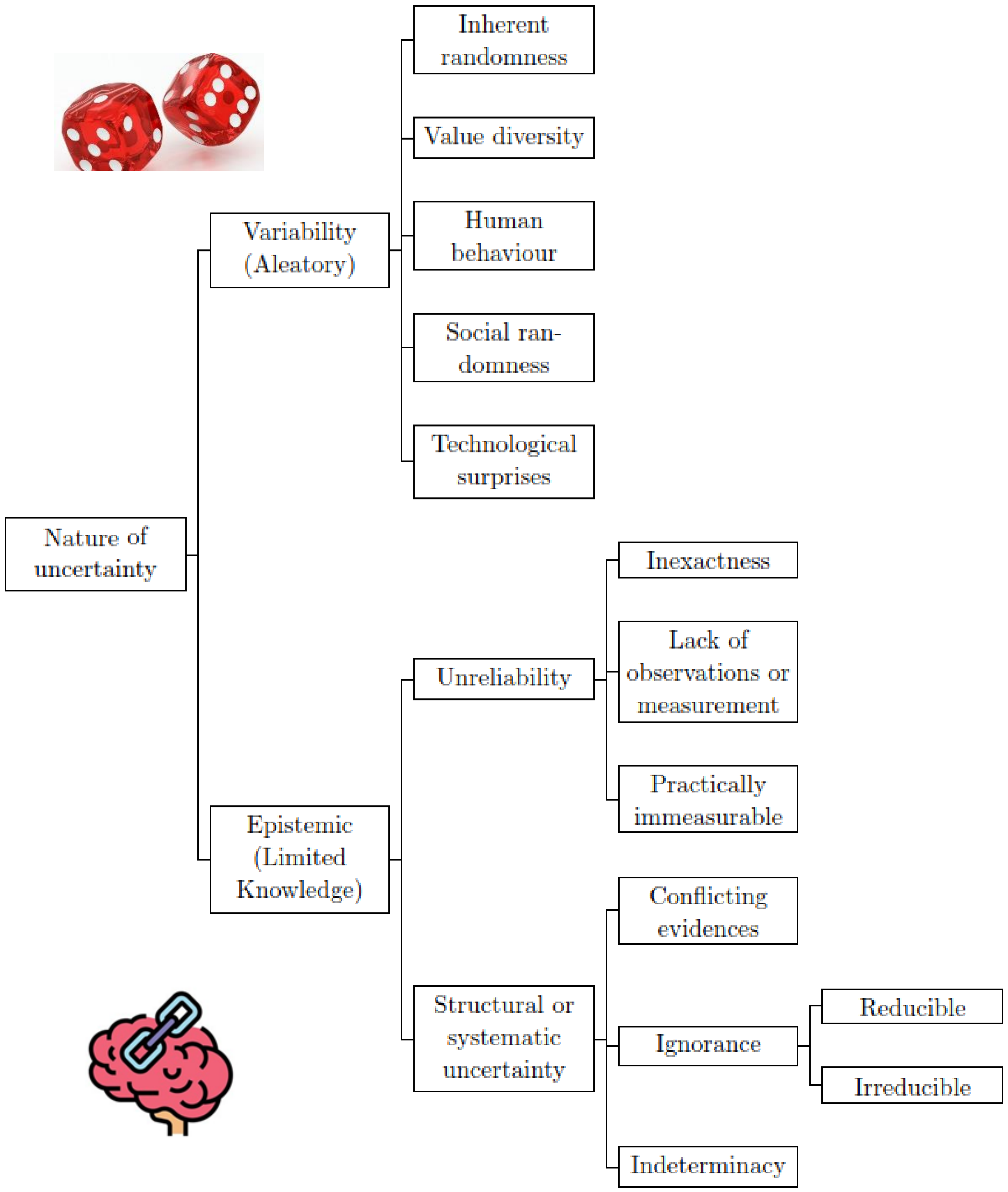
- “Any deviation from the unachievable ideal of completely deterministic knowledge
- of the relevant system” [Walker et al., 2003].
- “At a most fundamental level, uncertainty relates to a state of the human mind,
- i.e. lack of complete knowledge about something” [Stewart, 2005].
- “Incomplete information about a particular subject” [Ascough II et al., 2008].
- “Lack of confidence in knowledge related to a specific question” [Sigel et al., 2010].
- “In general, uncertainty can be defined as limited knowledge about the future, the
- past, or current events” [Walker et al., 2013a].

Important note!

