

Contracting and Compressing Shortest path Databases ICAPS 2021

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Monash University

Shortest Path Problem in Road Network

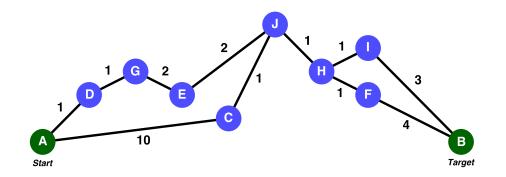


Road Network:

- A directed or undirected graph
 - A set of vertices: V.
 - A set of edges: $E \subseteq V \times V$.
 - Each edge $e \in E$ has a nonnegative weight w(e), which can represent travel time, distance and so on.

Shortest Path Problem:

- Given a start s and target t.
- Find an optimal path which minimize the sum of weight on each edge.



An example of undirected road network graph. The start and target are shown in green, and the weight of each edge are shown in the figure correspondingly.

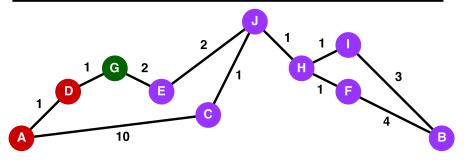


Compressed Path Databases:



- Compressed Path Databases:
 - Construction:
 - First move table:

Ordering	G	D	A	C	J	E	Н	F	В	I
G	*	D	D	E	E	E	Е	E	E	E
J	Е	E	E	С	*	Е	Н	Н	Н	H
I	Н	Н	Н	Н	Н	Н	Н	Н	В	*

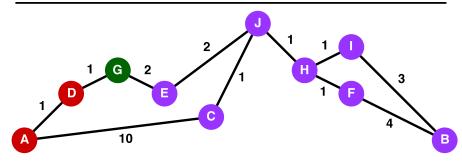


From the source node G, the optimal first move to any node colored red (resp. purple) is D (resp. E).



- Compressed Path Databases:
 - Construction:
 - First move table:
 - [A J]: indicates the optimal first move.

Ordering	G	D	Α	C	J	E	Н	F	В	I
G	*	D	D	E	E	Е	Е	E	E	E
J	E	Е	E	С	*	Е	Н	Н	Н	H
I	Н	Н	Н	Н	Н	Н	Н	Н	В	*

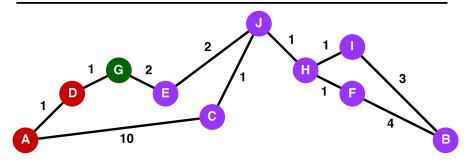


From the source node G, the optimal first move to any node colored red (resp. purple) is D (resp. E).



- Compressed Path Databases:
 - Construction:
 - First move table:
 - [A J]: indicates the optimal first move.
 - *: wildcard symbol [2].

Ordering	G	D	A	C	J	E	Н	F	В	I
G	*	D	D	E	E	E	Е	E	E	E
J	E	Е	Е	С	*	Е	Н	Н	Н	H
I	Н	Н	Н	Н	Н	Н	Н	Н	В	*



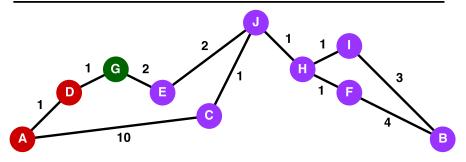
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Compressed Path Databases:

- Construction:
 - First move table:
 - [A J]: indicates the optimal first move.
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 - Compression:
 - Depth first search order [1].

Ordering	G	D	A	С	J	E	Н	F	В	I
G	*	D	D	E	Ε	E	Ε	E	E	E
J	Е	E	E	С	*	Е	Н	Н	Н	Н
I	Н	Н	Н	Н	Н	Н	Н	Н	В	*



From the source node G, the optimal first move to any node colored red (resp. purple) is D (resp. E).

