

Math 151 – Python Lab 7

November 6, 2022

0.1 MATH 151 Lab 7

Section Number: 568

Members:

- Brighton Sikarskie
- Colton Hesser
- Gabriel Gonzalez
- Gabriel Cuevas

```
[1]: from sympy import *  
from sympy.plotting import (plot, plot_parametric)
```

0.1.1 Question 1

1a

```
[2]: # piece wise function  
# 8 - x ** 2 if x < 0  
# 5 e ** ( -(x - 2) / 2) ** 2) + x if x >= 0  
# find critical vaules of the function  
x = Symbol('x')  
f_left = 8 - x ** 2  
f_right = 5 * exp(-(x - 2) / 2) ** 2) + x  
f_left_prime = diff(f_left, x)  
f_right_prime = diff(f_right, x)  
right_critical = [nsolve(f_right_prime, x, 2)] + [nsolve(f_right_prime, x, 5)]  
critical = right_critical  
points = [(x, f_right.subs(x, x)) for x in right_critical]  
p = Piecewise((f_left, x < 0), (f_right, x >= 0))  
  
print(f'The critical values for f(x) = {p} are x = {critical}. The points are_  
↪{points}.')
```

The critical values for $f(x) = \text{Piecewise}((8 - x^2, x < 0), (x + 5\exp(-(x/2 - 1)^2), \text{True}))$ are $x = [2.41784619385985, 4.78643377270907]$. The points are $[(2.41784619385985, x + 5\exp(-(x/2 - 1)^2)), (4.78643377270907, x + 5\exp(-(x/2 - 1)^2))]$.

1b

```
[3]: # find the absolute extrema of the function on the interval [-5, 5]
maxin_candidates = [-5, 5] + critical
yvals = [p.subs(x, i) for i in maxin_candidates]
maxy = max(yvals)
miny = min(yvals)
maxx = maxin_candidates[yvals.index(maxy)]
minx = maxin_candidates[yvals.index(miny)]
print(f'The absolute extrema of f(x) = {p} on the interval [-5, 5] are y =
↳ {maxy} and y = {miny}. The points are ({maxx}, {maxy}) and ({minx}, {miny}).
↳')
```

The absolute extrema of $f(x) = \text{Piecewise}((8 - x^2, x < 0), (x + 5\exp(-(x/2 - 1)^2), \text{True}))$ on the interval $[-5, 5]$ are $y = 7.20429639824017$ and $y = -17$. The points are $(2.41784619385985, 7.20429639824017)$ and $(-5, -17)$.

