# Math 151 – Python Lab 3

September 16, 2022

## 0.1 MATH 151 Lab 3

Section Number: 568

Members:

- Brighton Sikarskie
- Colton Hesser
- Gabriel Gonzalez
- Gabriel Cuevas

[1]:
```
from sympy import *
from sympy.plotting import (plot,plot_parametric)
```

### 0.1.1 Question 1

[2]:
```
x = symbols('x')
f = x ** 5 + 4 * x ** 3 - 2 * x ** 2 + 8 * x - 1
lower_limit = limit(f, x, 0)
upper_limit = limit(f, x, 1)
```

**1a**

[3]:
```
root = nsolve(f, x, 0)

print(f"To to show that there is a root to the function on the interval [0, 1]")
print(f"The limit of the function as x approaches 0 is {lower_limit}")
print(f"The limit of the function as x approaches 1 is {upper_limit}")
print(f"Therefore, there is a root to the function on the interval [0, 1] since␣
 ↪the Intermediate Value Theorem applies.")
print(f"IVT applies since the function is continuous on the interval [0, 1] and␣
 ↪the function has a different sign at the endpoints of the interval, meaning␣
 ↪if we let 'a' be the first endpoint and 'b' be the second endpoint and 'K'␣
 ↪is some number between 'a' and 'b' then in our case f(a) < K < f(b) and a <␣
 ↪c < b where c is the root of the function meaning f(c) = K and K = 0.")
print(f"We know that the function is continuous on the interval [0, 1] since␣
 ↪the function is a polynomial and all polynomials are continuous on their␣
 ↪domain.")
```

```
To to show that there is a root to the function on the interval [0, 1]
The limit of the function as x approaches 0 is -1
The limit of the function as x approaches 1 is 10
Therefore, there is a root to the function on the interval [0, 1] since the
Intermediate Value Theorem applies.
IVT applies since the function is continuous on the interval [0, 1] and the
function has a different sign at the endpoints of the interval, meaning if we
let 'a' be the first endpoint and 'b' be the second endpoint and 'K' is some
number between 'a' and 'b' then in our case f(a) < K < f(b) and a < c < b where
c is the root of the function meaning f(c) = K and K = 0.
We know that the function is continuous on the interval [0, 1] since the
function is a polynomial and all polynomials are continuous on their domain.
```

**1b**

[4]:
```python
root = nsolve(f, x, 0)
print(f"The root of the function {f} is {root}.")
```

```
The root of the function x**5 + 4*x**3 - 2*x**2 + 8*x - 1 is 0.128044891411745.
```

### 0.1.2 Question 2

**2a**

[5]:
```python
f1 = 2 * x - 3
break_point1_left = limit(f1, x, 3)

f2 = 4 * x - x ** 2
break_point1_right = limit(f2, x, 3)
break_point2_left = limit(f2, x, 4)

f3 = (x ** 2 - 6 * x + 8) / (x - 4)
break_point2_right = limit(f3, x, 4)
break_point3_left = limit(f3, x, 5)

f4 = exp((x - 4) * ln(3))
break_point3_right = limit(f4, x, 5)


print(f"Let f(x) = {f1} on the interval [0, 3], {f2} on the interval (3, 4],␣
↪{f3} on the interval (4, 5), and f{4} on the interval [5, oo).", end="\n\n")

print(f"The limit of f(x) as x approaches 3 from the left is␣
↪{break_point1_left}")
print(f"The limit of f(x) as x approaches 3 from the right is␣
↪{break_point1_right}")
print("Therefore, f(x) is continuous at x = 3", end = "\n\n")
```

```
print(f"The limit of f(x) as x approaches 4 from the left is␣
↪{break_point2_left}")
print(f"The limit of f(x) as x approaches 4 from the right is␣
↪{break_point2_right}")
print("Therefore, f(x) is not continuous at x = 4", end = "\n\n")

print(f"The limit of f(x) as x approaches 5 from the left is␣
↪{break_point3_left}")
print(f"The limit of f(x) as x approaches 5 from the right is␣
↪{break_point3_right}")
print("Therefore, f(x) is continuous at x = 5", end = "\n\n")
```

```
Let f(x) = 2*x - 3 on the interval [0, 3], -x**2 + 4*x on the interval (3, 4],
(x**2 - 6*x + 8)/(x - 4) on the interval (4, 5), and f4 on the interval [5, oo).

The limit of f(x) as x approaches 3 from the left is 3
The limit of f(x) as x approaches 3 from the right is 3
Therefore, f(x) is continuous at x = 3

The limit of f(x) as x approaches 4 from the left is 0
The limit of f(x) as x approaches 4 from the right is 2
Therefore, f(x) is not continuous at x = 4

The limit of f(x) as x approaches 5 from the left is 3
The limit of f(x) as x approaches 5 from the right is 3
Therefore, f(x) is continuous at x = 5
```

