Math 151 – Python Lab 7

November 3, 2022

0.1 MATH 151 Lab 7

Section Number: 568

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[1]:
```
from sympy import *
from sympy.plotting import (plot, plot_parametric)
```

0.1.1 Question 1

1a

[2]:
```
# piece wise function
# 8 - x ** 2 if x < 0
# 5 e ** ( -((x - 2) / 2) ** 2) + x if x >= 0
# find critical vaules of the function
x = Symbol('x')
f_left = 8 - x ** 2
f_right = 5 * exp(-((x - 2) / 2) ** 2) + x
f_left_prime = diff(f_left, x)
f_right_prime = diff(f_right, x)
left_critical = solve(f_left_prime, x)
right_critical = [nsolve(f_right_prime, x, 2)] + [nsolve(f_right_prime, x, 5)]
critical = left_critical + right_critical
points = [(x, f_left.subs(x, x)) for x in left_critical] + [(x, f_right.subs(x,
    x)) for x in right_critical]
p = Piecewise((f_left, x < 0), (f_right, x >= 0))

print(f'The critical values for f(x) = {p} are x = {critical}. The points are
    {points}.')
```

```
The critical values for f(x) = Piecewise((8 - x**2, x < 0), (x + 5*exp(-(x/2 -
1)**2), True)) are x = [0, 2.41784619385985, 4.78643377270907]. The points are
[(0, 8 - x**2), (2.41784619385985, x + 5*exp(-(x/2 - 1)**2)), (4.78643377270907,
x + 5*exp(-(x/2 - 1)**2))].
```

1b

[3]:
```
# find the absolute extrema of the function on the interval [-5, 5]
maxin_candidates = [-5, 5] + critical
yvals = [p.subs(x, i) for i in maxin_candidates]
maxy = max(yvals)
miny = min(yvals)
maxx = maxin_candidates[yvals.index(maxy)]
minx = maxin_candidates[yvals.index(miny)]
print(f'The absolute extrema of f(x) = {p} on the interval [-5, 5] are y = {maxy} and y = {miny}. The points are ({maxx}, {maxy}) and ({minx}, {miny}).')
```

```
The absolute extrema of f(x) = Piecewise((8 - x**2, x < 0), (x + 5*exp(-(x/2 -
1)**2), True)) on the interval [-5, 5] are y = 7.20429639824017 and y = -17. The
points are (2.41784619385985, 7.20429639824017) and (-5, -17).
```

