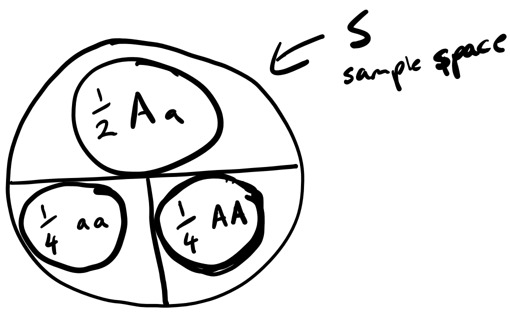
Conditional Probability

For an autosomal dominant trait the genotypes in the F2 occur in the following proportions:

$F_1: \quad Aa \times Aa$

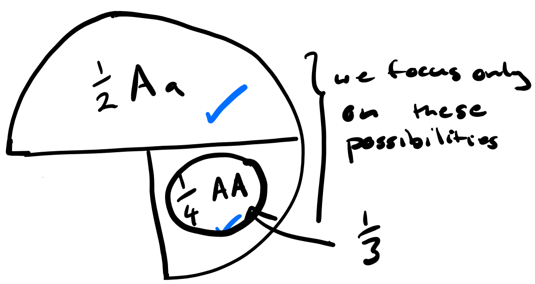
$F_2:$



Let us suppose a disease is determined by a single dominant A allele.

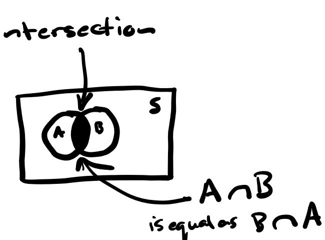
$$P(\underline{A}A \mid \text{affected})$$

(given: $\mid$)



The probability of event A conditional on event B:

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$



e.g. $$P(AA|\text{affected}) = \frac{P(AA \text{ and affected})}{P(\text{affected})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$$P(\text{affected} \mid \text{homozygous}) = \frac{\boxed{P(\text{affected} \cap \text{homo})}}{P(\text{homozygous})} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$$P(\text{homozygous} \mid \text{affected}) = \frac{\boxed{P(\text{homozygous} \cap \text{affected})}}{P(\text{affected})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

$$\therefore P(\text{affected} \mid \text{homozygous}) \neq P(\text{honozygous} \mid \text{affected})$$

Colorblind example

$$P(M \mid C)$$

$$P(M) = P(F) = 0.5$$

2.5% of people are colorblind (C) males (M)

$$P(M \cap C) = 0.025$$

0.5% of people are colorblind (C) females (F)

$$P(F \cap C) = 0.005$$

What is the probability that a male is colorbld?

$$P(C \mid M) = \frac{P(C \cap M)}{P(M)} = \frac{0.025}{0.5} = 0.005$$

What is the probability that a colorblind person is male?

$$P(M \mid C) = \frac{P(M \cap C)}{P(C)} = \frac{0.025}{0.03} \approx 0.833$$

i.e 83% of colorblind people are male

$$P(C) = P(M \cap C) + P(F \cap C) = 0.025 + 0.005 = 0.03$$

Check using large numbers

10,000 people → 5,000 males
10,000 people → 5,000 females

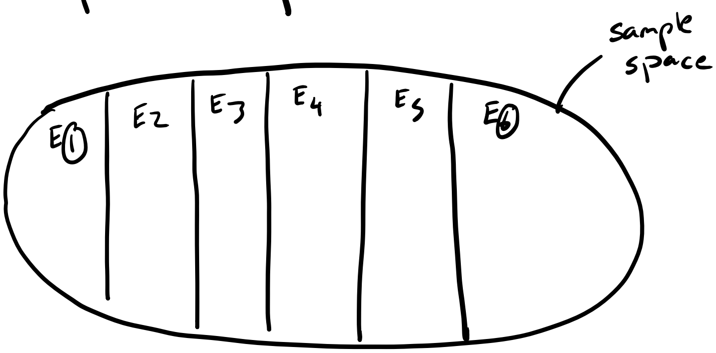
Fraction of 0.025 people, or 250 are colorblind males

Fraction of 0.005 people, or 50 are colorblind females

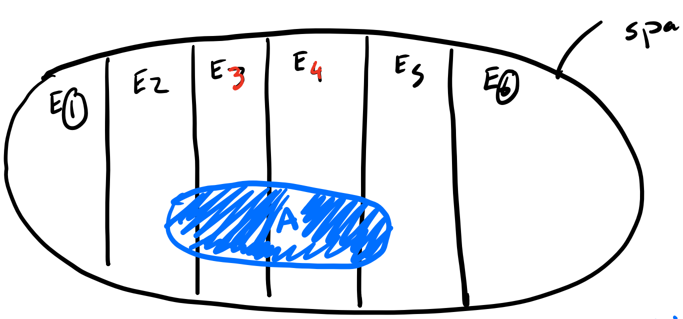
Total of 300 colorblind people

$$\frac{250}{300} = \frac{5}{6} = 0.833$$

Total probability



$$\sum_{i=1}^{i=6} P(E_i) = 1$$

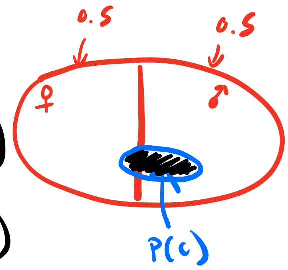


P(A) given the six different probabilities of E.