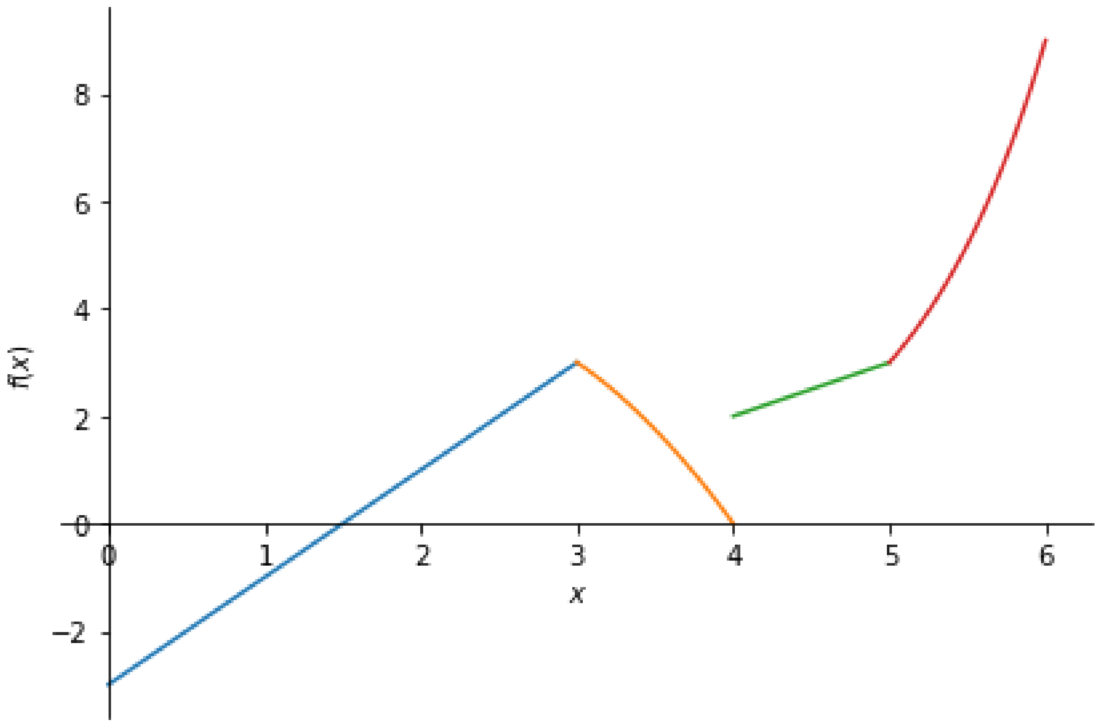
**2b**

[6]:

```
print("The graph of f(x) is shown below.")
print("The graph shows that f(x) is continuous at x = 3 and x = 5, but not at x␣
↪= 4.")

plot(
    (f1, (x, 0, 3)),
    (f2, (x, 3, 4)),
    (f3, (x, 4, 5)),
    (f4, (x, 5, 6)),
)
```

```
The graph of f(x) is shown below.
The graph shows that f(x) is continuous at x = 3 and x = 5, but not at x = 4.
```