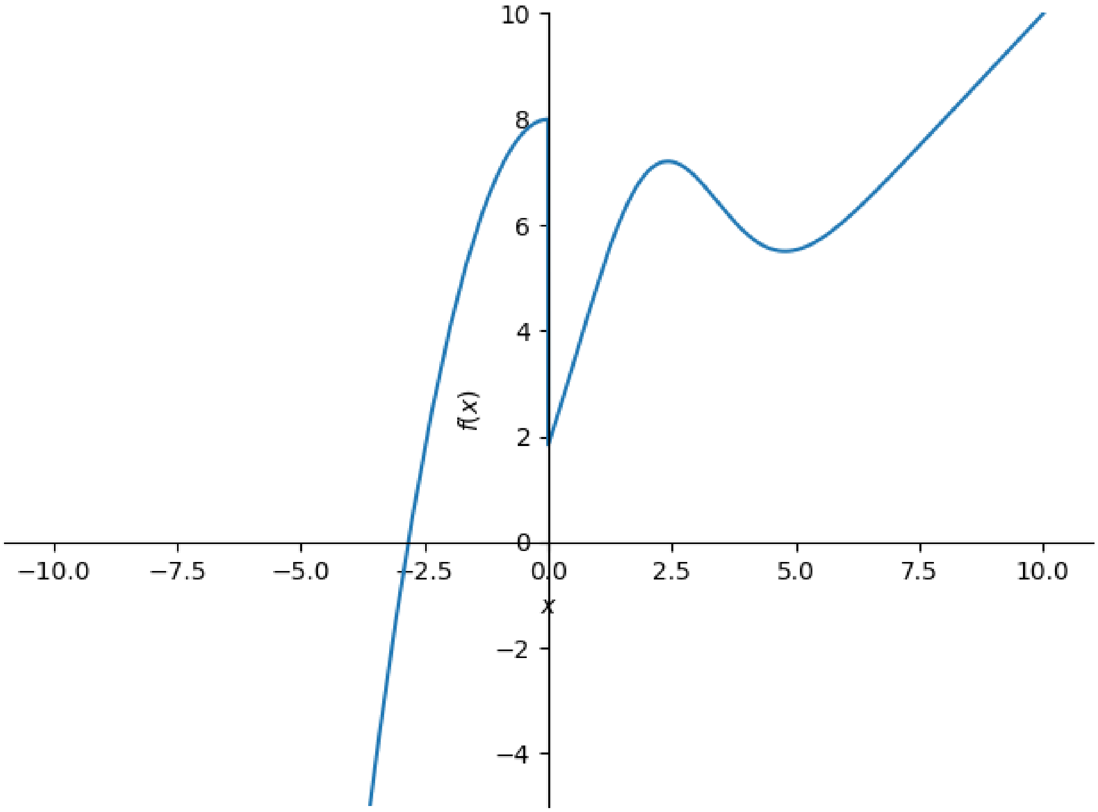
1c

[4]:
```
# find the absolute extrema of the function on the interval [-10, 10
maxin_candidates = [-10, 10] + critical
yvals = [p.subs(x, i) for i in maxin_candidates]
maxy = max(yvals)
miny = min(yvals)
maxx = maxin_candidates[yvals.index(maxy)]
minx = maxin_candidates[yvals.index(miny)]
print(f'The absolute extrema of f(x) = {p} on the interval [-10, 10] are y = {maxy} and y = {miny}. The points are ({maxx}, {maxy}) and ({minx}, {miny}).')
```

```
The absolute extrema of f(x) = Piecewise((8 - x**2, x < 0), (x + 5*exp(-(x/2 -
1)**2), True)) on the interval [-10, 10] are y = 5*exp(-16) + 10 and y = -92.
The points are (10, 5*exp(-16) + 10) and (-10, -92).
```

1d

[5]:
```
# plot the function on the interval [-10, 10]
main_plot = plot(p, (x, -10, 10), ylim=(-5, 10))
print(f'The plot of f(x) = {p} on the interval [-10, 10] is shown above.')
```



The plot of f(x) = Piecewise((8 - x**2, x < 0), (x + 5*exp(-(x/2 - 1)**2), True)) on the interval [-10, 10] is shown above.

0.1.2 Question 2

2a

[6]:
```
# v(r) = k (r_o - r) * r ** 2
# find the value of r in the interval [1/2 r_o, r_o]
# where v has an absolute maximum
k, r, r_o = symbols('k r r_o')
v = k * (r_o - r) * r ** 2
v_prime = diff(v, r)
r_crit = solve(v_prime, r)[1]
print(f'The absolute maximum of v(r) = {v} is at r = {r_crit}.')
```

```
The absolute maximum of v(r) = k*r**2*(-r + r_o) is at r = 2*r_o/3.
```

2b

[7]:
```
v_max = v.subs(r, r_crit)
point = (r_crit, v_max)
```

```
print(f'The absolute max of v(r) = {v} is {v_max} at r = {r_crit}. The point is␣
↪{point}.')
```

The absolute max of v(r) = k*r**2*(-r + r_o) is 4*k*r_o**3/27 at r = 2*r_o/3.
The point is (2*r_o/3, 4*k*r_o**3/27).