- Uncertainties are not merely caused by the lack of knowledge and information
- Uncertainty can exist in a situation where lots of information are available
- However, new knowledge can either increase or decrease uncertainty
- New information and knowledge on a complex process can unfold previously unknown uncertainties
- In this case, more knowledge clarifies that the processes are more complex or that our cognition is more limited than previously thought

Classification of uncertainty

- Three sources/dimensions of uncertainty related to model-based decision support exercises:
  - **Nature**:
    - Variability (Aleatory) – *Alea (Latin)* → *Dice*
    - Epistemic (Limited knowledge)



Different sources of the nature of uncertainty

Classification of uncertainty

- Three sources/dimensions of uncertainty related to model-based decision support exercises:
  - **Nature**:
    - Variability (Aleatory) – *Alea (Latin) → Dice*
    - Epistemic (Limited knowledge)
  - **Location:**

    The where the uncertainty manifests itself within the model complex (Models, Outcomes, Weights, etc.)
    - Internal (DM judgement and preferences)
    - External (lack of knowledge about outcomes)

Author(s)	Different categories	
French [1995]	Uncertainties expressed during modeling	What might happen or what can be done
		Meaning/ambiguity
		Related decisions
	Uncertainties expressed during exploration of the models	Physical randomness or lack of knowledge
		The evolution of future beliefs and performances
		Judgments
		The accuracy of calculations
	Uncertainty expressed during interpretation	Appropriateness of a descriptive model
		Appropriateness of a normative model
		the depth of analysis
Walker et al. [2003]	Context	
	Model	Structure Technology
	Input	Controllable Uncontrollable
	parameter Model outcome	
Stewart [2005]	Internal External	
Hawkins and Sutton [2009]	Internal variability of the system Model Uncertainty Scenario Uncertainty	
Marchau et al. [2010]	The value system(s) to be used to rank alternative policies The system models How the future will develop	
Kwakkel and Walker [2010]	External Forces Relations within a system Outcomes of interest Weights	
Walker et al. [2013b]	Context System model System Outcome Weights Weights on outcome	

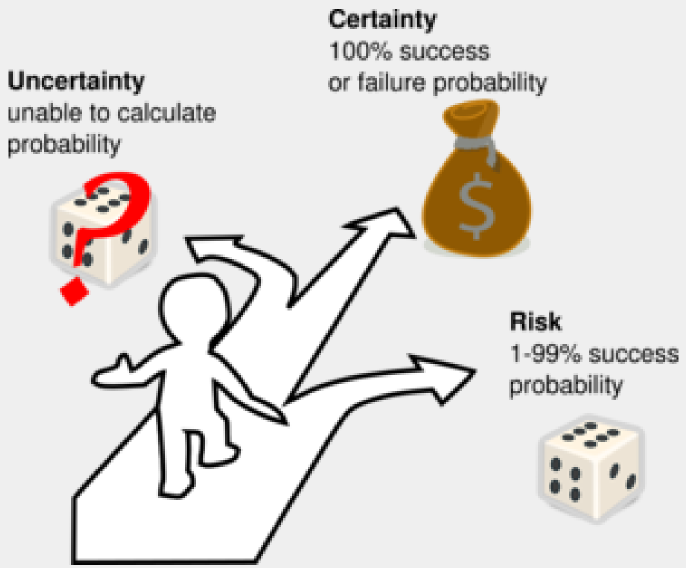
Different classifications of the location of uncertainty.

Classification of uncertainty

- Three sources/dimensions of uncertainty related to model-based decision support exercises:
  - **Nature**:
    - Variability (Aleatory)
    - Epistemic (Limited knowledge)
  - **Location:**

    The where the uncertainty manifests itself within the model complex (Models, Outcomes, Weights, etc.)
    - Internal (DM judgement and preferences)
    - External (lack of knowledge about outcomes)
  - **Depth or Level**:

    The where the uncertainty manifests itself along the spectrum between absolute certainty and total ignorance.



