Math 151- Python Lab 9

November 23, 2022

0.1 MATH 151 Lab 8

Section Number: 568

Members:

- Brighton Sikarskie
- Colton Hesser
- Gabriel Gonzalez
- Gabriel Cuevas

```
[1]: from sympy import *
     from sympy.plotting import (plot,plot_parametric)
```

0.1.1 Question 1

1a

```
[2]: # given: y = (1 + (26 / x))^x
     # rewrite in form y = f(x)/g(x)
     # f(x) = ln(1 + (26 / x))
     # g(x) = (1 / x)
     x = Symbol('x')
     f = ln(1 + (26 / x))
     g = (1 / x)
     y = f / g

     print(f"y = {(1 + (26/x))**x} can be rewritten as y = {exp(y)}")
     print(f"We can check for the limit of the exponent as x approaches infinity as␣
      ↪f(x) / g(x) where f(x) = ln(1 + (26 / x)) and g(x) = (1 / x)")
```

```
y = (1 + 26/x)**x can be rewritten as y = exp(x*log(1 + 26/x))
We can check for the limit of the exponent as x approaches infinity as f(x) /
g(x) where f(x) = ln(1 + (26 / x)) and g(x) = (1 / x)
```

1b

```
[3]: # find the limits of f and g as x -> oo
     f_lim = limit(f, x, oo)
     g_lim = limit(g, x, oo)
     print(f"The limit of {f} as x -> oo: {f_lim}")
```

1

```python
print(f"The limit of {g} as x -> oo: {g_lim}")
```

The limit of log(1 + 26/x) as x -> oo: 0
The limit of 1/x as x -> oo: 0

1c

```python
[4]:  # use L'Hopital's rule to find the limit of y as x -> oo
      print(f"Since the limit of {f} as x -> oo is {f_lim} and the limit of {g} as x␣
       ↪-> oo is {g_lim}, we can use L'Hopital's rule to find the limit of y as x ->␣
       ↪oo.")

      f_diff = diff(f, x)
      g_diff = diff(g, x)
      y_before = y
      y = simplify(f_diff / g_diff)

      print(f"For y = f(x) / g(x) where f(x) = {f} and g(x) = {g}, the derivative of␣
       ↪f(x) is {f_diff} and the derivative of g(x) is {g_diff}.")
      print(f"We can rewrite y = {y_before} as y = {f_diff} / {g_diff} or simplified␣
       ↪y = {y} in order to take the limit as stated by L'Hopital's rule.")

      numerator = numer(y)
      denominator = denom(y)
      print(f"The numerator of y is {numerator} and the denominator of y is␣
       ↪{denominator}.")

      numerator_lim = limit(numerator, x, oo)
      denominator_lim = limit(denominator, x, oo)

      print(f"The limit of the numerator as x -> oo is {numerator_lim} and the limit␣
       ↪of the denominator as x -> oo is {denominator_lim}.")

      numerator_diff = diff(numerator, x)
      denominator_diff = diff(denominator, x)
      print(f"The derivative of the numerator is {numerator_diff} and the derivative␣
       ↪of the denominator is {denominator_diff}.")

      numerator_diff_lim = limit(numerator_diff, x, oo)
      denominator_diff_lim = limit(denominator_diff, x, oo)

      print(f"The limit of the derivative of the numerator as x -> oo is␣
       ↪{numerator_diff_lim} and the limit of the derivative of the denominator as x␣
       ↪-> oo is {denominator_diff_lim}.")

      exponent = numerator_diff_lim / denominator_diff_lim
      print(f"The exponent of the limit of y as x -> oo is {exponent}.")
```

```
print(f"The limit of y as x -> oo is {exp(exponent)}.")
```

Since the limit of log(1 + 26/x) as x -> oo is 0 and the limit of 1/x as x -> oo
is 0, we can use L'Hopital's rule to find the limit of y as x -> oo.
For y = f(x) / g(x) where f(x) = log(1 + 26/x) and g(x) = 1/x, the derivative of
f(x) is -26/(x**2*(1 + 26/x)) and the derivative of g(x) is -1/x**2.
We can rewrite y = x*log(1 + 26/x) as y = -26/(x**2*(1 + 26/x)) / -1/x**2 or
simplified y = 26*x/(x + 26) in order to take the limit as stated by L'Hopital's
rule.
The numerator of y is 26*x and the denominator of y is x + 26.
The limit of the numerator as x -> oo is oo and the limit of the denominator as
x -> oo is oo.
The derivative of the numerator is 26 and the derivative of the denominator is
1.
The limit of the derivative of the numerator as x -> oo is 26 and the limit of
the derivative of the denominator as x -> oo is 1.
The exponent of the limit of y as x -> oo is 26.
The limit of y as x -> oo is exp(26).

