MATH 151 Lab 5

Section Number: 568

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In [1]:
```
from sympy import *
from sympy.plotting import (plot,plot_parametric)
%matplotlib inline
```

Question 1

1a

In [2]:
```
x = Symbol('x')
fo = exp(x) * (1 + x ** 2)
for i in range(1, 9):
    f = diff(fo, x, i).simplify()
    if i == 1:
        print(f'The {i}st derivative of {fo} is {f}')
    else:
        print(f'The {i}th derivative of {fo} is {f}')
```

```
The 1st derivative of (x**2 + 1)*exp(x) is (x**2 + 2*x + 1)*exp(x)
The 2th derivative of (x**2 + 1)*exp(x) is (x**2 + 4*x + 3)*exp(x)
The 3th derivative of (x**2 + 1)*exp(x) is (x**2 + 6*x + 7)*exp(x)
The 4th derivative of (x**2 + 1)*exp(x) is (x**2 + 8*x + 13)*exp(x)
The 5th derivative of (x**2 + 1)*exp(x) is (x**2 + 10*x + 21)*exp(x)
The 6th derivative of (x**2 + 1)*exp(x) is (x**2 + 12*x + 31)*exp(x)
The 7th derivative of (x**2 + 1)*exp(x) is (x**2 + 14*x + 43)*exp(x)
The 8th derivative of (x**2 + 1)*exp(x) is (x**2 + 16*x + 57)*exp(x)
```

1b

In [3]:

```python
n = symbols('n')
equation = 2 * n * x * exp(x) + (x ** 2 + 1) * exp(x) + (n ** 2 - n) * exp(x)
for i in range(1, 50):
    f = diff(fo, x, i).simplify()
    try:
        assert(equation.subs(n, i).simplify() == f)
    except AssertionError:
        print(f'The {i}th derivative of {fo} is {f}',
              f'and not {equation.subs(n, i).simplify()}')
        break
print(f'The nth derivative of {fo} can be found using {equation}')
```

```
The nth derivative of (x**2 + 1)*exp(x) can be found using 2*n*x*exp(x) + (n**2 - n)*exp(x) + (x**2 + 1)*e
xp(x)
```

1c

In [4]:

```python
print(f'The 50th derivative of {fo} is {diff(fo, x, 50).simplify()}')
print(f'Using our formula we can verify that the 50th derivative',
      f'of {fo} is {equation.subs(n, 50).simplify()}')
```

```
The 50th derivative of (x**2 + 1)*exp(x) is (x**2 + 100*x + 2451)*exp(x)
Using our formula we can verify that the 50th derivative of (x**2 + 1)*exp(x) is (x**2 + 100*x + 2451)*exp
(x)
```

Question 2

2a

In [5]:
```
k, t = symbols('k t')

y = cos(k * t)
y2 = diff(y, t, 2)
equation = 4 * y2 + 25 * y
solutions = solve(equation, k)

print(f'The values of k that make the function y = {y}',
      f'a solution to the differential equation {equation} are {solutions}')
```

```
The values of k that make the function y = cos(k*t) a solution to the differential equation -4*k**2*cos(k*
t) + 25*cos(k*t) are [-5/2, 5/2, pi/(2*t), 3*pi/(2*t)]
```

2b

In [6]:
```
for s in solutions:
    A, B = symbols('A B')
    y = A * sin(s * t) + B * cos(s * t)
    y2 = diff(y, t, 2)
    equation = 4 * y2 + 25 * y
    print(f'For k = {s}, every member of the family of functions',
          f'y = {y} is also a solution to the differential equation {equation}')
```

```
For k = -5/2, every member of the family of functions y = -A*sin(5*t/2) + B*cos(5*t/2) is also a solution
to the differential equation -25*A*sin(5*t/2) + 25*B*cos(5*t/2) + 25*(A*sin(5*t/2) - B*cos(5*t/2))
For k = 5/2, every member of the family of functions y = A*sin(5*t/2) + B*cos(5*t/2) is also a solution to
the differential equation 25*A*sin(5*t/2) + 25*B*cos(5*t/2) - 25*(A*sin(5*t/2) + B*cos(5*t/2))
For k = pi/(2*t), every member of the family of functions y = A is also a solution to the differential equ
ation 25*A
For k = 3*pi/(2*t), every member of the family of functions y = -A is also a solution to the differential
equation -25*A
```

Question 3

3a

In [7]:
```
t = symbols('t')
g = ((t - 2) / (2 * t + 1)) ** 9
g_diff = diff(g, t)
print(f'The derivative of {g} is {g_diff}')
```

```
The derivative of (t - 2)**9/(2*t + 1)**9 is -18*(t - 2)**9/(2*t + 1)**10 + 9*(t - 2)**8/(2*t + 1)**9
```

