but are occasionally used to convince a journal editor that nonsignificant results are worth publishing. Such actions are a response to *publication bias* by scientific journals that are more apt to publish only significant results (Dickersin 1990). Power has the basic form

$$\text{Power} = 1 - \beta \propto \frac{E\alpha\sqrt{n}}{\sigma}, \tag{6.10}$$

where E indicates the true effect size. The proportion sign in Equation 6.10, $\propto$, is indicative that power will depend in part on the measure of effect size being used, the type of statistical analysis one is conducting, and the particular testing procedure used in the analysis. Note that Equation 6.10 requires that we know σ or have a good estimate for it preceding analysis.

EXAMPLE 6.10 POWER ANALYSIS, SMOKING, AND ALZHEIMER'S

Surprisingly, the incidence of Alzheimer's disease has been negatively associated with moderate amounts of smoking (Van Duijn and Hoffman 1991, Graves et al. 1991, Brenner et al. 1993, Salib and Hillier 1997). Smoking may offer some protection because nicotine may reduce apoptosis (programmed cell death) of neurons (Larrick 1993).

Let us assume that previous to the start of the experiment concerning the effect of smoking on Alzheimer's, researchers were interested in being able to detect a 7% decrease in Alzheimer's incidence for subjects smoking 10–20 cigarettes a day, given that $\sigma = 45\%$. The investigators wanted to know if a sample size of 200 was sufficient to detect this effect (produce a significant result) given $\alpha = 0.05$.

Because we know σ, we will run the power analysis assuming that a one-sample z-test will be used for hypothesis testing. We are interested in the lower-tailed alternative hypothesis that smoking decreases the occurrence of Alzheimer's. Our hypotheses are H_0: $\mu \geq 0$ and H_A: $\mu < 0$. We note that we have a sample size large enough to assume a normal sampling distribution for $\bar{X}$, regardless of its underlying parent distribution.

A one-sample lower-tailed z-test would reject null at $\alpha = 0.05$ whenever z^* is less the lower-tailed critical value −1.645.

```
qnorm(.05)
[1] -1.644854
```

Solving for $\bar{x}$ in Equation 6.1, we have

$$\frac{\bar{x} - 0}{45/\sqrt{200}} < -1.644854 \quad \bar{x} < (45/\sqrt{200})\,1.644854 \quad \bar{x} < 5.234684.$$

The observed decrease in Alzheimer's as a result of smoking (effect size) has to be greater than 5.234684% for us to reject H_0 at $\alpha = 0.05$.

Given a significance level of 0.05, effect size of −7%, variance of 45^2, and a sample size of 200, power is obtained by finding

$$P(\bar{X} \leq -5.234684), \quad \text{where } \bar{X} \sim N(-7,\ 2025/200).$$

In **R**, we have

```
pnorm(-5.234684, -7, 45/sqrt(200))
[1] 0.7104792
```

Thus, $1 - \beta = 0.7104792$. In other words, there is a probability of 0.71 of rejecting the null hypothesis $\mu \geq 0$ if the true effect of smoking is a 7% decrease in Alzheimer's. This is the power of the experiment.

The relationship of type I error and power can be visualized in a number of ways. In one approach, a plot containing the null distribution is placed atop a plot containing a distribution that assumes H_A is true. The two distributions are then connected with a dashed vertical line representing the boundary for α and β. This is demonstrated for the Alzheimer's analysis in Figure 6.7.

```
Fig.6.7()##See Ch. 6 code
```

