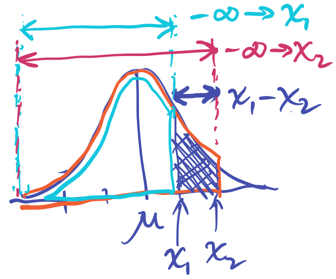
Ch 10: Normal Distribution (review)

10/15/2020

- Standard normal: $\mu = 0$, s.d. $= 1$ $\Rightarrow Z = \frac{Y-\mu}{\sigma}$ { Z-distribution, Z-score }
- Probabilities for normal distribution

$$\Pr[x_1 \le X \le x_2] = \Pr[X \le x_2] - \Pr[X \le x_1]$$

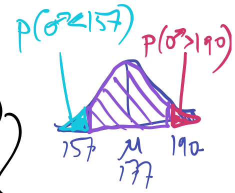


Ex 10.4: Height of astronauts @ NASA
Must be b/w 157.5 – 190.5 cm

Population: ♂ $\mu = 177.6$, sd $= 9.7$ cm
♀ $\mu = 163.2$, sd $= 10.1$ cm

Q: what is Prob. that ♂ or ♀ will NOT qualify?

$\Pr[♂ < 157.5] \cup \Pr[♂ > 190.5]$



(dqpr) ♂: pnorm(q_1, μ, sd) + pnorm(q_2, μ, sd, lower.tail=F) $\to$ = 0.11 $\to$ 11.1%

$\Pr[X \le q_1]$ $\Pr[X > q_2]$

♀: 0.289 $\to$ 29%

CORRECTED $\Rightarrow$ Equivalent: $1 - [$pnorm(q_2, μ, sd) − pnorm(q_1, μ, sd)$]$

$P[X \le q_2]$ $\Pr[X \le q_1]$

DEFAULT: lower.tail = F

- Distribution of sample means: If $Y \sim N$, then $\bar{Y} \sim N$

Population SEM: $\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} \Rightarrow Z = \frac{\bar{Y} - \mu}{\sigma_{\bar{Y}}} \Rightarrow \bar{Y} = \mu + \sigma_{\bar{Y}} Z$

Ex: Weight of babies at birth:
$n = 80$, $\mu = 3339$, $\sigma = 573$ g.

$\frac{\sigma}{\sqrt{n}} = \frac{573}{\sqrt{80}}$

① $\Pr[\bar{Y} > 3370]$ = pnorm(q = 3370, μ = 3339, sd = SEM, lower.tail=F)

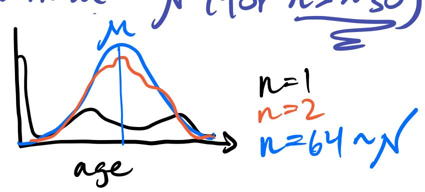
② $z = \frac{q - \mu}{SEM} = \frac{3370 - 3339}{SEM} \Rightarrow$ pnorm(z, 0, 1, lower.tail = F)

③ or equivalently $\Rightarrow$ pnorm($q - \mu$, 0, SEM, lower.tail = F)

- CLT: Σ or mean of random sample w/ large n from a non-normal population is approximately normal: $\sim N$ (for $n > \sim 30$)

⊛ Weak law of large numbers.

Ex 10.6 "Spanish" flu of 1918:



Ch 11: Inference for a Normal Population

- **Student's t-distribution**

  ⇒ sampling dist. for $\bar{Y} \sim N$ (CLT)

  BUT don't know $\sigma_{\bar{Y}}$ = SEM. for population!

  ∴ Best estimate is: $SE_{\bar{Y}} = \frac{s}{\sqrt{n}}$

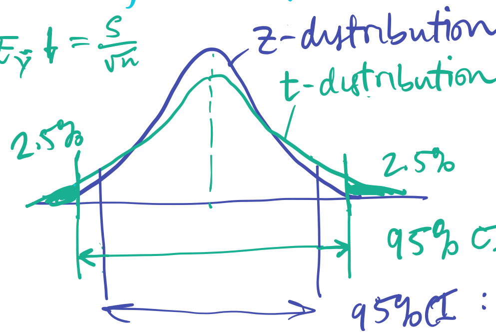
  ⊛ $t = \frac{\bar{Y} - \mu}{SE_{\bar{Y}}}$ "Student's t" w/ n−1 degrees of freedom

  ↳ Not a constant like $\sigma_{\bar{Y}}$ due to sampling error

  ∴ WIDER TAILS

  why n−1 = df? Recall: $s^2 = \frac{\Sigma (Y_i - \bar{Y})^2}{n-1} \Rightarrow \frac{(\bar{Y}_i - \mu)^2}{n-1}$

As $n\uparrow$, $SE_{\bar{Y}}\downarrow = \frac{s}{\sqrt{n}}$



$\alpha = 0.05$

e.g. If df = 5, $t_{crit} \approx \pm 2.78$ vs. $z = 1.96$.

