

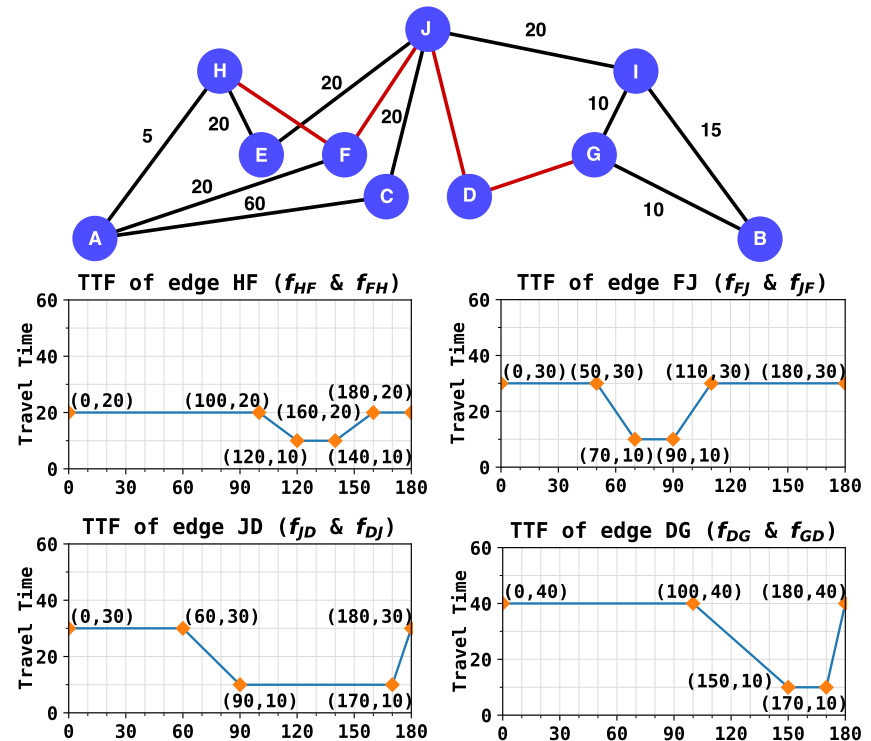
# Improving Time-Dependent Contraction Hierarchy ICAPS 2022

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## Time Dependent Road Network:

- A directed graph:
  - A set of vertices:  $V$ .
  - A set of edges:  $E \subseteq V \times V$ .
  - The travel cost of each edge  $e \in E$  is represented as a travel time function  $f$  (TTF).
    - Each TTF follows FIFO property (i.e.,  $f(t') + t' \geq f(t) + t \mid \forall e \in E \text{ and } \forall t' > t \in T$ ).
- Objective:
  - Given a start  $s$  and destination  $d$ .
  - Find the fastest path that minimize the travel time.



An example of an undirected time-dependent graph. TTFs of the red edges are shown below the graph, and the travel cost of the other edges are constant

## Compressed Path Databases (CPD) Heuristic [1]

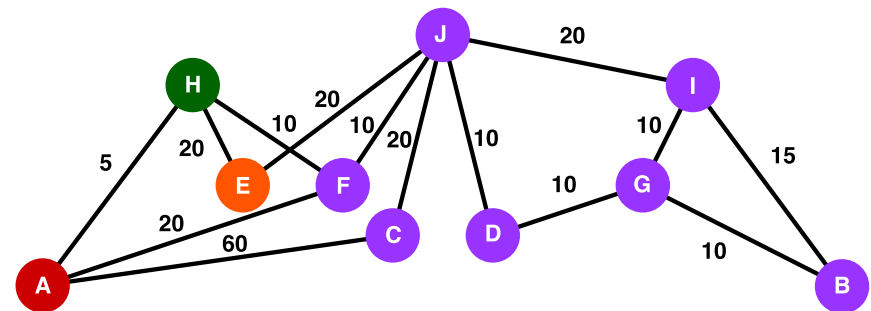
- Compressed Path Databases

### ■ Compressed Path Databases

#### — Construction:

##### ■ First move table:

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
J	F	F	F	*	I	D	D	D	C	E



From the source node H, the first move on the optimal path to any node are A (red), E (orange) and F (purple).

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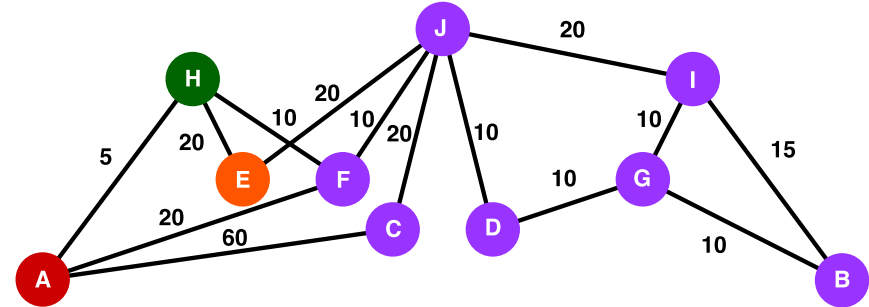
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- [A – J]: indicates the optimal first move.

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H	*	A	F	F	<b>F</b>	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
J	F	F	F	*	I	D	D	D	C	E



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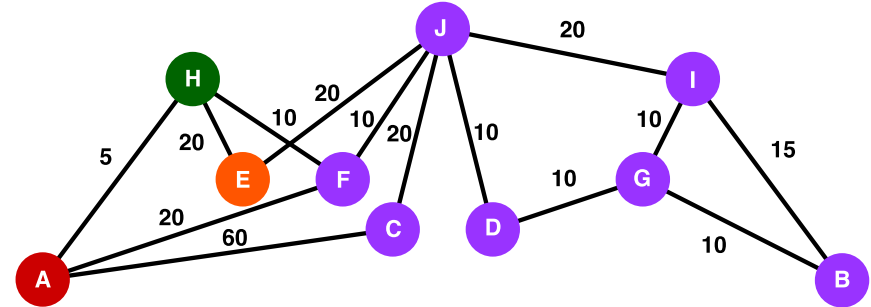
#### — Construction:

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Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
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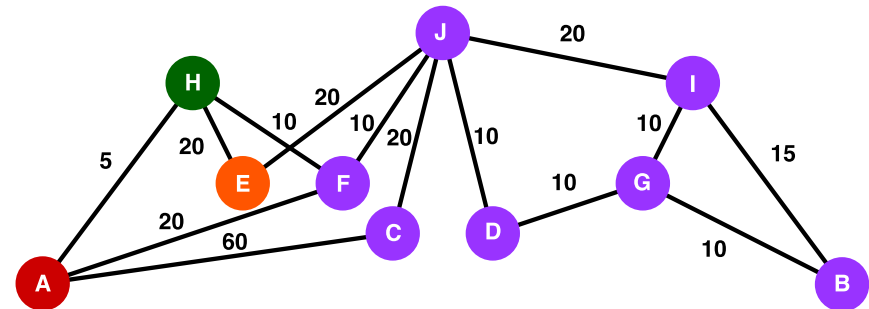
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- Depth first search order [3].

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A	H	*	H	H	H	H	H	H	H	H
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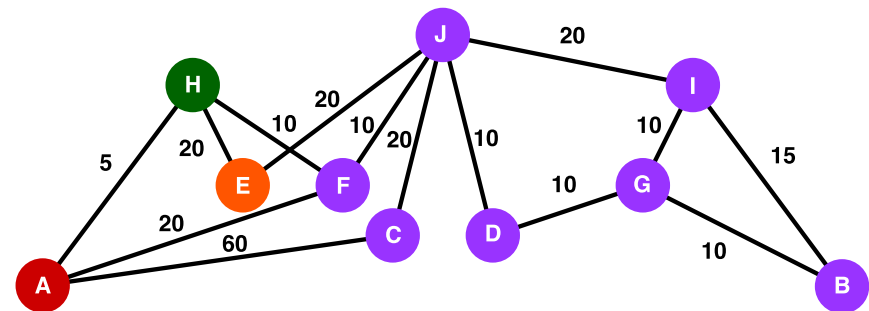
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(i.e. Row H: 1A; 3F; 10E).

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H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
J	F	F	F	*	I	D	D	D	C	E



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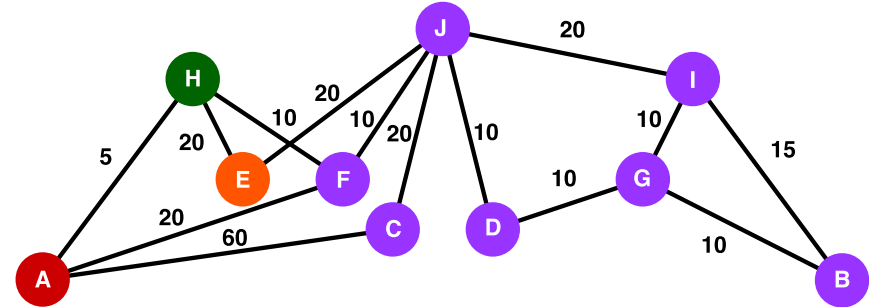
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Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
J	F	F	F	*	I	D	D	D	C	E



From the source node H, the first move on the optimal path to any node are A (red), E (orange) and F (purple).

#### – First move extraction:

- A binary search over compressed RLE string.

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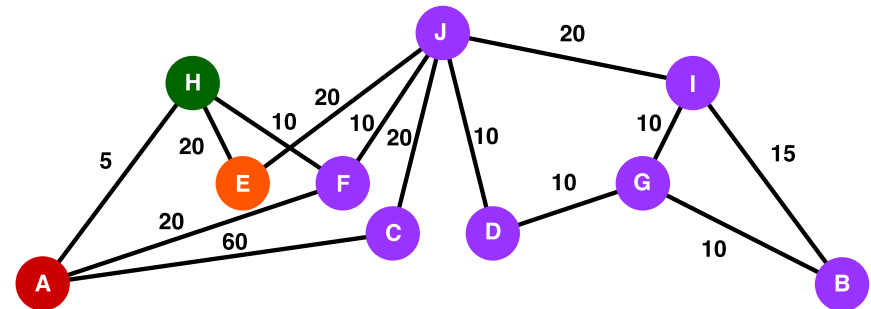
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H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
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### ■ Reverse Path Databases

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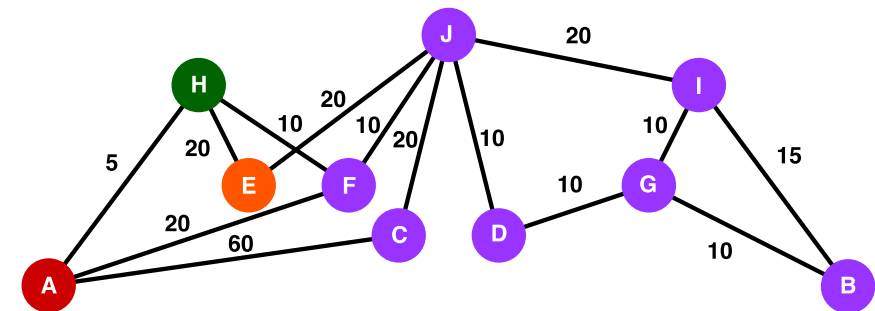
- A binary search over compressed RLE string.

### ■ Reverse Path Databases

#### – Construction:

- Reverse first move table [4].

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
J	F	F	F	*	I	D	D	D	C	E



From the source node H, the first move on the optimal path to any node are A (red), E (orange) and F (purple).

Ordering	H	A	F	J	I	B	G	D	C	E
H	H	H	H	F	J	G	D	J	J	H
A	A	A	H	F	J	G	D	J	J	H
J	F	H	J	J	J	G	D	J	J	J

A reverse first move table which records the for every  $d \in V$ , the first move on the shortest path from  $d$  to  $s$

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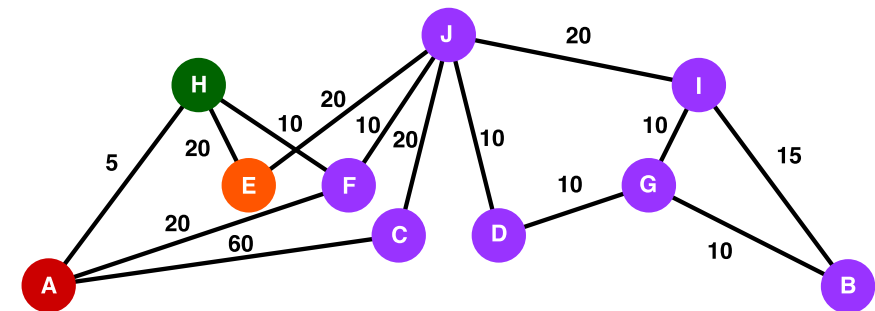
#### – Construction:

- Reverse first move table [4].

