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Beyond Pairwise Reasoning in Multi-Agent Path Finding

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[1]Monash University

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Multi-Agent Path Finding (MAPF)

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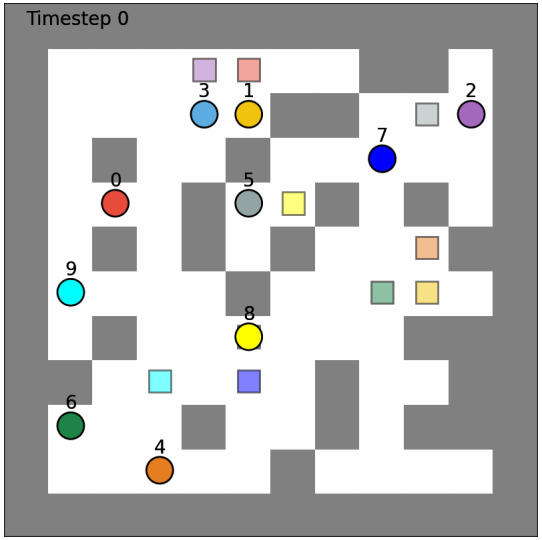

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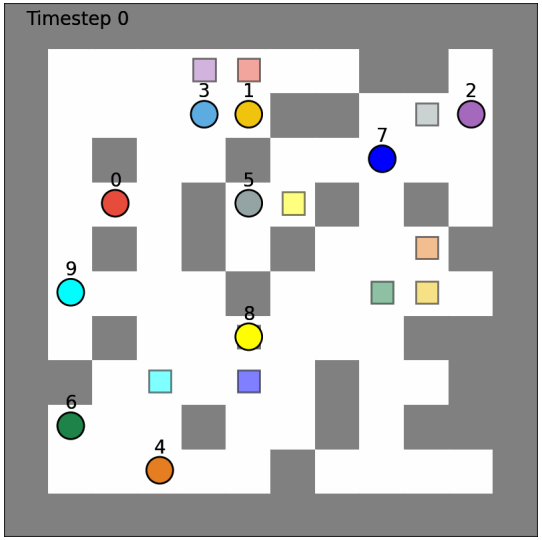
- Multi-Agent Path Finding:
  - Application:
    - Automated warehouse.
  - Environment:
    - 4-connected grid map.
    - Discretized timesteps.
  - Objectives:
    - Given a set of agents with source and destination.
    - Find a collision-free plan that minimizes the Sum of Individual Cost (SIC).



Related Work
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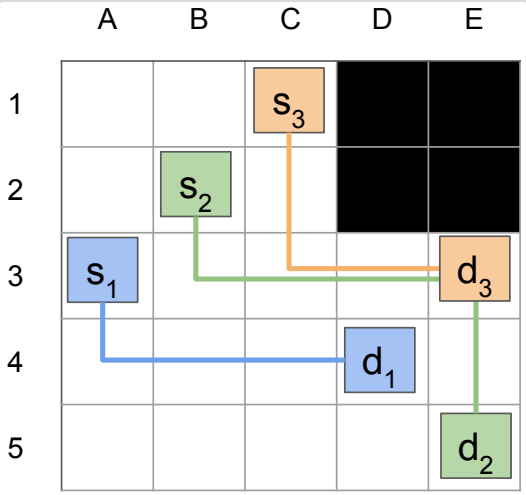

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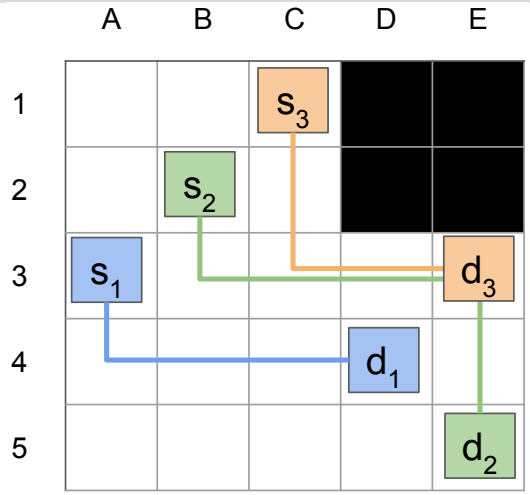