Background Communication Date



Compressed Path Databases (CPD) [1]

Compressed Path Databases

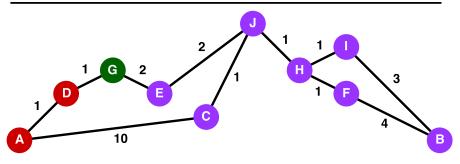
Construction:

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Compression:

- Depth first search order [1].
- Run length encoding [1] (i.e. Row G: 1D; 4E).

Ordering	G	D	Α	C	J	E	Н	F	В	I
G	*	D	D	E	E	E	E	E	E	E
J	E	Е	E	C	ж	Е	Н	Н	Н	Н
I	Н	Н	Н	Н	Н	Н	Н	Н	В	*



From the source node G, the optimal first move to any node colored red (resp. purple) is D (resp. E).

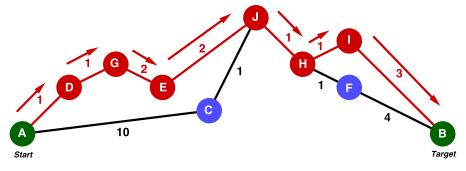


Compressed Path Databases

- Construction:
 - First move table:
 - [A J]: indicates the optimal first move.
 - *: wildcard symbol [2].
 - Compression:
 - Depth first search order [1].
 - Run length encoding [1] (i.e. Row G: 1D; 4E).

Query:

 From a given s, CPD recursively extract the optimal first move until reaches t.



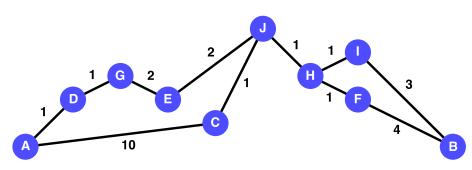
Exacting the shortest path from CPD, the shortest path from start to target is shown in red.



Contraction Hierarchy:



- Contraction Hierarchy:
 - Construction:
 - Apply a total lex order L.

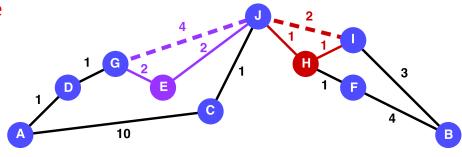


The lex order \boldsymbol{L} is the alphabetical order shown in the figure.



Contraction Hierarchy:

- Construction:
 - Apply a total lex order L.
 - Contraction:
 - W.r.t. L, choose the least node
 v from the graph.

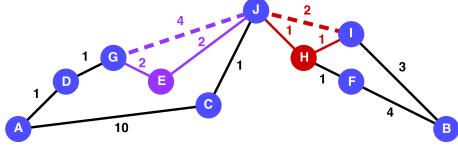


The result of contracting E (resp. H) in purple (resp. red). Dashed edges indicate shortcut edges.



Contraction Hierarchy:

- Construction:
 - Apply a total lex order *L*.
 - Contraction:
 - W.r.t. L, choose the least node
 v from the graph.
 - Add a shortcut edge (u, w) between each pair of inneighbour u and outneighbour w of v:
 - » $v <_L u \& v <_L w$.
 - $v \in sp(u,w)$.



The result of contracting E (resp. H) in purple (resp. red). Dashed edges indicate shortcut edges.

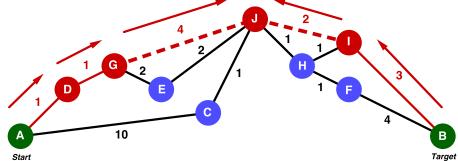
Background

Contraction Hierarchy (CH) [3]



Contraction Hierarchy:

- Construction:
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 - » v ∈ sp(u,w).



Bi-directional Dijkstra search from start and target, the shortest ch-path is shown in red.

– Query:

Bi-directional Dijkstra search.

