Math 151 – Python Lab 4

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0.1 MATH 151 Lab 4

Section Number: 568

Members:

- Brighton Sikarskie
- Colton Hesser
- Gabriel Gonzalez
- Gabriel Cuevas

[1]:
```python
from sympy import *
from sympy.plotting import (plot,plot_parametric)
%matplotlib inline
```

0.1.1 Question 1

1a

[2]:
```python
x = symbols('x')
f = (2 * x + 1) / (x**2 + 2)
# tangent line of f at x = 2

tangent = f.diff(x).subs(x, 2) * (x - 2) + f.subs(x, 2)
print(f"The tangent line of f(x) = {f} at x = 2 is {tangent}")
```

```
The tangent line of f(x) = (2*x + 1)/(x**2 + 2) at x = 2 is 23/18 - 2*x/9
```

1b

[3]:
```python
# normal line of f at x = 2
normal = -1 / f.diff(x).subs(x, 2) * (x - 2) + f.subs(x, 2)
print(f"The normal line of f(x) = {f} at x = 2 is {normal}")
```

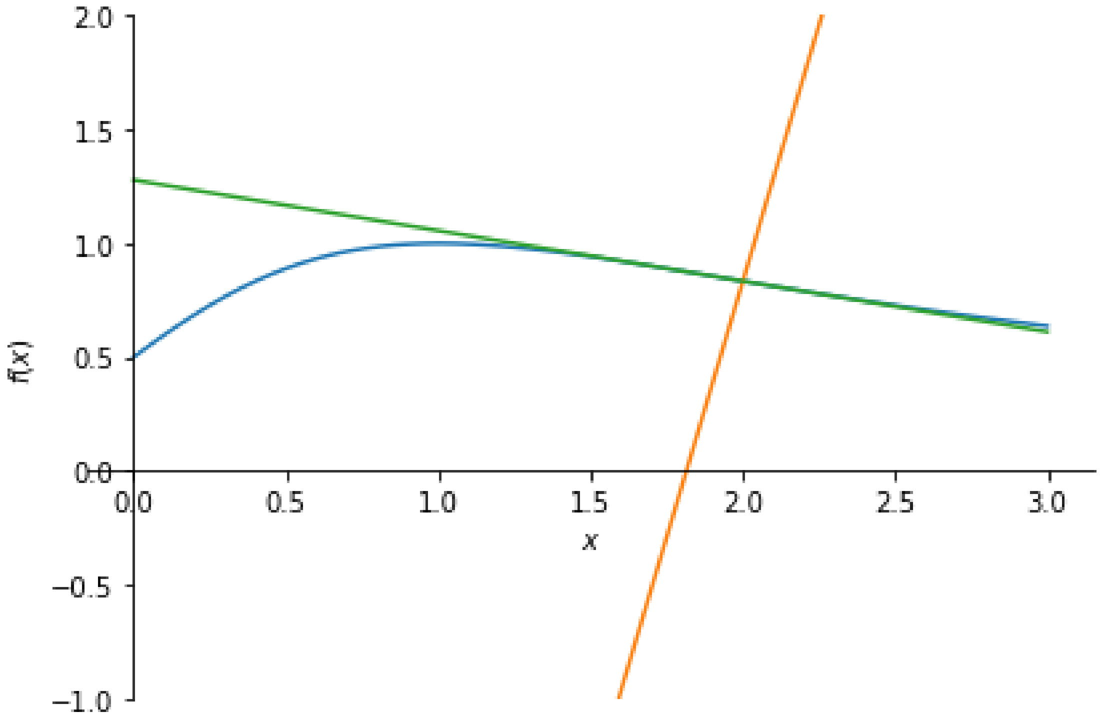
```
The normal line of f(x) = (2*x + 1)/(x**2 + 2) at x = 2 is 9*x/2 - 49/6
```

1c

[4]:
```python
print(f"The graphs of f(x) = {f}, the tangent line, and the normal line are␣
↪shown below.")
plot(
    (f, (x, 0, 3)),
```

```
    (normal, (x, 0, 3)),
    (tangent, (x, 0, 3)),
    ylim=(-1,2)
)
```

```
The graphs of f(x) = (2*x + 1)/(x**2 + 2), the tangent line, and the normal line
are shown below.
```