#### – First move extraction:

- Accessing the first move in  $O(1)$ .

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
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A	A	A	H	F	J	G	D	J	J	H
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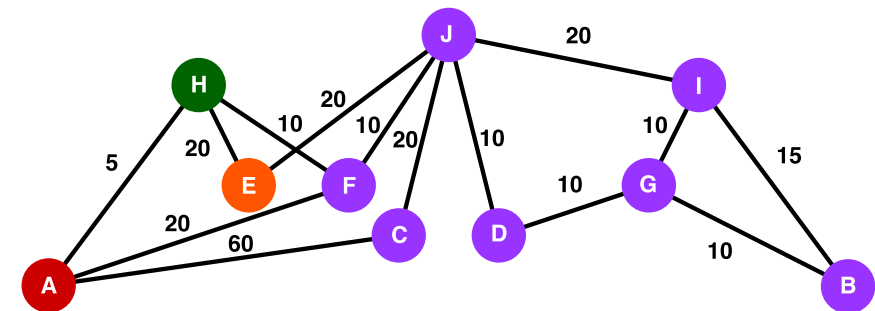
### ■ Reverse Path Databases

- Construction:
  - Reverse first move table [4].
- First move extraction:
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### ■ Heuristic:

- The shortest path extracted from CPD between  $s$  and  $d$  is a valid lower-bound.

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	F	F	F	F	F	F	E
A	H	*	H	H	H	H	H	H	H	H
J	F	F	F	*	I	D	D	D	C	E



From the source node H, the first move on the optimal path to any node are A (red), E (orange) and F (purple).

Ordering	H	A	F	J	I	B	G	D	C	E
H	H	H	H	F	J	G	D	J	J	H
A	A	A	H	F	J	G	D	J	J	H
J	F	H	J	J	J	G	D	J	J	J

A reverse first move table which records the for every  $d \in V$ , the first move on the shortest path from  $d$  to  $s$

## Time Dependent Contraction Hierarchy (TCH) [5]

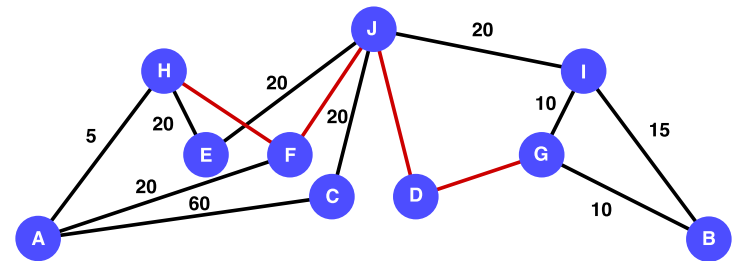
- Time Dependent Contraction Hierarchy:

## Time Dependent Contraction Hierarchy (TCH) [5]

- Time Dependent Contraction Hierarchy:

- Construction:

- Apply a total lex order  $L$ .



The lex order  $L$  is the alphabetical order shown in the figure.

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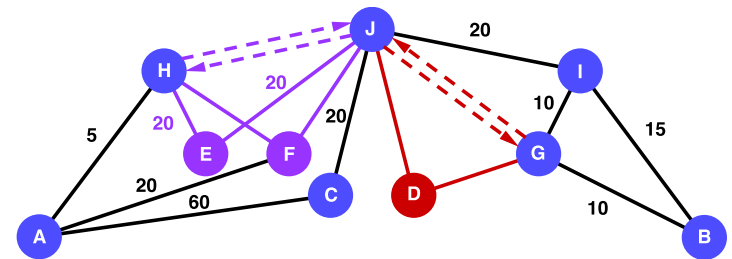
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- Apply a total lex order  $L$ .

#### ■ Contraction:

- W.r.t.  $L$ , choose the least node  $v$  from the graph.



We show the result for contracting nodes E and F in purple, and D in red. Dashed edge are the shortcut edges.

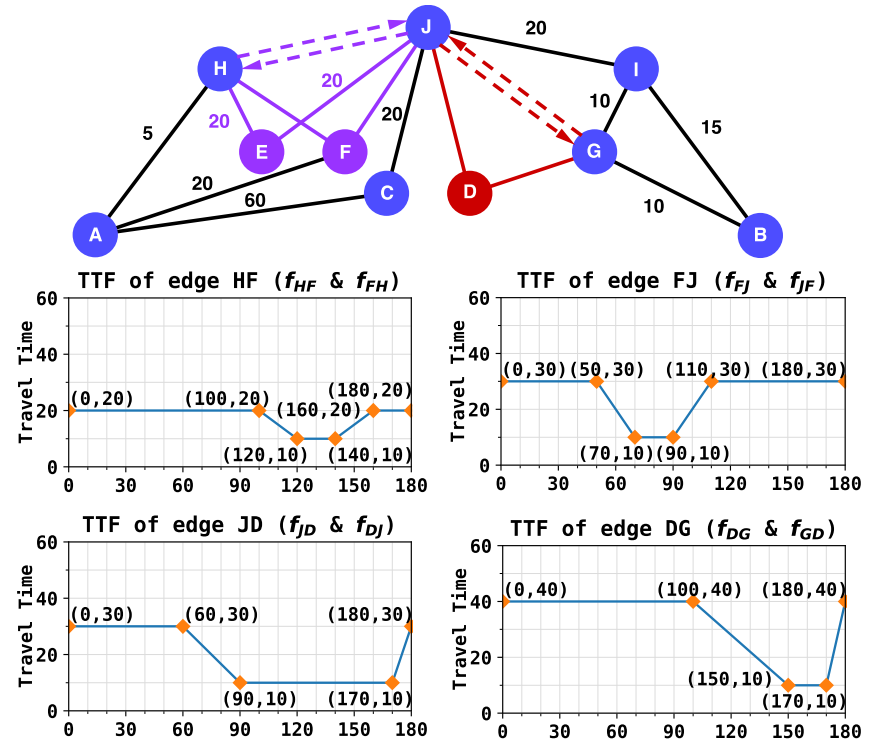


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#### Construction:

- Apply a total lex order  $L$ .
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  - W.r.t.  $L$ , choose the least node  $v$  from the graph.
  - Add a shortcut edge  $(u, w)$  between each pair of in-neighbour  $u$  and out-neighbour  $w$  of  $v$ :
    - »  $v <_L u$  &  $v <_L w$ .
    - »  $\exists t \in T, v \in sp(u, w, t)$ .



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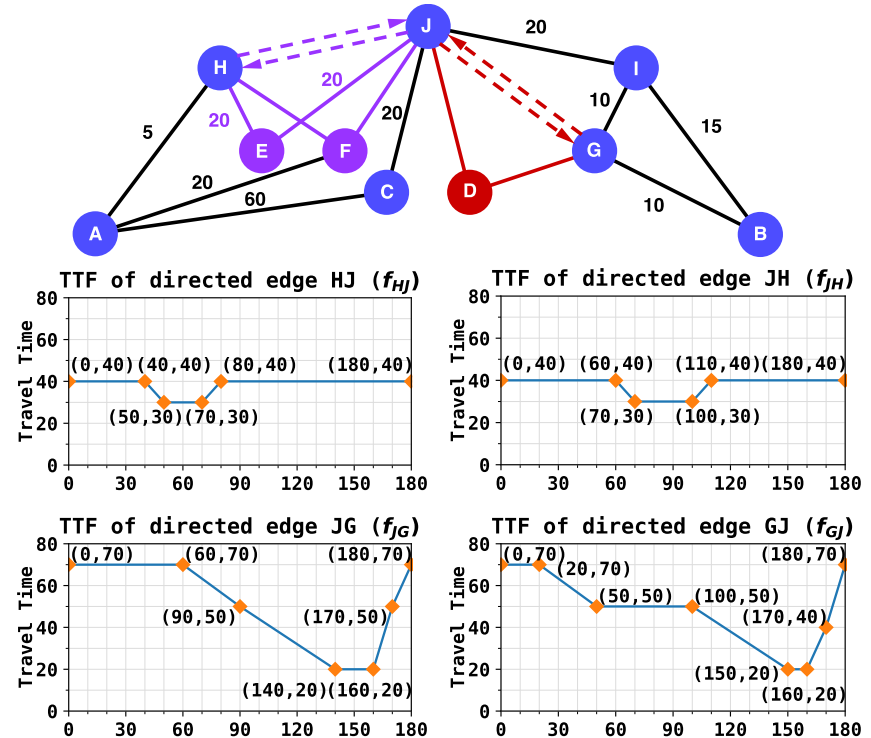
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  - $v <_L u$  &  $v <_L w$ .
  - $\exists t \in T, v \in sp(u, w, t)$ .

- When add a shortcut edge  $(u, w)$  the TTF is computed as

$$f_{uw} = f_{uv} \circ f_{vw} \text{ or } f_{uw} = \min(f_{uw}', f_{uv} \circ f_{vw}) \text{ if parallel edges existed.}$$

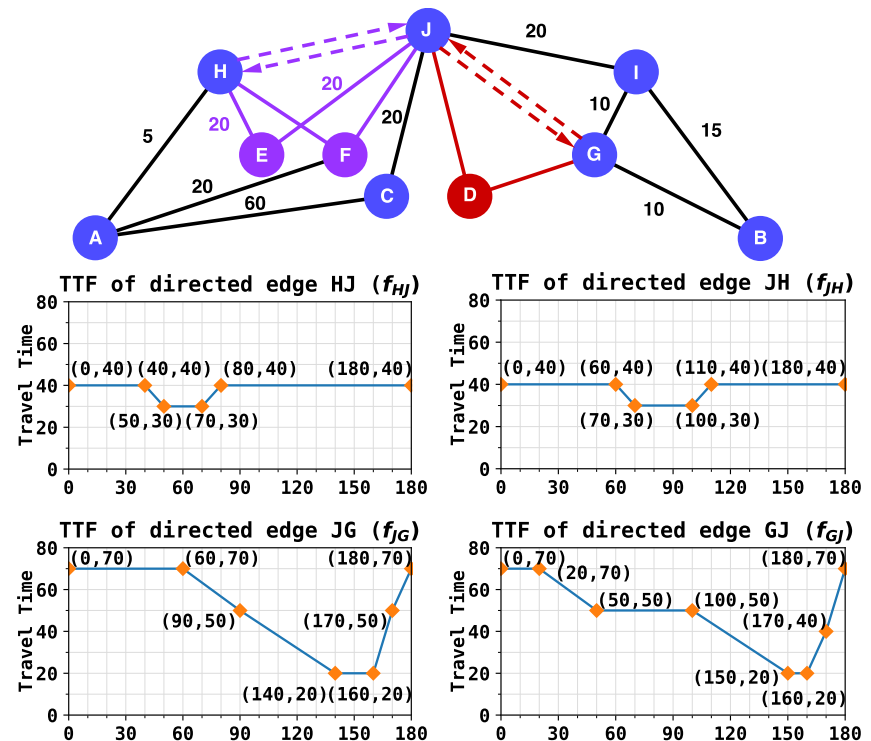


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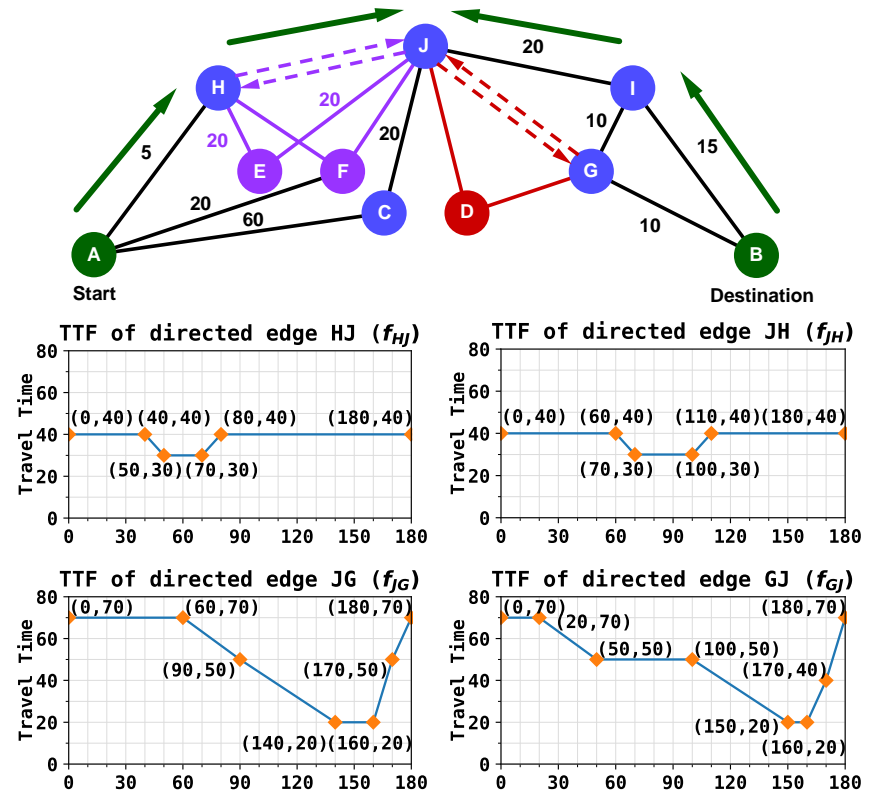