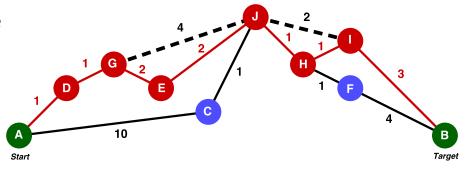


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 - » $v <_L u \& v <_L w$.
 - » v ∈ sp(u,w).

– Query:

- Bi-directional Dijkstra search.
- Unpack the path.



Unpacking the CH path, the final shortest path is shown in red.



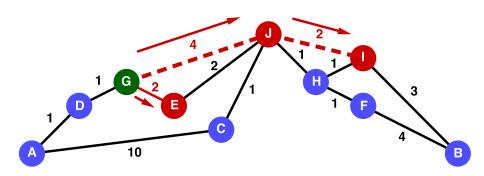
CH-based Compressed Path Databases (CH-CPD)

CH-CPD:



CH-based Compressed Path Databases (CH-CPD)

- CH-CPD:
 - Construction:
 - Modified Dijkstra:
 - Up-then-down policy.

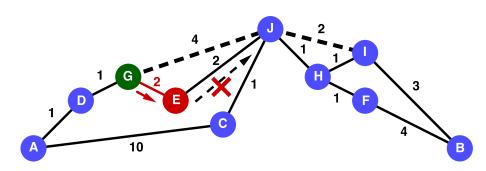


Deconstructing the ch-path gives following three cases: (i) up ch-path: <G, J>; (ii) up-down ch-path: <G, J, I>; (iii) down ch-path: <G, E>.



CH-based Compressed Path Databases (CH-CPD)

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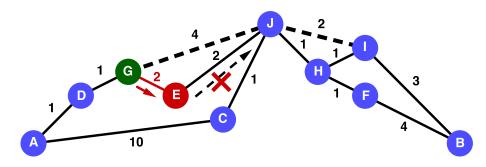


The up-then-down policy: the up successor J of E is pruned, because the predecessor G is lexically larger than E. (i.e., $G >_L E$).



CH-based Compressed Path Databases (CH-CPD)

- CH-CPD:
 - Construction:
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 - Distance table enhancement:



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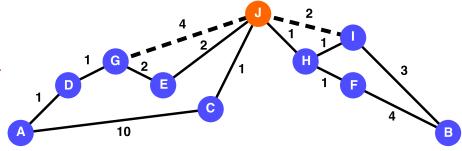


CH-based Compressed Path Databases (CH-CPD)

CH-CPD:

- Construction:
 - Modified Dijkstra:
 - Up-then-down policy.
 - Distance table enhancement:
 - Caching:
 - » Cache the distance table for top n% of CH-nodes.

Ordering	G	D	A	С	J	E	Н	F	В	I
FirstMove(J,_)	G	G	G	С	*	Е	Н	Н	I	I
d(J, _)	4	5	6	1	0	2	1	2	5	2



Distance table enhancement: We cache the all-pairs distance when compute the first moves for node J.

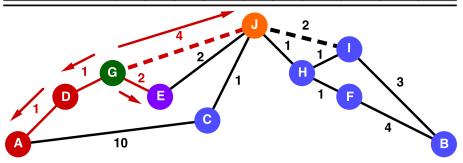


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- Construction:
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 - Up-then-down policy.
 - Distance table enhancement:
 - Caching:
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 - Pruning:

Ordering	G	D	A	С	J	Е	Н	F	В	I
d(J, L)	4	5	6	1	0	2	1	2	5	2
FirstMove(G,_)	*	D	D	-	J	Е	-	-	-	-
$g(G, \cline{L})$	0	1	2	∞	4	2	∞	∞	∞	∞



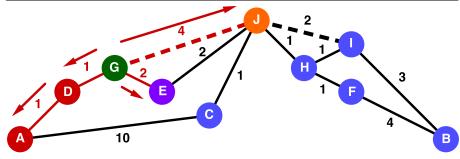


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- Construction:
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 - Caching:
 - » Cache the distance table for top n% of CH-nodes.
 - Pruning:
 - » Relax tentative distance.