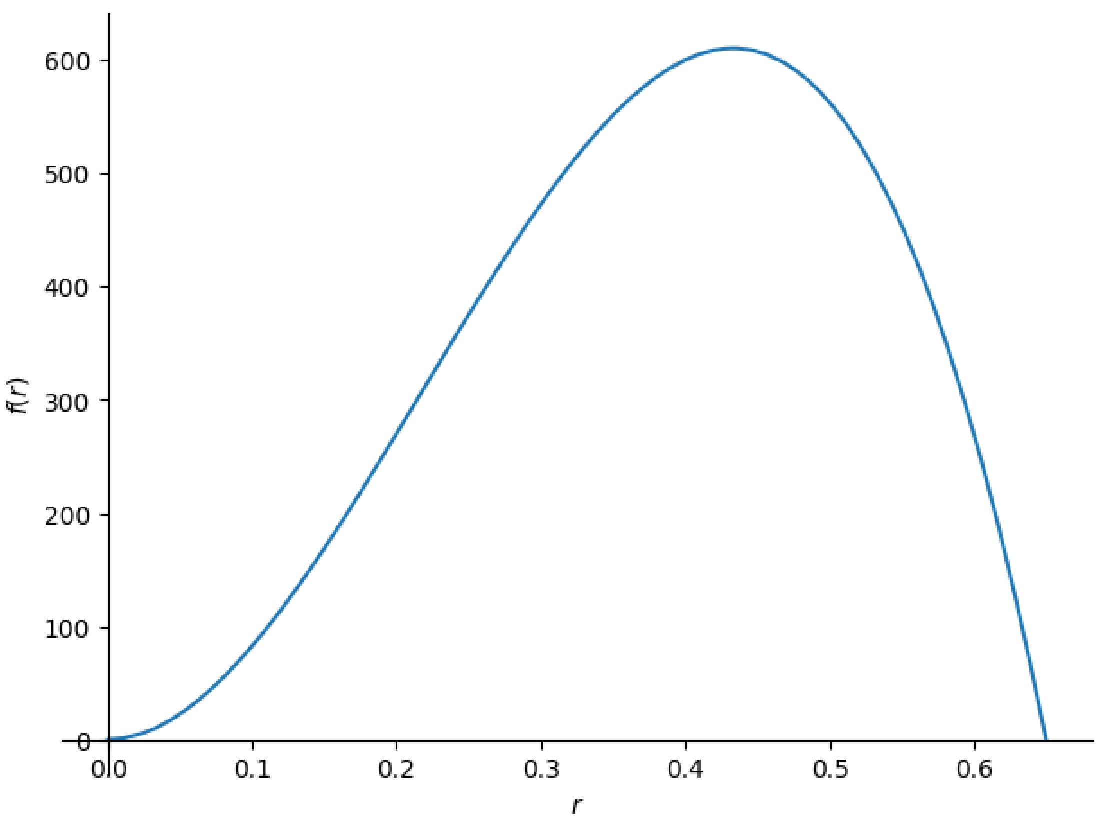
2c

[8]:
```
# what is the max vaule of the function and where does it occur
subs = [(r, r_crit), (r_o, 0.65), (k, 15000)]
v_max = v.subs(subs)
point = (r_crit.subs(subs), v_max)
print(f'The absolute max of v(r) = {v} is {v_max} at r = {r_crit}. The point is␣
↪{point}.')
```

The absolute max of v(r) = k*r**2*(-r + r_o) is 610.277777777778 at r = 2*r_o/3.
The point is (0.433333333333333, 610.277777777778).

2d

[9]:
```
# sketch the graph of v with the conditions from part c on the interval [0, r_o]
v_plot = plot(v.subs(subs[1:]), (r, 0, 0.65))
print(f'The plot of v(r) = {v.subs(subs[1:])} on the interval [0, r_o] is shown␣
↪above.')
```



The plot of v(r) = 15000*r**2*(0.65 - r) on the interval [0, r_o] is shown above.