Law of total probability

$$\boxed{P(A)=\sum_{i=1}^{n} P(A|E_i)\cdot P(E_i)}$$

if <u>1%</u> of females and <u>5%</u> of males are colorblind



$$P(c)=P(C|F)\cdot P(F)+P(C|M)\cdot P(M)$$

$$=(0.01)(0.5)+(0.05)\cdot(0.5)$$

$$=0.03$$

Often we know $P(A)$, $P(B)$ + $P(A|B)$

but what we want is $\underline{P(B|A)}$

① $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\underline{P(A \cap B)} = \underline{P(A|B) \cdot P(B)}$

$P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$\underline{P(B \cap A)} = \underline{P(B|A) \cdot P(A)}$

recall: $P(A \cap B) = P(B \cap A)$

$\therefore P(A|B) \cdot P(B) = \underline{P(B|A)} \cdot P(A)$

$$\boxed{P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}}$$

Baye's Theorem.

$$P(M|C) = \frac{P(C|M) \cdot P(M)}{P(C)}$$

$$= \frac{(0.05)(0.5)}{0.03}$$

$$= 0.83$$

Example of prenatal diagnosis for Down Syndrome

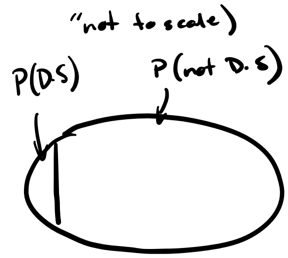
$$P(D.S \mid \text{positive result}) = \frac{P(\text{positive} \mid D.S) \cdot P(D.S)}{P(\text{positive})}$$

$P(\text{positive result} \mid D.S) = \underline{0.6}$

$P(D.S) = \underline{0.001}$

$P(\text{positive} \mid \text{not } D.S) = 0.05$

$1 - P(D.S) = P(\text{not } D.S) = 0.999$



$$\underline{P(\text{positive})} = P(\text{positive} \mid D.S) \cdot P(D.S) + \underline{P(\text{positive} \mid \text{not } D.S.)} \cdot \underline{P(\text{not } D.S)}$$

$$= 0.6\,(0.001) + 0.05\,(0.999)$$

$$= 0.05055$$

$$P(DS \mid \text{positive result}) = \frac{0.6 \times 0.001}{0.05055}$$

$$= 0.012$$

i.e there is a 1.2% chance that the fetus has down syndrome.

Let's consider large numbers.

test 1,000,000 fetuses

- 1,000 will have D.S ; 999 000 will not D.S

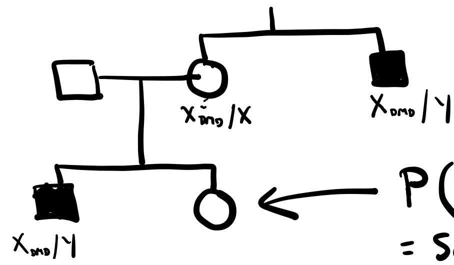
$\downarrow$; $\downarrow$ 5%

600 will test tve ; 49,950 will test tve

$\therefore$ there will be $600 + 49,950 = 50,550$

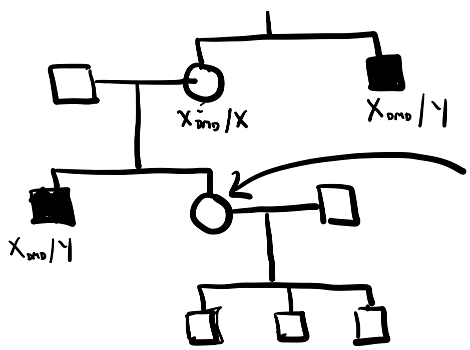
however $\frac{600}{50,550} = 1.2\%$

Apply Baye's thereom to pedigree analysis

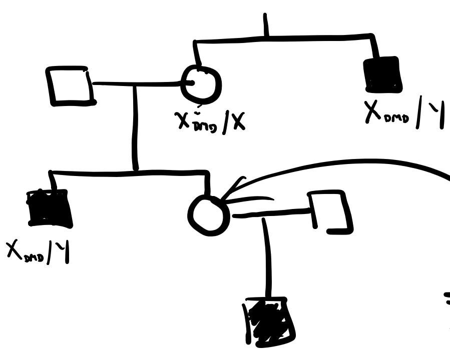


P(carrier)
= 50%

muscular dystrophy (DMD)
X recessive disorder X_{DMD}



P(carrier)



P(carrier)
= 1

prior

$$P(\text{carrier} \mid 3 \text{ unaffected sons}) = \frac{P(3 \text{ unaffected sons} \mid \text{carrier}) \cdot P(\text{carrier})}{P(3 \text{ unaffected sons})}$$

use law of total probability

$$P(3 \text{ unaffected}) = P(3 \text{ unaffected} \mid \text{carrier}) \cdot P(\text{carrier}) + P(3 \text{ unaffect} \mid \text{not}) \cdot P(\text{not})$$

$$= \underbrace{\left(\tfrac{1}{2}\right)\left(\tfrac{1}{8}\right)} + \underbrace{(1) \cdot \left(\tfrac{1}{2}\right)}$$

$$= \frac{1}{16} + \frac{1}{2}$$

$$= \frac{9}{16}$$

$$P(\text{carrier} \mid 3 \text{ unaffected sons}) = \frac{\left(\frac{1}{8}\right) \cdot \left(\frac{1}{2}\right)}{\frac{9}{16}}$$

$$= \frac{\frac{1}{16}}{\frac{9}{16}}$$

$$= \frac{1}{\cancel{16}} \times \frac{\cancel{16}}{9}$$

$$P(\text{carrier} \mid 3 \text{ unaffected sons}) = \frac{1}{9} \leftarrow \text{posterior probability}$$

we have updated our probability estimate w new information

$$P(\text{carrier} \mid 1 \text{ affected son}) = \frac{P(\text{affected} \mid \text{carrier}) \cdot P(\text{carrier})}{P(1 \text{ affected son})}$$

compute total probability