Discrete distributions

Bernoulli: p(success) for $x \in \{0,1\}$ for single trial

$f(x) = p^x (1-p)^{1-x}$ p is fixed.

coin toss: $p = 0.5$ $\underset{x=1}{H=1}$, $\underset{x=0}{T=0}$

$P(X=1) = f(1) = p^1(1-p)^{1-1} \Rightarrow p = 0.5$

$P(X=0) = f(0) = \underbrace{p^0}_{S}\underbrace{(1-p)^{1-0}}_{F} \Rightarrow p = 0.5$

What if 2 trials? $\Rightarrow$ **Binomial**:
- p fixed
- trials independent

X = # successes out of n trials

	A	B
A		
B		

$P[A] = p$
$P[B] = 1-p$

Under independence:

		# ways	
$x=2$	$P(A \cap A) = P(A)^2 = p^2$	1	$\binom{2}{2}$
$x=1$	$P(A \cap B) = P(A)P(B) = p(1-p)$	2	$\binom{2}{1}$
	$P(B \cap A) = P(B)P(A) = p(1-p)$		
$x=0$	$P(B \cap B) = P(B)^2 = (1-p)^2$	1	$\binom{2}{0}$

Binomial theorem

$(x+y)^2 = \underset{(1)}{x^2} + \underset{(2)}{2xy} + \underset{(1)}{y^2}$

$(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \dots \binom{n}{n}x^0 y^n$

$= \sum \binom{n}{k} x^{n-k} y^k$

PDF: $f(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

CDF: $F(x) = P(X \le x) = \sum_{x=0}^{x}$

$(x+y)^3 = \underset{(1)}{x^3} + \underset{(3)}{3x^2y} + \underset{(3)}{3y^2x} + \underset{(1)}{y^3}$

$x=0$: 1 $\qquad \binom{0}{k}$

$x=1$: 1 1 $\qquad \binom{1}{k}$

$x=2$: 1 2 1 $\qquad \binom{2}{k}$ $\quad k \in \{0,1,2\}$

$x=3$: 1 3 3 1 $\qquad \binom{3}{k}$

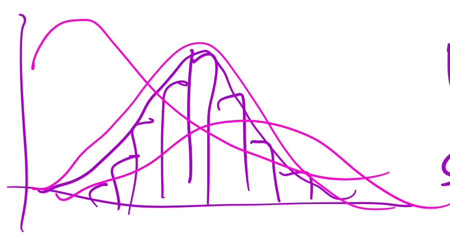
$x=4$: 1 4 6 4 1 $\qquad \binom{4}{k}$

Pascal's triangle

Binomial distribution

PDF : $f(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

CDF : $F(x) = P(X \le x) = \sum_{0}^{x} \binom{n}{x} p^x (1-p)^{n-x}$



large n : BIN $\to \mathcal{N}$

small p, large n : BIN $\to$ Poisson

$\lim_{n\to\infty}$ BIN, $p =$ small

Poisson : small $p \to$ right-skew
large $p \to$ more sym. (binom, Normal)