TIES483 Nonlinear Optimization

Lecture 15 (Part A)
Decision-making under Uncertainty (part 2)

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Babooshka Shavazipour, PhD
Babooshka.b.shavazipour@jyu.fi

Why must uncertainty be treated?

A major challenge is the requirement to accept, understand, and manage uncertainty, since:

1. Not all uncertainties can be eliminated;

2. Ignoring uncertainty limits our ability to make corrective action in the future and result in positions that could have been avoided;

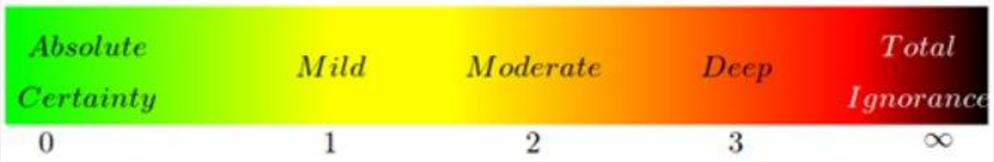
3. Ignoring uncertainty can throw away the opportunity of studying real-world problems, and/or lead to get some unsustainable approaches.

Dealing with uncertainty

- **Traditional** applied scientific works usually suppose that the existing uncertainties are caused by either
  - **Lack of knowledge** → **Uncertainty reduction** by increasing the information
  - **Random variation** → **Stochastic** processes and **statistical** analysis

Optimzation under uncertainty

- Mild uncertainty, which is commonly encountered by decision makers in short-term planning, is mostly treated by statistical approaches and probability theory while long-term programming, which largely involves deeper uncertainties, cannot deal with regular probability models and regular statistic approaches.
- We need to use some dynamic approaches which could be more robust, adaptive.



Moltiobjective optimzation under uncertainty

$$\text{Max } F(x,\omega) = (f_1(x,\omega), \dots, f_k(x,\omega))$$
s.t.
$$x \in X(\omega) = \{g_r(x,\omega) \le b_r(\omega), \text{r=}$$
$$1, \dots, R\}$$
$$x \in D, \omega \in \Omega$$

- $F(x,\omega) = (f_1(x,\omega), \dots, f_k(x,\omega))$ is a vector of k uncertain objectives,
- $g_r(x,\omega)$ and $b_r(\omega)$ describe uncertain constraints (defined on an uncertainty set Ω, could be discrete or continuous),
- (Ω, Ξ, p) is a probability space.
- $x = (x_1, \dots, x_n)$ is an n-dimensional decision vector.

- Stochastic process (Random parameters)
- Scenario-based (different solutions for different plausible scenarios)
- Robust (A robust solution which works well for the worst-case scenario)
- Fuzzy numbers/logic (Different membership functions)
- Dynamic process (Multi-period decision process)

Stochastic Multi-Objective problems (Mild uncertainty)

- Stochastic programming problems:
- "If in a problem some parameters take unknown values at the time of making a decision, and these parameters are random variables, then the resulting problem is called a stochastic programming problem" [Caballero et al., 2001].
- Therefore, the Stochastic Multi-Objective Optimization Problem (SMOOP) approach developed MOOP models in the presence of random parameters with a known or unknown probability distribution.
- Usually, assume that the probability distribution is known or can be approximated via sampling, tests, experiences and expertises, etc → Mild uncertainty
    - Note. They may fail in determining accurate values for the probability distribution

$$\textbf{Max } \boldsymbol{F}(\boldsymbol{x},\boldsymbol{\omega}) = (\boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{\omega}), \dots, \boldsymbol{f}_k(\boldsymbol{x},\boldsymbol{\omega}))$$

$$\textbf{s.t.}$$

$$\boldsymbol{x} \in \boldsymbol{X}(\boldsymbol{\omega}) = \{\boldsymbol{g}_r(\boldsymbol{x},\boldsymbol{\omega}) \le \boldsymbol{b}_r(\boldsymbol{\omega}),$$

$$\mathbf{r}= \mathbf{1}, \dots, \boldsymbol{R}$$

$$\boldsymbol{x} \in \boldsymbol{D}, \boldsymbol{\omega} \in \boldsymbol{\Omega}$$

- $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{\omega}) = (\boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{\omega}), \dots, \boldsymbol{f}_k(\boldsymbol{x},\boldsymbol{\omega}))$ is a vector of k random objectives (assume the joint distribution of the random variables is known),
- $\boldsymbol{X}(\boldsymbol{\omega}) = \{\boldsymbol{g}_r(\boldsymbol{x},\boldsymbol{\omega})$ and $\boldsymbol{b}_r(\boldsymbol{\omega})\}$ describes random constraints (defined on probability space (Ω, Ξ, p)),
- D is a deterministic convex set.

Caballero, R., et al. "Efficient solution concepts and their relations in stochastic multiobjective programming." *Journal of Optimization Theory and Applications* 110.1 (2001): 53-74.

Stochastic Multi-Objective problems (Mild uncertainty)
Solution approches

- Maximizing k stochastic objectives under R stochastic constraints
- Usually transform to an equivalent deterministic problem
  - Scalarization (the stochastic transformation): First, transforms to a stochastic single-objective problem (e.g Stochastic Goal Prpgramming);
  - Non-scalarization (the multiobjective transformation): First, transforms to a deterministic MOP (e.g STRANGE, PROMISE)
  - Expected value efficient solutions

$$\max\ (\mathbb{E}[\boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{\omega})], \ldots, \mathbb{E}[\boldsymbol{f}_k(\boldsymbol{x},\boldsymbol{\omega})])$$
$$\text{s.t. } \boldsymbol{x} \in \boldsymbol{X}(\boldsymbol{\omega}), \boldsymbol{x} \in \boldsymbol{D}, \boldsymbol{\omega} \in \boldsymbol{\Omega}$$

- For both transformations, random constraints have to be addressed first
- In many other situations, the probability distribution is unknown and information about possible outcomes is limited (moderate and deep uncertainty)
- → We need different approaches such as DMDU, Robust or Scenario-based methods

- Abdelaziz, Fouad Ben. "Solution approaches for the multiobjective stochastic programming." European Journal of Operational Research 216.1 (2012): 1-16.

- Gutjahr, Walter J., and Alois Pichler. "Stochastic multi-objective optimization: a survey on non-scalarizing methods." *Annals of Operations Research* 236.2 (2016): 475-499.

How can we deal with uncertainty?