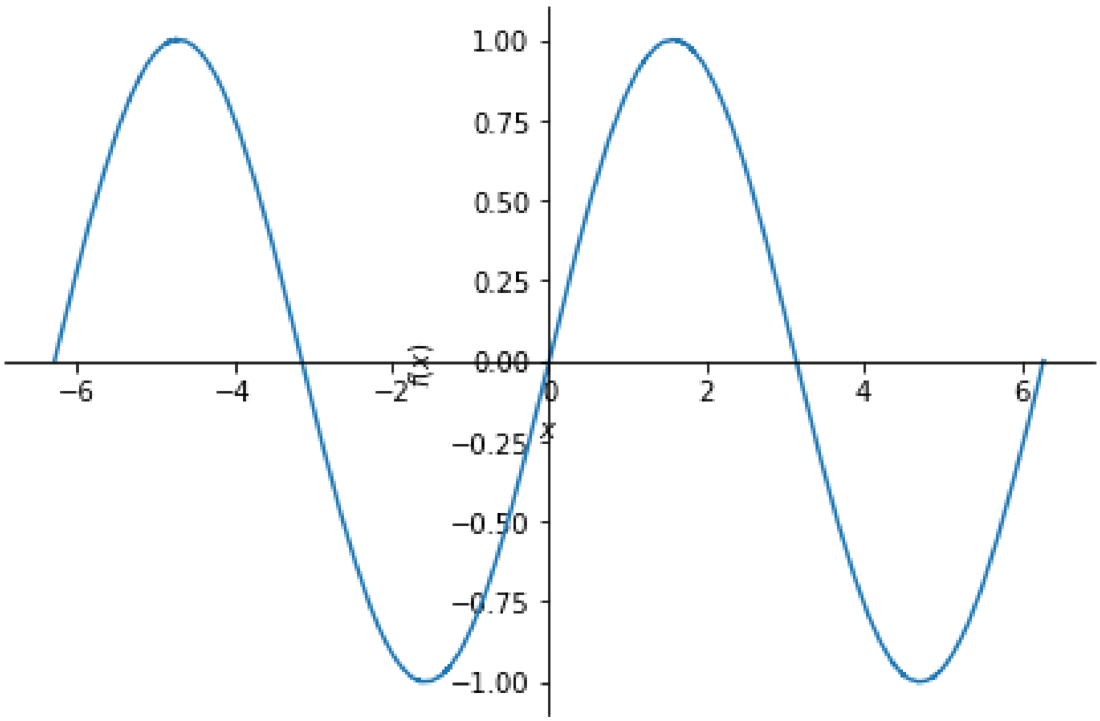
[4]: <sympy.plotting.plot.Plot at 0x7f400daf31c0>

0.1.2 Question 2

2a

[5]:
```
y = sin(x) - 1 / 1000 * sin(1000 * x)
print(f"The graph of y = {y} on the interval [-2pi, 2pi] is shown below.")
plot(y, (x, -2 * pi, 2 * pi))
print(f"My estimate for the slope of the graph near x = 0 is {1/2}")
```

```
The graph of y = sin(x) - 0.001*sin(1000*x) on the interval [-2pi, 2pi] is shown
below.
```