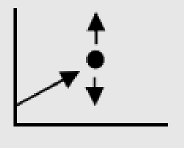
<table>
<tr><td>Shavazipour [2018]</td><td rowspan="9">Uncertainty of degree 0 (complete certainty)</td><td colspan="2">Mild uncertainty (First-degree)</td><td colspan="2">Moderate uncertainty (Second-degree)</td><td>Deep uncertainty (Third-degree)</td><td rowspan="9">Infinity degree of uncertainty (total ignorance)</td></tr>
<tr><td>Walker et al. [2013a]</td><td>Level 1
(A clear
enough future)</td><td>Level 2
(Alternate futures
with probabilities)</td><td>Level 3
(Alternate futures
with ranking)</td><td>Level 4
(Multiplicity
of futures)</td><td>Level 5
(Unknown
future)</td></tr>
<tr><td rowspan="2">Walker et al. [2003]</td><td colspan="2" rowspan="2">statistical uncertainty</td><td rowspan="2">scenario uncertainty</td><td colspan="2">Recognized ignorance</td></tr>
<tr><td>Reducible</td><td>Irreducible</td></tr>
<tr><td>Kwakkel et al. [2010]</td><td colspan="2">Shallow Uncertainty (Level 1)</td><td>Medium Uncertainty (Level 2)</td><td>Deep Uncertainty (Level 3)</td><td>Recognized Ignorance (Level 4)</td></tr>
<tr><td>Courtney [2001]</td><td>Level 1 (so low that the traditional methods that employ point forecasts can be used with great success)</td><td>Level 2 (manager can identify a set of distinct possible outcomes, one of which will occur)</td><td colspan="2">Level 3
(manager can bound
the range of
possible outcomes)</td><td>Level 4
(analysis cannot
even bound the range
of possibilities)</td></tr>
<tr><td>Morgan et al. [1992]</td><td colspan="2">Uncertainties can be treated through probabilities</td><td colspan="3">Uncertainties cannot be treated probabilistically</td></tr>
<tr><td>Quade [1989]</td><td colspan="4">Stochastic uncertainty</td><td>Real uncertainty</td></tr>
<tr><td>Knight [1921]</td><td colspan="4">Risk</td><td>Uncertainty</td></tr>
</table>

Different classifications of the depth of uncertainty.

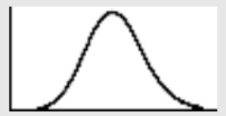
Classification of uncertainty

- ➢ We use the term degree of uncertainty as depth or level of uncertainty.
- ➢ Uncertainty of degree ‘0’ as absolute certainty or deterministic knowledge.
- ➢ Uncertainty of infinity degree as total ignorance.

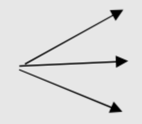
Degrees of Uncertainty



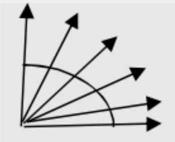
- **Absolute Certainty (clear enough future – deterministic)**



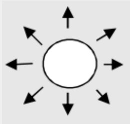
- **Mild uncertainty (1st):** Outcomes can be enumerated, and probabilities (or probability distribution) are specified (probable futures).



- **Moderate uncertainty (2nd):** Outcomes can be enumerated but probabilities are difficult to specify generally (few possible/plausible futures).
- **Deep uncertainty (3rd):** Outcomes cannot be completely enumerated, so that, probabilities are not definable (many plausible futures).



- No nothing



Shavazipour, B., & Stewart, T. J. (2019). Multi-objective optimisation under deep uncertainty. *Operational Research*, 1-29.