3b

```
In [8]: g_simple = simplify(g_diff)
        print(f'The simplified derivative of {g} is {g_simple}')
```

```
The simplified derivative of (t - 2)**9/(2*t + 1)**9 is 45*(t - 2)**8/(2*t + 1)**10
```

3c

```
In [9]: horizontal_tangent_lines = solve(g_diff, t)[0]
        print(f'The horizontal tangent line of {g} is at',
              f't = {horizontal_tangent_lines}')
        print(f'The equation of the horizontal tangent line',
              f'of {g} is x = {g.subs(t, horizontal_tangent_lines)}')
```

```
The horizontal tangent line of (t - 2)**9/(2*t + 1)**9 is at t = 2
The equation of the horizontal tangent line of (t - 2)**9/(2*t + 1)**9 is x = 0
```

3d

```
In [10]: t = symbols('t')
         f = (2 * t + 1) ** 5 * (t ** 2 - t + 2) ** 4
         f_diff = diff(f, t)
         print(f'The derivative of {f} is {f_diff}')
```

```
The derivative of (2*t + 1)**5*(t**2 - t + 2)**4 is (2*t + 1)**5*(8*t - 4)*(t**2 - t + 2)**3 + 10*(2*t +
1)**4*(t**2 - t + 2)**4
```

3e

```
In [11]: print(f'The simplified derivative of {f} is {simplify(f_diff)}')
```

```
The simplified derivative of (2*t + 1)**5*(t**2 - t + 2)**4 is (2*t + 1)**4*(t**2 - t + 2)**3*(10*t**2 - 1
0*t + 4*(2*t - 1)*(2*t + 1) + 20)
```

3f

```
In [12]: print(f'The factored derivative of {f} is {factor(f_diff)}')
```

```
The factored derivative of (2*t + 1)**5*(t**2 - t + 2)**4 is 2*(2*t + 1)**4*(t**2 - t + 2)**3*(13*t**2 - 5
*t + 8)
```

3g

```
In [13]: print(f'The simplified version of the derivative of {f} that',
               f'would be best used for finding horizontal tangent lines',
               f'is {factor(f_diff)} since it is the simplest form of the',
               f'derivative of {f}.')
         print(f'The horizontal tangent lines of {f} are at t = {solve(f_diff, t)}')
```

```
The simplified version of the derivative of (2*t + 1)**5*(t**2 - t + 2)**4 that would be best used for fin
ding horizontal tangent lines is 2*(2*t + 1)**4*(t**2 - t + 2)**3*(13*t**2 - 5*t + 8) since it is the simp
lest form of the derivative of (2*t + 1)**5*(t**2 - t + 2)**4.
The horizontal tangent lines of (2*t + 1)**5*(t**2 - t + 2)**4 are at t = [-1/2, 5/26 - sqrt(391)*I/26, 5/
26 + sqrt(391)*I/26, 1/2 - sqrt(7)*I/2, 1/2 + sqrt(7)*I/2]
```

Question 4

4a

```
In [14]: mu, theta, W = symbols('μ θ W')
         F = (mu * W) / (mu * sin(theta) + cos(theta))

         print(f'The rate of change of F = {F} with respect',
               f'to {theta} is {diff(F, theta)}')
```

```
The rate of change of F = W*μ/(μ*sin(θ) + cos(θ)) with respect to θ is W*μ*(-μ*cos(θ) + sin(θ))/(μ*sin(θ)
+ cos(θ))**2
```

4b

```
In [15]: formula = diff(F, theta)
         print(f'The formula for when the rate of change of',
               f'F = {F} equals 0 is {formula}')
         print(f'When the rate of change of F = {F} equals 0',
               f', we get {theta} = {solve(formula, theta)}')
```

The formula for when the rate of change of F = W*μ/(μ*sin(θ) + cos(θ)) equals 0 is W*μ*(-μ*cos(θ) + sin(θ))/(μ*sin(θ) + cos(θ))**2
When the rate of change of F = W*μ/(μ*sin(θ) + cos(θ)) equals 0 , we get θ = [2*atan((sqrt(μ**2 + 1) - 1)/μ), -2*atan((sqrt(μ**2 + 1) + 1)/μ)]