0.1.3 Question 3

3a

[10]:
```
x = Symbol('x')
f = atan(x)
g = acot(x)
f_prime = diff(f, x)
g_prime = diff(g, x)
f_g_prime = f_prime + g_prime

print(f"The derivative of [f(x) + g(x)] is f'(x) + g'(x) if we apply the sum␣
↪rule. f(x) = {f} and g(x) = {g}. f'(x) = {f_prime} and g'(x) = {g_prime}.␣
↪Therefore, f'(x) + g'(x) = {f_g_prime}.")
```

The derivative of [f(x) + g(x)] is f'(x) + g'(x) if we apply the sum rule. f(x) = atan(x) and g(x) = acot(x). f'(x) = 1/(x**2 + 1) and g'(x) = -1/(x**2 + 1). Therefore, f'(x) + g'(x) = 0.

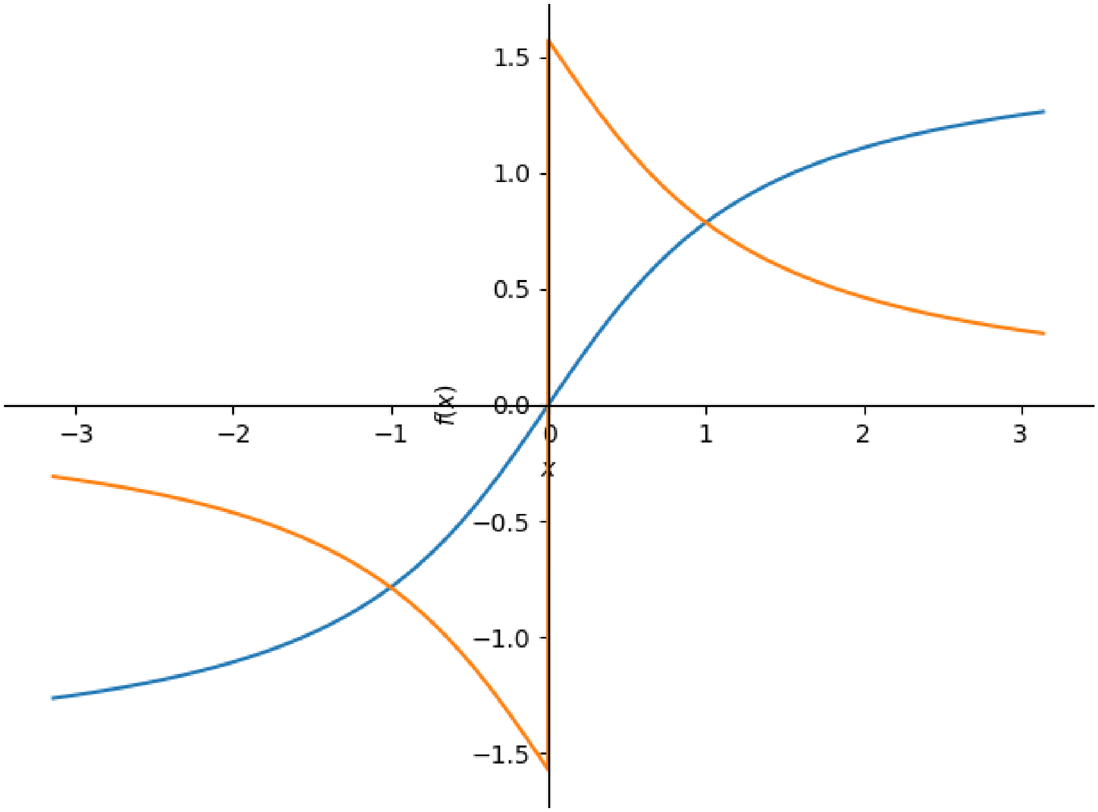
3b

[11]:
```
print(f'Since the derivative of f(x) + g(x) is 0 we can conclude that f(x) +␣
↪g(x) is a constant function or we cannot take the derivative of the function.
↪')
```