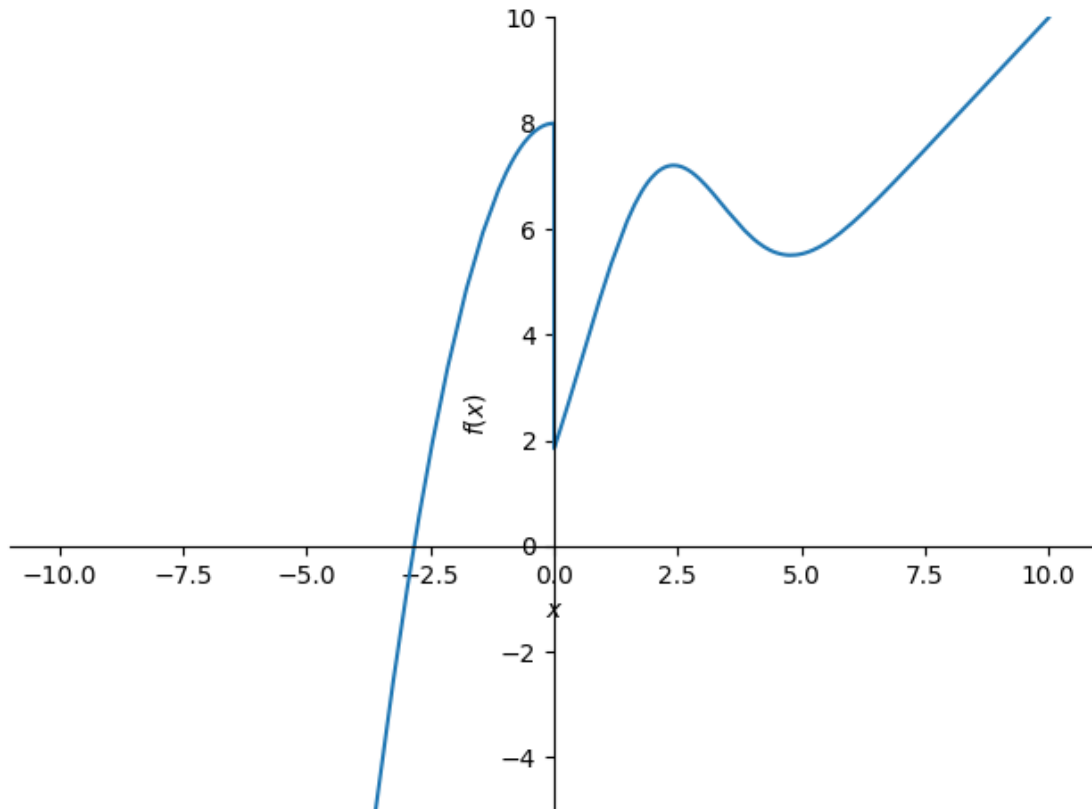
1c

```
[4]: # find the absolute extrema of the function on the interval [-10, 10]
maxin_candidates = [-10, 10] + critical
yvals = [p.subs(x, i) for i in maxin_candidates]
maxy = max(yvals)
miny = min(yvals)
maxx = maxin_candidates[yvals.index(maxy)]
minx = maxin_candidates[yvals.index(miny)]
print(f'The absolute extrema of f(x) = {p} on the interval [-10, 10] are y =
↳ {maxy} and y = {miny}. The points are ({maxx}, {maxy}) and ({minx}, {miny}).
↳')
```

The absolute extrema of $f(x) = \text{Piecewise}((8 - x^2, x < 0), (x + 5\exp(-(x/2 - 1)^2), \text{True}))$ on the interval $[-10, 10]$ are $y = 5\exp(-16) + 10$ and $y = -92$. The points are $(10, 5\exp(-16) + 10)$ and $(-10, -92)$.

1d

```
[5]: # plot the function on the interval [-10, 10]
main_plot = plot(p, (x, -10, 10), ylim=(-5, 10))
print(f'The plot of f(x) = {p} on the interval [-10, 10] is shown above.')
```



The plot of $f(x) = \text{Piecewise}((8 - x^2, x < 0), (x + 5\exp(-(x/2 - 1)^2), \text{True}))$ on the interval $[-10, 10]$ is shown above.

0.1.2 Question 2

2a

```
[6]: # v(r) = k (r_o - r) * r ** 2
      # find the value of r in the interval [1/2 r_o, r_o]
      # where v has an absolute maximum
      k, r, r_o = symbols('k r r_o')
      v = k * (r_o - r) * r ** 2
      v_prime = diff(v, r)
      r_crit = solve(v_prime, r)[1]
      print(f'The absolute maximum of v(r) = {v} is at r = {r_crit}.')
```

The absolute maximum of $v(r) = k*r**2*(-r + r_o)$ is at $r = 2*r_o/3$.

2b

```
[7]: v_max = v.subs(r, r_crit)
      point = (r_crit, v_max)
```

```
print(f'The absolute max of  $v(r) = \{v\}$  is  $\{v_{\max}\}$  at  $r = \{r_{\text{crit}}\}$ . The point is  $\hookrightarrow \{point\}$ .'
```

The absolute max of $v(r) = k*r**2*(-r + r_o)$ is $4*k*r_o**3/27$ at $r = 2*r_o/3$.
The point is $(2*r_o/3, 4*k*r_o**3/27)$.