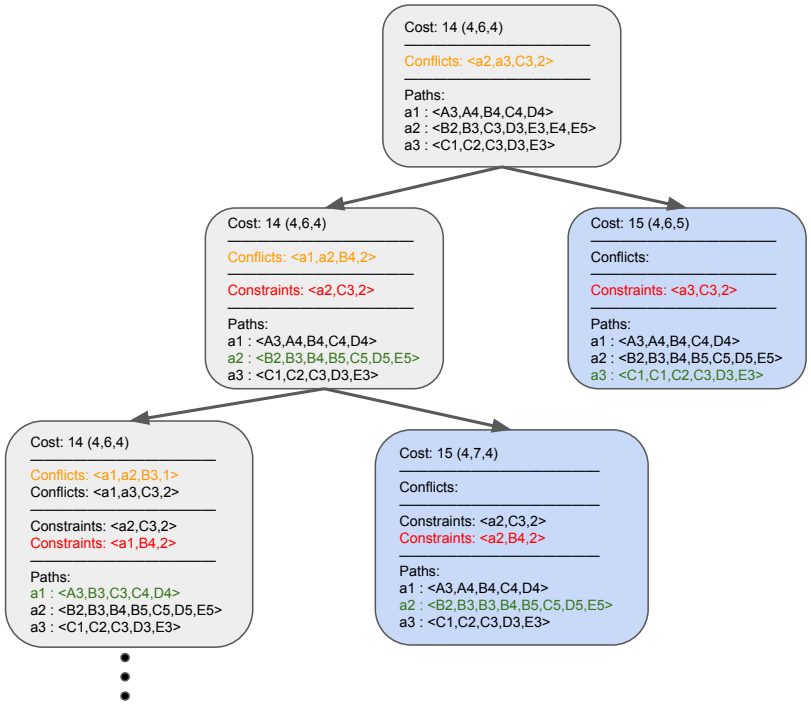

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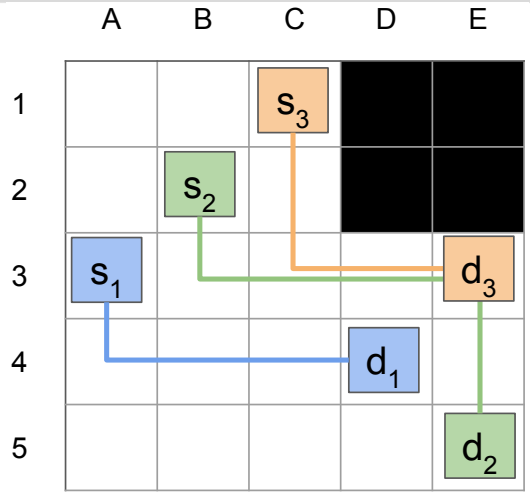
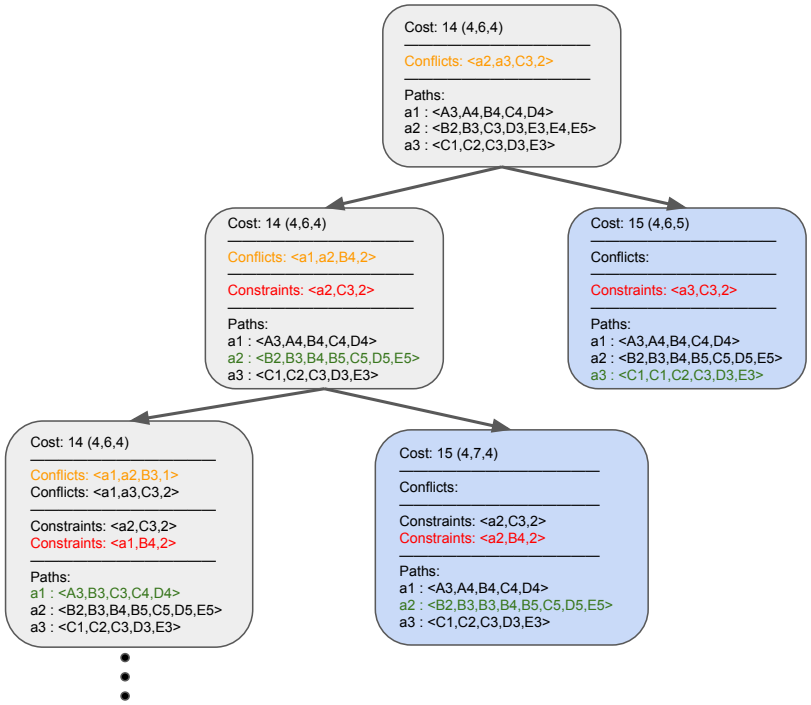
  - Recent enhancements:
    - Pairwise heuristics:
      - Cardinal [2] and WDG [3] heuristic.
    - Pairwise symmetry reasoning:
      - Rectangle, Target, Corridor [4,5] and Mutex [6] reasoning.
    - Pairwise conflict prioritization:
      - F-aware prioritization [7].





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- CT nodes:
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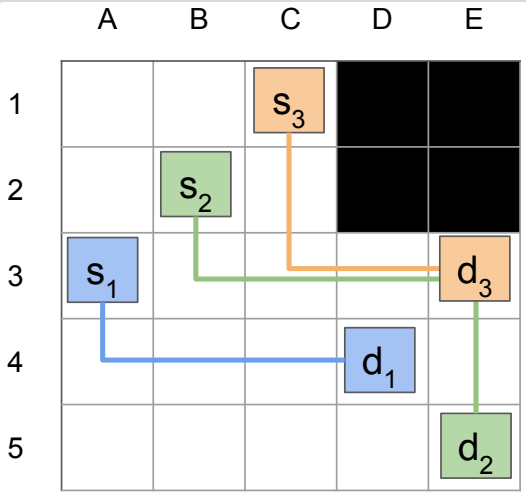

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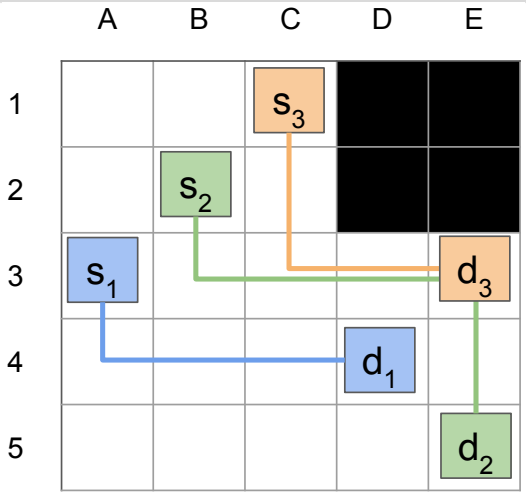

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- How can we detect conflict clusters?

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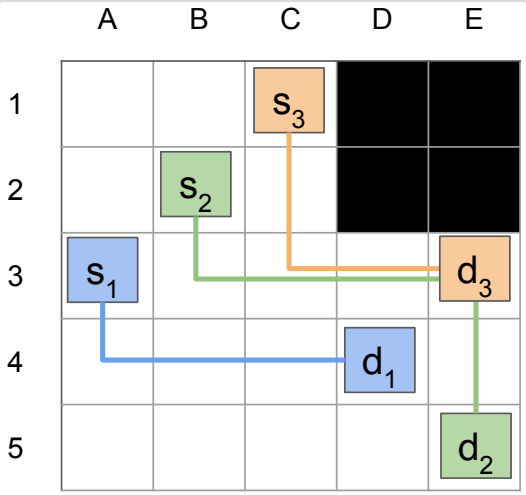
- Detecting conflict clusters
  - Mutex propagation:



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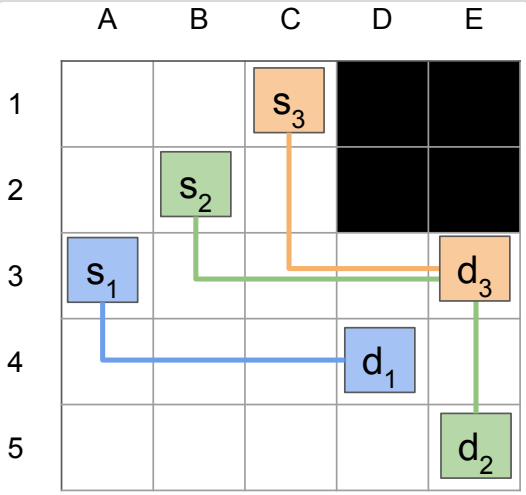

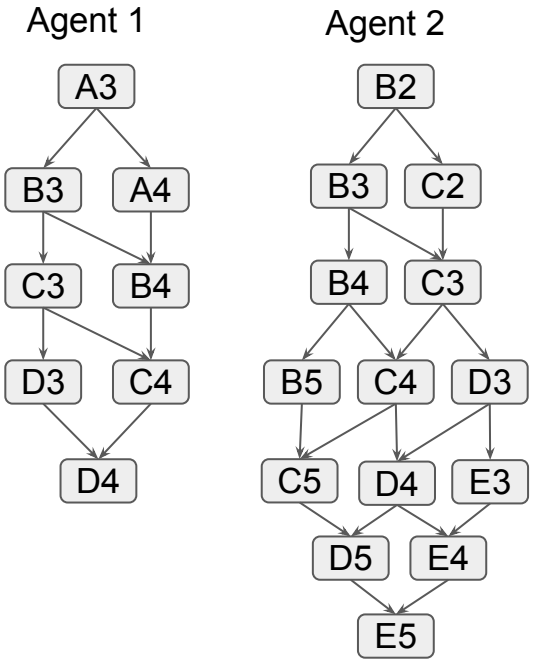

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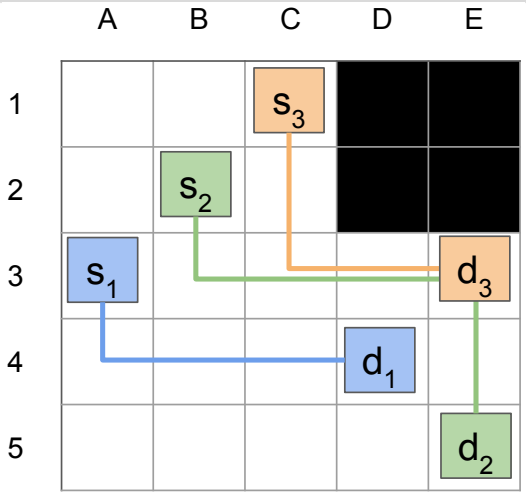

