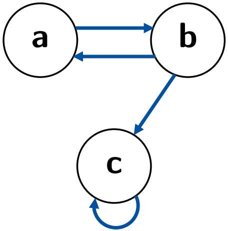
Bayesian estimation and testing for idiographic networks

Björn Siepe & Daniel Heck Philipps-Universität Marburg 12.09.2023

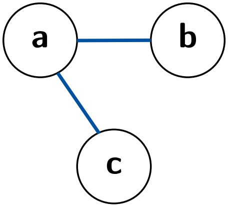
Dynamic Networks

Dynamic Networks

Temporal



Contemporaneous



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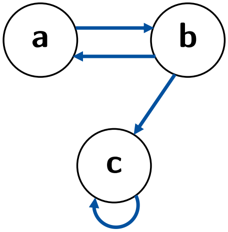

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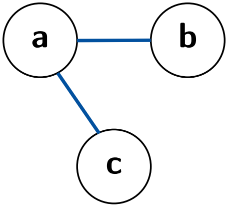

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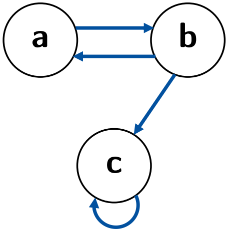

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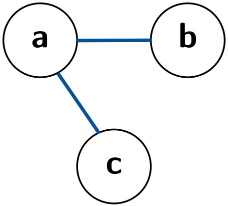
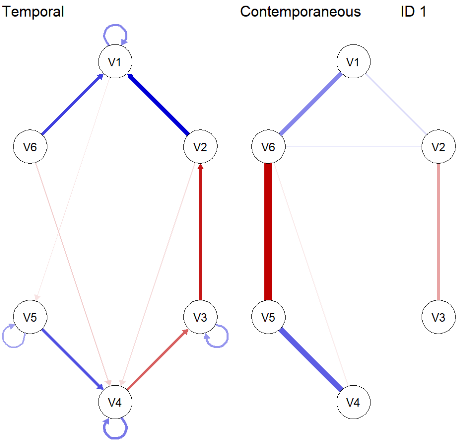


- Issues: Questionable performance in typical psychological data (Hoekstra et al., 2022; Mansueto et al., 2020)
- This presentation: New ways to estimate and compare idiographic networks in a Bayesian framework (Siepe and Heck, 2023)

Heterogeneity?



Bayesian gVAR Estimation

Gibbs sampler in R package BGGM (Williams and Mulder, 2021)

Temporal Network

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{y}_{t-1} + \boldsymbol{\epsilon}_t$$

$$\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Theta}^{-1})$$

Prior:

$$\beta_{ij} \overset{i.i.d.}{\sim} \mathcal{N}(0, s_\beta)$$

Contemporaneous Network

$$\rho_{ij} = \frac{-\boldsymbol{\Theta}_{ij}}{\sqrt{\boldsymbol{\Theta}_{ii}\boldsymbol{\Theta}_{jj}}}.$$

Prior:

$$\rho \sim \text{Beta}\left(\frac{\delta}{2}, \frac{\delta}{2}\right)$$

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- DGP: Real-world examples with six and eight nodes & dense network