1d

```
[5]: y_lim = limit(y, x, oo)
     print(f"Using the limit function in sympy, the limit of y as x -> oo is {y_lim}.
      ↪")
     print(f"We can see that the limit of y as x -> oo is {exp(exponent)} and the␣
      ↪limit of y as x -> oo using the limit function in sympy is {y_lim}.")
     print(f"Since they are the same value, we can conclude that the limit of y as x␣
      ↪-> oo is {exp(exponent)}.")
```

Using the limit function in sympy, the limit of y as x -> oo is 26.
We can see that the limit of y as x -> oo is exp(26) and the limit of y as x ->
oo using the limit function in sympy is 26.
Since they are the same value, we can conclude that the limit of y as x -> oo is
exp(26).

0.1.2 Question 2

2a

```
[6]: """
     Suppose a photograph with a width of 42 inches and a
     height of 50 inches is placed in the blue frame of the

     round billboard shown. The margins between the rect-
     angular frame and the picture are 10 inches at the top

     and bottom and 4 inches on the sides. What would the
     radius of the billboard be that would fit a photograph
     of such dimensions?
```

3

```python
"""

# given:
# width = 42 inches
# height = 50 inches
# top and bottom margins = 10 inches
# left and right margins = 4 inches

# the outer most rectangle
w = 42 + (2 * 4)
h = 50 + (2 * 10)
# from the center of the rectangle to the top right corner
# get the distance, which is the radius
d = sqrt((w / 2)**2 + (h / 2)**2)
print(f"The radius of the billboard is {d} inches.")
```

The radius of the billboard is 43.0116263352131 inches.

2b

```python
"""
A billboard with radius 55 inches is designed like the
figure to the left. Determine the dimensions, a and b,
of the largest picture that could fit in the frame.
"""

# R^2 = (b / 2)^2 + (a / 2)^2
# R^2 = 1/4 (b^2 + a^2)
# 4R^2 = b^2 + a^2
# b = sqrt(4R^2 - a^2)
a = Symbol('a')
b = sqrt(4 * (55 ** 2) - a ** 2)
# A = (a - 8)(b - 20)
A = (a - 8) * (b - 20)
A_diff = diff(A, a)
a_ = nsolve(A_diff, a, 70)
b = b.subs(a, a_)
a = a_

# check the radius
assert(sqrt((a / 2)**2 + (b / 2)**2) == 55)

print(f"The dimensions of the largest picture that could fit in the frame are␣
 ␣{a} inches by {b} inches.")
```

The dimensions of the largest picture that could fit in the frame are
74.4148590423809 inches by 81.0088189872101 inches.