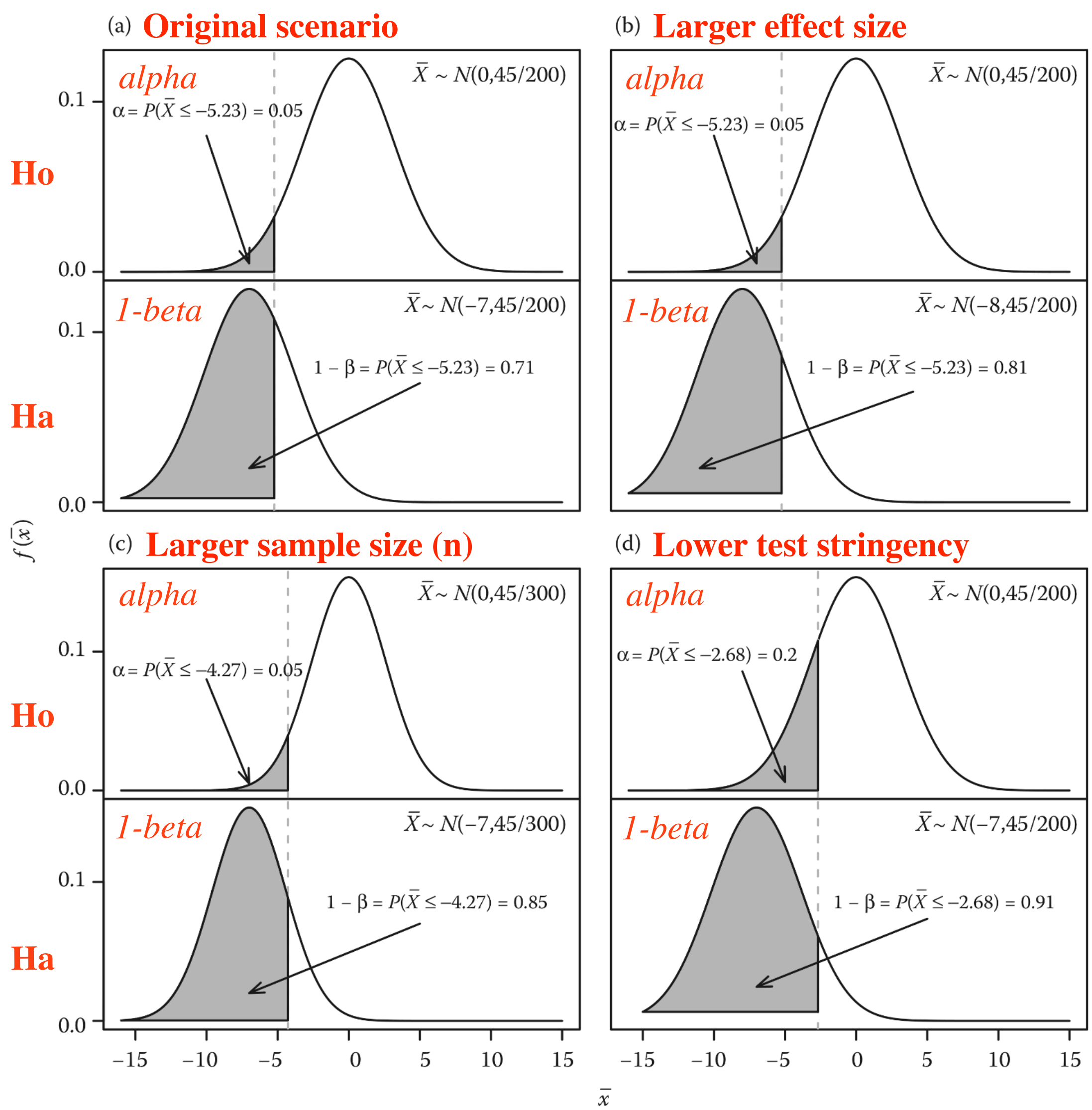

FIGURE 6.7
Illustration of type I error and power for Example 6.10. For (a) through (d), the top figure illustrates the sampling distribution for $\bar{X}$ assumed by the null hypothesis and shows (shades) α, while the bottom figure plots the distribution for a particular value of the alternative and shows power, 1 – β. (a) provides an initial example (effect size = –7, α = 0.05, σ = 45, n = 200, and power = 0.71). Figures (b) through (d) demonstrate ways to increase power: (b) increase effect size (effect size = –8, α = 0.05, σ = 45, n = 200, and power = 0.81), (c) increase n (effect size = –7, α = 0.05, σ = 45, n = 300, and power = 0.85), and (d) increase α (effect size = –7, α = 0.2, σ = 45, n = 200, and power = 0.91).