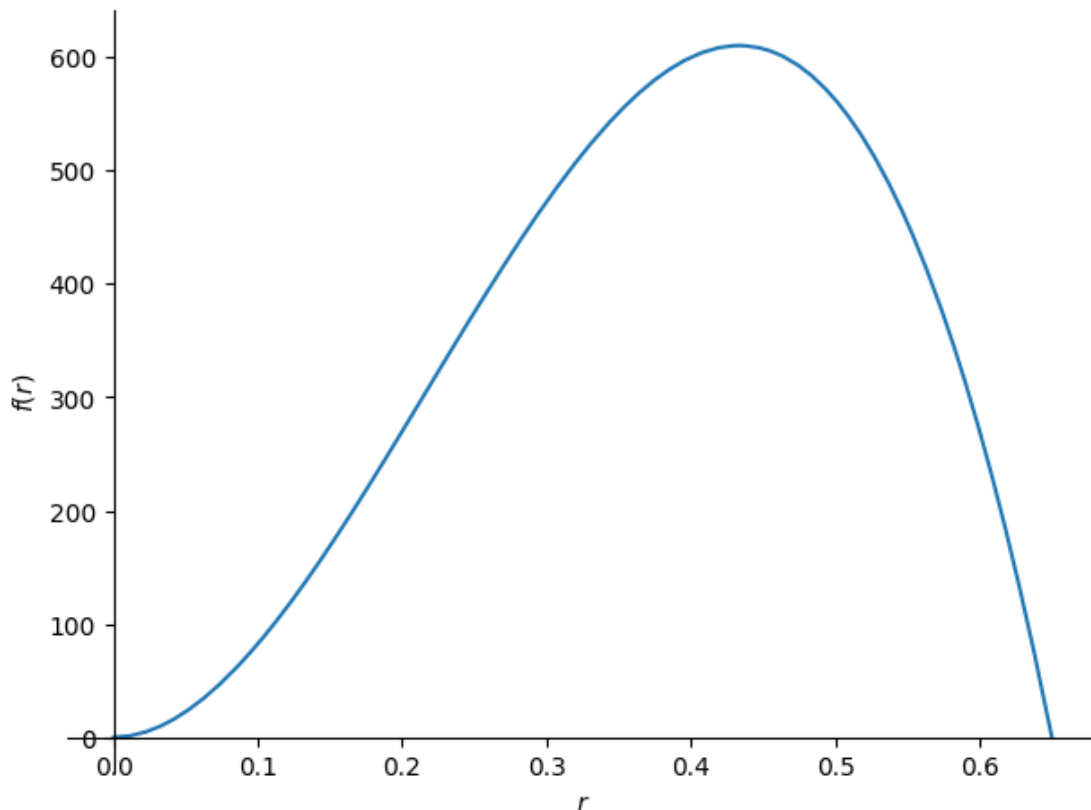
2c

```
[8]: # what is the max vaule of the function and where does it occur
subs = [(r, r_crit), (r_o, 0.65), (k, 15000)]
v_max = v.subs(subs)
point = (r_crit.subs(subs), v_max)
print(f'The absolute max of  $v(r) = \{v\}$  is  $\{v_{\max}\}$  at  $r = \{r_{\text{crit}}\}$ . The point is  $\hookrightarrow \{point\}$ .'
```

The absolute max of $v(r) = k*r**2*(-r + r_o)$ is 610.2777777777778 at $r = 2*r_o/3$.
The point is $(0.4333333333333333, 610.2777777777778)$.

2d

```
[9]: # sketch the graph of  $v$  with the conditions from part c on the interval  $[0, r_o]$ 
v_plot = plot(v.subs(subs[1:]), (r, 0, 0.65))
print(f'The plot of  $v(r) = \{v.subs(subs[1:])\}$  on the interval  $[0, r_o]$  is shown  $\hookrightarrow$  above.'
```



The plot of $v(r) = 15000r^2(0.65 - r)$ on the interval $[0, r_o]$ is shown above.