0.1.3 Question 3

3a

```
[8]: # R_2 = 2R_1
     # V = 1/3*pi*h*(R_1^2 + R_2^2 + R_1*R_2)
     # SA = pi*(R_1 + R_2)*sqrt((R_2-R_1)^2 + h^2)+pi*(R_1^2 + R_2^2)

     # V = 590
     # minimize SA
     R_1, R_2, h = symbols('R_1 R_2 h')
     V = 1/3 * pi * h * (R_1**2 + R_2**2 + R_1 * R_2)
     SA = pi * (R_1 + R_2) * sqrt((R_2 - R_1)**2 + h**2) + pi * (R_1**2 + R_2**2)
     # R_2 = 2 * R_1
     V = V.subs(R_2, 2 * R_1)
     h_ = solve(V - 590, h)[0]
     SA = SA.subs(h, h_).subs(R_2, 2 * R_1)
     SA_diff = diff(SA, R_1)

     # plot the optimization function on R_1 x = [0, 5] with y [300, 800]
     plot(SA, (R_1, 0, 5), ylim=(300, 800))
     print(f"The graph of the optimization function is shown above.")
```
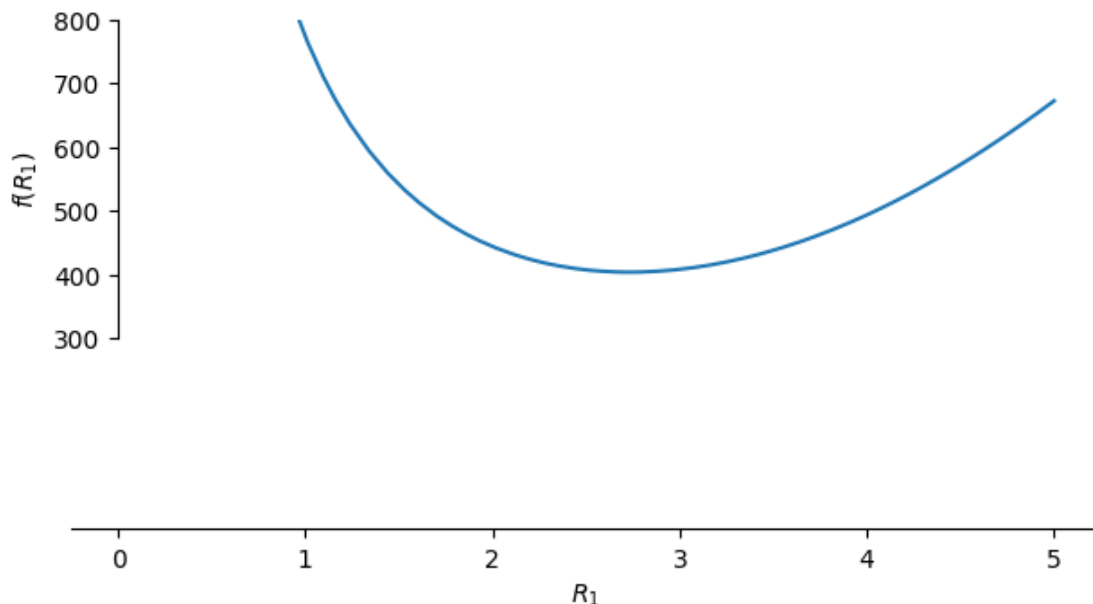

The graph of the optimization function is shown above.

3b

```
[9]: R_1_ = nsolve(SA_diff, R_1, 3)
     R_2_ = 2 * R_1_
```

5

```
h_ = h_.subs(R_1, R_1_).subs(R_2, R_2_)
print(f"The radius of the top of the cone is {R_1_} inches, the radius of the␣
 ↪bottom of the cone is {R_2_} inches, and the height of the cone is {h_}.")
```

The radius of the top of the cone is 2.73235307040265 inches, the radius of the
bottom of the cone is 5.46470614080531 inches, and the height of the cone is
10.7808181056803.

0.1.4 Question 4

4a

```
[10]: # given f''(x) = 5 / (x + 1)^2
      # f'(0) = 3
      # f(0) = 9
      # find f'(x) and f(x)

      f_double_prime = 5 / ((x + 1) ** 2)
      f_prime = integrate(f_double_prime, x)

      # solve for C
      C = symbols('C')
      C = solve(f_prime.subs(x, 0) - 3 + C, C)[0]
      f_prime = f_prime + C

      print(f"The anti-derivative of f''(x) is f'(x) = {f_prime}.")

      f = integrate(f_prime, x)

      # solve for C
      C = symbols('C')
      C = solve(f.subs(x, 0) - 9 + C, C)[0]
      f = f + C

      print(f"The anti-derivative of f'(x) is f(x) = {f}.")
```

The anti-derivative of f''(x) is f'(x) = 8 - 5/(x + 1).
The anti-derivative of f'(x) is f(x) = 8*x - 5*log(x + 1) + 9.

4b

```
[11]: # f(1) = f(4) = 10
      # find f(x)

      f_prime_prime = 5 / ((x + 1) ** 2)
      f_prime = integrate(f_prime_prime, x)
      x, C, D = symbols('x C D')
      f = integrate(f_prime, x) + C*x + D
```

```python
# system of equations
eq1 = f.subs(x, 1) - 10
eq2 = f.subs(x, 4) - 10

# solve eq1 in terms of C
eq1 = solve(eq1, C)[0]

# plug eq1 into eq2
eq2 = eq2.subs(C, eq1)

# solve eq2 in terms of D
eq2 = solve(eq2, D)[0]

# plug eq1 and eq2 into f
f = f.subs(C, eq1)
f = f.subs(D, eq2)

print(f"The function f(x) is {f.simplify()}.")
```

The function f(x) is -5*x*log(2)/3 + 5*x*log(5)/3 - 5*log(x + 1) - 5*log(5)/3 + 20*log(2)/3 + 10.