4c

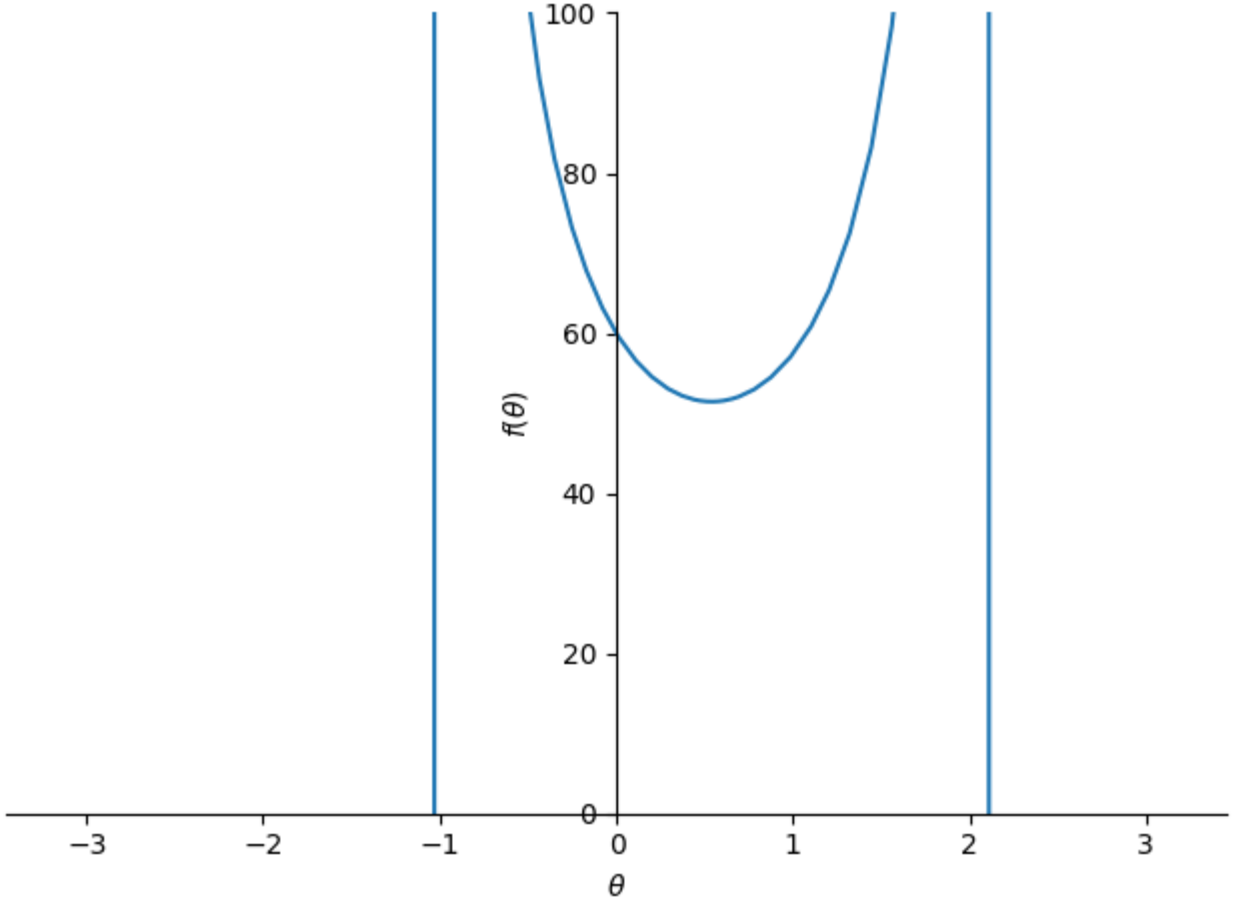
In [16]:

```
F = F.subs(W, 100).subs(mu, 0.6)

print(f'The graph shows below shows the rate of change',
      f'of F = {F} with respect to {theta} when W = 100',
      f'and μ = 0.6')
print(f'I estimate that the rate of change of F = {F}',
      f'with respect to {theta} is 0 when θ = 0.5')

plot(F, (theta, -pi, pi), ylim=(0, 100))
```

```
The graph shows below shows the rate of change of F = 60.0/(0.6*sin(θ) + cos(θ)) with respect to θ when W = 100 and μ = 0.6
I estimate that the rate of change of F = 60.0/(0.6*sin(θ) + cos(θ)) with respect to θ is 0 when θ = 0.5
```



Out[16]: <sympy.plotting.plot.Plot at 0x7ff4cb3aa7d0>

4d

In [17]:
```
print(f'I can verify my estimate of 0.5 being the value of',
      f'{theta} when {formula} equals 0 by solving for {theta}',
      f'in the formula {formula} and getting',
      f'{solve(formula, theta)[0]}')
```

```
I can verify my estimate of 0.5 being the value of θ when W*μ*(-μ*cos(θ) + sin(θ))/(μ*sin(θ) + cos(θ))**2
equals 0 by solving for θ in the formula W*μ*(-μ*cos(θ) + sin(θ))/(μ*sin(θ) + cos(θ))**2 and getting 2*ata
n((sqrt(μ**2 + 1) - 1)/μ)
```