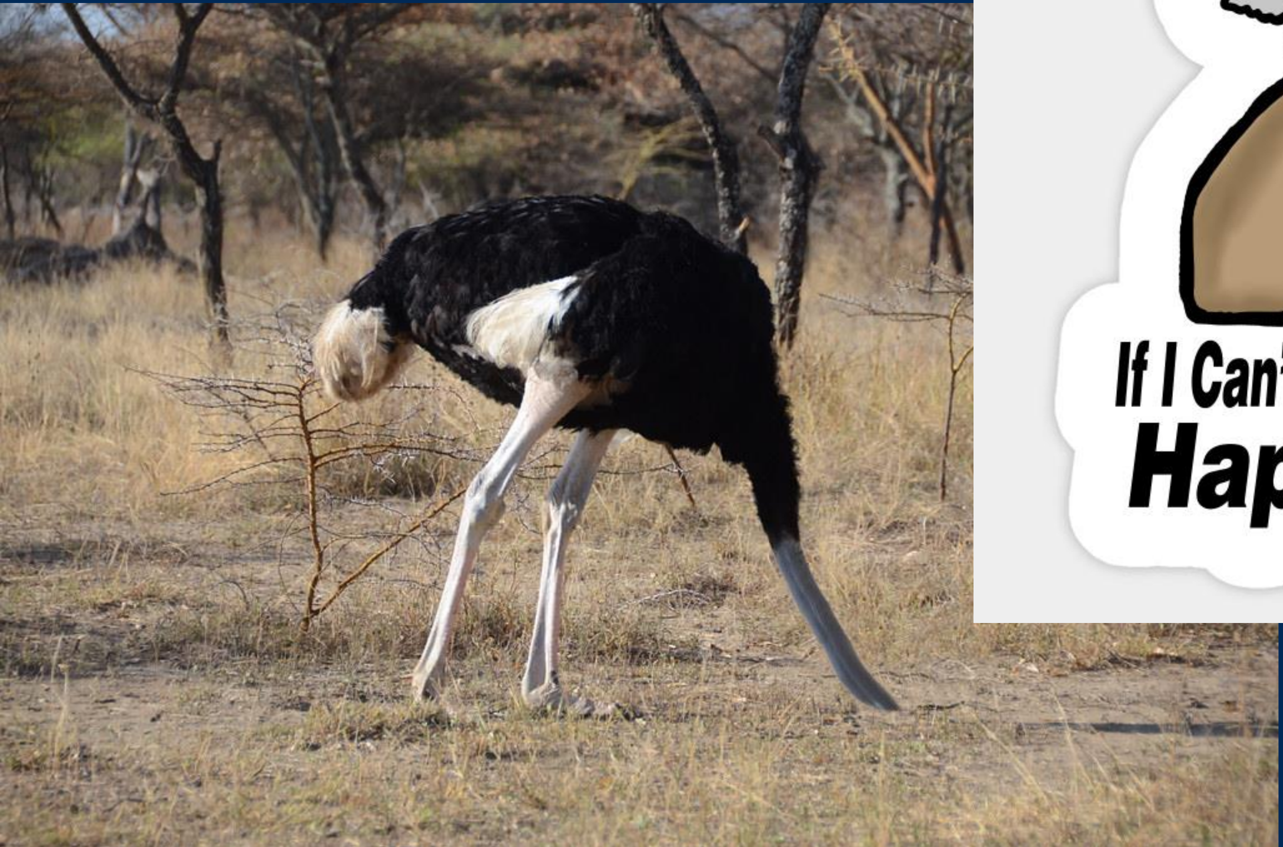
A more general definition of Deep Uncertainty

- A situation in which the relevant actors do not know or cannot agree upon:
  - how likely or plausible various future states are
  - how the system works (or would work)
  - how to value the various outcomes of interest

* Walker, Warren E., Robert J. Lempert, and Jan H. Kwakkel (2013). "Deep Uncertainty", entry (pp. 395-402) in Saul I. Gass and Michael C. Fu (eds.), Encyclopedia of Operations Research and Management Science., 3 rd Edition, New York: Springer.

Dealing with uncertainty

Why must uncertainty be treated?



Dealing with uncertainty

Why must uncertainty be treated?

- Many people have ignored uncertainty for many years
- Still, some try to avoid the higher degrees of uncertainty
- However, they must face this challenge eventually

Why must uncertainty be treated?

COVID-19 Pandemic (2019-?)

- **In 2018**: The probability of an extreme epidemics was < **0.5 percent in a year**
- **31.12.2019**: The first COVID-19 case is reported
- **31.08.2021**: the probability of a pandemic with similar impact to COVID-19 is **about 2% in any year***
- **15.09.2021:** most countries removed the restrictions and said the pandemic was over
- **10.11.2021: 250 million** cases, **> 5 million death, …**
- **21.11.2021:** a new variant has found in South Africa (Omicron)
- **14.12.2021:** many restrictions came back
- **01.02.2022:** many goverments say we are entering the endemic Covid. (Do we??)
- **21.02.2022**: **> 424 million** cases, **> 5.89 million death, + 4996 death since yesterday**

* Marani, M., Katul, G. G., Pan, W. K., & Parolari, A. J. (2021). Intensity and frequency of extreme novel epidemics. *Proceedings of the National Academy of Sciences*, *118*(35).

Why must uncertainty be treated?

A major challenge is the requirement to accept, understand, and manage uncertainty, since:

1. Not all uncertainties can be eliminated;

2. Ignoring uncertainty limits our ability to make corrective action in the future and result in positions that could have been avoided;

3. Ignoring uncertainty can throw away the opportunity of studying real-world problems, and/or lead to get some unsustainable approaches.

Dealing with uncertainty

- **Traditional** applied scientific works usually suppose that the existing uncertainties are caused by either
  - **Lack of knowledge** → **Uncertainty reduction** by increasing the information
  - **Random variation** → **Stochastic** processes and **statistical** analysis

Optimzation under uncertainty

- Mild uncertainty, which is commonly encountered by decision makers in short-term planning, is mostly treated by statistical approaches and probability theory while long-term programming, which largely involves deeper uncertainties, cannot deal with regular probability models and regular statistic approaches.
- We need to use some dynamic approaches which could be more robust, adaptive.

