Lab2

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Introduction

This report relates to finding an appropriate representation for nonlinear model fitting. Given three sets of data, we were expected to find a fit for a model type of $y = ln(ax)$ using the method of minimizing the derivative of the error function given by: $E = \sum_{i=0}^{N}(y_i - ln(ax))^2$. By creating a Python script, the process of iterating a solution to find the best value of a can be determined so long as an initial condition, a_n, is provided and many of these conditions are tried until a valid sequence of iterations produces a valid a value.

Methods

The first step in this lab was to find the appropriate functions to use for $f(a)$ and $f'(a)$ where $f(a)$ and $f'(a)$ are given below:

$$f(a) = \frac{\delta}{\delta a}\sum_{i=0}^{N}(y_i - ln(ax_i))^2$$

$$f'(a) = \frac{\delta}{\delta a}f(a)$$

Then, the value of a is iterated by using the function given below:

$$a_{n+1} = a_n - \frac{f(a)}{f'(a)} \tag{1}$$

This iteration of a_n is used until a predetermined threshold, T, is reached where $|a_{n+1} - a_n| < T$, and at this point the value of a to be used in the nonlinear model is considered to be the final value of a_{n+1} so long as this value is within an arbitrary threshold (mine was 0.001) of the previous approximation for a_n, otherwise, the process continues iterating.

The above methods of determining $f(a)$, $f'(a)$, and a were implemented for the three data sets, A, B, and C, in Python code as explained below in Results.

Data Set	Initial a	Final a	Iterations
A	11	6.711	12
B	20	18.996	3
C	0.2	0.290	4

Table 1: Values for iterations of nonlinear models of the three data sets

Data Set	a_{min}	a_{max}
A	0.002	11
B	0.002	31.3
C	0.002	0.4

Table 2: Values for ranges of acceptable guesses for the three data sets

Results

The functions $f(a)$ and $f'(a)$ were found for the model $y = ln(ax)$ as shown below:

$$f(a) = \frac{\delta}{\delta a} \sum_{i=0}^{N} (y_i - ln(ax_i))^2$$

$$f(a) = \frac{\delta}{\delta a} (y_i - ln(ax_i))^2$$

$$f(a) = -2(y_i - ln(a * x_i))(\frac{1}{a})$$

$$f(a) = -2 * \frac{y_i - ln(a * x_i)}{a} \tag{2}$$

$$f'(a) = \frac{\delta}{\delta a} f(a)$$

$$f'(a) = -2 * \frac{y_i - ln(a * x_i)}{a}$$

$$f'(a) = 2 * (\frac{1}{a^2} + \frac{y_i}{a^2} - \frac{ln(a * x_i)}{a^2})$$

$$f(a) = 2 * \frac{1 + y_i - ln(a * x_i)}{a^2} \tag{3}$$

The above functions (2) and (3) were used to determine the value of a for all three nonlinear models in data sets A, B, and C along with the iterative function (1). Using this technique in Python resulted in the values of a listed in Table 1 for the three data sets.

These values were then used to generate the model $y = ln(ax)$ for each data set and the resulting best fit line was graphed over the original data as shown in Figures 1, 2, and 3.

In the process of finding values of a that satisfied the nonlinear model, a range of initial a values was found as shown in Table 2. Values above or below these values either result in a_{n+1} that do not converge to a valid value of a.