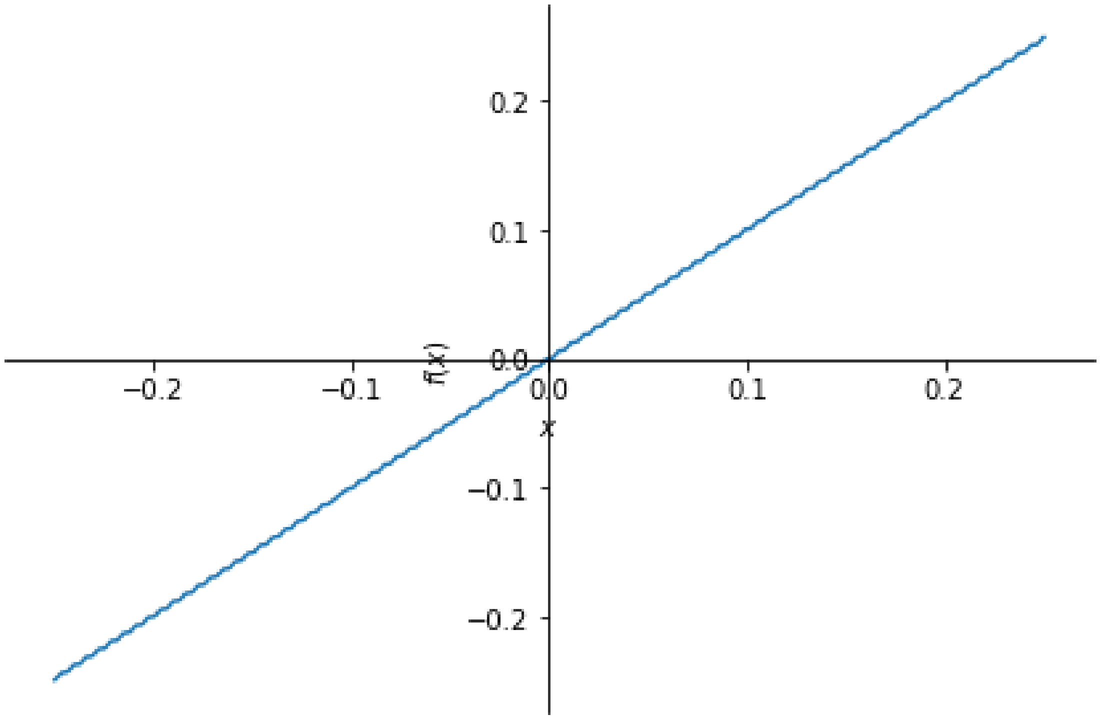


My estimate for the slope of the graph near x = 0 is 0.5

2b

```
[6]: print(f"The graph of y = {y} on the interval [-0.25, 0.25] is shown below.")
plot(y, (x, -0.25, 0.25))
print(f"My estimate for the slope of the graph near x = 0 is {0.1/0.1}")
```

The graph of y = sin(x) - 0.001*sin(1000*x) on the interval [-0.25, 0.25] is shown below.



My estimate for the slope of the graph near x = 0 is 1.0

2c

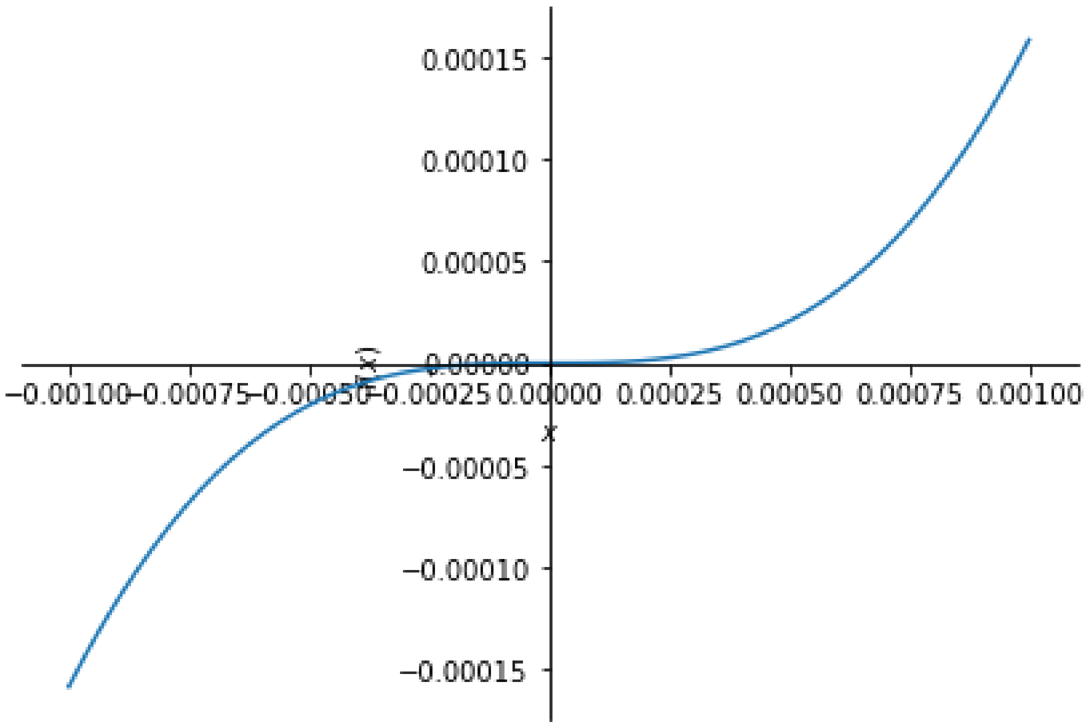
[7]:
```
slope = y.diff(x).subs(x, 0)
print(f"The slope of the graph of y = {y} at x = 0 is {slope}")
```