Ordering	G	D	A	С	J	E	Н	F	В	I
d(J, L)	4	5	6	1	0	2	1	2	5	2
FirstMove(G,_)	*	D	D	-	J	E	-	-	-	_
$g(G, {}_{ extsf{-}})$	0	1	2	5	4	2	5	6	9	6



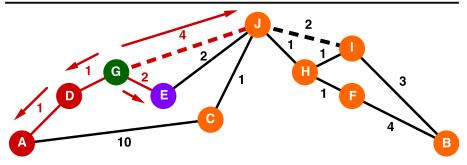


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 - » Update first move table.

Ordering	G	D	A	С	J	E	Н	F	В	I
$d(\mathtt{J}, \mathtt{L})$	4	5	6	1	0	2	1	2	5	2
FirstMove(G,_)	*	D	D	J	J	E	J	J	J	J
$g(G, {}_{ extsf{-}})$	0	1	2	5	4	2	5	6	9	6



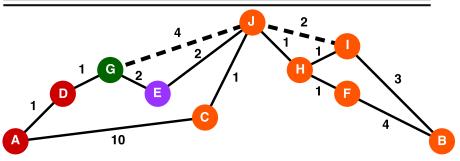


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Ordering	G	D	A	С	J	E	Н	F	В	I
G	*	D	D	J	J	E	J	J	J	J
J	G	G	G	С	*	E	Н	Н	Ι	I
I	J	J	J	J	J	J	Н	Н	В	*

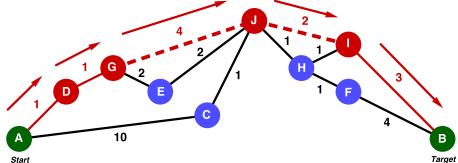




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Exacting the shortest ch-path from CH-CPD, the shortest ch-path from start to target is shown in red.

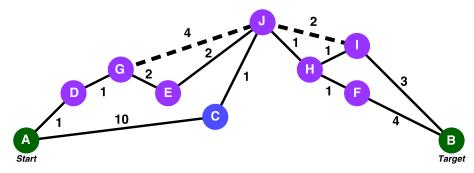
– Query:

Extract and unpack the CH-path from CH-CPD.





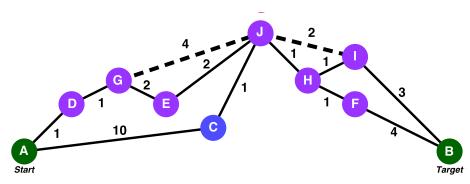
- Construction:
 - Select top n% of CH nodes to construct a CH-CPD.



The partial CH-CPD nodes are D-J which shown in purple color.



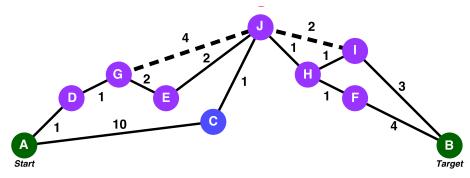
- Construction:
 - Select top n% of CH nodes to construct a CH-CPD.
- Bi-directional CPD search:



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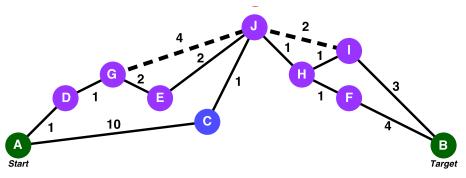
- Construction:
 - Select top n% of CH nodes to construct a CH-CPD.
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 - Incremental exploration:
 - A* search:
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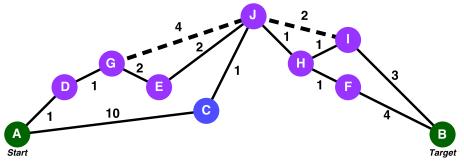
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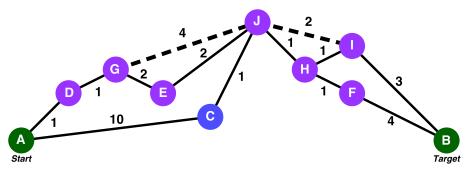
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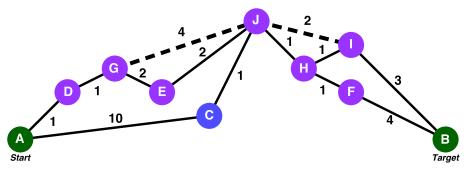
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 - $-V_s \& V_t$.
 - Shortest path (sp).



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 - Pruning:



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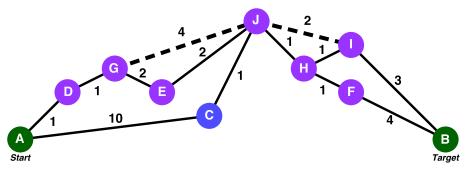
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Pruning:

Distance-based pruning:

»
$$g(s, v_s) + landmark(v_s, v_t) + g(v_t, t) \ge |sp|$$
.



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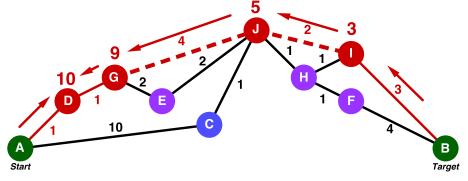


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Pruning:

- Distance-based pruning:
 - » $g(s, v_s) + landmark(v_s, v_t) + g(v_t, t) ≥ |sp|.$
- Cost caching.



Bi-directional CPD search caches distance on each node when extracts the path from I to D.

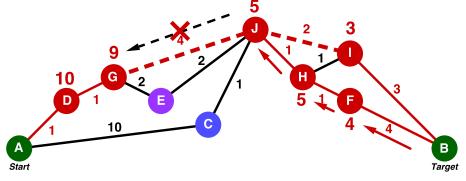


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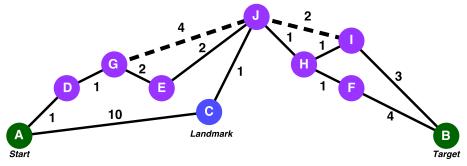
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 - » $g(s, v_s) + landmark(v_s, v_t) + g(v_t, t) ≥ |sp|.$
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The path extraction from F to D terminates at J, because the cached distance g(B, J) < g(B, H) + d(H, J).



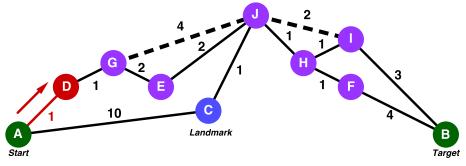
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 - Running example:



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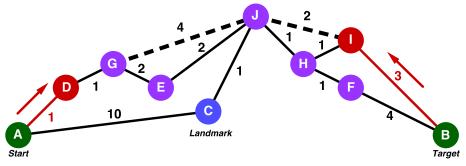
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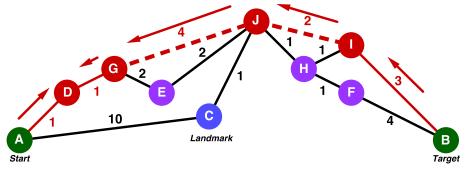
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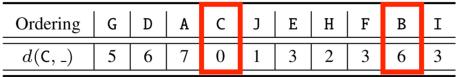


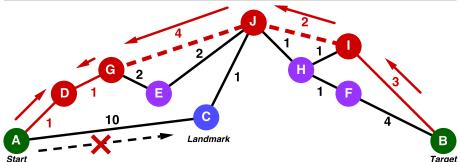
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Partial CH-CPD