95% CI for $t_{2(\alpha), df} \to t_{2(0.05), df}$

95% CI: $z = \pm 1.96$.

⇒ CI expresses precision of an estimate, follows t-dist. for $n < 30$

"Critical value" of t-statistic for n−1 d.f. at significance level α:

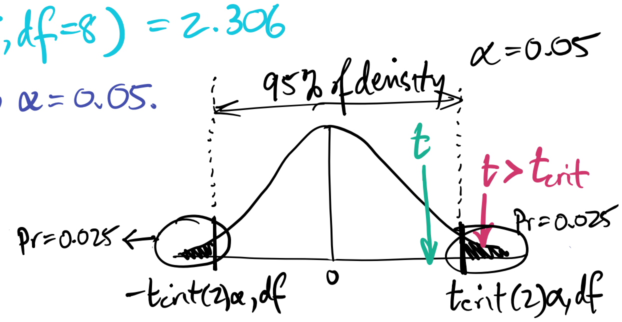
$$-t_{crit(2)\alpha, df} < \frac{\bar{Y} - \mu}{SE_{\bar{Y}}} < t_{crit(2)\alpha, df} \Rightarrow \bar{Y} = \pm t_{crit} \cdot SE_{\bar{Y}} + \mu$$

$$\left.\bar{Y} - t_{crit} \cdot SE_{\bar{Y}} < \mu < \bar{Y} + t_{crit} \cdot SE_{\bar{Y}}\right\} \text{ CI for } \mu$$

e.g. t_{crit} = qt(0.025, df=8, lower.tail=F)
= qt(0.975, df=8) = 2.306

The significance cutoff is $\alpha = 0.05$.

⊛ If the sample statistic $t_{obs} > t_{crit}$, then $p < \alpha$ and we reject H_0.



ch 11.3 One-sample t-test.

H_0 : True mean $= \mu_0$

H_A : " " $\neq \mu_0$

Assumptions:
- Random sample
- " variable $\sim N$

$\Rightarrow$ compare sample to pop $t = \dfrac{\bar{Y} - \mu}{SE_{\bar{Y}}}$ THIS IS THE t-statistic

⊗ sampling distribution of t gives Pr[obs. data] under H_0.

Ex 11.3 Human body temp. 37°C ≈ 98.6°F (36.8)

$n = 25$ vs $n = 130$

$p > 0.05$ $p < 0.05$ REJECT H_0

since $\uparrow n \Rightarrow \downarrow SE_{\bar{Y}} = \dfrac{s}{\sqrt{n}}$

ONE-SAMPLE t-test
- compare to Exp. value of pop. param

⊗ t.test(body_temp, mu = 98.6)
↳ data for your sample = VECTOR.

H_0 : Mean body temp = 98.6
H_A : " " " $\neq 98.6$
} TWO-SIDED TEST

{ computes d.f. from the sample data
{ can also directly access p-value & 95% CI

{ my_ttest$p.value
{ my_ttest$conf.int

$n = 25$: $t_{obs} = \dfrac{\bar{Y} - \mu_0}{SE_{\bar{Y}}} = \dfrac{98.52 - \mu_0}{0.136} = -0.56$

p-value $= \underbrace{Pr[t < -0.56]}_{\text{left tail}} + \underbrace{Pr[t > 0.56]}_{\text{right tail}} = 2 * Pr[t > 0.56] = 0.58$ DO NOT REJECT H_0

$n = 130$: $t_{obs} = \dfrac{98.25 - \mu_0}{0.064} = -5.44 \Rightarrow$ p-value $= 1.6 \cdot 10^{-5}$ REJECT H_0

$t_{24} \approx t_{130} \approx N$ BUT $\bar{Y} - \mu_0$ was larger & $SE_{\bar{Y}}$ was smaller

$\therefore t_{obs(130)} >> t_{obs(24)}$