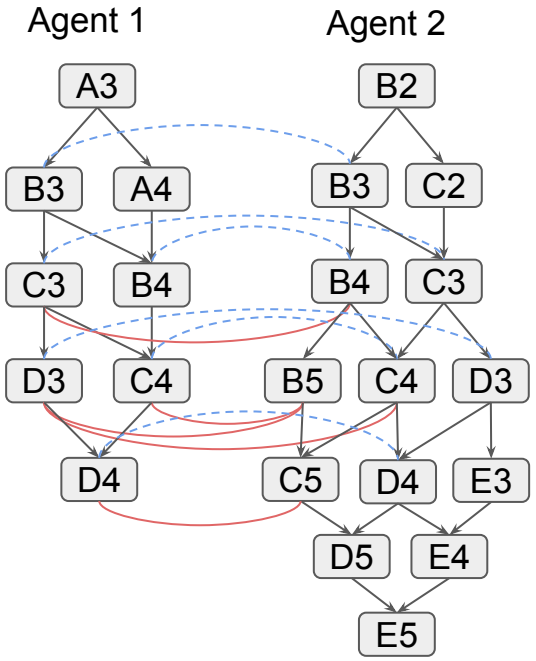

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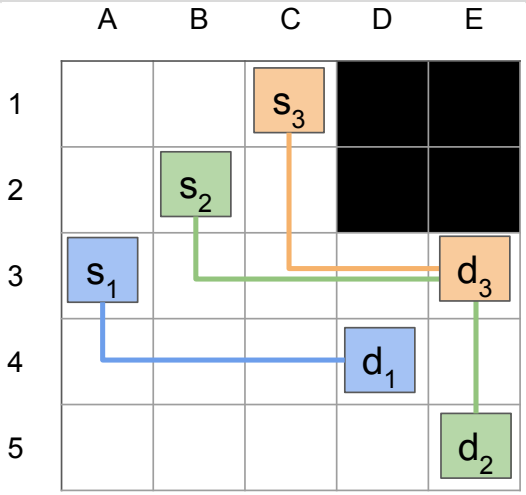
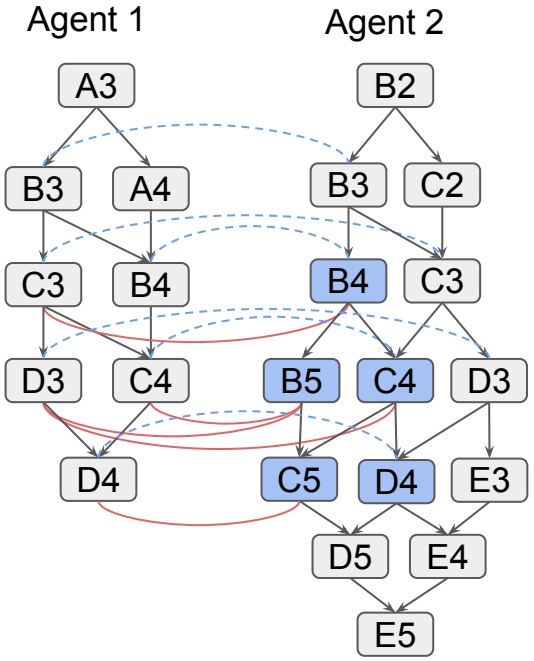
    - Given a pair of MDD_i and MDD_j for agents a_i and a_j, a MDD node n_i at level t from MDD_i is incompatible with MDD_j iff n_i is mutex with all MDD nodes at level t from MDD_j.





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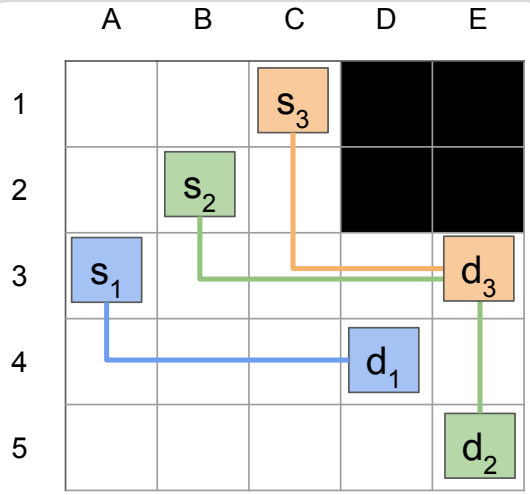
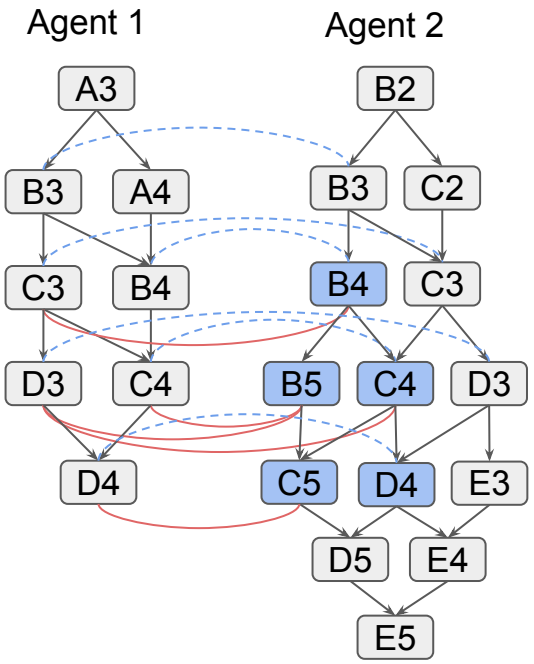
  - Naïve approach:
    - Select a random agent a_i and its MDD_i .
    - Exhaustively perform mutex propagation with other agent a_j , and remove the incompatible MDD_i nodes.
    - Until MDD_i become empty.