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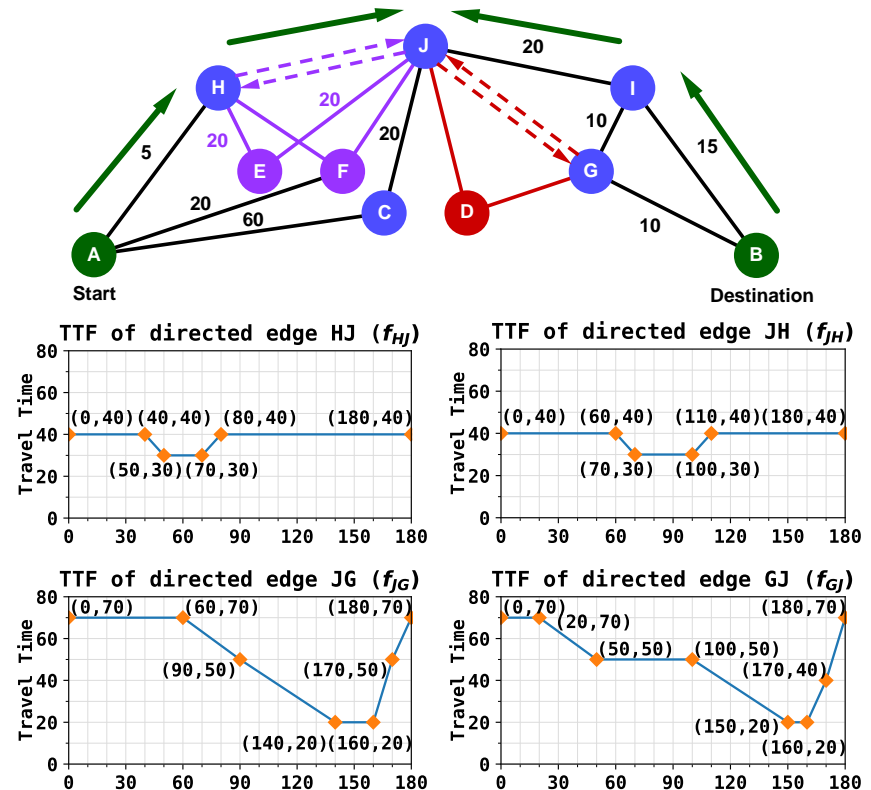


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      - » Run a time-dependent Dijkstra search considering only the outgoing edges in  $E^+$ .

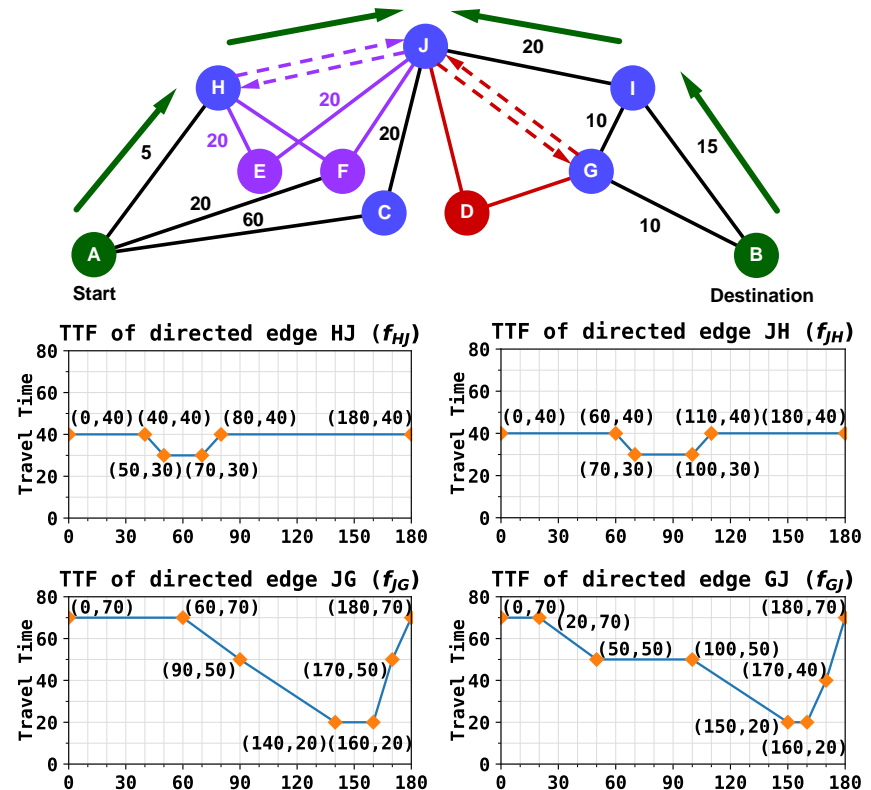


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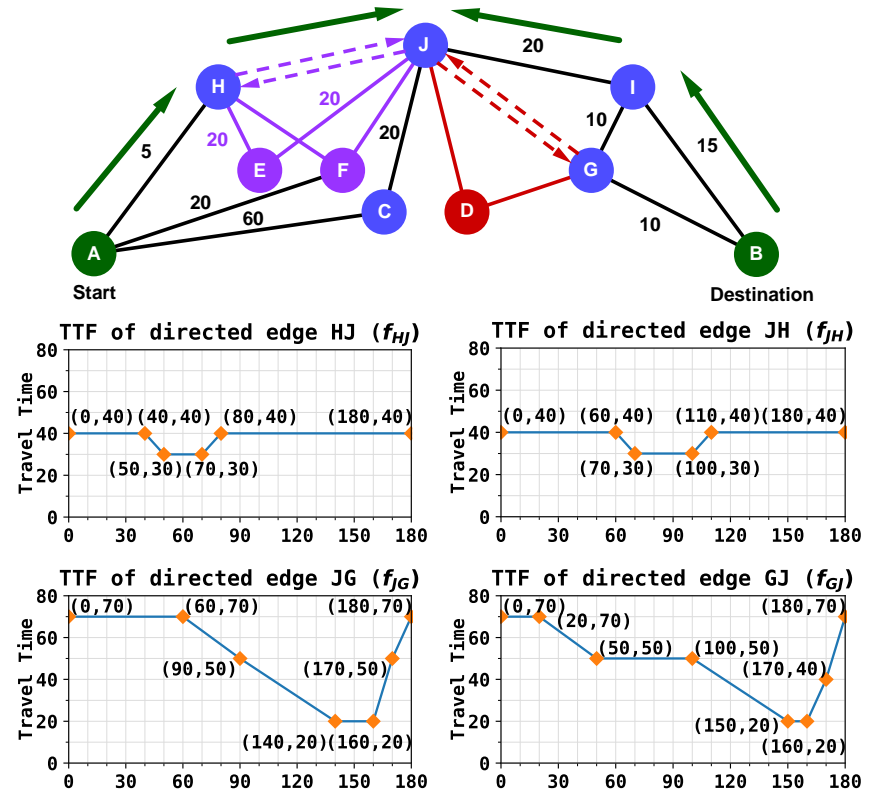


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  - When the search meet at  $k$ :
    - » Compute the upper-bound
      - $fwd(s,k) + upper(k,d)$
    - » Compute the lower-bound
      - $fwd(s,k) + lower(k,d)$

