Binomial - a bunch of independent trials (e.g. coin toss)
- p(success) is fixed

A single trial = "Bernoulli trial"

$p = 0.5 \qquad f(x) = p^x (1-p)^{1-x}, \quad 0 \le p \le 1, \quad x \in \{0, 1\}$

so $P(X=1) = 0.5^1 \times 0.5^0 = 0.5$

$P(X=0) = 0.5^0 \times 0.5^1 = 0.5$

What if there are 2 trials?

Under independence: let's call $Pr[A] = p$, $P[B] = 1-p$

	A	B
A		
B		

$P(A \cap A) = p(A)^2 = p^2$

$P(A \cap B) = p(A) \cdot P(B)$ } 2 possibilities: $\binom{2}{1}$

$\quad '' \quad = p(B) \cdot P(A)$

$P(B \cap B) = p(B)^2$

$$P(X=x) = \underbrace{\binom{2}{x}}_{\#\text{ ways to get } x \text{ "successes"}} \underbrace{p^x (1-p)^{2-x}}_{1 \text{ trial}}, \quad x \in \{0, 1, 2\}$$

$x=0: B \cap B$
$x=1: A \cap B \cap B \cap A$
$x=2: A \cap A$

$= (1-p)^2 + 2p(1-p) + p^2 \Rightarrow$ Binomial expansion

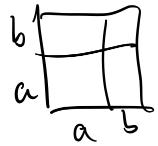
$n=2 \qquad (x+y)^2 = x^2 + 2xy + y^2$

$n=3 \qquad (x+y)^3 = x^3 + 3xy^2 + 3yx^2 + 1y^3$

$n=n \qquad (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum \binom{n}{k} x^k y^{n-k}$ (symmetrical)

$\binom{n}{k}$: Binomial coefficient = # ways to get a combination

Pascal's Triangle



$(a+b)^2 \longrightarrow$

$n=0$	1	$\binom{0}{k}$
$n=1$	1 1	$\binom{1}{k}$
$n=2$	1 2 1	$\binom{2}{k}$
$n=3$	1 3 3 1	$\binom{3}{k}$
$n=4$	1 4 6 4 1	$\binom{4}{k}$
		$\vdots$

Binomial Distribution

$f(x) = P(X=x) = \binom{n}{x} p^x (1-p)^{1-x}$ PDF

$F(x) = P(X \le x) = \sum \binom{n}{x} p^{x} (1-p)^{1-x}$ $x = \{0, 1, \ldots\}$ CDF

Survival $= P(X > x) = 1 -$ CDF

Large N $\rightarrow$ Normal

Small p $\rightarrow$ Poisson, for large n lim as $n \to \infty$, $p \to 0$

Bernoulli : P(1 success) out of 1 trial.

Binomial : P(X success) out of n trials

Geometric : P(1 success) after X UNSUCCESSFUL trials

$$P(X=x) = \underset{\uparrow \text{ 1 success}}{p}\,\underbrace{(1-p)^x}_{x \text{ failures}}$$

NB : P(r successes) after x failures (last trial is rth success)

$$P(X=x) = \binom{n-1}{x} p^r (1-p)^x$$

r = success
x = failure