Moltiobjective optimzation under uncertainty

$$\text{Max } F(x,\omega) = (f_1(x,\omega), \dots, f_k(x,\omega))$$

s.t.

$$x \in X(\omega) = \{g_r(x,\omega) \le b_r(\omega), \text{r=} 1, \dots, R\}$$

$$x \in D, \omega \in \Omega$$

- $F(x,\omega) = (f_1(x,\omega), \dots, f_k(x,\omega))$ is a vector of k uncertain objectives,
- $g_r(x,\omega)$ and $b_r(\omega)$ describe uncertain constraints (defined on an uncertainty set Ω, could be discrete or continuous),
- (Ω, Ξ, p) is a probability space.
- $x = (x_1, \dots, x_n)$ is an n-dimensional decision vector.

- Stochastic process (Random parameters)
- Scenario-based (different solutions for different plausible scenarios)
- Robust (A robust solution which works well for the worst-case scenario)
- Fuzzy numbers/logic (Different membership functions)
- Dynamic process (Multi-period decision process)

Stochastic Multi-Objective problems (Mild uncertainty)

- Stochastic programming problems:
- "If in a problem some parameters take unknown values at the time of making a decision, and these parameters are random variables, then the resulting problem is called a stochastic programming problem" [Caballero et al., 2001].
- Therefore, the Stochastic Multi-Objective Optimization Problem (SMOOP) approach developed MOOP models in the presence of random parameters with a known or unknown probability distribution.
- Usually, assume that the probability distribution is known or can be approximated via sampling, tests, experiences and expertises, etc → Mild uncertainty
    - Note. They may fail in determining accurate values for the probability distribution

$$\text{Max } \boldsymbol{F}(\boldsymbol{x},\boldsymbol{\omega}) = (\boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{\omega}), \dots, \boldsymbol{f}_k(\boldsymbol{x},\boldsymbol{\omega}))$$

s.t.

$$\boldsymbol{x} \in \boldsymbol{X}(\boldsymbol{\omega}) = \{\boldsymbol{g}_r(\boldsymbol{x},\boldsymbol{\omega}) \le \boldsymbol{b}_r(\boldsymbol{\omega}), \quad r = 1, \dots, R$$

$$\boldsymbol{x} \in \boldsymbol{D}, \boldsymbol{\omega} \in \boldsymbol{\Omega}$$

- $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\omega}) = (\boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{\omega}), \dots, \boldsymbol{f}_k(\boldsymbol{x},\boldsymbol{\omega}))$ is a vector of k random objectives (assume the joint distribution of the random variables is known),
- $\boldsymbol{X}(\boldsymbol{\omega}) = \{\boldsymbol{g}_r(\boldsymbol{x},\boldsymbol{\omega})$ and $\boldsymbol{b}_r(\boldsymbol{\omega})\}$ describes random constraints (defined on probability space (Ω, Ξ, p)),
- D is a deterministic convex set.

Caballero, R., et al. "Efficient solution concepts and their relations in stochastic multiobjective programming." *Journal of Optimization Theory and Applications* 110.1 (2001): 53-74.

Stochastic Multi-Objective problems (Mild uncertainty)
Solution approches

- Maximizing k stochastic objectives under R stochastic constraints
- Usually transform to an equivalent deterministic problem
    - Scalarization (the stochastic transformation): First, transforms to a stochastic single-objective problem (e.g Stochastic Goal Prpgramming);
    - Non-scalarization (the multiobjective transformation): First, transforms to a deterministic MOP (e.g STRANGE, PROMISE)
    - Expected value efficient solutions

$$\max\ (\mathbb{E}[\boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{\omega})], \dots, \mathbb{E}[\boldsymbol{f}_k(\boldsymbol{x},\boldsymbol{\omega})])$$
$$\text{s.t. } \boldsymbol{x} \in \boldsymbol{X}(\boldsymbol{\omega}),\ \boldsymbol{x} \in \boldsymbol{D},\ \boldsymbol{\omega} \in \boldsymbol{\Omega}$$

- For both transformations, random constraints have to be addressed first
- In many other situations, the probability distribution is unknown and information about possible outcomes is limited (moderate and deep uncertainty)
- → We need different approaches such as DMDU, Robust or Scenario-based methods

- Abdelaziz, Fouad Ben. "Solution approaches for the multiobjective stochastic programming." European Journal of Operational Research 216.1 (2012): 1-16.
- Gutjahr, Walter J., and Alois Pichler. "Stochastic multi-objective optimization: a survey on non-scalarizing methods." *Annals of Operations Research* 236.2 (2016): 475-499.