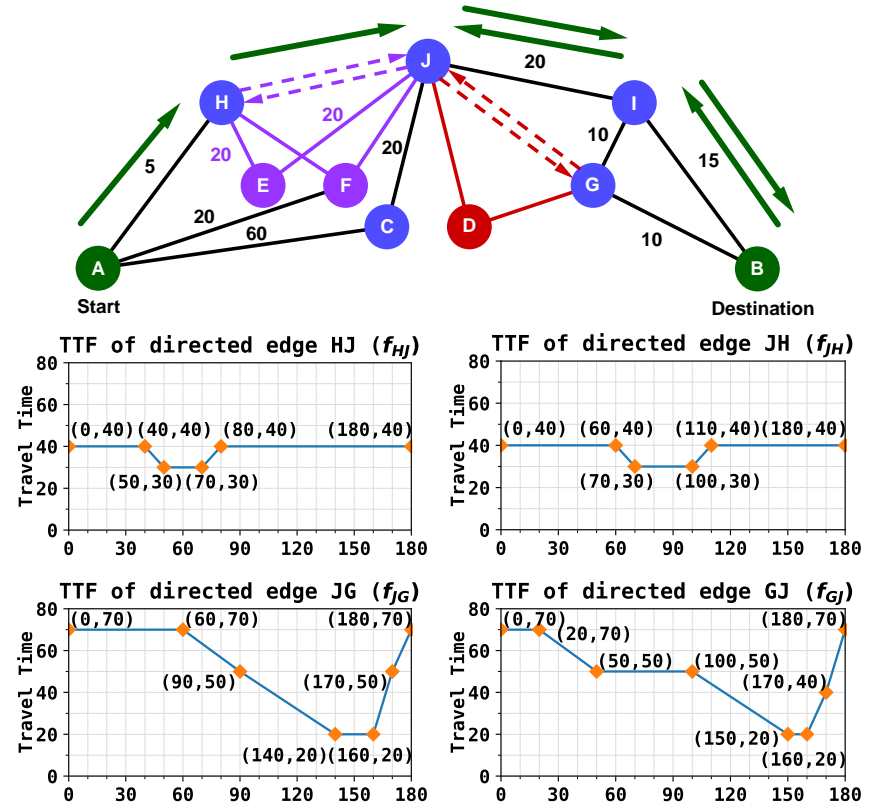


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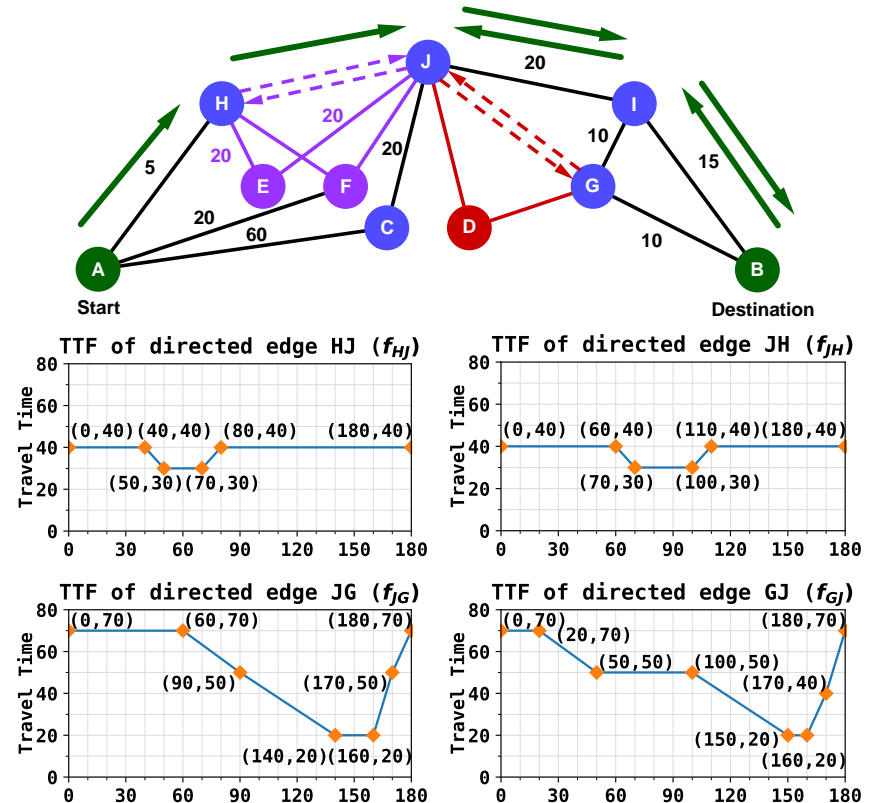
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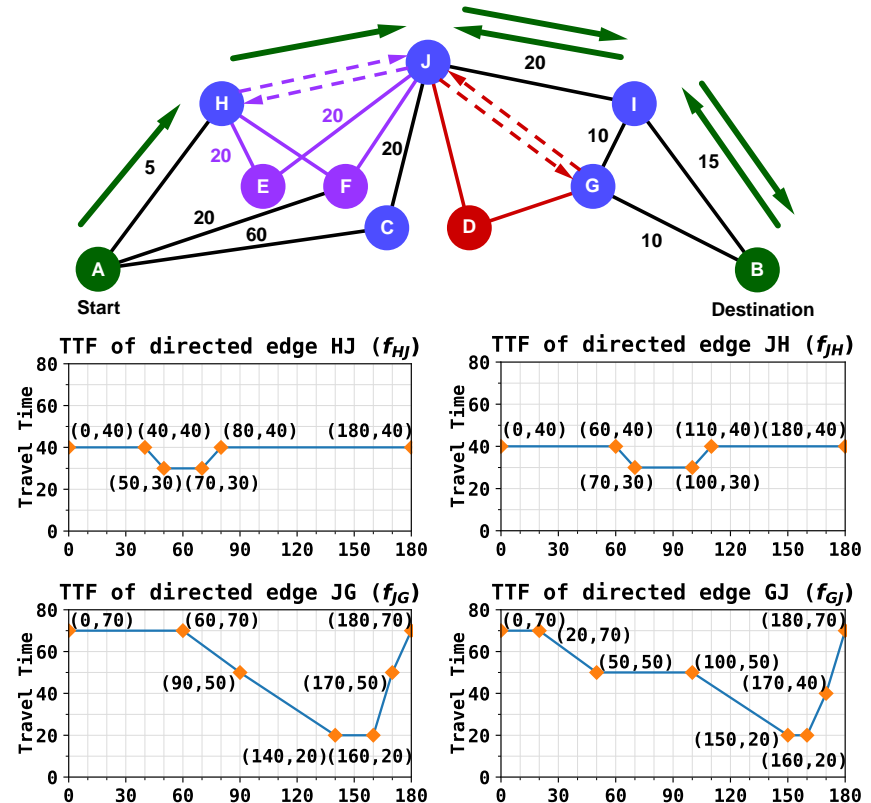


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- Extract the path and unpack.



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# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:

# Our Approach

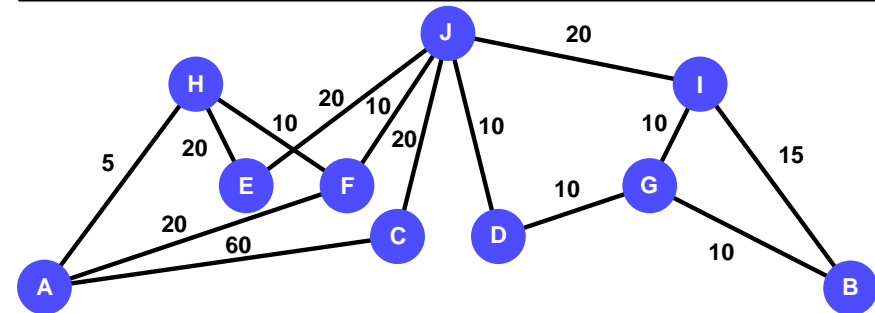
## Improving TCH using Heuristic

- Combining BTCH with landmark:

- Bi-directional search:

- landmark heuristic [6]:

Ordering	H	A	F	J	I	B	G	D	C	E
$d(A, -), d(-, A)$	5	0	15	25	45	55	45	35	45	25
$d(B, -), d(-, B)$	50	55	40	30	15	0	10	20	50	50
$d(J, -), d(-, J)$	20	25	10	0	20	30	20	10	20	20



An example of static graph, where the travel cost of each edge is the free-flow cost of corresponding TTF.

# Our Approach

## Improving TCH using Heuristic

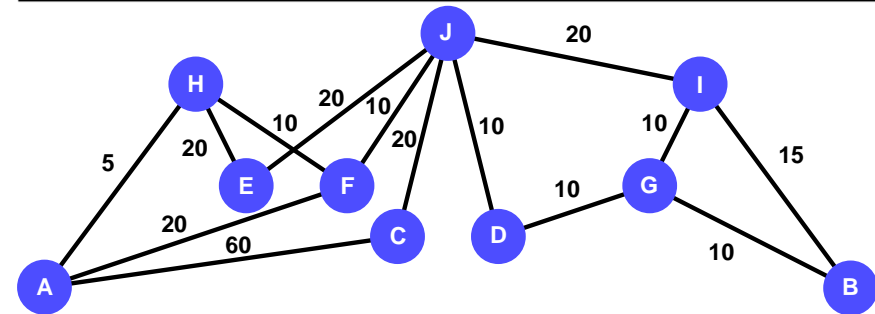
### ■ Combining BTCH with landmark:

#### — Bi-directional search:

##### ■ landmark heuristic [6]:

- $\text{landmark}(v_i, v_j) = \max_{l \in L} \{ \max (d(v_i, l) - d(v_j, l), d(l, v_i) - d(l, v_j)) \}$

Ordering	H	A	F	J	I	B	G	D	C	E
$d(A, -), d(-, A)$	5	0	15	25	45	55	45	35	45	25
$d(B, -), d(-, B)$	50	55	40	30	15	0	10	20	50	50
$d(J, -), d(-, J)$	20	25	10	0	20	30	20	10	20	20



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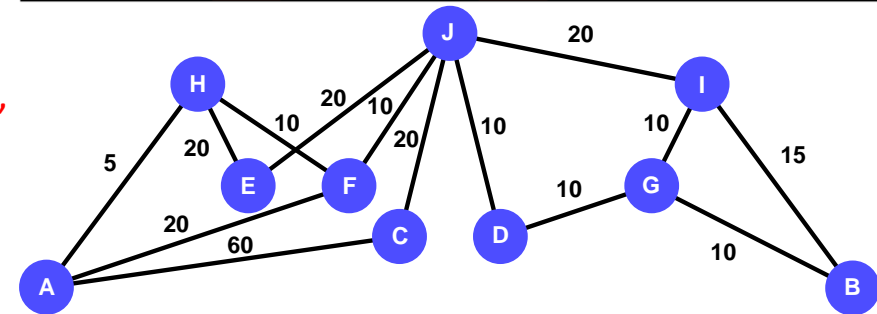
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- E.g.,  $\text{landmark}(H, I) = \max(|5 - 45|, |50 - 15|, |20 - 20|) = 35$

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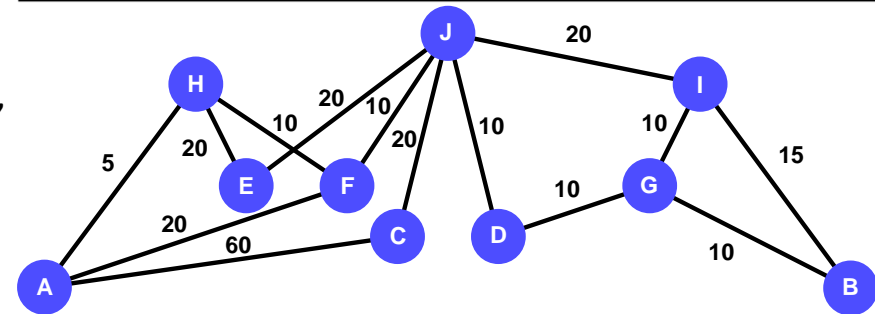
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- E.g.,  $\text{landmark}(H, I) = \max(|5 - 45|, |50 - 15|, |20 - 20|) = 35$

#### – Forward search:

##### ■ Heuristic:

- Directly reuse the lower-bound computed by backward search.

Ordering	H	A	F	J	I	B	G	D	C	E
$d(A, -), d(-, A)$	5	0	15	25	45	55	45	35	45	25
$d(B, -), d(-, B)$	50	55	40	30	15	0	10	20	50	50
$d(J, -), d(-, J)$	20	25	10	0	20	30	20	10	20	20



An example of static graph, where the travel cost of each edge is the free-flow cost of corresponding TTF.

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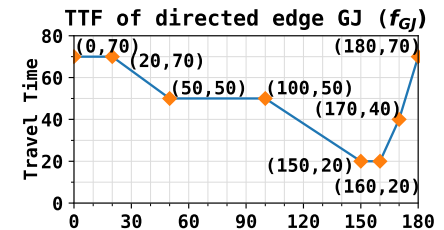
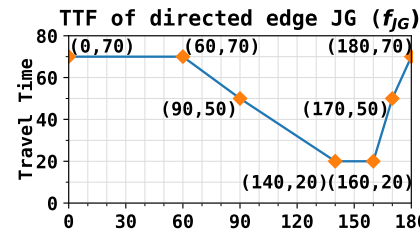
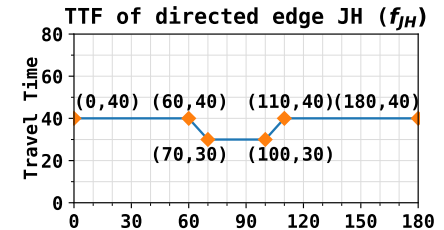
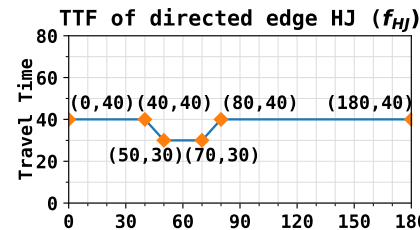
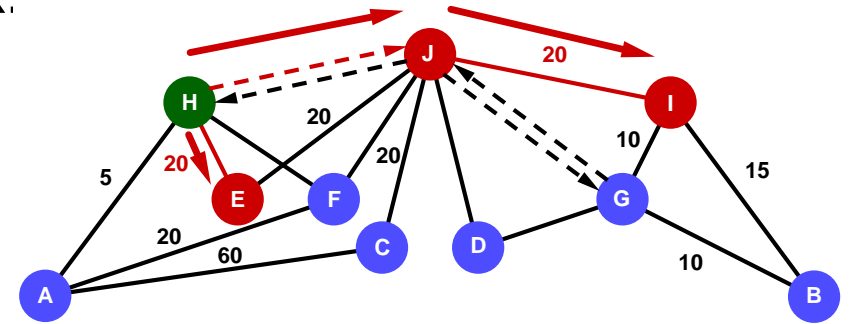
- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:



# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.