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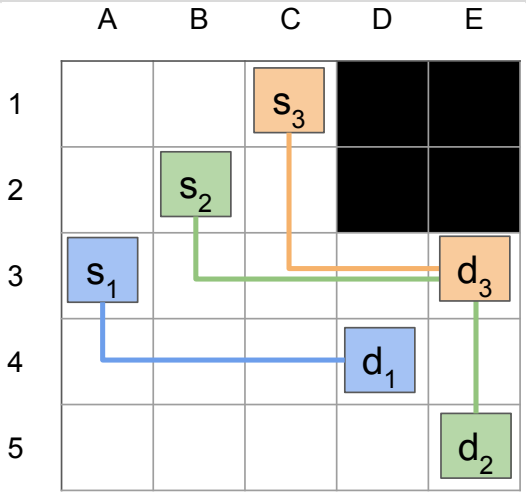

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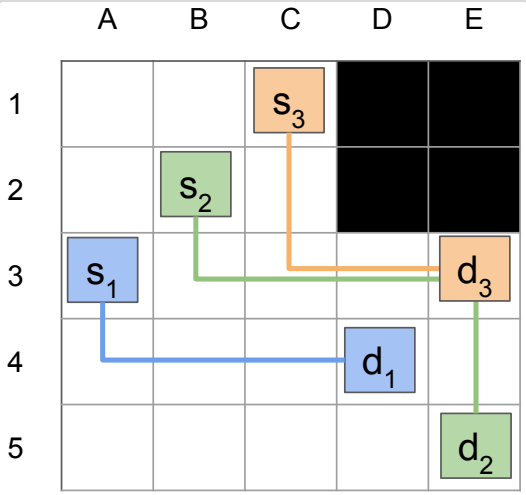

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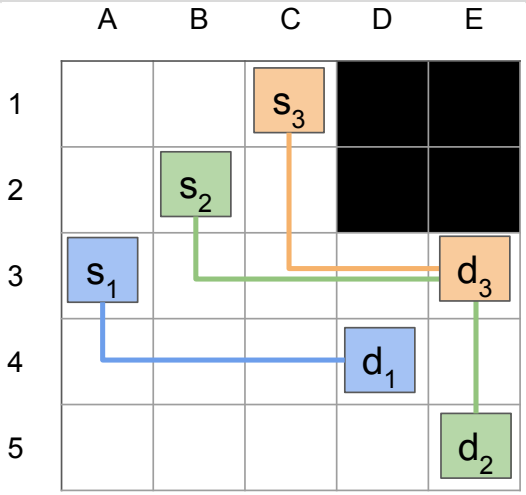

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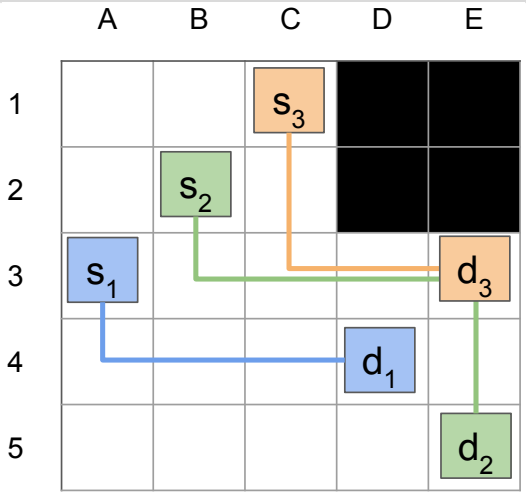

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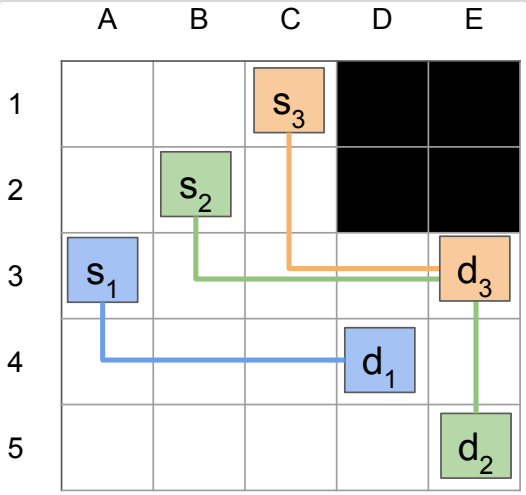

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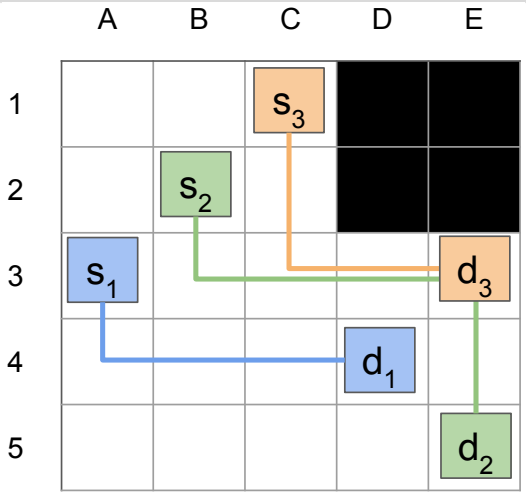

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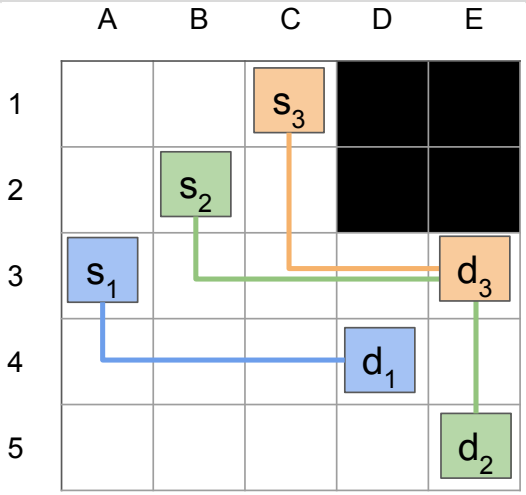

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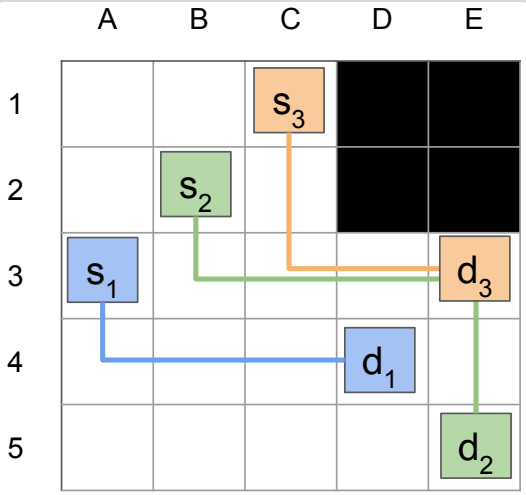

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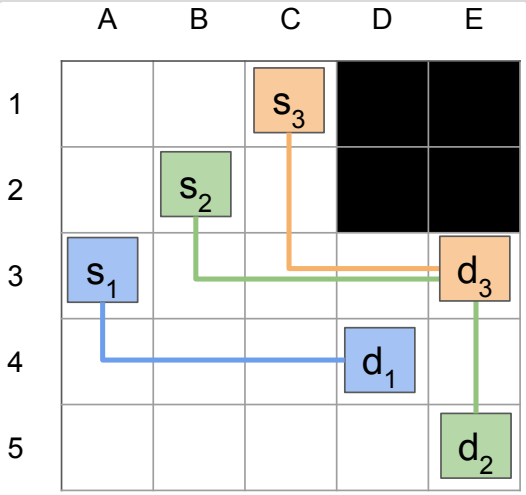