0.1.3 Question 3

3a

```
[10]: x = Symbol('x')
      f = atan(x)
      g = acot(x)
      f_prime = diff(f, x)
      g_prime = diff(g, x)
      f_g_prime = f_prime + g_prime

      print(f"The derivative of [f(x) + g(x)] is f'(x) + g'(x) if we apply the sum
      ↳rule. f(x) = {f} and g(x) = {g}. f'(x) = {f_prime} and g'(x) = {g_prime}.
      ↳Therefore, f'(x) + g'(x) = {f_g_prime}." )
```

The derivative of $[f(x) + g(x)]$ is $f'(x) + g'(x)$ if we apply the sum rule. $f(x) = \arctan(x)$ and $g(x) = \operatorname{acot}(x)$. $f'(x) = 1/(x^2 + 1)$ and $g'(x) = -1/(x^2 + 1)$. Therefore, $f'(x) + g'(x) = 0$.

3b

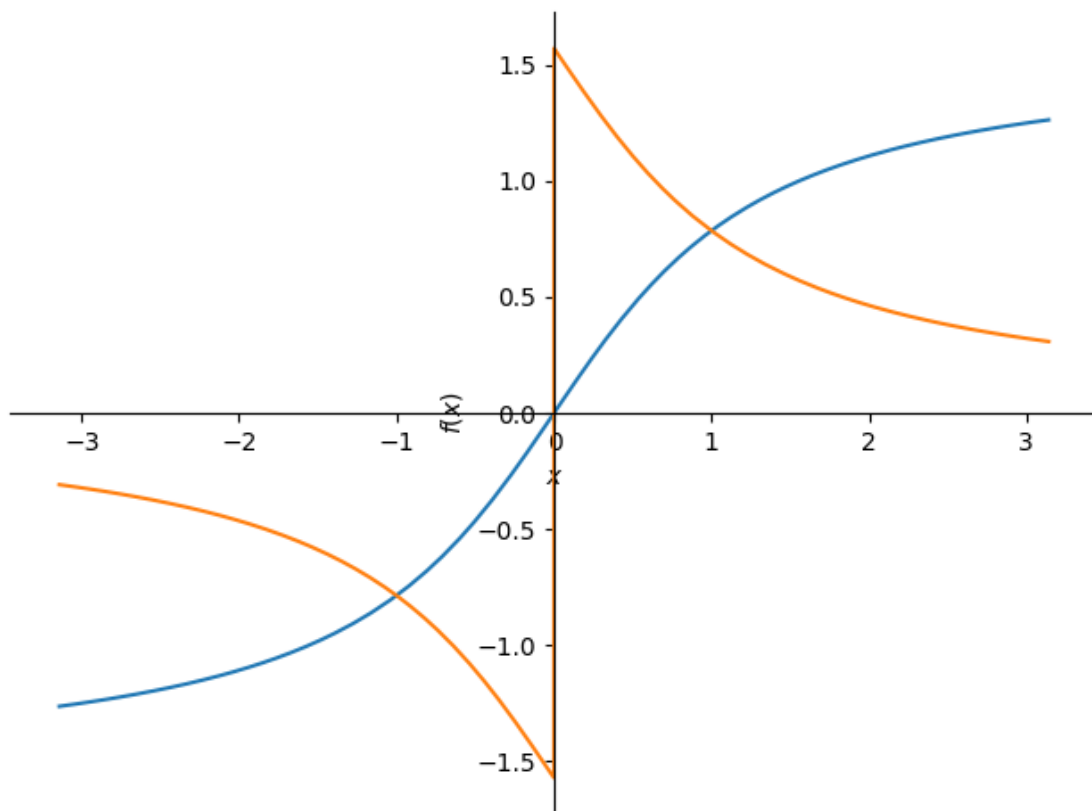
```
[11]: print(f'Since the derivative of f(x) + g(x) is 0 we can conclude that f(x) +
      ↳g(x) is a constant function or we cannot take the derivative of the function.
      ↳')
```

Since the derivative of $f(x) + g(x)$ is 0 we can conclude that $f(x) + g(x)$ is a constant function or we cannot take the derivative of the function.