How can we deal with uncertainty?

General approaches

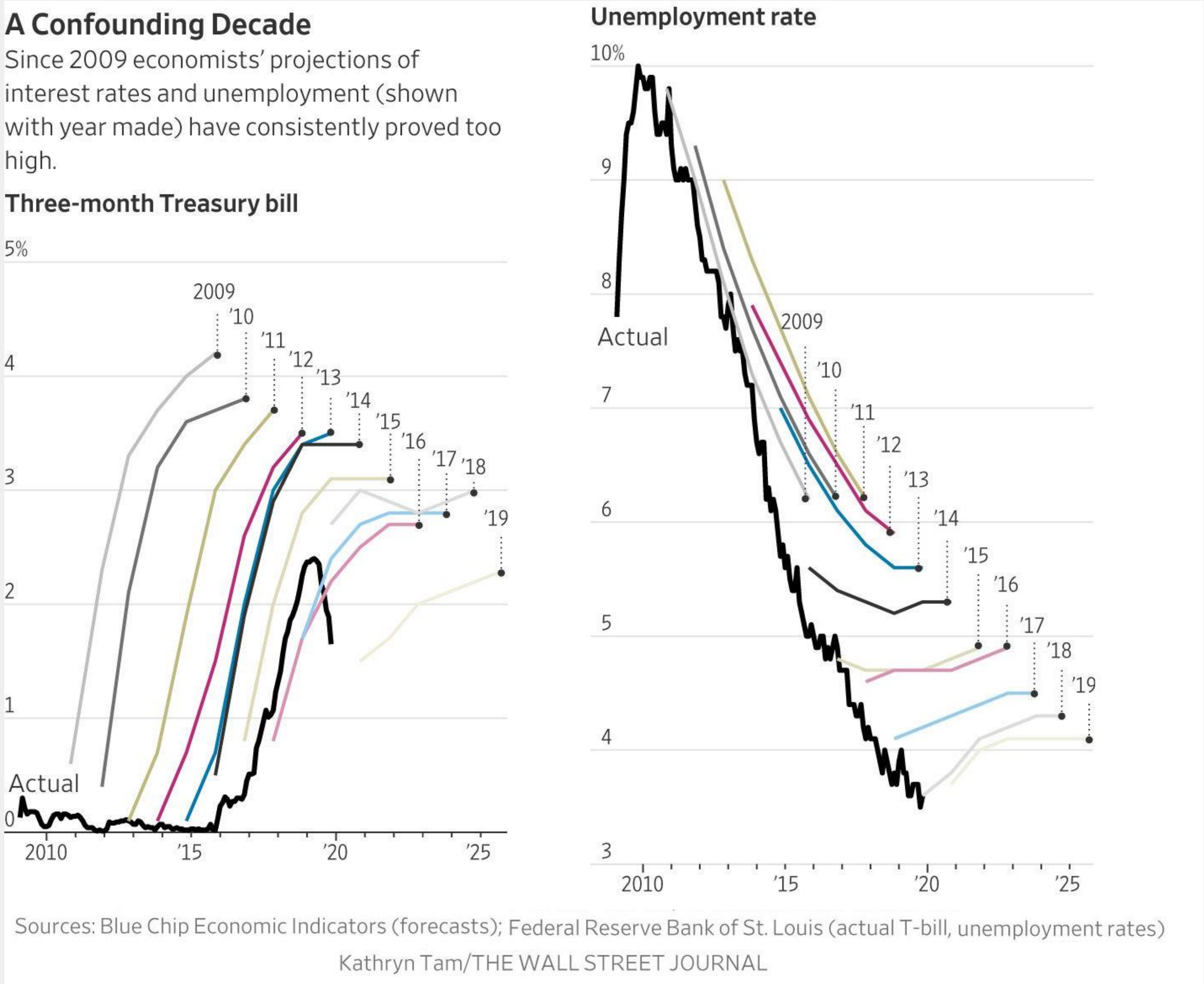
1. The predict and act approaches:
   a. Assume the future is knowable
      → Predict the future
      → Find the optimal decision the single predicted future.

Bad News: Most of the predictions are failed.