The slope of the graph of y = sin(x) - 0.001*sin(1000*x) at x = 0 is 0

2d

[8]:
```
print(f"The graph of y = {y} on the interval [-0.001, 0.001] is shown below.")
plot(y, (x, -0.001, 0.001))
print(f"If I zoom in, I can see that the slope of the graph of y = {y} at x = 0␣
↪is {slope}")
```

The graph of y = sin(x) - 0.001*sin(1000*x) on the interval [-0.001, 0.001] is shown below.



If I zoom in, I can see that the slope of the graph of y = sin(x) - 0.001*sin(1000*x) at x = 0 is 0

0.1.3 Question 3

3a

[9]:
```
t = symbols('t')
v = 100_000 * (1 - (1 / 60) * t) ** 2
# the average rate of change from 0 to 10
average_rate_of_change = (v.subs(t, 10) - v.subs(t, 0)) / 10
print(f"The average rate of change of v(t) = {v} from t = 0 to t = 10 is␣
↪{average_rate_of_change}")
```

The average rate of change of v(t) = 100000*(1 - 0.0166666666666667*t)**2 from t = 0 to t = 10 is -3055.55555555555

3b

[10]:
```
# the instantaneous rate of change of v with respect to t
instantaneous_rate_of_change = v.diff(t)
print(f"The instantaneous rate of change of v(t) = {v} with respect to t is␣
↪{instantaneous_rate_of_change}")
```

The instantaneous rate of change of v(t) = 100000*(1 - 0.0166666666666667*t)**2 with respect to t is 55.5555555555556*t - 3333.33333333333

3c

[11]:
```python
# the instantaneous rate of change of v with respect to t at t = 10
instantaneous_rate_of_change_at_10 = instantaneous_rate_of_change.subs(t, 10)
print(f"The instantaneous rate of change of v(t) = {v} with respect to t at t =␣
↪10 is {instantaneous_rate_of_change_at_10}")
```

```
The instantaneous rate of change of v(t) = 100000*(1 - 0.0166666666666667*t)**2
with respect to t at t = 10 is -2777.77777777778
```

3d

[12]:
```python
print(f"The average rate of change is {average_rate_of_change} and the␣
↪instantaneous rate of change at 10 is {instantaneous_rate_of_change_at_10}.")
print("The average rate of change is less than the instantaneous rate of change␣
↪meaning that the average rate of change is in fact an average.")
print("If we calculated more instantaneous rate of changes and averaged them␣
↪together we would most likely get something very similar to the average rate␣
↪of change.")
import numpy as np

step = 0.1
lst = np.arange(0, 10 + step, step)
average = 0
for x in lst:
    average += instantaneous_rate_of_change.subs(t, x)
average /= len(lst)

print(f"The average instantaneous rate of change of v(t) = {v} from t = 0 to t␣
↪= 10 with a step of {step} is {average} confirming that the average rate of␣
↪change is in fact an average while the instantaneous rate of change is the␣
↪rate of change at a given point.")
```

```
The average rate of change is -3055.55555555555 and the instantaneous rate of
change at 10 is -2777.77777777778.
The average rate of change is less than the instantaneous rate of change meaning
that the average rate of change is in fact an average.
If we calculated more instantaneous rate of changes and averaged them together
we would most likely get something very similar to the average rate of change.
The average instantaneous rate of change of v(t) = 100000*(1 -
0.0166666666666667*t)**2 from t = 0 to t = 10 with a step of 0.1 is
-3055.55555555556 confirming that the average rate of change is in fact an
average while the instantaneous rate of change is the rate of change at a given
point.
```