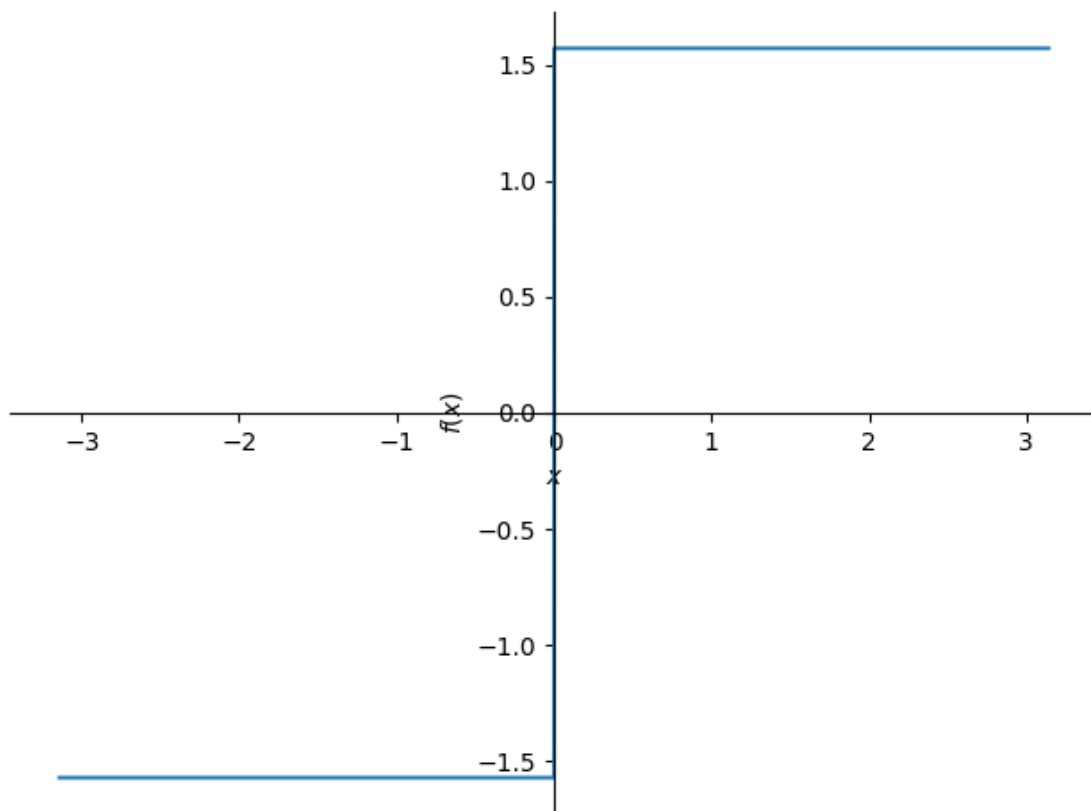
3c

```
[12]: f_plot = plot(f, (x, -pi, pi), show=False)
      g_plot = plot(g, (x, -pi, pi), show=False)
      f_plot.extend(g_plot)
      f_plot.show()

      print(f'The graph shows f(x) = {f} in blue and g(x) = {g} in orange.')
      plot(atan(x)+acot(x), (x, -pi, pi))
      print(f'The graph above shows f(x) + g(x) = {f + g} in blue.')
      print(f'This graph above shows that the left side of the graph is a constant
      ↳negative value and the right side is a constant positive value.')
      plot(pi/2, (x, -pi, pi))
      print(f'The graph above shows that the graph of f(x) + g(x) is a constant
      ↳function if we look at when x > 0. This is also the graph that symbolab and
      ↳desmos show.')
```