General approaches

1. The predict and act approaches:

   a. Assume the future is knowable

   → Predict the future

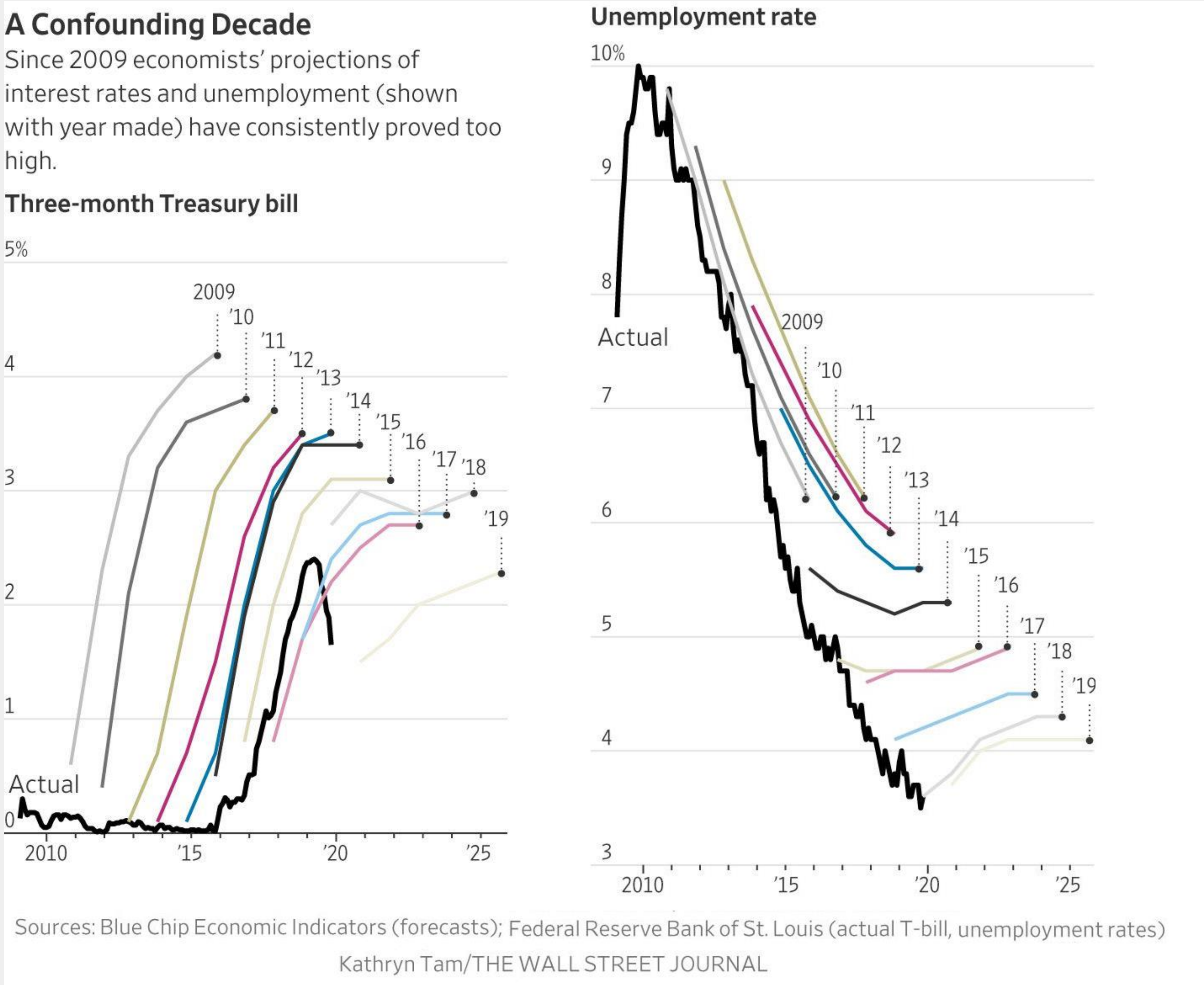
   → Find the optimal decision for the single predicted future.

Bad News: Most of the predictions are failed.

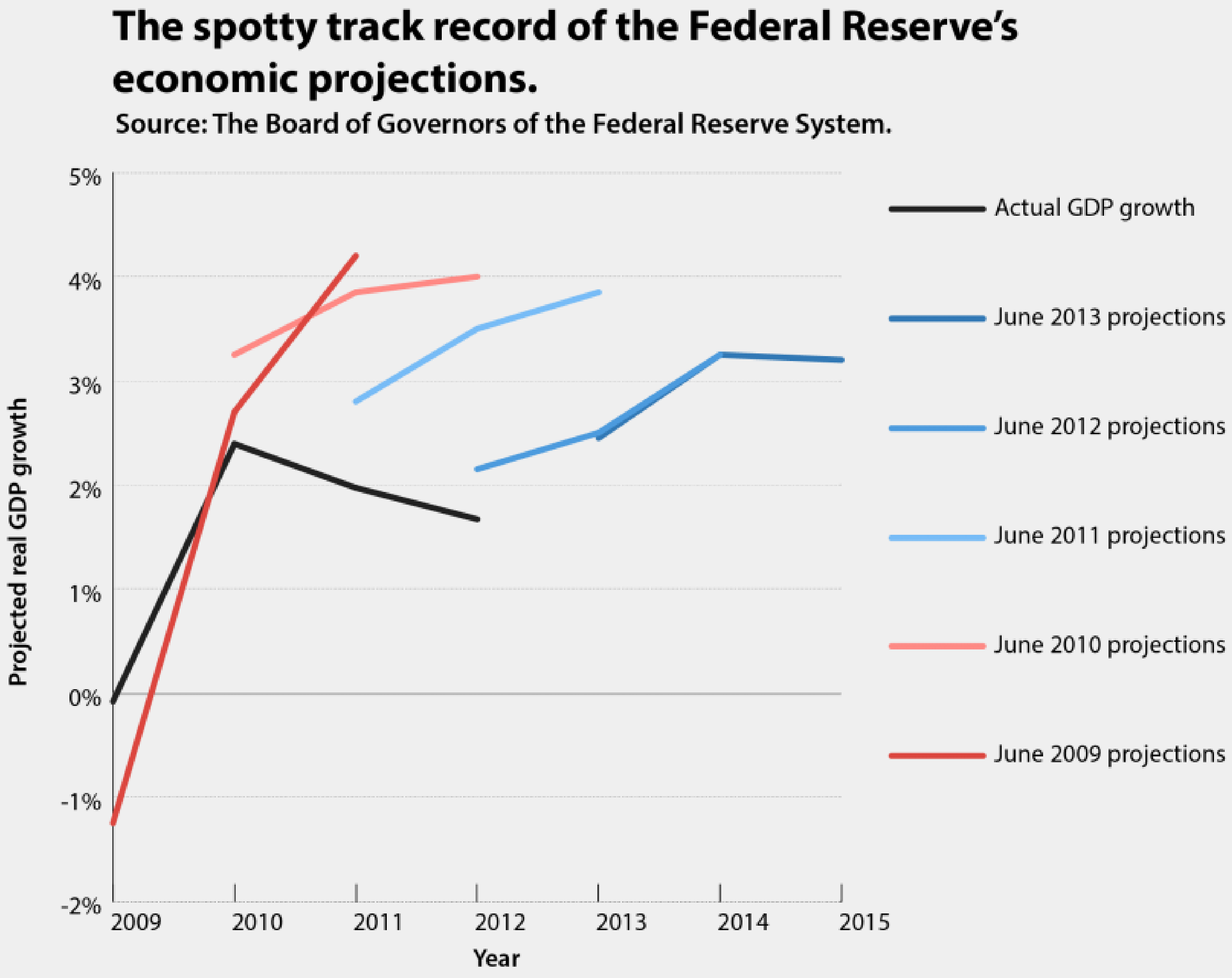
Benjamin Disraeli (Former British Prime Minister - **1804 - 1881)**

