1 Constraint Graph Weighting Schemes

Let $G = (V, E)$ denote the Sudoku constraint graph. Each vertex $i \in V$ corresponds to a cell. An edge $(i, j) \in E$ exists if cells i and j are peers (i.e., share a row, column, or box).

Let:

$$D_i \subseteq \{1, \ldots, 9\}$$

be the candidate domain of cell i,

$$|D_i|$$

its domain size, and

$$k_{ij} = |D_i \cap D_j|$$

the domain overlap size between cells i and j.

Let $\varepsilon > 0$ be a small constant used for numerical stability.

1.1 Binary Weighting

$$w_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{otherwise} \end{cases}$$

This corresponds to the unweighted Sudoku constraint graph.

1.2 Overlap Weighting

$$w_{ij} = \frac{|D_i \cap D_j|}{9}$$

Equivalently,

$$w_{ij} = \frac{k_{ij}}{9}.$$

Weights are proportional to shared candidate ambiguity.

1.3 Inverse Overlap Weighting

$$w_{ij} = \frac{1}{|D_i \cap D_j| + \varepsilon}$$

Equivalently,

$$w_{ij} = \frac{1}{k_{ij} + \varepsilon}.$$

Smaller overlap implies stronger constraint coupling.

1.4 Expected Fraction Weighting

$$w_{j\to i} = \frac{|D_i \cap D_j|}{|D_i|\,|D_j|}$$

Equivalently,

$$w_{j\to i} = \frac{k_{ij}}{|D_i|\,|D_j|}.$$

This represents the expected fraction of i's domain eliminated if j is assigned uniformly at random.

Note that this expression is symmetric in i and j.

1.5 Directional Target Fraction Weighting

$$w_{j\to i} = \frac{|D_i \cap D_j|}{|D_i|} \cdot \frac{1}{|D_j|^{\alpha}}$$

Equivalently,

$$w_{j\to i} = \frac{k_{ij}}{|D_i|\,|D_j|^{\alpha}},$$

where $\alpha \geq 0$ controls emphasis on source tightness.

This weighting is generally asymmetric.

1.6 Directional Information Weighting

$$w_{j\to i} = \frac{|D_i \cap D_j|}{|D_i|} \log\left(\frac{9}{|D_j|}\right)$$

Equivalently,

$$w_{j\to i} = \frac{k_{ij}}{|D_i|} \log\left(\frac{9}{|D_j|}\right).$$

This treats domain size as an inverse measure of information.

2 Spectral Graph Construction

Let A denote the weighted adjacency matrix:

$$A_{ij} = w_{ij}.$$

Define the degree matrix D:

$$D_{ii} = \sum_j A_{ij}.$$

The combinatorial Laplacian is defined as:

$$L = D - A.$$