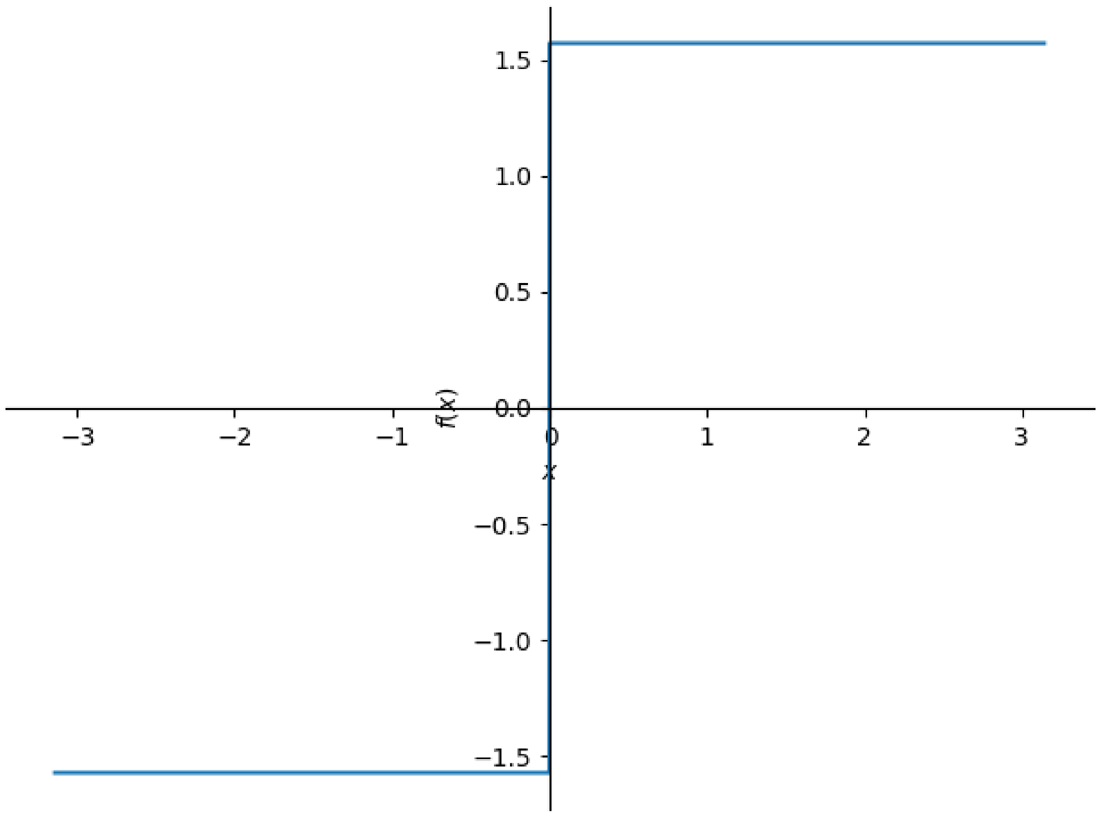
Since the derivative of f(x) + g(x) is 0 we can conclude that f(x) + g(x) is a constant function or we cannot take the derivative of the function.