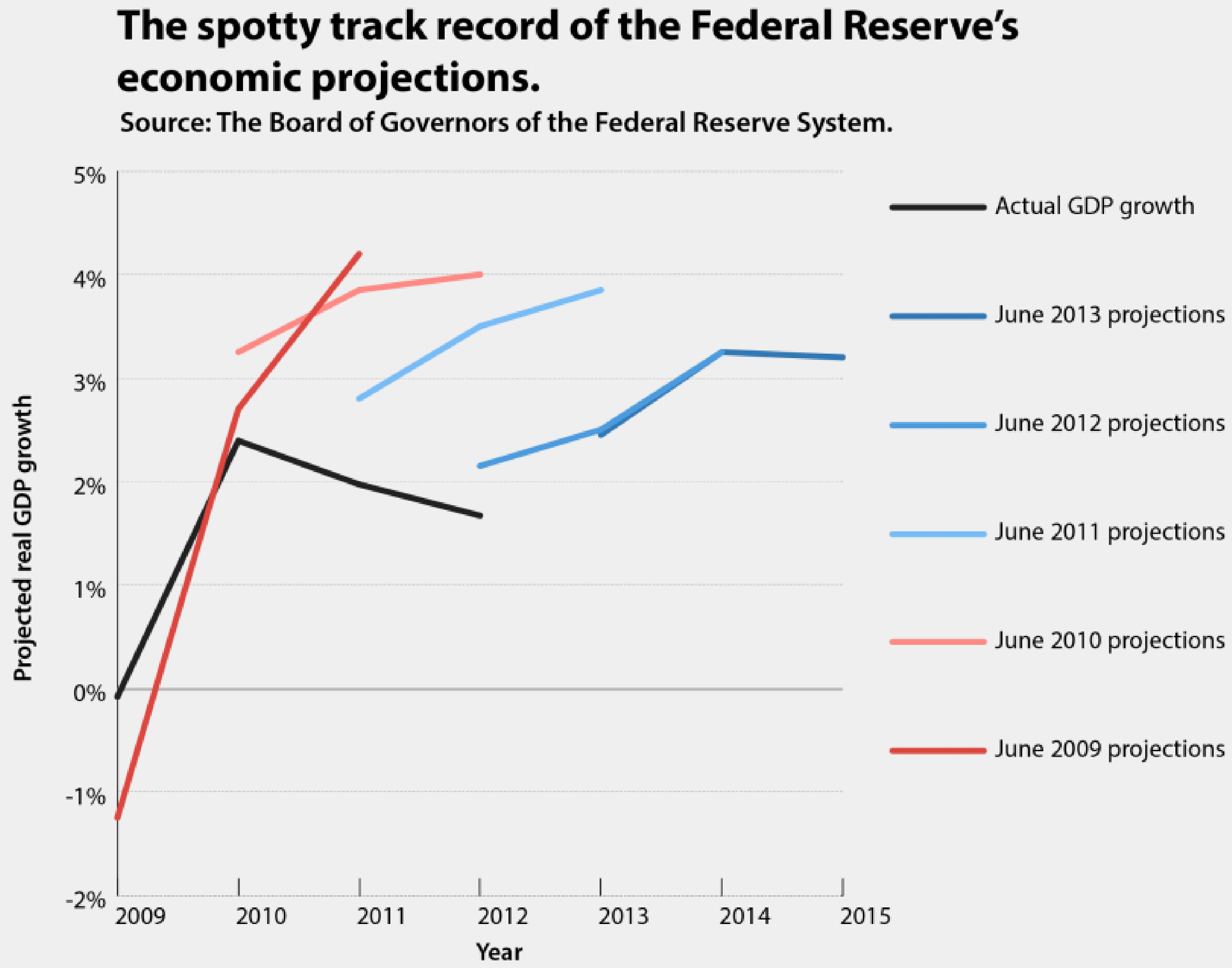
Benjamin Disraeli (Former British Prime Minister - **1804 - 1881**)