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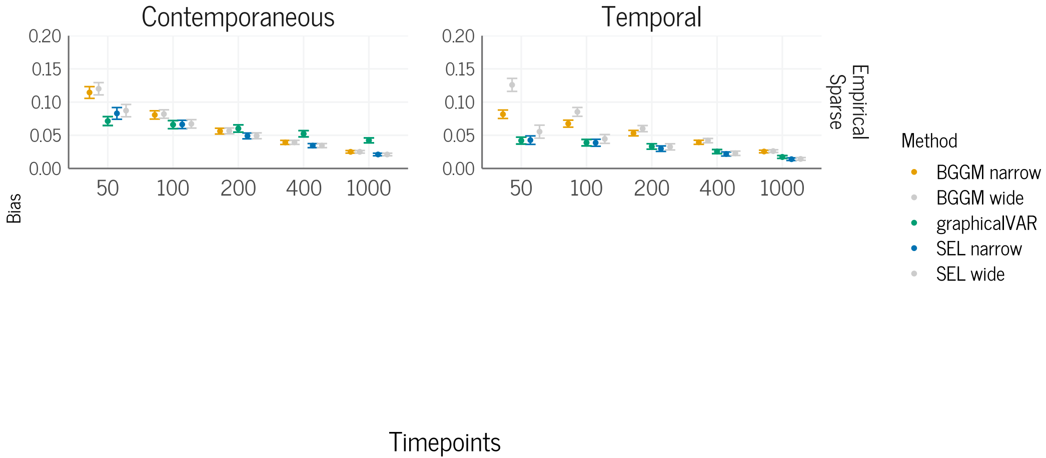
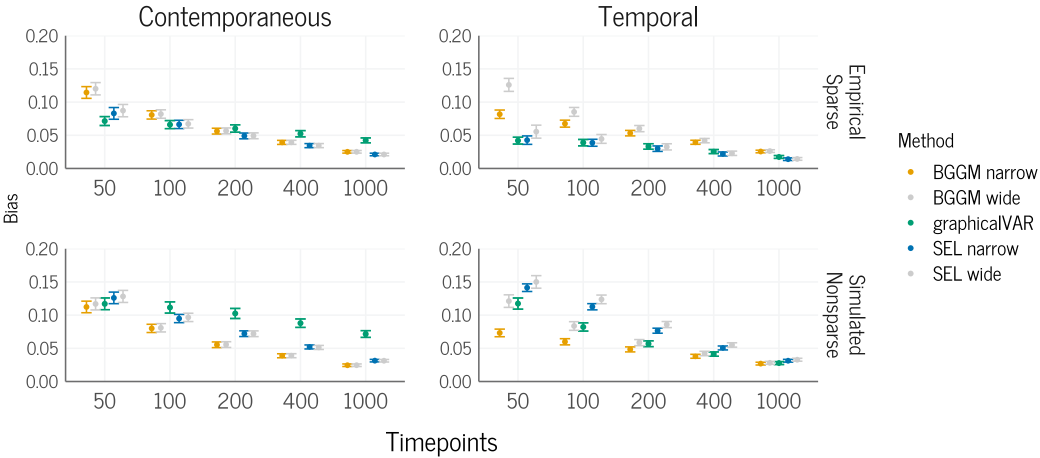
  $\}$ Wide & narrow priors
- 1000 Monte Carlo replications

Simulation 1: Results



Simulation 1: Results



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Idea of the Test

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- Enough evidence that differences between networks are more than estimation uncertainty? (Williams et al., 2020)

$$
\begin{array}{cccc}
B_{a} & B_{b} & D & ||D||_F \\
\begin{pmatrix} 0.3 & 0.5 & 0.5 \\ 0.5 & 0.3 & 1 \\ 0.3 & 0.1 & 0.3 \end{pmatrix} - & \begin{pmatrix} 0.1 & 0.5 & 0.2 \\ 0.5 & 0.2 & 0.3 \\ 0.3 & 0.1 & 0.1 \end{pmatrix} = & \begin{pmatrix} 0.2 & 0 & 0.3 \\ 0 & 0.1 & 0.7 \\ 0.3 & 0 & 0.2 \end{pmatrix} \longrightarrow & 0.87
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- Randomly draw matrix pairs from posterior of two networks
  - ➥ Obtain reference distribution of uncertainty
- Comparison of empirical norm with reference distribution for temporal and contemporaneous network
- Decision: Heuristic rule

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$$\begin{pmatrix} 0.3 & 0.5 & 0 \\ -0.5 & 0.3 & 1 \\ 0.3 & 0 & 0.3 \end{pmatrix} \xrightarrow{\text{Largest} \times \{1.4,\, 1.6\}} \begin{pmatrix} 0.3 & 0.5 & 0 \\ -0.5 & 0.3 & \color{red}{1.4} \\ 0.3 & 0 & 0.3 \end{pmatrix}$$

$$\begin{pmatrix} 0.3 & 0.5 & 0 \\ -0.5 & 0.3 & 1 \\ 0.3 & 0 & 0.3 \end{pmatrix} \xrightarrow{\text{All} \pm \{0.05,\, 0.10,\, 0.15\}} \begin{pmatrix} \color{red}{0.2} & \color{red}{0.6} & \color{red}{-0.1} \\ \color{red}{-0.4} & \color{red}{0.4} & \color{red}{1.1} \\ \color{red}{0.1} & \color{red}{0.1} & \color{red}{0.2} \end{pmatrix}$$

$$\begin{pmatrix} \color{blue}{0.3} & \color{red}{0.5} & \color{gray}{0} \\ \color{blue}{-0.5} & \color{red}{0.3} & \color{gray}{1} \\ \color{blue}{0.3} & \color{red}{0} & \color{gray}{0.3} \end{pmatrix} \xrightarrow{\text{Permute Columns}} \begin{pmatrix} \color{gray}{0} & \color{blue}{0.3} & \color{red}{0.5} \\ \color{gray}{1} & \color{blue}{-0.5} & \color{red}{0.3} \\ \color{gray}{0.3} & \color{blue}{0.3} & \color{red}{0} \end{pmatrix}$$