[6]: <sympy.plotting.plot.Plot at 0x7fb67a9b35b0>

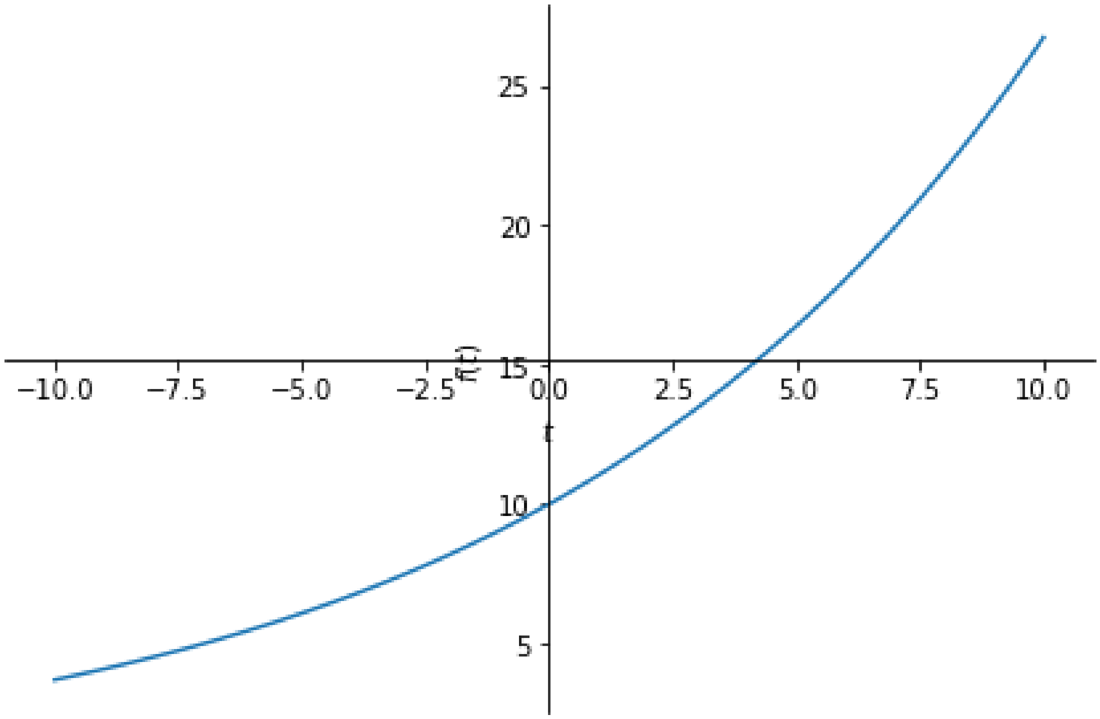
### 0.1.3 Question 3

**3a**

[7]:
```
(K, P, t) = symbols('K P t')
r = 0.1
P = 10
p = (K * P) / (P + (K - P) * exp(-r * t))

print(f"The graph of P(t) = {p} where K = 1000 is shown below.")
plot(p.subs(K, 1000))
```

```
The graph of P(t) = 10*K/((K - 10)*exp(-0.1*t) + 10) where K = 1000 is shown
below.
```




[7]: <sympy.plotting.plot.Plot at 0x7fb63bb93160>

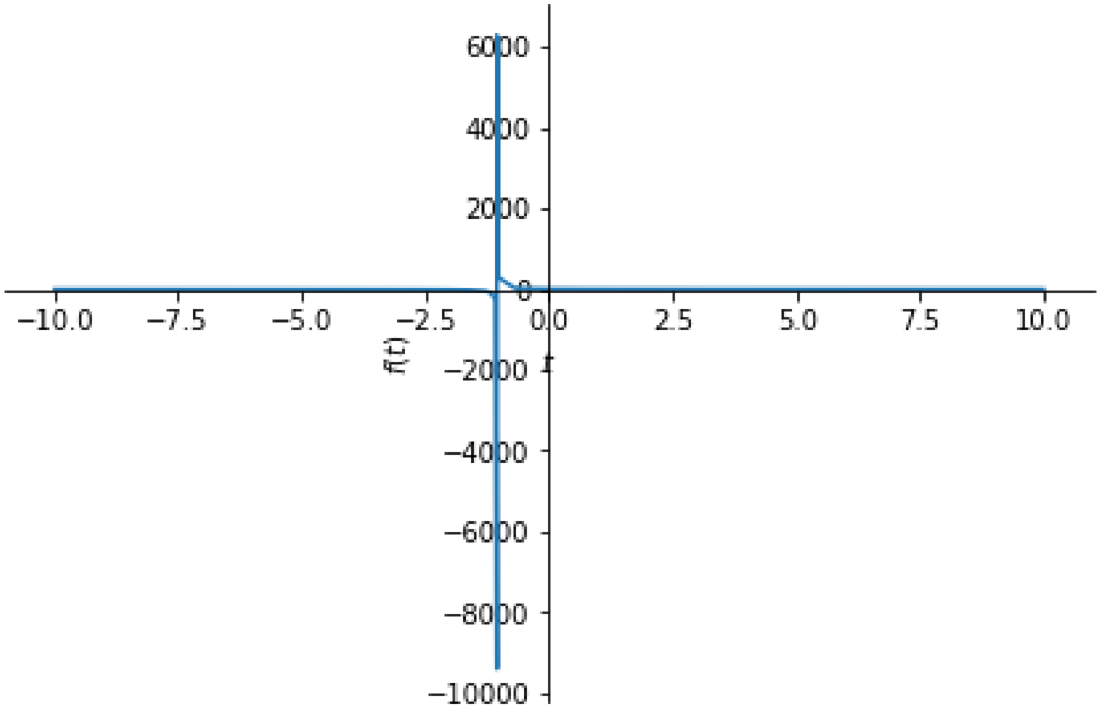
[8]:
```
print(f"The limit of P(t) as t approaches infinity is {limit(p.subs(K, 1000),␣
↪t, oo)}")
```