What we anticipate seldom occurs; what we least expected generally happens

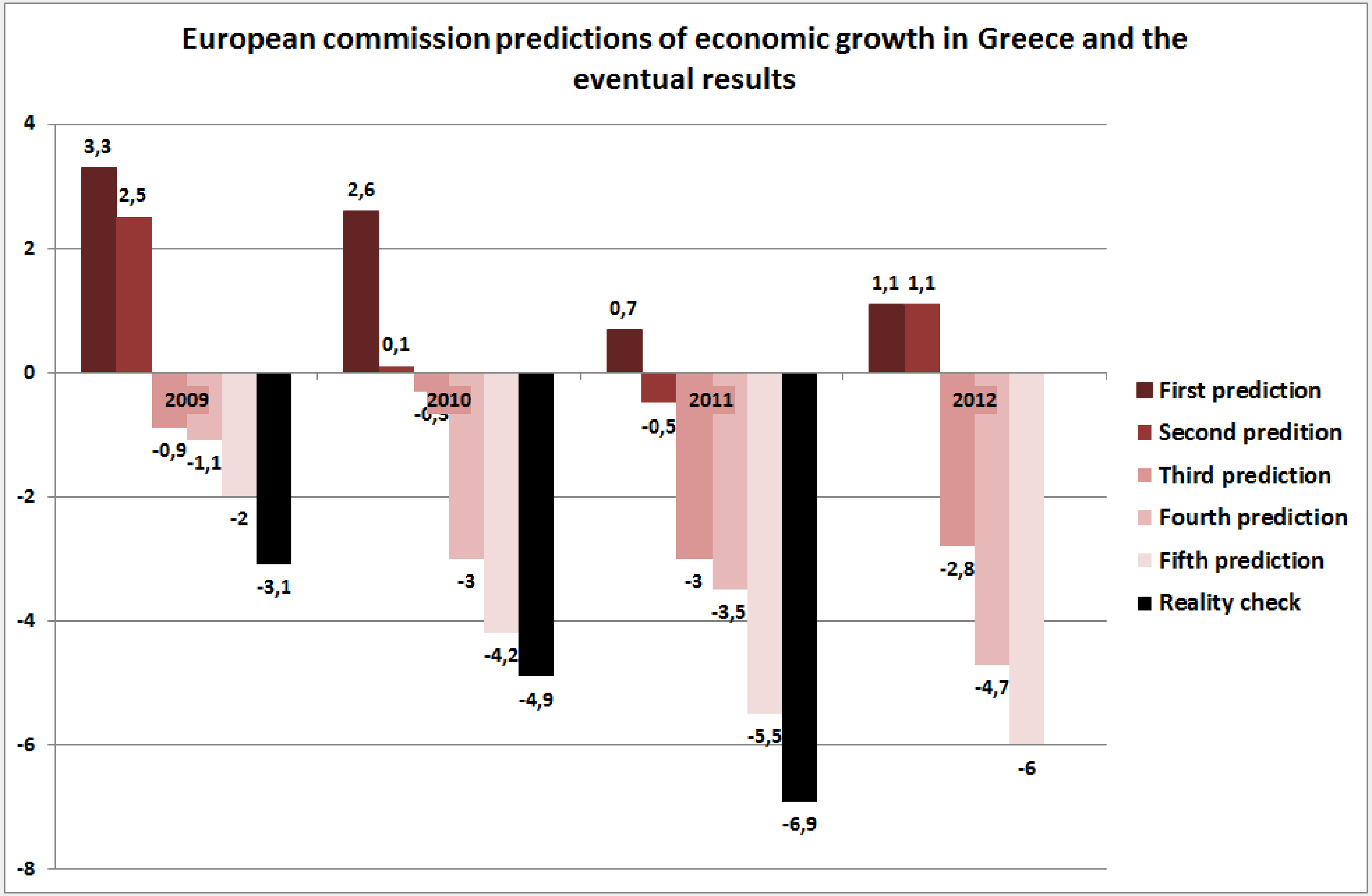
Some evidences: Economists failures



Some evidences: Economists failures



Some evidences: Economists failures



How can we deal with uncertainty?

General approaches

1. The predict and act approach:

   b. Assume the future will (probabilistically) look like the past

      → Stochastic/Probabilistic models

      → Find trend-based decision for the average/most probable scenario

(Like driving while looking only through rear-view mirror)

Some more evidences:

- Scandic Hotels
- Delta Air Lines

- Amazon
- Tesla

Traditional Approaches for Dealing with Uncertainty

[COVID-19]

Traditional applied scientific works usually suppose that the existing uncertainties are caused by either

Lack of knowledge → **Uncertainty reduction** by increasing the information

Random variation → **Stochastic** processes and **statistical** analysis

Nevertheless, some real problems involve deep uncertainty (e.g., uncertainties about the future)

- May not be reduced by gathering more information,
- Nor are they statistical in nature.

Dealing with uncertainty in practice

➢ In practice, for complex problems (e.g., strategic planning) uncertainties are ***incompletely*** understood and potential outcomes ***not enumerable***

➢ Probabilities and derived concepts (Expectation, Variance) are ***not properly defined***

The point.

In some real problems, ***probability and statistics are not sufficient to represent our entire knowledge*** and, therefore, some ***supplementary tools*** should be used in addition to probabilistic methods.

- Under such circumstances, a decision
  - Needs to relatively seize our objective(s), and
  - To be **robust** (i.e., perform satisfactorily under a broad variety of futures)