What we anticipate seldom occurs; what we least expected generally happens

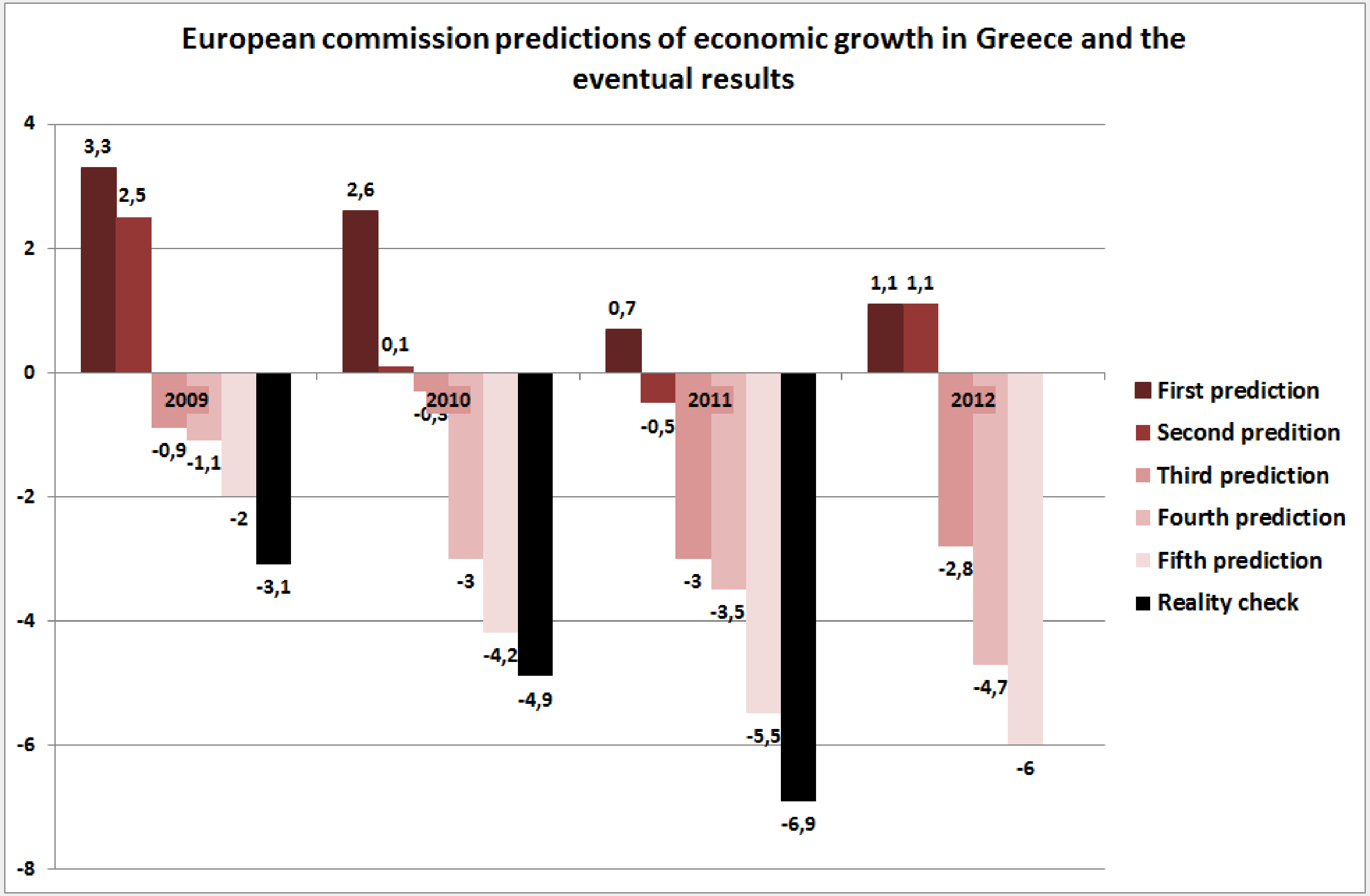
Some evidences: Economists failures



Some evidences: Economists failures



Some evidences: Economists failures



How can we deal with uncertainty?

General approaches

1. The predict and act approach:
   b. Assume the future will (probabilistically) look like the past
      → Stochastic/Probabilistic models
      → Find trend-based decision for the average/most probable scenario

(Like driving while looking only through rear-view mirror)

Some more evidences:

- Scandic Hotels
- Delta Air Lines

- Amazon
- Tesla

Traditional Approaches for Dealing with Uncertainty

[COVID-19]

Traditional applied scientific works usually suppose that the existing uncertainties are caused by either

Lack of knowledge → **Uncertainty reduction** by increasing the information

Random variation → **Stochastic** processes and **statistical** analysis

Nevertheless, some real problems involve deep uncertainty (e.g., uncertainties about the future)

- May not be reduced by gathering more information,
- Nor are they statistical in nature.