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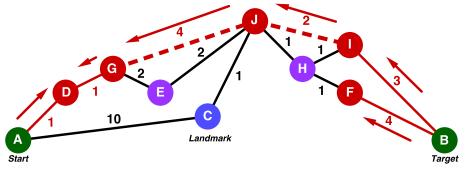




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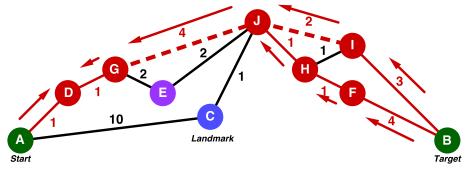
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 - » Landmark heuristics [4].
 - $-V_s \& V_t$.
 - Shortest path (sp).
 - Pruning:
 - Distance-based pruning:
 - » $g(s, v_s) + landmark(v_s, v_t) + g(v_t, t) \ge |sp|$.
 - Cost caching.
 - Running example:



The partial CH-CPD nodes are D-J which shown in purple color.



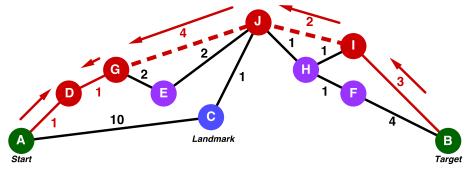
- Construction:
 - Select top n% of CH nodes to construct a CH-CPD.
- Bi-directional CPD search:
 - Incremental exploration:
 - A* search:
 - » Search, & Search, .
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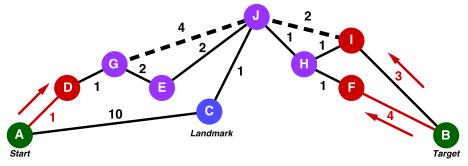
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Experimental Results



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 - Benchmarks:
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- Preprocessing cost:

	Build Time (Mins)								Memory (MB)						
Map Type		(CH-CP	D		Competitors		CH-CPD					Competitors		
	20%	40%	60%	80%	100%	CPD	СН	20%	40%	60%	80%	100%	CPD	CH	
NY Distance	0.36	0.73	1.18	1.87	2.95	8.76	0.24	70	104	183	271	338	219	29	
NY Travel Time	0.27	0.96	1.79	2.24	3.00	11.03	0.16	63	88	156	222	277	188	28	

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– Query performance:

		Average Runtime (µs)											
Map Type		(CH-CPE)		Competitors							
	20%	40%	60%	80%	100%	CPD	СН	CH+L	PHL	SHP			
NY Distance	23.17	20.83	17.19	13.67	11.42	26.38	38.64	25.58	25.36	26.76			
NY Travel Time	19.16	18.82	14.83	12.85	9.53	18.23	25.27	20.06	18.32	14.93			

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 - Real-world road network from DIMACS challenge [5].
- Preprocessing cost:

	Build Time (Mins)								Memory (MB)						
Map Type	CH-CPD					Compe	CH-CPD					Competitors			
	20%	40%	60%	80%	100%	CPD	СН	20%	40%	60%	80%	100%	CPD	СН	
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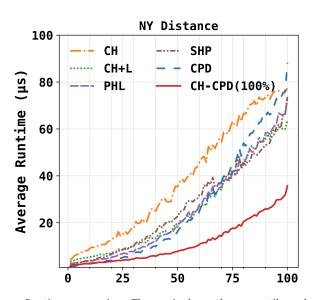
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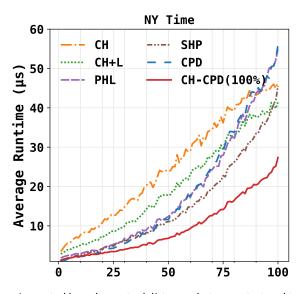
Experimental Results



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Runtime comparison. The x-axis shows the percentile ranks of path queries sorted based on actual distances between start and target.

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Thank you for listening