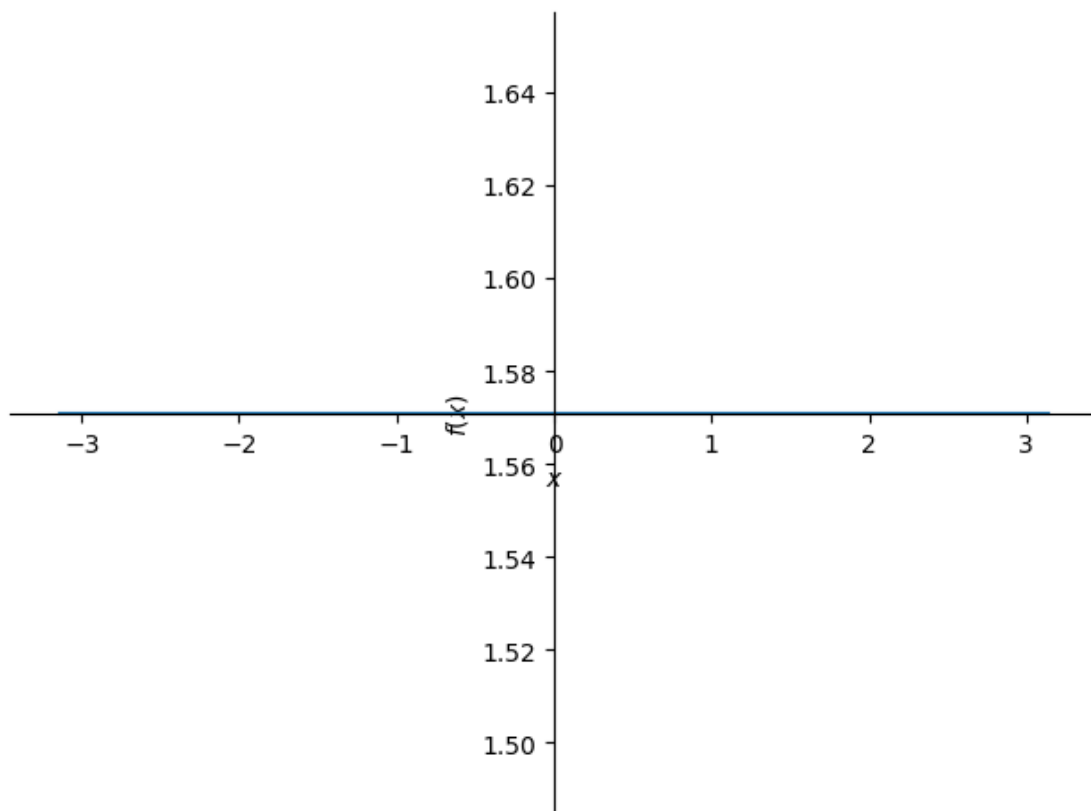


The graph shows $f(x) = \text{atan}(x)$ in blue and $g(x) = \text{acot}(x)$ in orange.



The graph above shows $f(x) + g(x) = \text{acot}(x) + \text{atan}(x)$ in blue.

This graph above shows that the left side of the graph is a constant negative value and the right side is a constant positive value.



The graph above shows that the graph of $f(x) + g(x)$ is a constant function if we look at when $x > 0$. This is also the graph that symbolab and desmos show.

3d

```
[13]: print(f'It makes sense that the graph of  $f(x) + g(x)$  would look like this since
↳ the graph of  $f(x) = \{f\}$  is a reflection of the graph of  $g(x) = \{g\}$  over the
↳  $x$ -axis. The graph of  $f(x) + g(x)$  is the sum of the two graphs. We also know
↳ that  $\text{acot}(x) = \pi/2 - \text{atan}(x)$  so  $\text{acot}(x) + \text{atan}(x) = \pi/2$  which is shown by
↳ the graph.')
```

It makes sense that the graph of $f(x) + g(x)$ would look like this since the graph of $f(x) = \text{atan}(x)$ is a reflection of the graph of $g(x) = \text{acot}(x)$ over the x -axis. The graph of $f(x) + g(x)$ is the sum of the two graphs. We also know that $\text{acot}(x) = \pi/2 - \text{atan}(x)$ so $\text{acot}(x) + \text{atan}(x) = \pi/2$ which is shown by the graph.