Dealing with uncertainty in practice

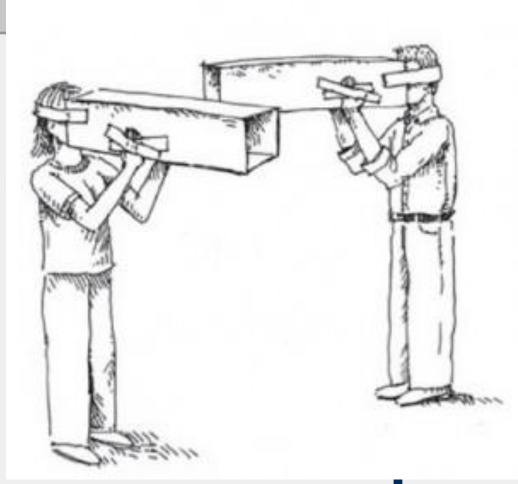
➢ In practice, for complex problems (e.g., strategic planning) uncertainties are ***incompletely*** understood and potential outcomes ***not enumerable***

➢ Probabilities and derived concepts (Expectation, Variance) are ***not properly defined***

The point.

In some real problems, ***probability and statistics are not sufficient to represent our entire knowledge*** and, therefore, some ***supplementary tools*** should be used in addition to probabilistic methods.

- Under such circumstances, a decision
  - Needs to relatively seize our objective(s), and
  - To be **robust** (i.e., perform satisfactorily under a broad variety of futures)



How can we deal with uncertainty?

General approaches

1. The predict and act approaches:
    a. Assume the future is knowable

    b. Assume the future will (probabilistically) look like the past

2. Robust approches (consider future variabilities):

Robust approaches:

2.1. Resistance (Min-Max robustness):

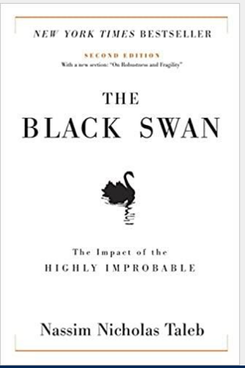
Plan for the worst-case scenario in the future.

Bad news: it is significantly costly

But what would be the worst case then?

From which perspective?

The worst-case or several worst cases?



Worst news: the plan may not work well because of ***Surprises*** or ***Black Swans*** (Painful events).

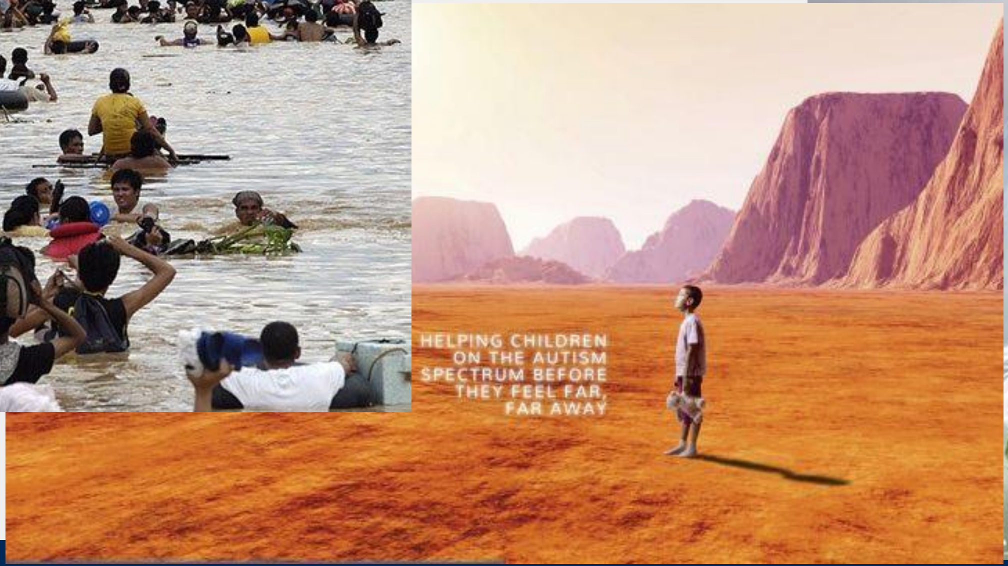
Robust approaches:

2.2. Resilience: Concentrate on recovering the system, but, without considering plausible future events.

- Accepts short-term pain and negative system performance but focuses on recovery, whenever, a different scenario has happened.

It might be too late then!

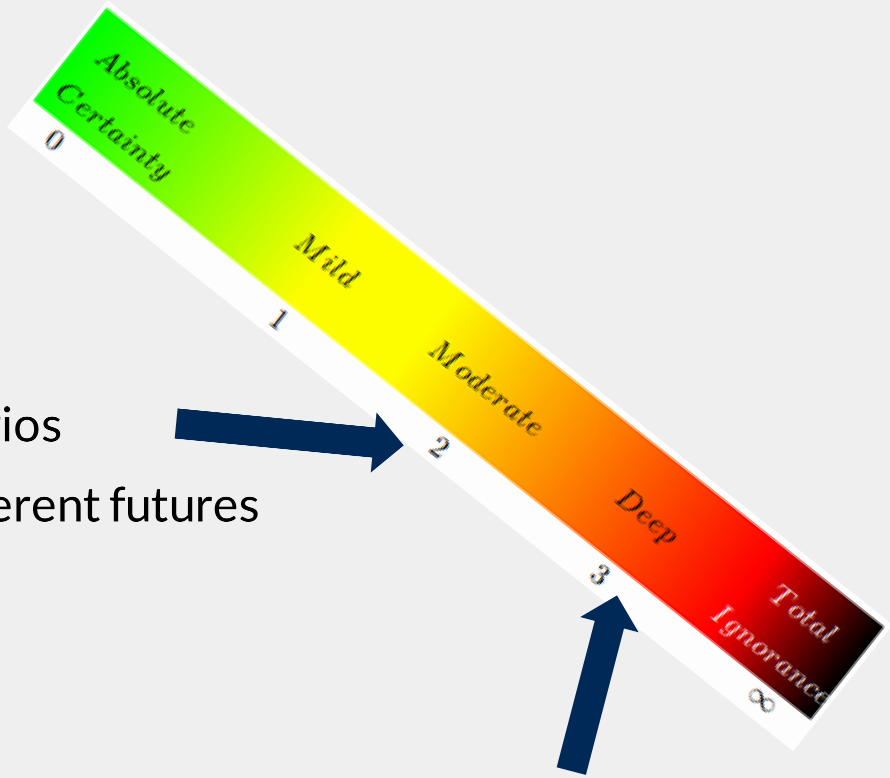
(Good Luck ! God will help you!)



Robust approaches:

2.3. Static robustness (considering different futures):

- Look for a decision/policy that will do well in a few scenarios
- Decrease vulnerability in the widest possible range of different futures



- But is it enough?

- What if the experts do not know and/or cannot agree on what the future might bring? Specially in long-term planning.