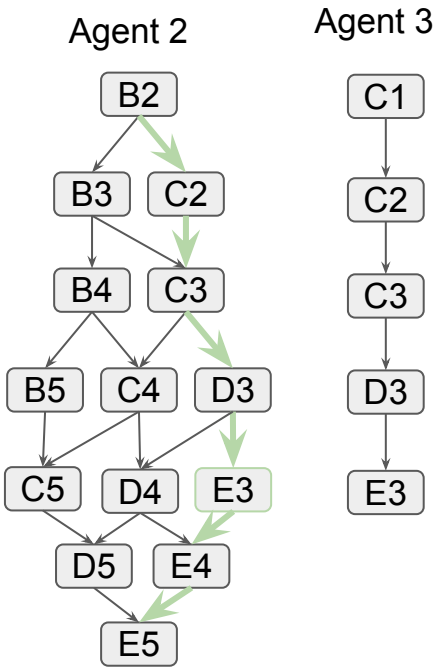

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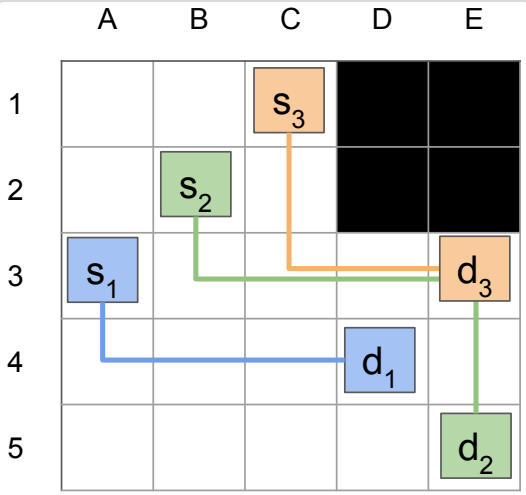

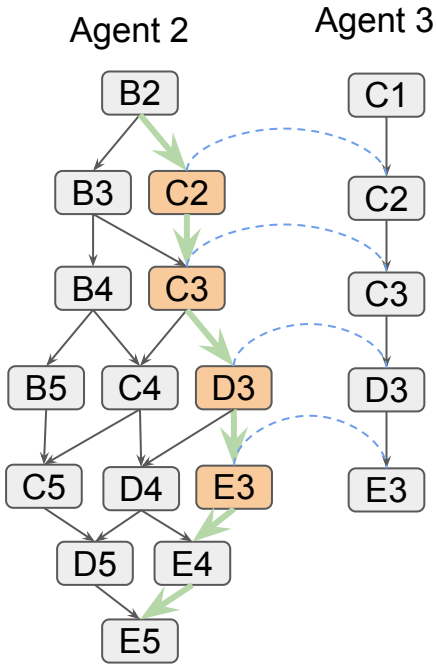

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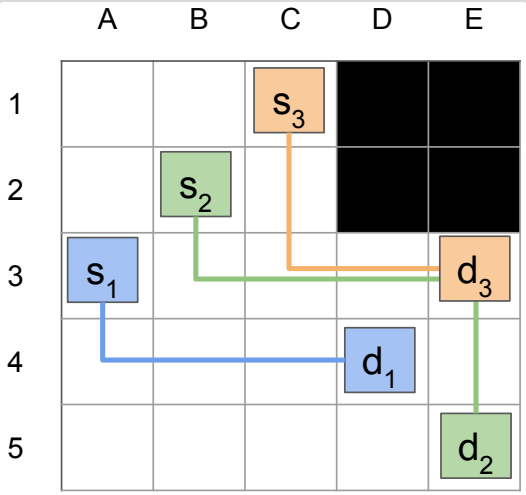

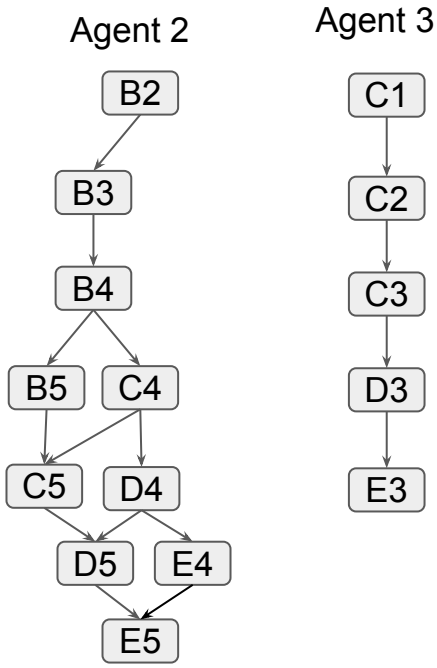

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      - If MDD_m becomes empty:
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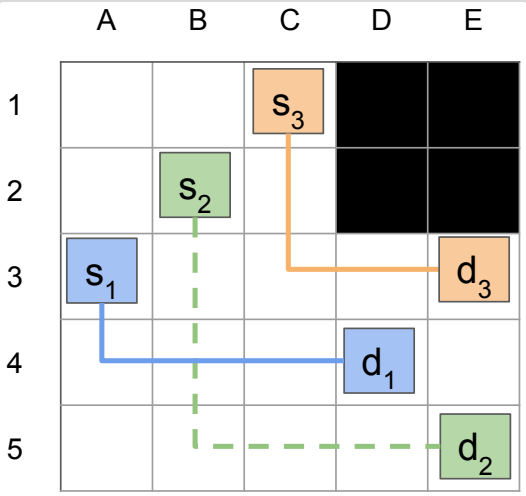
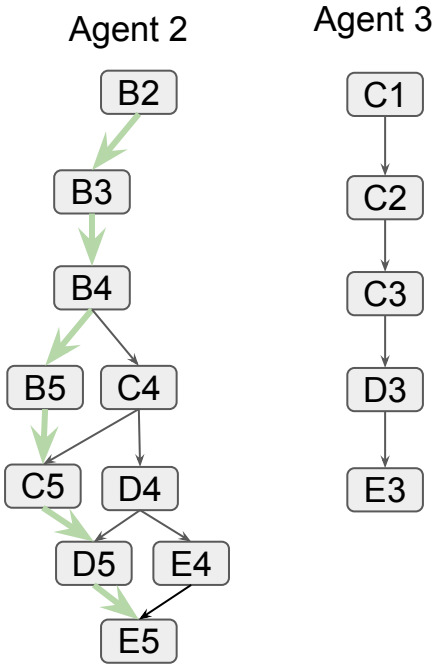
        - » Exclude these agents in a cluster.
      - Even if MDD_m are not empty:
        - » We can adapt p_m as a bypass!