The limit of P(t) as t approaches infinity is 1000

**3b**

[9]:
```
print(f"The graph of P(t) = {p} where K = 1 is shown below.")
plot(p.subs(K, 1))
```

The graph of P(t) = 10*K/((K - 10)*exp(-0.1*t) + 10) where K = 1 is shown below.




[9]: <sympy.plotting.plot.Plot at 0x7fb63b9ef3d0>

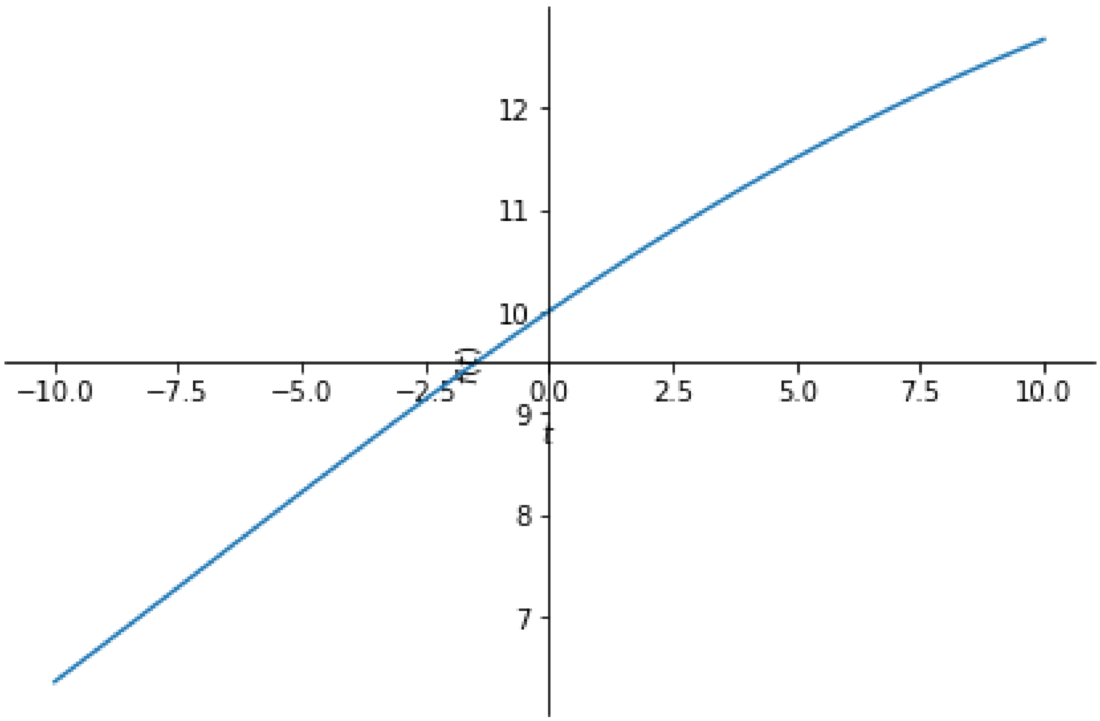
[10]:
```
print(f"The limit of P(t) as t approaches infinity is {limit(p.subs(K, 1), t,␣
↪oo)}")
```

The limit of P(t) as t approaches infinity is 1

**3c**

[11]:
```
print(f"The graph of P(t) = {p} where K = 15 is shown below.")
plot(p.subs(K, 15))
```

The graph of P(t) = 10*K/((K - 10)*exp(-0.1*t) + 10) where K = 15 is shown below.




[11]: <sympy.plotting.plot.Plot at 0x7fb63b9a0b50>

[12]:
```
print(f"The limit of P(t) as t approaches infinity is {limit(p.subs(K, 15), t,␣
↪oo)}")
```

```
The limit of P(t) as t approaches infinity is 15
```

### 3d

- As t goes to infinity the population size approaches K
- The population given a value of K will approach K