- The world is continuously changing!

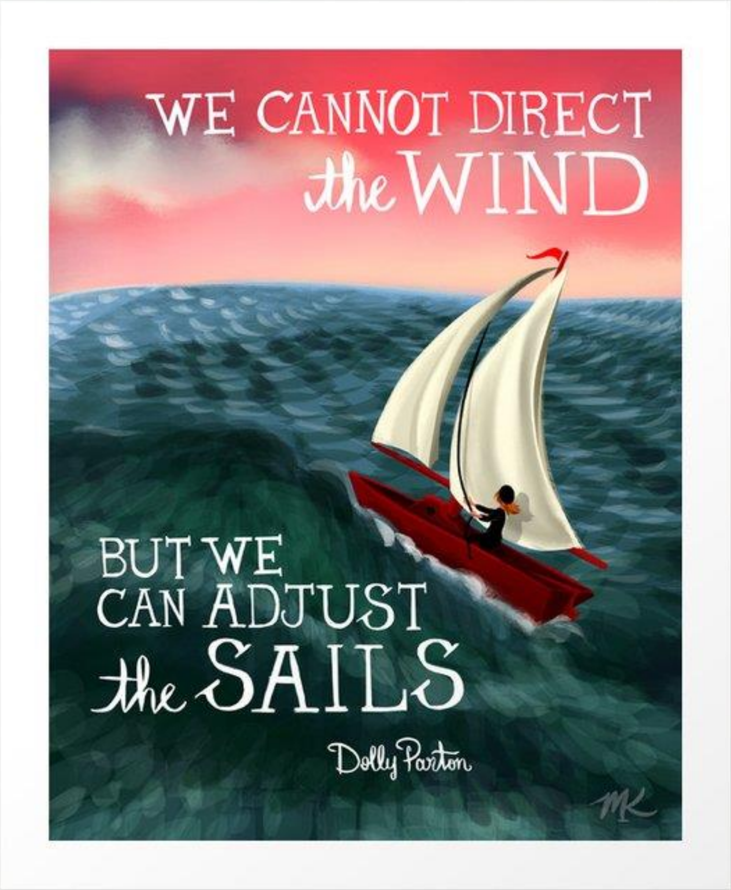
Robust approaches:

2.4. The monitor and adapt (dynamic robustness) approach:

- Assume the future cannot be predicted and implement dynamic adaptive policies.
- Plan for continuous change and adaptation regarding upcoming situations

Basic principles:

- Indeed, we do not produce forecasts
- Instead, we seek robust decisions that perform well in many different realizations of the future (not an optimal in one scenario)
- Explore the vulnerabilities/fragilities of the decisions and identify the adaptation/contingency plans
- Monitor the progress and update/adapt whenever is needed

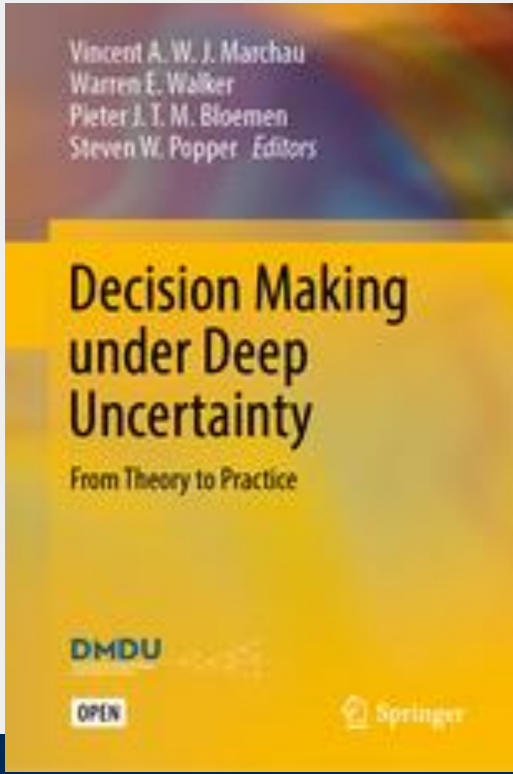


Decision Making under Deep Uncertainty (DMDU)

- DMDU methods aim to support decision-makers to find the robust, adaptive decisions
- Exploratory Modeling* approaches (e.g., Scenario Discovery) to identify vulnerabilities
- Robust Decision Making (RDM)** is an iterative analytic process to testing alternatives robustness
- Dynamic Adaptive Policy Pathways¤ support DMs to identify, compare and choose adaptations and contingency plans for multiple scenarios
- …

Open Access book: https://link.springer.com/book/10.1007/978-3-030-05252-2

DMDU society https://www.deepuncertainty.org/



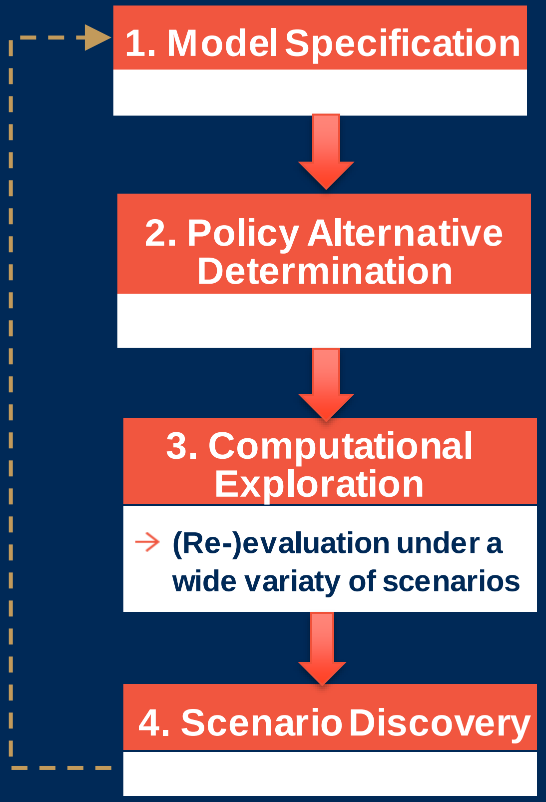
* S. Bankes, W..E. Walker, and J.H. Kwakkel (2013). "Exploratory Modeling and Analysis", entry (pp. 532-537) in S. Gass, and M..Fu (eds.), Encyclopedia of Operations Research and Management Science., 3rd Edition, New York: Springer.

** Lempert et al. (2006), 'A general, analytic method for generating robust strategies and narrative scenarios', Management Science 52(4), 514–528.

