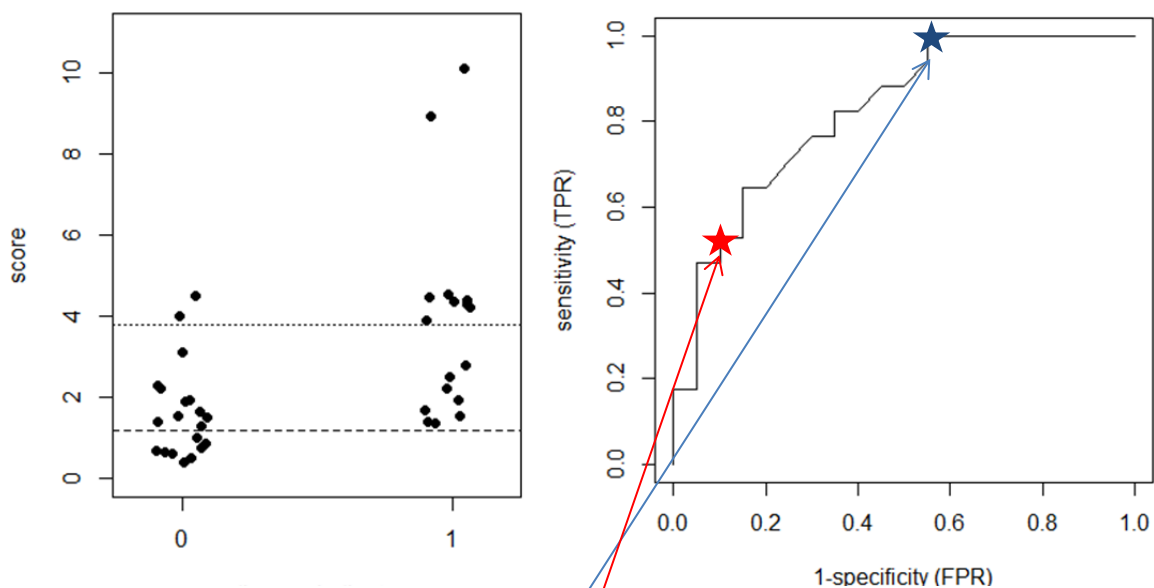


In-class exercise week 7 solution

Topic: Diagnostic tests: Sensitivity, Specificity, Posteriori Probability

1) Sensitivity, Specificity, ROC curve

A paper from D.G. Altman and JM Bland (BMJ 1994; 309:188) gives an introduction to diagnostic tests and receiver operation characteristic (ROC) plots. As an example they have used a test for graft reaction (german: Abstossungsreaktion) in case of bone marrow transplantation. The test relies on an index for the reaction of lymphocytes and is shown in the plot below for patients who showed no graft vs the marrow (left point column) and patients who showed developed an reaction vs the bone marrow (right point column).



We want to use the index as a test for graft vs the host disease.

- Use the left plot to determine the sensitivity and specificity of the test when using
 - the indicated thresholds 1.2 as cut-off
 - the indicated thresholds 3.8 as cut-off
 to discriminate between a positive test result (patient will probably show a graft reaction) and a negative test result.

$$Sens_{1.2} = \frac{17}{17} = 1$$

$$Spec_{1.2} = \frac{8}{20} = 0.40$$

$$Sens_{3.8} = \frac{9}{17} = 0.53$$

$$Spec_{3.8} = \frac{18}{20} = 0.90$$

- Indicate the position of your calculated results in the ROC curve on the right side
- How would an ideal ROC curve look like?

For a perfect test we have no overlap between the continuous measure in healthy and diseased subjects. We can find a threshold which yields sensitivity and specificity of 100%. If we shift the threshold specificity (or sensitivity) remains at 100% while sensitivity (or specificity) goes down. Hence the ROC curve of a perfect test goes straight up through the upper left corner (1,1) and then horizontally to the right.

2) Predictive value of a test: posteriori probability

Notation:

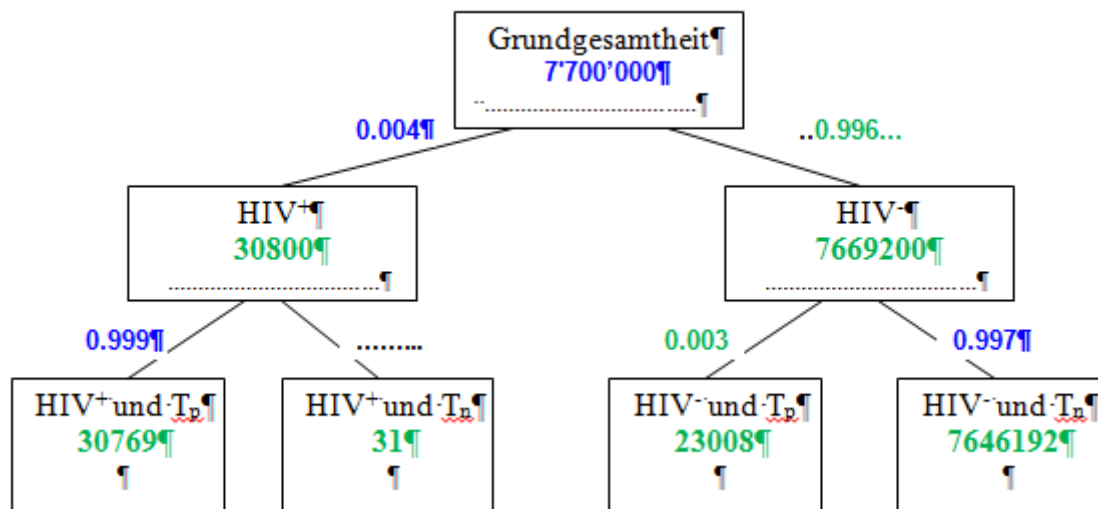
HIV⁺: HIV positive (diseased)

HIV⁻: HIV negative (healthy)

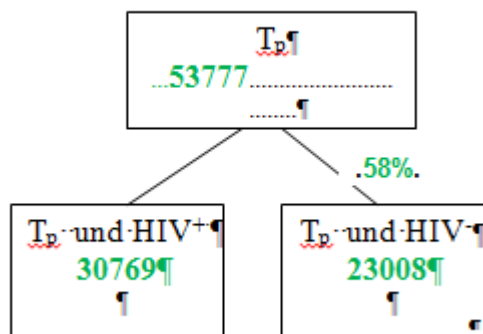
T₊ : Test is positive

T₋ : Test is negative

Fill in the missing numbers:



How large is the probability $P(\text{HIV}^+ | T_+)$ that a randomly selected Swiss person, who made an AIDS-Test and had a positive result, has AIDS?



$$P(\text{HIV}^+ | T_p) = P(\text{HIV}^+ \text{ given } T_p) =$$

$$P(\text{HIV}^+ | T_p) = \frac{P(T_p \cap \text{HIV}^+)}{P(T_{pos})} = \frac{30769}{53777} = 58\%$$