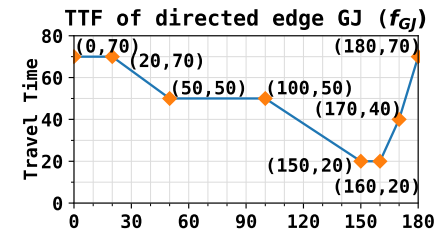
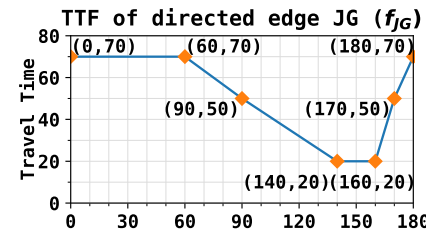
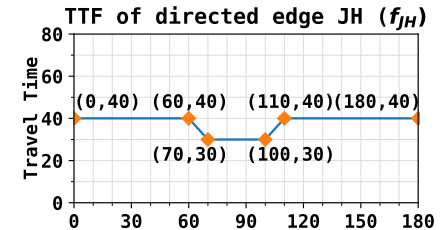
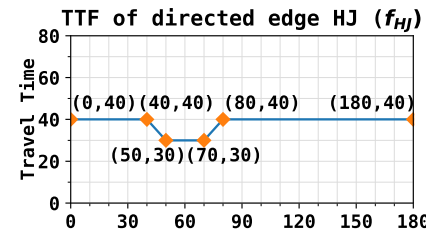
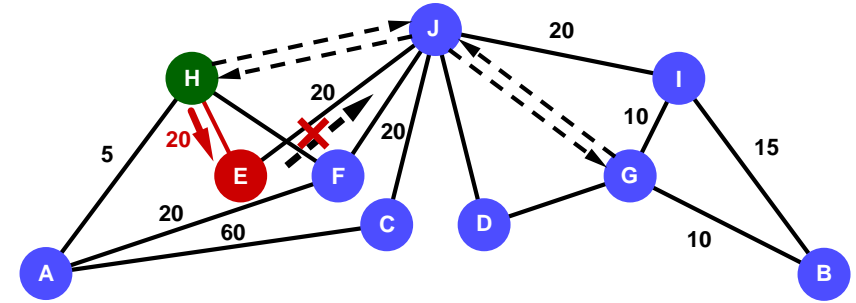


Deconstructing the TCH-path gives following three cases: (i) up TCH-path:  $\langle H, J \rangle$ ; (ii) up-down TCH-path:  $\langle H, J, I \rangle$ ; (iii) down TCH-path:  $\langle H, E \rangle$ .

# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.

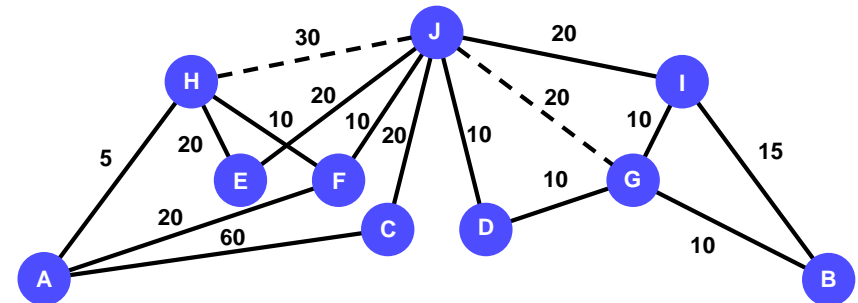


The up-then-down policy: the up successor J of E is pruned, because the predecessor H is lexically larger than E. (i.e.,  $h >_L E$ ).

# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy
  - Heuristic:
    - TCH-CPD heuristic.



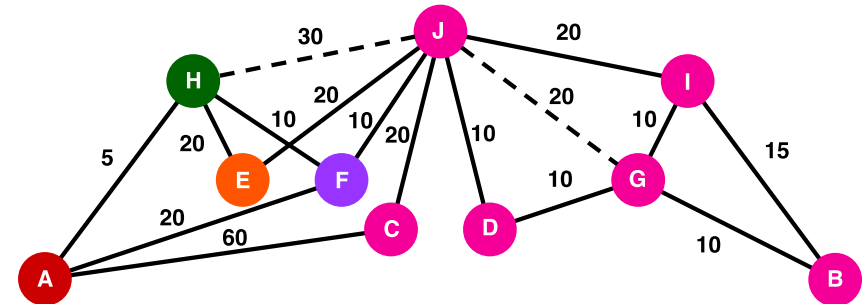
An example of static TCH, where the travel cost of each edge is the free-flow cost of corresponding TTF.

# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
      - Construction:
        - » Run a modified Dijkstra search to compute first move on the optimal TCH-path.

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	J	J	J	J	J	J	E
A	H	*	H	F	F	F	F	F	F	H
J	H	F	F	*	I	G	G	D	C	E



From the source node H, the optimal first move to any node are A (red), E (orange), F (purple) and J (pink)

# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:

– Forward TCH Search:

- Up-then-Down Policy.

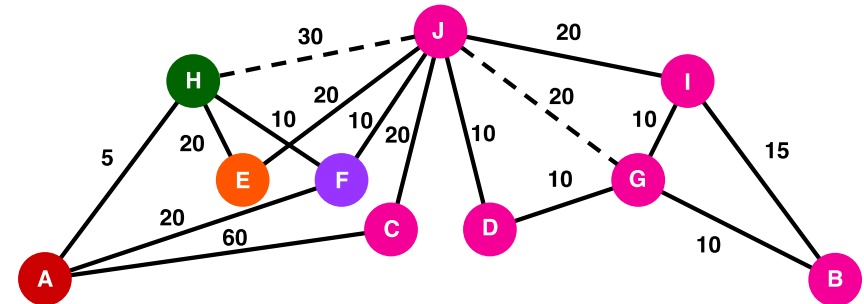
– Heuristic:

- TCH-CPD heuristic.

– Construction:

- » Run a modified Dijkstra search to compute first move on the optimal TCH-path.
- » Compress by following the same procedures as CPD.

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	J	J	J	J	J	J	E
A	H	*	H	F	F	F	F	F	F	H
J	H	F	F	*	I	G	G	D	C	E



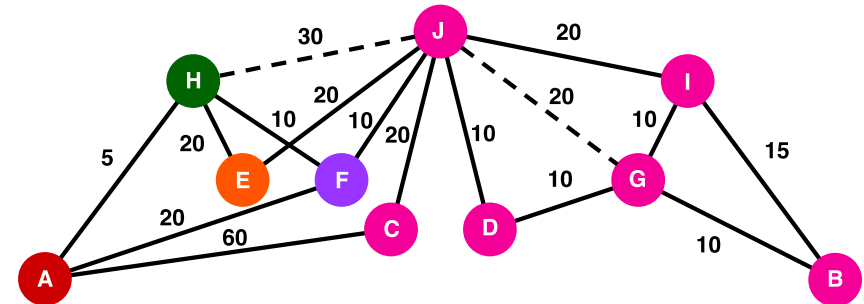
From the source node H, the optimal first move to any node are A (red), E (orange), F (purple) and J (pink)

# Our Approach

## Improving TCH using Heuristic

- Combing BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
  - **Enhancement:**

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	J	J	J	J	J	J	E
A	H	*	H	F	F	F	F	F	F	H
J	H	F	F	*	I	G	G	D	C	E



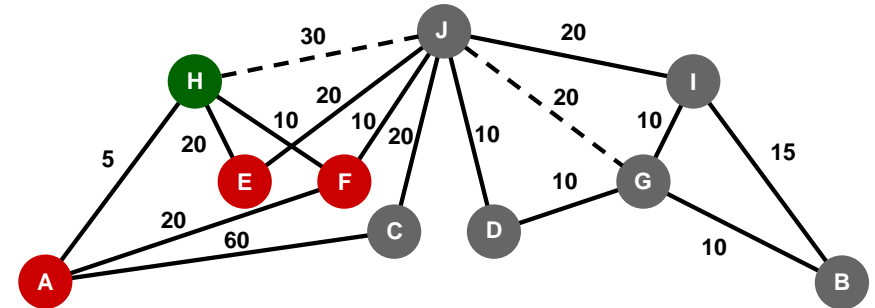
From the source node H, the optimal first move to any node are A (red), E (orange), F (purple) and J (pink)

# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
  - Enhancement:
    - Downward successor pruning:
      - Reachability Oracle.

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	T	T	F	F	F	F	F	F	T
A	F	*	F	F	F	F	F	F	F	F
J	T	T	T	*	T	T	T	T	T	T

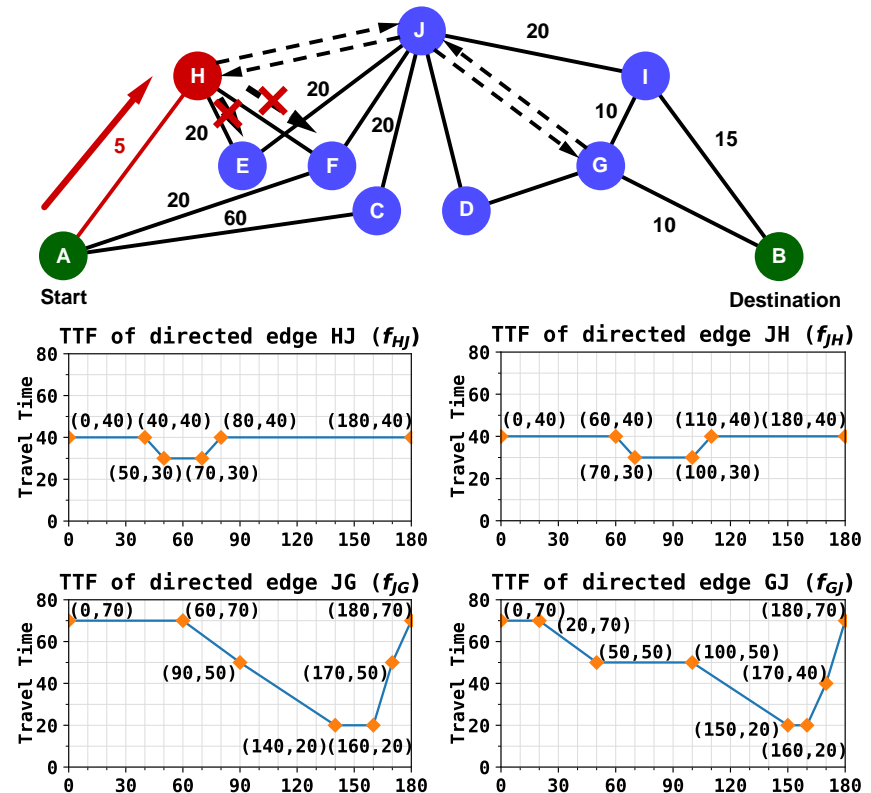


From the source node H, the downward reachable and non-reachable nodes are colored in red and grey respectively.

# Our Approach

## Improving TCH using Heuristic

- Combining BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
  - **Enhancement:**
    - **Downward successor pruning:**
      - Reachability Oracle.
      - **During the search, prune the unreachable down successors.**



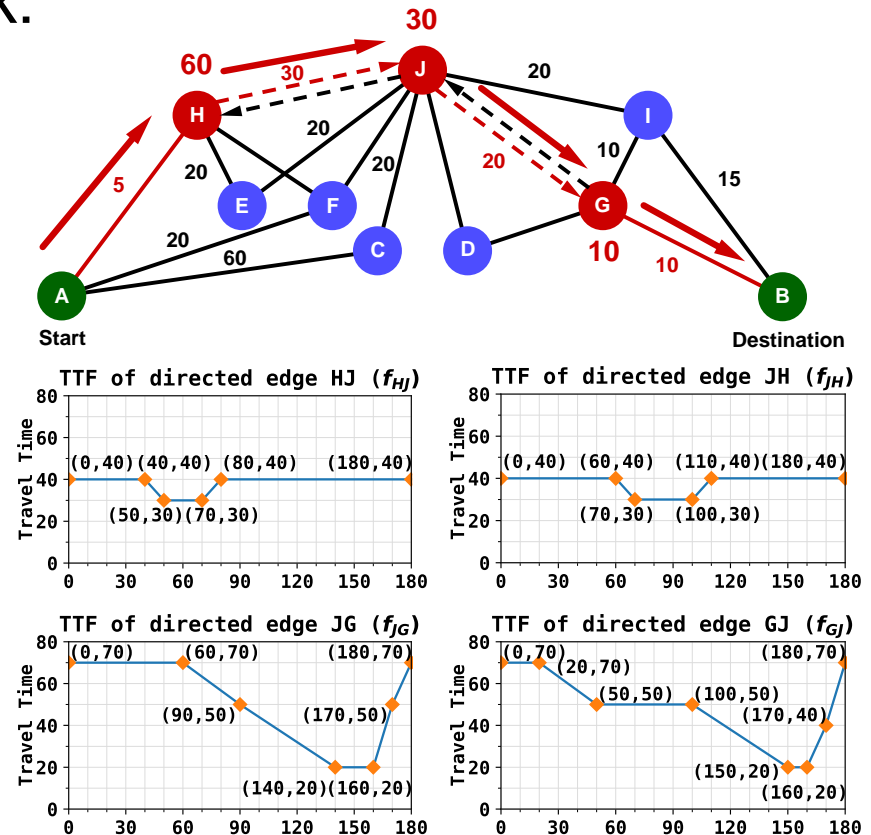
The down successor E and F are pruned, because there exists no down path from E or F can reach the destination B.