¤ M. Haasnoot, J.H. Kwakkel, and W.E. Walker (2013). "Dynamic Adaptive Policy Pathways: A New Method for Crafting Robust Decisions for a Deeply Uncertain World", Global Environmental Change, Vol. 23, No. 2, pp. 485–498.

Robust Decision Making framework (RDM)

RDM

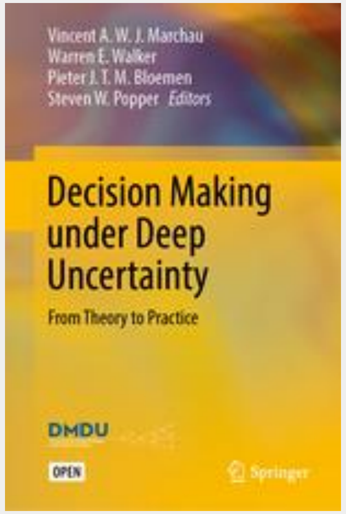


Lempert et al (2006)*

* *Lempert et al. (2006), 'A general, analytic method for generating robust strategies and narrative scenarios', Management Science* 52(4), 514–528.

Decision-making under deep uncertainty (DMDU)

- Triditional MORDM techniques
- Consider one nominal/reference scenario
- Use Evolutionary algorithms to generate solutions for the single-scenario MOOP
- Randomly generate a wide range of plausible scenarios (1000-10’000)
- Re-evaluate the solutions in all random scenarios
- Select solutions which provide better performances in a wider range of scenarios (robust)
- More recently, these methods integrated by robust multiobjective optimization methods from mathematical programming literature (see, e.g., https://www.sciencedirect.com/science/article/pii/S1364815221001778)



Open Access book: https://link.springer.com/book/10.1007/978-3-030-05252-2

Robustness in (mathematical) optimization problems

- Robust optimisation (RO) is one of the more recent approaches to treat uncertainty in optimization problems.
- **However,** its origin goes back to the use of worst-case analysis to deal with uncertainty in the foundation of the modern decision theory in the 1950s.
- **RO**, generally, help DMs in dealing with **two** different types of uncertainty:
  a) **The uncertainty of feasibility** (i.e. the feasibility of a solution is affected by uncertainty), or
  b) **The uncertainty of optimality** (i.e. the optimality of a solution is affected by uncertainty).
- In **RO**, mainly based on the worst-case/min-max robustness, the DM attempts to capture a solution that will be feasible (robust feasible solution) or optimal (robust optimal solution) for any realization of the uncertainty in a given set.
- Generally, this means that a robust feasible/optimal solution is a solution which retains its feasibility/optimality even if some of the decision parameters are changed.
- Various RO models have been developed, still, the robust solutions usually are (too) conservative/risk-averse.
- Also, in many complex real-world problems, identification of the worst-case scenario is difficult and even the worst-case feasible solutions might be failed by surprises.

Multiobjective Robust Optimization

- MOO → Pareto/effient solution
- Robust Optimization → Robust/stable solutions
- MORO → Robust effient solutions
- There are different definitions:
  - flimsily robustness
  - (locally) highly robustness
  - point-based minmax robustness
  - hull-based minmax robustness
  - set-based minmax robustness
  - light robustness

Some references:

- Ide, J. and Schöbel, A., 2016. Robustness for uncertain multi-objective optimization: a survey and analysis of different concepts. *OR spectrum*, *38*(1), pp.235-271.

- Botte, M. and Schöbel, A., 2019. Dominance for multi-objective robust optimization concepts. *European Journal of Operational Research*, *273*(2), pp.430-440.

- Zhou-Kangas, Y. and Miettinen, K., 2019. Decision making in multiobjective optimization problems under uncertainty: balancing between robustness and quality. *OR Spectrum*, *41*(2), pp.391-413.

- Krüger, C., Schöbel, A. and Wiecek, M.M., 2017. The Robustness Gap for Uncertain Multiobjective Optimization.

Two-stage decision-making structure (A dynamic robust approach)

➢ **Robust decision-making:**

- Seeks good performance under each scenario (especially the worst-case scenario) where performance is typically based on multiple criteria. → too risk-averse, often having solutions
- Dominated by a bad scenario

➢ **Dynamic decision-making:**

- More realistic → **Multi-stage programming** . . .

➢ **Suggesteded methodology:**

- **A dynamic-robust approach** → two-stage scenario-based structure to dealing with deep uncertainty in MCDM/MOO problems.

Scenario Planning VS. Statistical Expectation

➢ Generally, to dealing with uncertainty about the future the Here-and-Now decision problems are faced and the decision must be taken before some parameters are known.

➢ These uncertainties are almost impossible to reduce by gathering more information and are not statistical in nature.

➢ It seems logical that corrective action, named recourse, had better be done once the unknown parameters are known. Meaning that some penalties must be paid for any shortfalls (deviation from the goals).

➢ Scenarios provide us a framework to though and critical conversation about possible futures