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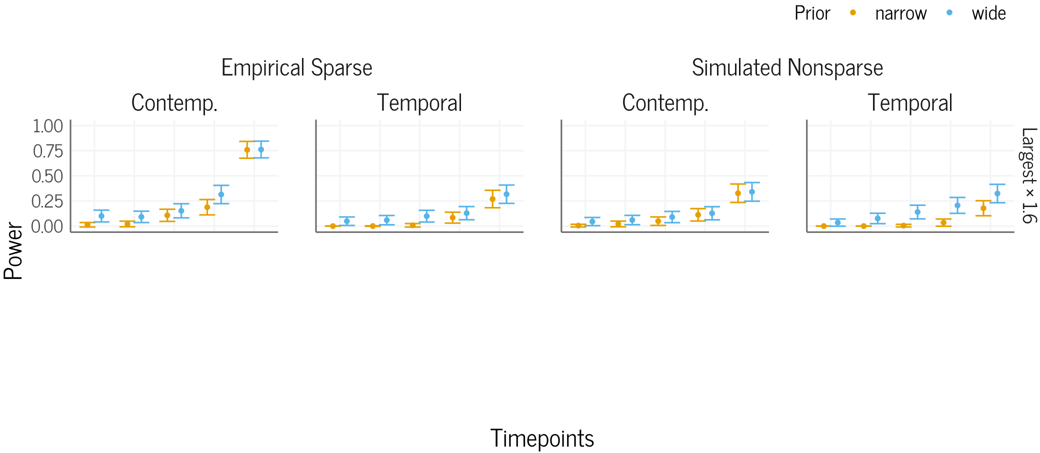

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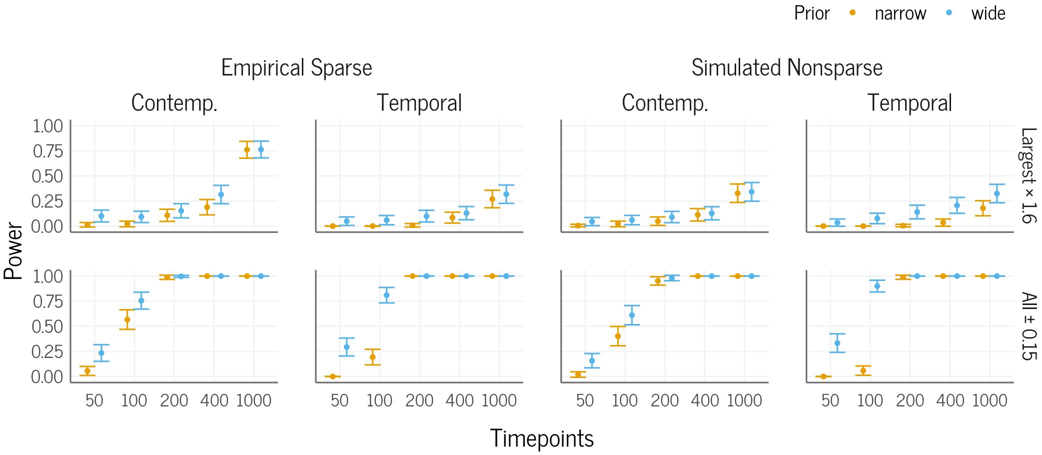
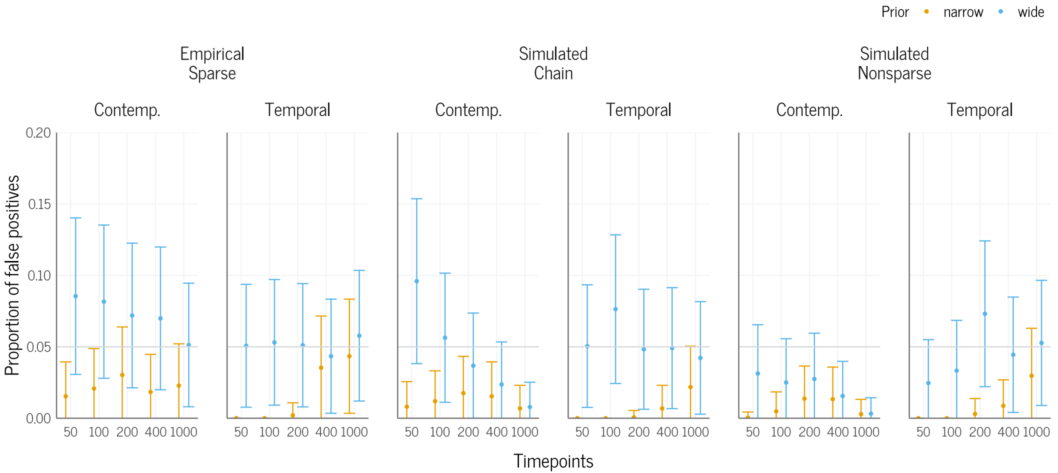


False Positive Rate



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  - Another very recent invariance test in Hoekstra et al., 2023
  - Potential use in intra-individual comparisons

References

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Get in Touch

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