# Our Approach

## Improving TCH using Heuristic

- Combing BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
  - **Enhancement:**
    - Downward successor pruning:
      - Reachability Oracle.
      - During the search, prune the unreachable down successors.
    - **Cost Caching.**

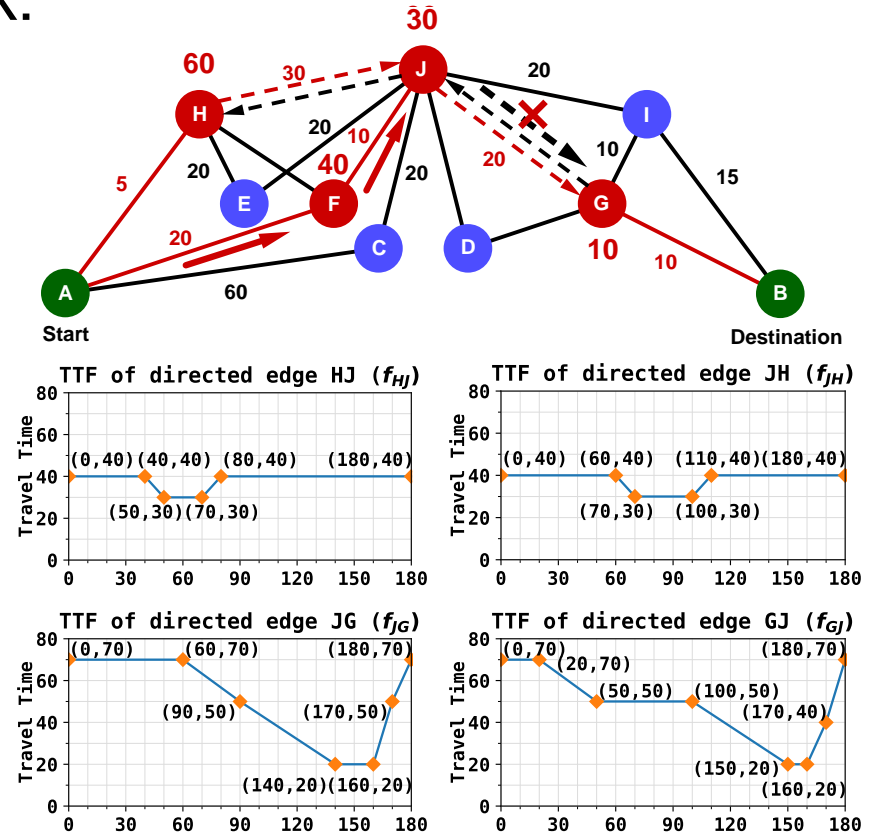


When compute the heuristic, we cache distance on each node when extracts the path from H to B.

# Our Approach

## Improving TCH using Heuristic

- Combing BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
  - **Enhancement:**
    - Downward successor pruning:
      - Reachability Oracle.
      - During the search, prune the unreachable down successors.
    - **Cost Caching.**



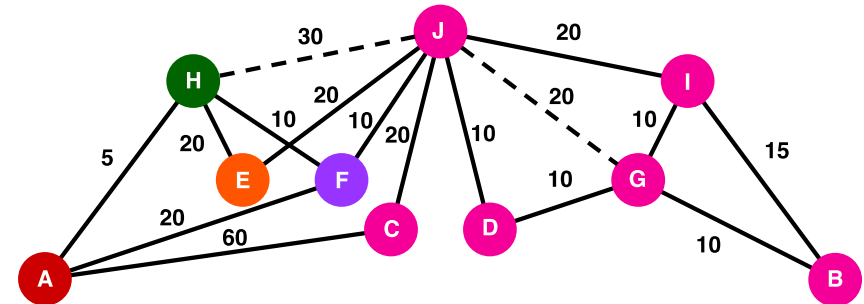
When compute heuristic for node F, the path extraction terminate at J as we already recorded the distance from J to B.

# Our Approach

## Improving TCH using Heuristic

- Combing BTCH with landmark:
- Forward TCH Search with CPD-based Heuristic:
  - Forward TCH Search:
    - Up-then-Down Policy.
  - Heuristic:
    - TCH-CPD heuristic.
  - **Enhancement:**
    - Downward successor pruning:
      - Reachability Oracle.
      - During the search, prune the unreachable down successors.
    - Cost Caching.
    - **Reverse TCH path database.**

Ordering	H	A	F	J	I	B	G	D	C	E
H	*	A	F	J	J	J	J	J	J	E
A	H	*	H	F	F	F	F	F	F	H
J	H	F	F	*	I	G	G	D	C	E



From the source node H, the optimal first move to any node are A (red), E (orange), F (purple) and J (pink)

Ordering	H	A	F	J	I	B	G	D	C	E
H	H	H	H	H	J	G	J	J	J	H
A	A	A	H	F	J	G	J	J	J	H
J	J	H	J	J	J	G	J	J	J	J

A reverse first move table which records the for every  $d \in V$ , the first move on the shortest path from  $d$  to  $s$

# Our Approach

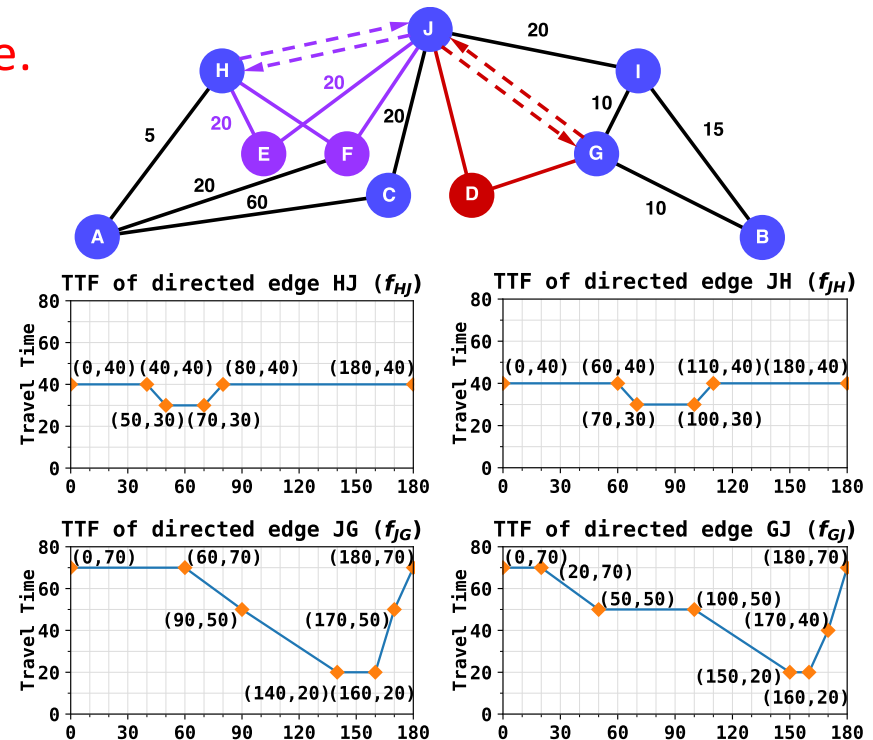
## Improving TCH by splitting the time domain

- Drawbacks of TCH:

# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
  - TTF stores all interpolate points for entire  $T$ , thus are harder to evaluate.

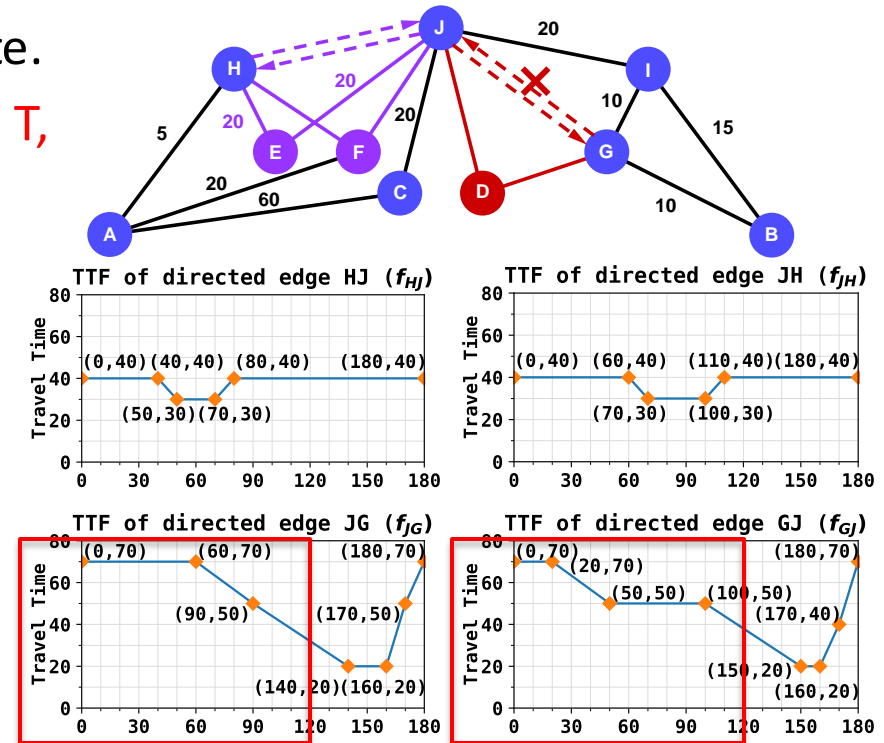


We show the result for contracting nodes E and F in purple, and D in red. Dashed edge are the shortcut edges and their corresponding TTFs are shown in the figure below.

# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
  - TTF stores all interpolate points for entire  $T$ , thus are harder to evaluate.
  - Shortcut edge are added for entire  $T$ , may be unnecessary for  $T' \subseteq T$ .

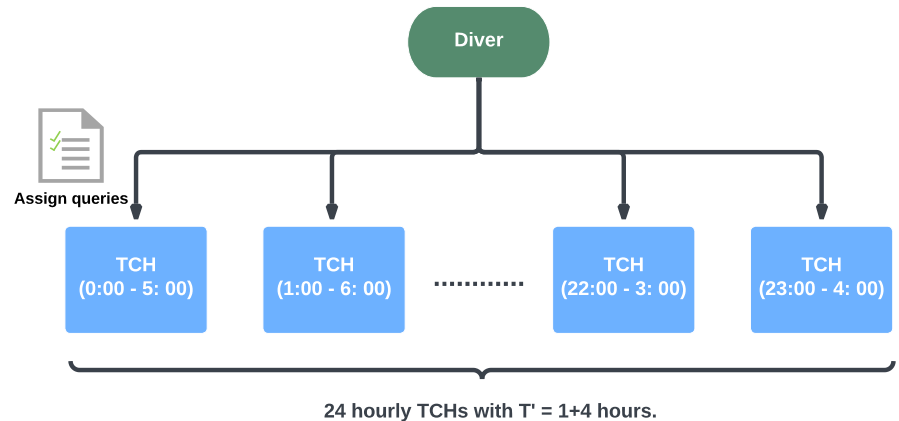


Assume a TCH is built for  $T = [0, 120]$ , contracting the node D cannot add the shortcut JG, as  $\langle J, D, G \rangle$  is a shorter path than  $\langle J, G \rangle$  (i.e.,  $20 + 10 < \min(f_{JD} \circ f_{DG})$ ).

# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
- Single layer TCH (STCH):

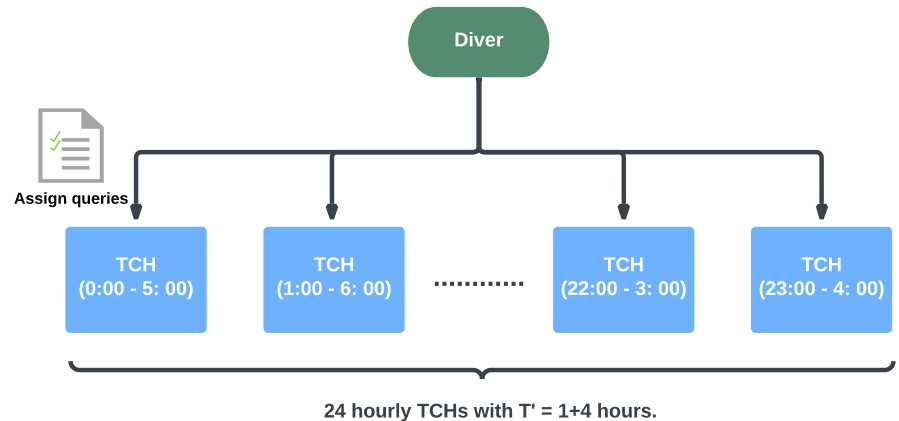




# Our Approach

## Improving TCH by splitting the time domain

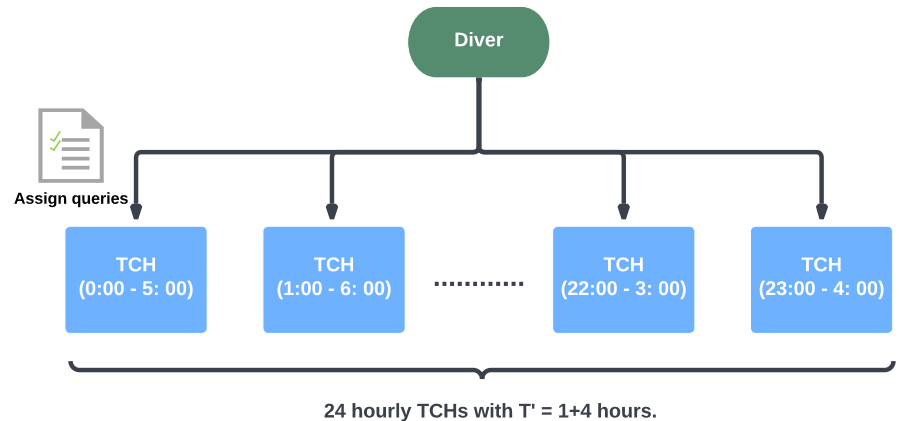
- Drawbacks of TCH:
- Single layer TCH (STCH):
  - Divide the time domain  $T$  into  $n$  buckets.
  - Built a TCH for each bucket:
    - $T' = T/n + U$ .



# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
- Single layer TCH (STCH):
  - Divide the time domain  $T$  into  $n$  buckets.
  - Built a TCH for each bucket:
    - $T' = T/n + U$ .
  - Query:
    - Assign queries to the corresponding TCH.



# Our Approach

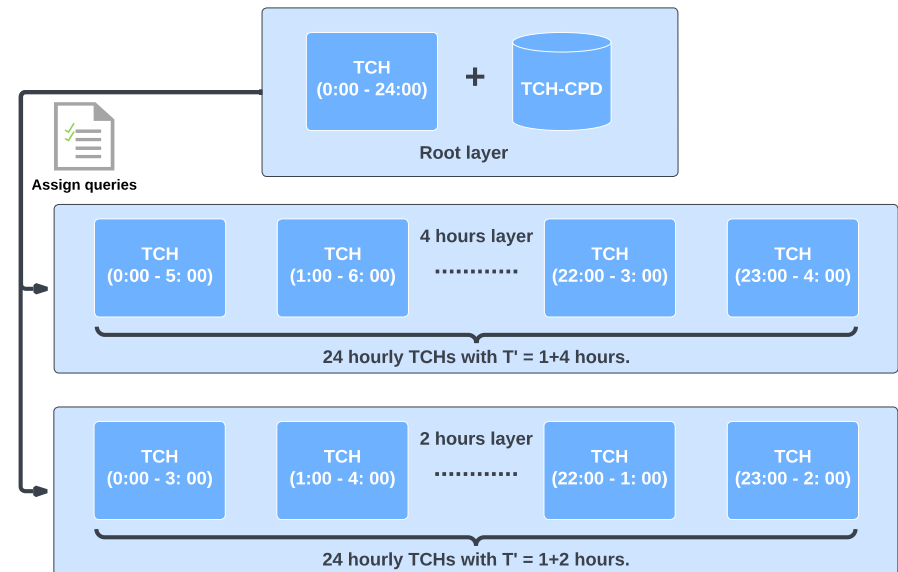
## Improving TCH by splitting the time domain

- Drawbacks of TCH:
- Single layer TCH (STCH):
- Multi layer TCH (MTCH):

# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
- Single layer TCH (STCH):
- Multi layer TCH (MTCH):
  - Root layer:
    - Build a full TCH and TCH-CPD.

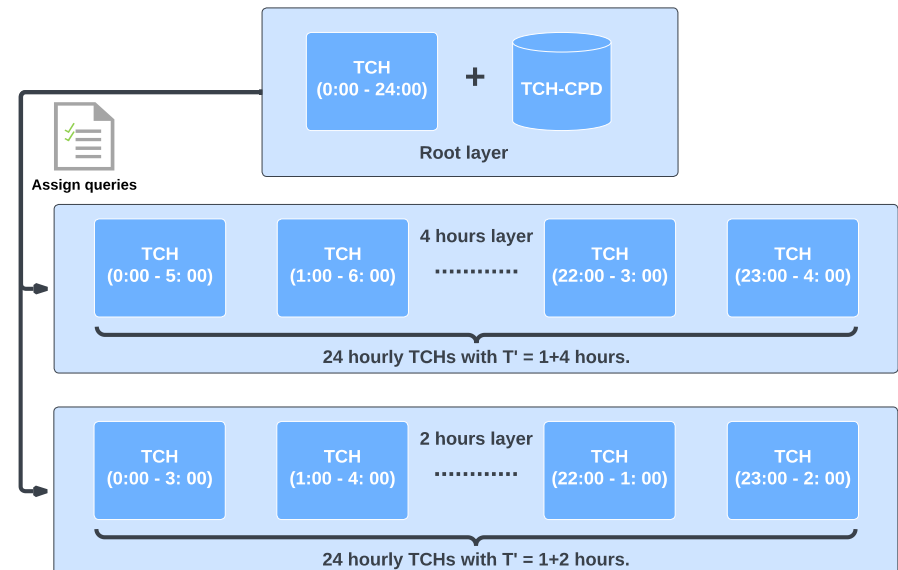


# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
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- Multi layer TCH (MTCH):
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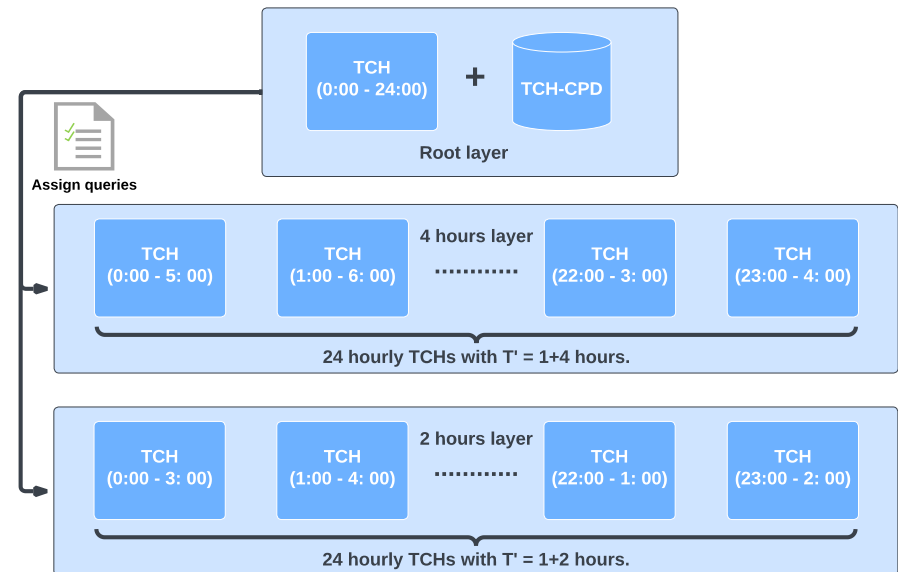
- For each layer under root layer:
  - Build STCH with any arbitrary U.



# Our Approach

## Improving TCH by splitting the time domain

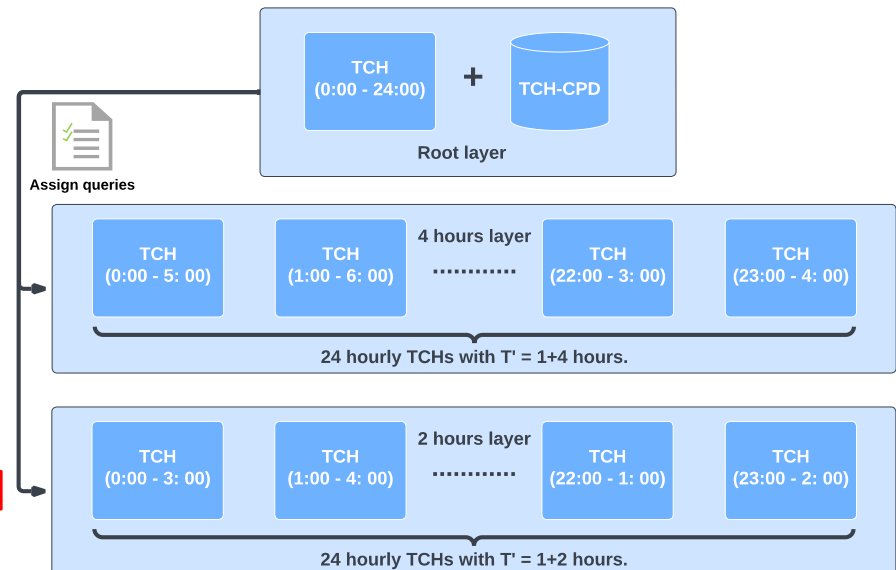
- Drawbacks of TCH:
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  - Query:
    - TCH-CPD to compute an upper-bound  $U(s,d)$ .