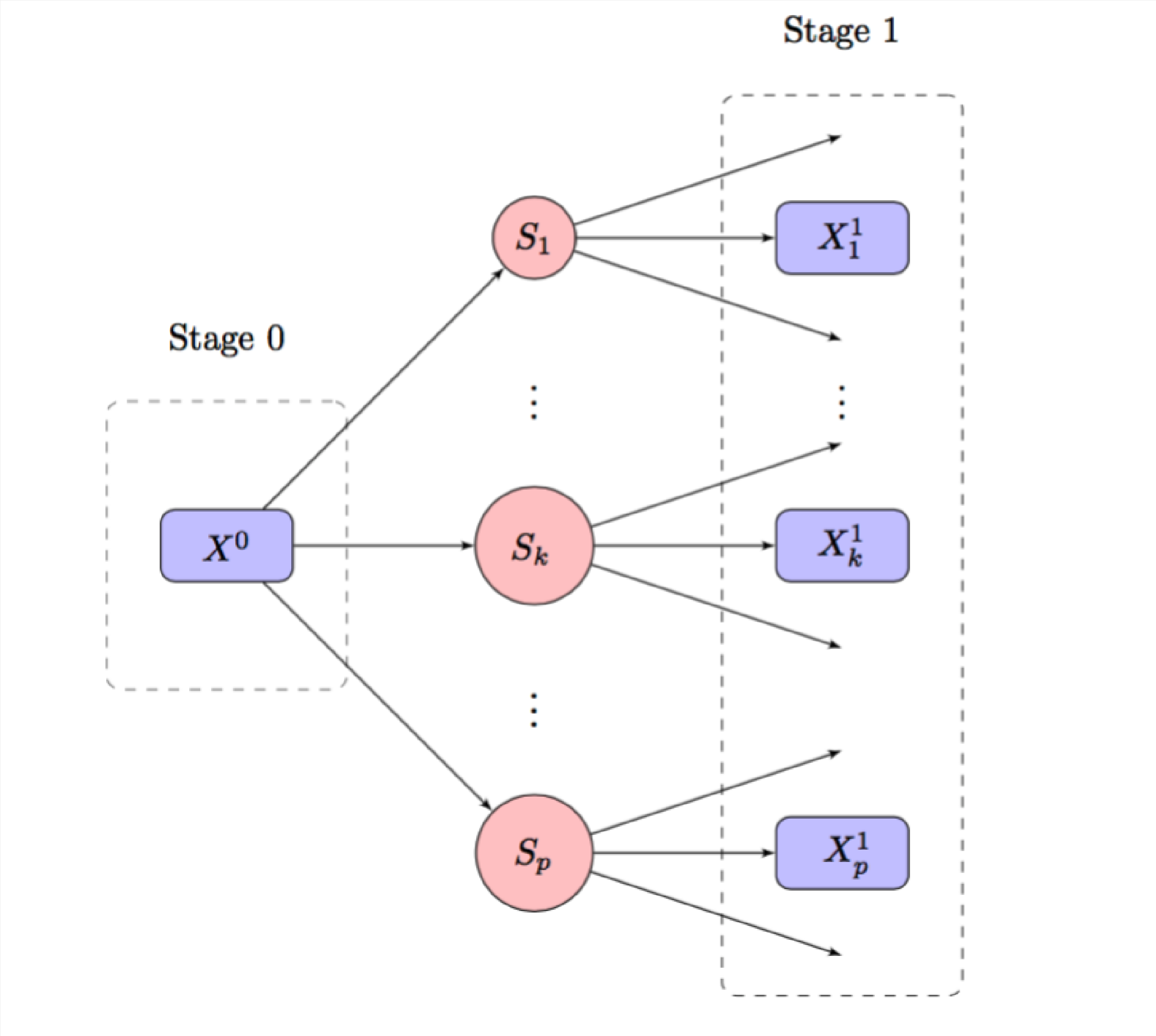
➢ **Scenario: Descriptions of plausible futures in which the outcomes of decisions will emerge**

➢ So we were looking for a scenario-based structure for multi-objective optimization under deep uncertainty

Two-stage decision-making process with p scenarios

Stage 0: An ***initial decision*** is made before any scenario revelation (Scenario free decision).

Stage 1: A ***Contingent/recourse*** decision to be taken if the scenario k is revealed (Scenario dependent decision).



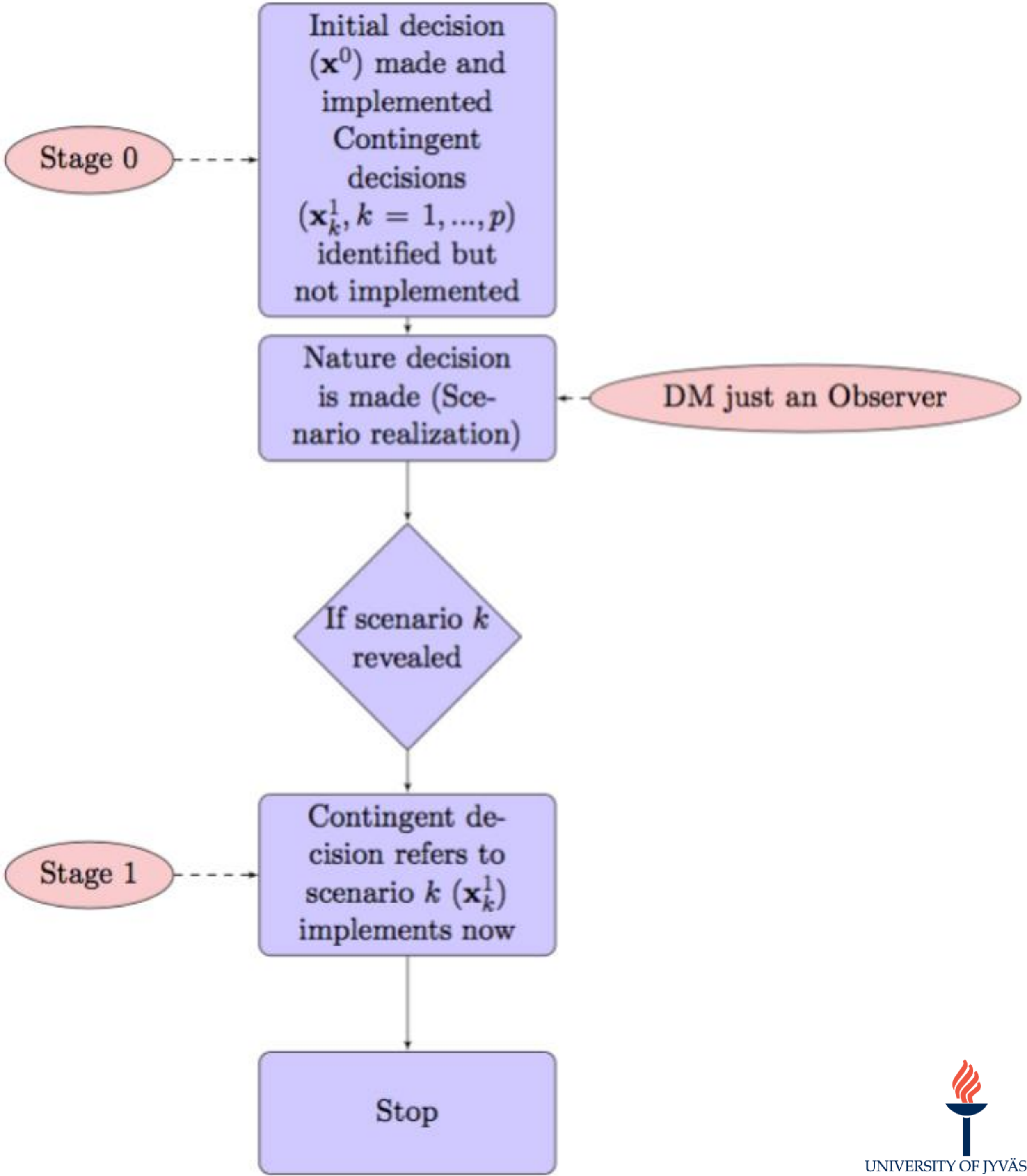
Shavazipour, B. and Stewart, T.J., 2021. Multi-objective optimisation under deep uncertainty. *Operational Research*, 21, 2459–2487. https://doi.org/10.1007/s12351-019-00512-1.