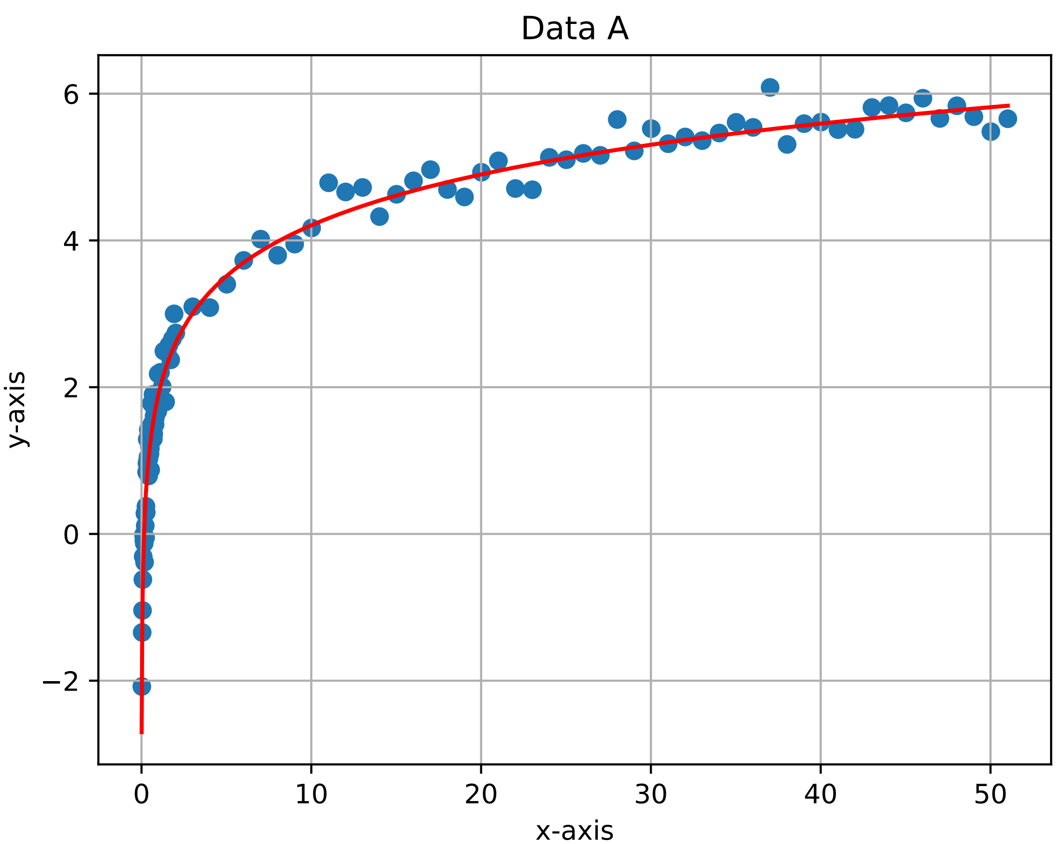

Figure 1: Best fit line produced and overlain on data set A

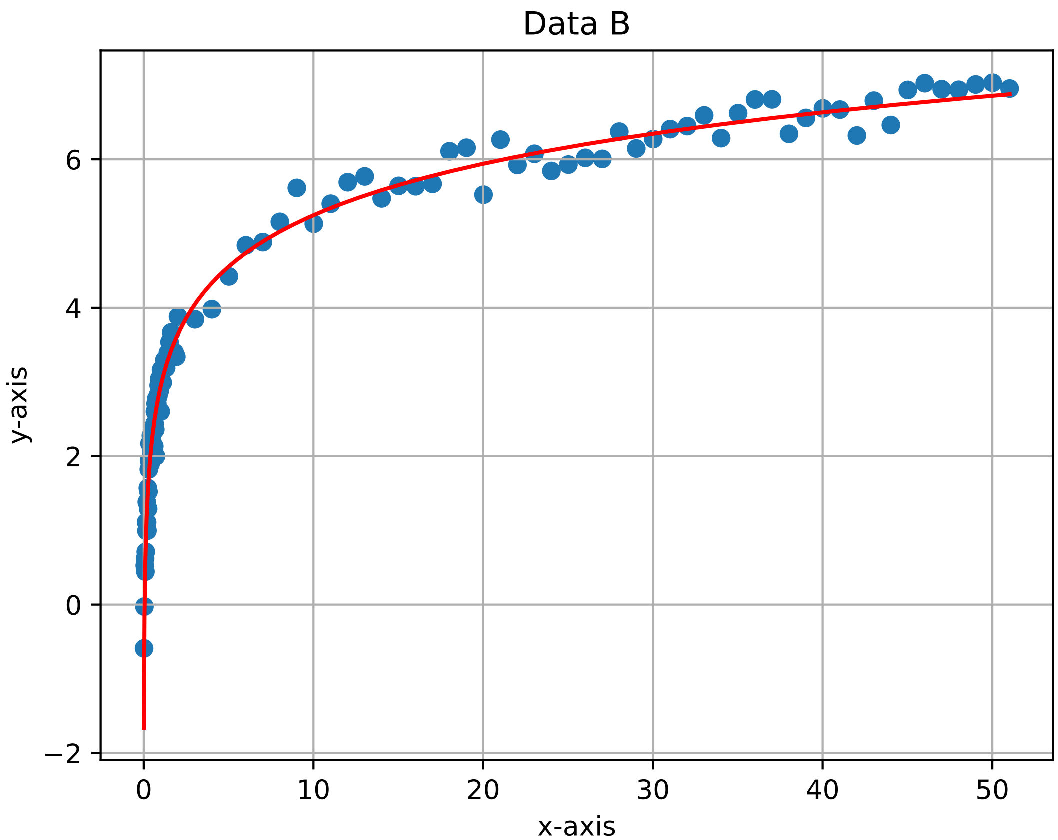

Figure 2: Best fit line produced and overlain on data set B

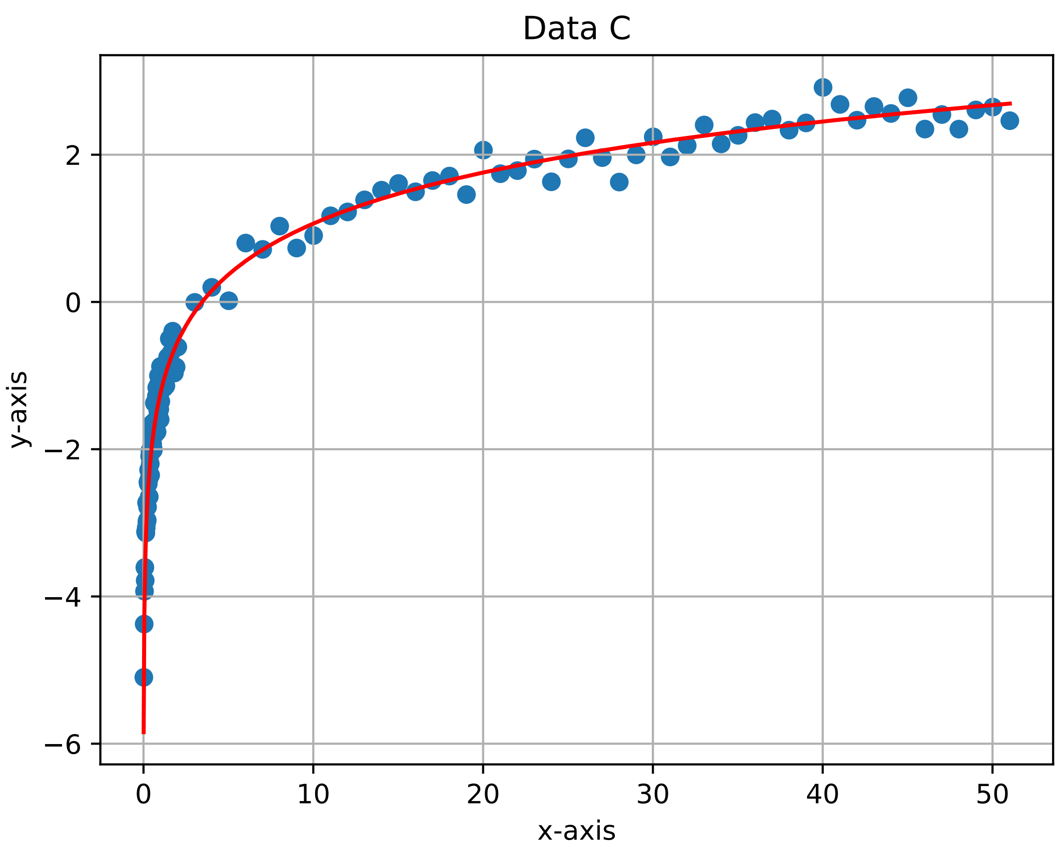

Figure 3: Best fit line produced and overlain on data set C

Conclusion

In this lab, we learned how to use the error equations and its first two derivatives to find approximate answers for nonlinear models of data. The limits of the model that I made were demonstrated, as well as the accuracy of the approximation reached. The model $y = ln(ax)$ proved to be useful in achieving an approximate function of the data presented from the three data sets.