# Our Approach

## Improving TCH by splitting the time domain

- Drawbacks of TCH:
- Single layer TCH (STCH):
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  - Root layer:
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    - Build STCH with any arbitrary U.
  - Query:
    - TCH-CPD to compute an upper-bound  $U(s,d)$ .
    - Assign queries to the TCH with minimal  $T'$  that covers  $[t, t+U(s,d)]$



# Our Approach

## Experimental Results

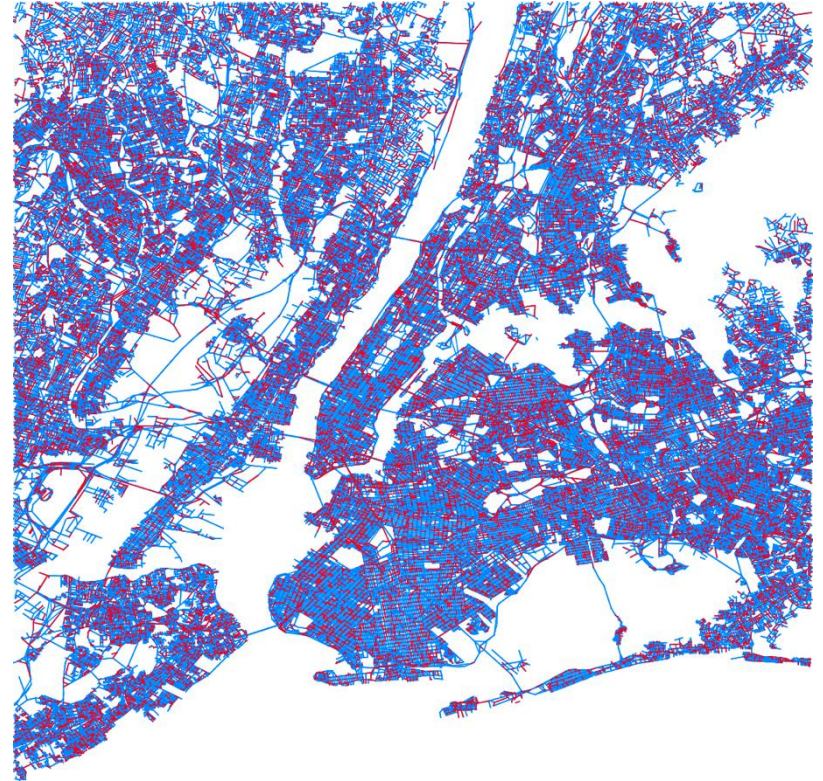
- **Experimental Results:**



# Our Approach

## Experimental Results

- Experimental Results:
  - Benchmarks:
    - Real-world time-dependent road network [7].



Real-world dataset: New York. The edges that have TTF are colored in red.

# Our Approach

## Experimental Results

- Experimental Results:
  - Preprocessing cost:

Map	#V	#E	Build Time (Mins)								Memory (MB)							
			TCH				STCH		MTCH		TCH				STCH		MTCH	
			-	CPD	TCPD	RTPD	-	TCPD	-	TCPD	-	CPD	TCPD	RTPD	-	TCPD	-	TCPD
NY	96k	260k	1.72	2.92	2.94	3.21	2.99	31.87	9.04	95.12	269	346	353	9596	1279	3286	3193	9198

# Our Approach

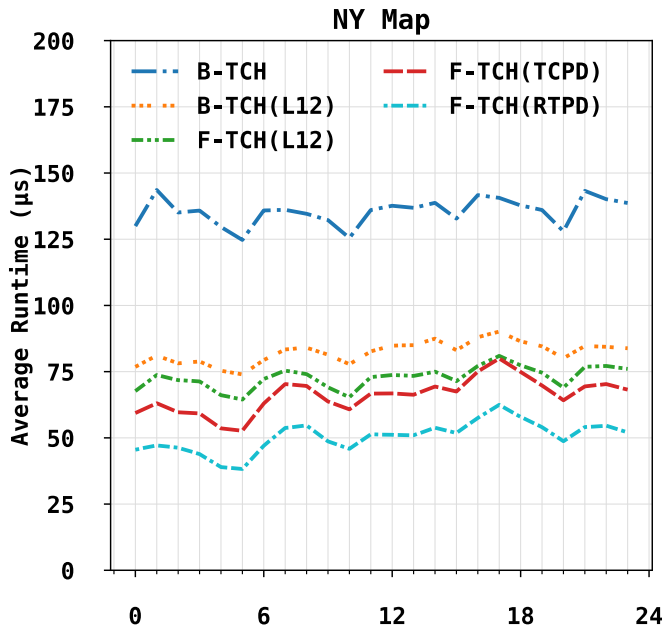
## Experimental Results

- Experimental Results:
  - Preprocessing cost:

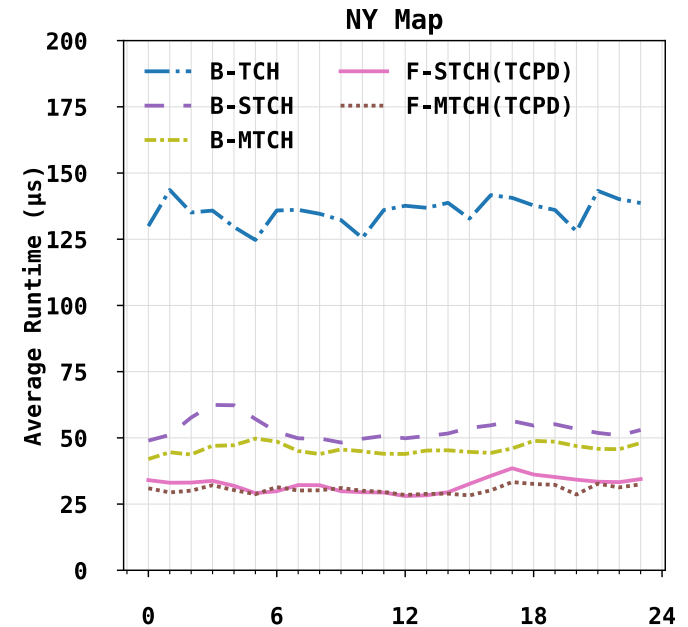
Map	#V	#E	Build Time (Mins)								Memory (MB)							
			TCH				STCH		MTCH		TCH				STCH		MTCH	
			-	CPD	TCPD	RTPD	-	TCPD	-	TCPD	-	CPD	TCPD	RTPD	-	TCPD	-	TCPD
NY	96k	260k	1.72	2.92	2.94	3.21	2.99	31.87	9.04	95.12	269	346	353	9596	1279	3286	3193	9198

- Query performance:

### Heuristic Search



### Time Domain Splitting



# Our Approach

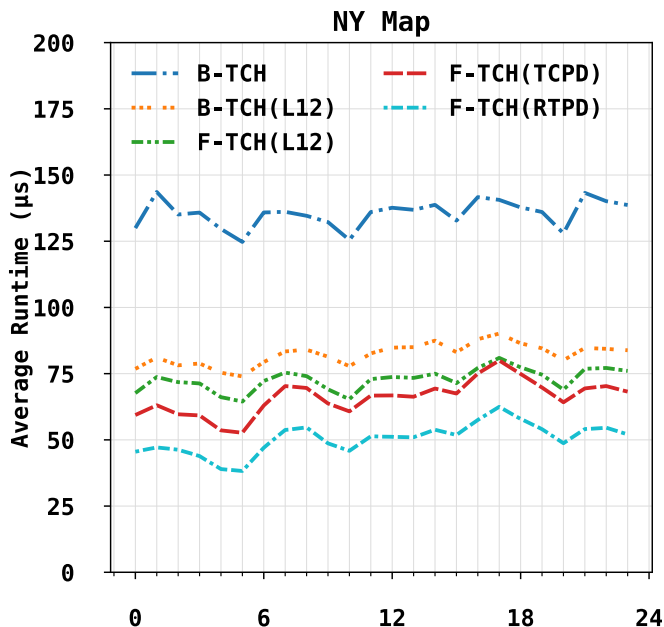
## Experimental Results

- Experimental Results:
  - Preprocessing cost:

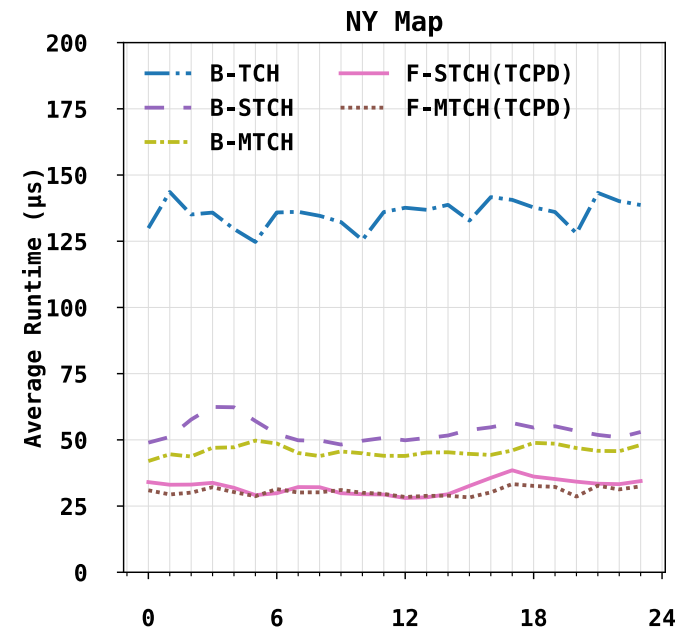
Map	#V	#E	Build Time (Mins)								Memory (MB)							
			TCH				STCH		MTCH		TCH				STCH		MTCH	
			-	CPD	TCPD	RTPD	-	TCPD	-	TCPD	-	CPD	TCPD	RTPD	-	TCPD	-	TCPD
NY	96k	260k	1.72	2.92	2.94	3.21	2.99	31.87	9.04	95.12	269	346	353	9596	1279	3286	3193	9198

- Query performance:

### Heuristic Search



### Time Domain Splitting



# Our Approach

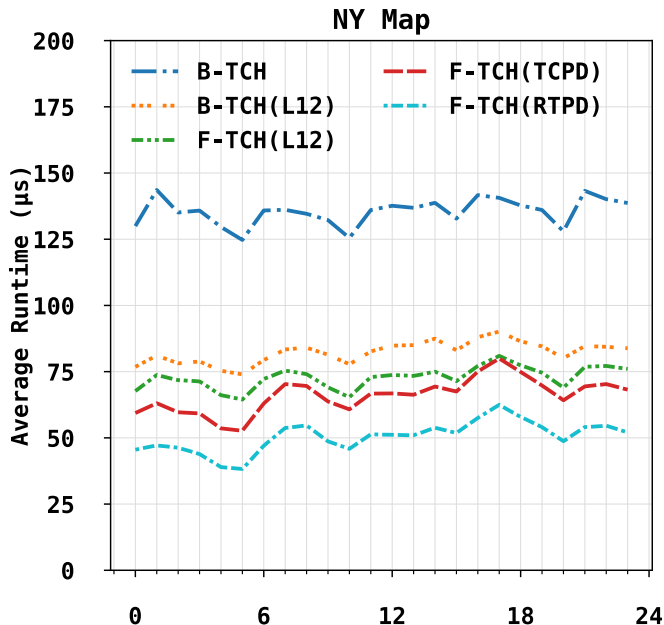
## Experimental Results

- Experimental Results:
  - Preprocessing cost:

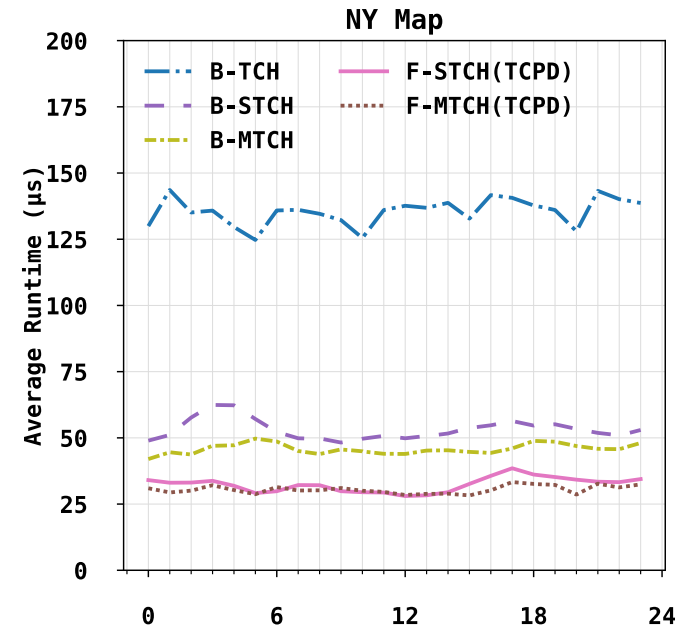
Map	#V	#E	Build Time (Mins)								Memory (MB)							
			TCH				STCH		MTCH		TCH				STCH		MTCH	
			-	CPD	TCPD	RTPD	-	TCPD	-	TCPD	-	CPD	TCPD	RTPD	-	TCPD	-	TCPD
NY	96k	260k	1.72	2.92	2.94	3.21	2.99	31.87	9.04	95.12	269	346	353	9596	1279	3286	3193	9198

- Query performance:

### Heuristic Search



### Time Domain Splitting



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Thank you for listening