Our Approach
Cluster Heuristic and Bypass (CHBP)

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- Cluster Heuristic and Bypass
  - Given a CT node:
    - Exclude the agents that are involved in pairwise heuristic (e.g., WDG).
    - Iteratively select an agent a_m with minimal number of conflicts:
      - Use current path p_m *as an indicator:*
        - » Perform the mutex propagation with other agent a_c that conflicts with p_m, and remove the incompatible nodes.
        - » If p_m becomes unavailable in MDD_m , Switch p_m to anther path in MDD_m with minimal number of conflicts.
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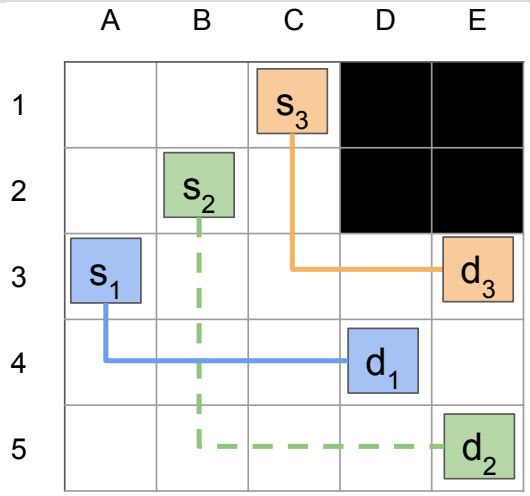

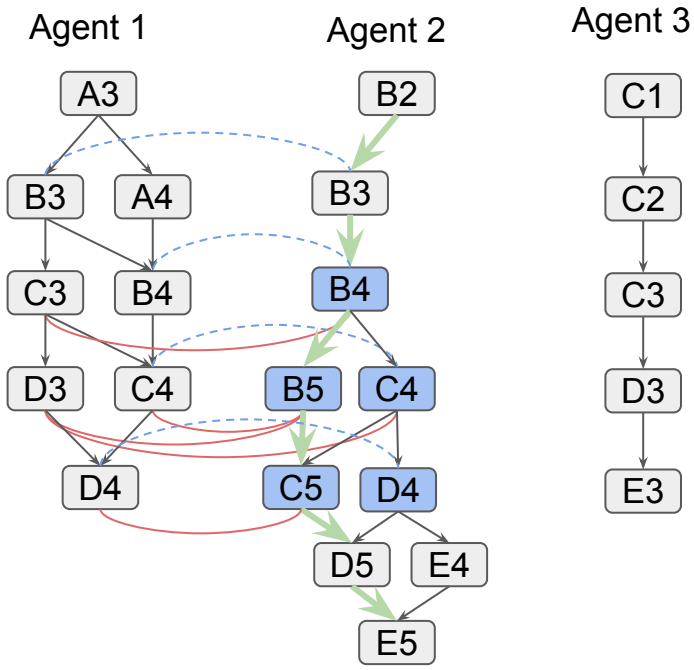

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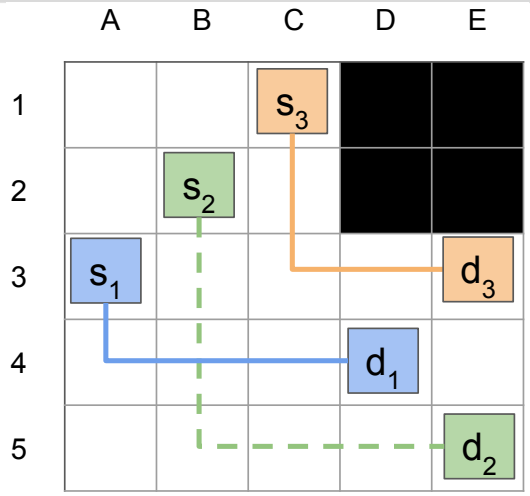
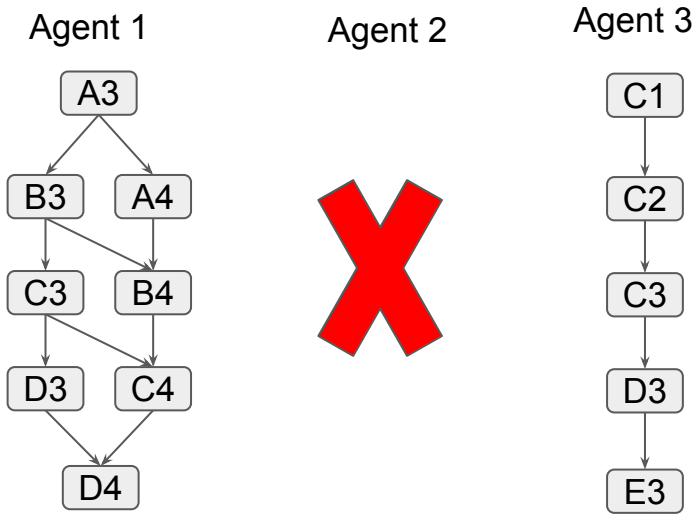
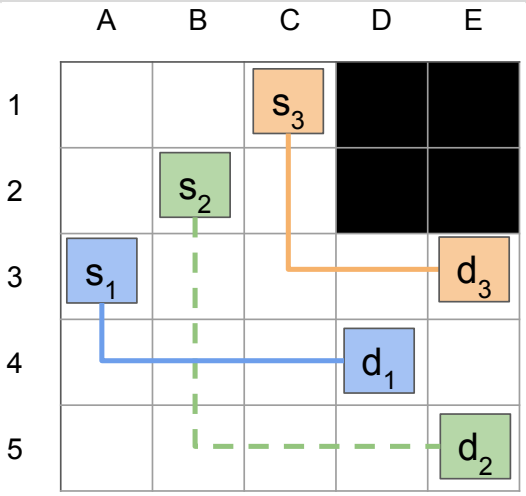




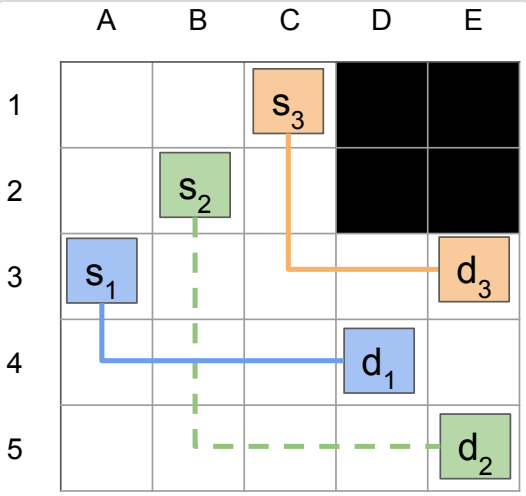
Our Approach
Cluster Heuristic and Bypass (CHBP)

- Cluster Heuristic and Bypass
  - Given a CT node:
  - Enhancement:
    - Speed up technique for CHBP.



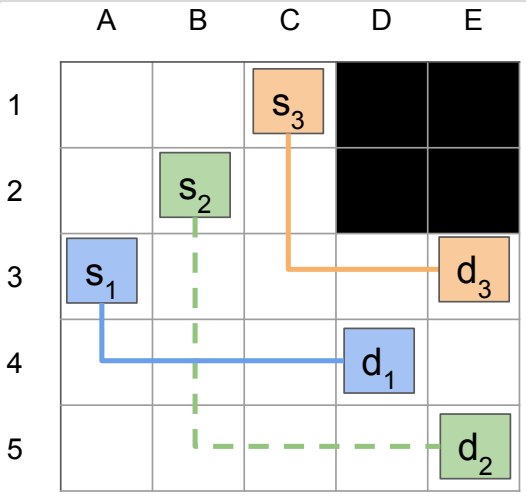
Our Approach
Cluster Heuristic and Bypass (CHBP)

- Cluster Heuristic and Bypass
  - Given a CT node:
  - Enhancement:
    - Speed up technique for CHBP.
    - Solving each cluster C.