The schematic of two-stage decision-making process

- Models are not used to produce forecasts

- Identify the reasonable initial decision (which is compensable, whatever happens in the future) + suitable contingent scenario-dependent decision(s) for every single plausible scenario, implemented after scenario revelation.

- Suitable to be used in ***prescriptive analytics***



➢ We are looking one step ahead, postpone part of the decision and leave room for possible adaptation later (after scenario realisation).

➢ We identified suitable adaptive plans for every plausible scenario in advance.

➢ A robust decision in our philosophy (dynamic-robust decision) is a split decision containing two subgroups of decisions:

The initial decision(s) followed by recourse decision(s)

➢ Unlike the regular robust solution, the initial decision does not need to be optimal or even feasible under conditions of all scenarios. Instead, it must be good enough, with beneficial foresight, to lead us to the optimal aggregations of this initial decision(s) and the following recourse decisions.

Mathematical Formulation

$$
\begin{aligned}
&Opt_{(\mathbf{x}^0,\mathbf{x}_k^1)} \quad \mathbf{F} = f_{ik}^1(\mathbf{x}^0,\mathbf{x}_k^1);\ k = 1,\ldots,p; \quad i = 1,\ldots,m;\\
&s.t. \qquad u_r^0(\mathbf{x}^0) \le 0, \qquad r = 1,\ldots,R_0;\\
&\qquad\quad\ u_r^1(\mathbf{x}^0,\mathbf{x}_k^1) \le 0, \quad k = 1,\ldots,p;\\
&\qquad\qquad\qquad\qquad\qquad\quad r = R_0+1,\ldots,R_0+\cdots+R_{k-1}+R_k;
\end{aligned}
$$

where $\mathbf{x}^0 = (x_1^0,\ldots,x_n^0) \in \mathbf{X}^0$ is an n-dimensional initial scenario-free decision variable vector which was made in stage ‘0’ before scenario k happens and $\mathbf{X}_0$ is an initial decision space.

$\mathbf{x}_k^1 = (x_{1k}^1,\ldots,x_{nk}^1) \in \mathbf{X}^1(\mathbf{x}^0,k), (k = 1,\ldots,p)$; is an n-dimensional contingent scenario-dependent decision vector which is taken in stage ‘1’ if scenario k is revealed and $\mathbf{X}^1(\mathbf{x}^0,k)$ is a contingent decision space when scenario k is unfolded.

$f_{ik}^1(\mathbf{x}^0,\mathbf{x}_k^1), (k = 1,\ldots,p; i = 1,\ldots,m)$, is ith meta-criterion/objective includes scenario-free performances, if scenario k is revealed. In fact, they indicate *preferences regarding criterion i under conditions of scenario k*.

$u_r^0(\mathbf{x}^0)$ is the set of inequality constraints in stage ‘0’.

$u_r^1(\mathbf{x}^0,\mathbf{x}_k^1)$ is the set of inequality constraints in stage ‘1’.

The problem consists in optimising $m \times p$ objectives under $(R_0 + R_1 + \cdots + R_p)$ constraints. Both objectives and constraints can be linear or non-linear.

Shavazipour, B. and Stewart, T.J., 2021. Multi-objective optimisation under deep uncertainty. *Operational Research*, 21, 2459–2487.

Some advantages of the two-stage framework

➢ Make a reasonable decision at the first stage (under uncertainty), followed by an adaptive decision after scenario realization.

➢ Ability to treat the higher degrees of uncertainty specially when there is not enough information about the probability space and distributions.

➢ An opportunity for DM to plan for adaptive decisions while the consequences of the initial decision after the realization of each plausible scenario are considered.

➢ Provides feasible solutions in the conditions that previous models failed (empty intersection between the scenarios).

➢ Provide a less risk-averse solution and in the meantime provides the same, or even better, robustness compares to typical robust approaches (Dynamic-Robust).

➢ Can be extended to multiple stages (>2)

Example.
SBMOO for quarantine during the pandemics: The case study of COVID-19

- This crisis should be a wake-up call to address long-term vulnerabilities (deep uncertainty).

- Objective functions:
  1. **Min Economic crises** (unemployment salary, bankruptcy (e.g. 2500 £ per person in UK)
  2. **Min Social crises** (unemployment, individual stress)
  3. **Min the length of quarantine**
  4. **Min The number of deaths**
  5. **Min the number of infected**
- Scenarios:
  1. **The best-case e.g. the virus mutates and actually dies out**
  2. **The weather may help** (warmness in summer may decrease the speed of spread).
  3. **Enjoy a summer break before a second wave in the fall.**
  4. **Effective antiviral medicine is found**
  5. **The worst-case scenario, almost all people are infected and a lot of them are died**

Example (cont.).
SBMOO for quarantine during the pandemics: The case study of COVID-19

Decision variables:

1. **Initial** (e.g., (not) lockdown a city; quarantine the returnees; monitor persons with positive corona test)
2. **Recourse** (e.g., when (close) open the entries)

Constraints:

- ***Surge critical care bed capacity***
- **Limitations in lockdown**

Sources:
https://www.weforum.org/agenda/2020/03/why-lockdowns-work-epidemics-coronavirus-covid19/
https://www.nytimes.com/2020/03/20/opinion/sunday/coronavirus-outcomes.html
Dr Tara C. Smith, an epidemiologist at Kent State University, said: “I’m not pessimistic. I think this can work.” She thinks it will take eight weeks of social distancing to have a chance to slow the virus, and success will depend on people changing behaviors and on hospitals not being overrun.
In the worst-case scenario, will social services collapse in some areas? Will order fray? Gun sales are increasing because some people expect chaos and crime.

Conclusions

- Uncertainty is always present in decision making
- In reality, we are surrounded by higher degrees of uncertainty
- Ignoring uncertainty is a terrible idea
- We cannot predict the future, but we can be prepared for Black Swans
- Dynamic robust and adaptive decisions are good ways of dealing with deep uncertainty
  - Get implementation under way
  - Allow adaptations of decision over time as new information arrived, new solutions are developed, values change, and other external events take place
  - Enable learning from experience over time

Take-home message:

“There are things that we know that we know;
there are things that we know that we don’t know;
there are things that we don’t know that we don’t know.”
– Donald Rumsfeld

- To make a robust, adaptive decision we need to consider the things that we know that we don’t know and preparing for the things that we don’t know that we don’t know.

Be prepared for the future to save the world!

Thank you