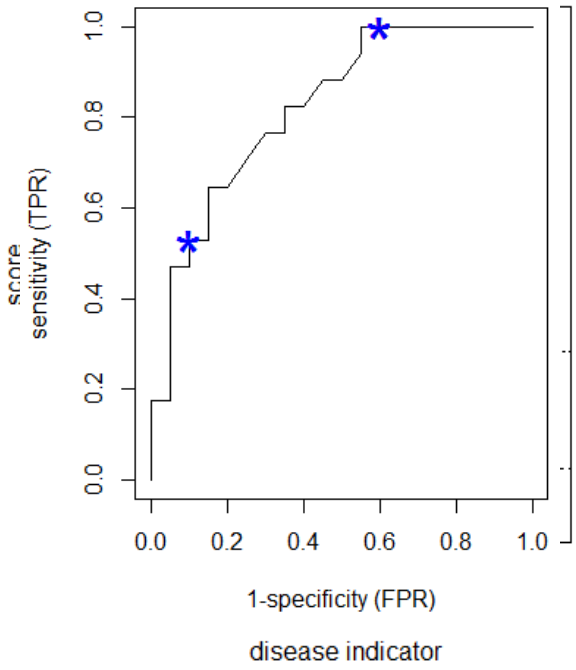
In-class exercise - Solution
Topic: Sensitivity, Specificity, ROC curve

1) Sensitivity, Specificity, ROC curve

A paper from D.G. Altman and JM Bland (BMJ 1994; 309:188) gives an introduction to diagnostic tests and receiver operation characteristic (ROC) plots. As an example they have used a test for graft reaction (german: Abstossungsreaktion) in case of bone marrow transplantation. The test relies on a score for the reaction of lymphocytes and is shown in the plot below for patients who showed no graft vs the marrow (left point column) and patients who showed developed an reaction vs the bone marrow (right point column).



We want to use the score as a diagnostic test for graft vs the host disease.

a) Use the left plot to determine the sensitivity and specificity of the test when using
- the indicated thresholds 1.2 as cut-off
- the indicated thresholds 3.8 as cut-off

to discriminate between a positive test result (patient will probably show a graft reaction) and a negative test result.

$$Sens_{1.2} = \frac{17}{17} = 1$$

$$Spez_{1.2} = \frac{8}{20} = 0.40$$

$$Sens_{3.8} = \frac{9}{17} = 0.53$$

$$Spez_{3.8} = \frac{18}{20} = 0.90$$

b) Indicate the position of your calculated results in the ROC curve on the right side

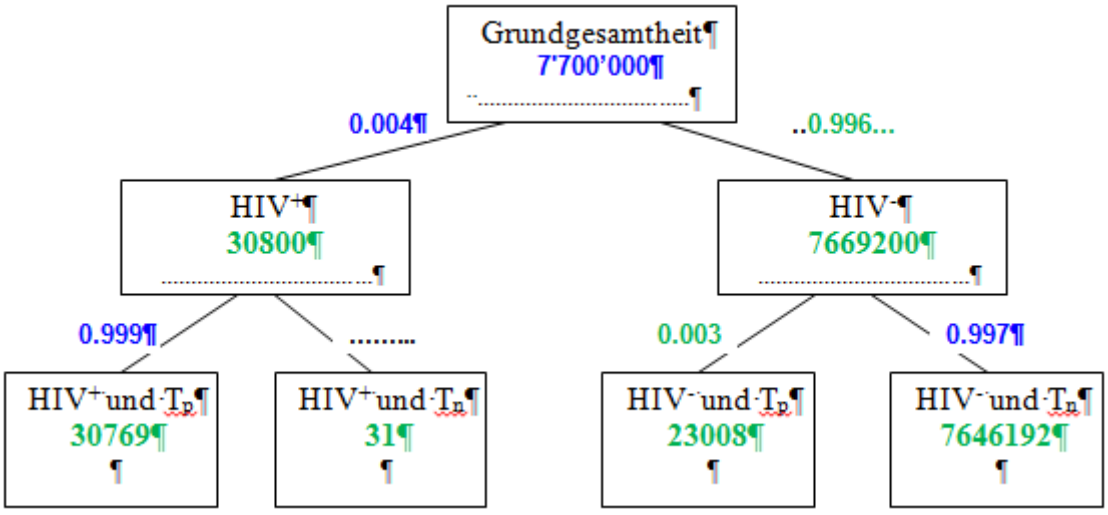
c) How would an ideal ROC curve look like?
For a perfect test we have no overlap between the continuous measure in healthy and diseased subjects. We can find a threshold which yields sensitivity and specificity of 100%. If we shift the threshold specificity (or sensitivity) remains at 100% while sensitivity (or specificity) goes down. Hence the ROC curve of a perfect test goes straight up through the upper left corner (1,1) and then horizontally to the right.

d) Which threshold would you use? Give reasons.
it depends which error is worse –if sensitivity is most important but specificity should also be not to bad a cutoff of 1.4 instead of 1.2 would be good, since only one diseased person will be classified as healthy (sensitivity goes form 17/17 down to 16/17) while 3 more non-diseased people will be correctly classified (specificity goes from 8/20 up to 11/20).

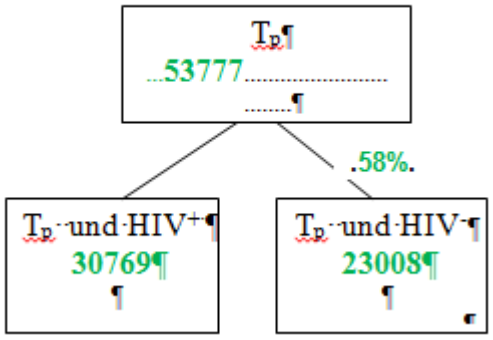
2) Predictive value of a test: posteriori probability

Notation:
HIV^+: HIV positive (diseased)
HIV^-: HIV negative (healthy)
T_+ : Test is positive
T_- : Test is negative

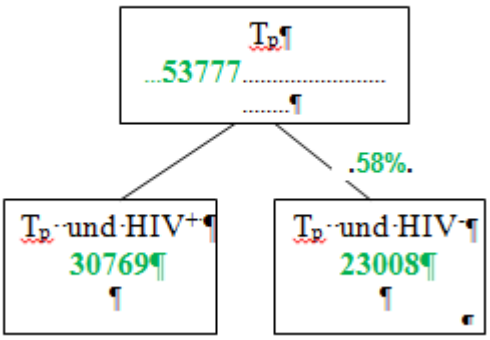
Fill in the missing numbers:



How la idomly selected Swiss person, who ma has AIDS?



How large is he probability P(HIV$^+$ | T+) that a randomly selected Swiss person, who made an AIDS-Test and had a positive result, has AIDS?



P(HIV$^+$ | T_p) = P(HIV$^+$ given T_p) =

$$P\left(HIV^+ \mid T_p\right) = \frac{P\left(T_p \cap HIV^+\right)}{P(T_{pos})} = \frac{30769}{53777} = 58\%$$