Our Approach
Cluster Heuristic and Bypass (CHBP)

- Cluster Heuristic and Bypass
  - Given a CT node:
  - Enhancement:
    - Speed up technique for CHBP.
    - Solving each cluster C.
    - Memoization:
      - Caching the results of each cluster.
      - Caching the results of mutex propagation.



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- Experimental Results
  - Dataset (selected):
    - Empty map (empty-32-32): The number of agents is set to 50,70,...,150.
    - Warehouse map (warehouse-10-20-10-2-1): The number of agents is set to 30,50,...,130.

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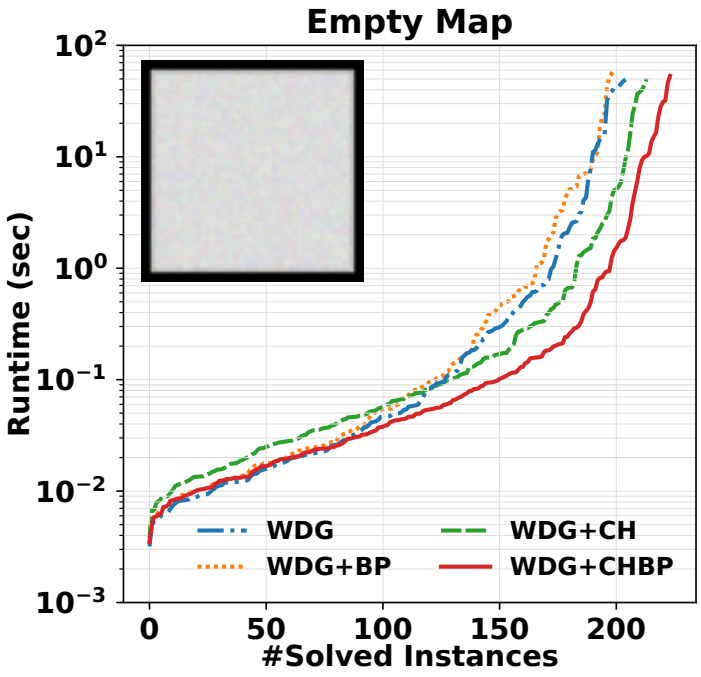
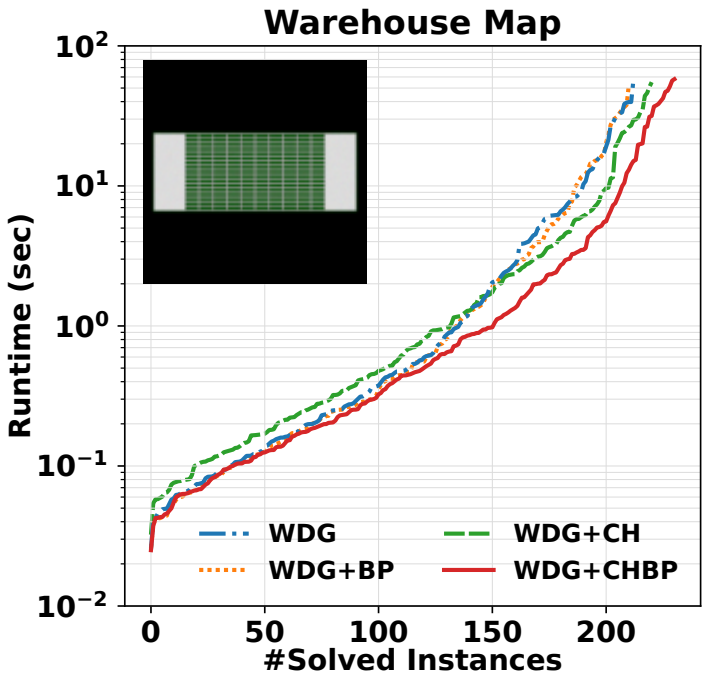
- Experimental Results
  - Dataset (selected):
  - Success rate:





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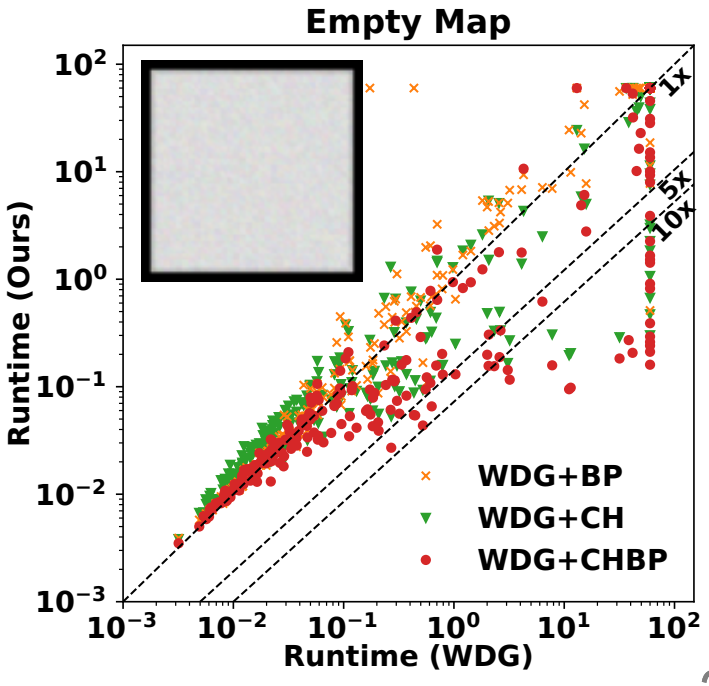
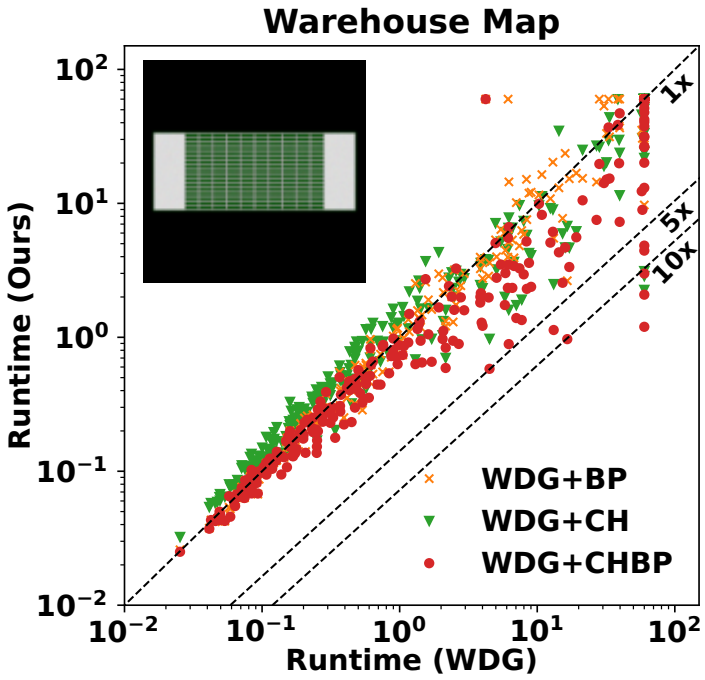
- Experimental Results
  - Dataset (selected):
  - Success rate:
  - Runtime:





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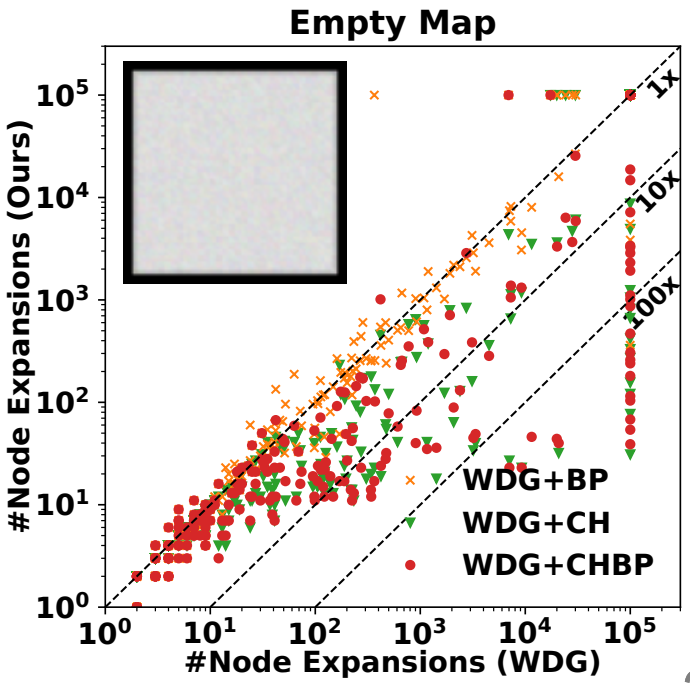
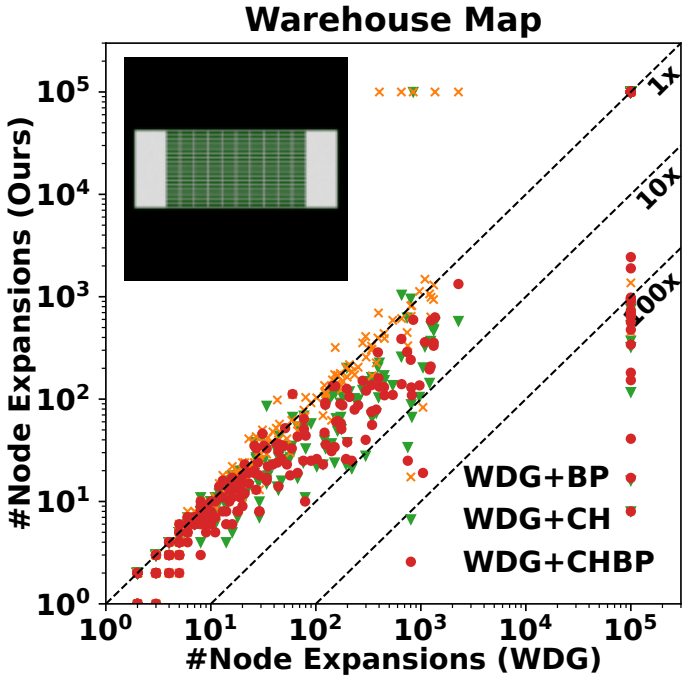
- Experimental Results
  - Dataset (selected):
  - Success rate:
  - Runtime:
  - Node expansion:





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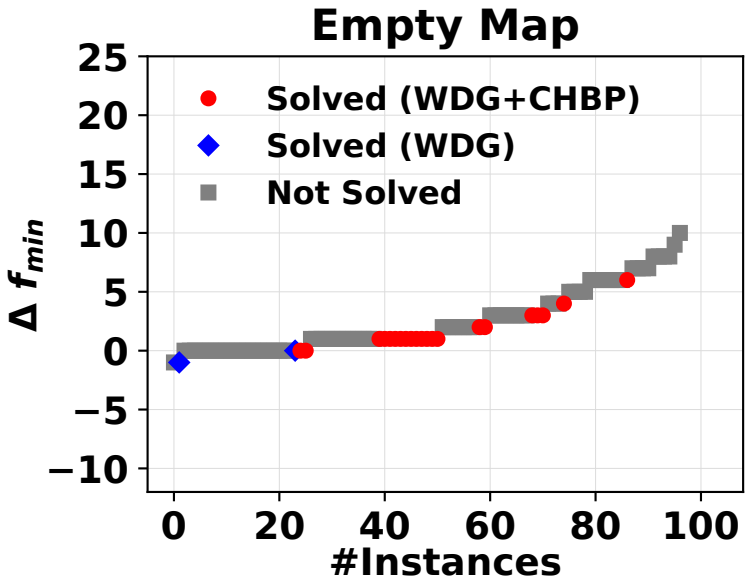
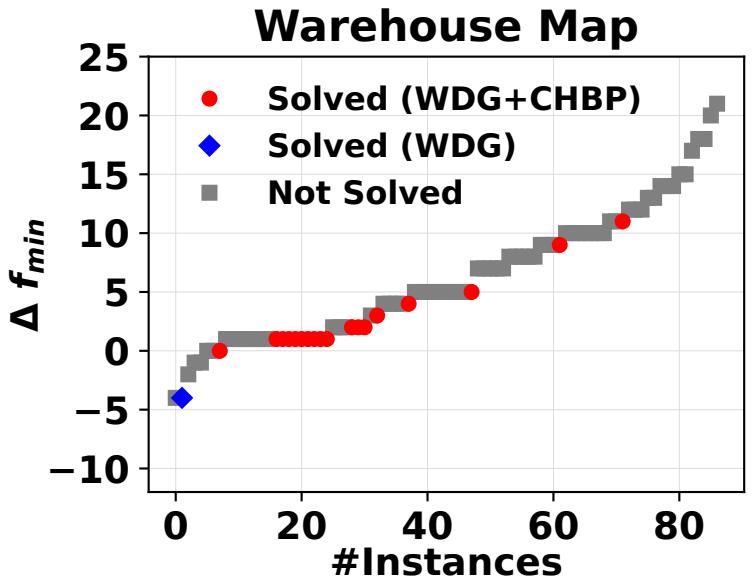
- Experimental Results
  - Dataset (selected):
  - Success rate:
  - Runtime:
  - Node expansion:
  - Δf_{min} in the open list:





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[7] Boyarski, E., Felner, A., Le Bodic, P., Harabor, D. D., Stuckey, P. J., & Koenig, S. (2021). f-Aware Conflict Prioritization & Improved Heuristics For Conflict-Based Search. In *AAAI* (Vol. 35, No. 14, pp. 12241-12248).

Thank you for listening