3c

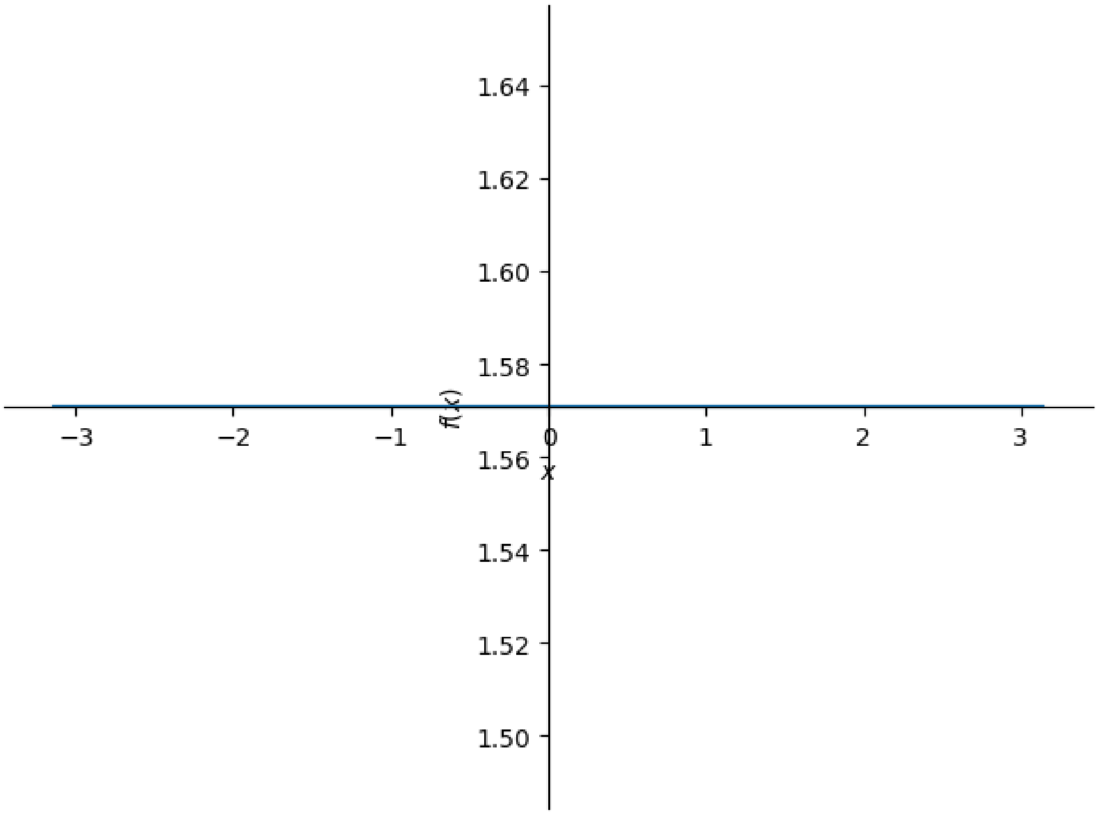
[12]:
```
f_plot = plot(f, (x, -pi, pi), show=False)
g_plot = plot(g, (x, -pi, pi), show=False)
f_plot.extend(g_plot)
f_plot.show()
print(f'The graph shows f(x) = {f} in blue and g(x) = {g} in orange.')
plot(atan(x)+acot(x), (x, -pi, pi))
print(f'The graph above shows f(x) + g(x) = {f + g} in blue.')
print(f'This graph above shows that the left side of the graph is a constant␣
↪negative value and the right side is a constant positive value.')
plot(pi/2, (x, -pi, pi))
print(f'The graph above shows that the graph of f(x) + g(x) is a constant␣
↪function if we look at when x > 0. This is also the graph that symbolab and␣
↪desmos show.')
```



The graph shows f(x) = atan(x) in blue and g(x) = acot(x) in orange.



The graph above shows f(x) + g(x) = acot(x) + atan(x) in blue.
This graph above shows that the left side of the graph is a constant negative value and the right side is a constant positive value.



The graph above shows that the graph of f(x) + g(x) is a constant function if we look at when x > 0. This is also the graph that symbolab and desmos show.

3d

[13]:
```
print(f'It makes sense that the graph of f(x) + g(x) would look like this since the graph of f(x) = {f} is a reflection of the graph of g(x) = {g} over the x-axis. The graph of f(x) + g(x) is the sum of the two graphs. We also know that acot(x) = pi/2 - atan(x) so acot(x) + atan(x) = pi/2 which is shown by the graph.')
```

```
It makes sense that the graph of f(x) + g(x) would look like this since the
graph of f(x) = atan(x) is a reflection of the graph of g(x) = acot(x) over the
x-axis. The graph of f(x) + g(x) is the sum of the two graphs. We also know that
acot(x) = pi/2 - atan(x) so acot(x) + atan(x) = pi/2 which is shown by the
graph.
```